Heat wave simulation - supplemental files

Debora

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Definition of heat waves: "According to the recommendation of IPCC, we refer to local climatology and define heat wave as the period of at least five consecutive days with a daily maximum temperature over 5 °C higher than that of the local summertime climatology." (Li et al., 2018)

1. Identify climatology: The first step involves calculating the average daily maximum temperature (T_{max}) for the months of June, July, and August across the years 1961 to 1990. This average serves as the local temperature threshold, representing the standard "normal" for summer in that specific area.
2. Identify warm days: A warm day is identified whenever the daily maximum temperature exceeds the established baseline by 5°C or more. These are days where the heat significantly deviates from historical norms.
3. Identify heat wave: A heat wave is officially recorded only when at least five consecutive warm days occur. Frequency: The number of times these multi-day events happen. Duration: The total length of the continuous hot wave.

Li, Z., Huang, G., Huang, W., Lin, Q., Liao, R., & Fan, Y. (2018). Future changes of temperature and heat waves in Ontario, Canada. Theoretical and Applied Climatology, 132(3-4), 1029–1038. https://doi.org/10.1007/s00704-017-2123-8

Historical data to obtain average Tmax (climatology)

Define years and station

```r
years <- 1961:1990
station <- "CYYZ"
```

Collect temperature data from all years (1961:1990)

```
##         date month_day temp_c  diel_sd
## 1 1961-06-01     06-01   21.3 2.661317
## 2 1961-06-02     06-02   24.1 3.392308
## 3 1961-06-03     06-03   23.0 4.263903
## 4 1961-06-04     06-04   26.3 5.235574
## 5 1961-06-05     06-05   28.5 5.103322
## 6 1961-06-06     06-06   26.3 3.876041
```

```r
years <- 1961:1990
station <- "CYYZ"
```

Overall mean of historical daily maxima

```r
summer_climatology_mean <- round(mean(all_data$temp_c, na.rm = TRUE), 2)
summer_climatology_mean
```

```
## [1] 25.38
```

Finding recent heat waves

Define station and date range

Download observations

```
obs <- riem_measures(
  station = station,
  date_start = start_date,
  date_end = end_date
)

obs$temp_c <- (obs$tmpf - 32) * 5 / 9
obs$date <- as.Date(obs$valid)
obs$month <- format(obs$date, "%m")
```

Keep only summer months: June, July, August

```
obs <- obs[obs$month %in% c("06", "07", "08"), ]
```

Daily max temperatures

```
daily_max <- aggregate(temp_c ~ date, data = obs, FUN = max)
```

Identify 5+ consecutive days 5°C above historical average tmax

Keep only summer months: June, July, August

```
over <- daily_max$temp_c > summer_climatology_mean + 5
```

Run-length encoding: groups consecutive TRUEs and FALSEs together

```
runs <- rle(over)
```

Ending positions of each run

```
ends <- cumsum(runs$lengths)
```

Calculate the starting positions of each run

```
starts <- ends - runs$lengths + 1
```

Find runs where condition was TRUE for 5 or more consecutive days

```
heatwave_indices <- which(runs$values & runs$lengths >= 5)
```

Get start and end dates of heatwaves

```
heatwave_periods <- data.frame(
  start_date = daily_max$date[starts[heatwave_indices]],
  end_date   = daily_max$date[ends[heatwave_indices]],
  duration   = runs$lengths[heatwave_indices]
)
```

View result

```
print(heatwave_periods)
```

```
##   start_date   end_date duration
## 1 2021-08-22 2021-08-26        5
## 2 2022-07-19 2022-07-24        6
```

