人工智能的数学基础

华东师范大学 数学科学学院 黎芳(教授) 2019年9月4日

Chapter0(Cont.) 梯度下降法的缺点

在数学最优化中，Rosenbrock函数是一个用来测试最优化算法性能的非凸函数，由Howard Harry Rosenbrock在1960年提出。也称为Rosenbrock山谷或Rosenbrock香蕉函数，也简称为香蕉函数。 Rosenbrock函数的定义如下：
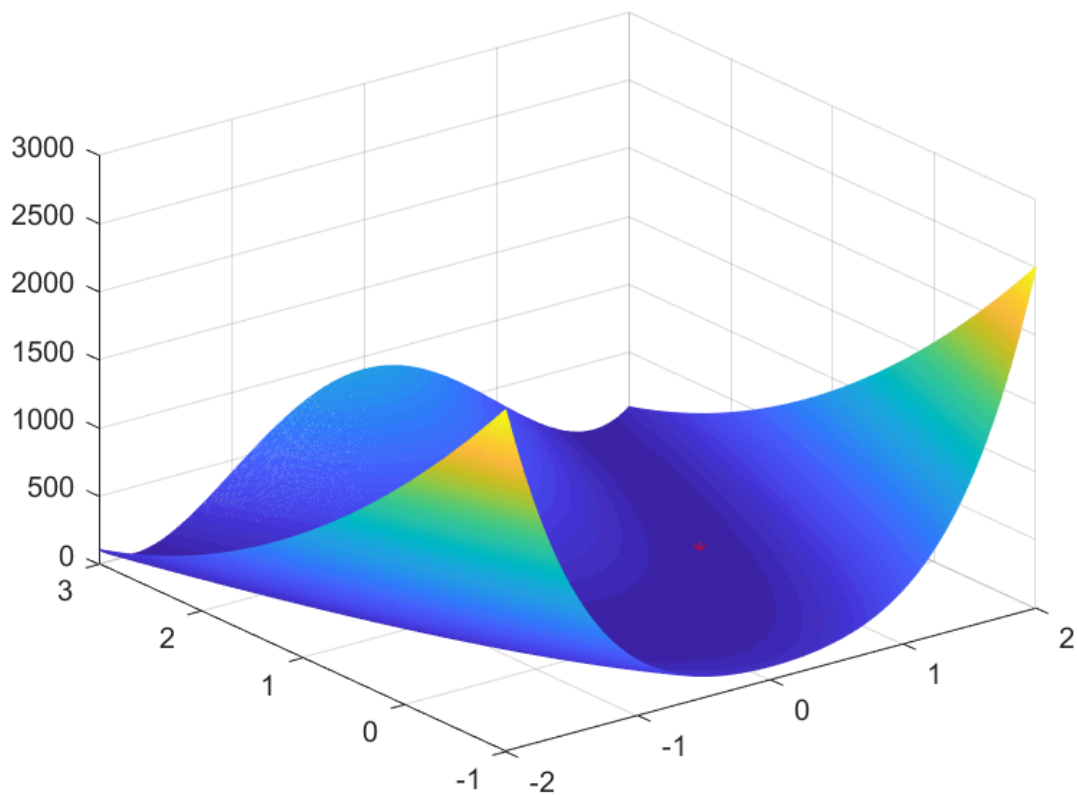
$$f(x, y) = (1 - x)^2 + 100(y - x^2)^2.$$

Rosenbrock函数的每个等高线大致呈抛物线形，其全域最小值也位在抛物线形的山谷中（香蕉型山谷）。很容易找到这个山谷，但由于山谷内的值变化不大，要找到全域的最小值相当困难。
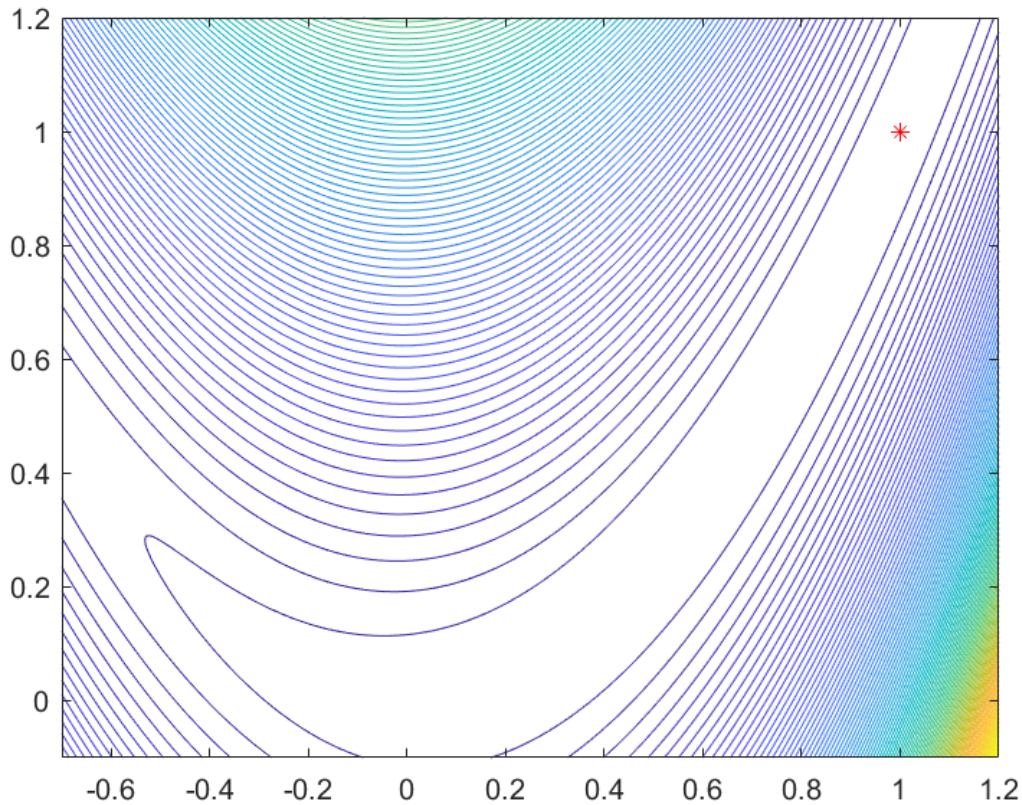
其全域最小值位于点(1,1)，数值为0。一般形式

$$f(x, y) = (a - x)^2 + b(y - x^2)^2.$$

```
[x,y] = meshgrid(-2:0.01:2,-1:0.01:3);
f = (1-x).^2+100*(y-x.^2).^2;
figure,mesh(x,y,f)
hold on; plot(1,1,'r*')
```

```
[x,y] = meshgrid(-0.7:0.01:1.2,-0.1:0.01:1.2);
f = (1-x).^2+100*(y-x.^2).^2;
figure,contour(x,y,f,100);
axis([-0.7 1.2 -0.1 1.2])
hold on; plot(1,1,'r*')
```


梯度下降法的缺点:

- 靠近极小值时收敛速度减慢。
- 直线搜索时可能会产生一些问题。
- 可能会"之字形"地下降。

```
[x1,x2] = meshgrid(-0.7:0.01:1.2,-0.1:0.01:1.2);
f = (1-x1).^2+100*(x2-x1.^2).^2;
figure,contour(x1,x2,f,100);
axis([-0.7 1.2 -0.1 1.2])
hold on; plot(1,1,'m*')

x0 = [-0.5;0.5]; % initial point
x = x0;
iter = 0;
gradf = [2*(x(1)-1)+400*x(1)*(x(1)^2-x(2));200*(x(2)-x(1).^2)];
xs(1,:) = x;
```

```matlab
while norm(gradf)>eps&&iter<20000
    iter = iter+1;
    d   = -gradf;
    fun = @(alpha)((1-x(1)-alpha*d(1))^2+100*(x(2)+alpha*d(2)-(x(1)+alpha*d(1))^2)^2);
    alpha = fminsearch(fun,0.01);
%     alpha = 0.001;
    x = x-alpha*gradf;
    gradf = [2*(x(1)-1)+400*x(1)*(x(1)^2-x(2));200*(x(2)-x(1).^2)];

    fun_value = (1-x(1)).^2+100*(x(2)-x(1)).^2;
    fprintf('iter_num = %3d  norm_grad = %2.6f  fun_value = %2.6f\n'...
        ,iter,norm(gradf),fun_value);
    xs(iter+1,:)=x;
end
```

```
iter_num =    1  norm_grad = 2.053141  fun_value = 101.177412
iter_num =    2  norm_grad = 7.827414  fun_value = 10.772247
iter_num =    3  norm_grad = 2.377651  fun_value = 10.749852
iter_num =    4  norm_grad = 5.380124  fun_value = 5.797181
iter_num =    5  norm_grad = 2.482290  fun_value = 5.737036
iter_num =    6  norm_grad = 4.341071  fun_value = 3.279781
iter_num =    7  norm_grad = 2.577411  fun_value = 3.221682
iter_num =    8  norm_grad = 3.696628  fun_value = 1.920884
iter_num =    9  norm_grad = 2.685743  fun_value = 1.869535
iter_num =   10  norm_grad = 3.212483  fun_value = 1.248042
iter_num =   11  norm_grad = 2.795968  fun_value = 1.200796
iter_num =   12  norm_grad = 2.839255  fun_value = 1.015434
iter_num =   13  norm_grad = 2.932489  fun_value = 0.973330
iter_num =   14  norm_grad = 2.528035  fun_value = 1.093162
iter_num =   15  norm_grad = 3.086155  fun_value = 1.057181
iter_num =   16  norm_grad = 2.272434  fun_value = 1.406239
iter_num =   17  norm_grad = 3.251548  fun_value = 1.373859
iter_num =   18  norm_grad = 2.035916  fun_value = 1.899606
iter_num =   19  norm_grad = 3.487602  fun_value = 1.873839
iter_num =   20  norm_grad = 1.821143  fun_value = 2.549605
iter_num =   21  norm_grad = 3.803595  fun_value = 2.527871
iter_num =   22  norm_grad = 1.606478  fun_value = 3.334182
iter_num =   23  norm_grad = 4.545904  fun_value = 3.351529
iter_num =   24  norm_grad = 1.379887  fun_value = 4.346919
iter_num =   25  norm_grad = 2.012069  fun_value = 5.132431
iter_num =   26  norm_grad = 0.387410  fun_value = 4.905454
iter_num =   27  norm_grad = 1.804504  fun_value = 4.940627
iter_num =   28  norm_grad = 0.378784  fun_value = 4.740693
iter_num =   29  norm_grad = 1.328290  fun_value = 4.794796
iter_num =   30  norm_grad = 0.367167  fun_value = 4.650065
iter_num =   31  norm_grad = 1.466993  fun_value = 4.691515
iter_num =   32  norm_grad = 0.352761  fun_value = 4.538432
iter_num =   33  norm_grad = 1.838496  fun_value = 4.537493
iter_num =   34  norm_grad = 0.335609  fun_value = 4.355457
iter_num =   35  norm_grad = 2.090701  fun_value = 4.298752
iter_num =   36  norm_grad = 0.313972  fun_value = 4.105038
iter_num =   37  norm_grad = 2.612710  fun_value = 3.879666
iter_num =   38  norm_grad = 0.284346  fun_value = 3.659894
iter_num =   39  norm_grad = 1.552395  fun_value = 3.633373
iter_num =   40  norm_grad = 0.273800  fun_value = 3.505751
iter_num =   41  norm_grad = 1.449575  fun_value = 3.483975
iter_num =   42  norm_grad = 0.267793  fun_value = 3.367204
iter_num =   43  norm_grad = 1.068959  fun_value = 3.376050
iter_num =   44  norm_grad = 0.260514  fun_value = 3.291077
iter_num =   45  norm_grad = 1.201495  fun_value = 3.286358
iter_num =   46  norm_grad = 0.258625  fun_value = 3.192171
```

