

Reductions

$P = \{ \text{polynomial solvable time} \}$

$NP = \{ \text{Non-Deterministic, verifiable in poly-time} \}$

$A \rightarrow NP\text{-completă dacă } \rightarrow A \in NP$

$\rightarrow \forall B \in NP \Rightarrow B \leq_p A$

\downarrow
se reduce în timp polinomial

$f: B \rightarrow A$ a. p. $\forall w \in B \Leftrightarrow f(w) \in A$

$3SAT \rightsquigarrow \text{clacă} \rightsquigarrow A$

① $SAT \rightsquigarrow 3SAT$

$$x_1 \vee x_2 \vee \dots \vee x_n = (x_1 \vee x_2 \vee x_A) \wedge [x_A \Leftrightarrow (x_3 \vee \dots \vee x_n)]$$

$$x_1 \vee x_2 \vee x_3 \vee x_n = (x_1 \vee x_2 \vee x_A) \wedge (x_A \Leftrightarrow (x_3 \vee x_n)) \quad \text{NF}$$

$$= (x_1 \vee x_2 \vee x_A) \wedge [(x_A \Rightarrow (x_3 \vee x_n)) \wedge ((x_3 \vee x_n) \Rightarrow x_A)]$$

$$= (x_1 \vee x_2 \vee x_A) \wedge [(x_A \vee x_3 \vee x_n) \wedge (x_3 \wedge x_n \vee x_A)]$$

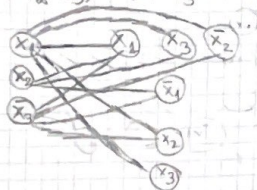
$$\stackrel{2}{=} (x_1 \vee x_2 \vee x_A) \wedge [(\bar{x}_A \vee x_3 \vee x_n) \wedge ((x_A \vee \bar{x}_3) \wedge (x_A \vee \bar{x}_n))]$$

$$\stackrel{2}{=} (x_1 \vee x_2 \vee x_A) \wedge (\bar{x}_A \vee x_3 \vee x_n) \wedge (x_A \vee \bar{x}_3 \vee x_n) \wedge (x_A \vee \bar{x}_n \vee x_n)$$

3SAT

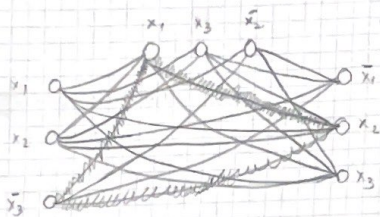
2) Clacă \rightarrow subgraf complet

$$(x_1 \vee x_2 \vee \bar{x}_3) \wedge (x_1 \vee x_3 \vee \bar{x}_2) \wedge (\bar{x}_1 \vee x_2 \vee x_3)$$



Regula

- nu se unesc din aceeași clacă
- se unesc doar ce nu e opus (nu unim x_1 cu \bar{x}_1)



$$\begin{aligned} x_1 &= T \\ x_2 &= T \\ x_3 &= F \end{aligned}$$

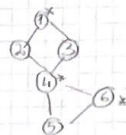
Sauze pe SAT $(x_1 \vee x_2) \wedge (\bar{x}_1 \vee \bar{x}_2) \wedge (x_1 \vee x_3)$



$$\begin{aligned} x_1 &= T \\ x_2 &= T \\ x_3 &= T \end{aligned}$$

3) Vertex Cover

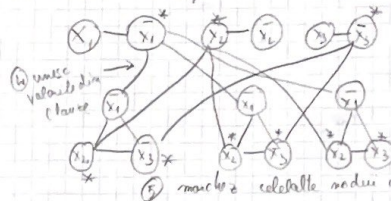
o alegere noduri care să traverseze toate muchiile



$$K = 3 \{1, 4, 6\} \checkmark$$

$$(x_1 \vee x_2 \vee \bar{x}_3) \wedge (\bar{x}_1 \vee x_2 \vee x_3) \wedge (\bar{x}_1 \vee x_2 \vee x_3)$$

1) Notăz toate muchiile de literele



2) unesc variabile din fiecare muchie

2) Notăz clauze ca noduri - se fac subgrafi complete

3) aleg interpretare corectă (satisfiabilă)

$$\begin{aligned} x_1 &= F \\ x_2 &= T \\ x_3 &= F \end{aligned}$$

$$K = m + n - 2$$

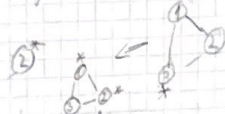
$$K = 3 + 3 - 2 = 4$$

6) Vertex Cover și Max Independent Set

MIS = mulțime de cardinalitate k care să conțină noduri fără muchii



VC \Rightarrow Inversa sa sa facem MIS



1) VC \Rightarrow MIS
VC = $\langle G, k \rangle$
G = $\langle V, E \rangle$

MIS = $\langle G', m-k \rangle$

G' = V \setminus VC

$\forall u, v \in G'$ au put avea muchii în graf pt că excludem coverul (care conține toate nodurile cu muchii)

