1191214 杨鸿嘉.

卷积

unit Impulse Response

$$y[n]=\sum_{k=-\infty}^{+\infty}x[k]\underline{h[n-k]}$$

$$y(t)=\int_{-\infty}^{+\infty}x(\tau)h(t-\tau)d\tau$$

傅里叶级数.

$$x(t)=\sum_{k=-\infty}^{+\infty}a_ke^{jk\omega_0t}=\sum_{k=-\infty}^{+\infty}a_ke^{jk\frac{2\pi}{T}t}$$ synthesis

可能周期

$$a_k=\frac{1}{T}\int_T x(t)e^{-jk\omega_0t}dt=\frac{1}{T}\int_T x(t)e^{-jk\frac{2\pi}{T}t}dt$$ analysis

$$x[n]=\sum_{k=\langle N\rangle}a_ke^{jk\omega_0n}=\sum_{k=\langle N\rangle}a_ke^{jk\frac{2\pi}{N}n}$$

$$a_k=\frac{1}{N}\sum_{n=\langle N\rangle}x[n]e^{-jk\omega_0n}=\frac{1}{N}\sum_{n=\langle N\rangle}x[n]e^{-jk\frac{2\pi}{N}n}$$

均为周期 $a_k=a_{k+N}$

连续傅级数性质.

$x(t)\xleftrightarrow{FS}a_k$ $y(t)\xleftrightarrow{FS}b_k$ $\omega_0=\frac{2\pi}{T}$

线性	$Ax(t)+By(t)$	Aa_k+Bb_k
时移	$x(t-t_0)$	$a_ke^{-jk\omega_0t_0}$
频移	$e^{jM\omega_0t}x(t)$	a_{k-M}
共轭	$x^*(t)$	a^*_{-k}
时间反转	$x(-t)$	a_{-k}
时域尺度变换	$x(\alpha t)$, $\alpha>0$, $T=\frac{T}{\alpha}$	a_k
周期卷积	$\int_T x(\tau)y(t-\tau)d\tau$	Ta_kb_k
相乘	$x(t)y(t)$	$\sum_{l=-\infty}^{+\infty}a_lb_{k-l}$
微分	$\frac{dx(t)}{dt}$	$jk\omega_0a_k$
积分	$\int_{-\infty}^{t}x(t)dt$	$(\frac{1}{jk\omega_0})a_k$

仅当 $a_0=0$ 才为有限值且周期

帕斯瓦尔 $$\frac{1}{T}\int_T|x(t)|^2dt=\sum_{k=-\infty}^{+\infty}|a_k|^2$$

离散傅级数性质

$x[n]\xleftrightarrow{FS}a_k$ → 周期也为N ← $y[n]\xleftrightarrow{FS}b_k$. $\omega_0=\frac{2\pi}{N}$.

线性	$Ax[n]+By[n]$	Aa_k+Bb_k
时移	$x[n-n_0]$	$a_ke^{-jk\omega_0n_0}$
频移	$e^{jM\omega_0n}x[n]$	a_{k-M}
共轭	$x^*[n]$	a^*_{-k}
时间反转	$x[-n]$	a_{-k}
时域尺度变换	$x[\frac{n}{m}]$ n是m倍数	$\frac{1}{m}a_k$ 周期mN.
周期卷积	$\sum_{r=\langle N\rangle}x[r]y[n-r]$	Na_kb_k
相乘	$x[n]y[n]$	$\sum_{l=\langle N\rangle}a_lb_{k-l}$
一阶差分	$x[n]-x[n-1]$	$(1-e^{-jk\omega_0})a_k$
求和	$\sum_{k=-\infty}^{n}x[k]$	$(\frac{1}{1-e^{-jk\omega_0}})a_k$

仅当 $a_0=0$ 才有限且周期

$$\frac{1}{N}\sum_{n=\langle N\rangle}|x[n]|^2=\sum_{k=\langle N\rangle}|a_k|^2$$

实信号共轭对称性.