Function to get mean, sd, and autocorrelation for 15-day window

```
get_stats_around_heatwave <- function(mid_date, daily_max) {
  start_window <- mid_date - 7
  end_window <- mid_date + 7

  # Subset data
  window_data <- daily_max[daily_max$date >= start_window & daily_max$date <= end_window, ]

  # Calculate stats
  mean_temp <- mean(window_data$temp_c, na.rm = TRUE)
  sd_temp <- sd(window_data$temp_c, na.rm = TRUE)
  acf_temp <- acf(window_data$temp_c, lag.max = 1, plot = FALSE)$acf[2]  # lag-1 autocorrelation

  # Return with start and end date
  return(data.frame(
    start_date = start_window,
    end_date = end_window,
    mean = round(mean_temp, 2),
    sd = round(sd_temp, 2),
    autocorrelation = round(acf_temp, 2)
  ))
}
```

Calculate midpoint of each heatwave

```
mid_dates <- as.Date(heatwave_periods$start_date + floor((heatwave_periods$end_date - heatwave_periods$
```

Apply to each midpoint

```
results <- do.call(rbind, lapply(mid_dates, get_stats_around_heatwave, daily_max = daily_max))
```

View result

```
print(results)
```

```
##   start_date   end_date  mean   sd autocorrelation
## 1 2021-08-17 2021-08-31 29.47 2.29            0.28
## 2 2022-07-14 2022-07-28 29.33 3.06            0.53
```

Get stats around historical data with the same timeframe of heatwaves

Reference heatwave midpoints

```
mid_dates_reference <- as.Date(c("2021-08-24", "2022-07-21"))
```

Generate 15-day windows for each midpoint across all years

```
get_windows <- function(mid_date, years) {
  md <- format(mid_date, "%m-%d")
  lapply(years, function(yr) {
    ref <- as.Date(paste0(yr, "-", md))
    seq(ref - 7, ref + 7, by = "1 day")
  })
}
```

Apply for both midpoints and combine

```
windows1 <- get_windows(mid_dates_reference[1], years)
windows2 <- get_windows(mid_dates_reference[2], years)
windows_all <- unique(do.call(c, c(windows1, windows2)))
```

Filter for those dates

```
selected_data <- all_data[all_data$date %in% windows_all, ]
```

Ensure date column is in Date format

```
selected_data$date <- as.Date(selected_data$date)
```

Extract year, month, and day

```
selected_data$year <- format(selected_data$date, "%Y")
selected_data$month_day <- format(selected_data$date, "%m-%d")
```

Define periods

```
periods <- list(
  c("07-14", "07-28"),
  c("08-17", "08-30")
)
```

Get all unique years

```
years <- unique(format(selected_data$date, "%Y"))
```

Loop over each year and period

```
for (yr in years) {
  for (p in periods) {
    start_date <- as.Date(paste0(yr, "-", p[1]))
    end_date <- as.Date(paste0(yr, "-", p[2]))

    # Subset the data for this year and period
    subset_data <- selected_data[selected_data$date >= start_date &
                                 selected_data$date <= end_date, ]

    temps <- subset_data$temp_c
    if (length(temps) >= 2) {
      all_means <- c(all_means, mean(temps, na.rm = TRUE))
      all_sds <- c(all_sds, sd(temps, na.rm = TRUE))
      all_acfs <- c(all_acfs, acf(temps, lag.max = 1, plot = FALSE)$acf[2])
    }
  }
}
```

Final summary

```
final_mean <- round(mean(all_means, na.rm = TRUE), 2)
final_sd <- round(mean(all_sds, na.rm = TRUE), 2)
final_acf <- round(mean(all_acfs, na.rm = TRUE), 2)
```

Print

```
cat("Mean of means:", final_mean, "\n")
```

```
## Mean of means: 25.82
```

```
cat("Mean of SDs:", final_sd, "\n")
```

```
## Mean of SDs: 2.96
```

```
cat("Mean of autocorrelations:", final_acf, "\n")
```

```
## Mean of autocorrelations: 0.41
```

MIMICRY FUNCTION

```r
mimicry <- function(x, y) {
  xs <- sort(x, index.return = TRUE)
  ys <- sort(y, index.return = TRUE)
  zs <- sort(ys$ix, index.return = TRUE)
  z <- xs$x[zs$ix]
  return(z)
}
```

