Signals and Systems

Southern University of Science and Technology

Fall 2021

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Education and Employment：

- Assistant Professor, SUSTech, 2020 - now
- Marie Curie Fellow, UCL, 2018 - 2020
- PhD, Beijing Institute of Technology, 2013 - 2018
- BEng, Beijing Institute of Technology, 2009 - 2013

Awards and Professional Activities：

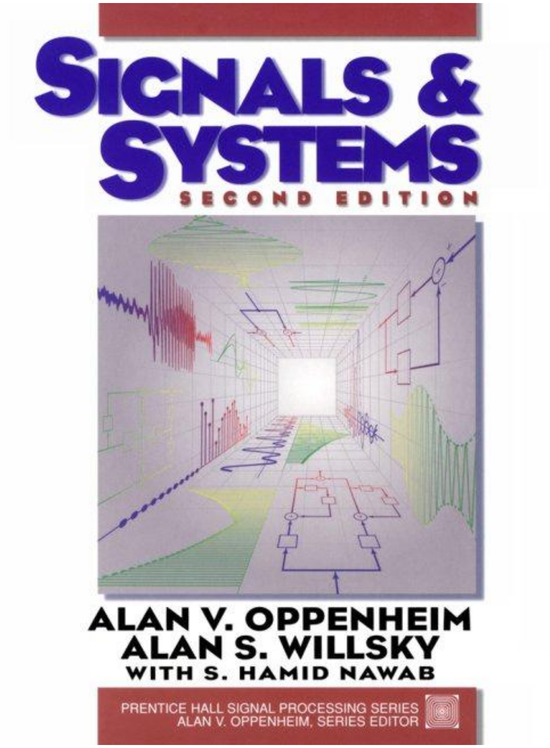
- Best Doctoral Thesis Award, Chinese Institute of Electronics, 2019
- Marie Curie Individual Fellowship, EU H2020, 2018
- Exemplary Reviewer for IEEE TWC, TCOM, and COMML
- Academic Chair of IEEE ComSoc ISAC Emerging Technology Initiative (ISAC-ETI)
- Associate Editor for IEEE Comml
- Lead Guest Editor for IEEE JSAC Special Issue on ISAC
- Workshop Co-Chairs and Special Session Organizers for ICC , ICASSP, SPAWC

Research Interests：

- Integrated Sensing and Communication (ISAC)
- V2X Network and Intelligent Transportation
- MmWave Signal Processing

Scope of Lecture

- "Signals and Systems", Oppenheim, Willsky and Nawab, 2nd Edition, 1997, Prentice-Hall.



- This course teaches Chapters 1 to 8.
  - Roughly two weeks for one chapter
  - Middle-term exam for Chapters 1 to 4
  - Final exam for all

Textbook reading is crucial, as I cannot cover every detail in slides

Three Pillars

Lectures (Tutorials)

Matlab Labs

YOU

100%

Assignment/Quiz 10%

Mid-term Exam 30%

Final Exam 30%

Lab Reports

Project Report & Presentation

30%

Class Schedules

SUSTech S&S 2021 ...
群号：891613641

- Lab Sessions – **Start from the first week**
- Instructor: Dr. Guang Wu (吴光)
- Tutorials – **Start from week 3 (一教111, Monday-Thursday, 21:00 - 22:00)**
- Teaching Assistant (TA):
  - 卢仕航 (lush2021@mail.sustech.edu.cn)
  - 董宇翔 (12132113@mail.sustech.edu.cn)
  - 李柯 (lik2021@mail.sustech.edu.cn)
- Assignment: Every week (**no for week 1**)
- Submit assignment in **softcopy to Blackboard system**
- Deadline: Next Friday, 12pm.

Signals and Systems

- What is a signal?
- What is a system?

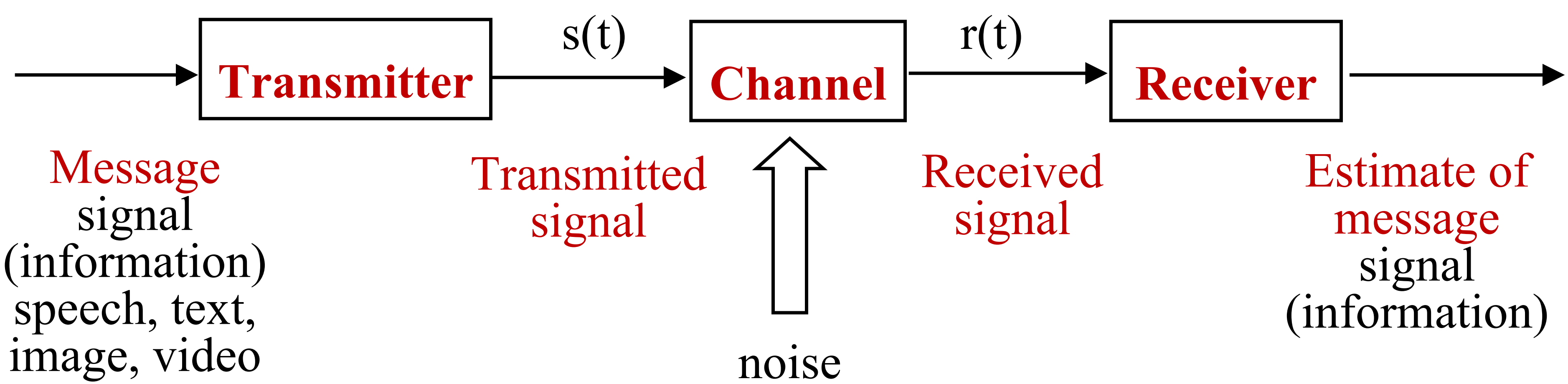
Signals and Systems

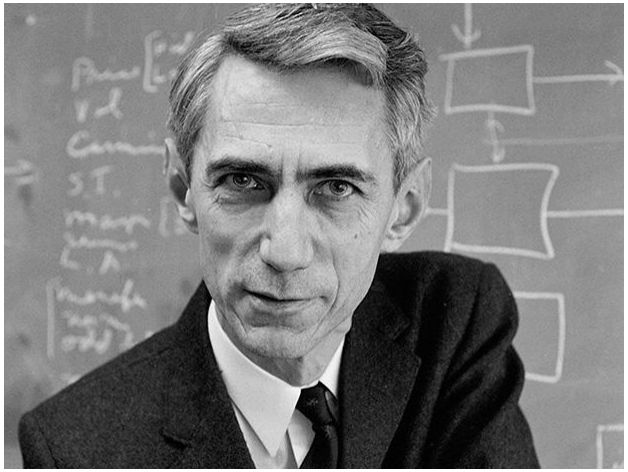
- **Signal:** everything which carries information
- **System:** everything which processes input signal and generate output signal

Communication Signals & Systems

Can you find any example of signals and systems when making a phone call?

- Transmitter, channel and receiver are all systems.
- Each system has one input signal and one output signal.



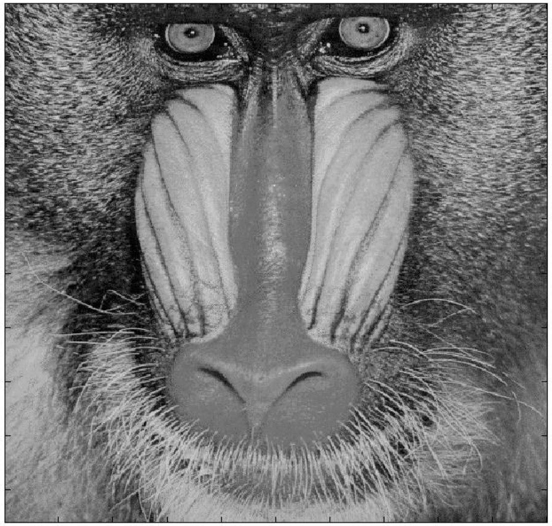


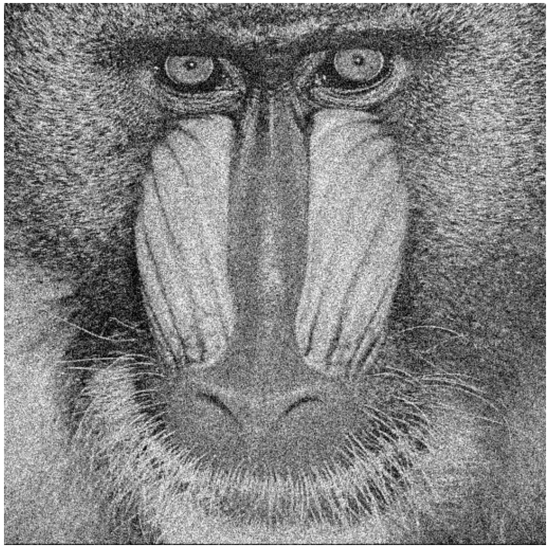
C. E. Shannon

"The fundamental problem of communication is that of reproducing at one point either exactly or approximately a message selected at another point."