x(t)实信号 $\begin{cases}a_k=a^*_{-k}\\ Re\{a_k\}=Re\{a_{-k}\}\\ Im\{a_k\}=-Im\{a_{-k}\}\\ |a_k|=|a_{-k}|\\ \measuredangle a_k=-\measuredangle a_{-k}\end{cases}$

x(t)实偶 a_k 实偶

x(t) 实奇 a_k 纯虚奇

x(t)实信号 $Ev\{x(t)\}\xleftrightarrow{FS}Re\{a_k\}$

$Od\{x(t)\}\xleftrightarrow{FS}jIm\{a_k\}$

$$H(j\omega)=\int_{-\infty}^{+\infty}h(t)e^{-j\omega t}dt$$ ← 频率响应. frequency response

$$x(t)=\sum_{k=-\infty}^{+\infty}a_ke^{jk\omega_0t}$$

$$y(t)=\sum_{k=-\infty}^{+\infty}a_kH(jk\omega_0)e^{jk\omega_0t}$$

$$H(e^{j\omega})=\sum_{n=-\infty}^{+\infty}h[n]e^{-j\omega n}$$

$$x[n]=\sum_{k=\langle N\rangle}a_ke^{jk\frac{2\pi}{N}n}$$

$$y[n]=\sum_{k=\langle N\rangle}a_kH(e^{j2\pi\frac{k}{N}})e^{jk\frac{2\pi}{N}n}$$

傅里叶变换

$$x(t)=\frac{1}{2\pi}\int_{-\infty}^{+\infty}X(j\omega)e^{j\omega t}d\omega$$

$$X(j\omega)=\int_{-\infty}^{+\infty}x(t)e^{-j\omega t}dt$$

周期信号

$$X(j\omega)=\sum_{k=-\infty}^{+\infty}2\pi a_k\delta(\omega-k\omega_0)$$

傅里叶变换性质

$x(t)\xleftrightarrow{F}X(j\omega)$ $y(t)\xleftrightarrow{F}Y(j\omega)$

线性	$ax(t)+by(t)$	$aX(j\omega)+bY(j\omega)$
时移	$x(t-t_0)$	$e^{-j\omega t_0}X(j\omega)$
频移	$e^{j\omega_0t}x(t)$	$X(j(\omega-\omega_0))$
共轭	$x^*(t)$	$X^*(-j\omega)$
时间反转	$x(-t)$	$X(-j\omega)$
时间与频率尺度变换	$x(at)$	$\frac{1}{\lvert a\rvert}X(\frac{j\omega}{a})$
卷积	$x(t)*y(t)$	$X(j\omega)Y(j\omega)$
相乘	$x(t)y(t)$	$\frac{1}{2\pi}\int_{-\infty}^{+\infty}X(j\theta)Y(j(\omega-\theta))d\theta$
时域微分	$\frac{d}{dt}x(t)$	$j\omega X(j\omega)$
积分	$\int_{-\infty}^{t}x(t)dt$	$\frac{1}{j\omega}X(j\omega)+\pi X(0)\delta(\omega)$
频域微分	$tx(t)$	$j\frac{d}{d\omega}X(j\omega)$

帕斯瓦尔 $$\int_{-\infty}^{+\infty}|x(t)|^2dt=\frac{1}{2\pi}\int_{-\infty}^{+\infty}|X(j\omega)|^2d\omega$$

energy-density spectrum

实信号共轭对称性(傅变)

x(t)实信号 $\begin{cases}X(j\omega)=X^*(-j\omega)\\ Re\{X(j\omega)\} 偶\\ Im\{X(j\omega)\} 奇\\ |X(j\omega)| 偶\\ \measuredangle X(j\omega) 奇.\end{cases}$

x(t)实偶 → X(jω)实偶

x(t) 实奇 → X(jω) 纯虚奇.