MIMICRY SIMULATION FUNCTION

```r
simulate_temp_mimicry <- function(n_days, mean_temp, target_autocorr, target_sd,
                                  lower_thresh = min(selected_data$temp_c),
                                  upper_thresh = max(selected_data$temp_c), tol = 100) {
  # Step 1: White noise
  x <- rnorm(n_days, mean = mean_temp, sd = target_sd)

  # Step 2: Truncate out-of-bounds values
  icount <- 0
  while ((any(x > upper_thresh) || any(x < lower_thresh)) && icount < tol) {
    icount <- icount + 1
    for (i in 1:length(x)) {
      if (x[i] > upper_thresh || x[i] < lower_thresh) {
        x[i] <- rnorm(1, mean = mean_temp, sd = target_sd)
      }
    }
  }
  if (icount >= tol) {
    x[x > upper_thresh] <- upper_thresh
    x[x < lower_thresh] <- lower_thresh
  }

  # Step 3: Standardize and rescale
  x <- scale(x)[, 1]
  noise <- x * target_sd + mean_temp

  # Step 4: Generate AR(1) process
  max_attempts <- 1000
  attempt <- 0
  best_diff <- Inf
  best_gs <- NULL

  while (attempt < max_attempts) {
    attempt <- attempt + 1
    gs <- numeric(n_days)
    gs[1] <- rnorm(1)
    crand <- rnorm(n_days)
    for (i in 2:n_days) {
      gs[i] <- target_autocorr * gs[i - 1] + crand[i] * sqrt(target_sd) * sqrt(1 - target_autocorr^2)
    }
    ac_try <- acf(gs, lag.max = 1, plot = FALSE)$acf[2]
    diff <- abs(ac_try - target_autocorr)
    if (diff < 0.01) break
    if (diff < best_diff) {
      best_diff <- diff
      best_gs <- gs
    }
  }

  if (attempt >= max_attempts) {
    warning("Max attempts reached. Using best approximation.")
    gs <- best_gs
  }

  # Step 5: Apply mimicry
  z <- mimicry(noise, gs)
  return(round(z, 1))
}
```

RUN SIMULATIONS

```r
n_days <- 15
set.seed(123)
```

Function to count clusters of 5+ consecutive days > threshold

```r
count_clusters <- function(temp, threshold) {
  exceed <- temp > threshold
  rle_res <- rle(exceed)
  sum(rle_res$values & rle_res$lengths >= 5)
}
```

Number of simulations and days

```r
n_sim <- 1000
```

Threshold = Ambient mean of max + 5BC

```r
temp_threshold <- summer_climatology_mean + 5
```

Prepare list to store results

```r
cluster_counts <- list()
simulation_data <- list()
```

New values after 40% and 60% increase in autocorrelation

```r
auto_40 <- sqrt(0.6^2 * 1.40)
auto_60 <- sqrt(0.6^2 * 1.60)
```

Create the simulation_scenarios data frame

```r
simulation_scenarios <- data.frame(
  Scenario = c("Ambient", "Ambient+", "Ambient+Autocorrel+40", "Ambient+Autocorrel+60"),
  Mean = c(summer_climatology_mean, summer_climatology_mean+3.5, summer_climatology_mean+3.5, summer_cl
  #historical mean of max daily temps or mean+climate change warming
  SD = c(final_sd), #historical standard deviation
  Autocorrelation = c(0.6, 0.6, auto_40, auto_60),
  #historical autocorrelation of max daily temps + projected increase in autocorrelation for eastern Ca
  #fig. 3 https://doi.org/10.1038/s41586-021-03943-z
  stringsAsFactors = FALSE
)
```

