Lab 3. Fourier Series Representation of Periodic Signals

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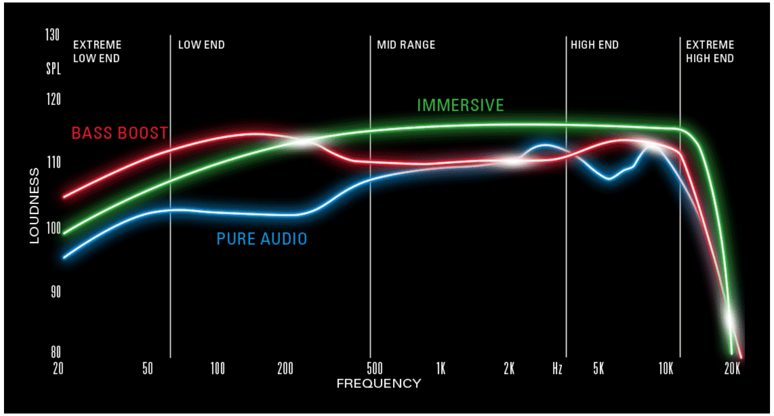
Fall 2020

Overview

- In this tutorial, you will learn
  1. How to calculate the frequency response of DT LTI system (frequency domain)



  2. How to calculate the output of CT LTI system via Matlab (time domain)
  3. How to calculate the DTFS of signal via Matlab

Calculating the frequency response of DT LTI system

Complex Exponentials

- The *Only* Eigenfunctions of *Any* LTI Systems

$$x(t) = e^{st} \longrightarrow \boxed{h(t)} \longrightarrow y(t) = \int_{-\infty}^{+\infty} h(\tau) e^{s(t-\tau)} d\tau$$

$$= \left[\int_{-\infty}^{+\infty} h(\tau) e^{-s\tau} d\tau\right] e^{st}$$

$$= \underbrace{H(s)}_{\text{eigenvalue}}\underbrace{e^{st}}_{\text{eigenfunction}}$$

For each s (each ω), eigenvalue eigenfunction

$H(s)\ or\ H\left(e^{j\omega}\right) -$ frequency response

$s = j\omega$ - purely imaginary,
i.e. signals of the form $e^{j\omega t}$

DT LTI System

$$e^{j\omega n}$$

$$H(e^{j\omega})e^{j\omega n}$$

$$a_0 e^{j0\underline{\left(\frac{2\pi}{N}\right)}n}$$

$$a_1 e^{j1\left(\frac{2\pi}{N}\right)n}$$

$$a_2 e^{j2\left(\frac{2\pi}{N}\right)n}$$

$$\dots$$

$$a_{N-1} e^{j(N-1)\left(\frac{2\pi}{N}\right)n}$$

$$a_0 H\left(e^{j0\underline{\left(\frac{2\pi}{N}\right)}}\right)e^{j0\underline{\left(\frac{2\pi}{N}\right)}n}$$

$$a_1 H\left(e^{j1\left(\frac{2\pi}{N}\right)}\right)e^{j1\left(\frac{2\pi}{N}\right)n}$$

$$a_2 H\left(e^{j2\left(\frac{2\pi}{N}\right)}\right)e^{j2\left(\frac{2\pi}{N}\right)n}$$

$$\dots$$

$$a_{N-1} H\left(e^{j(N-1)\left(\frac{2\pi}{N}\right)}\right)e^{j(N-1)\left(\frac{2\pi}{N}\right)n}$$

LTI System by Difference Equation

- Reading assignment: textbook 2.4

- Causal DT LTI system can be specified by a linear constant-coefficient difference equation

$$\sum_{k=0}^{K} a_k y[n-k] = \sum_{m=0}^{M} b_m x[n-m]$$

- For example:
  - $y[n]=0.5x[n]+x[n-1]+2x[n-2]$; h[n]=? Finite Impulse Response (FIR)
  - $y[n]-0.8y[n-1]=2x[n]$; h[n]=? Infinite Impulse Response (IIR)

- Causal DT LTI system is uniquely specified by two vectors $A=[a_0\ a_1\ a_2\ \ldots\ a_K]$ and $B=[b_0\ b_1\ \ldots\ b_M]$
  - A=[1] B=[0.5 1 2]
  - A=[1 -0.8] B=[2]

Frequency Response: freqz()

