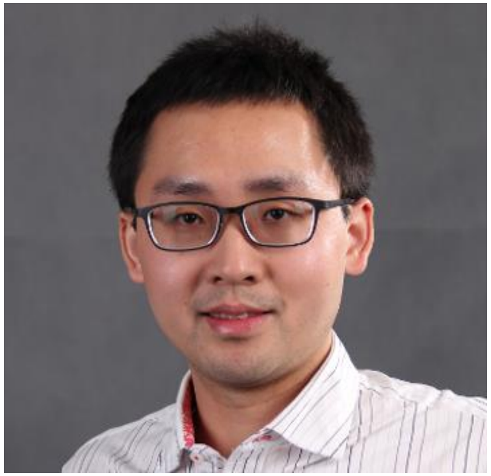
Signals and Systems

Department of Electrical & Electronic Engineering

Southern University of Science and Technology

Autumn 2020



WANG Rui

- USTC - CSE - BEng
- HKUST - ECE - PhD
- Huawei - Senior Research Engineer
- SUSTech – EEE - Associate Professor

Research Interests:

- Wireless communications: 5G, VLC, mmWave and etc.
- Cloud and edge computing
- Stochastic optimization, Reinforcement learning, convex optimization and etc.

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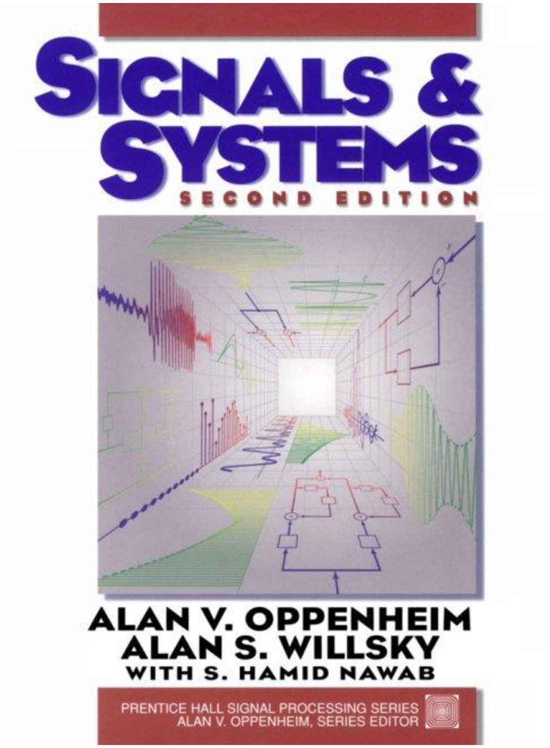
Scope of Lecture

- "Signals and Systems", Oppenheim, Willsky and Nawab, 2nd Edition, 1997, Prentice-Hall.



- This course teaches Chapters 1 to 10.
  - Roughly two weeks for one chapter
  - Middle-term exam for Chapters 1 to 4
  - Chapters 6, 8, 9, 10 in a short manner
  - Final exam for all

Textbook reading is crucial, as I cannot cover every detail in slides

Three Pillars

Lectures (Tutorial)

Matlab Labs

YOU

100%

Assignment/Quiz 20%

Mid-term Exam 25%

Final Exam 25%

Lab Reports

Project Report & Presentation

30%

Class Schedules

- **Lab Session – Starts from the first week**
- **Instructor: Dr. Guang Wu (吴光)**

- **Tutorials – TBD**
- **Every week (no for week 1)**

群名称:Signals and Systems, 2020...
群　号:883107947

Practice is Important

- **Which taste of 粽子 do you like? Salty or sweet**
- **How can a southern Chinese get used to sweet 粽子?**

- **Assignment: Every week (no for week 1)**
- **Submit assignment in softcopy after one week.**

Signals and Systems

- **Signals:** everything which carries information
- **Systems:** everything which processes input signal and generate output signal

Slides partly extracted from “Signals and Systems”, Lecture Notes by Prof. Qing Hu, MIT, 2004, and Prof. Linshan Lee, NTU, 2009

Communication Signals & Systems

Can you find any example of signals and systems when making a phone call?

- Transmitter, channel and receiver are all systems.
- Each system has one input signal and one output signal.

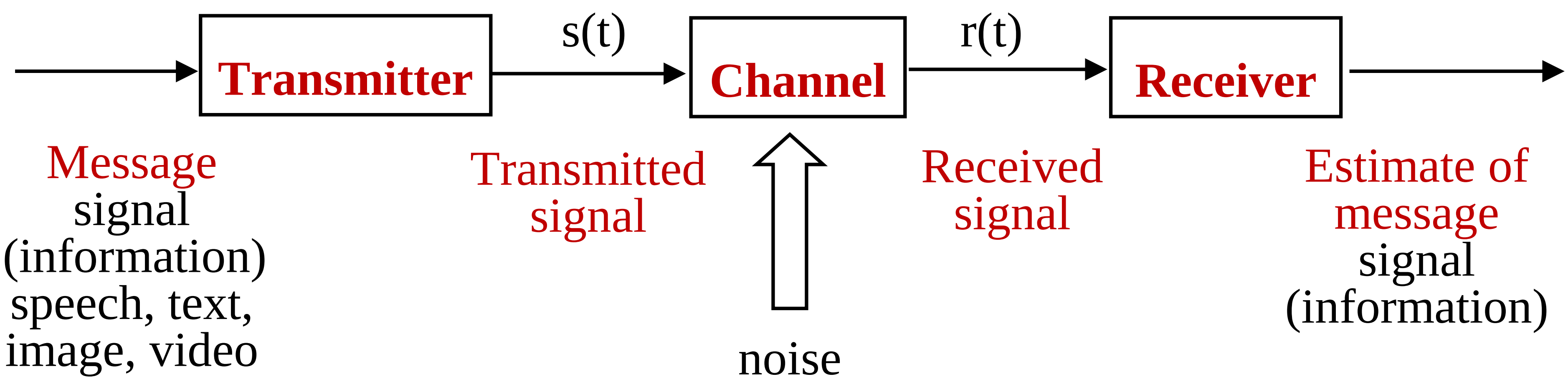
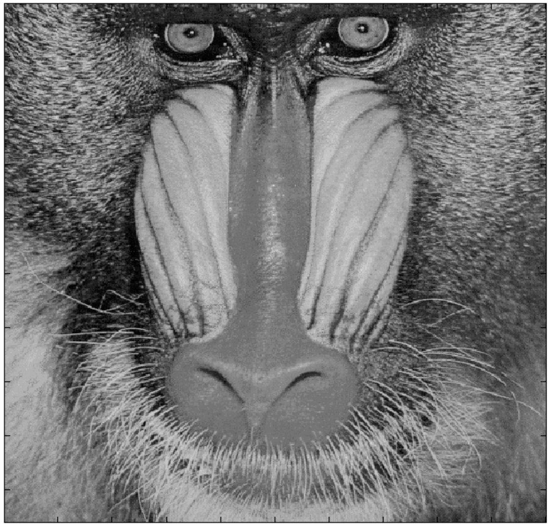
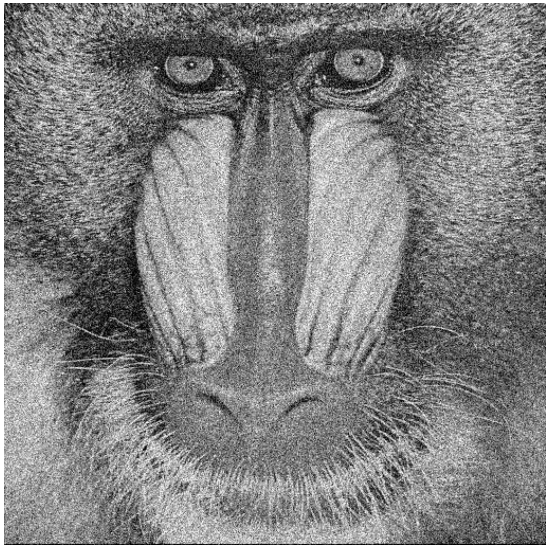


