第四次作业

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3.3　$x(t) = 2 + \cos(\frac{2\pi}{3}t) + 4\sin(\frac{5\pi}{3}t)$

$= 2 + \frac{1}{2}e^{j\frac{2\pi}{3}t} + \frac{1}{2}e^{j(-\frac{2\pi}{3})t} - 2je^{j\frac{5\pi}{3}t} + 2je^{-j\frac{5\pi}{3}t}$

$= 2 + \frac{1}{2}e^{2j\frac{\pi}{3}t} + \frac{1}{2}e^{-2j\frac{\pi}{3}t} - 2je^{5j\frac{\pi}{3}t} + 2je^{-5j\frac{\pi}{3}t}$

so $w_0 = \frac{\pi}{3}$

$a_0 = 2$　$a_2 = a_{-2} = \frac{1}{2}$　$a_5 = a_{-5} = -2j$

3.21　$x(t) = a_1 e^{jwt} + a_{-1}e^{-jwt} + a_5 e^{5jwt} + a_{-5}e^{-5jwt}$

$w = \frac{2\pi}{8} = \frac{\pi}{4}$

so $x(t) = -2je^{\frac{\pi j}{4}t} - je^{-\frac{\pi j}{4}t} + 2e^{\frac{5j\pi t}{4}} + 2e^{-\frac{5j\pi t}{4}}$

$= -2\sin(\frac{\pi}{4}t) + 4\cos(\frac{5\pi}{4}t)$

$= 2\cos(\frac{\pi}{4}t + \frac{\pi}{2}) + 4\cos(\frac{5\pi}{4}t)$

3.22 (a).

fig b:　$x(t) = \begin{cases} t+2 & -2<t<-1 \\ 1 & -1<t<1 \\ 2-t & 1<t<2 \end{cases}$

$T = 6$, $a_0 = \frac{1}{2}$

so $a_k = \begin{cases} 0 & \text{when } k \text{ is even} \\ \frac{6}{\pi^2 k^2}\sin(\frac{\pi k}{2})\sin(\frac{\pi k}{6}) & \text{when } k \text{ is odd} \end{cases}$

(d). $T=2$, $w_0 = \frac{2\pi}{2} = \pi$.　for $0 \le t < 2$: $x(t) = \delta(t) - 2\delta(t-1)$

$a_0 = \frac{1}{T}\int_T x(t)\,dt = \frac{1}{2}\int_0^2 [\delta(t) - 2\delta(t-1)]\,dt = -\frac{1}{2}$

$a_k = \frac{1}{T}\int_T x(t)e^{-jkw_0 t}\,dt = \frac{1}{2}\int_0^2 (\delta(t) - 2\delta(t-1))\cdot e^{-jkw_0 t}\,dt$

$= \frac{1}{2}\int_0^2 \delta(t)\,dt - \int_0^2 e^{-jk\pi}\delta(t-1)\,dt$

$= \frac{1}{2} - (-1)^k$　$(k \neq 0)$

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(f). $x(t) = \begin{cases} 2 & 0<x<1 \\ 1 & 1<x<2 \\ 0 & 2<x<3 \end{cases}$

$T = 3$. $w_0 = \frac{2\pi}{3}$ $\quad a_0 = 1$

$$a_0 = \frac{1}{T}\int_T x(t)dt = \frac{1}{3}\int_0^1 2dt + \frac{1}{3}\int_1^2 dt = 1$$

$$a_k = \frac{1}{T}\int_T x(t) e^{-jkw_0t} dt = \frac{1}{3}\int_0^1 2e^{-jkw_0t} dt + \frac{1}{3}\int_1^2 e^{-jkw_0t} dt$$

$$= \frac{1}{jk\pi}\left(1 - e^{-jk\frac{2\pi}{3}}\right) + \frac{1}{2jk\pi}\left(e^{-jk\frac{2\pi}{3}} - e^{-\frac{4jk\pi}{3}}\right)$$

$$= \frac{2}{k\pi} e^{-jk\pi/3} \cdot \sin\frac{k\pi}{3} + \frac{1}{k\pi} e^{-jk\pi} \sin\frac{k\pi}{3}$$

$$= \frac{\sin(k\pi/3)}{k\pi/3}\left(\frac{2}{3}e^{-jk\pi/3} + \frac{1}{3}e^{-jk\pi}\right) \quad (k \neq 0)$$

3.24 (a). $a_0 = \frac{1}{2}\left(\int_0^1 t dt + \int_1^2 (2-t) dt\right) = \frac{1}{2}$

(b). the coefficients b_k of $\frac{dx(t)}{dt}$ is:

$$b_0 = \frac{1}{2}\int_0^1 dt - \frac{1}{2}\int_1^2 dt = 0$$

$$b_k = \frac{1}{2}\int_0^1 e^{-j\pi kt} dt - \frac{1}{2}\int_1^2 e^{-j\pi kt} dt$$

$$= \frac{1}{j\pi k}\left(1 - \frac{1}{e^{j\pi k}}\right)$$

so for $\frac{dx(t)}{dt}$: $\frac{dx(t)}{dt} = \sum_{k=-\infty}^{\infty} \frac{1-e^{-jk\pi}}{jk\pi} e^{jkw_0t}$, $w_0 = \pi$ $(k \neq 0)$

(c). $\frac{dx(t)}{dt} \xrightarrow{FS} b_k = j\pi k a_k$

so $a_k = \frac{1}{j\pi k} b_k = \frac{1}{j^2\pi^2k^2}\left(1 - \frac{1}{e^{j\pi k}}\right) = -\frac{1}{\pi^2k^2}\left(1 - \frac{1}{e^{j\pi k}}\right)$

3.25 (a). $T = \frac{1}{2}$, $w_0 = \frac{2\pi}{T} = 4\pi$

so $x(t) = \frac{1}{2}\left(e^{j4\pi t} + e^{-j4\pi t}\right) = \sum_{k=-\infty}^{\infty} a_k e^{jk(4\pi)t}$

then we can get $a_1 = a_{-1} = \frac{1}{2}$

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(b). similar to (a). $y(t) = \frac{1}{2j}\left(e^{j4\pi t} - e^{-j4\pi t}\right) = \sum_{k=-\infty}^{\infty} b_k e^{jk(4\pi)t}$

here $b_1 = -\frac{1}{2}j$ $b_{-1} = \frac{1}{2}j$

(c). $z(t) = x(t)y(t) \Rightarrow c_k = a_k * b_k$

$$= \left\{\frac{1}{2}\delta[k+1] + \frac{1}{2}\delta[k-1]\right\} * \left\{\frac{1}{2}j\delta[k+1] - \frac{1}{2}j\delta[k-1]\right\}$$

$$= \frac{1}{4}j\delta[k+2] - \frac{1}{4}j\delta[k-2]$$

so $c_{-2} = \frac{1}{4}j$ $c_2 = -\frac{1}{4}j$

(d). $z(t) = x(t)y(t) = \cos(4\pi t)\sin(4\pi t)$

$= \frac{1}{2}\sin(8\pi t)$

$= \frac{1}{4j}\left(e^{j8\pi t} - e^{-j8\pi t}\right)$

$c_2 = -\frac{1}{4}j$ $c_{-2} = \frac{1}{4}j$ which is the same as (c)