$Ev\{x(t)\}\xleftrightarrow{F}Re\{X(j\omega)\}$

$Od\{x(t)\}\xleftrightarrow{F}jIm\{X(j\omega)\}$

基本傅变换对

信号	傅变换	傅系数(若为周期的)
$\sum_{k=-\infty}^{+\infty}a_ke^{jk\omega_0t}$	$2\pi\sum_{k=-\infty}^{+\infty}a_k\delta(\omega-k\omega_0)$	a_k
$e^{jk\omega_0t}$	$2\pi\delta(\omega-k\omega_0)$	$a_1=1$ $a_k=0(k\neq1)$
$\cos\omega_0t$	$\pi[\delta(\omega-\omega_0)+\delta(\omega+\omega_0)]$	$a_1=a_{-1}=\frac{1}{2}$ $a_k=0$(其余)
$\sin\omega_0t$	$\frac{\pi}{j}[\delta(\omega-\omega_0)-\delta(\omega+\omega_0)]$	$a_1=-a_{-1}=\frac{1}{2}$ $a_k=0$
$x(t)=1$	$2\pi\delta(\omega)$	$a_0=1$ $a_k=0$ (对任意T>0)
$\sum_{n=-\infty}^{+\infty}\delta(t-nT)$	$\frac{2\pi}{T}\sum_{k=-\infty}^{+\infty}\delta(\omega-\frac{2\pi k}{T})$	$a_k=\frac{1}{T}$ 所有k.
$x(t)=\begin{cases}1 & \lvert t\rvert<T_1\\ 0 & T_1<\lvert t\rvert\le\frac{T}{2}\end{cases}$ $x(t+T)=x(t)$	$\sum_{k=-\infty}^{+\infty}\frac{2\sin k\omega_0T_1}{k}\delta(\omega-k\omega_0)$	$a_k=\frac{\omega_0T_1}{\pi}sinc(\frac{k\omega_0T_1}{\pi})=\frac{(\sin k\omega_0T_1)}{k\pi}$ $(sinc(\theta)=\frac{\sin\pi\theta}{\pi\theta})$
$x(t)=\begin{cases}1 & \lvert t\rvert<T_1\\ 0 & \lvert t\rvert>T_1\end{cases}$ $u(t+T_1)-u(t-T_1)$	$\frac{2\sin\omega T_1}{\omega}$	—
$\frac{\sin Wt}{\pi t}$	$X(j\omega)=\begin{cases}1 & \lvert\omega\rvert<W\\ 0 & \lvert\omega\rvert>W\end{cases}$ $u(\omega+W)-u(\omega-W)$	—

$\delta(t)$	1	—
$u(t)$	$\frac{1}{j\omega}+\pi\delta(\omega)$	—
$\delta(t-t_0)$	$e^{-j\omega t_0}$	—
$e^{-at}u(t)$, $Re\{a\}>0$	$\frac{1}{a+j\omega}$	—
$te^{-at}u(t)$, $Re\{a\}>0$	$\frac{1}{(a+j\omega)^2}$	—
$\frac{t^{n-1}}{(n-1)!}e^{-at}u(t)$. $Re\{a\}>0$	$\frac{1}{(a+j\omega)^n}$	—

infinite 无限的

waveform 波形

computation 计算

even 偶

odd 奇

$x(at+b)$ 先由b再由a.

$e^{j\omega_0t}$	$e^{j\omega_0n}$
均周期	仅 $\frac{\omega_0}{2\pi}$ 有理才周期
ω_0不同 信号不同	ω_0差2π 相同
基波频率ω_0	基波频率$\frac{\omega_0}{m}$
$T=\begin{cases}\omega_0=0, 无定义\\ \omega_0\neq0, \frac{2\pi}{\omega_0}\end{cases}$	$N=\begin{cases}\omega_0=0, 无定义\\ \omega_0\neq0, m\frac{2\pi}{\omega_0}\end{cases}$

成谐波关系、

$\phi_k(t)=e^{jk\omega_0t}$

$\phi_k[n]=e^{jk\frac{2\pi}{N}n}$. N个

Memoryless / with memory

Invertability invertible

causality stability

Time invariance linearity

$$u(t)=\int_{-\infty}^{t}\delta(\tau)d\tau=\int_{\infty}^{0}\delta(t-\sigma)d\sigma=\int_0^{\infty}\delta(t-\sigma)d\sigma$$

$$\delta(t)=\frac{du(t)}{dt}$$

$$\delta[n]=u[n]-u[n-1]$$

$$u[n]=\sum_{m=-\infty}^{n}\delta[m]=\sum_{k=-\infty}^{n}\delta[n-k]=\sum_{k=0}^{n}\delta[n-k]$$

记忆：仅取决于该时刻的输入. 因果：取决于现在及过去的输入.