Shannon, C. E. (1948). A mathematical theory of communication. The Bell system technical journal, 27(3), 379-423.

Image Processing





More examples of signals

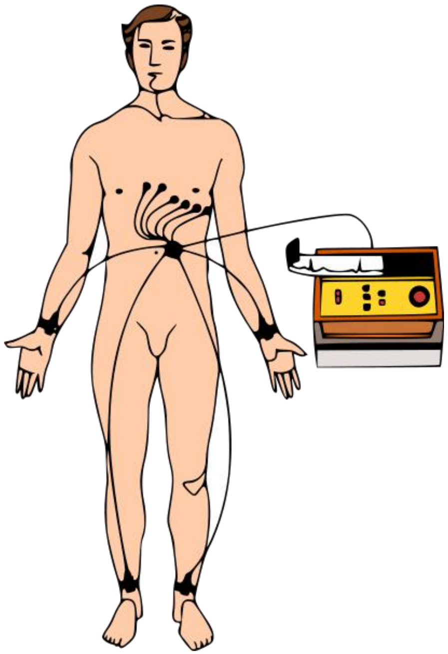
- Electrical signals – voltages and currents in a circuit
- Acoustic signals – audio or speech signals
- Video signals – movie
- Biological signals – sequence of bases in a gene
- We will treat noise as unwanted signals.

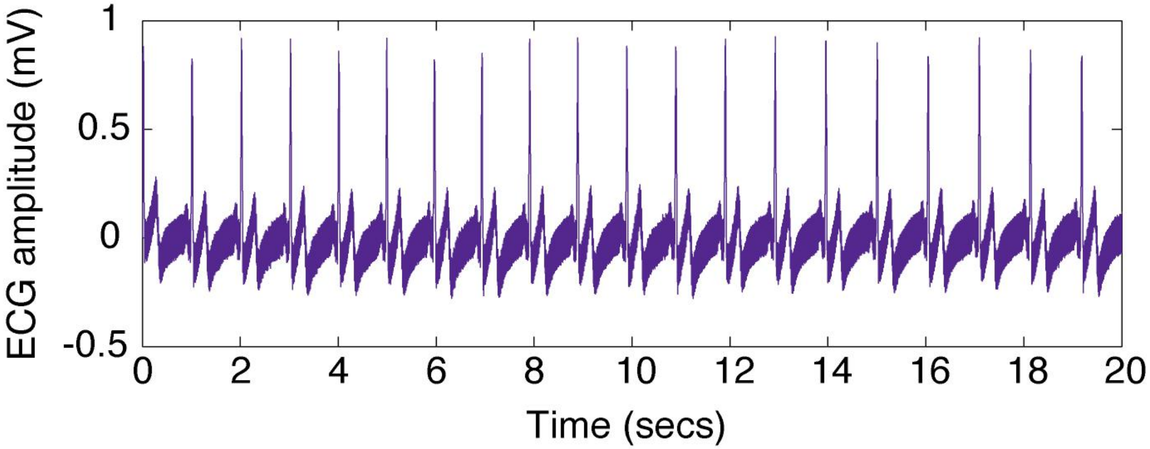
Signals and Systems from Our Point of View

- Signals are variables that carry information, like function.
- Systems process input signals to produce output signals.
- The course is about using mathematical techniques to analyze and synthesize systems which process signals.

Independent Variable of Signals

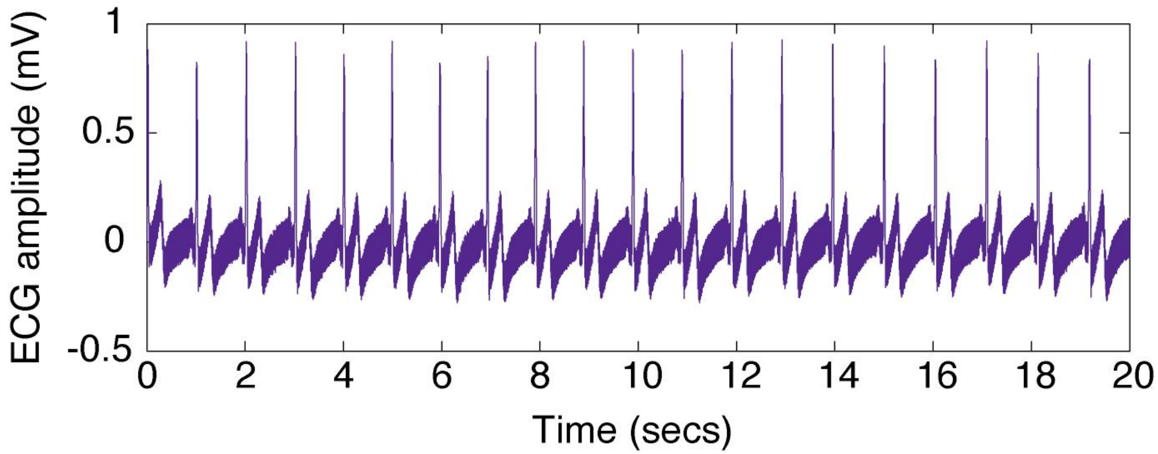
- Time is often the independent variable.
- Example: the electrical activity of the heart recorded with chest electrodes –– the electrocardiogram (ECG).

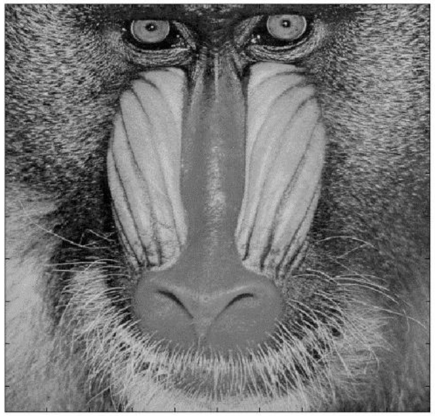




Signal Classification 1: Dimension of Independent Variable

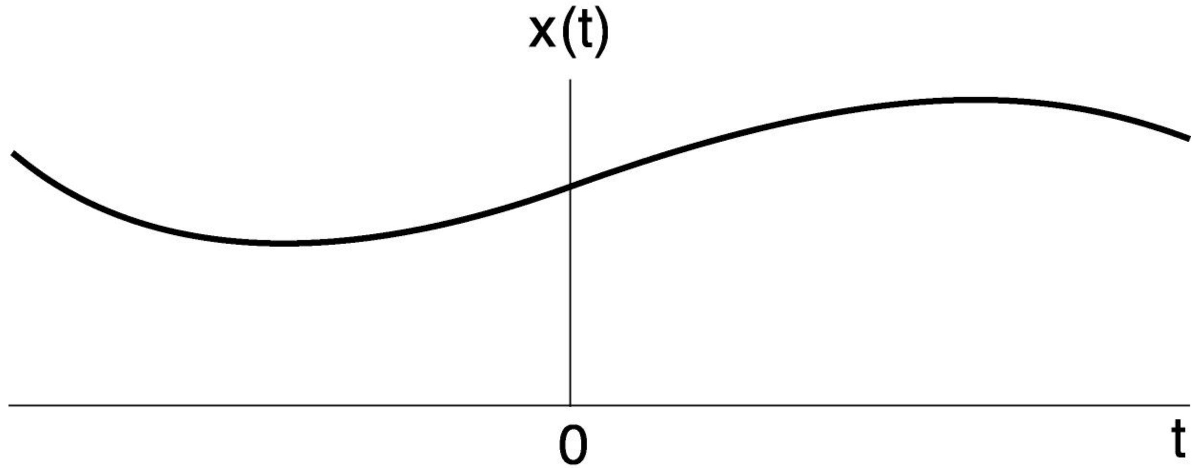
- An independent variable can be 1-D (*t* in the ECG), 2-D (*x*, *y* in an image), or 3-D (*x*, *y*, *t* in an video).