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iter_num =  47  norm_grad = 0.881020  fun_value = 3.207047
iter_num =  48  norm_grad = 0.251168  fun_value = 3.138817
iter_num =  49  norm_grad = 1.073309  fun_value = 3.139719
iter_num =  50  norm_grad = 0.251807  fun_value = 3.057410
iter_num =  51  norm_grad = 0.770812  fun_value = 3.073979
iter_num =  52  norm_grad = 0.252834  fun_value = 3.013290
iter_num =  53  norm_grad = 0.671710  fun_value = 3.032410
iter_num =  54  norm_grad = 0.247285  fun_value = 2.980103
iter_num =  55  norm_grad = 0.733572  fun_value = 2.996391
iter_num =  56  norm_grad = 0.248599  fun_value = 2.939392
iter_num =  57  norm_grad = 0.642676  fun_value = 2.957912
iter_num =  58  norm_grad = 0.243144  fun_value = 2.908521
iter_num =  59  norm_grad = 0.703899  fun_value = 2.924583
iter_num =  60  norm_grad = 0.244782  fun_value = 2.870565
iter_num =  61  norm_grad = 0.613686  fun_value = 2.888510
iter_num =  62  norm_grad = 0.239111  fun_value = 2.841952
iter_num =  63  norm_grad = 0.677373  fun_value = 2.857498
iter_num =  64  norm_grad = 0.241186  fun_value = 2.806132
iter_num =  65  norm_grad = 0.589752  fun_value = 2.823547
iter_num =  66  norm_grad = 0.235401  fun_value = 2.779350
iter_num =  67  norm_grad = 0.654874  fun_value = 2.794522
iter_num =  68  norm_grad = 0.237863  fun_value = 2.745426
iter_num =  69  norm_grad = 0.569094  fun_value = 2.762399
iter_num =  70  norm_grad = 0.231935  fun_value = 2.720248
iter_num =  71  norm_grad = 0.632122  fun_value = 2.735032
iter_num =  72  norm_grad = 0.234566  fun_value = 2.688169
iter_num =  73  norm_grad = 0.552597  fun_value = 2.704669
iter_num =  74  norm_grad = 0.228775  fun_value = 2.664189
iter_num =  75  norm_grad = 0.610891  fun_value = 2.678707
iter_num =  76  norm_grad = 0.231390  fun_value = 2.633905
iter_num =  77  norm_grad = 0.534893  fun_value = 2.649847
iter_num =  78  norm_grad = 0.234228  fun_value = 2.609611
iter_num =  79  norm_grad = 0.486025  fun_value = 2.626198
iter_num =  80  norm_grad = 0.228907  fun_value = 2.590070
iter_num =  81  norm_grad = 0.521495  fun_value = 2.605630
iter_num =  82  norm_grad = 0.231562  fun_value = 2.566754
iter_num =  83  norm_grad = 0.476449  fun_value = 2.582874
iter_num =  84  norm_grad = 0.226485  fun_value = 2.547766
iter_num =  85  norm_grad = 0.511250  fun_value = 2.563014
iter_num =  86  norm_grad = 0.229221  fun_value = 2.525225
iter_num =  87  norm_grad = 0.464843  fun_value = 2.540919
iter_num =  88  norm_grad = 0.232142  fun_value = 2.505724
iter_num =  89  norm_grad = 0.431223  fun_value = 2.521668
iter_num =  90  norm_grad = 0.235143  fun_value = 2.488262
iter_num =  91  norm_grad = 0.404768  fun_value = 2.504326
iter_num =  92  norm_grad = 0.238022  fun_value = 2.472283
iter_num =  93  norm_grad = 0.384557  fun_value = 2.488418
iter_num =  94  norm_grad = 0.233329  fun_value = 2.458324
iter_num =  95  norm_grad = 0.399338  fun_value = 2.474083
iter_num =  96  norm_grad = 0.236130  fun_value = 2.442675
iter_num =  97  norm_grad = 0.378479  fun_value = 2.458442
iter_num =  98  norm_grad = 0.238895  fun_value = 2.428057
iter_num =  99  norm_grad = 0.362708  fun_value = 2.443871
iter_num = 100  norm_grad = 0.242031  fun_value = 2.414147
iter_num = 101  norm_grad = 0.349913  fun_value = 2.430090
iter_num = 102  norm_grad = 0.237848  fun_value = 2.401712
iter_num = 103  norm_grad = 0.358816  fun_value = 2.417382
iter_num = 104  norm_grad = 0.240353  fun_value = 2.388145
iter_num = 105  norm_grad = 0.343639  fun_value = 2.403704
iter_num = 106  norm_grad = 0.242684  fun_value = 2.375189
iter_num = 107  norm_grad = 0.336319  fun_value = 2.390892
iter_num = 108  norm_grad = 0.239853  fun_value = 2.363189
iter_num = 109  norm_grad = 0.340609  fun_value = 2.378707
iter_num = 110  norm_grad = 0.241216  fun_value = 2.350589
iter_num = 111  norm_grad = 0.330314  fun_value = 2.365940
```

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iter_num = 112    norm_grad = 0.237283    fun_value = 2.338946
iter_num = 113    norm_grad = 0.336674    fun_value = 2.354026
iter_num = 114    norm_grad = 0.239412    fun_value = 2.326399
iter_num = 115    norm_grad = 0.328644    fun_value = 2.341545
iter_num = 116    norm_grad = 0.236277    fun_value = 2.314797
iter_num = 117    norm_grad = 0.332068    fun_value = 2.329675
iter_num = 118    norm_grad = 0.237352    fun_value = 2.302603
iter_num = 119    norm_grad = 0.326249    fun_value = 2.317472
iter_num = 120    norm_grad = 0.235012    fun_value = 2.291044
iter_num = 121    norm_grad = 0.326653    fun_value = 2.305637
iter_num = 122    norm_grad = 0.234978    fun_value = 2.279202
iter_num = 123    norm_grad = 0.322968    fun_value = 2.293702
iter_num = 124    norm_grad = 0.233419    fun_value = 2.267687
iter_num = 125    norm_grad = 0.325385    fun_value = 2.282125
iter_num = 126    norm_grad = 0.234148    fun_value = 2.255893
iter_num = 127    norm_grad = 0.318555    fun_value = 2.270216
iter_num = 128    norm_grad = 0.231394    fun_value = 2.244727
iter_num = 129    norm_grad = 0.322884    fun_value = 2.258887
iter_num = 130    norm_grad = 0.232859    fun_value = 2.232983
iter_num = 131    norm_grad = 0.318025    fun_value = 2.247231
iter_num = 132    norm_grad = 0.230835    fun_value = 2.221862
iter_num = 133    norm_grad = 0.319262    fun_value = 2.235926
iter_num = 134    norm_grad = 0.231139    fun_value = 2.210463
iter_num = 135    norm_grad = 0.316349    fun_value = 2.224517
iter_num = 136    norm_grad = 0.229851    fun_value = 2.199387
iter_num = 137    norm_grad = 0.314392    fun_value = 2.213229
iter_num = 138    norm_grad = 0.228929    fun_value = 2.188332
iter_num = 139    norm_grad = 0.313362    fun_value = 2.202060
iter_num = 140    norm_grad = 0.228363    fun_value = 2.177301
iter_num = 141    norm_grad = 0.313245    fun_value = 2.191007
iter_num = 142    norm_grad = 0.228156    fun_value = 2.166292
iter_num = 143    norm_grad = 0.308949    fun_value = 2.179848
iter_num = 144    norm_grad = 0.226314    fun_value = 2.155603
iter_num = 145    norm_grad = 0.310597    fun_value = 2.169023
iter_num = 146    norm_grad = 0.226802    fun_value = 2.144642
iter_num = 147    norm_grad = 0.303030    fun_value = 2.157871
iter_num = 148    norm_grad = 0.230430    fun_value = 2.133578
iter_num = 149    norm_grad = 0.291738    fun_value = 2.146860
iter_num = 150    norm_grad = 0.231754    fun_value = 2.123116
iter_num = 151    norm_grad = 0.290044    fun_value = 2.136543
iter_num = 152    norm_grad = 0.230820    fun_value = 2.113010
iter_num = 153    norm_grad = 0.289053    fun_value = 2.126323
iter_num = 154    norm_grad = 0.230241    fun_value = 2.102923
iter_num = 155    norm_grad = 0.288755    fun_value = 2.116197
iter_num = 156    norm_grad = 0.230015    fun_value = 2.092854
iter_num = 157    norm_grad = 0.283869    fun_value = 2.105924
iter_num = 158    norm_grad = 0.227445    fun_value = 2.083141
iter_num = 159    norm_grad = 0.284892    fun_value = 2.095984
iter_num = 160    norm_grad = 0.227891    fun_value = 2.073111
iter_num = 161    norm_grad = 0.281397    fun_value = 2.085899
iter_num = 162    norm_grad = 0.226021    fun_value = 2.063432
iter_num = 163    norm_grad = 0.283719    fun_value = 2.076140
iter_num = 164    norm_grad = 0.227143    fun_value = 2.053441
iter_num = 165    norm_grad = 0.281524    fun_value = 2.066237
iter_num = 166    norm_grad = 0.225936    fun_value = 2.043799
iter_num = 167    norm_grad = 0.279988    fun_value = 2.056424
iter_num = 168    norm_grad = 0.225068    fun_value = 2.034175
iter_num = 169    norm_grad = 0.279091    fun_value = 2.046700
iter_num = 170    norm_grad = 0.224530    fun_value = 2.024570
iter_num = 171    norm_grad = 0.278825    fun_value = 2.037063
iter_num = 172    norm_grad = 0.224323    fun_value = 2.014984
iter_num = 173    norm_grad = 0.274011    fun_value = 2.027280
iter_num = 174    norm_grad = 0.221731    fun_value = 2.005744
iter_num = 175    norm_grad = 0.280028    fun_value = 2.018036
iter_num = 176    norm_grad = 0.218362    fun_value = 1.996506
```