时不变性：$y(t)=x(\frac{t}{3})$. $x_2(t)=x_1(t-t_0)$

$y_2(t)=x_2(\frac{t}{3})=x_1(\frac{t}{3}-t_0)\neq$

~~$y_1(t)=x_1$~~ $y_1(t-t_0)=x_1(\frac{t-t_0}{3})$

线性：$y(ax_1(t)+bx_2(t))\overset{?}{=}ay_1(t)+by_2(t)$

卷积性质. 记忆: $t\neq0$ $h(t)=0$. $y(t)=h(0)x(t)$

对 $n\neq0$ $h[n]=0$ $y[n]=h[0]x[n]$

因果： $k>n$. $h[n-k]=0$ ∴ $n<0$, $h[n]=0$

$$y[n]=\sum_{k=0}^{\infty}h[k]x[n-k]$$

$t<0$. $h(t)=0$. $$y(t)=\int_{-\infty}^{t}x(\tau)h(t-\tau)d\tau=\int_0^{\infty}h(\tau)x(t-\tau)d\tau.$$

稳定. $\sum_{k=-\infty}^{\infty}|h[k]|<\infty$ $\int_{-\infty}^{+\infty}|h(\tau)|d\tau<\infty$

可逆 $h_1[n]*h_2[n]=\delta[n]$

信号能量 $E_\infty=\lim_{T\to\infty}\int_{-T}^{T}|x(t)|^2dt$. $E_\infty=\sum_{n=-\infty}^{\infty}|x[n]|^2$

平均功率 $P_\infty=\lim_{T\to\infty}\frac{1}{2T}\int_{-T}^{T}|x(t)|^2dt$ $P_\infty=\lim_{N\to\infty}\frac{1}{2N+1}\sum_{n=-N}^{N}|x[n]|^2$

卷积性质 $$\frac{d}{dt}[x(t)*h(t)]=x(t)*\frac{d}{dt}h(t)=\frac{d}{dt}x(t)*h(t)$$

$$\int_{-\infty}^{t}[x(\tau)*h(\tau)]d\tau=x(t)*\int_{-\infty}^{t}h(\tau)d\tau=[\int_{-\infty}^{t}x(\tau)d\tau]*h(t)$$

若 $x(t)*h(t)=y(t)$ 则 $x(t-t_1)*h(t-t_2)=y(t-t_1-t_2)$

傅变换频率响应函数 $y(t)=x(t)*h(t)\xleftrightarrow{F}Y(j\omega)=\boxed{H(j\omega)}X(j\omega)$

$$\sum_{k=0}^{N}a_k\frac{d^ky(t)}{dt^k}=\sum_{k=0}^{M}b_k\frac{d^kx(t)}{dt^k}\Rightarrow H(j\omega)=\frac{Y(j\omega)}{X(j\omega)}=\frac{\sum_{k=0}^{M}b_k(j\omega)^k}{\sum_{k=0}^{N}a_k(j\omega)^k}$$

$x(t)=e^{st}\to y(t)=H(s)e^{st}$

eigenvalue / system function

eigenfunction.

$$\boxed{x(t)=x(t)*\delta(t)}$$

$$\delta(t)=\delta(t)*\delta(t)$$

$$\boxed{\int_{-\infty}^{+\infty}\delta(\tau)d\tau=1}$$

任意信号g(t). $g(0)=\int_{-\infty}^{+\infty}g(\tau)\delta(\tau)d\tau$

$f(t)\delta(t)=f(0)\delta(t)$

$x(t)*\delta(t-t_0)=x(t-t_0)$

$a_0=\frac{2\pi}{T}$

$$\sum_{n=0}^{N-1}\alpha^n = \begin{cases} N & \alpha=1 \\ \frac{1-\alpha^N}{1-\alpha} & \alpha\neq 1 \end{cases} \qquad \sum_{n=0}^{\infty} n\alpha^n = \frac{\alpha}{(1-\alpha)^2} \quad |\alpha|<1$$

$$\sum_{n=0}^{\infty}\alpha^n = \frac{1}{1-\alpha}, \quad |\alpha|<1 \qquad \sum_{n=k}^{\infty}\alpha^n = \frac{\alpha^k}{1-\alpha}, \quad |\alpha|<1$$