Image Processing





More examples of signals

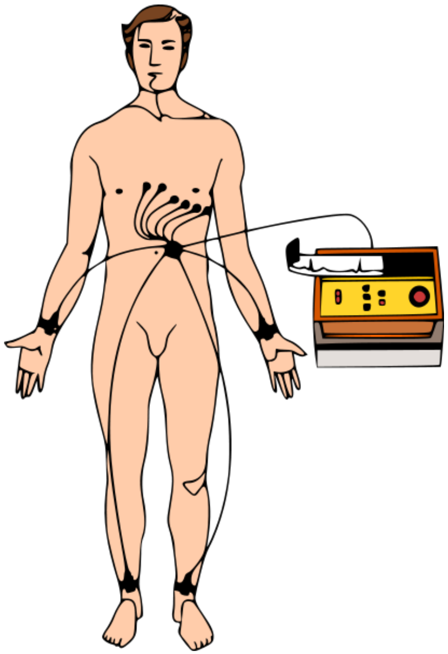
- Electrical signals – voltages and currents in a circuit
- Acoustic signals – audio or speech signals
- Video signals – movie
- Biological signals – sequence of bases in a gene
- We will treat noise as unwanted signals.

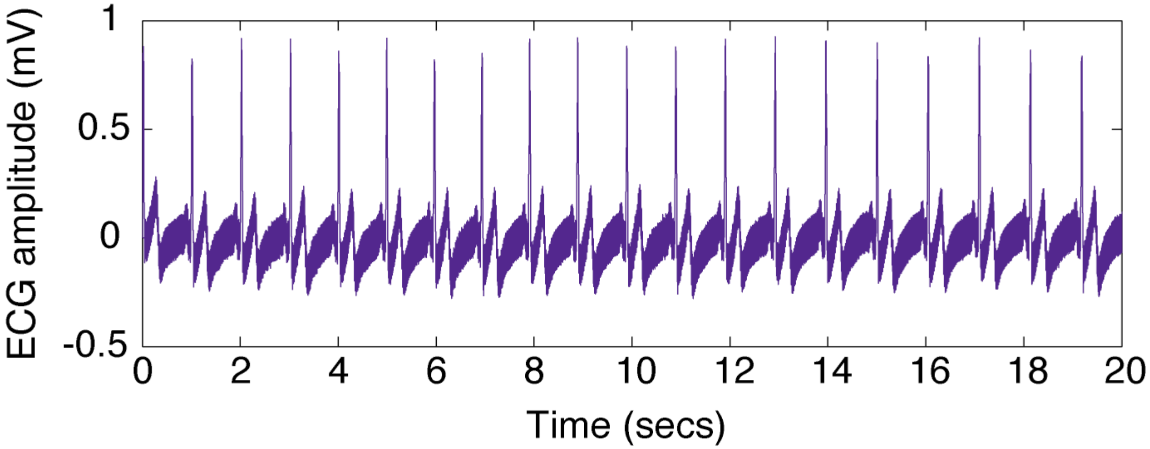
Signals and Systems from Our Point of View

- Signals are variables that carry information, like function.
- Systems process input signals to produce output signals.
- The course is about using mathematical techniques to analyze and synthesize systems which process signals.

Independent Variable of Signals

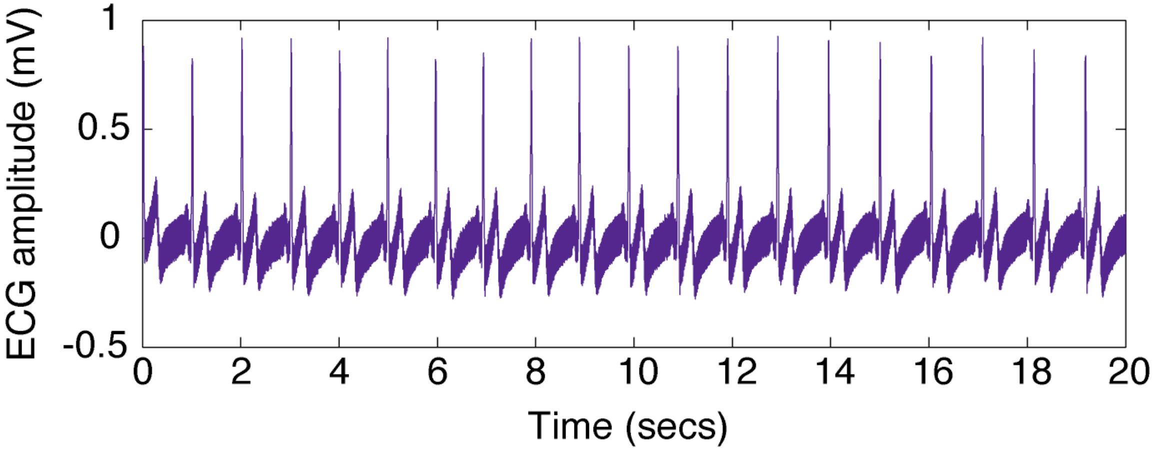
- Time is often the independent variable.
- Example: the electrical activity of the heart recorded with chest electrodes — the electrocardiogram (ECG).

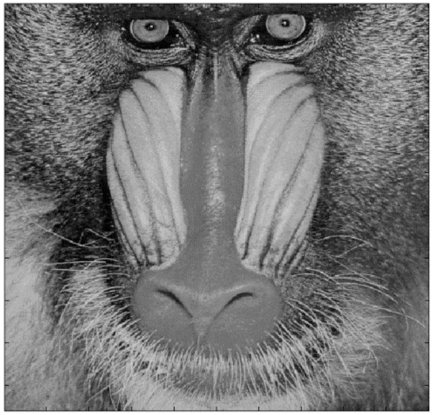




Signal Classification 1: Dimension of Independent Variable

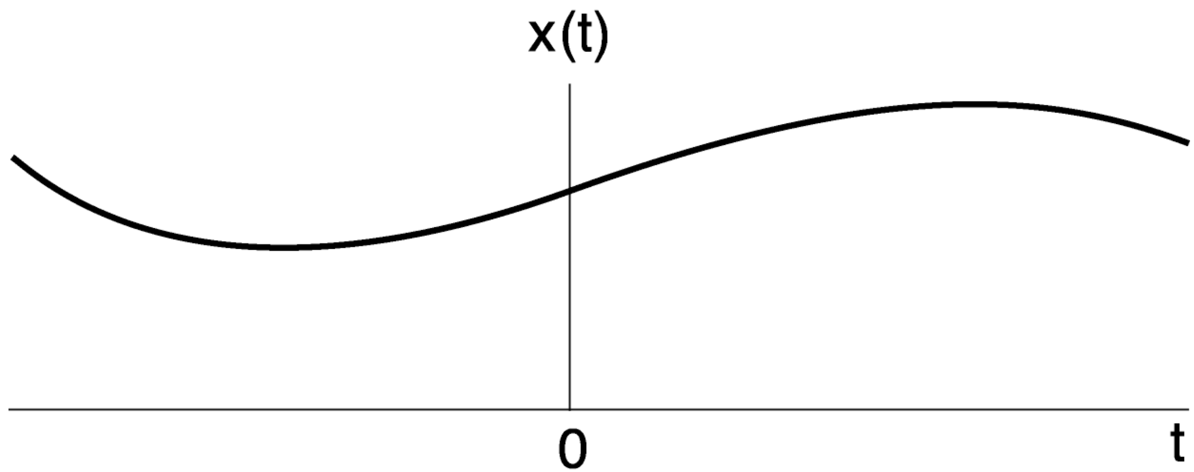
- An independent variable can be 1-D (*t* in the ECG), 2-D (*x*, *y* in an image), or 3-D (*x*, *y*, *t* in an video).