Loop through each scenario

```r
for (i in 1:nrow(simulation_scenarios)) {
  scenario <- simulation_scenarios$Scenario[i]
  mean_temp <- simulation_scenarios$Mean[i]
  sd_temp <- simulation_scenarios$SD[i]
  autocorr <- simulation_scenarios$Autocorrelation[i]

  # Run simulations
  sims <- replicate(n_sim, simulate_temp_mimicry(n_days, mean_temp, autocorr, sd_temp), simplify = FALS

  # Store first 6 simulations for plotting
  for (j in 1:6) {
    sim_df <- data.frame(
      Day = 1:n_days,
      Temp = sims[[j]],
      Scenario = scenario,
      Simulation = paste("Sim", j)
    )
    simulation_data[[length(simulation_data) + 1]] <- sim_df
  }

  # Count clusters of 5+ days > threshold
  cluster_counts[[scenario]] <- sapply(sims, count_clusters, threshold = temp_threshold)
}
```

Summary table

```r
cluster_summary <- data.frame(
  Scenario = names(cluster_counts),
  Simulations_with_clusters = sapply(cluster_counts, function(x) sum(x > 0))
)
```

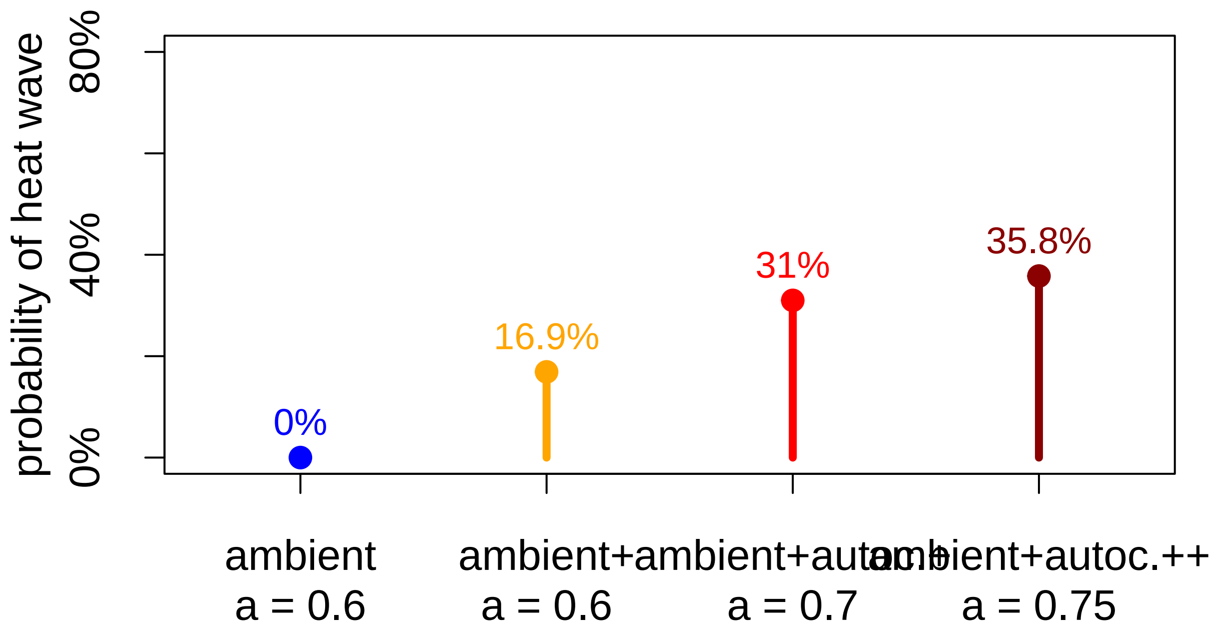
Print result

```
##                                          Scenario Simulations_with_clusters
## Ambient                                   Ambient                         0
## Ambient+                                 Ambient+                       169
## Ambient+Autocorrel+40 Ambient+Autocorrel+40                       310
## Ambient+Autocorrel+60 Ambient+Autocorrel+60                       358
```

Calculate probability

```
probability <- cluster_summary$Simulations_with_clusters / 1000
```

Plot


probability of heat wave
ambient
a = 0.6
ambient+
a = 0.6
ambient+autoc.+
a = 0.7
ambient+autoc.++
a = 0.75