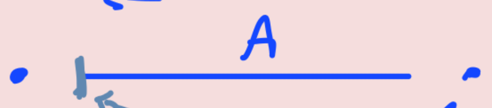
← lower higher →



$x \in lb(A)$
lower bound
$\leq$ cu toate elem din A
margine inf.

$y \in ub(A)$
upper bound
$\geq$ cu toate elem din A
margine sup.

min(A), belongs to set

$x \in A \cap B$
$\Rightarrow x \in A$ and $x \in B$

- supremum

$x :=$ (is defined as) $\sup(A)$

1. $x \geq a, \forall a \in A; \Rightarrow x \in ub(A)$
2. if u is an upper bound for A, then $x \leq u$

The supremum is the least upper bound

$\sup(A) := \min(ub(A))$

- ex: $A = [0, 1)$

$lb(A) = (-\infty, 0]$
$ub(A) = [1, +\infty)$

$\min(A) = 0$

$\nexists \max(A)$; the ub and the set have no elements in common

- **infimum**

$x := \inf(A)$

1. $x \in lb(A)$
2. if u is a lower bound of $A \Rightarrow x \geq 1$ (largest lower bound)

- $\sup(A) = 1$
- $\inf(A) = \min(A) = 0$

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example: ① $A = \{\frac{1}{n} \mid n \in \mathbb{N}\} = \{1, \frac{1}{2}, \frac{1}{3}, \dots\}$

↳ discrete set

- $\sup(A) = 1 = \max(A)$
- $\inf(A) = 0$, $\nexists \min(A)$

$lb(A) = (-\infty, 0]$

② $A = \{x \in \mathbb{Q} \mid x^2 \leq 2\} = [-\sqrt{2}, \sqrt{2}] \cap \mathbb{Q}$

$\sup(A) = \sqrt{2}$, $\nexists \max(A)$

$\inf(A) = -\sqrt{2}$, $\nexists \min(A)$

Completeness Axiom

every set that is bounded upper / lower has a supremum / infimum

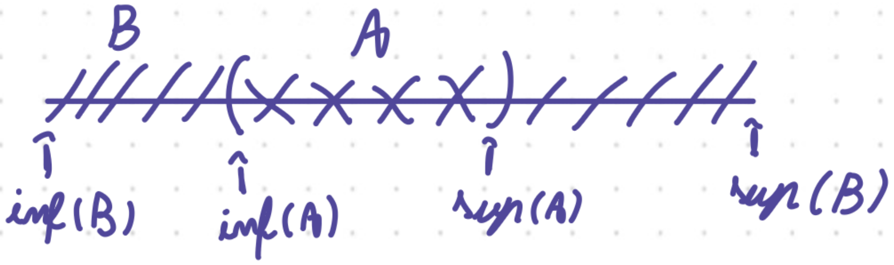
$\forall \varepsilon > 0, \exists x \in A$ such as $\sup(A) - \varepsilon < x$

$\forall \varepsilon > 0, \exists x \in A$ such as $x < \inf(A) + \varepsilon$

$\sup(A) - \varepsilon \notin ub(A)$

$\inf(A) + \varepsilon \notin lb(A)$

- Proposition -



$$\inf(B) \le \inf(A) \le \sup(A) \le \sup(B)$$

$$\sup(A \cup B) = \max\{\sup(A), \sup(B)\}$$

$$\inf(A \cup B) = \min\{\inf(A), \inf(B)\}$$

- Dacă avem ∞. Definition -

$\overline{\mathbb{R}} : \mathbb{R} \cup \{-\infty, \infty\}$

$\forall x \in \mathbb{R}, -\infty < x < \infty$

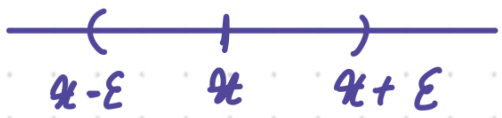
if a set A is not bounded above $\Rightarrow \sup(A) := \infty$

below $\Rightarrow \inf(A) := -\infty$

- neighborhood (vecinity) -

a set $V \subseteq \mathbb{R}$ is a -‖- of $x \in \mathbb{R}$ if:

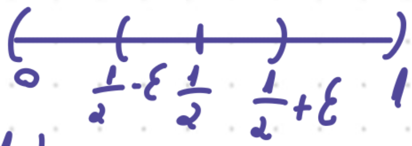
$$\boxed{\exists \varepsilon > 0 \text{ such that } (x - \varepsilon, x + \varepsilon) \subseteq V}$$



- ex: $(0,1) \notin V_{(0)}$

$\forall \varepsilon > 0, (-\varepsilon, \varepsilon) \not\subset (0,1)$

$(0,1) \in \mathcal{V}(\frac{1}{2})$



$\varepsilon = \frac{1}{3}$ $(\varepsilon \leq \frac{1}{2})$

$(0,1) \cup \{100\} \in \mathcal{V}(\frac{1}{2})$

- Interior

  int(A) -> toată vecinătatea e în set

  $\text{int}(0,1) = (0,1)$

- Closure

  $\text{cl}(0,1) = [0,1]$

check lecture notes