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iter_num = 177   norm_grad = 0.283891   fun_value = 2.008586
iter_num = 178   norm_grad = 0.220085   fun_value = 1.986681
iter_num = 179   norm_grad = 0.278341   fun_value = 1.998790
iter_num = 180   norm_grad = 0.217355   fun_value = 1.977480
iter_num = 181   norm_grad = 0.278531   fun_value = 1.989302
iter_num = 182   norm_grad = 0.217344   fun_value = 1.967996
iter_num = 183   norm_grad = 0.279375   fun_value = 1.979901
iter_num = 184   norm_grad = 0.217650   fun_value = 1.958532
iter_num = 185   norm_grad = 0.275829   fun_value = 1.970370
iter_num = 186   norm_grad = 0.215856   fun_value = 1.949387
iter_num = 187   norm_grad = 0.277927   fun_value = 1.961137
iter_num = 188   norm_grad = 0.216771   fun_value = 1.939964
iter_num = 189   norm_grad = 0.270612   fun_value = 1.951561
iter_num = 190   norm_grad = 0.219400   fun_value = 1.930558
iter_num = 191   norm_grad = 0.266083   fun_value = 1.942306
iter_num = 192   norm_grad = 0.216893   fun_value = 1.921803
iter_num = 193   norm_grad = 0.267245   fun_value = 1.933354
iter_num = 194   norm_grad = 0.217464   fun_value = 1.912748
iter_num = 195   norm_grad = 0.263889   fun_value = 1.924258
iter_num = 196   norm_grad = 0.215581   fun_value = 1.904027
iter_num = 197   norm_grad = 0.266141   fun_value = 1.915459
iter_num = 198   norm_grad = 0.216754   fun_value = 1.895010
iter_num = 199   norm_grad = 0.258792   fun_value = 1.906295
iter_num = 200   norm_grad = 0.218689   fun_value = 1.886081
iter_num = 201   norm_grad = 0.257469   fun_value = 1.897554
iter_num = 202   norm_grad = 0.217863   fun_value = 1.877502
iter_num = 203   norm_grad = 0.256649   fun_value = 1.888884
iter_num = 204   norm_grad = 0.217342   fun_value = 1.868940
iter_num = 205   norm_grad = 0.251192   fun_value = 1.880059
iter_num = 206   norm_grad = 0.219872   fun_value = 1.860193
iter_num = 207   norm_grad = 0.250312   fun_value = 1.871564
iter_num = 208   norm_grad = 0.219292   fun_value = 1.851813
iter_num = 209   norm_grad = 0.244676   fun_value = 1.862901
iter_num = 210   norm_grad = 0.221241   fun_value = 1.843290
iter_num = 211   norm_grad = 0.242473   fun_value = 1.854497
iter_num = 212   norm_grad = 0.219650   fun_value = 1.835157
iter_num = 213   norm_grad = 0.240710   fun_value = 1.846158
iter_num = 214   norm_grad = 0.218381   fun_value = 1.827038
iter_num = 215   norm_grad = 0.244551   fun_value = 1.838120
iter_num = 216   norm_grad = 0.215429   fun_value = 1.819076
iter_num = 217   norm_grad = 0.245857   fun_value = 1.829953
iter_num = 218   norm_grad = 0.216338   fun_value = 1.810777
iter_num = 219   norm_grad = 0.242439   fun_value = 1.821622
iter_num = 220   norm_grad = 0.214007   fun_value = 1.802844
iter_num = 221   norm_grad = 0.244581   fun_value = 1.813582
iter_num = 222   norm_grad = 0.215497   fun_value = 1.794578
iter_num = 223   norm_grad = 0.242015   fun_value = 1.805379
iter_num = 224   norm_grad = 0.213743   fun_value = 1.786675
iter_num = 225   norm_grad = 0.239904   fun_value = 1.797241
iter_num = 226   norm_grad = 0.212298   fun_value = 1.778786
iter_num = 227   norm_grad = 0.243282   fun_value = 1.789387
iter_num = 228   norm_grad = 0.208821   fun_value = 1.771078
iter_num = 229   norm_grad = 0.243352   fun_value = 1.781362
iter_num = 230   norm_grad = 0.208849   fun_value = 1.763061
iter_num = 231   norm_grad = 0.243860   fun_value = 1.773400
iter_num = 232   norm_grad = 0.209157   fun_value = 1.755061
iter_num = 233   norm_grad = 0.239798   fun_value = 1.765288
iter_num = 234   norm_grad = 0.206562   fun_value = 1.747396
iter_num = 235   norm_grad = 0.241120   fun_value = 1.757449
iter_num = 236   norm_grad = 0.207388   fun_value = 1.739431
iter_num = 237   norm_grad = 0.237943   fun_value = 1.749463
iter_num = 238   norm_grad = 0.211041   fun_value = 1.731307
iter_num = 239   norm_grad = 0.233571   fun_value = 1.741530
iter_num = 240   norm_grad = 0.207981   fun_value = 1.723869
iter_num = 241   norm_grad = 0.234660   fun_value = 1.733876
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iter_num = 242   norm_grad = 0.208767   fun_value = 1.716108
iter_num = 243   norm_grad = 0.231136   fun_value = 1.726065
iter_num = 244   norm_grad = 0.211844   fun_value = 1.708228
iter_num = 245   norm_grad = 0.225319   fun_value = 1.718234
iter_num = 246   norm_grad = 0.212747   fun_value = 1.700623
iter_num = 247   norm_grad = 0.226668   fun_value = 1.710792
iter_num = 248   norm_grad = 0.208478   fun_value = 1.693486
iter_num = 249   norm_grad = 0.226684   fun_value = 1.703300
iter_num = 250   norm_grad = 0.208540   fun_value = 1.686003
iter_num = 251   norm_grad = 0.227054   fun_value = 1.695863
iter_num = 252   norm_grad = 0.208880   fun_value = 1.678534
iter_num = 253   norm_grad = 0.227787   fun_value = 1.688481
iter_num = 254   norm_grad = 0.203990   fun_value = 1.671537
iter_num = 255   norm_grad = 0.231425   fun_value = 1.681216
iter_num = 256   norm_grad = 0.201009   fun_value = 1.664358
iter_num = 257   norm_grad = 0.232199   fun_value = 1.673810
iter_num = 258   norm_grad = 0.201506   fun_value = 1.656886
iter_num = 259   norm_grad = 0.233377   fun_value = 1.666461
iter_num = 260   norm_grad = 0.196657   fun_value = 1.649901
iter_num = 261   norm_grad = 0.235462   fun_value = 1.659109
iter_num = 262   norm_grad = 0.197859   fun_value = 1.642357
iter_num = 263   norm_grad = 0.233234   fun_value = 1.651627
iter_num = 264   norm_grad = 0.196493   fun_value = 1.635112
iter_num = 265   norm_grad = 0.236149   fun_value = 1.644389
iter_num = 266   norm_grad = 0.192518   fun_value = 1.628080
iter_num = 267   norm_grad = 0.238768   fun_value = 1.637078
iter_num = 268   norm_grad = 0.193865   fun_value = 1.620538
iter_num = 269   norm_grad = 0.237208   fun_value = 1.629649
iter_num = 270   norm_grad = 0.192959   fun_value = 1.613274
iter_num = 271   norm_grad = 0.236076   fun_value = 1.622278
iter_num = 272   norm_grad = 0.192283   fun_value = 1.606026
iter_num = 273   norm_grad = 0.235361   fun_value = 1.614965
iter_num = 274   norm_grad = 0.191832   fun_value = 1.598797
iter_num = 275   norm_grad = 0.235057   fun_value = 1.607707
iter_num = 276   norm_grad = 0.191606   fun_value = 1.591585
iter_num = 277   norm_grad = 0.230538   fun_value = 1.600330
iter_num = 278   norm_grad = 0.194634   fun_value = 1.584188
iter_num = 279   norm_grad = 0.226681   fun_value = 1.593107
iter_num = 280   norm_grad = 0.192252   fun_value = 1.577354
iter_num = 281   norm_grad = 0.227927   fun_value = 1.586120
iter_num = 282   norm_grad = 0.192981   fun_value = 1.570261
iter_num = 283   norm_grad = 0.224855   fun_value = 1.579004
iter_num = 284   norm_grad = 0.191070   fun_value = 1.563458
iter_num = 285   norm_grad = 0.226813   fun_value = 1.572120
iter_num = 286   norm_grad = 0.192242   fun_value = 1.556399
iter_num = 287   norm_grad = 0.224468   fun_value = 1.565109
iter_num = 288   norm_grad = 0.190769   fun_value = 1.549628
iter_num = 289   norm_grad = 0.222511   fun_value = 1.558149
iter_num = 290   norm_grad = 0.189533   fun_value = 1.542872
iter_num = 291   norm_grad = 0.225519   fun_value = 1.551416
iter_num = 292   norm_grad = 0.185878   fun_value = 1.536307
iter_num = 293   norm_grad = 0.228613   fun_value = 1.544634
iter_num = 294   norm_grad = 0.187546   fun_value = 1.529256
iter_num = 295   norm_grad = 0.223026   fun_value = 1.537569
iter_num = 296   norm_grad = 0.189767   fun_value = 1.522284
iter_num = 297   norm_grad = 0.217819   fun_value = 1.530633
iter_num = 298   norm_grad = 0.191796   fun_value = 1.515441
iter_num = 299   norm_grad = 0.216468   fun_value = 1.523942
iter_num = 300   norm_grad = 0.190867   fun_value = 1.508897
iter_num = 301   norm_grad = 0.215445   fun_value = 1.517298
iter_num = 302   norm_grad = 0.190162   fun_value = 1.502367
iter_num = 303   norm_grad = 0.214739   fun_value = 1.510701
iter_num = 304   norm_grad = 0.189676   fun_value = 1.495852
iter_num = 305   norm_grad = 0.214347   fun_value = 1.504150
iter_num = 306   norm_grad = 0.189406   fun_value = 1.489352
```