- We focus on 1-D for mathematical simplicity but the results can be extended to 2-D or even higher dimensions.

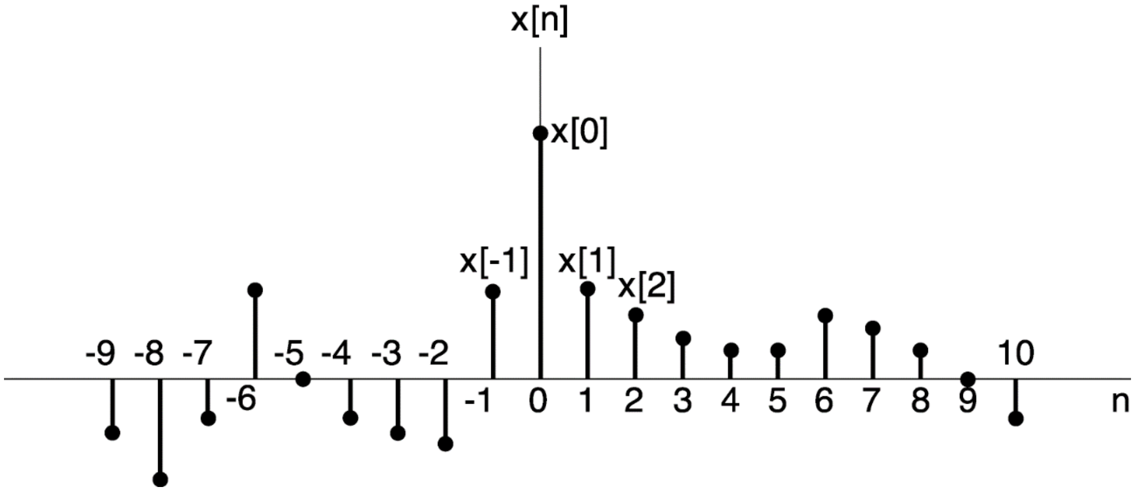
Signal Classification 2: Continuous-time (CT) Signals



- Independent variable is continuous
- Most of the signals in the physical world are CT signals.
- E.g. voltage & current, pressure, temperature, velocity, etc.

Notation: x(t)

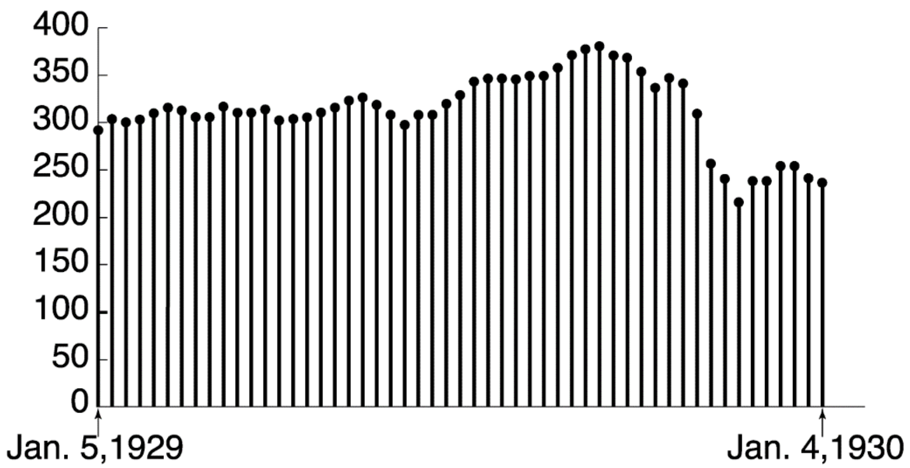
Discrete-time (DT) Signals



- Independent variable is integer
- Examples of DT signals: DNA sequence, population of the *n*-th generation of certain species