- freqz(): generate system frequency response

```
[H omega] = freqz(b, a, N);
```

$$H\left(e^{j\omega_k}\right) \quad \omega_k = \left(\frac{\pi}{N}\right)k, 0 \le k \le N-1$$

```
[H omega] = freqz(b, a, N, 'whole');
```

$$H\left(e^{j\omega_k}\right) \quad \omega_k = \left(\frac{2\pi}{N}\right)k, 0 \le k \le N-1$$

Example

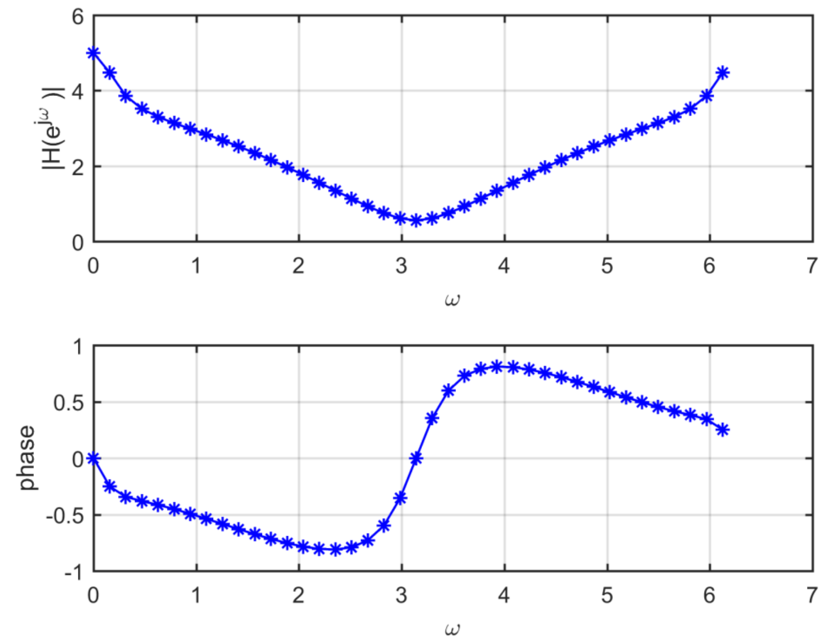
- Consider LTI System: y[n]-0.8y[n-1]=2x[n]-x[n-2]
  - Define the vector of coefficients:

```
A=[1 -0.8];
B=[2 0 -1];
```

  - Plot the frequency response:

```
[H omega] = freqz(B, A, 40, 'whole');
subplot(211);plot(omega, abs(H), '*-');
xlabel('\omega');
ylabel('|H(e^{j\omega} )|'); grid;
subplot(212);plot(omega, angle(H), '*-');
xlabel ('\omega');
ylabel('phase'); grid;
```

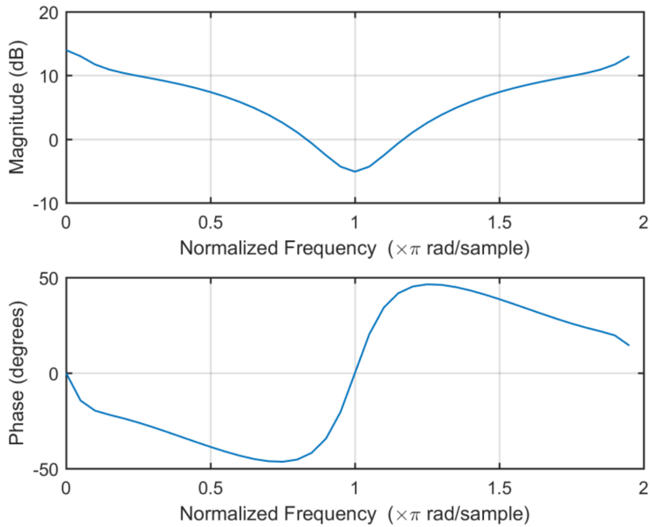
$H\left(e^{j0\left(\frac{2\pi}{N}\right)}\right)$ …… $H\left(e^{j(N-1)\left(\frac{2\pi}{N}\right)}\right)$

