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Exam on Dynamical Systems
June 6, 2022

1. (1p) Consider the map

$$f : (0, \infty) \to \mathbb{R}, \quad f(x) = \frac{x^2 + 5}{2x} \ .$$

(a) Prove that it has a unique fixed point, denoted η^*.
(b) Using the linearization method, prove that η^* is an attractor.
(c) Using the stair-step (cob-web) diagram, estimate the basin of attraction of η^*.

2. (2p) Consider the following planar system

$$\dot{x} = -y(x^2 + y^2), \quad \dot{y} = x(x^2 + y^2).$$

a) Find its equilibria.
b) Decide whether the equilibrium point $(0, 0)$ is hyperbolic or not.
c) Verify that $\varphi(t, 1, 0) = (\cos t, \sin t)$, $\varphi(t, 2, 0) = (2\cos 4t, 2\sin 4t)$ for all $t \in \mathbb{R}$. Find $\varphi(t, 3, 0)$.
d) Find a first integral.
e) Represent its phase portrait.
f) What remarkable property have the solutions of this system?

3. (2p) Consider the system

$$\dot{x} = ax - 5y, \ \dot{y} = x - 2y \ .$$

(a) For what values of the real parameter a the system has a center at the origin?
(b) For $a = 0$ find the general solution of this system and specify its type and stability.