Notation: $x[n]$

Many Human-made Signals are DT

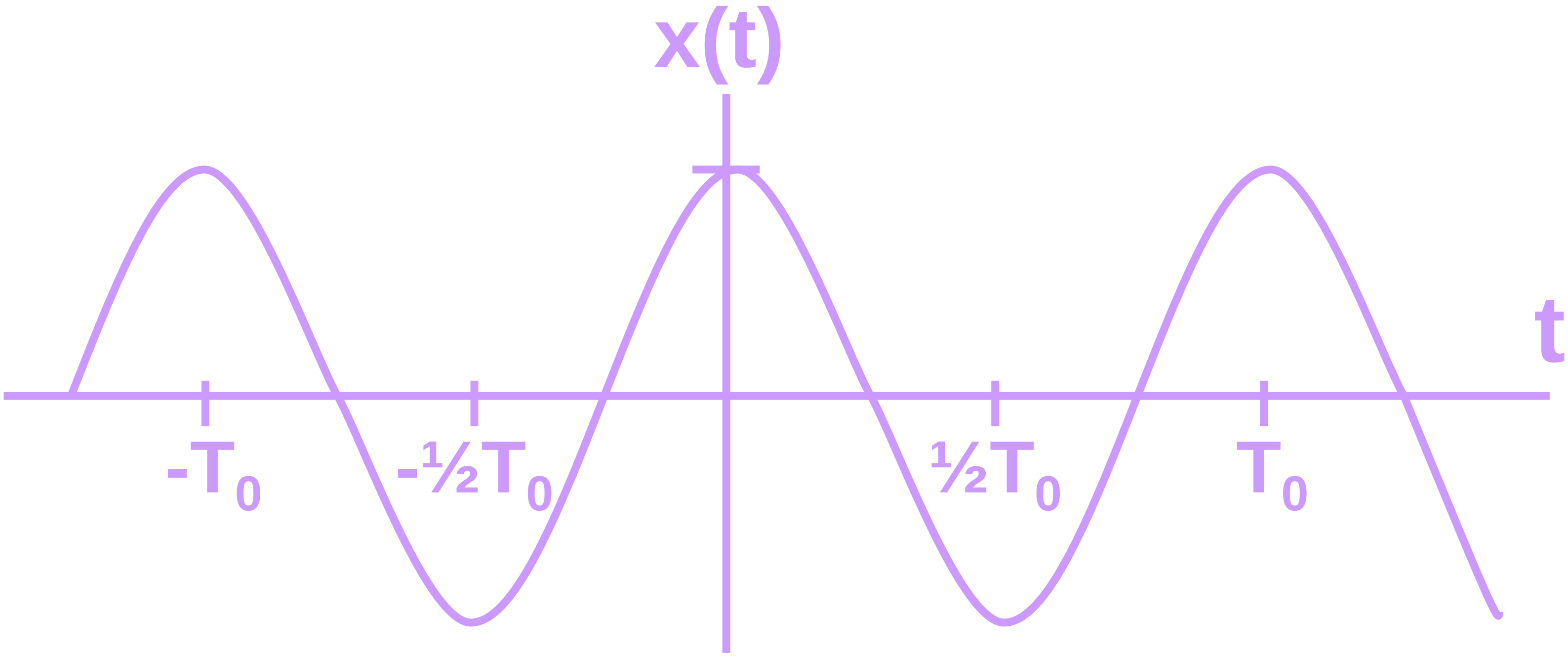


Weekly Dow-Jones industrial average

Digital image

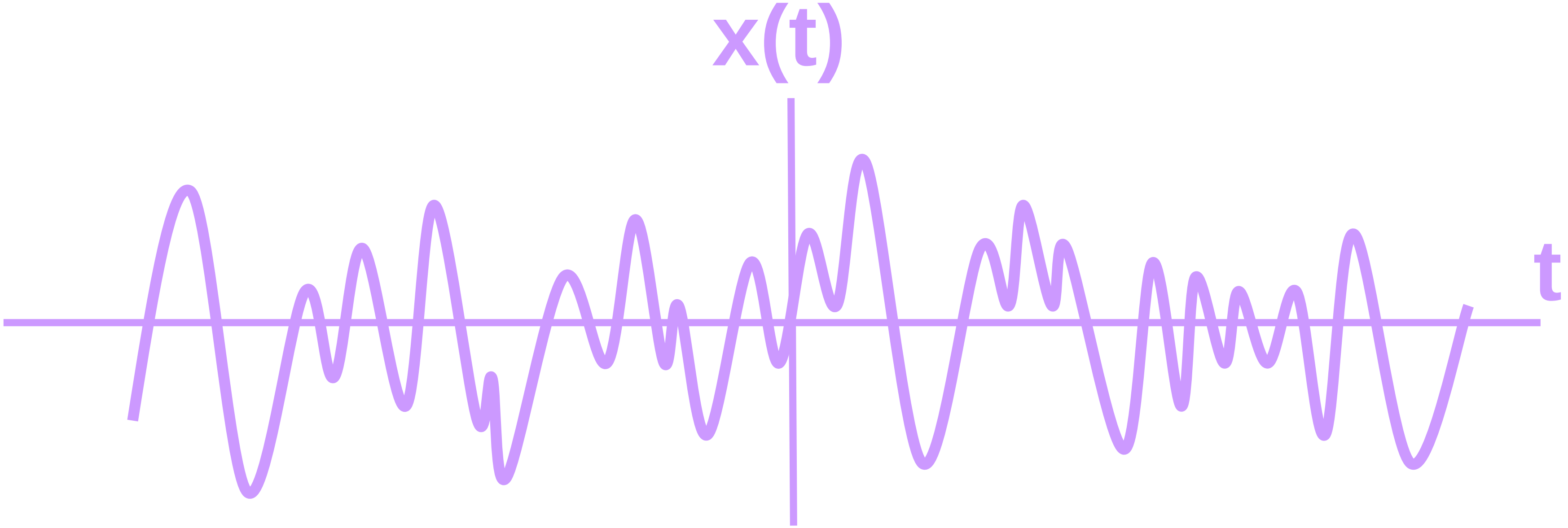
- Why DT? — Can be processed by modern digital computers and digital signal processors (DSPs).

Signal Classification 3: Deterministic Signal



- Each value of the signal is fixed, and can be determined by a mathematical expression, rule, or table.

Signal Classification 3: Random Signal



- Signal value at any time instance is a random variable.

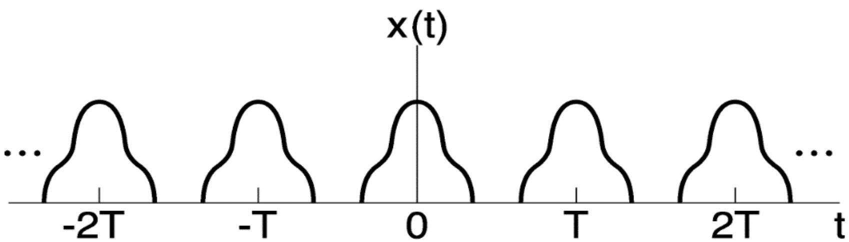
Signal Classification 4: Periodic / Aperiodic

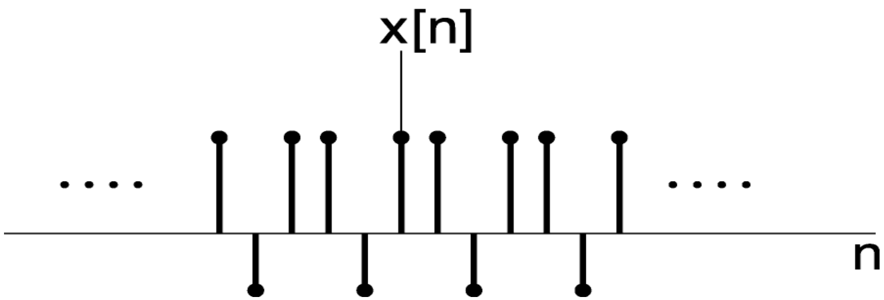
- Periodic Signals

  CT: $x(t) = x(t + T)$, T : *period*

  $x(t) = x(t + mT)$, m: *integer*

  DT: $x[n] = x[n + N]=x[n + mN]$, N: period





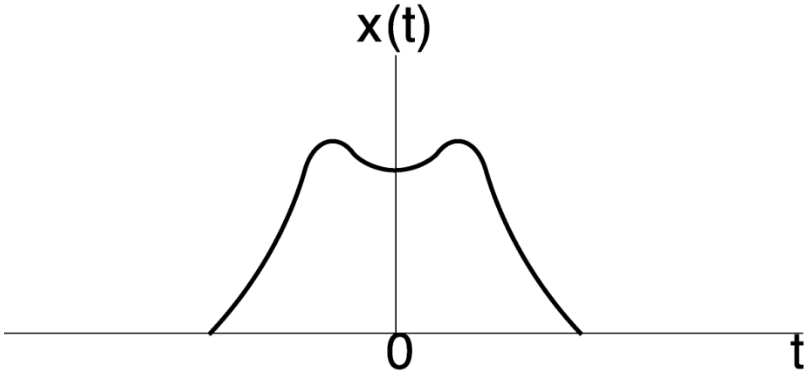
- Fundamental period: the smallest positive period

- Aperiodic: NOT period

Signal Classification 5: Even / Odd

- Even and Odd Signals

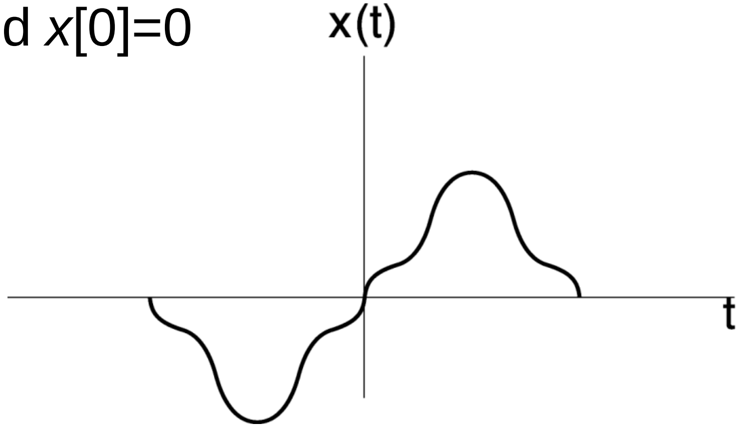
Even $x(t) = x(-t)$ *or* $x[n] = x[-n]$



Example: cos(t)

Odd $x(t) = -\,x(-t)$ *or* $x[n] = -\,x[-n]$

$x(0)=0$, and $x[0]=0$



Example: sin(t)