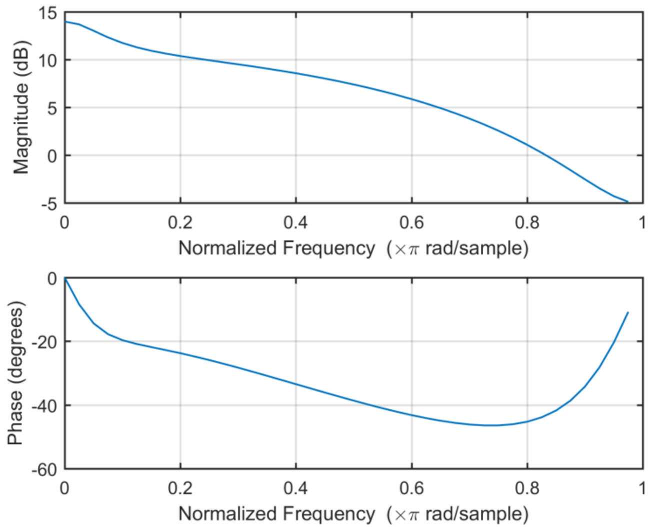


Search 'Greek Letters and Special Characters' in Matlab Help Documents

```
A=[1 -0.8];
B=[2 0 -1];
figure;
freqz(B,A,40, 'whole')
```

```
A=[1 -0.8];
B=[2 0 -1];
figure;
freqz(B,A,40)
```





Time to Call the Roll

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原标题：就问你怕不怕！川大老师研发人脸识别无人机课堂点名

“上课的学生不知道有这节目，Spurise!”因独特的教学方式，学术圈网红教授，四川大学计算机系主任，魏骁勇又“作妖”了。这次搞了个升级版基于人脸识别的上课点名——无人机点名+巡堂。除了增加教学的趣味和参与度，也让学生在参与的过程中学会发现和解决新问题。

从利用徒手劈砖技能讲解物理原理，到“刷脸神器”打考勤，“刷女神器”帮联谊。魏骁勇教授6年来，一直变着花样的带给课堂惊喜。5月23日下午，在四川大学江安校区的教室里，魏

Calculating the output of CT LTI system

CT LTI System by Differential Equation

- Reading assignment: textbook 2.4.1
- Causal CT LTI system can be specified by a linear constant-coefficient differential equation

$$\sum_{k=0}^{K} a_k \frac{d^k y(t)}{dt^k} = \sum_{m=0}^{M} b_m \frac{d^m x(t)}{dt^m}$$

- Coefficient vectors:
  - $A = [a_K, a_{K-1}, \dots a_0]$
  - $B = [b_M, b_{M-1}, \dots b_0]$

- CT LTI system by differential equation
  - $\sum_{k=0}^{K} a_k \frac{d^k y(t)}{dt^k} = \sum_{m=0}^{M} b_m \frac{d^m x(t)}{dt^m}$
  - $A = [a_K, a_{K-1}, \dots a_0]$
  - $B = [b_M, b_{M-1}, \dots b_0]$

- DT LTI system by difference equation
  - $\sum_{k=0}^{K} a_k y[n-k] = \sum_{m=0}^{M} b_m x[n-m]$
  - $A = [a_0, a_1, \dots a_K]$
  - $B = [b_0, b_1, \dots b_M]$

Multiple Choice(single)

Points: 1

设置

Coefficient vector A and B of system 0.3y(t)+dy(t)/dt = 3x(t) are:

A A=[1, 0.3]; B=3;

B A=[0.3, 1]; B=3;

C A=3; B = [1, 0.3];

D A=3; B = [0.3, 1];

$$\sum_{k=0}^{K} a_k \frac{d^k y(t)}{dt^k} = \sum_{m=0}^{M} b_m \frac{d^m x(t)}{dt^m}$$

$$A = [a_K, a_{K-1}, \dots a_0]$$

$$B = [b_M, b_{M-1}, \dots b_0]$$

提交

- Differential Equation and Transfer Function

$$\sum_{k=0}^{K} a_k \frac{d^k y(t)}{dt^k} = \sum_{m=0}^{M} b_m \frac{d^m x(t)}{dt^m}$$