- We focus on 1-D for mathematical simplicity but the results can be extended to 2-D or even higher dimensions.

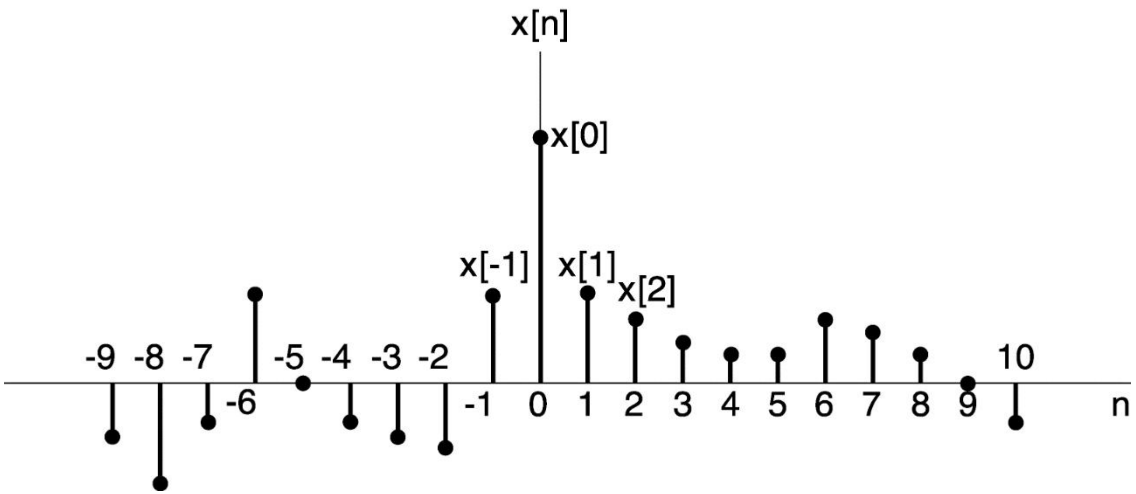
Signal Classification 2: CT/DT
Continuous-time (CT) Signals



- Independent variable is continuous
- Most of the signals in the physical world are CT signals.
- E.g. voltage & current, pressure, temperature, velocity, etc.

Notation: x(t)

Discrete-time (DT) Signals



- Independent variable is integer
- Examples of DT signals: DNA sequence, population of the *n*-th generation of certain species