```
iter_num = 307   norm_grad = 0.214267   fun_value = 1.497646
iter_num = 308   norm_grad = 0.189354   fun_value = 1.482868
iter_num = 309   norm_grad = 0.209813   fun_value = 1.491006
iter_num = 310   norm_grad = 0.191402   fun_value = 1.476284
iter_num = 311   norm_grad = 0.208087   fun_value = 1.484545
iter_num = 312   norm_grad = 0.190112   fun_value = 1.470011
iter_num = 313   norm_grad = 0.206664   fun_value = 1.478130
iter_num = 314   norm_grad = 0.189051   fun_value = 1.463751
iter_num = 315   norm_grad = 0.205531   fun_value = 1.471758
iter_num = 316   norm_grad = 0.188211   fun_value = 1.457506
iter_num = 317   norm_grad = 0.209332   fun_value = 1.465610
iter_num = 318   norm_grad = 0.185921   fun_value = 1.451376
iter_num = 319   norm_grad = 0.211035   fun_value = 1.459372
iter_num = 320   norm_grad = 0.181909   fun_value = 1.445382
iter_num = 321   norm_grad = 0.214269   fun_value = 1.453164
iter_num = 322   norm_grad = 0.178688   fun_value = 1.439292
iter_num = 323   norm_grad = 0.217962   fun_value = 1.446929
iter_num = 324   norm_grad = 0.175453   fun_value = 1.433157
iter_num = 325   norm_grad = 0.220858   fun_value = 1.440602
iter_num = 326   norm_grad = 0.176874   fun_value = 1.426597
iter_num = 327   norm_grad = 0.215558   fun_value = 1.434028
iter_num = 328   norm_grad = 0.179285   fun_value = 1.420083
iter_num = 329   norm_grad = 0.211474   fun_value = 1.427620
iter_num = 330   norm_grad = 0.176866   fun_value = 1.414052
iter_num = 331   norm_grad = 0.216435   fun_value = 1.421563
iter_num = 332   norm_grad = 0.174358   fun_value = 1.407981
iter_num = 333   norm_grad = 0.216481   fun_value = 1.415277
iter_num = 334   norm_grad = 0.174309   fun_value = 1.401706
iter_num = 335   norm_grad = 0.216872   fun_value = 1.409037
iter_num = 336   norm_grad = 0.174442   fun_value = 1.395448
iter_num = 337   norm_grad = 0.213329   fun_value = 1.402697
iter_num = 338   norm_grad = 0.172513   fun_value = 1.389422
iter_num = 339   norm_grad = 0.214362   fun_value = 1.396546
iter_num = 340   norm_grad = 0.172983   fun_value = 1.383199
iter_num = 341   norm_grad = 0.215754   fun_value = 1.390441
iter_num = 342   norm_grad = 0.173650   fun_value = 1.376991
iter_num = 343   norm_grad = 0.213201   fun_value = 1.384236
iter_num = 344   norm_grad = 0.172232   fun_value = 1.371015
iter_num = 345   norm_grad = 0.211018   fun_value = 1.378077
iter_num = 346   norm_grad = 0.171007   fun_value = 1.365053
iter_num = 347   norm_grad = 0.213347   fun_value = 1.372101
iter_num = 348   norm_grad = 0.172176   fun_value = 1.358897
iter_num = 349   norm_grad = 0.207549   fun_value = 1.365886
iter_num = 350   norm_grad = 0.174097   fun_value = 1.352797
iter_num = 351   norm_grad = 0.202637   fun_value = 1.359811
iter_num = 352   norm_grad = 0.176116   fun_value = 1.346794
iter_num = 353   norm_grad = 0.201719   fun_value = 1.353967
iter_num = 354   norm_grad = 0.175493   fun_value = 1.341044
iter_num = 355   norm_grad = 0.201075   fun_value = 1.348162
iter_num = 356   norm_grad = 0.175051   fun_value = 1.335310
iter_num = 357   norm_grad = 0.200702   fun_value = 1.342397
iter_num = 358   norm_grad = 0.174787   fun_value = 1.329589
iter_num = 359   norm_grad = 0.196230   fun_value = 1.336516
iter_num = 360   norm_grad = 0.176652   fun_value = 1.323771
iter_num = 361   norm_grad = 0.194672   fun_value = 1.330809
iter_num = 362   norm_grad = 0.175518   fun_value = 1.318221
iter_num = 363   norm_grad = 0.193374   fun_value = 1.325141
iter_num = 364   norm_grad = 0.174572   fun_value = 1.312684
iter_num = 365   norm_grad = 0.196695   fun_value = 1.319665
iter_num = 366   norm_grad = 0.172052   fun_value = 1.307262
iter_num = 367   norm_grad = 0.197611   fun_value = 1.314088
iter_num = 368   norm_grad = 0.172652   fun_value = 1.301615
iter_num = 369   norm_grad = 0.194460   fun_value = 1.308399
iter_num = 370   norm_grad = 0.170520   fun_value = 1.296218
iter_num = 371   norm_grad = 0.195854   fun_value = 1.302895
```

```
iter_num = 372   norm_grad = 0.171443   fun_value = 1.290600
iter_num = 373   norm_grad = 0.193234   fun_value = 1.297280
iter_num = 374   norm_grad = 0.169662   fun_value = 1.285230
iter_num = 375   norm_grad = 0.195115   fun_value = 1.291849
iter_num = 376   norm_grad = 0.170918   fun_value = 1.279642
iter_num = 377   norm_grad = 0.192987   fun_value = 1.286307
iter_num = 378   norm_grad = 0.169464   fun_value = 1.274299
iter_num = 379   norm_grad = 0.195383   fun_value = 1.280948
iter_num = 380   norm_grad = 0.166163   fun_value = 1.269087
iter_num = 381   norm_grad = 0.198795   fun_value = 1.275592
iter_num = 382   norm_grad = 0.163247   fun_value = 1.263812
iter_num = 383   norm_grad = 0.202132   fun_value = 1.270187
iter_num = 384   norm_grad = 0.160025   fun_value = 1.258514
iter_num = 385   norm_grad = 0.208027   fun_value = 1.264790
iter_num = 386   norm_grad = 0.157682   fun_value = 1.253056
iter_num = 387   norm_grad = 0.211020   fun_value = 1.259211
iter_num = 388   norm_grad = 0.158876   fun_value = 1.247274
iter_num = 389   norm_grad = 0.206573   fun_value = 1.253444
iter_num = 390   norm_grad = 0.156838   fun_value = 1.241847
iter_num = 391   norm_grad = 0.210204   fun_value = 1.247947
iter_num = 392   norm_grad = 0.153214   fun_value = 1.236482
iter_num = 393   norm_grad = 0.216464   fun_value = 1.242394
iter_num = 394   norm_grad = 0.150274   fun_value = 1.230907
iter_num = 395   norm_grad = 0.222792   fun_value = 1.236670
iter_num = 396   norm_grad = 0.152188   fun_value = 1.224785
iter_num = 397   norm_grad = 0.215148   fun_value = 1.230611
iter_num = 398   norm_grad = 0.149530   fun_value = 1.219247
iter_num = 399   norm_grad = 0.222268   fun_value = 1.224978
iter_num = 400   norm_grad = 0.146470   fun_value = 1.213573
iter_num = 401   norm_grad = 0.229730   fun_value = 1.219125
iter_num = 402   norm_grad = 0.148368   fun_value = 1.207277
iter_num = 403   norm_grad = 0.220426   fun_value = 1.212894
iter_num = 404   norm_grad = 0.145594   fun_value = 1.201642
iter_num = 405   norm_grad = 0.228755   fun_value = 1.207137
iter_num = 406   norm_grad = 0.142468   fun_value = 1.195811
iter_num = 407   norm_grad = 0.239722   fun_value = 1.201135
iter_num = 408   norm_grad = 0.139421   fun_value = 1.189633
iter_num = 409   norm_grad = 0.250399   fun_value = 1.194703
iter_num = 410   norm_grad = 0.141178   fun_value = 1.182640
iter_num = 411   norm_grad = 0.237012   fun_value = 1.187815
iter_num = 412   norm_grad = 0.138424   fun_value = 1.176513
iter_num = 413   norm_grad = 0.248906   fun_value = 1.181508
iter_num = 414   norm_grad = 0.140432   fun_value = 1.169580
iter_num = 415   norm_grad = 0.233699   fun_value = 1.174682
iter_num = 416   norm_grad = 0.142547   fun_value = 1.163185
iter_num = 417   norm_grad = 0.224761   fun_value = 1.168452
iter_num = 418   norm_grad = 0.140232   fun_value = 1.157492
iter_num = 419   norm_grad = 0.229428   fun_value = 1.162574
iter_num = 420   norm_grad = 0.141139   fun_value = 1.151367
iter_num = 421   norm_grad = 0.224951   fun_value = 1.156531
iter_num = 422   norm_grad = 0.139876   fun_value = 1.145602
iter_num = 423   norm_grad = 0.227470   fun_value = 1.150687
iter_num = 424   norm_grad = 0.140284   fun_value = 1.139635
iter_num = 425   norm_grad = 0.220766   fun_value = 1.144674
iter_num = 426   norm_grad = 0.138486   fun_value = 1.134026
iter_num = 427   norm_grad = 0.227330   fun_value = 1.138998
iter_num = 428   norm_grad = 0.139871   fun_value = 1.127997
iter_num = 429   norm_grad = 0.218294   fun_value = 1.133010
iter_num = 430   norm_grad = 0.137502   fun_value = 1.122543
iter_num = 431   norm_grad = 0.225733   fun_value = 1.127429
iter_num = 432   norm_grad = 0.139109   fun_value = 1.116561
iter_num = 433   norm_grad = 0.217685   fun_value = 1.121543
iter_num = 434   norm_grad = 0.136965   fun_value = 1.111150
iter_num = 435   norm_grad = 0.222951   fun_value = 1.115991
iter_num = 436   norm_grad = 0.138059   fun_value = 1.105321
```

```
iter_num = 437   norm_grad = 0.215933   fun_value = 1.110205
iter_num = 438   norm_grad = 0.136158   fun_value = 1.099950
iter_num = 439   norm_grad = 0.222085   fun_value = 1.104748
iter_num = 440   norm_grad = 0.137481   fun_value = 1.094167
iter_num = 441   norm_grad = 0.212746   fun_value = 1.098984
iter_num = 442   norm_grad = 0.135003   fun_value = 1.088946
iter_num = 443   norm_grad = 0.219669   fun_value = 1.093615
iter_num = 444   norm_grad = 0.136521   fun_value = 1.083209
iter_num = 445   norm_grad = 0.211314   fun_value = 1.087947
iter_num = 446   norm_grad = 0.134275   fun_value = 1.078027
iter_num = 447   norm_grad = 0.219090   fun_value = 1.082668
iter_num = 448   norm_grad = 0.136021   fun_value = 1.072334
iter_num = 449   norm_grad = 0.208377   fun_value = 1.077020
iter_num = 450   norm_grad = 0.138073   fun_value = 1.066941
iter_num = 451   norm_grad = 0.199810   fun_value = 1.071705
iter_num = 452   norm_grad = 0.140164   fun_value = 1.061804
iter_num = 453   norm_grad = 0.192879   fun_value = 1.066645
iter_num = 454   norm_grad = 0.137581   fun_value = 1.057186
iter_num = 455   norm_grad = 0.199794   fun_value = 1.061952
iter_num = 456   norm_grad = 0.135047   fun_value = 1.052423
iter_num = 457   norm_grad = 0.205404   fun_value = 1.057057
iter_num = 458   norm_grad = 0.136593   fun_value = 1.047218
iter_num = 459   norm_grad = 0.198300   fun_value = 1.051898
iter_num = 460   norm_grad = 0.134291   fun_value = 1.042487
iter_num = 461   norm_grad = 0.204564   fun_value = 1.047077
iter_num = 462   norm_grad = 0.136048   fun_value = 1.037319
iter_num = 463   norm_grad = 0.194838   fun_value = 1.041916
iter_num = 464   norm_grad = 0.137626   fun_value = 1.032411
iter_num = 465   norm_grad = 0.189876   fun_value = 1.037075
iter_num = 466   norm_grad = 0.135719   fun_value = 1.027886
iter_num = 467   norm_grad = 0.191940   fun_value = 1.032432
iter_num = 468   norm_grad = 0.136336   fun_value = 1.023130
iter_num = 469   norm_grad = 0.190985   fun_value = 1.027742
iter_num = 470   norm_grad = 0.135870   fun_value = 1.018512
iter_num = 471   norm_grad = 0.190318   fun_value = 1.023086
iter_num = 472   norm_grad = 0.135507   fun_value = 1.013910
iter_num = 473   norm_grad = 0.189933   fun_value = 1.018462
iter_num = 474   norm_grad = 0.135246   fun_value = 1.009323
iter_num = 475   norm_grad = 0.189828   fun_value = 1.013871
iter_num = 476   norm_grad = 0.135088   fun_value = 1.004752
iter_num = 477   norm_grad = 0.186660   fun_value = 1.009231
iter_num = 478   norm_grad = 0.133816   fun_value = 1.000317
iter_num = 479   norm_grad = 0.190370   fun_value = 1.004781
iter_num = 480   norm_grad = 0.135052   fun_value = 0.995657
iter_num = 481   norm_grad = 0.184347   fun_value = 1.000123
iter_num = 482   norm_grad = 0.132732   fun_value = 0.991373
iter_num = 483   norm_grad = 0.188525   fun_value = 0.995732
iter_num = 484   norm_grad = 0.134143   fun_value = 0.986746
iter_num = 485   norm_grad = 0.183100   fun_value = 0.991140
iter_num = 486   norm_grad = 0.132034   fun_value = 0.982490
iter_num = 487   norm_grad = 0.187782   fun_value = 0.986810
iter_num = 488   norm_grad = 0.133640   fun_value = 0.977895
iter_num = 489   norm_grad = 0.182885   fun_value = 0.982279
iter_num = 490   norm_grad = 0.131716   fun_value = 0.973667
iter_num = 491   norm_grad = 0.184874   fun_value = 0.977936
iter_num = 492   norm_grad = 0.132334   fun_value = 0.969219
iter_num = 493   norm_grad = 0.183876   fun_value = 0.973547
iter_num = 494   norm_grad = 0.131850   fun_value = 0.964901
iter_num = 495   norm_grad = 0.183140   fun_value = 0.969188
iter_num = 496   norm_grad = 0.131463   fun_value = 0.960598
iter_num = 497   norm_grad = 0.182661   fun_value = 0.964858
iter_num = 498   norm_grad = 0.131171   fun_value = 0.956310
iter_num = 499   norm_grad = 0.182435   fun_value = 0.960559
iter_num = 500   norm_grad = 0.130974   fun_value = 0.952036
iter_num = 501   norm_grad = 0.179197   fun_value = 0.956212
```

