第七次作业

2021年11月11日 星期四　　下午8:21

4.14　　　　4.14 / 25 / 31 / 33 / 35

$\mathcal{F}^{-1}\{(1+j\omega)X(j\omega)\} = Ae^{-2t}u(t)$

$\mathcal{F}\{Ae^{-2t}u(t)\} = \frac{A}{j\omega+2}$

$\Rightarrow (1+j\omega)X(j\omega) = \frac{A}{j\omega+2}$

$X(j\omega) = \frac{A}{j\omega+1} - \frac{A}{j\omega+2}$

$x(t) = Ae^{-t}u(t) - Ae^{-2t}u(t)$

$\int_{-\infty}^{\infty}|X(j\omega)|^2 d\omega = 2\pi\int_{-\infty}^{\infty}|x(t)|^2 dt$

$\int_{-\infty}^{\infty}|A^2e^{-2t}u(t) - 2A^2e^{-3t}u(t) + A^2e^{-4t}u(t)|dt = 1$

$\int_0^{\infty}(A^2e^{-2t} - 2A^2e^{-3t} + A^2e^{-4t})dt = 1$

$\Rightarrow A = \pm\sqrt{12}$

$x(t)$ 是非负，故 $A = \sqrt{12}$

$x(t) = \sqrt{12}e^{-t}u(t) - \sqrt{12}e^{-2t}u(t)$

4.25 (a) 令 $y(t) = x(t+1)$，则 $y(t)$ 为实偶信号

$Y(j\omega) = |Y(j\omega)|e^{j\arg Y(j\omega)}$，$\arg Y(j\omega) = 0$

而 $Y(j\omega) = X(j\omega)e^{j\omega} = |X(j\omega)|e^{j\arg X(j\omega)}\cdot e^{j\omega}$

$\Rightarrow \arg X(j\omega) = -\omega$

(b). $X(j0) = \int_{-\infty}^{\infty}x(t)e^{-j0t}dt = \int_{-\infty}^{\infty}x(t)dt = 7$

(c). $\int_{-\infty}^{\infty}X(j\omega)d\omega = \int_{-\infty}^{\infty}X(j\omega)e^{j\omega t}d\omega \quad (t=0)$

$= 2\pi x(0) = 4\pi$

(d). 令 $Y(j\omega) = \frac{2\sin\omega}{\omega}e^{j2\omega}$

则 $y(t) = \begin{cases}1, & -3<t<-1\\ 0 & 其它\end{cases}$

$\int_{-\infty}^{\infty}X(j\omega)\frac{2\sin\omega}{\omega}e^{j2\omega}d\omega = \int_{-\infty}^{\infty}X(j\omega)Y(j\omega)d\omega$

$= 2\pi(x(t) * y(t)) \quad (t=0)$

$= 7\pi$

(e). $\int_{-\infty}^{\infty}|X(j\omega)|^2 d\omega$

$= 2\pi\int_{-\infty}^{\infty}|x(t)|^2 dt$

$= 2\pi\int_{-\infty}^{\infty}|x(t+1)|^2 dt$

$= 2\pi\times 2\cdot\left(\int_0^1 (t+1)^2 dt + \int_1^2 2^2 dt\right)$

$= 25\frac{\pi}{3}$　　　　$\frac{3}{2}+2$　$\frac{7}{2}\times 2\cdot 2$

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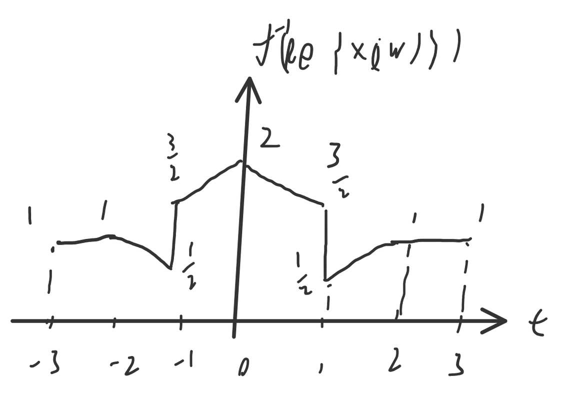
(f). $\mathcal{F}^{-1}\{Re\{X(j\omega)\}\} = \mathcal{E}v\{x(t)\} = \frac{1}{2}[x(t)+x(-t)]$



4.31 (a).