Notation: $x[n]$

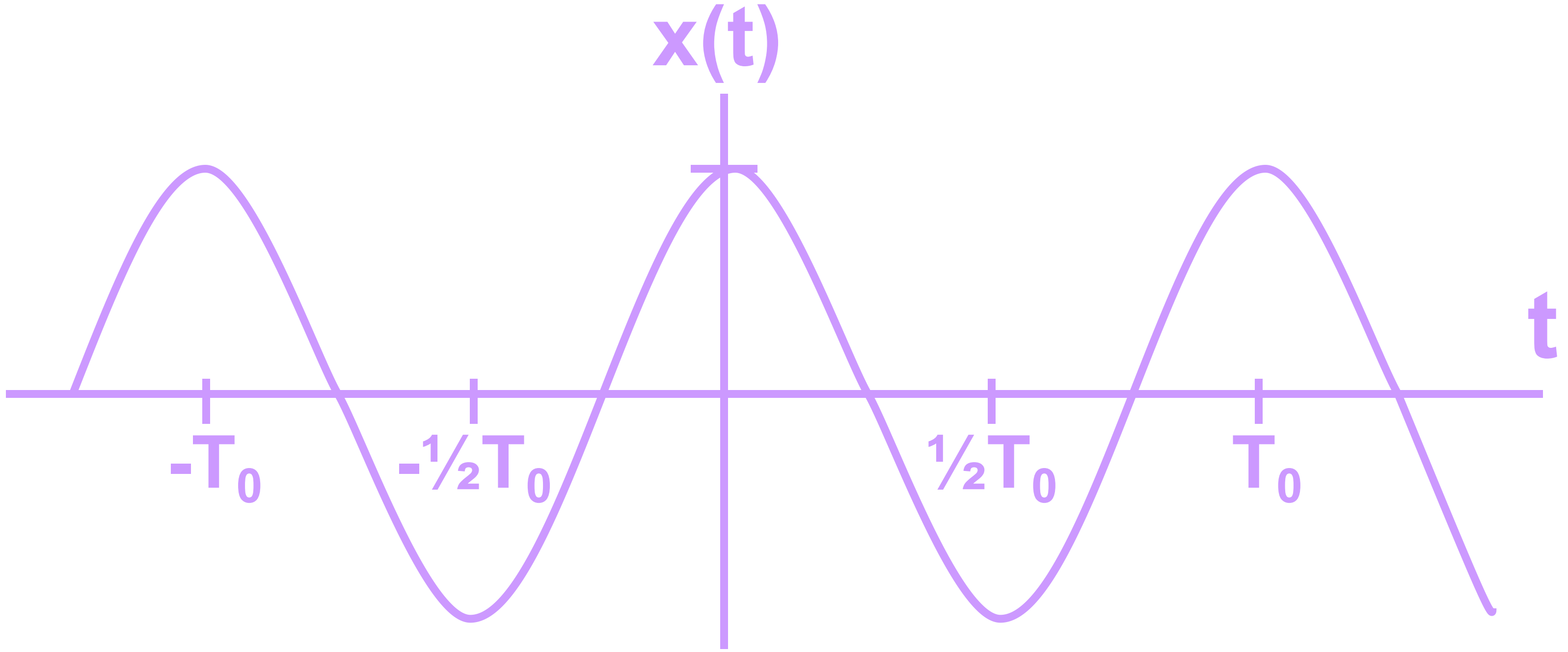
Many Human-made Signals are DT

Weekly Dow-Jones industrial average

Digital image

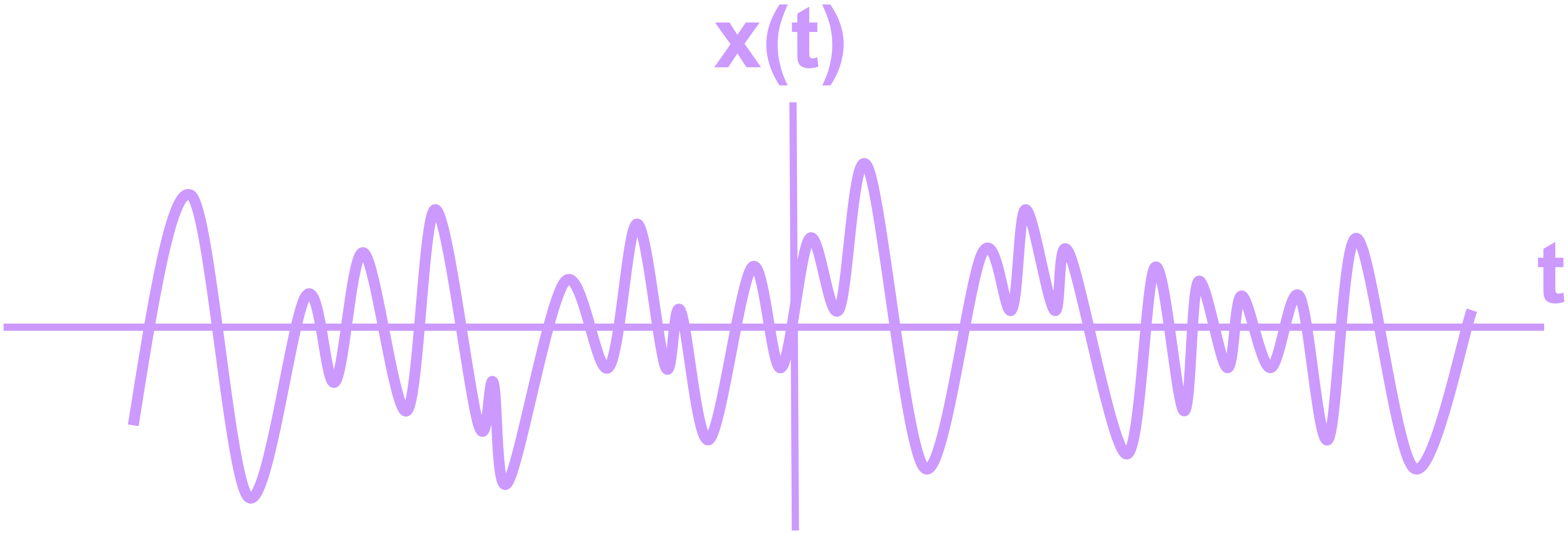
- Why DT? — Can be processed by modern digital computers and digital signal processors (DSPs).

Signal Classification 3: Deterministic /Random
Deterministic Signal



- Each value of the signal is fixed, and can be determined by a mathematical expression, rule, or table.

Signal Classification 3: Random Signal



- Signal value at any time instance is a random variable.

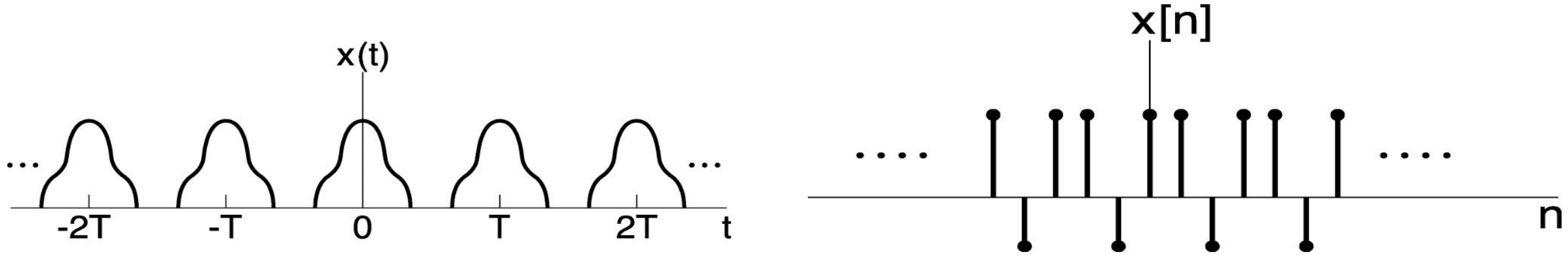
Signal Classification 4: Periodic / Aperiodic

- Periodic Signals

  CT: $x(t) = x(t + T)$, T : *period*

  $x(t) = x(t + mT)$, m: *integer*

  DT: $x[n] = x[n + N]=x[n + mN]$, N: period



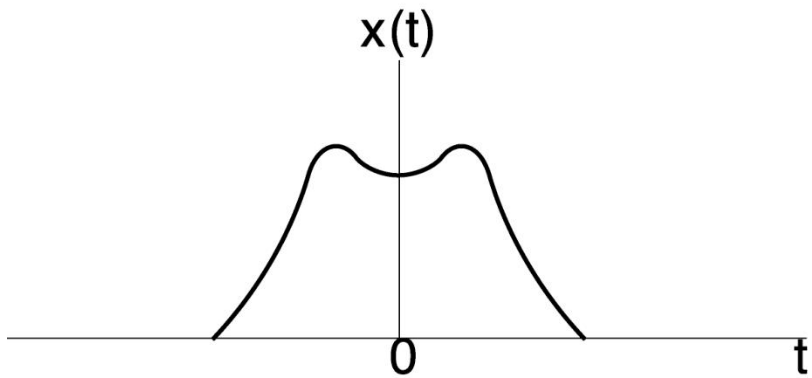
- Fundamental period: the smallest positive period

- Aperiodic: NOT periodic

Signal Classification 5: Even / Odd

- Even and Odd Signals

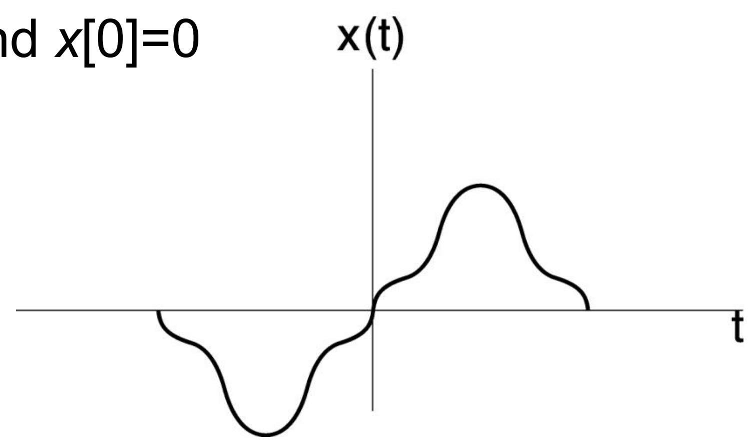
Even $x(t) = x(-t)$ *or* $x[n] = x[-n]$



Example: cos(t)

Odd $x(t) = -\ x(-t)$ *or* $x[n] = -\ x[-n]$

$x(0)=0$, and $x[0]=0$



Example: sin(t)

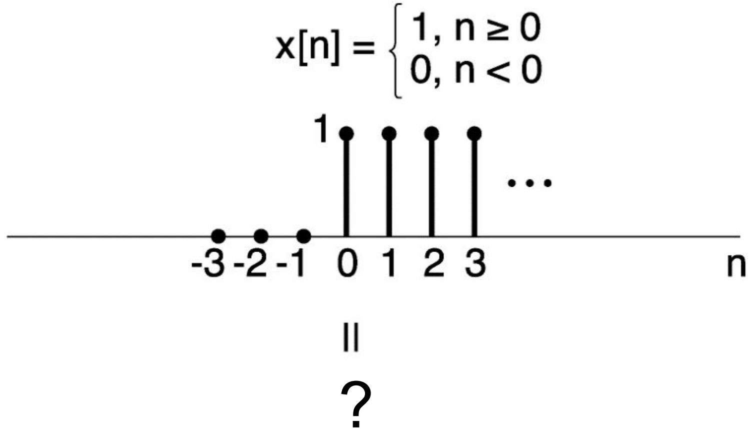
- Any signals can be expressed as a sum of *Even* and *Odd* signals. That is:

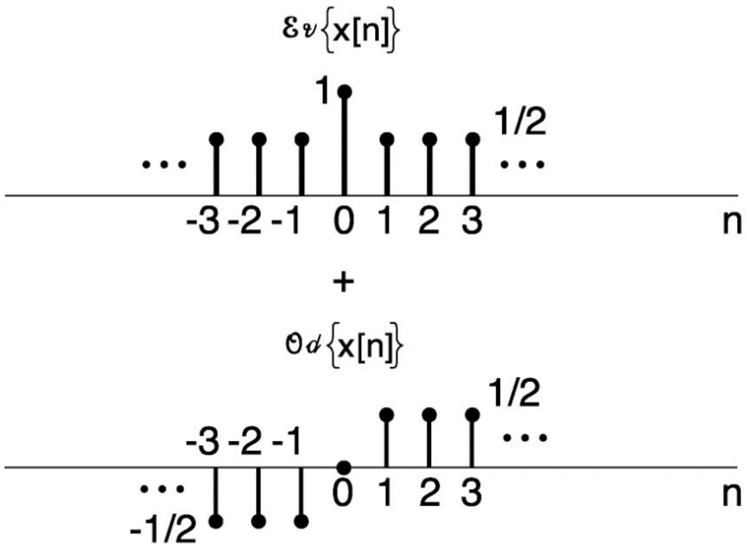
$x(t) = x_{even}(t) + x_{odd}(t),$

where:

$x_{even}(t) = [x(t) + x(-t)]/2,$

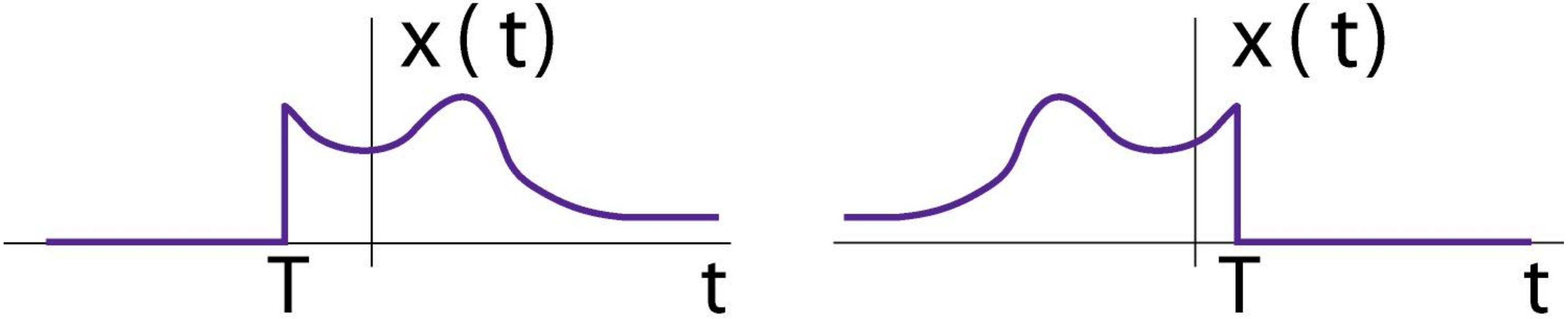
$x_{odd}(t) = [x(t) - x(-t)]/2.$



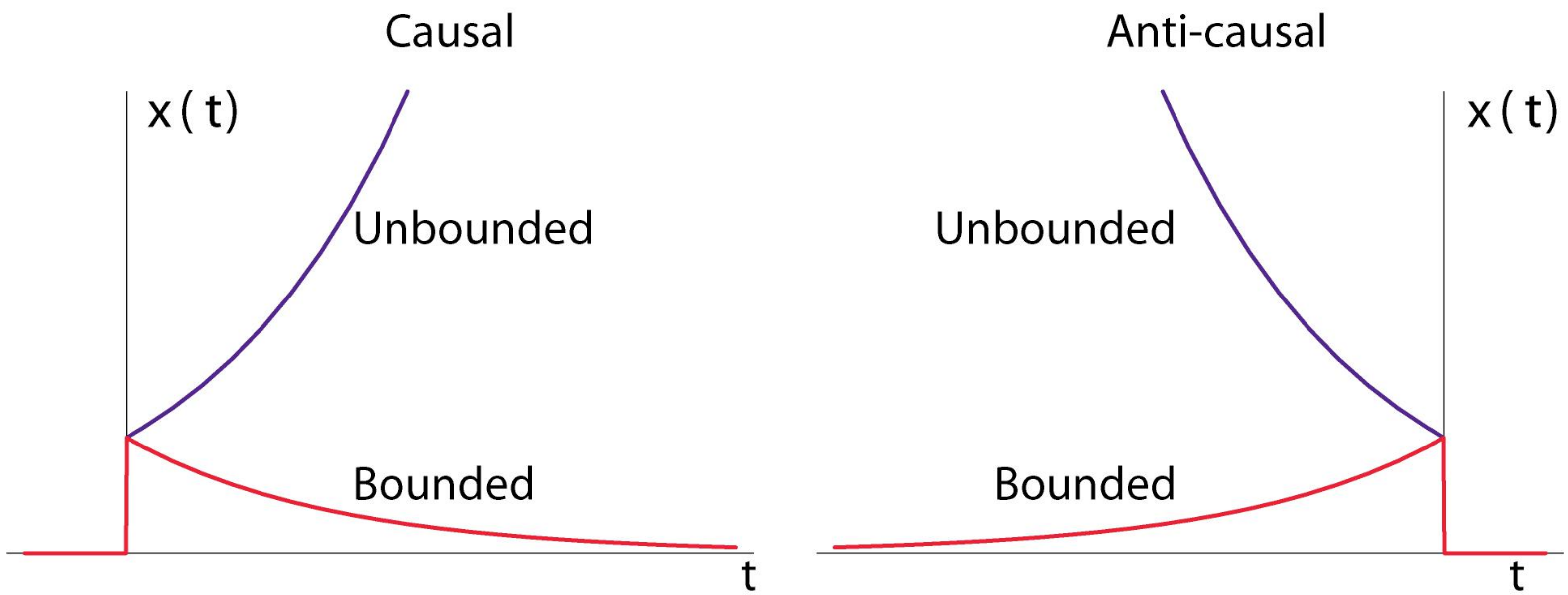


Signal Classification 6: Right- and Left-Sided

- A right-sided signal is zero for $t < T$, and
- A left-sided signal is zero for $t > T$, where T can be positive or negative.