```
iter_num = 502   norm_grad = 0.129653   fun_value = 0.947892
iter_num = 503   norm_grad = 0.182651   fun_value = 0.952043
iter_num = 504   norm_grad = 0.130834   fun_value = 0.943533
iter_num = 505   norm_grad = 0.176604   fun_value = 0.947677
iter_num = 506   norm_grad = 0.132865   fun_value = 0.939253
iter_num = 507   norm_grad = 0.171809   fun_value = 0.943475
iter_num = 508   norm_grad = 0.130693   fun_value = 0.935355
iter_num = 509   norm_grad = 0.173921   fun_value = 0.939461
iter_num = 510   norm_grad = 0.131517   fun_value = 0.931223
iter_num = 511   norm_grad = 0.172971   fun_value = 0.935396
iter_num = 512   norm_grad = 0.131014   fun_value = 0.927226
iter_num = 513   norm_grad = 0.172238   fun_value = 0.931356
iter_num = 514   norm_grad = 0.130604   fun_value = 0.923242
iter_num = 515   norm_grad = 0.171716   fun_value = 0.927342
iter_num = 516   norm_grad = 0.130286   fun_value = 0.919271
iter_num = 517   norm_grad = 0.171403   fun_value = 0.923354
iter_num = 518   norm_grad = 0.130060   fun_value = 0.915313
iter_num = 519   norm_grad = 0.171297   fun_value = 0.919392
iter_num = 520   norm_grad = 0.129925   fun_value = 0.911367
iter_num = 521   norm_grad = 0.168099   fun_value = 0.915376
iter_num = 522   norm_grad = 0.128434   fun_value = 0.907555
iter_num = 523   norm_grad = 0.171604   fun_value = 0.911538
iter_num = 524   norm_grad = 0.129889   fun_value = 0.903517
iter_num = 525   norm_grad = 0.168780   fun_value = 0.907571
iter_num = 526   norm_grad = 0.128554   fun_value = 0.899730
iter_num = 527   norm_grad = 0.169478   fun_value = 0.903708
iter_num = 528   norm_grad = 0.128776   fun_value = 0.895835
iter_num = 529   norm_grad = 0.167098   fun_value = 0.899792
iter_num = 530   norm_grad = 0.127637   fun_value = 0.892072
iter_num = 531   norm_grad = 0.168176   fun_value = 0.895977
iter_num = 532   norm_grad = 0.128028   fun_value = 0.888203
iter_num = 533   norm_grad = 0.166204   fun_value = 0.892110
iter_num = 534   norm_grad = 0.127067   fun_value = 0.884464
iter_num = 535   norm_grad = 0.167667   fun_value = 0.888343
iter_num = 536   norm_grad = 0.127633   fun_value = 0.880620
iter_num = 537   norm_grad = 0.166082   fun_value = 0.884524
iter_num = 538   norm_grad = 0.126841   fun_value = 0.876905
iter_num = 539   norm_grad = 0.164708   fun_value = 0.880729
iter_num = 540   norm_grad = 0.126144   fun_value = 0.873202
iter_num = 541   norm_grad = 0.166732   fun_value = 0.877033
iter_num = 542   norm_grad = 0.126966   fun_value = 0.869395
iter_num = 543   norm_grad = 0.162462   fun_value = 0.873208
iter_num = 544   norm_grad = 0.124971   fun_value = 0.865834
iter_num = 545   norm_grad = 0.167978   fun_value = 0.869627
iter_num = 546   norm_grad = 0.123055   fun_value = 0.862194
iter_num = 547   norm_grad = 0.169866   fun_value = 0.865882
iter_num = 548   norm_grad = 0.123691   fun_value = 0.858353
iter_num = 549   norm_grad = 0.165735   fun_value = 0.862023
iter_num = 550   norm_grad = 0.121978   fun_value = 0.854736
iter_num = 551   norm_grad = 0.171036   fun_value = 0.858389
iter_num = 552   norm_grad = 0.119656   fun_value = 0.851079
iter_num = 553   norm_grad = 0.176742   fun_value = 0.854649
iter_num = 554   norm_grad = 0.117099   fun_value = 0.847320
iter_num = 555   norm_grad = 0.182947   fun_value = 0.850761
iter_num = 556   norm_grad = 0.118713   fun_value = 0.843135
iter_num = 557   norm_grad = 0.175059   fun_value = 0.846623
iter_num = 558   norm_grad = 0.120646   fun_value = 0.839136
iter_num = 559   norm_grad = 0.169514   fun_value = 0.842715
iter_num = 560   norm_grad = 0.118654   fun_value = 0.835531
iter_num = 561   norm_grad = 0.173206   fun_value = 0.839019
iter_num = 562   norm_grad = 0.119787   fun_value = 0.831653
iter_num = 563   norm_grad = 0.168182   fun_value = 0.835164
iter_num = 564   norm_grad = 0.117968   fun_value = 0.828072
iter_num = 565   norm_grad = 0.172289   fun_value = 0.831517
iter_num = 566   norm_grad = 0.119249   fun_value = 0.824221
```

```
iter_num = 567   norm_grad = 0.167725   fun_value = 0.827713
iter_num = 568   norm_grad = 0.117580   fun_value = 0.820666
iter_num = 569   norm_grad = 0.169359   fun_value = 0.824056
iter_num = 570   norm_grad = 0.118021   fun_value = 0.816935
iter_num = 571   norm_grad = 0.168287   fun_value = 0.820364
iter_num = 572   norm_grad = 0.117541   fun_value = 0.813310
iter_num = 573   norm_grad = 0.167436   fun_value = 0.816697
iter_num = 574   norm_grad = 0.117137   fun_value = 0.809698
iter_num = 575   norm_grad = 0.166803   fun_value = 0.813054
iter_num = 576   norm_grad = 0.116807   fun_value = 0.806098
iter_num = 577   norm_grad = 0.166384   fun_value = 0.809435
iter_num = 578   norm_grad = 0.116551   fun_value = 0.802512
iter_num = 579   norm_grad = 0.166175   fun_value = 0.805840
iter_num = 580   norm_grad = 0.116367   fun_value = 0.798938
iter_num = 581   norm_grad = 0.166177   fun_value = 0.802268
iter_num = 582   norm_grad = 0.116258   fun_value = 0.795376
iter_num = 583   norm_grad = 0.163464   fun_value = 0.798661
iter_num = 584   norm_grad = 0.115210   fun_value = 0.791919
iter_num = 585   norm_grad = 0.166737   fun_value = 0.795191
iter_num = 586   norm_grad = 0.116235   fun_value = 0.788293
iter_num = 587   norm_grad = 0.161458   fun_value = 0.791570
iter_num = 588   norm_grad = 0.114295   fun_value = 0.784951
iter_num = 589   norm_grad = 0.165074   fun_value = 0.788145
iter_num = 590   norm_grad = 0.115440   fun_value = 0.781352
iter_num = 591   norm_grad = 0.160257   fun_value = 0.784571
iter_num = 592   norm_grad = 0.113657   fun_value = 0.778033
iter_num = 593   norm_grad = 0.164242   fun_value = 0.781189
iter_num = 594   norm_grad = 0.114936   fun_value = 0.774459
iter_num = 595   norm_grad = 0.159834   fun_value = 0.777660
iter_num = 596   norm_grad = 0.113289   fun_value = 0.771164
iter_num = 597   norm_grad = 0.164224   fun_value = 0.774323
iter_num = 598   norm_grad = 0.114720   fun_value = 0.767615
iter_num = 599   norm_grad = 0.157265   fun_value = 0.770780
iter_num = 600   norm_grad = 0.116136   fun_value = 0.764207
iter_num = 601   norm_grad = 0.152287   fun_value = 0.767396
iter_num = 602   norm_grad = 0.117913   fun_value = 0.760880
iter_num = 603   norm_grad = 0.147324   fun_value = 0.764105
iter_num = 604   norm_grad = 0.119305   fun_value = 0.757666
iter_num = 605   norm_grad = 0.144001   fun_value = 0.760922
iter_num = 606   norm_grad = 0.121244   fun_value = 0.754484
iter_num = 607   norm_grad = 0.141118   fun_value = 0.757806
iter_num = 608   norm_grad = 0.119465   fun_value = 0.751551
iter_num = 609   norm_grad = 0.141568   fun_value = 0.754782
iter_num = 610   norm_grad = 0.119695   fun_value = 0.748506
iter_num = 611   norm_grad = 0.142148   fun_value = 0.751775
iter_num = 612   norm_grad = 0.120007   fun_value = 0.745470
iter_num = 613   norm_grad = 0.139677   fun_value = 0.748711
iter_num = 614   norm_grad = 0.118472   fun_value = 0.742563
iter_num = 615   norm_grad = 0.143595   fun_value = 0.745808
iter_num = 616   norm_grad = 0.116975   fun_value = 0.739629
iter_num = 617   norm_grad = 0.144189   fun_value = 0.742798
iter_num = 618   norm_grad = 0.117241   fun_value = 0.736592
iter_num = 619   norm_grad = 0.141833   fun_value = 0.739738
iter_num = 620   norm_grad = 0.115914   fun_value = 0.733675
iter_num = 621   norm_grad = 0.145698   fun_value = 0.736828
iter_num = 622   norm_grad = 0.114102   fun_value = 0.730753
iter_num = 623   norm_grad = 0.148240   fun_value = 0.733845
iter_num = 624   norm_grad = 0.115251   fun_value = 0.727639
iter_num = 625   norm_grad = 0.144905   fun_value = 0.730752
iter_num = 626   norm_grad = 0.113584   fun_value = 0.724735
iter_num = 627   norm_grad = 0.147716   fun_value = 0.727803
iter_num = 628   norm_grad = 0.111026   fun_value = 0.721847
iter_num = 629   norm_grad = 0.150762   fun_value = 0.724798
iter_num = 630   norm_grad = 0.112215   fun_value = 0.718692
iter_num = 631   norm_grad = 0.148229   fun_value = 0.721696
```

