第八次作业

2021年11月15日 星期一　　下午8:55

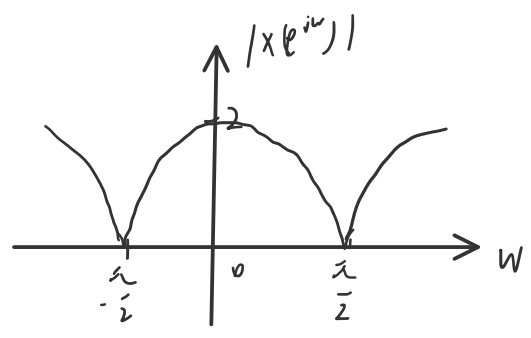
5.2 (a).

$$X(e^{jw}) = \sum_{n=-\infty}^{\infty} (\delta[n-1] + \delta[n+1]) e^{-jwn}$$

$$= \sum_{n=-\infty}^{\infty} \delta[n-1] e^{-jwn} + \sum_{n=-\infty}^{\infty} \delta[n+1] e^{-jwn}$$

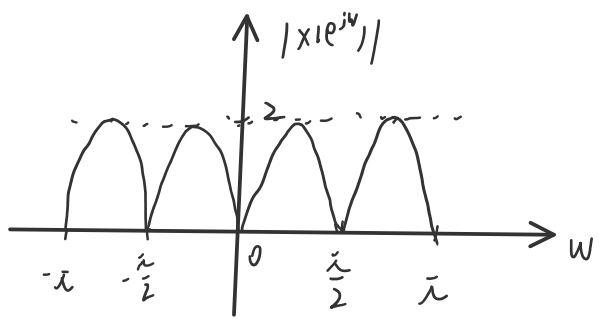
$$= e^{-jw} + e^{jw} = 2\cos w$$

$$|X(e^{jw})| = |2\cos w|$$



(b). $X(e^{jw}) = \sum_{n=-\infty}^{\infty} \delta[n+2] e^{-jwn} - \sum_{n=-\infty}^{\infty} \delta[n-2] e^{-jwn}$

$$= e^{j2w} - e^{-j2w} = 2j\sin 2w$$



5.5. $x[n] = \frac{1}{2\pi} \int_{-\pi}^{\pi} |X(e^{jw})| e^{j\arg X(e^{jw})} e^{jwn} dw$

$$= \frac{1}{2\pi} \int_{-\frac{3}{4}}^{\frac{3}{4}} 1 \cdot e^{-j\frac{3}{2}w} dw$$

$$= \frac{1}{2\pi} \frac{1}{j(n-\frac{3}{2})} e^{j(n-\frac{3}{2})w} \Big|_{-\frac{3}{4}}^{\frac{3}{4}}$$

$$= \frac{\sin(\frac{3}{4}n - \frac{3\pi}{8})}{\pi(n-\frac{3}{2})}$$

当 $n = 4k+\frac{3}{2}$ 时 $x[n] = 0$，但 n 为整数 $n \neq 4k+\frac{3}{2}$

或当 $n = \pm\infty$ 时 $x[n] = \pm\infty$

5.15 令 $x[n] = \frac{\sin w_c n}{\pi n}$

则 $X(e^{jw}) = u(w+w_c) - u(w-w_c)$

$$Y(e^{jw}) = \frac{1}{2\pi} \int_{2\pi} X(e^{j\theta}) X(e^{j(w-\theta)}) d\theta$$

当 $w_c < \frac{\pi}{2}$ 时 在 $(-\pi, \pi)$ 内 $Y(e^{jw})$ 为

$$\Rightarrow Y(e^{jw}) = \begin{cases} -\frac{1}{2\pi}|w| + \frac{w_c}{\pi}, & 0 \le |w| \le 2w_c \\ 0, & 2w_c < |w| \le \pi \end{cases}$$

因 $Y(e^{jw}) = \frac{1}{2}$ 故 $2w_c > \pi$.