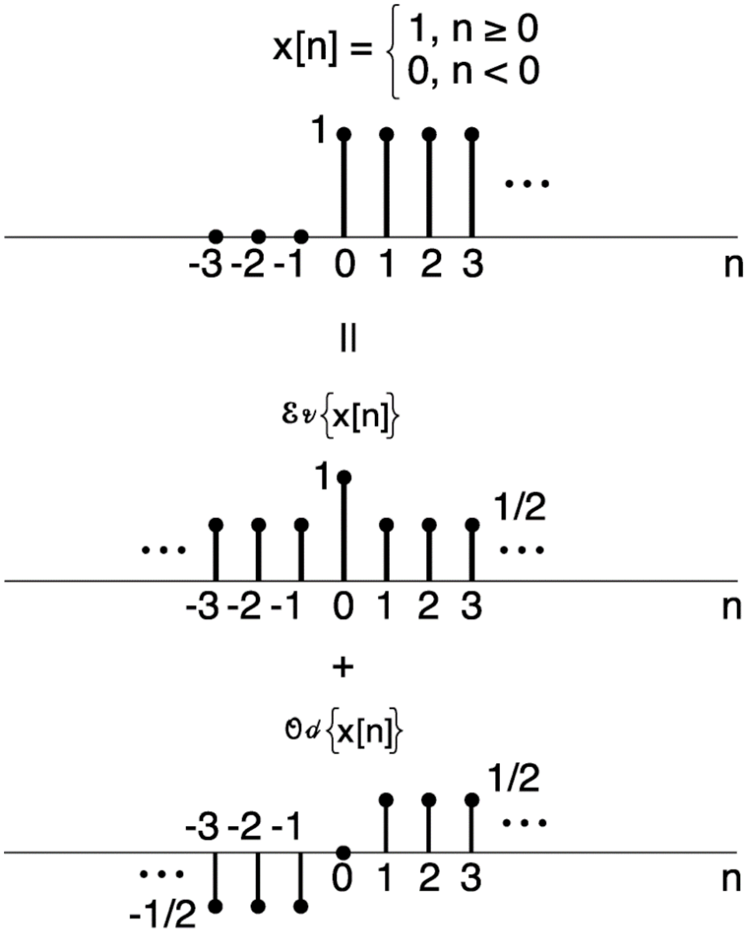
- Any signals can be expressed as a sum of *Even* and *Odd* signals. That is:

$$x(t) = x_{even}(t) + x_{odd}(t),$$

where:

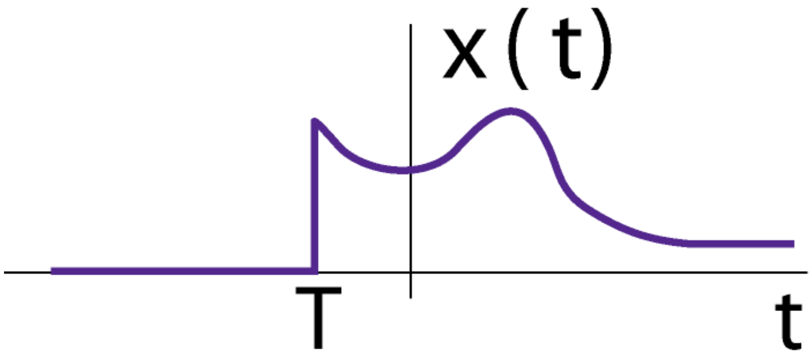
$$x_{even}(t) = [x(t) + x(-t)]/2,$$

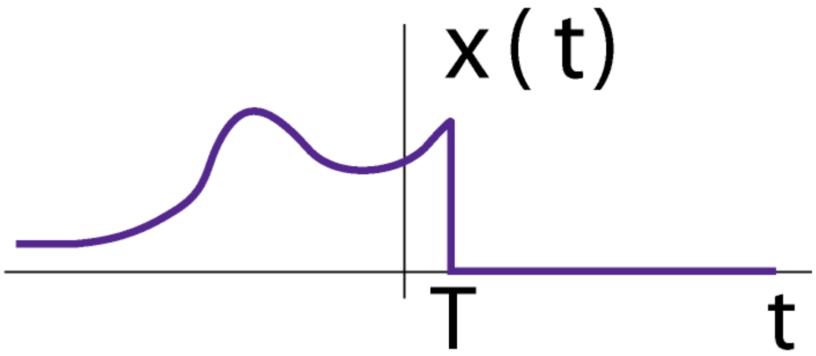
$$x_{odd}(t) = [x(t) - x(-t)]/2.$$



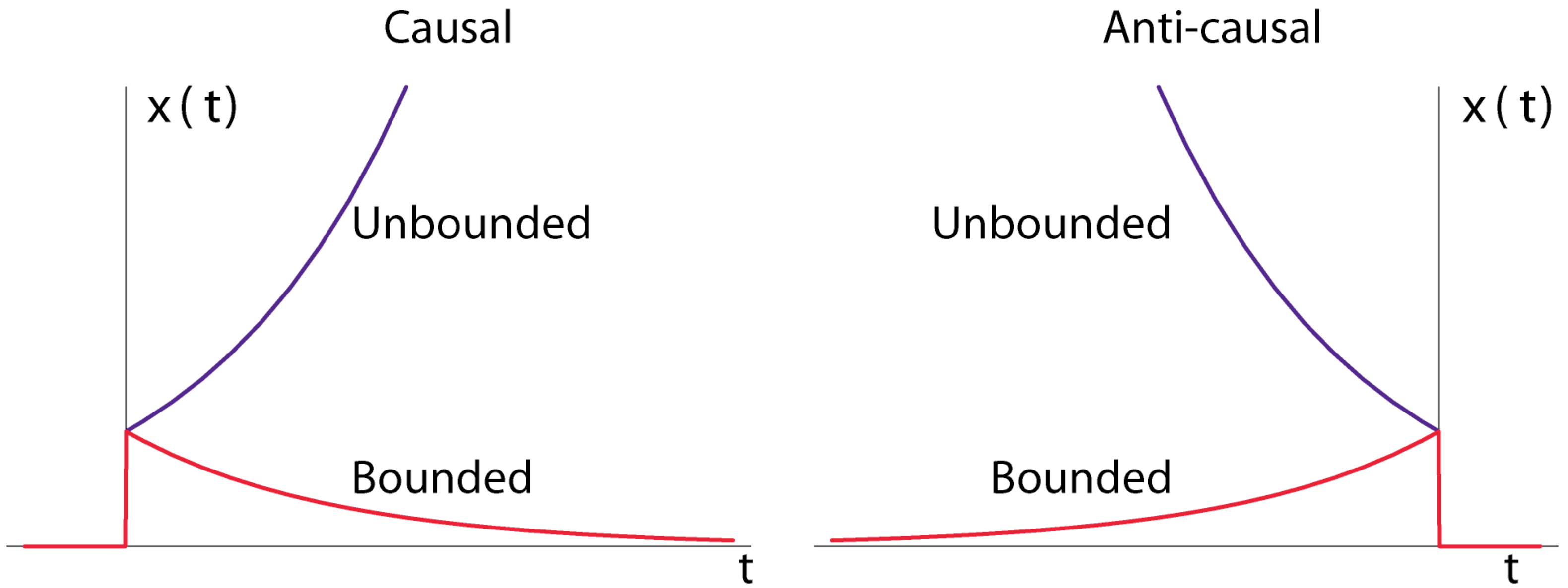
Signal Classification 6: Right- and Left-Sided

- A right-sided signal is zero for $t < T$, and
- A left-sided signal is zero for $t > T$, where T can be positive or negative.