```
iter_num = 632   norm_grad = 0.111070   fun_value = 0.715729
iter_num = 633   norm_grad = 0.148760   fun_value = 0.718671
iter_num = 634   norm_grad = 0.111209   fun_value = 0.712685
iter_num = 635   norm_grad = 0.146574   fun_value = 0.715607
iter_num = 636   norm_grad = 0.110209   fun_value = 0.709741
iter_num = 637   norm_grad = 0.150223   fun_value = 0.712673
iter_num = 638   norm_grad = 0.107801   fun_value = 0.706837
iter_num = 639   norm_grad = 0.155206   fun_value = 0.709681
iter_num = 640   norm_grad = 0.105582   fun_value = 0.703829
iter_num = 641   norm_grad = 0.159294   fun_value = 0.706560
iter_num = 642   norm_grad = 0.106718   fun_value = 0.700531
iter_num = 643   norm_grad = 0.155662   fun_value = 0.703322
iter_num = 644   norm_grad = 0.105487   fun_value = 0.697468
iter_num = 645   norm_grad = 0.157563   fun_value = 0.700199
iter_num = 646   norm_grad = 0.105957   fun_value = 0.694268
iter_num = 647   norm_grad = 0.154359   fun_value = 0.697003
iter_num = 648   norm_grad = 0.104856   fun_value = 0.691228
iter_num = 649   norm_grad = 0.156619   fun_value = 0.693920
iter_num = 650   norm_grad = 0.105440   fun_value = 0.688051
iter_num = 651   norm_grad = 0.153806   fun_value = 0.690765
iter_num = 652   norm_grad = 0.104457   fun_value = 0.685034
iter_num = 653   norm_grad = 0.156442   fun_value = 0.687722
iter_num = 654   norm_grad = 0.105164   fun_value = 0.681881
iter_num = 655   norm_grad = 0.151333   fun_value = 0.684560
iter_num = 656   norm_grad = 0.103467   fun_value = 0.678959
iter_num = 657   norm_grad = 0.156829   fun_value = 0.681597
iter_num = 658   norm_grad = 0.101176   fun_value = 0.675974
iter_num = 659   norm_grad = 0.163769   fun_value = 0.678512
iter_num = 660   norm_grad = 0.102840   fun_value = 0.672605
iter_num = 661   norm_grad = 0.155902   fun_value = 0.675207
iter_num = 662   norm_grad = 0.100674   fun_value = 0.669643
iter_num = 663   norm_grad = 0.163246   fun_value = 0.672164
iter_num = 664   norm_grad = 0.098493   fun_value = 0.666525
iter_num = 665   norm_grad = 0.170504   fun_value = 0.668920
iter_num = 666   norm_grad = 0.099864   fun_value = 0.663004
iter_num = 667   norm_grad = 0.161501   fun_value = 0.665458
iter_num = 668   norm_grad = 0.101729   fun_value = 0.659690
iter_num = 669   norm_grad = 0.154599   fun_value = 0.662244
iter_num = 670   norm_grad = 0.099798   fun_value = 0.656776
iter_num = 671   norm_grad = 0.160371   fun_value = 0.659250
iter_num = 672   norm_grad = 0.101185   fun_value = 0.653551
iter_num = 673   norm_grad = 0.151401   fun_value = 0.656038
iter_num = 674   norm_grad = 0.102442   fun_value = 0.650520
iter_num = 675   norm_grad = 0.148272   fun_value = 0.653081
iter_num = 676   norm_grad = 0.101344   fun_value = 0.647710
iter_num = 677   norm_grad = 0.150489   fun_value = 0.650230
iter_num = 678   norm_grad = 0.101942   fun_value = 0.644770
iter_num = 679   norm_grad = 0.147715   fun_value = 0.647310
iter_num = 680   norm_grad = 0.100954   fun_value = 0.641981
iter_num = 681   norm_grad = 0.147717   fun_value = 0.644452
iter_num = 682   norm_grad = 0.100849   fun_value = 0.639131
iter_num = 683   norm_grad = 0.147886   fun_value = 0.641612
iter_num = 684   norm_grad = 0.100798   fun_value = 0.636293
iter_num = 685   norm_grad = 0.145664   fun_value = 0.638747
iter_num = 686   norm_grad = 0.099985   fun_value = 0.633533
iter_num = 687   norm_grad = 0.148667   fun_value = 0.635984
iter_num = 688   norm_grad = 0.100843   fun_value = 0.630648
iter_num = 689   norm_grad = 0.144155   fun_value = 0.633109
iter_num = 690   norm_grad = 0.099295   fun_value = 0.627978
iter_num = 691   norm_grad = 0.147437   fun_value = 0.630380
iter_num = 692   norm_grad = 0.100244   fun_value = 0.625114
iter_num = 693   norm_grad = 0.143285   fun_value = 0.627541
iter_num = 694   norm_grad = 0.098808   fun_value = 0.622463
iter_num = 695   norm_grad = 0.146869   fun_value = 0.624845
iter_num = 696   norm_grad = 0.099858   fun_value = 0.619621
```

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iter_num = 697   norm_grad = 0.143038   fun_value = 0.622040
iter_num = 698   norm_grad = 0.098521   fun_value = 0.616990
iter_num = 699   norm_grad = 0.144456   fun_value = 0.619338
iter_num = 700   norm_grad = 0.098876   fun_value = 0.614234
iter_num = 701   norm_grad = 0.143536   fun_value = 0.616611
iter_num = 702   norm_grad = 0.098477   fun_value = 0.611555
iter_num = 703   norm_grad = 0.142776   fun_value = 0.613901
iter_num = 704   norm_grad = 0.098131   fun_value = 0.608885
iter_num = 705   norm_grad = 0.142172   fun_value = 0.611207
iter_num = 706   norm_grad = 0.097835   fun_value = 0.606226
iter_num = 707   norm_grad = 0.141720   fun_value = 0.608530
iter_num = 708   norm_grad = 0.097588   fun_value = 0.603576
iter_num = 709   norm_grad = 0.141420   fun_value = 0.605870
iter_num = 710   norm_grad = 0.097391   fun_value = 0.600936
iter_num = 711   norm_grad = 0.141269   fun_value = 0.603225
iter_num = 712   norm_grad = 0.097242   fun_value = 0.598305
iter_num = 713   norm_grad = 0.141267   fun_value = 0.600596
iter_num = 714   norm_grad = 0.097143   fun_value = 0.595685
iter_num = 715   norm_grad = 0.141414   fun_value = 0.597984
iter_num = 716   norm_grad = 0.097094   fun_value = 0.593074
iter_num = 717   norm_grad = 0.139216   fun_value = 0.595347
iter_num = 718   norm_grad = 0.096276   fun_value = 0.590537
iter_num = 719   norm_grad = 0.139645   fun_value = 0.592765
iter_num = 720   norm_grad = 0.096319   fun_value = 0.587945
iter_num = 721   norm_grad = 0.140226   fun_value = 0.590200
iter_num = 722   norm_grad = 0.096413   fun_value = 0.585363
iter_num = 723   norm_grad = 0.138474   fun_value = 0.587610
iter_num = 724   norm_grad = 0.095740   fun_value = 0.582854
iter_num = 725   norm_grad = 0.139333   fun_value = 0.585075
iter_num = 726   norm_grad = 0.095927   fun_value = 0.580291
iter_num = 727   norm_grad = 0.137875   fun_value = 0.582516
iter_num = 728   norm_grad = 0.095350   fun_value = 0.577800
iter_num = 729   norm_grad = 0.139015   fun_value = 0.580011
iter_num = 730   norm_grad = 0.095633   fun_value = 0.575256
iter_num = 731   norm_grad = 0.135371   fun_value = 0.577443
iter_num = 732   norm_grad = 0.094333   fun_value = 0.572847
iter_num = 733   norm_grad = 0.139136   fun_value = 0.575005
iter_num = 734   norm_grad = 0.095487   fun_value = 0.570259
iter_num = 735   norm_grad = 0.133272   fun_value = 0.572426
iter_num = 736   norm_grad = 0.096870   fun_value = 0.567765
iter_num = 737   norm_grad = 0.129666   fun_value = 0.569979
iter_num = 738   norm_grad = 0.095381   fun_value = 0.565483
iter_num = 739   norm_grad = 0.133696   fun_value = 0.567671
iter_num = 740   norm_grad = 0.093403   fun_value = 0.563172
iter_num = 741   norm_grad = 0.137950   fun_value = 0.565289
iter_num = 742   norm_grad = 0.094730   fun_value = 0.560622
iter_num = 743   norm_grad = 0.132639   fun_value = 0.562769
iter_num = 744   norm_grad = 0.092863   fun_value = 0.558328
iter_num = 745   norm_grad = 0.137118   fun_value = 0.560415
iter_num = 746   norm_grad = 0.094268   fun_value = 0.555797
iter_num = 747   norm_grad = 0.132099   fun_value = 0.557924
iter_num = 748   norm_grad = 0.092495   fun_value = 0.553519
iter_num = 749   norm_grad = 0.136836   fun_value = 0.555598
iter_num = 750   norm_grad = 0.090500   fun_value = 0.551176
iter_num = 751   norm_grad = 0.141845   fun_value = 0.553172
iter_num = 752   norm_grad = 0.091796   fun_value = 0.548565
iter_num = 753   norm_grad = 0.135516   fun_value = 0.550595
iter_num = 754   norm_grad = 0.093361   fun_value = 0.546072
iter_num = 755   norm_grad = 0.131076   fun_value = 0.548164
iter_num = 756   norm_grad = 0.091773   fun_value = 0.543828
iter_num = 757   norm_grad = 0.133975   fun_value = 0.545858
iter_num = 758   norm_grad = 0.092660   fun_value = 0.541412
iter_num = 759   norm_grad = 0.129854   fun_value = 0.543456
iter_num = 760   norm_grad = 0.091179   fun_value = 0.539183
iter_num = 761   norm_grad = 0.132966   fun_value = 0.541177
```

