第六次作业

2021年11月3日 星期三　　上午9:22

4.5 $x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(jw)e^{jwt}dw$

$= \frac{1}{2\pi}\int_{-\infty}^{\infty} |X(jw)| e^{j\arg X(jw)} e^{jwt}dw$

$= \frac{1}{2\pi}\int_{-3}^{3} 2e^{-j\frac{3}{2}w}\cdot e^{jw}\cdot e^{jwt}dw$

$= -\frac{1}{j\pi(t-\frac{3}{2})}\left[e^{j3(t-\frac{3}{2})} - e^{-j3(t-\frac{3}{2})}\right]$

$= \frac{-2\sin[3(t-\frac{3}{2})]}{\pi(t-\frac{3}{2})}$

当 $3(t-\frac{3}{2}) = k\pi$, $k = \pm1, \pm2 \dots$ 时, $x(t) = 0$ 即 $t = k\frac{\pi}{3} + \frac{3}{2}$

4.21 b. $x(t) = e^{-3|t|}\sin 2t = e^{-3|t|}\frac{e^{-2jt} - e^{2jt}}{2j}$

$X(t) = \int_{-\infty}^{\infty} x(t)e^{-jwt}dt$

$= \int_{-\infty}^{0} x(t)e^{-jwt}dt + \int_{0}^{\infty} x(t)e^{-jwt}dt$

$= \int_{-\infty}^{0} e^{3t}\frac{e^{-2jt} - e^{2jt}}{2j}e^{-jwt}dt + \int_{0}^{\infty} e^{-3t}\frac{e^{-2jt} - e^{2jt}}{2j}e^{-jwt}dt$

$= \frac{1}{2j}\left[\int_{-\infty}^{0}\left(e^{(3-2j)t} - e^{(3+2j)t}\right)\cdot e^{-jwt}dt + \int_{0}^{\infty}\left(e^{(-3-2j)t} - e^{(-3+2j)t}\right)\cdot e^{-jwt}dt\right]$

$= \frac{1}{2j}\left(\frac{1}{3-2j-jw} - \frac{1}{3+2j-jw} + \frac{1}{3+2j+jw} - \frac{1}{3-2j+jw}\right)$

$= \frac{2}{(jw+3)^2+4} - \frac{2}{(jw-3)^2+4}$

(g). $X(t) = \int_{-\infty}^{\infty} x(t)e^{-jwt}dt$

$= \int_{-2}^{-1}(-1)\cdot e^{-jwt}dt + \int_{-1}^{1} t\cdot e^{-jwt}dt + \int_{1}^{2} e^{-jwt}dt$

$= \frac{1}{jw}(e^{jw} - e^{2jw}) - \frac{1}{jw}(e^{jw} + e^{-jw}) - \frac{1}{w^2}(e^{jw} - e^{-jw}) + \frac{1}{jw}(e^{-jw} - e^{-j2w})$

$= \frac{2j}{w}\left(\cos 2w - \frac{\sin w}{w}\right)$

(h). $x(t) = \sum_{k=-\infty}^{\infty}\left[2\delta(t-2k) + \delta(t-1-2k)\right]$

$= 2\sum_{k=-\infty}^{\infty}\delta(t-2k) + \sum_{k=-\infty}^{\infty}(t-1-2k)$

对 $y(t) = \sum_{k=-\infty}^{\infty}\delta(t-2k) \Rightarrow Y(jw) = \pi\sum_{k=-\infty}^{\infty}\delta(w-k\pi)$

$X(jw) = \pi\sum_{k=-\infty}^{\infty}\delta(w-k\pi)[2+(-1)^k]$

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c. $x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(jw)e^{jwt}dw$

$= \frac{1}{2\pi}\int_{-\infty}^{\infty}|X(jw)| e^{j\arg X(jw)} e^{jwt}dw$

$= \frac{1}{2\pi}\int_{-1}^{0} -w e^{-j3w}e^{jwt}dw + \frac{1}{2\pi}\int_{0}^{1} w e^{-j3w}e^{jwt}dw$

$= \frac{1}{\pi}\left[\frac{\sin(t-3)}{t-3} + \frac{\cos(t-3)-1}{(t-3)^2}\right]$

e. $x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(jw)e^{jwt}dw$

$= \frac{1}{2\pi}\left(\int_{-3}^{-2} -e^{jwt}dw + \int_{-2}^{-1}(w+1)e^{jwt}dw + \int_{1}^{2}(w-1)e^{jwt}dw + \int_{2}^{3} e^{jwt}dw\right)$

$= \frac{1}{2\pi}\left(\frac{1}{jt}(e^{-j3t} - e^{-j2t}) + \frac{e^{-j2t}}{jt} + \frac{1}{t^2}(e^{-jt} - e^{-j2t}) + \frac{e^{j2t}}{jt} + \frac{1}{t^2}(e^{j2t} - e^{jt}) + \frac{1}{jt}(e^{j3t} - e^{j2t})\right)$

$= \frac{\cos 3t}{j\pi t} + \frac{\sin t - \sin 2t}{j\pi t^2}$

4.27 $X(jw) = \int_{-\infty}^{\infty} x(t)e^{-jwt}dt$

$= \int_{1}^{2} e^{-jwt}dt - \int_{2}^{3} e^{-jwt}dt$

$= \frac{e^{-jw} - e^{-2jw}}{jw} - \frac{e^{-2jw} - e^{-3jw}}{jw}$

$= \frac{2\sin(\frac{w}{2})}{w}e^{-\frac{3}{2}jw}(1 - e^{-jw})$

(b). $a_k = \frac{1}{T}\int_T \tilde{x}(t)e^{-j\frac{2\pi}{3}kt}dt$

$= \frac{1}{2}\left(\int_{1}^{2} e^{-\frac{2\pi}{2}jkt}dt - \int_{2}^{3} e^{-\frac{2\pi}{2}jkt}dt\right)$

$= \frac{\sin(k\pi/2)}{k\pi}e^{-\frac{3}{2}\pi jk}(1 - e^{-jk\pi})$

得 $a_k = \frac{1}{T}X(j\frac{2\pi k}{T})$