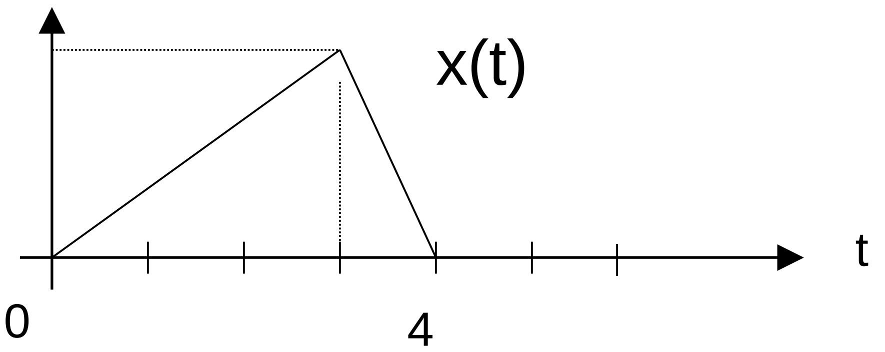
Classification 7: Bounded and Unbounded

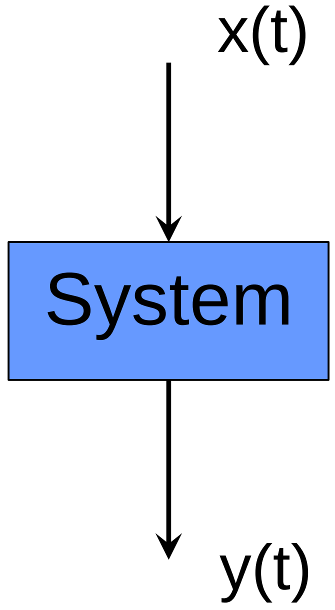


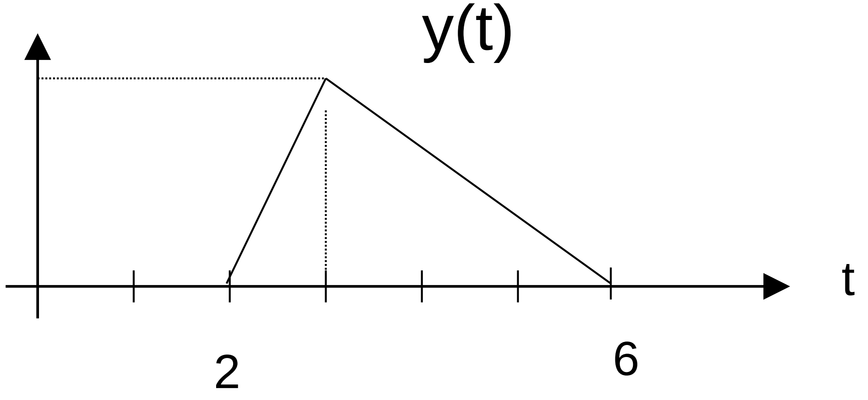
- Bounded signal: the absolute value of signal is bounded.
- Unbounded signal: otherwise

$$\exists C, |x(t)| \leq C \ \forall t$$

Transformation of a Signal







Transformation of a Signal

- Time Shift

$$x(t) \rightarrow x(t - t_0) \quad , \quad x[n] \rightarrow x[n - n_0]$$

- Time Reversal

$$x(t) \rightarrow x(-t) \quad , \quad x[n] \rightarrow x[-n]$$

- Time Scaling

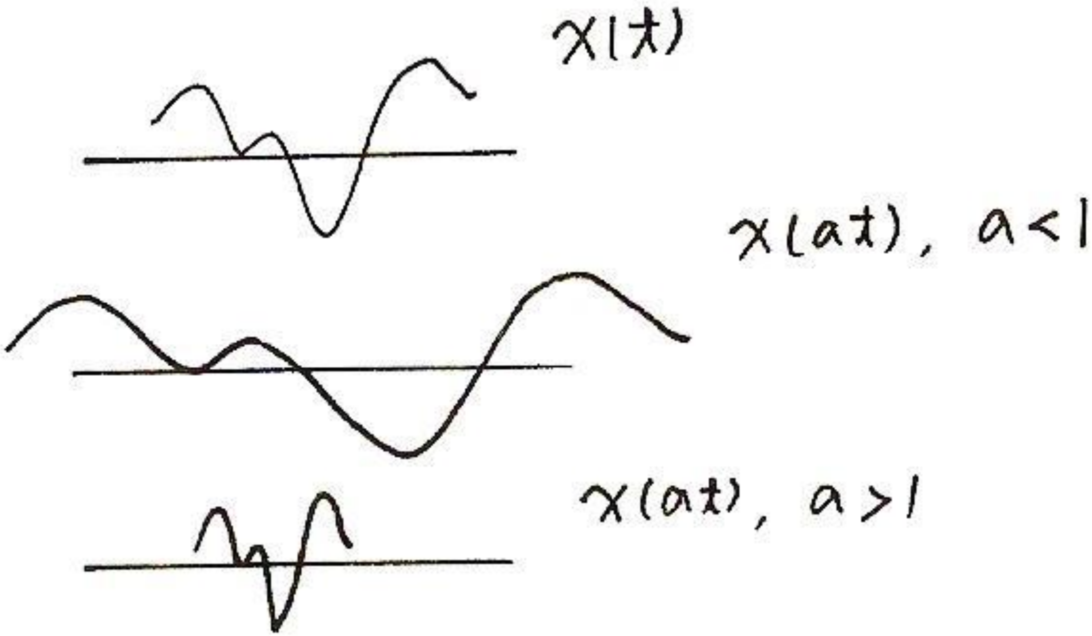
$$x(t) \rightarrow x(at) \quad , \quad x[n] \rightarrow ?$$