2) MIS \Rightarrow VC

MIS = $\langle G, k \rangle$

G = $\langle V, E \rangle$

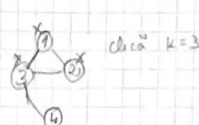
VC = $G' \cup k$

G' = V \setminus MIS

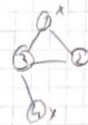
$\forall u, v \in G'$ trebuie să aibă muchii pt că nu conține noduri indep

De asemenea, asigurăm toate muchiile pt că e opusul unei set independente

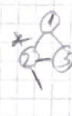
7) MIS \rightarrow Clacă



clacă k=3



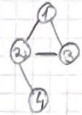
MIS k=2



clacă k=3
MIS k=1



Putem construi opusul Grafului



clacă k=3
1, 2, 3?



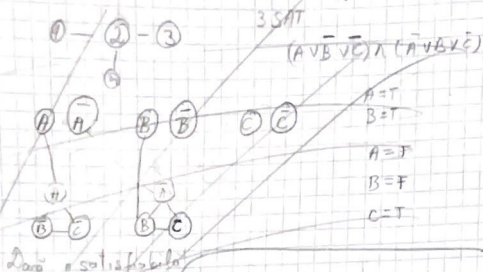
MIS cu k=2
1, 3, 4?

8) SUBSET SUM \rightarrow 3SAT

$(x_1 \vee x_2 \vee x_3) \wedge (\bar{x}_2 \vee \bar{x}_3 \vee x_4) \wedge (x_1 \vee \bar{x}_1 \vee \bar{x}_3)$