$$A = [a_K, a_{K-1}, \dots a_0]$$
$$B = [b_M, b_{M-1}, \dots b_0]$$

$$\sum_{k=0}^{K} a_k s^k Y(s) = \sum_{m=0}^{M} b_m s^m X(s)$$

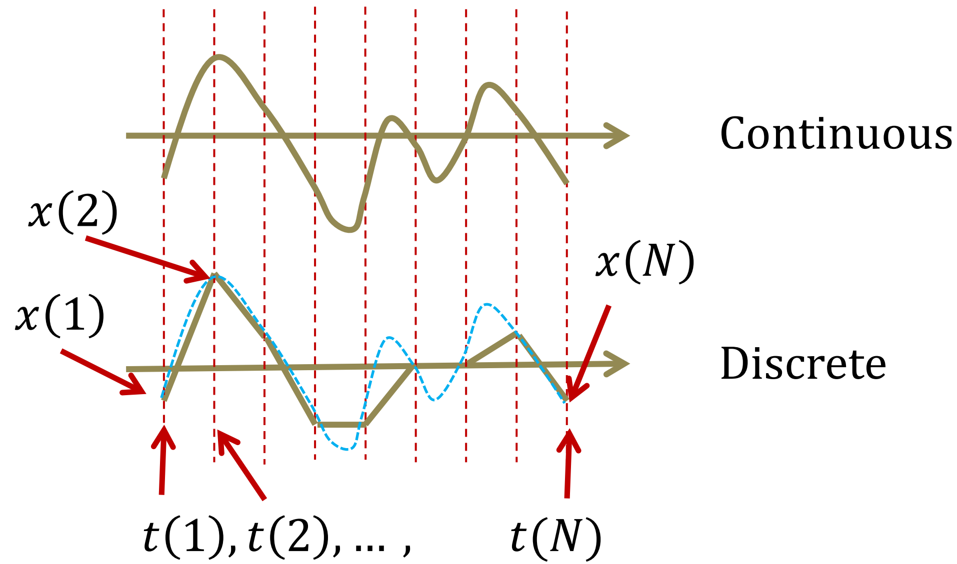
$$\boxed{s = j\omega}$$

$$H(s) = \frac{Y(s)}{X(s)} = \frac{\sum_{m=0}^{M} b_m s^m}{\sum_{k=0}^{K} a_k s^k}$$

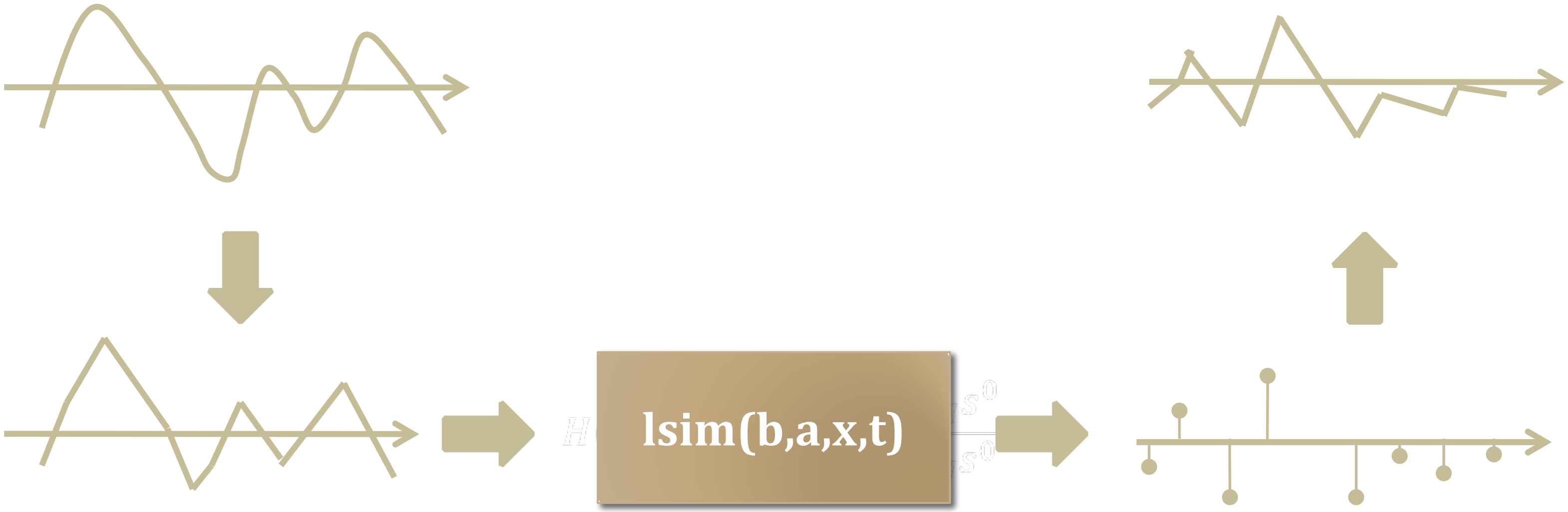
System function
or Transfer function

$$H(s) = \frac{B(s)}{A(s)} = \frac{b_M s^M + b_{M-1} s^{M-1} + \cdots + b_1 s^1 + b_0}{a_K s^K + a_{K-1} s^{K-1} + \cdots + a_1 s^1 + a_0}$$