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iter_num = 762   norm_grad = 0.092141   fun_value = 0.536785
iter_num = 763   norm_grad = 0.129127   fun_value = 0.538803
iter_num = 764   norm_grad = 0.090752   fun_value = 0.534573
iter_num = 765   norm_grad = 0.132471   fun_value = 0.536551
iter_num = 766   norm_grad = 0.091797   fun_value = 0.532192
iter_num = 767   norm_grad = 0.128882   fun_value = 0.534204
iter_num = 768   norm_grad = 0.090489   fun_value = 0.529996
iter_num = 769   norm_grad = 0.132483   fun_value = 0.531978
iter_num = 770   norm_grad = 0.091628   fun_value = 0.527633
iter_num = 771   norm_grad = 0.126722   fun_value = 0.529620
iter_num = 772   norm_grad = 0.092861   fun_value = 0.525360
iter_num = 773   norm_grad = 0.122840   fun_value = 0.527375
iter_num = 774   norm_grad = 0.094485   fun_value = 0.523140
iter_num = 775   norm_grad = 0.119111   fun_value = 0.525197
iter_num = 776   norm_grad = 0.095888   fun_value = 0.520993
iter_num = 777   norm_grad = 0.116817   fun_value = 0.523094
iter_num = 778   norm_grad = 0.094624   fun_value = 0.519001
iter_num = 779   norm_grad = 0.117216   fun_value = 0.521047
iter_num = 780   norm_grad = 0.094787   fun_value = 0.516941
iter_num = 781   norm_grad = 0.117705   fun_value = 0.519012
iter_num = 782   norm_grad = 0.094999   fun_value = 0.514888
iter_num = 783   norm_grad = 0.115692   fun_value = 0.516941
iter_num = 784   norm_grad = 0.093881   fun_value = 0.512915
iter_num = 785   norm_grad = 0.116328   fun_value = 0.514926
iter_num = 786   norm_grad = 0.094171   fun_value = 0.510876
iter_num = 787   norm_grad = 0.114511   fun_value = 0.512877
iter_num = 788   norm_grad = 0.093157   fun_value = 0.508915
iter_num = 789   norm_grad = 0.117793   fun_value = 0.510925
iter_num = 790   norm_grad = 0.091663   fun_value = 0.506954
iter_num = 791   norm_grad = 0.119803   fun_value = 0.508918
iter_num = 792   norm_grad = 0.092545   fun_value = 0.504865
iter_num = 793   norm_grad = 0.116948   fun_value = 0.506840
iter_num = 794   norm_grad = 0.091149   fun_value = 0.502916
iter_num = 795   norm_grad = 0.119113   fun_value = 0.504854
iter_num = 796   norm_grad = 0.092105   fun_value = 0.500841
iter_num = 797   norm_grad = 0.116447   fun_value = 0.502797
iter_num = 798   norm_grad = 0.090796   fun_value = 0.498905
iter_num = 799   norm_grad = 0.118780   fun_value = 0.500832
iter_num = 800   norm_grad = 0.091834   fun_value = 0.496844
iter_num = 801   norm_grad = 0.116286   fun_value = 0.498796
iter_num = 802   norm_grad = 0.090602   fun_value = 0.494921
iter_num = 803   norm_grad = 0.116372   fun_value = 0.496812
iter_num = 804   norm_grad = 0.090584   fun_value = 0.492938
iter_num = 805   norm_grad = 0.116544   fun_value = 0.494839
iter_num = 806   norm_grad = 0.090607   fun_value = 0.490962
iter_num = 807   norm_grad = 0.116803   fun_value = 0.492876
iter_num = 808   norm_grad = 0.090673   fun_value = 0.488993
iter_num = 809   norm_grad = 0.114698   fun_value = 0.490883
iter_num = 810   norm_grad = 0.089622   fun_value = 0.487096
iter_num = 811   norm_grad = 0.117506   fun_value = 0.488978
iter_num = 812   norm_grad = 0.087743   fun_value = 0.485210
iter_num = 813   norm_grad = 0.120321   fun_value = 0.487030
iter_num = 814   norm_grad = 0.088829   fun_value = 0.483153
iter_num = 815   norm_grad = 0.116183   fun_value = 0.484983
iter_num = 816   norm_grad = 0.090152   fun_value = 0.481147
iter_num = 817   norm_grad = 0.114410   fun_value = 0.483031
iter_num = 818   norm_grad = 0.089254   fun_value = 0.479276
iter_num = 819   norm_grad = 0.115159   fun_value = 0.481129
iter_num = 820   norm_grad = 0.089554   fun_value = 0.477346
iter_num = 821   norm_grad = 0.113569   fun_value = 0.479197
iter_num = 822   norm_grad = 0.088742   fun_value = 0.475487
iter_num = 823   norm_grad = 0.114471   fun_value = 0.477314
iter_num = 824   norm_grad = 0.089116   fun_value = 0.473570
iter_num = 825   norm_grad = 0.113051   fun_value = 0.475402
iter_num = 826   norm_grad = 0.088383   fun_value = 0.471724
```

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iter_num = 827   norm_grad = 0.114109   fun_value = 0.473539
iter_num = 828   norm_grad = 0.088833   fun_value = 0.469820
iter_num = 829   norm_grad = 0.112848   fun_value = 0.471646
iter_num = 830   norm_grad = 0.088176   fun_value = 0.467985
iter_num = 831   norm_grad = 0.111688   fun_value = 0.469764
iter_num = 832   norm_grad = 0.087567   fun_value = 0.466157
iter_num = 833   norm_grad = 0.112960   fun_value = 0.467929
iter_num = 834   norm_grad = 0.088120   fun_value = 0.464272
iter_num = 835   norm_grad = 0.111947   fun_value = 0.466065
iter_num = 836   norm_grad = 0.087580   fun_value = 0.462456
iter_num = 837   norm_grad = 0.111025   fun_value = 0.464212
iter_num = 838   norm_grad = 0.087084   fun_value = 0.460646
iter_num = 839   norm_grad = 0.112523   fun_value = 0.462405
iter_num = 840   norm_grad = 0.087750   fun_value = 0.458780
iter_num = 841   norm_grad = 0.109358   fun_value = 0.460531
iter_num = 842   norm_grad = 0.089130   fun_value = 0.456924
iter_num = 843   norm_grad = 0.107247   fun_value = 0.458716
iter_num = 844   norm_grad = 0.087930   fun_value = 0.455205
iter_num = 845   norm_grad = 0.107644   fun_value = 0.456950
iter_num = 846   norm_grad = 0.088105   fun_value = 0.453426
iter_num = 847   norm_grad = 0.108116   fun_value = 0.455192
iter_num = 848   norm_grad = 0.088322   fun_value = 0.451652
iter_num = 849   norm_grad = 0.106237   fun_value = 0.453404
iter_num = 850   norm_grad = 0.087248   fun_value = 0.449950
iter_num = 851   norm_grad = 0.106827   fun_value = 0.451664
iter_num = 852   norm_grad = 0.087529   fun_value = 0.448188
iter_num = 853   norm_grad = 0.107497   fun_value = 0.449932
iter_num = 854   norm_grad = 0.087857   fun_value = 0.446431
iter_num = 855   norm_grad = 0.105824   fun_value = 0.448170
iter_num = 856   norm_grad = 0.086893   fun_value = 0.444746
iter_num = 857   norm_grad = 0.106612   fun_value = 0.446456
iter_num = 858   norm_grad = 0.087286   fun_value = 0.443001
iter_num = 859   norm_grad = 0.105089   fun_value = 0.444711
iter_num = 860   norm_grad = 0.086404   fun_value = 0.441326
iter_num = 861   norm_grad = 0.105999   fun_value = 0.443013
iter_num = 862   norm_grad = 0.086865   fun_value = 0.439593
iter_num = 863   norm_grad = 0.104613   fun_value = 0.441285
iter_num = 864   norm_grad = 0.086058   fun_value = 0.437929
iter_num = 865   norm_grad = 0.105648   fun_value = 0.439604
iter_num = 866   norm_grad = 0.086589   fun_value = 0.436206
iter_num = 867   norm_grad = 0.104391   fun_value = 0.437893
iter_num = 868   norm_grad = 0.085852   fun_value = 0.434553
iter_num = 869   norm_grad = 0.105557   fun_value = 0.436228
iter_num = 870   norm_grad = 0.086458   fun_value = 0.432843
iter_num = 871   norm_grad = 0.104423   fun_value = 0.434533
iter_num = 872   norm_grad = 0.085788   fun_value = 0.431200
iter_num = 873   norm_grad = 0.103373   fun_value = 0.432848
iter_num = 874   norm_grad = 0.085165   fun_value = 0.429563
iter_num = 875   norm_grad = 0.104712   fun_value = 0.431207
iter_num = 876   norm_grad = 0.085869   fun_value = 0.427869
iter_num = 877   norm_grad = 0.103776   fun_value = 0.429537
iter_num = 878   norm_grad = 0.085308   fun_value = 0.426243
iter_num = 879   norm_grad = 0.102915   fun_value = 0.427875
iter_num = 880   norm_grad = 0.084789   fun_value = 0.424622
iter_num = 881   norm_grad = 0.104441   fun_value = 0.426259
iter_num = 882   norm_grad = 0.085601   fun_value = 0.422944
iter_num = 883   norm_grad = 0.101322   fun_value = 0.424575
iter_num = 884   norm_grad = 0.086591   fun_value = 0.421291
iter_num = 885   norm_grad = 0.100648   fun_value = 0.422959
iter_num = 886   norm_grad = 0.086138   fun_value = 0.419708
iter_num = 887   norm_grad = 0.100037   fun_value = 0.421351
iter_num = 888   norm_grad = 0.085724   fun_value = 0.418131
iter_num = 889   norm_grad = 0.099487   fun_value = 0.419751
iter_num = 890   norm_grad = 0.085348   fun_value = 0.416558
iter_num = 891   norm_grad = 0.098995   fun_value = 0.418158