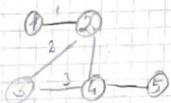
	x_1	x_2	x_3	x_4	C_1	C_2	C_3	
x_1	1	0	0	0	1	0	1	1000101
\bar{x}_1	1	0	0	0	0	0	0	1000111
x_2	0	1	0	0	1	0	0	10010
\bar{x}_2	0	1	0	0	0	1	1	1010
x_3	0	0	1	0	1	1	0	100
\bar{x}_3	0	0	1	0	0	0	1	100
x_4	0	0	0	1	0	1	0	100
\bar{x}_4	0	0	0	1	0	0	0	100
C_1	1	0	0	0	1	0	0	100
C_2	0	1	0	0	1	0	0	100
C_3	0	0	1	0	1	1	0	100
C_4	0	0	0	1	0	1	0	100
C_5	0	0	0	1	0	0	0	100
C_6	0	0	0	1	0	0	0	100
C_7	0	0	0	1	0	0	0	100
C_8	0	0	0	1	0	0	0	100
C_9	0	0	0	1	0	0	0	100
C_{10}	0	0	0	1	0	0	0	100
C_{11}	0	0	0	1	0	0	0	100
C_{12}	0	0	0	1	0	0	0	100
C_{13}	0	0	0	1	0	0	0	100
C_{14}	0	0	0	1	0	0	0	100
C_{15}	0	0	0	1	0	0	0	100
C_{16}	0	0	0	1	0	0	0	100
C_{17}	0	0	0	1	0	0	0	100
C_{18}	0	0	0	1	0	0	0	100
C_{19}	0	0	0	1	0	0	0	100
C_{20}	0	0	0	1	0	0	0	100
C_{21}	0	0	0	1	0	0	0	100
C_{22}	0	0	0	1	0	0	0	100
C_{23}	0	0	0	1	0	0	0	100
C_{24}	0	0	0	1	0	0	0	100
C_{25}	0	0	0	1	0	0	0	100
C_{26}	0	0	0	1	0	0	0	100
C_{27}	0	0	0	1	0	0	0	100
C_{28}	0	0	0	1	0	0	0	100
C_{29}	0	0	0	1	0	0	0	100
C_{30}	0	0	0	1	0	0	0	100
C_{31}	0	0	0	1	0	0	0	100
C_{32}	0	0	0	1	0	0	0	100
C_{33}	0	0	0	1	0	0	0	100
C_{34}	0	0	0	1	0	0	0	100
C_{35}	0	0	0	1	0	0	0	100
C_{36}	0	0	0	1	0	0	0	100
C_{37}	0	0	0	1	0	0	0	100
C_{38}	0	0	0	1	0	0	0	100
C_{39}	0	0	0	1	0	0	0	100
C_{40}	0	0	0	1	0	0	0	100
C_{41}	0	0	0	1	0	0	0	100
C_{42}	0	0	0	1	0	0	0	100
C_{43}	0	0	0	1	0	0	0	100
C_{44}	0	0	0	1	0	0	0	100
C_{45}	0	0	0	1	0	0	0	100
C_{46}	0	0	0	1	0	0	0	100
C_{47}	0	0	0	1	0	0	0	100
C_{48}	0	0	0	1	0	0	0	100
C_{49}	0	0	0	1	0	0	0	100
C_{50}	0	0	0	1	0	0	0	100
C_{51}	0	0	0	1	0	0	0	100
C_{52}	0	0	0	1	0	0	0	100
C_{53}	0	0	0	1	0	0	0	100
C_{54}	0	0	0	1	0	0	0	100
C_{55}	0	0	0	1	0	0	0	100
C_{56}	0	0	0	1	0	0	0	100
C_{57}	0	0	0	1	0	0	0	100
C_{58}	0	0	0	1	0	0	0	100
C_{59}	0	0	0	1	0	0	0	100
C_{60}	0	0	0	1	0	0	0	100
C_{61}	0	0	0	1	0	0	0	100
C_{62}	0	0	0	1	0	0	0	100
C_{63}	0	0	0	1	0	0	0	100
C_{64}	0	0	0	1	0	0	0	100
C_{65}	0	0	0	1	0	0	0	100
C_{66}	0	0	0	1	0	0	0	100
C_{67}	0	0	0	1	0	0	0	100
C_{68}	0	0	0	1	0	0	0	100
C_{69}	0	0	0	1	0	0	0	100
C_{70}	0	0	0	1	0	0	0	100
C_{71}	0	0	0	1	0	0	0	100
C_{72}	0	0	0	1	0	0	0	100
C_{73}	0	0	0	1	0	0	0	100
C_{74}	0	0	0	1	0	0	0	100
C_{75}	0	0	0	1	0	0	0	100
C_{76}	0	0	0	1	0	0	0	100
C_{77}	0	0	0	1	0	0	0	100
C_{78}	0	0	0	1	0	0	0	100
C_{79}	0	0	0	1	0	0	0	100
C_{80}	0	0	0	1	0	0	0	100
C_{81}	0	0	0	1	0	0	0	100
C_{82}	0	0	0	1	0	0	0	100
C_{83}	0	0	0	1	0	0	0	100
C_{84}	0	0	0	1	0	0	0	100
C_{85}	0	0	0	1	0	0	0	100
C_{86}	0	0	0	1	0	0	0	100
C_{87}	0	0	0	1	0	0	0	100
C_{88}	0	0	0	1	0	0	0	100
C_{89}	0	0	0	1	0	0	0	100
C_{90}	0	0	0	1	0	0	0	100
C_{91}	0	0	0	1	0	0	0	100
C_{92}	0	0	0	1	0	0	0	100
C_{93}	0	0	0	1	0	0	0	100
C_{94}	0	0	0	1	0	0	0	100
C_{95}	0	0	0	1	0	0	0	100
C_{96}	0	0	0	1	0	0	0	100
C_{97}	0	0	0	1	0	0	0	100
C_{98}	0	0	0	1	0	0	0	100
C_{99}	0	0	0	1	0	0	0	100
C_{100}	0	0	0	1	0	0	0	100

7 Graf G . Există drum de lungime $> k$



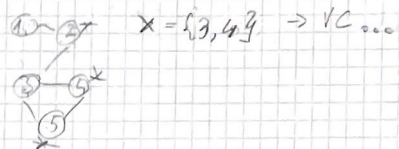
Notăm că DNP
 Dacă avem un set de muchii și un graf
 $G=(V,E)$ putem verifica că are
 mai mult de k muchii drumul

Reducție la Hamiltonian Path (margem din nod în nod și revărm
 muchiile) \Rightarrow HP



Luăm $HP=(G,V)$ unde V sunt nodurile
 incluse în drumul Hamiltonian
 Luăm $LP=(G',K)$ unde $G=G'$ și $K=|V|-1$

8 1 $G=(V,E)$ $\exists x \in V$ cu $|x| \leq k$ și $\forall (a,b) \in E$
 fie $a \in x$, $b \in x$ sau $a, b \notin x$



2 $G=(V,E)$, $k \in \mathbb{Z}$ MIS

\Rightarrow Fie $G=(V,E)$ și x un cover pt G . Construim
 $G'=(V \setminus x, E)$. Atunci MIS are soluția G' cu
 $K = m - |x|$, $m = |V|$.

Fie $u, v \in G'$. Aș putea avea muchie între u și v
 pentru că altfel ar fi fost în cover

\Leftarrow Fie $G=(V,E)$ și x un MIS. Construim G'
 cu $V' = V \setminus x$. G' este un vertex cover.
 Fie $u, v \in G'$. Ele trebuie să aibă muchie pt că
 altfel erau MIS.