第十一次作业

2021年12月9日 星期四　　下午8:31

4.50.

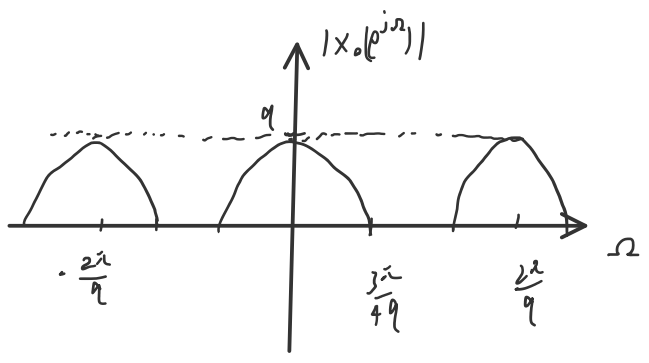
$$X_0(e^{j\Omega}) = X(e^{j\Omega})H_0(e^{j\Omega})$$

$$x[n] = \frac{\sin(\frac{3\pi}{4}n)}{\pi n}$$

$$X(e^{j\Omega}) = \sum_{n=-\infty}^{\infty} x[n]e^{-j\Omega n} = \sum_{n=-\infty}^{\infty} \frac{\sin(\frac{3\pi}{4}n)}{\pi n} e^{-j\Omega n}$$

$$X(e^{j\Omega}) = \begin{cases} 1 & |\Omega| < \frac{3\pi}{4} \\ 0 & \frac{3\pi}{4} < |\Omega| < \pi \end{cases} \quad (T = 2\pi)$$

$$|X(e^{j\Omega})| = |X(e^{j\Omega q})| \cdot \left| \frac{\sin(\Omega\frac{q}{2})}{\sin(\frac{\Omega}{2})} \right|$$



(b). to get the ideal interpolation, we need to discard components which are not centered at multiples of 2π.

In addition, we need to do some corrections to get correct magnitude and phase distortion.

$$H(e^{j\Omega}) = \begin{cases} \frac{\sin(\frac{\Omega}{2})\cdot e^{j\Omega\frac{q}{2}}}{\sin(\frac{q}{2}\cdot\Omega)} & |\Omega| < \frac{W}{q} \\ 0 & \frac{W}{q} \le |\Omega| < 2\pi - \frac{W}{q} \end{cases} \quad (T = 2\pi)$$

将 $H(e^{j\Omega})$ 代入 q. w, 即可得结果

(i). $H(e^{j\Omega}) = \begin{cases} \frac{\sin(\frac{\Omega}{2})\cdot e^{j\Omega}}{\sin(\Omega)} & |\Omega| < \frac{3\pi}{8} \\ 0 & \frac{3\pi}{8} < |\Omega| < \frac{13\pi}{8} \end{cases}$ $(W = \frac{3\pi}{4}, \ q = 2)$

(ii). $H(e^{j\Omega}) = \begin{cases} \frac{\sin(\frac{\Omega}{2})\cdot e^{2j\Omega}}{\sin(2\Omega)} & |\Omega| < \frac{3\pi}{16} \\ 0 & \frac{3\pi}{16} < |\Omega| < \frac{29\pi}{16} \end{cases}$ $(W = \frac{3\pi}{4}, \ q = 4)$

4.51. passband: $100\pi < W < 200\pi$

so $\Omega_a = 100\pi T_s$

$\Omega_b = 200\pi T_s$

$$|H_0(jw)| = \left| \frac{2\sin(w\frac{T_s}{2})}{w} \right|$$

when $w = 100\pi \Rightarrow \frac{2\sin(50\pi)}{100\pi T_s} < 1.1$

when $w = 200\pi \Rightarrow \frac{2\sin(100\pi)}{200\pi T_s} > 0.9$

so $T_s(100\pi) < 0.78\pi$

$\max T_s = 0.002\pi$

$\min W_3 = 200\pi$

$\max W_4 = \frac{2\pi}{T_s} - 200\pi$

$= \frac{2\pi}{0.002\pi} - 200\pi = 800\pi$

$\Omega_a = 100\pi T_s = 0.2\pi$

$\Omega_b = 200\pi T_s = 0.4\pi$

so $\min W_1 = 200\pi$

$\max W_2 = \frac{1}{2}\cdot\frac{2\pi}{T_s} = 400\pi$

no overlap.