3, 7, 8, 10

7. Let $n \in \mathbb{N}$. Consider the relation ρ_n on $\mathbb{Z}$, defined by:

$$x \, \rho_n \, y \Leftrightarrow n \mid (x-y)$$

ρ_n - equivalence rel. and $\mathbb{Z}/\rho_n$

R S T

R: $\forall x \in \mathbb{Z}$, $x \, \rho_n \, x = n \mid (x-x) = 0$.

$n \mid 0, \forall n \in \mathbb{N}$

S. $x \, \rho_n \, y = n \mid (x-y)$

$x - y = k \cdot n$, $k \in \mathbb{Z}$

$\Rightarrow \underline{y - x = -k \cdot n}$

$\Rightarrow n \mid (y-x) = y \, \rho_n \, x$

T. $x \, \rho_n \, y = n \mid (x-y)$

$y \, \rho_n \, z = n \mid (y-z)$

$$\left.\begin{array}{l} x - y = k \cdot n, \, k \in \mathbb{Z} \\ y - z = l \cdot n, \, l \in \mathbb{Z} \end{array}\right\} + \Rightarrow x - y + y - z =$$

$$= x - z = k \cdot n + l \cdot n$$

$$= \underline{(k+l)n}$$

$\Rightarrow n \mid (x-z) = x \, \rho_n \, z$

$R + S + T \Rightarrow \rho_n$ - equivalence relation

$\mathbb{Z}/\rho_n = ?$

let $[a] = \{x \in \mathbb{Z} \mid x \equiv a \pmod{n}\}$

$\Rightarrow x = a + kn,\ k \in \mathbb{Z}$

$\mathbb{Z}/\rho_n = \{[0], [1], [2], \dots [n-1]\}$

$n = 0$: the relation is trivial because $n \mid (x-y)$ is not possible.

$\mathbb{Z}/\rho_0$ can't be determined

$n = 1 : \forall x, y \in \mathbb{Z},\ 1 \mid (x-y) = TRUE$

$\mathbb{Z}/\rho_1 = \mathbb{Z}$

8. $\{\{1\}, \{2\}, \{3\}\}$ - partition

$\{(1,1), (2,2), (3,3)\}$ - pair

$\{\{1,2\}, \{3\}\}$

$\Delta_M \cup \{(1,2), (2,1)\}$

$\{\{1,3\}, \{2\}\}$

$\Delta_M \cup \{(1,3), (3,1)\}$

$\{\{2,3\}, \{1\}\}$

$\Delta_M \cup \{(2,3), (3,2)\}$

$\{\{1,2,3\}\}$

$\Delta_M \cup \{(1,2), (2,1), (1,3), (3,1), (2,3), (3,2)\}$

10. $m \, r \, n \iff \exists a \in \mathbb{N}: m = 2^a n$

$m \, s \, n \iff (m = n$ or $m = n^2$ or $n = m^2)$

r

R: $m \, r \, m \iff m = 2^a m$

if $a = 0$ TRUE

S. $m \, r \, n = n \, r \, m$

$m = 2^a n \Rightarrow n = 2^{-a} m$

2^{-a} not an integer $\Rightarrow$ FALSE

$\Rightarrow r$ in not an equivalence rel.

s

$m = n$ or $m = n^2$ or $n = m^2$

R: $m \, s \, m \iff m = m$ TRUE

S: $m \, s \, n \iff m = n^2$

$n \, s \, m \iff n = m^2$

$n \neq m^2$ in general $\Rightarrow$ FALSE

$\Rightarrow s$ is not an equivalence rel.

3. Reflexitivity only

$A = \{1, 2, 3\}$

let relation $R = \underbrace{\Delta_M}_{\text{reflexive}} \cup \{(1,2)\}$

Symmetry only

$A = \{1, 2, 3\}$

let relation $S = \{(1,2), (2,1)\}$

Transitivity only

$A = \{1, 2, 3\}$

Let rel. $T = \{(1,2), (2,3), (1,3)\}$

$+ \quad \Rightarrow$