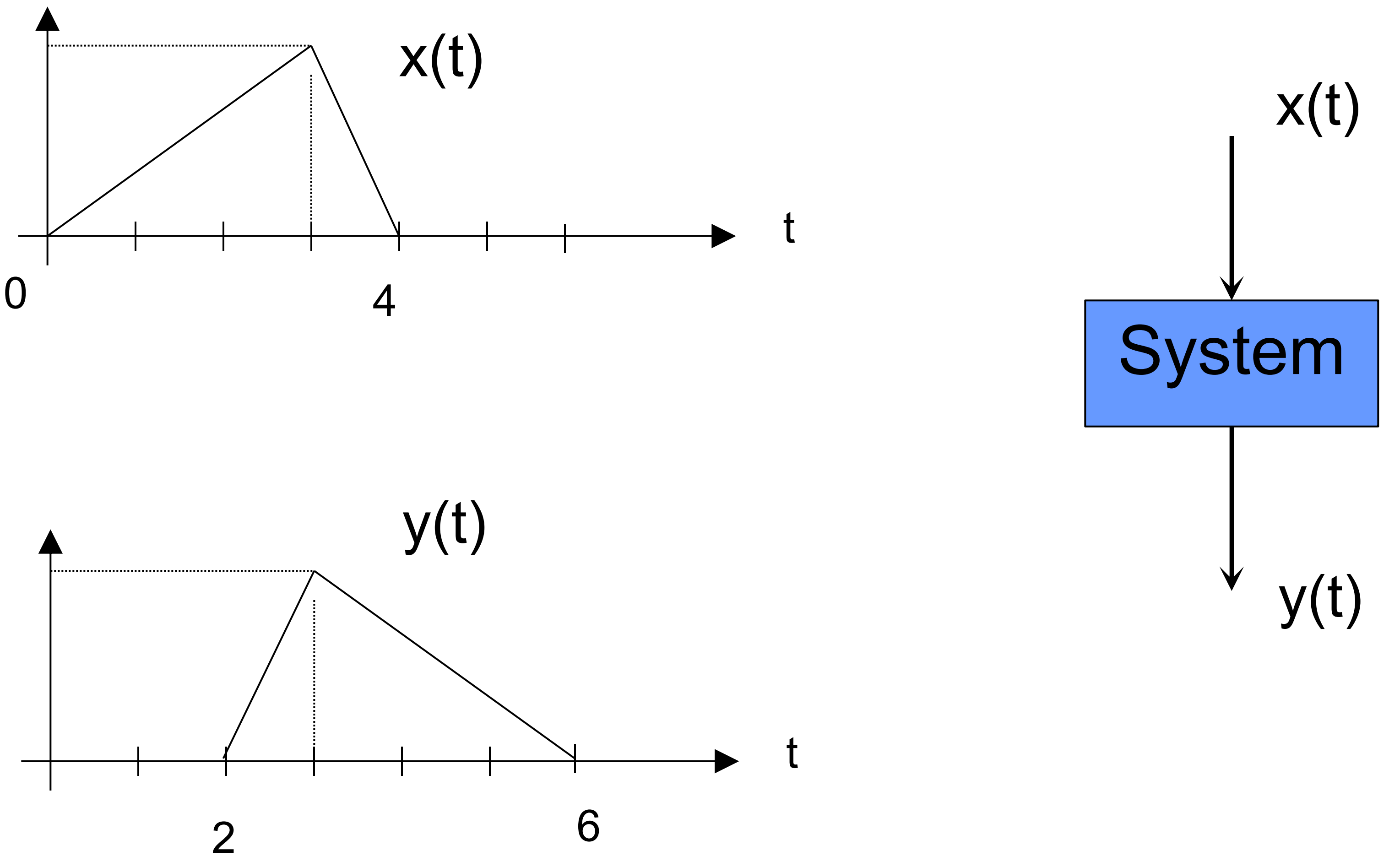
Classification 7: Bounded and Unbounded



- Bounded signal: the absolute value of signal is bounded.
- Unbounded signal: otherwise

$$\exists C,\ |x(t)| \leq C\ \forall t$$

Transformation of a Signal



Transformation of a Signal

- Time Shift

$$x(t) \to x(t - t_0) \quad , \quad x[n] \to x[n - n_0]$$

- Time Reversal

$$x(t) \to x(-t) \quad , \quad x[n] \to x[-n]$$

- Time Scaling

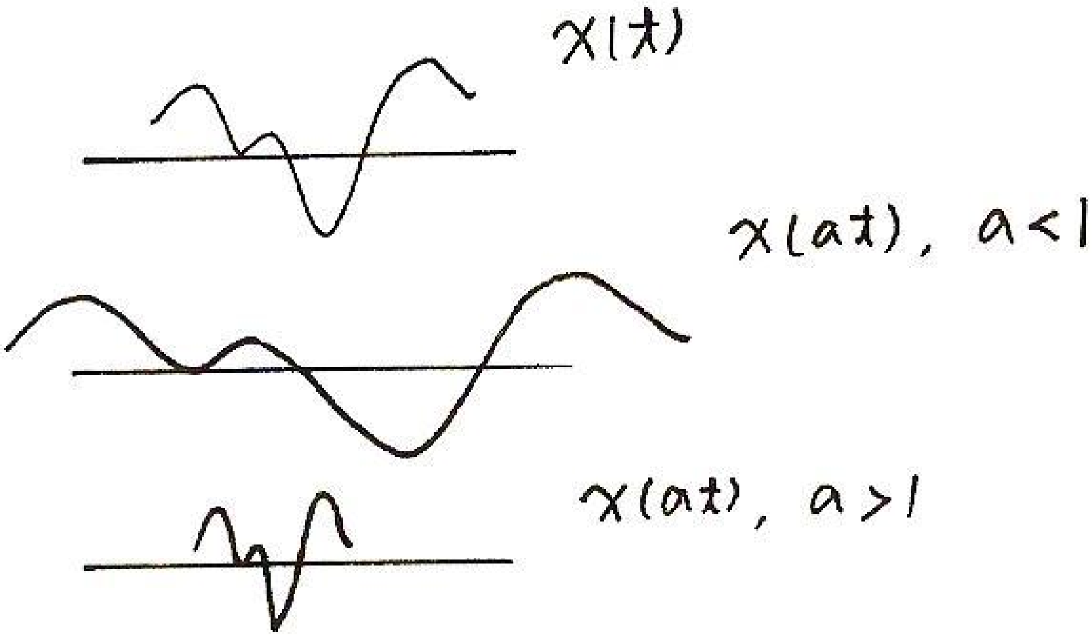
$$x(t) \to x(at) \quad , \quad x[n] \to ?$$

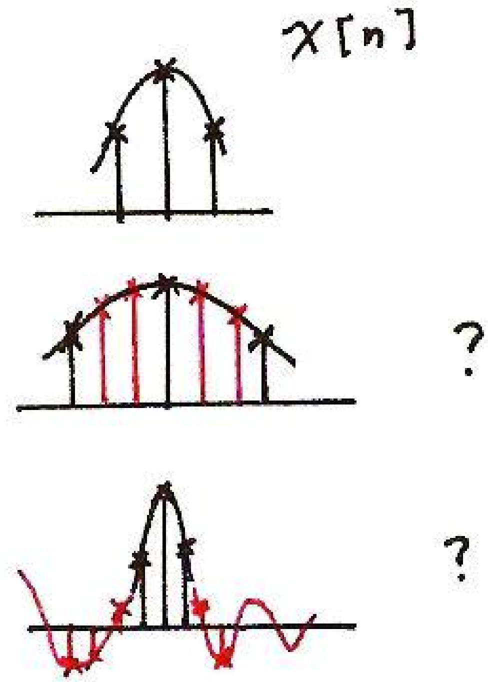
- Combination

$$x(t) \to x(at + b) \quad , \quad x[n] \to ?$$

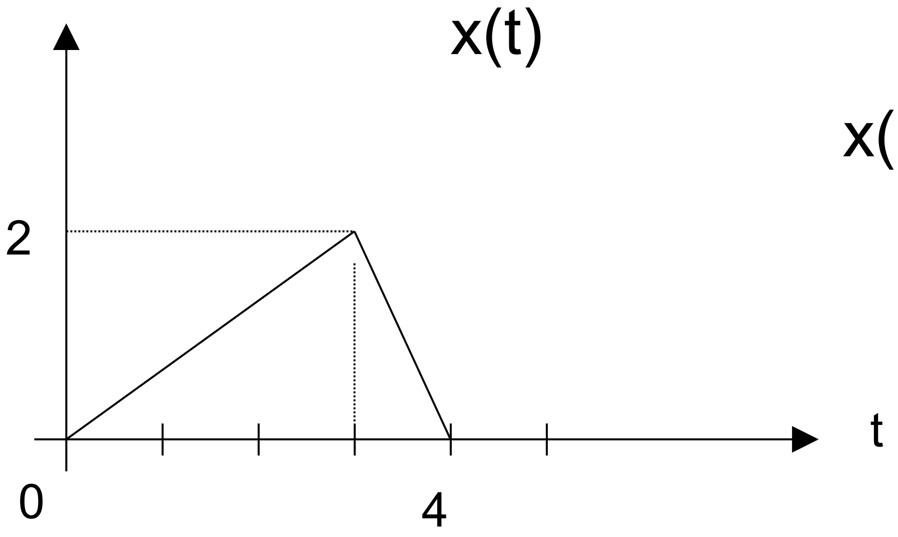
Transformation of a Signal

Time Scaling

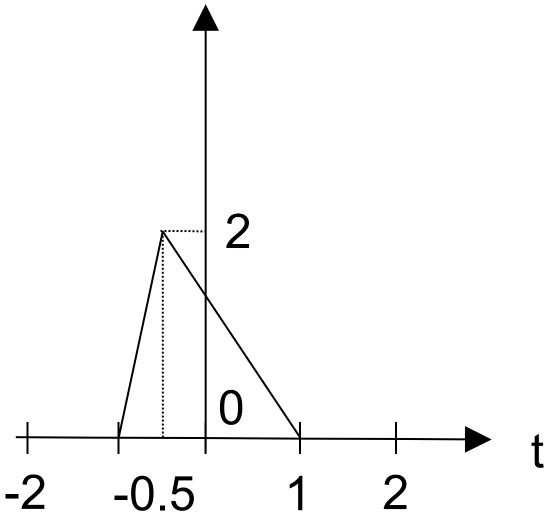




Class problem



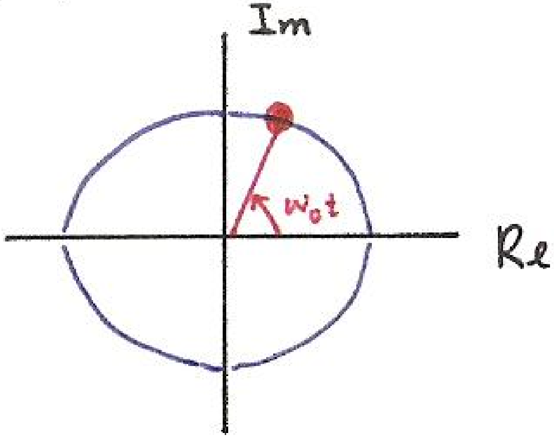
x(-2t+2) ?