$X(j\omega) = \pi(\delta(\omega+1) + \delta(\omega-1))$

$h_1(t) \xrightarrow{FT} H_1(j\omega) = \pi\delta(\omega) + \frac{1}{j\omega}$

$h_2(t) \xrightarrow{FT} H_2(j\omega) = -2 + \frac{5}{j\omega+2}$

$h_3(t) \xrightarrow{FT} H_3(j\omega) = \frac{2}{(j\omega+1)^2}$

$Y_1(j\omega) = H_1(j\omega)X(j\omega)$

$= (\pi\delta(\omega) + \frac{1}{j\omega})\times\pi(\delta(\omega+1) + \delta(\omega-1))$

$= \frac{\pi}{j}(-\delta(\omega+1) + \delta(\omega-1))$

$y_1(t) = \sin t$

$Y_2(j\omega) = j\pi(\delta(\omega+1) - \delta(\omega-1))$

$y_2(t) = \sin t$

$Y_3(j\omega) = \frac{\pi}{j}(\delta(\omega-1) - \delta(\omega+1))$

$y_3(t) = \sin t$

$\therefore$ 对 $x(t)$ 有相同响应

(b). 令 $h_4(t) = \frac{1}{2}(h_1(t) + h_2(t))$

$Y_4(j\omega) = H_4(j\omega)X(j\omega) = \frac{1}{2}Y_1(j\omega) + \frac{1}{2}Y_2(j\omega)$

$y_4 = \sin t$

可见，由 h_1 h_2 h_3 2(相加除2的单位冲激响应的LTI 响应相同

对 cost 响应不能唯一指定 LTI 系统。

4.33 (a) $H(j\omega) = \frac{Y(j\omega)}{X(j\omega)} = \frac{2}{-\omega^2+6j\omega+8} = \frac{1}{j\omega+2} - \frac{1}{j\omega+4}$

$h(t) = e^{-2t}u(t) - e^{-4t}u(t)$

(b). $X(j\omega) = \frac{1}{(j\omega+2)^2}$

$Y(j\omega) = H(j\omega)X(j\omega) = \frac{1}{(j\omega+2)^3} - \frac{1}{(j\omega+4)(j\omega+2)^2}$

$= \frac{1}{4(j\omega+2)} - \frac{1}{4(j\omega+4)} - \frac{1}{2(j\omega+2)^2} + \frac{1}{(j\omega+2)^3}$

$y(t) = \frac{1}{4}e^{-2t}u(t) - \frac{1}{4}e^{-4t}u(t) - \frac{1}{2}te^{-2t}u(t) + \frac{1}{2}t^2e^{-2t}u(t)$

(c). $H(j\omega) = \frac{Y(j\omega)}{X(j\omega)} = \frac{-2\omega^2-2}{-\omega^2+\sqrt{2}j\omega+1} = 2 - \frac{\sqrt{2}-j\sqrt{2}}{j\omega+\frac{\sqrt{2}-j\sqrt{2}}{2}} - \frac{\sqrt{2}+j\sqrt{2}}{j\omega+\frac{\sqrt{2}+j\sqrt{2}}{2}}$

$h(t) = \mathcal{F}^{-1}(H(j\omega))$

$= 2\delta(t) - (\sqrt{2}-j\sqrt{2})e^{\frac{(-1+j)}{\sqrt{2}}t}u(t) - (\sqrt{2}+j\sqrt{2})e^{\frac{-(1+j)}{\sqrt{2}}t}u(t)$

4.35 (a) $|H(j\omega)| = \left|\frac{a-j\omega}{a+j\omega}\right| = \frac{\sqrt{a^2+\omega^2}}{\sqrt{a^2+\omega^2}} = 1$

$\measuredangle H(j\omega) = -\arctan\frac{\omega}{a} - \arctan\frac{\omega}{a} = -2\arctan\frac{\omega}{a}$

$H(j\omega) = -1 + \frac{2a}{j\omega+a}$

$h(t) = \mathcal{F}^{-1}\{H(j\omega)\} = -\delta(t) + 2ae^{-at}u(t)$

(b). $H(j\omega) = \frac{1-j\omega}{1+j\omega}$

$x(t) = \cos\frac{t}{\sqrt{3}} + \cos t + \cos\sqrt{3}t$

$\omega_1 = \frac{1}{\sqrt{3}}$　　$\omega_2 = 1$　　$\omega_3 = \sqrt{3}$

$\Rightarrow -2\arctan\omega_1 = -\frac{\pi}{3}$

$-2\arctan\omega_2 = -\frac{\pi}{2}$

$-2\arctan\omega_3 = -\frac{2\pi}{3}$

故 $y(t) = \cos(\frac{t}{\sqrt{3}} - \frac{\pi}{3}) + \cos(t - \frac{\pi}{2}) + \cos(\sqrt{3}t - \frac{2\pi}{3})$