$e^{j\omega t} = \cos\omega t + j\sin\omega t$

$\sin\omega t = \frac{1}{2j}(e^{j\omega t} - e^{-j\omega t})$

$\cos\omega t = \frac{1}{2}(e^{j\omega t} + e^{-j\omega t})$

$\sin(at+b) = \frac{1}{2j}(e^{jat}e^{jb} - e^{-jat}e^{-jb})$

$\sin(at-b) = \frac{1}{2j}(e^{jat}e^{-jb} - e^{-jat}e^{jb})$

$\cos(at+b) = \frac{1}{2}(e^{jat}e^{jb} + e^{-jat}e^{-jb})$

$\cos(at-b) = \frac{1}{2}(e^{jat}e^{-jb} + e^{-jat}e^{jb})$

$x(t) \xleftrightarrow{F} X(j\omega)$

$\Downarrow$

$X(jt) \xleftrightarrow{F} 2\pi x(-\omega)$

积化和差

$\sin a\sin b = \frac{1}{2}[\sin(a+b) + \sin(a-b)]$

$\cos a \sin b = \frac{1}{2}[\sin(a+b) - \sin(a-b)]$

$\cos a\cdot\cos b = \frac{1}{2}[\cos(a+b) + \cos(a-b)]$

$\sin a\cdot\sin b = -\frac{1}{2}[\cos(a+b) + \cos(a-b)]$

和差化积

$\sin x + \sin y = 2\sin\frac{x+y}{2}\cos\frac{x-y}{2}$

$\sin x - \sin y = 2\cos\frac{x+y}{2}\sin\frac{x-y}{2}$

$\cos x + \cos y = 2\cos\frac{x+y}{2}\cos\frac{x-y}{2}$

$\cos x - \cos y = -2\sin\frac{x+y}{2}\sin\frac{x-y}{2}$

$\sin(a+b) = SaCb + CaSb$

$\cos(a+b) = CaCb - SaSb$

$\sum_{n=-2}^{7} e^{\frac{j\pi n}{2}} = e^{-\frac{j2\pi}{2}}\sum_{n=0}^{9} e^{\frac{j\pi n}{2}} = -(1+j)$

$\sum_{n=0}^{\infty}(\frac{1}{2})^n\cos(\frac{\pi}{2}n) = \frac{1}{2}\sum_{n=0}^{\infty}(\frac{1}{2})^n e^{\frac{j\pi n}{2}} + \frac{1}{2}\sum_{n=0}^{\infty}(\frac{1}{2})^n e^{-\frac{j\pi n}{2}}$

分部积分

$\int f(x)g'(x)dx = f(x)g(x) - \int f'(x)g(x)dx$

$z = x + jy$

$\parallel$

$z = re^{j\theta}$

$x = Re\{z\} \quad y = Im\{z\} \qquad e^{j\pi} = -1$

$r = |z| = \sqrt{x^2+y^2}$

$\theta = \measuredangle z = \arctan(\frac{y}{x})$

$z^* = x - jy = re^{-j\theta}$

$z = re^{j\theta} = r\angle\theta \qquad z_1 z_2 = r_1 r_2 \angle\theta_1+\theta_2$

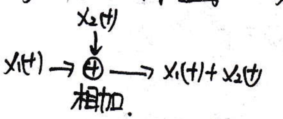
$\frac{z_1}{z_2} = \frac{r_1}{r_2}\angle\theta_1-\theta_2 \qquad \frac{1}{z} = \frac{1}{r}\angle-\theta \qquad \sqrt{z} = \sqrt{r}\angle\frac{\theta}{2}$

实函数奇偶分解

$f(x) = \underset{\underset{\frac{f(x)+f(-x)}{2}}{\parallel}}{Ev\{f(x)\}} + \underset{\underset{\frac{f(x)-f(-x)}{2}}{\parallel}}{Od\{f(x)\}}$

微分差分方程一阶框图

$x(t) \xrightarrow{a} ax(t)$



相加.

$x(t) \longrightarrow \boxed{D} \longrightarrow \frac{dx(t)}{dt}$

微分.

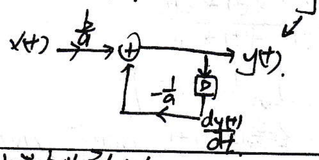
$x(t) \longrightarrow \boxed{S} \longrightarrow \int_{-\infty}^{t} x(\tau)d\tau$

积分.