Exponential Signals

- A very important class of signals is presented as:
  CT signals of the form $x(t) = e^{j\omega t}$
  DT signals of the form $x[n] = e^{j\omega n}$
- For both *exponential* CT and DT signals, x is a complex quantity and has:
  a real and imaginary part [i.e., *Cartesian form*], or equivalently
  a magnitude and a phase angle [i.e., *polar form*].
- We will use whichever form that is convenient.

$$x(t) = e^{j\omega_0 t}$$

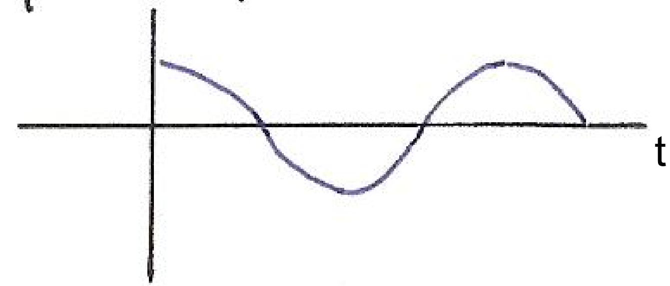


Euler's relation

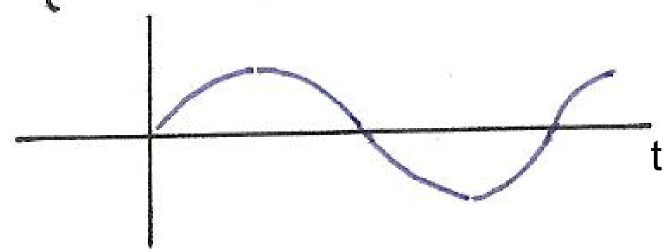
$$e^{jx} = \cos x + j \sin x$$

$\omega_0 t$ is defined as phase

$$Re\{e^{j\omega_0 t}\} = \cos \omega_0 t$$



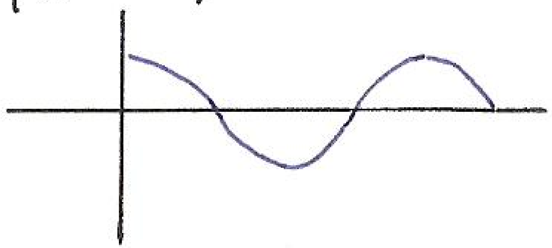
$$Im\{e^{j\omega_0 t}\} = \sin \omega_0 t$$



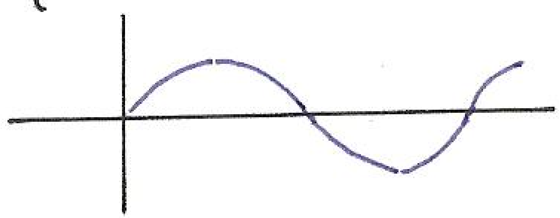
Real and imaginary parts are periodic signals with the same period, but out of phase (90^o phase difference)

$$x(t) = e^{j\omega_0 t} = \cos(\omega_0 t) + j\sin(\omega_0 t)$$

$Re\{e^{jw_0t}\} = \cos w_0 t$



$Im\{e^{jw_0t}\} = \sin w_0 t$



-Fundamental (angular) frequency: $|\omega_0|$

-Fundamental period: $T_0 = \dfrac{2\pi}{|\omega_0|}$

-In CT, $e^{j\omega_0 t}$ **always periodic**

-larger ω_0 **=> higher frequency**

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$$x[n] = e^{j\omega_0 n} = cos\omega_0 n + \mathrm{j}\ \sin\omega_0 n$$

Is it periodic?
Larger ω_0 => higher frequency?

$$e^{j\pi n} = (e^{j\pi})^n = (-1)^n$$

$$e^{j2\pi n} = (e^{j2\pi})^n = (1)^n = 1$$

$$x[n] = e^{j\omega_0 n} = cos\omega_0 n + \mathrm{j}\ \mathrm{sin}\omega_0 n$$

Is it periodic?
Larger ω_0 => higher frequency?

$$e^{j\omega_0 n} = e^{j\omega_0 (n+N)} = e^{j\omega_0 n}$$

$$\Rightarrow e^{j\omega_0 N} = 1 \Rightarrow \omega_0 N = 2\pi m \Rightarrow \frac{\omega_0}{2\pi} = \frac{m}{N}$$

$\dfrac{\omega_0}{2\pi}$ should be a rational number!

Periodicity Properties of DT Complex Exponentials

Important difference between $e^{j\omega_0 n}$ and $e^{j\omega_0 t}$:

- $e^{j\omega_0 n}$ is periodic w.r.t. ω_0

$$\boxed{e^{j(\omega_0+m\cdot 2\pi)n} = e^{j\omega_0 n}\cdot e^{jm\cdot 2\pi n} = e^{j\omega_0 n}}$$

- However, $e^{j\omega_0 t}$ *is aperiodic* $w.r.t.\,\omega_0$

$$\boxed{\forall x \neq 0,\ e^{j(\omega_0+x)t} = e^{j\omega_0 t}e^{jxt} \neq e^{j\omega_0 t}}$$