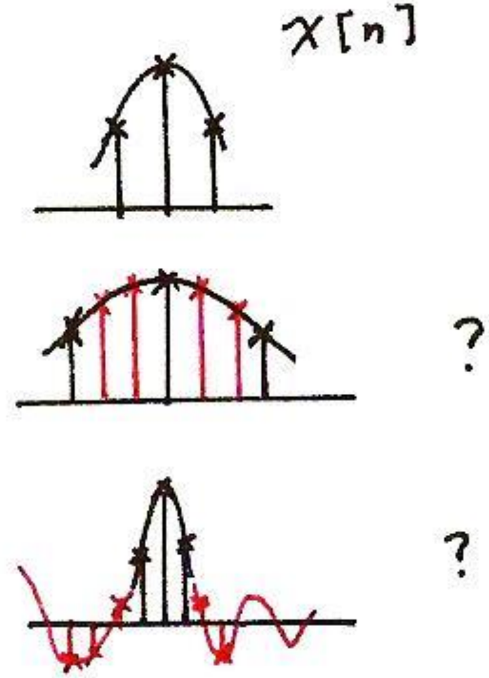
- Combination

$$x(t) \rightarrow x(at + b) \quad , \quad x[n] \rightarrow ?$$

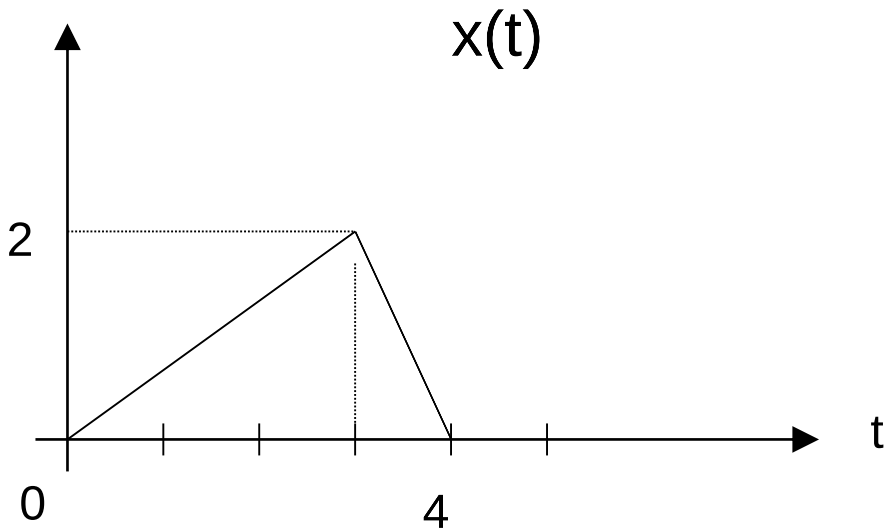
Transformation of a Signal

Time Scaling





Class problem

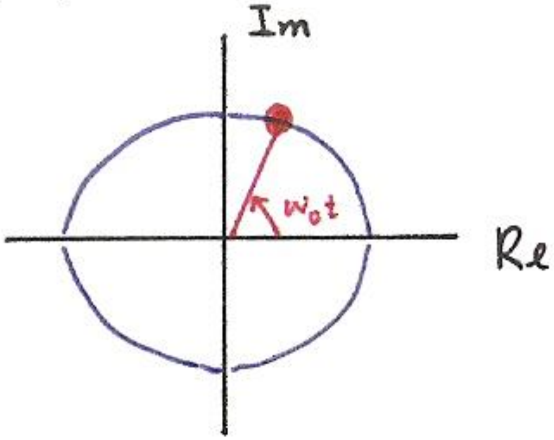


x(-2t+2) ?

Exponential Signals

- A very important class of signals is presented as:
  - CT signals of the form $x(t) = e^{j\omega t}$
  - DT signals of the form $x[n] = e^{j\omega n}$
- For both *exponential* CT and DT signals, x is a complex quantity and has:
  - a real and imaginary part [i.e., *Cartesian form*], or equivalently
  - a magnitude and a phase angle [i.e., *polar form*].
- We will use whichever form that is convenient.

$$x(t) = e^{j\omega_0 t}$$

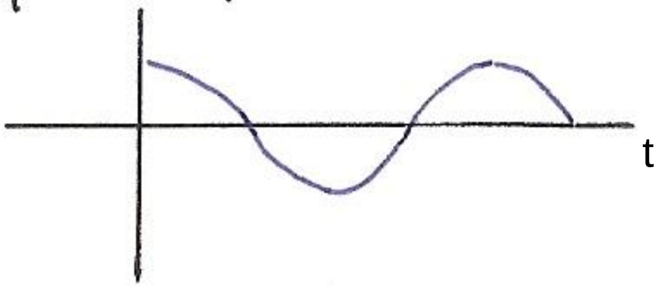


Euler's relation

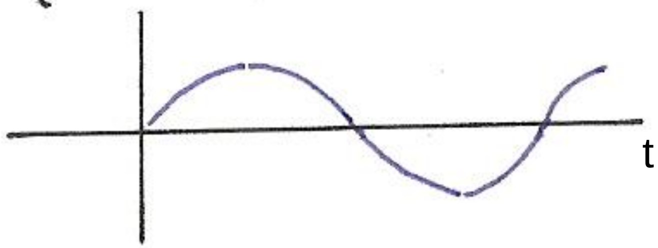
$$e^{jx} = \cos x + j \sin x$$

$\omega_0 t$ is defined as phase

$$Re\{e^{j\omega_0 t}\} = \cos\omega_0 t$$



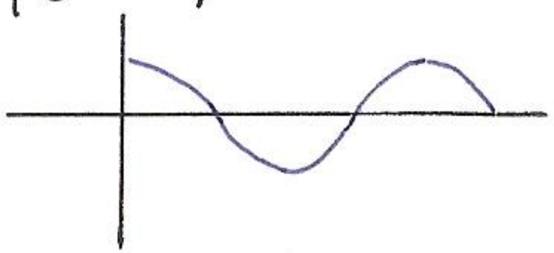
$$Im\{e^{j\omega_0 t}\} = \sin\omega_0 t$$



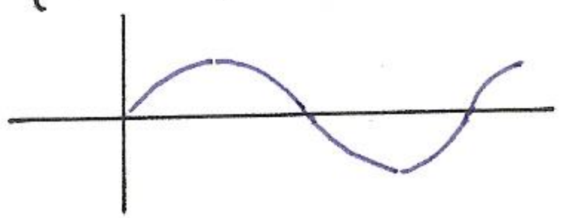
Real and imaginary parts are periodic signals with the same period, but out of phase (90^o phase difference)

$$x(t) = e^{j\omega_0 t} = \cos(\omega_0 t) + j\sin(\omega_0 t)$$

$Re\{e^{j\omega_0 t}\} = \cos\omega_0 t$



$Im\{e^{j\omega_0 t}\} = \sin\omega_0 t$



-Fundamental (angular) frequency: ω_0

-Fundamental period: $T_0 = \dfrac{2\pi}{\omega_0}$

-In CT, $e^{j\omega_0 t}$ **always periodic**

-larger ω_0 **=> higher frequency**

$$x[n] = e^{j\omega_0 n} = cos\omega_0 n + \mathrm{j}\ \sin\omega_0 n$$

Is it periodic?

Larger ω_0 => higher frequency?

$$e^{j\pi n} = (e^{j\pi})^n = (-1)^n$$

$$e^{j2\pi n} = (e^{j2\pi})^n = (1)^n = 1$$

Periodicity Properties of DT Complex Exponentials

Important difference between $e^{j\omega_0 n}$ and $e^{j\omega_0 t}$:

- $e^{j\omega_0 n}$ is periodic w.r.t. ω_0

$$\boxed{e^{j(\omega_0+m\cdot 2\pi)n} = e^{j\omega_0 n}\cdot e^{jm\cdot 2\pi n} = e^{j\omega_0 n}}$$

- However, $e^{j\omega_0 t}$ *is aperiodic w.r.t.* ω_0

$$\boxed{\forall x \neq 0, e^{j(\omega_0+x)t} = e^{j\omega_0 t}e^{jxt} \neq e^{j\omega_0 t}}$$

