Planning and Decision Making - Assignment 5

Fleet Management

Due date: Monday, January 8, 2024
Submit by Brightspace before midnight

Course: RO47005 Planning & Decision Making, TU Delft, CoR
Contact: Andrei-Carlo Papuc, email: a.c.papuc@tudelft.nl

Deliver a PDF with the answers to all questions in this assignment on Brightspace before the due date. You may do the assignments in collaboration with other students, but **every student needs to submit their own solutions.** The solutions to this assignment will be discussed in the Q&A session in the same week as the submission deadline. The assignments will not be graded (only pass/fail), so no personal feedback is provided. Questions can be answered in the Q&A sessions, by email or preferably on the Brightspace forum.

1 Traveling Salesman Problem (3 pt)

Consider an undirected complete graph $G = (V, E)$ with costs $c(e)$ at the arcs, which fulfill the triangular inequality, i.e. for each $u, v, w \in V, c(uv) <= c(uw) + c(wv)$ (intuitively this means that the shortest path between two nodes is the straight path). In this exercise, we are interested in finding a solution to the traveling salesman problem (TSP). We know that the problem is NP-Hard, so we are not seeking the optimal solution. Rather, this exercise guides you to find a polynomial algorithm that finds a solution whose cost is no larger than twice the optimal one. For this, consider the following definition and fact:

- Definition: A spanning tree of the graph $G = (V, E)$ is $T = (V, E')$, with $E' \subseteq E$, such that T is connected and has no cycles.
- Assume the existence of a polynomial algorithm (named "Kruskal algorithm") that finds the spanning tree with minimum total cost. For this exercise, you can use this algorithm as a black box.
- Denote that T is the optimal spanning tree.

Question 1.1 (1pt) Show that the cost of the optimal TSP is greater or equal to the cost of the arcs in T.

Question 1.2 (1pt) Take the tree T. Explain, on an intuitive description level, how you can visit all the nodes by touring each arc exactly twice. (no proves needed)

Question 1.3 (0.5pt) Propose how to build a circuit on the original graph, based on the path built in Question 1.2. A circuit is a non-empty path in which the first vertex is equal to the last vertex.

Question 1.4 (0.5pt) Conclude that such a circuit fulfills the aimed property, i.e., it's cost is no larger than twice the cost of the optimal solution.

2 Assignment Problem (4 pt)

Consider three robots $R = [r_1, r_2, r_3]$ and four tasks $T = [t_1, \dots, t_4]$, where each robot can only execute one task and each task can only be assigned to a maximum of one robot. Consider c_{ij} the cost of assigning robot r_i to task t_j given by the following matrix.

c_{ij}	t_1	t_2	t_3	t_4
r_1	2	1	0	4
r_2	3	5	1	3
r_3	2	3	4	5

Consider the joint cost function $C = \sum_{1 \leq i \leq 3} c_{i\sigma(i)}$, where $\sigma(i)$ is the index of the task assigned to robot i. Only three tasks are assigned $(1 \leq i \leq 3)$.

Question 2.1 (1.5pt) Formalize the problem at hand as an ILP or a LP (only one of the two is required).

Question 2.2 (1.0pt) What is the difference between the two approaches (ILP and LP)?

Question 2.3 (1.5pt) Solve the problem using the Hungarian algorithm on paper.

3 Vehicle Routing Problem (Coding exercise) (3 pt)

In this exercise we step-wise solve a vehicle routing problem (VRP) by seperating it in multiple traveling salesman problems (TSP). To do so, we first cluster all locations. Second, we model the TSPs as an integer-linear program (ILP) and solve it using a freely available solver. We operate in the euclidean plane.
To guide this exercise we provide you a jupyter-notebook, containing all needed data, pointers, additional information and guidance.

Question 3.1 (2.5pt) Complete all gaps, indicated by ".............", within the provided jupyter-notebook. Add the final result graph to your report.

Question 3.2 (0.5pt) How would this approach fail, if each customer additional has a time window in which he or she needs to be served. There is more than one correct answer to this question, but one is sufficient.