$\frac{dy(t)}{dt} + ay(t) = bx(t)$

$\Downarrow$

$y(t) = -\frac{1}{a}\frac{dy(t)}{dt} + \frac{b}{a}x(t)$



线性常系数差分方程 $y[n] - \frac{1}{2}y[n-1] = x[n]$

$\downarrow$

$h[n] - \frac{1}{2}h[n-1] = \delta[n]$

由初始松弛 condition of initial rest $h[n] = 0, \ n<0$

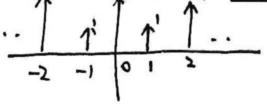
$\therefore h[0] = \frac{1}{2}h[-1] + \delta[0] = 1$

$h[1] = \frac{1}{2}h[0] + \delta[1] = \frac{1}{2} + 0 = \frac{1}{2}$

$h[2] = \frac{1}{2}h[1] + \delta[2] = (\frac{1}{2})^2$

$\vdots$

$\therefore h[n] = (\frac{1}{2})^n u[n]$



利用对偶性求傅变. $g(t) = \frac{2}{1+t^2}$

$\because x(t) = e^{-|t|} \xleftrightarrow{F} X(j\omega) = \frac{2}{1+\omega^2}$

$\therefore e^{-|t|} = \frac{1}{2\pi}\int_{-\infty}^{+\infty}(\frac{2}{1+\omega^2})e^{j\omega t}d\omega$

$\therefore 2\pi e^{-|t|} = \int_{-\infty}^{+\infty}(\frac{2}{1+\omega^2})e^{j\omega t}d\omega$

以 $-t$ 换 t

$\therefore 2\pi e^{-|t|} = \int_{-\infty}^{+\infty}(\frac{2}{1+\omega^2})e^{-j\omega t}d\omega$

$\therefore 2\pi e^{-|\omega|} = \int_{-\infty}^{+\infty}(\frac{2}{1+t^2})e^{-j\omega t}dt$

$\therefore \frac{2}{1+t^2} \xrightarrow{F} 2\pi e^{-|\omega|}$

单位冲激偶 unit doublet.

$\frac{d\delta(t)}{dt} = \delta'(t) = u_1(t)$

$\frac{dx(t)}{dt} = x(t) * u_1(t)$

$\frac{d^2x(t)}{dt^2} = \frac{d}{dt}(\frac{dx(t)}{dt}) = x(t) * u_1(t) * u_1(t)$

$u_2(t) = u_1(t) * u_1(t)$

$u_k(t) = \underbrace{u_1(t) * \cdots u_1(t)}_{k次}$

$\int_{-\infty}^{+\infty} u_1(\tau)d\tau = 0.$

$u_{-k}(t) = u_{-1}(t) * \cdots u_{-1}(t) = \int_{-\infty}^{t} u_{-(k-1)}(\tau)d\tau \quad k>0$

$\int_{-\infty}^{\infty}\delta'(t)dt = 0 \qquad \int_{-\infty}^{t}\delta'(\tau)d\tau = \delta(t)$

$\int_{-\infty}^{\infty} x(t)\delta^n(t)dt = (-1)^n x^n(0)$

$x(t) * \delta^k(t) = x^k(t)$

$x(t) * u(t) = \int_{-\infty}^{t} x(\tau)d\tau$

$u_2(t) = u(t) * u(t) = \int_{-\infty}^{t} u(\tau)d\tau$

$\parallel$

$tu(t)$ 单位斜坡函数. unit ramp function

$u_{-k}(t) = u(t) * \cdots u(t) = \int_{-\infty}^{t} u_{-(k-1)}(\tau)d\tau$

$u_{-k}(t) = \frac{t^{k-1}}{(k-1)!}u(t) \qquad \begin{pmatrix} \delta(t) = u_0(t) \\ u(t) = u_{-1}(t) \end{pmatrix}$

$u(t) * u_1(t) = \delta(t)$

$u_{-1}(t) * u_1(t) = u_0(t)$

$u_k(t) * u_r(t) = u_{k+r}(t)$

解线性常微分方程 $\frac{dy(t)}{dt} + 2y(t) = x(t)$ ① 输入 $x(t) = Ke^{3t}u(t)$

$y(t) = y_p(t) + y_h(t)$

$\swarrow \qquad \searrow$

$\frac{dy(t)}{dt} + 2y(t) = x(t)$ ③ $\qquad \frac{dy(t)}{dt} + 2y(t) = 0.$ ④

$y_p(t) = Ye^{3t}$ ②

①②→③ $\Rightarrow Y = \frac{K}{5} \quad \therefore y_p(t) = \frac{K}{5}e^{3t} \quad t>0$