即 $Y(e^{jw})$ 当 $w = \pi$ 时为 $2(-\frac{1}{2\pi}\pi + \frac{w_c}{\pi}) = -1 + \frac{2w_c}{\pi}$

即 $w_c = \frac{3}{4}\pi$

5.21 (a). $x[n] = u[n-2] - u[n-6]$

$$= \delta[n-2] + \delta[n-3] + \delta[n-4] + \delta[n-5]$$

$$X(jw) = e^{-j2w} + e^{-j3w} + e^{-j4w} + e^{-j5w} = \frac{e^{-j2w}(1-e^{-j4w})}{1-e^{-jw}}$$

(b). $x[n] = (\frac{1}{2})^{-n} u[-n-1] = (\frac{1}{2}) \cdot (\frac{1}{2})^{-n-1} u[-n-1]$

令 $x_1[n] = \frac{1}{2} \cdot (\frac{1}{2})^{n-1} u[n-1]$

得 $X_1(e^{jw}) = \frac{\frac{1}{2}e^{-jw}}{1-\frac{1}{2}e^{-jw}}$

$$X(e^{jw}) = X_1(e^{-jw}) = \frac{\frac{1}{2}e^{jw}}{1-\frac{1}{2}e^{jw}}$$

(c). $x[n] = (\frac{1}{3})^{|n|} u[-n-2] = (\frac{1}{3})^{-n} u[-n-2]$ $(n>0$ 时 $-n-2<0)$

$$x_1[n] = (\frac{1}{3})^{n} u[n-2] = \frac{1}{9}(\frac{1}{3})^{n-2} u[n-2]$$

$$X_1(jw) = \frac{1}{9} \frac{e^{-j2w}}{1-\frac{1}{3}e^{-jw}}$$

$$X(e^{jw}) = X_1(e^{-jw}) = \frac{1}{9} \cdot \frac{e^{j2w}}{1-\frac{1}{3}e^{jw}}$$

(d). $X_1[n] = 2^{-n}\sin(-\frac{\pi}{4}n) u[n] = -(\frac{1}{2})^n \sin(\frac{\pi}{4}n) u[n]$

$$= -\frac{1}{2j}(\frac{1}{2}e^{j\frac{\pi}{4}})^n u[n] + \frac{1}{2j}(\frac{1}{2}e^{-j\frac{\pi}{4}})^n u[n]$$

$$X_1(jw) = -\frac{1}{2j} \cdot \frac{1}{1-\frac{1}{2}e^{j\frac{\pi}{4}}e^{-jw}} + \frac{1}{2j} \cdot \frac{1}{1-\frac{1}{2}e^{-j\frac{\pi}{4}}e^{-jw}}$$

$$X(jw) = X_1(-jw) = -\frac{1}{2j} \cdot \frac{1}{1-\frac{1}{2}e^{j\frac{\pi}{4}}e^{jw}} + \frac{1}{2j} \cdot \frac{1}{1-\frac{1}{2}e^{-j\frac{\pi}{4}}e^{jw}}$$

(e). 令 $x_1[n] = (\frac{1}{2})^{|n|}$ $x_2[n] = \cos(\frac{\pi}{8}n - \frac{\pi}{8})$

$$X_1(e^{jw}) = \frac{1-(\frac{1}{2})^2}{1-2\times\frac{1}{2}\cos w + \frac{1}{4}} = \frac{3}{5-4\cos w}$$

$$X_2(e^{jw}) = \pi e^{-j\frac{\pi}{8}} \delta(w-\tfrac{\pi}{8}) + \pi e^{j\frac{\pi}{8}} \delta(w+\tfrac{\pi}{8})$$

$$X(e^{jw}) = \frac{1}{2\pi} \int_{-\pi}^{\pi} X_1(e^{j\theta}) X_2(e^{j(w-\theta)}) d\theta$$

$$= \frac{\frac{3}{2}e^{-j\frac{\pi}{8}}}{5-4\cos(w-\frac{\pi}{8})} + \frac{\frac{3}{2}e^{j\frac{\pi}{8}}}{5-4\cos(w+\frac{\pi}{8})}$$

(f). $x[n] = -3\delta[n+3] - 2\delta[n+2] - \delta[n+1] + \delta[n-1] + 2\delta[n-2] + 3\delta[n-3]$

$$X(e^{jw}) = -6\cos 3w - 4\cos 2w - 2\cos w$$

(h). 由 $\sin x, \cos x$ 的傅里叶变换公式可得

$$X(e^{jw}) = j\pi \sum_{l=-\infty}^{\infty} \{\delta(w+\tfrac{\pi}{3}-2\pi l) - \delta(w-\tfrac{\pi}{3}-2\pi l)\}$$

$$+ \pi \sum_{l=-\infty}^{\infty} \{\delta(w+\tfrac{2\pi}{3}-2\pi l) + \delta(w-\tfrac{\pi}{3}-2\pi l)\}$$