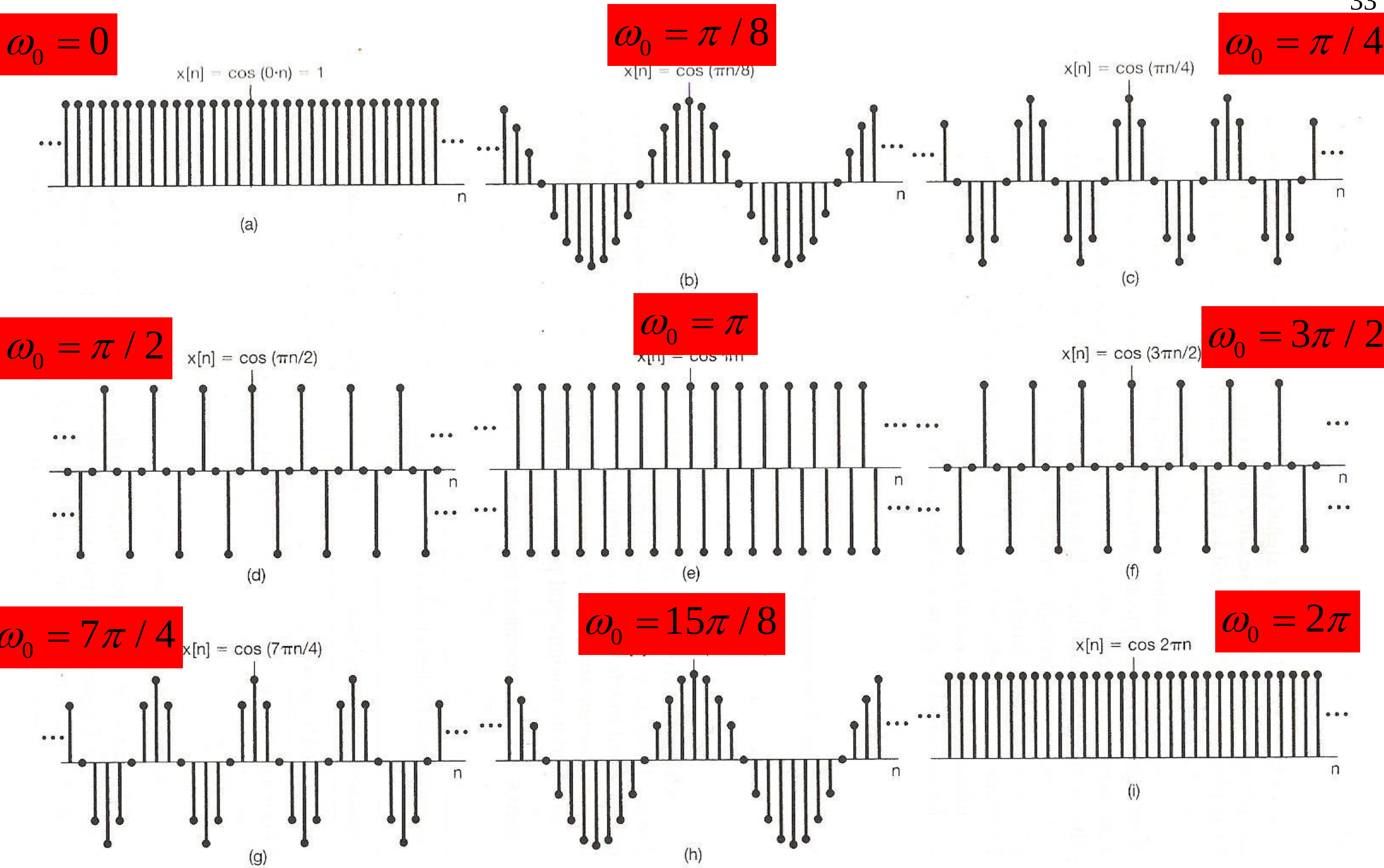


Figure 1.27 Discrete-time sinusoidal sequences for several different frequencies.

- We need only consider a frequency interval of length 2π, and on most cases, we use the interval: $0 \le \omega_0 < 2\pi$, or $-\pi \le \omega_0 < \pi$

- $e^{j\omega_0 n}$ does **not** have a continually increasing rate of oscillation as ω_0 is increased.

  lowest-frequency (slowly varying): ω_0 near 0, 2π, …, or 2k • π

  highest-frequency (rapid variation): ω_0 near ± π, …, or (2k+1) • π

$$e^{j(2k+1)\pi n} = e^{j\pi n} = (e^{j\pi})^n = (-1)^n$$

$$e^{j2\pi n} = (e^{j2\pi})^n = (1)^n = 1$$

Harmonically Related Signal Sets

- A set of periodic exponentials which have a common period.

$$\{\phi_k(t) = e^{jk\omega_0 t}, k = 0, \pm 1, \pm 2, ...\}$$

Fundamental (Angular) Frequency : $|k\omega_0|$

Fundamental Period: $\dfrac{2\pi}{|k\omega_0|}$

Common Period: $\dfrac{2\pi}{|\omega_0|}$

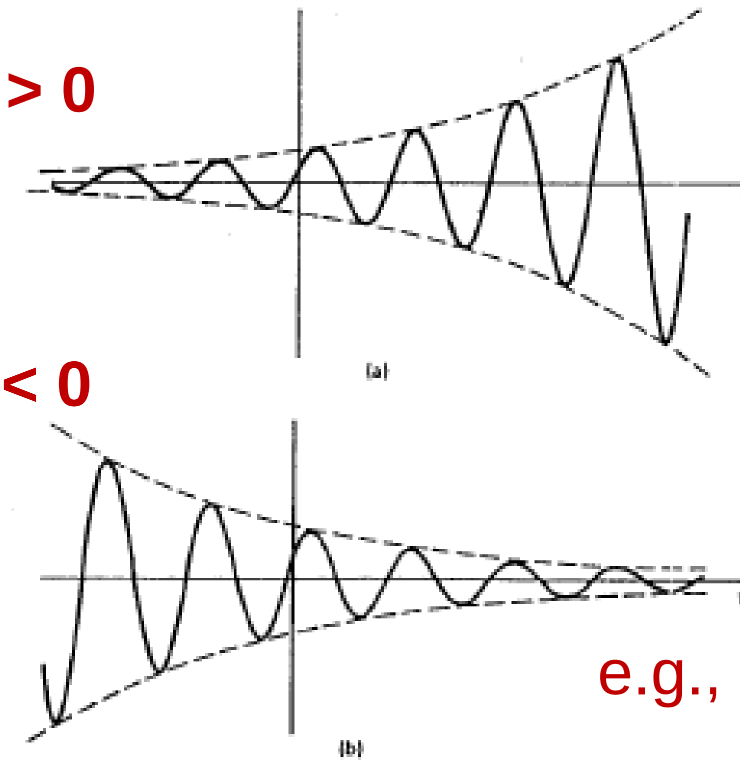
General Complex Exponential Signals - CT

- General format (C and a are complex numbers)

$$x(t) = Ce^{at} = |C| e^{j\theta} \cdot e^{(r+j\omega_0)t} = |C| e^{rt} \cdot e^{j(\omega_0 t+\theta)}$$

$r > 0$

$r < 0$



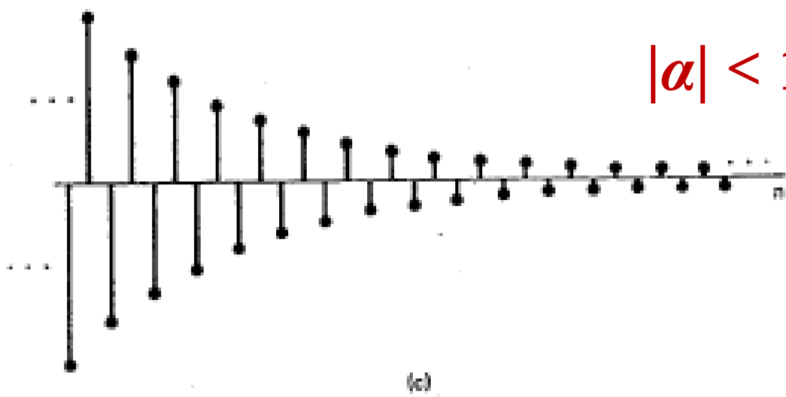
e.g., damped sinusoids

General Complex Exponential Signals - DT

- General format (C and α are complex numbers)

$$x[n] = C\alpha^n = |C|e^{j\vartheta} \cdot |\alpha|^n e^{j\omega_0 n} = |C||\alpha|^n e^{j(\omega_0 n + \vartheta)}$$

$|\alpha| < 1$



$|\alpha| > 1$