设 $y_h(t) = Ae^{st}$ 代入④. $s = -2$

$\therefore y(t) = Ae^{-2t} + \frac{K}{5}e^{3t} \quad t>0.$

初始松弛. $t<0. \ x(t) = 0 \quad \therefore t<0 \quad y(t) = 0$

以 $y(0) = 0$ 代入 $\quad A = -\frac{K}{5}$

$\therefore y(t) = \begin{cases} \frac{K}{5}[e^{3t} - e^{-2t}] & t>0 \\ 0 & t<0 \end{cases}$

$(\delta[n] + \delta[n-1]) * (\delta[n] + \delta[n-1])$

$= \delta[n] + 2\delta[n-1] + \delta[n-2]$

$\int_{-\infty}^{\infty}\omega X(j\omega)d\omega = 0$

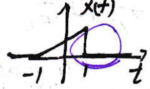
$\Downarrow$

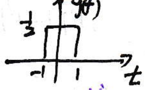
$\frac{dx(t)}{dt} = \frac{1}{2\pi}\int_{-\infty}^{+\infty} j\omega X(j\omega)e^{j\omega t}d\omega$

$\therefore \frac{dx(t)}{dt}\Big|_{t=0} = 0$

$\alpha(t) = \sum_{k=-\infty}^{\infty}[2\delta(t-2k) + \delta(t-1-2k)]$

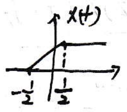
$x(t) = \begin{cases} 0 & |t|>1 \\ \frac{t+1}{2} & -1\leq t\leq 1 \end{cases}$

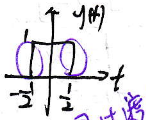




$\alpha(t) = \int_{-\infty}^{t} y(t)dt - u(t-1)$ 阶跃无连线

$x(t) = \begin{cases} 0 & t<-\frac{1}{2} \\ t+\frac{1}{2} & -\frac{1}{2}\leq t\leq\frac{1}{2} \\ 1 & t>\frac{1}{2} \end{cases}$



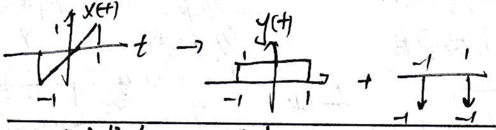


$x(t) = \int_{-\infty}^{t} y(t)dt.$

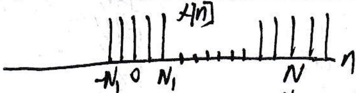
$= \frac{2\sin\frac{\omega}{2}}{j\omega^2} + \pi\delta(\omega).$

平滑过渡有连线

$\frac{1}{(a+j\omega)(b+j\omega)} = \frac{1}{b-a}[\frac{1}{a+j\omega} - \frac{1}{b+j\omega}]$



直流偏移 $-\frac{1}{2}$ 傅系数 $a_k = \begin{cases} 0, & k\neq 0 \\ -\frac{1}{2}, & k=0 \end{cases}$



$a_k = \frac{1}{N}\sum_{n=-N_1}^{N_1} e^{-jk\frac{2\pi}{N}\cdot n}$ 令 $m = n+N_1$

$a_k = \frac{1}{N}\sum_{m=0}^{2N_1} e^{-jk\frac{2\pi}{N}(m-N_1)}$

$= \frac{1}{N}e^{jk(\frac{2\pi}{N})N_1}\sum_{m=0}^{2N_1} e^{-jk\frac{2\pi}{N}m}$

$= \frac{1}{N}e^{jk\frac{2\pi}{N}N_1}\left(\frac{1-e^{-jk2\pi(2N_1+1)N}}{1-e^{-jk\frac{2\pi}{N}}}\right)$

$= \frac{1}{N}\frac{\sin(\frac{2\pi k(N_1+\frac{1}{2})}{N})}{\sin(\frac{\pi k}{N})} \qquad k\neq 0. \pm N, \pm 2N.$

$a_k = \frac{2N_1+1}{N} \quad k = 0, \pm N, \pm 2N.$

知 $h(t)$ 求 $H(j\omega)$ 离散. $\sum_{-\infty}^{-1} + \sum_{0}^{\infty}$

连续 $\int_{-\infty}^{0} \quad \int_{0}^{\infty}$

$\sum_{n=-\infty}^{\infty}(-1)^n\delta(t-n) = (\delta(t) - \delta(t-1))_{0\sim 2内}$