Computational Logic - Seminar 14 | 20.01

Homework - Logic Circuits | 913 | FÎNTÎNĂ OLIVIA

Exercise : 2.8

Problem Statement: For the following Boolean function, draw the corresponding logic circuit using derived gates, simplify the function and draw the logic circuits associated to all simplified forms of the initial function using only basic gates.

$$f_8(x, y, z) = x(\bar{y} \uparrow z) \vee \bar{x}(\bar{y} \oplus z) \vee y(x \oplus \bar{z})$$

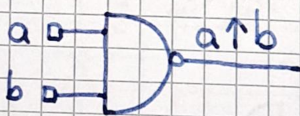
Solution:

We'll start by defining the derived gates as their name, symbol and associated boolean function:

XOR

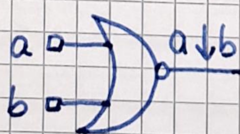

$a \oplus b = \bar{a}b \vee a\bar{b}$

→ true when a - false and b - true, or a - true and b - false

NAND


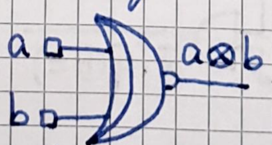
$a \uparrow b = \overline{ab} = \bar{a} \vee \bar{b}$

→ true when a - false or b - false (negated and)

NOR


$a \downarrow b = \overline{a \vee b} = \bar{a}\bar{b}$

→ true when a - false and b - false (negated or)

NXOR


$a \otimes b = \bar{a}\bar{b} \vee ab = (\bar{a} \vee b)(\bar{b} \vee a)$

→ true when (a - false or b - true) and (b - false or a - true) (negated XOR)

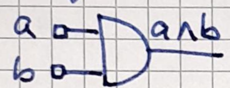
Computational Logic - Seminar 14	20.01
Homework - Logic Circuits 2.8 913	FINȚINĂ OLIVIA

∝ The next step is identifying the gates we need to use in order to represent f ⟹

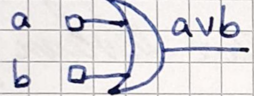
$$f_8(x,y,z) = x(\overline{y} \uparrow z) \lor \overline{x}(\overline{y} \oplus z) \lor y(x \oplus \overline{z})$$

AND, NAND GATE, OR GATE, XOR GATE, OR GATE, XOR GATE, NOT

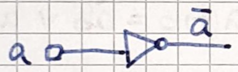
∝ The basic gates and their symbols are:

AND 

: conjunction $(a \land b)$

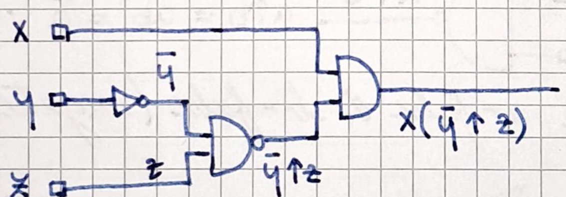
OR 

: disjunction $(a \lor b)$

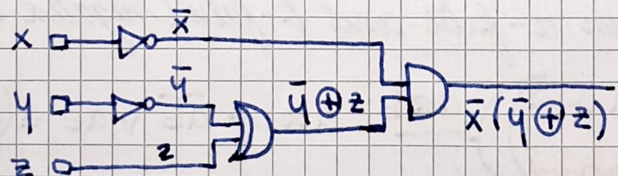
NOT 

: negation $(\sim\neg a)$

I. $x(\overline{y} \uparrow z)$:

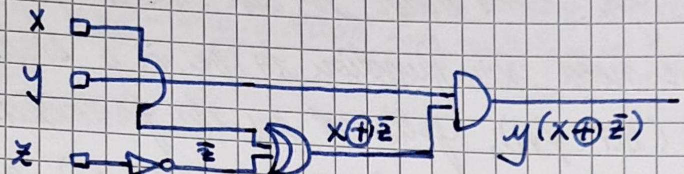


II. $\overline{x}(\overline{y} \oplus z)$

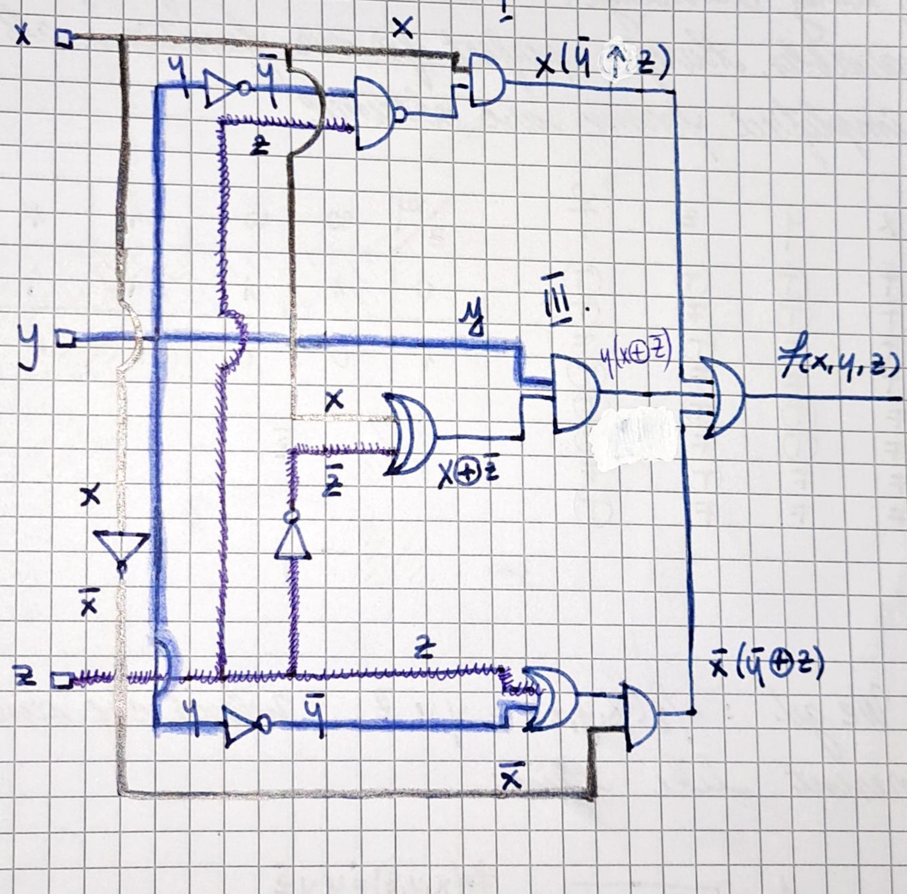


Computational Logic - Seminar 14 | 20.01
Homework - Logic Circuits 2.8 | 913 | FÎNTÎNĂ OLIVIA

III. $y(x \oplus \bar{z})$



I, II, III =>



Computational Logic - Seminar 14 | 20.01

Homework - Logic Circuits | 913 - FÎNTÎNĂ OLIVIA

Page 4

The next step is <u>simplification</u>: in order to use the simplification algorithm, we need our function to be in a disjunctive canonical form (DCF(f_8)), followed by the factorization process (the set of maximal monoms (M(f_8)), after which we select the central monoms (C(f_8)). At last, by using Karnaugh (which can be used for up to 4 variables, therefore ~~perfect for~~ perfectly suited to our function), all simplified forms are obtained.

$$f(x,y,z) = x(\bar{y} \uparrow z) \vee \bar{x}(\bar{y} \oplus z) \vee y(x \oplus \bar{z}) \xrightarrow[\text{of } \uparrow]{\text{replacement}}$$

$$f(x,y,z) = x(\bar{\bar{y}} \vee \bar{z}) \vee \bar{x}(\bar{y} \oplus z) \vee y(x \oplus \bar{z}) \Rightarrow$$

$$f(x,y,z) = x(y \vee \bar{z}) \vee \bar{x}(\bar{y} \oplus z) \vee y(x \oplus \bar{z}) \xrightarrow[\text{of } \oplus]{\text{replacement}}$$

$$f(x,y,z) = x(y \vee \bar{z}) \vee \bar{x}(yz \vee \overline{yz}) \vee y(xz \vee \overline{xz}) \xrightarrow[\text{distributivity}]{\text{using}}$$

$$f(x,y,z) = (xy \vee x\bar{z}) \vee (\bar{x}yz \vee \overline{xyz}) \vee (xyz \vee \bar{x}y\bar{z}) \Rightarrow$$

$$f(x,y,z) = xy \vee x\bar{z} \vee \bar{x}yz \vee \overline{xyz} \vee xyz \vee \bar{x}y\bar{z} \Rightarrow$$

$$f(x,y,z) = xy\bar{z} \vee xyz \vee x\bar{y}\bar{z} \vee x\overline{yz} \vee \overline{xyz} \vee xyz \vee \bar{x}y\bar{z} \Rightarrow$$

$$f(x,y,z) = xyz \vee xy\bar{z} \vee \bar{x}yz \vee x\bar{y}z \vee \overline{xyz} \vee \bar{x}y\bar{z} \vee x\overline{yz}$$

$max_1 = y$

$max_2 = m_4 \vee m_5 \vee m_7 \vee m_6 = x$

$max_3 = m_0 \vee m_4 \vee m_2 \vee m_6 = \bar{z}$

~~max~~

x \ yz	00	01	11	10
0	m_0		m_3	m_2
1	m_4	m_5	m_7	m_6

Computational Logic – Seminar 14		20.01
Homework – Logic Circuits 2.8	913	Fintină Olivia

Page 5

$M(f) = \{ max_1, max_2, max_3 \} \to$ set of maximal monoms

$C(f) = \{ max_1, max_2, max_3 \} \to$ set of central monoms

$M(f) = C(f) \Rightarrow$

$f_8^{\delta}(x,y,z) = max_1 \vee max_2 \vee max_3$

$= y \vee x \vee \bar{z} \Rightarrow$

$= x \vee y \vee \bar{z}$

The simplified function can be represented: