partial 1 midterm (16 nov)
partial 2 final exam (18 ian)

notă = 1 + p1 + p2 + Bonus points (seminar, projects)

① Relations

↳ relational database

2 sets (not necessarily of the same type) with a connection

Def 1 $r = (A, B, R)$

A – domain
B – codomain
R – graph of relation r
$R \subseteq A \times B$ (A times B) $= \{(a, b) \mid a \in A, b \in B\}$
$A = B \Rightarrow r$ – homogeneous

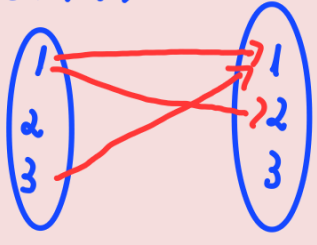
Def 2 $r = (A, B, R)$; $X \subseteq A$

$r(X) = \{b \in B \mid \exists x \in X : x \, r \, b\}$

if $x \in X \Rightarrow r\langle x \rangle = r(\{x\})$

$$r(X) = \bigcup_{x \in X} r\langle x \rangle$$

Ex: $r = (A, B, R)$
$A = \{1, 2, 3\}$, $B = \{1, 2\}$
$R = \{(1,1); (1,2); (3,1)\}$



$0 = (A, B, \emptyset)$ -> void relation

$u = (A, B, A \times B)$ -> universal relation

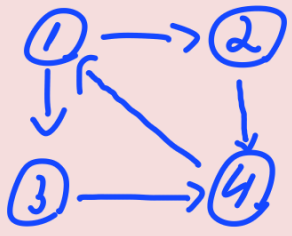
$\delta_A = (A, A, \Delta A)$
$\Delta_A = \{(a, a) \mid a \in A\}$ } -> equality relation

every function is a relation

$r([1,2]) = \{b \in \mathbb{R} \mid \exists a \in [1,2] : a \, r \, b\}$
$a \leq b$

$= [1, +\infty)$
$\downarrow$
b

graphs have relations



Functions

fiecare elem in domeniu $\xrightarrow{\text{rel.}}$ cu un singur elem in codomeniu

$(a, b) \in F \iff f(a) = b$

ex:
R: $|r\langle a \rangle| = 1, \forall a \in A$ function!
S: $s\langle 2 \rangle = \emptyset, |s\langle a \rangle| = 0$ not func

T: $t\langle 1\rangle = \{1, 2\}, |t\langle 1\rangle| = 2$

Equivalence relations

homogeneous relation $r = (A, A, R)$ on A is called:

1) reflexive ($\mathfrak{r}$): $\forall x \in A, x \mathfrak{r} x$

ex: $f(x) = x$

divisibility

2) transitive: $x, y, z \in A$; $x \mathfrak{r} y$ and $y \mathfrak{r} z \Rightarrow x \mathfrak{r} z$

ex: $x \leq y$ and $y \leq z \Rightarrow x \leq z$ (for every case!!)

non-transitive: $x \perp y$ and $y \perp z \neq x \perp z$

$x, y, z \Rightarrow x \parallel y$ and $y \parallel z \neq x \parallel z$

symmetric: $x, y \in A$; $x \mathfrak{r} y \Rightarrow y \mathfrak{r} x$

a) equality rel $\Rightarrow$ equivalence rel

b) similarity of Δ ↗ equivalence rel

c)

d) let ρ_m be rel on $\mathbb{Z}$:

$x \rho_m y \Leftrightarrow x \equiv y \pmod m$

Partitions - family / set of sets

reuniunea tuturor submultimilor $A_i = A$

$A = \{1, 2, 3, 4, 5\}$

$A_1 = \{1, 2, 3\}$; $A_2 = \{4\}$; $A_3 = \{5\}$

Quotient set

2 elem related if they are in the same subset

$r \in E(A) \Rightarrow A/r \in P(A)$

ex: a) $A/r = \{ r<a> \mid a \in A \} = \{ \{1, 2\}; \{3\} \} \in P(A)$

$r<1> = \{1, 2\}$

$r<2> = \{1, 2\}$

$r<3> = \{3\}$

b) $x \, r_\pi \, y \overset{def}{\Leftrightarrow} \exists i \in I : x, y \in A_i$

$R_\pi = \{(1,1), (2,2), (2,3), (3,2), (3,3), (4,4)\}$

$\{1\}$ $\{2, 3\}$ $\{4\}$