```

```
iter_num = 892   norm_grad = 0.085009   fun_value = 0.414991
iter_num = 893   norm_grad = 0.100900   fun_value = 0.416611
iter_num = 894   norm_grad = 0.083387   fun_value = 0.413468
iter_num = 895   norm_grad = 0.102806   fun_value = 0.415047
iter_num = 896   norm_grad = 0.084418   fun_value = 0.411828
iter_num = 897   norm_grad = 0.100139   fun_value = 0.413419
iter_num = 898   norm_grad = 0.082882   fun_value = 0.410315
iter_num = 899   norm_grad = 0.102140   fun_value = 0.411870
iter_num = 900   norm_grad = 0.083968   fun_value = 0.408686
iter_num = 901   norm_grad = 0.099611   fun_value = 0.410258
iter_num = 902   norm_grad = 0.085256   fun_value = 0.407080
iter_num = 903   norm_grad = 0.097060   fun_value = 0.408671
iter_num = 904   norm_grad = 0.086297   fun_value = 0.405509
iter_num = 905   norm_grad = 0.096234   fun_value = 0.407133
iter_num = 906   norm_grad = 0.085693   fun_value = 0.404012
iter_num = 907   norm_grad = 0.095469   fun_value = 0.405603
iter_num = 908   norm_grad = 0.085134   fun_value = 0.402520
iter_num = 909   norm_grad = 0.097131   fun_value = 0.404119
iter_num = 910   norm_grad = 0.083571   fun_value = 0.401062
iter_num = 911   norm_grad = 0.097175   fun_value = 0.402600
iter_num = 912   norm_grad = 0.083571   fun_value = 0.399544
iter_num = 913   norm_grad = 0.097271   fun_value = 0.401088
iter_num = 914   norm_grad = 0.083604   fun_value = 0.398031
iter_num = 915   norm_grad = 0.097422   fun_value = 0.399583
iter_num = 916   norm_grad = 0.083673   fun_value = 0.396523
iter_num = 917   norm_grad = 0.095299   fun_value = 0.398047
iter_num = 918   norm_grad = 0.084924   fun_value = 0.394987
iter_num = 919   norm_grad = 0.094829   fun_value = 0.396559
iter_num = 920   norm_grad = 0.084573   fun_value = 0.393523
iter_num = 921   norm_grad = 0.094411   fun_value = 0.395077
iter_num = 922   norm_grad = 0.084258   fun_value = 0.392063
iter_num = 923   norm_grad = 0.094042   fun_value = 0.393603
iter_num = 924   norm_grad = 0.083978   fun_value = 0.390608
iter_num = 925   norm_grad = 0.093722   fun_value = 0.392135
iter_num = 926   norm_grad = 0.083732   fun_value = 0.389157
iter_num = 927   norm_grad = 0.093449   fun_value = 0.390674
iter_num = 928   norm_grad = 0.083520   fun_value = 0.387711
iter_num = 929   norm_grad = 0.093223   fun_value = 0.389219
iter_num = 930   norm_grad = 0.083341   fun_value = 0.386270
iter_num = 931   norm_grad = 0.093043   fun_value = 0.387771
iter_num = 932   norm_grad = 0.083196   fun_value = 0.384833
iter_num = 933   norm_grad = 0.092910   fun_value = 0.386330
iter_num = 934   norm_grad = 0.083083   fun_value = 0.383400
iter_num = 935   norm_grad = 0.092822   fun_value = 0.384895
iter_num = 936   norm_grad = 0.083004   fun_value = 0.381972
iter_num = 937   norm_grad = 0.092780   fun_value = 0.383467
iter_num = 938   norm_grad = 0.082957   fun_value = 0.380549
iter_num = 939   norm_grad = 0.092784   fun_value = 0.382046
iter_num = 940   norm_grad = 0.082944   fun_value = 0.379130
iter_num = 941   norm_grad = 0.092835   fun_value = 0.380631
iter_num = 942   norm_grad = 0.082964   fun_value = 0.377715
iter_num = 943   norm_grad = 0.090602   fun_value = 0.379184
iter_num = 944   norm_grad = 0.083855   fun_value = 0.376283
iter_num = 945   norm_grad = 0.089454   fun_value = 0.377770
iter_num = 946   norm_grad = 0.082935   fun_value = 0.374924
iter_num = 947   norm_grad = 0.090678   fun_value = 0.376400
iter_num = 948   norm_grad = 0.083903   fun_value = 0.373500
iter_num = 949   norm_grad = 0.089604   fun_value = 0.374997
iter_num = 950   norm_grad = 0.083040   fun_value = 0.372150
iter_num = 951   norm_grad = 0.088597   fun_value = 0.373602
iter_num = 952   norm_grad = 0.084712   fun_value = 0.370718
iter_num = 953   norm_grad = 0.088244   fun_value = 0.372228
iter_num = 954   norm_grad = 0.084406   fun_value = 0.369363
iter_num = 955   norm_grad = 0.087931   fun_value = 0.370860
iter_num = 956   norm_grad = 0.084135   fun_value = 0.368013
```
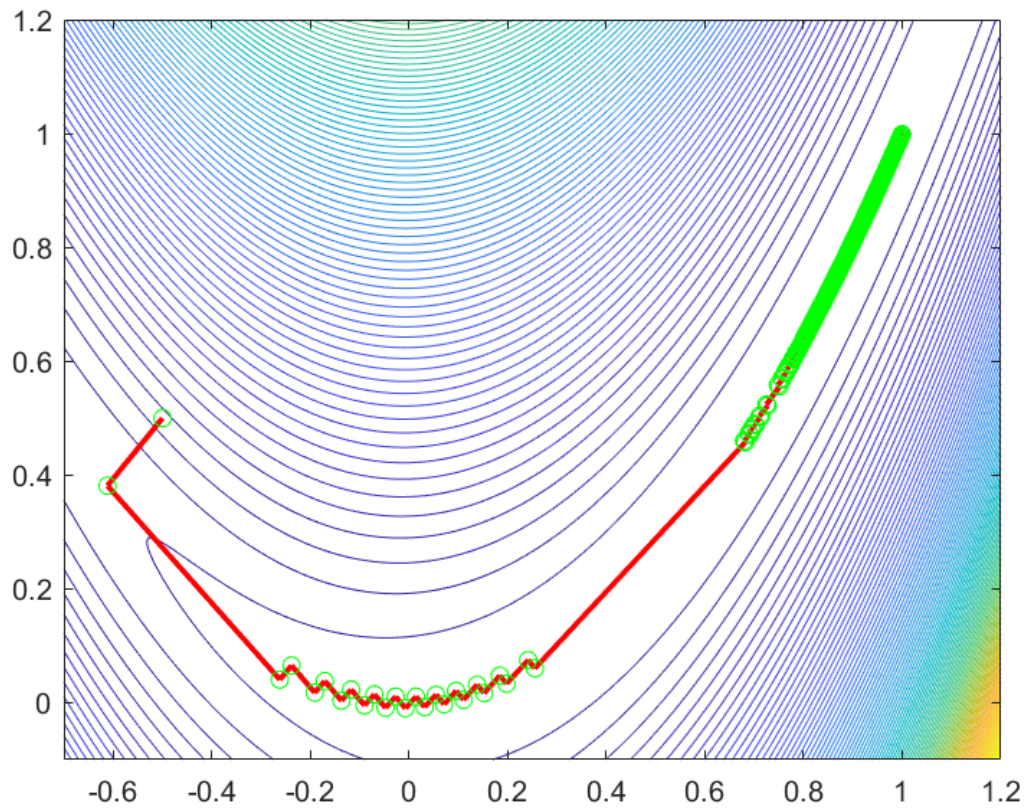
```
iter_num = 957   norm_grad = 0.087659   fun_value = 0.369498
iter_num = 958   norm_grad = 0.083900   fun_value = 0.366666
iter_num = 959   norm_grad = 0.087427   fun_value = 0.368142
iter_num = 960   norm_grad = 0.083698   fun_value = 0.365323
iter_num = 961   norm_grad = 0.087233   fun_value = 0.366792
iter_num = 962   norm_grad = 0.083532   fun_value = 0.363985
iter_num = 963   norm_grad = 0.087078   fun_value = 0.365448
iter_num = 964   norm_grad = 0.083399   fun_value = 0.362651
iter_num = 965   norm_grad = 0.086961   fun_value = 0.364110
iter_num = 966   norm_grad = 0.083299   fun_value = 0.361321
iter_num = 967   norm_grad = 0.086883   fun_value = 0.362778
iter_num = 968   norm_grad = 0.083234   fun_value = 0.359995
iter_num = 969   norm_grad = 0.086842   fun_value = 0.361452
iter_num = 970   norm_grad = 0.083202   fun_value = 0.358673
iter_num = 971   norm_grad = 0.084493   fun_value = 0.360092
iter_num = 972   norm_grad = 0.083461   fun_value = 0.357348
iter_num = 973   norm_grad = 0.084789   fun_value = 0.358784
iter_num = 974   norm_grad = 0.083761   fun_value = 0.356028
iter_num = 975   norm_grad = 0.085127   fun_value = 0.357482
iter_num = 976   norm_grad = 0.081697   fun_value = 0.354790
iter_num = 977   norm_grad = 0.085229   fun_value = 0.356179
iter_num = 978   norm_grad = 0.081791   fun_value = 0.353485
iter_num = 979   norm_grad = 0.085369   fun_value = 0.354882
iter_num = 980   norm_grad = 0.081918   fun_value = 0.352183
iter_num = 981   norm_grad = 0.085549   fun_value = 0.353590
iter_num = 982   norm_grad = 0.082081   fun_value = 0.350885
iter_num = 983   norm_grad = 0.085768   fun_value = 0.352304
iter_num = 984   norm_grad = 0.079858   fun_value = 0.349671
iter_num = 985   norm_grad = 0.087688   fun_value = 0.351045
iter_num = 986   norm_grad = 0.078900   fun_value = 0.348412
iter_num = 987   norm_grad = 0.088686   fun_value = 0.349769
iter_num = 988   norm_grad = 0.077119   fun_value = 0.347179
iter_num = 989   norm_grad = 0.090462   fun_value = 0.348497
iter_num = 990   norm_grad = 0.075723   fun_value = 0.345924
iter_num = 991   norm_grad = 0.092442   fun_value = 0.347216
iter_num = 992   norm_grad = 0.074273   fun_value = 0.344657
iter_num = 993   norm_grad = 0.093928   fun_value = 0.345910
iter_num = 994   norm_grad = 0.074994   fun_value = 0.343301
iter_num = 995   norm_grad = 0.091347   fun_value = 0.344554
iter_num = 996   norm_grad = 0.076139   fun_value = 0.341956
iter_num = 997   norm_grad = 0.089244   fun_value = 0.343232
iter_num = 998   norm_grad = 0.077364   fun_value = 0.340633
iter_num = 999   norm_grad = 0.086907   fun_value = 0.341925
iter_num = 1000   norm_grad = 0.078225   fun_value = 0.339342
iter_num = 1001   norm_grad = 0.085970   fun_value = 0.340656
iter_num = 1002   norm_grad = 0.077523   fun_value = 0.338115
iter_num = 1003   norm_grad = 0.087266   fun_value = 0.339425
iter_num = 1004   norm_grad = 0.075993   fun_value = 0.336913
iter_num = 1005   norm_grad = 0.089362   fun_value = 0.338199
iter_num = 1006   norm_grad = 0.074819   fun_value = 0.335690
iter_num = 1007   norm_grad = 0.089608   fun_value = 0.336935
iter_num = 1008   norm_grad = 0.074930   fun_value = 0.334420
iter_num = 1009   norm_grad = 0.089902   fun_value = 0.335678
iter_num = 1010   norm_grad = 0.075071   fun_value = 0.333154
iter_num = 1011   norm_grad = 0.088134   fun_value = 0.334397
iter_num = 1012   norm_grad = 0.074003   fun_value = 0.331942
iter_num = 1013   norm_grad = 0.090554   fun_value = 0.333178
iter_num = 1014   norm_grad = 0.072850   fun_value = 0.330721
iter_num = 1015   norm_grad = 0.090537   fun_value = 0.331911
iter_num = 1016   norm_grad = 0.072801   fun_value = 0.329457
iter_num = 1017   norm_grad = 0.0905...
```

```
hold on;plot(xs(:,1),xs(:,2),'-r','LineWidth',2);drawnow
hold on;plot(xs(:,1),xs(:,2),'go');drawnow
```

```
x
```

```
x = 2×1
    1.0000
    1.0000
```

```
% compare
fun = @(x)100*(x(2) - x(1)^2)^2 + (1 - x(1))^2;
x0 = [-1.2,1];
x = fminsearch(fun,x0)
```

```
x = 1×2
    1.0000    1.0000
```