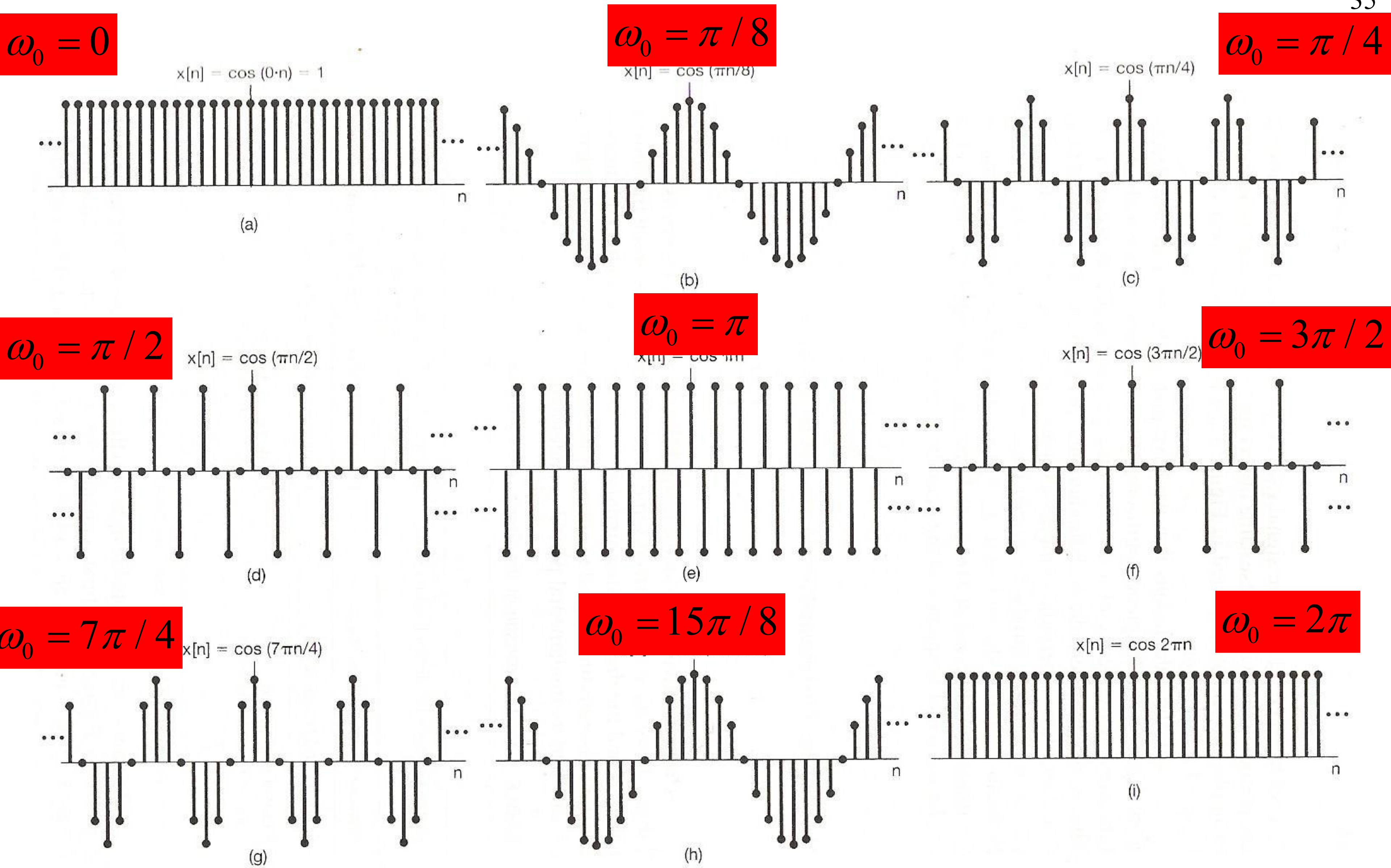


Figure 1.27 Discrete-time sinusoidal sequences for several different frequencies.

- We need only consider a frequency interval of length 2π, and on most cases, we use the interval: $0 \le \omega_0 < 2\pi$, or $-\pi \le \omega_0 < \pi$

- $e^{j\omega_0 n}$ does **not** have a continually increasing rate of oscillation as ω_0 is increased.

  lowest-frequency (slowly varying): ω_0 near 0, 2π, …, or 2k • π

  highest-frequency (rapid variation): ω_0 near ± π, …, or (2k+1) • π

$$e^{j(2k+1)\pi n} = e^{j\pi n} = (e^{j\pi})^n = (-1)^n$$

$$e^{j2\pi n} = (e^{j2\pi})^n = (1)^n = 1$$

Harmonically Related Signal Sets

- A set of periodic exponentials which have a common period.

$$\{\phi_k(t) = e^{jk\omega_0 t}, k = 0, \pm 1, \pm 2, \ldots\}$$

Fundamental (Angular) Frequency : $|k\omega_0|$

Fundamental Period: $\dfrac{2\pi}{|k\omega_0|}$

Common Period: $\dfrac{2\pi}{|\omega_0|}$

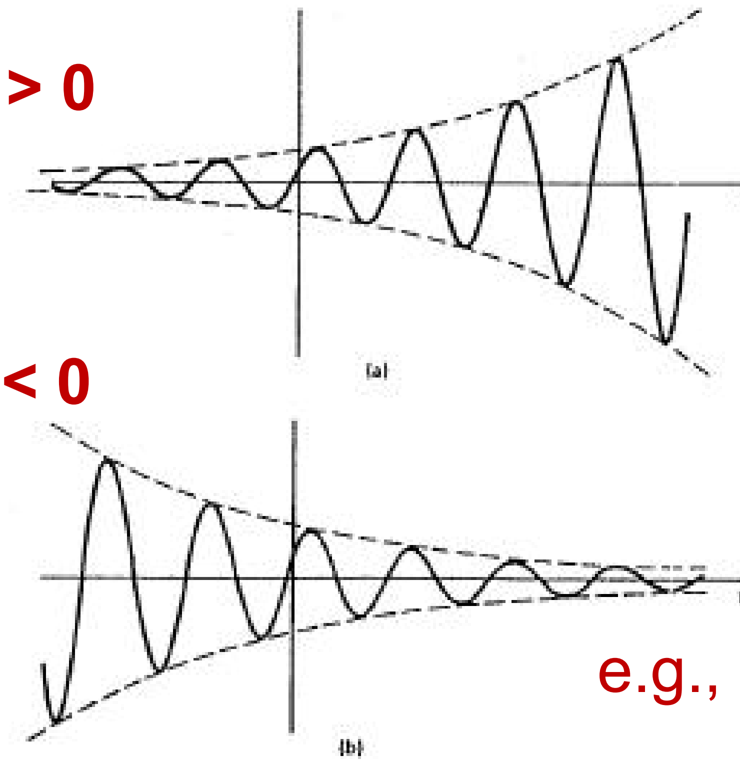
General Complex Exponential Signals - CT

- General format (C and a are complex numbers)

$$x(t) = Ce^{at} = |C| e^{j\theta} \cdot e^{(r+j\omega_0)t} = |C| e^{rt} \cdot e^{j(\omega_0 t+\theta)}$$

$r > 0$

$r < 0$



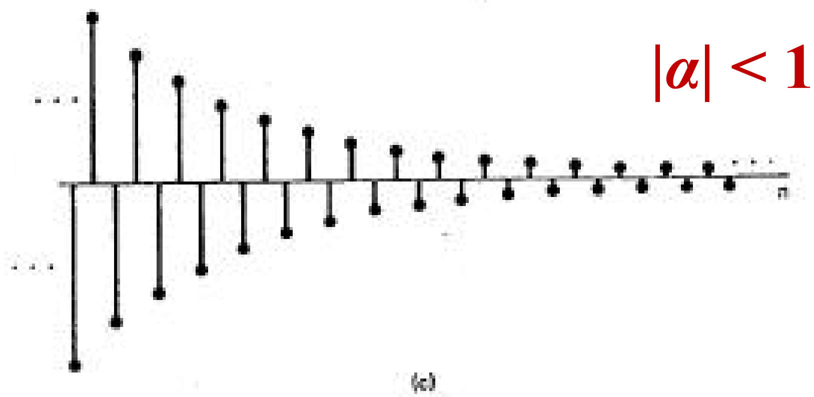
e.g., damped sinusoids

General Complex Exponential Signals - DT

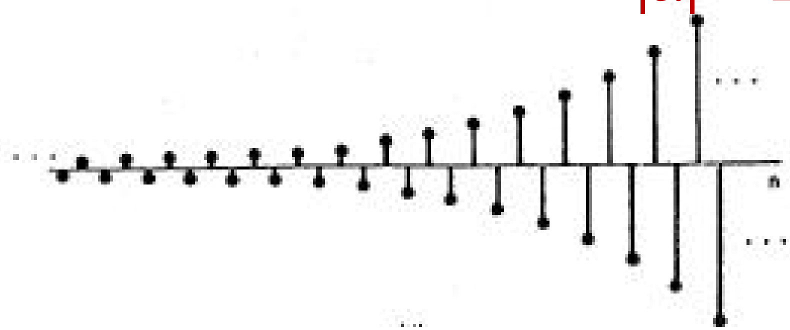
- General format (C and α are complex numbers)

$$x[n] = C\alpha^n = |C| e^{j\vartheta} \cdot |\alpha|^n e^{j\omega_0 n} = |C||\alpha|^n e^{j(\omega_0 n + \vartheta)}$$

$|\alpha| < 1$



$|\alpha| > 1$



Summary of week 1

- Meaning of signals and systems
- How to describe signals?
- Transformation of a signal
- Signal properties
- Periodic complex exponential signal
  - Harmonically related signal set