1. $a, b \in \mathbb{R}$ ; $a > 0$

S: nonempty and bounded above

let $M = \sup(S)$

$\forall x \in S,\ x \leq M \mid \cdot a$

$ax \leq aM \mid + b$

$ax + b \leq aM + b$

$\Rightarrow aM + b \in ub(ax + b)$

$\forall \varepsilon > 0, \exists x \in S$ s.t $ax + b > aM + b - \varepsilon$

$aM + b \geq ax + b > aM + b - \varepsilon$

$\Rightarrow \exists z \in A$ s.t $z > a \sup(S) + b - \varepsilon$

$\Rightarrow \sup(A) = a \cdot \sup(S) + b$

$\Rightarrow \sup(ax + b) = a \cdot \sup(S) + b$

2. $U \in \mathcal{V}(a)$

$V \in \mathcal{V}(b)$

$U \in \mathcal{V}(a) \Leftrightarrow \exists \varepsilon$ s.t $[a - \varepsilon, a + \varepsilon] \subset U$

$V \in \mathcal{V}(b) \Leftrightarrow \exists \varepsilon$ s.t $[b - \varepsilon, b + \varepsilon] \subset V$

$\Rightarrow$ let $r = \frac{|a - b|}{2}$ (distance of a and b /2)

$U = (a - r, a + r)$

$V = (b - r, b + r)$

$a + r = \frac{a+b}{2}$

$b - r = \frac{a+b}{2}$

$a + b = b - r \Rightarrow U \cap V = \emptyset$

3. $A = (0,1) \cap \mathbb{Q}$

inf (A): highest lower bound of the set,

$\forall x \in A,\ x \geq 0 \Rightarrow \inf(A) = \max(lb(A))$

$lb(A) = (-\infty; 0]$

$\Rightarrow \inf(A) = 0$

sup (A): lowest upper bound of the set,

$\forall x \in A,\ x \leq \sup(A)$

$\sup(A) = \min(ub(A))$

$ub(A) = [1, +\infty) \Rightarrow$

$\Rightarrow \sup(A) = 1$

int (A):

A only has rationals in (0,1).

$\nexists (a,b)$ s.t every point in $(a,b) \in A$ (irrationals)

$\Rightarrow A = \emptyset$

cl (A):

$\forall x \in A, \exists \varepsilon \in A$ s.t $x - \varepsilon \in A$ or $x + \varepsilon \in A \Rightarrow$

$\Rightarrow \mathrm{cl}\, A = [0,1]$

for any rational nr in A, there is a neighborhood that intersects A