- How to simulate CT systems via Matlab?
  - lsim(): generate sampled output according to sampled input signal and CT system function:
    - Syntax: lsim(B, A, x, t)
  - Coefficient vector:
    - $A = [a_K, a_{K-1}, \ldots a_0]$
    - $B = [b_M, b_{M-1}, \ldots b_0]$
  - Sampled input signal
    - Vector of sampling time: t
    - Vector of sampled value: x



- lsim(B,A,x,t)

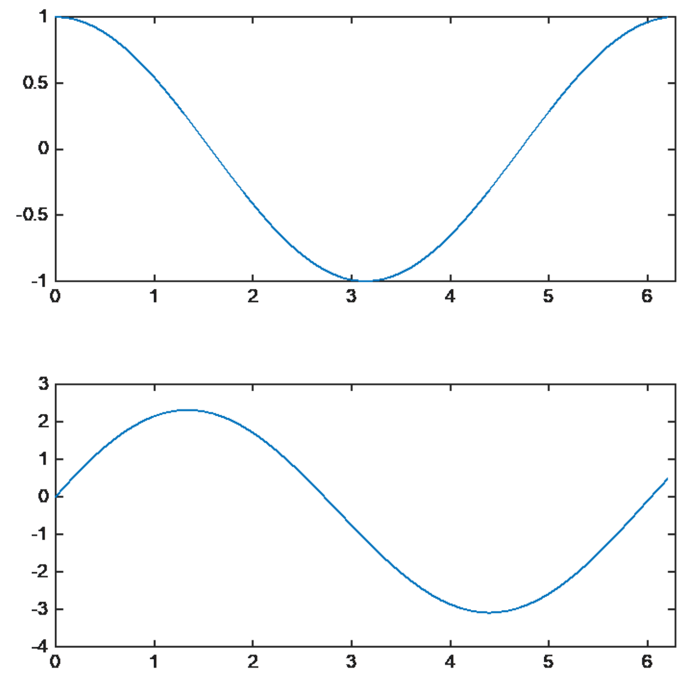


- Consider LTI System: $0.3y(t)+dy(t)/dt = 3x(t)$

```
A=[1 0.3];
B=3;
```

- Sample the input signal x=cos(t):

```
t=0:0.1:2*pi;
x=cos(t);
y=lsim(B,A,x,t)';
subplot(2,1,1), plot(t,x);
xlim([0 2*pi]);
subplot(2,1,2), plot(t,y);
xlim([0 2*pi]);
```



Lab Assignment 3 – part (a)

- Read tutorial 3.1, 3.2, 3.3 (and 2.3) by yourself
- 3.8 part (a)(b) & 3.9
- Submission: TBA in next week

Several parts of this exercise require you to generate vectors which should be purely real, but have very small imaginary parts due to roundoff errors. Use *real* to remove these residual imaginary parts from these vectors.

Hints – A correction

- 3.9(c)

Advanced Problems

(c). Analytically calculate the CTFS for the square wave **x2**. You may find it helpful to first find a relationship between the signal $x_2(t)$ and the signal $s(t)$ defined in Eq. (3.9). Use the ten lowest frequency nonzero CTFS coefficients of **x2** to create the first 5 harmonic components individually. For example if you have the positive and

$$s(t) = \begin{cases} 1, & |t| < T/4, \\ 0, & T/4 \le |t| \le T/2 \end{cases} \qquad (3.9)$$

Eq. (3.9) is provided in section 3.7

CTFS coefficients a_k given by

$$a_k = \frac{\sin(\pi k/2)}{\pi k}.$$

Example 3.5 in your textbook by Oppenheim – 2nd Edition

Hint: Properties of CTFS – Linearity, Time Shifting, ……