$$Z_1 = Z_0 W_1^T + b_1 \quad , W_1 \in \mathbb{R}^{30\times 13}$$

$$Z_2 = \frac{1}{1+e^{-Z_1}}$$

$$Z_3 = Z_2 W_2^T + b_2$$

$$\hat{Y} = Z_3$$

$$L(\hat{Y}, Y) = \frac{1}{2}\sum_{i=1}^{N}(\hat{Y}_i - Y_i)^2$$

Z_3^i 表示 Z_3 的第 i 组向量.

$$\frac{\partial L}{\partial W_2} = \frac{1}{2}\sum_{i=1}^{N}\frac{\partial L}{\partial Z_3^i}\cdot\frac{\partial Z_3^i}{\partial W_2}$$

$$\frac{\partial L}{\partial Z_3^i} = 2(Z_3^i - Y^i)$$

$$\frac{\partial Z_3^i}{\partial W_2} = Z_2^i$$

$$\Rightarrow \frac{\partial L}{\partial W_2} = \sum_{i=1}^{N}(Z_3 - Y)^T Z_2$$

$$\frac{\partial L}{\partial W_2} = \frac{1}{2}\sum_{i=1}^{N}\frac{\partial L}{\partial Z_3^i}\cdot\frac{\partial Z_3^i}{\partial b_2}$$

$$\frac{\partial Z_3^i}{\partial b_2} = 1$$

$$\Rightarrow \frac{\partial L}{\partial W_2} = \sum_{i=1}^{N}(Z_3 - Y)^T$$

$$\frac{\partial L}{\partial W_1} = \frac{1}{2}\sum_{i=1}^{N}\frac{\partial L}{\partial Z_1^i}\cdot\frac{\partial Z_1^i}{\partial W_1}$$

$$\frac{\partial L}{\partial Z_1^i} = \frac{\partial L}{\partial Z_3^i}\cdot\frac{\partial Z_3^i}{\partial Z_2^i}\cdot\frac{\partial Z_2^i}{\partial Z_1^i}$$

$$\frac{\partial Z_3^i}{\partial Z_2^i} = W$$

$$\frac{\partial Z_2^i}{\partial Z_1^i} = \frac{e^{Z_1}}{e^{2Z_1}+2e^{Z_1}+1}$$

$$\frac{\partial Z_1^i}{\partial W_1} = Z_0^i$$

$$\Rightarrow \frac{\partial L}{\partial W_1} = \frac{1}{2}\frac{\partial L}{\partial Z_1}^T\frac{\partial Z_1}{\partial W_1}$$

$$\frac{\partial L}{\partial b_1} = \frac{1}{2}\sum_{i=1}^{N}\frac{\partial L}{\partial Z_1^i}\cdot\frac{\partial Z_1^i}{\partial b_1}$$

$$\frac{\partial Z_1^i}{\partial b_1} = 1$$

$$\Rightarrow \frac{\partial L}{\partial b_1} = \frac{1}{2}\sum_{i=1}^{N}\frac{\partial L}{\partial Z_1^i}$$