L 变

①性质 线性: $\sum a_i f_i(t) \to \sum a_i F_i(s)$

时移: $f(t-t_0) \to e^{-st_0}F(s)$
频移 $e^{s_0 t}f(t) \to F(s-s_0)$ } 单边周期signal: $f(t)=\sum_0^\infty f_1(t-kT)u(t-kT) \Rightarrow F(s)=\sum F_1(s)e^{-kTs}=\frac{F_1(s)}{1-e^{-Ts}}$.

尺度: $f(at) \to \frac{1}{|a|}F(\frac{s}{a})$

conj: $f^*(t) \to F^*(s^*)$ 相乘: $f_1(t)f_2(t) \to \frac{F_1 * F_2}{2\pi j}$

卷积: $f_1 * f_2 \to F_1F_2$ 微分(t): $\frac{df(t)}{dt} \to sF(s)-f(0)$, $\int_0^t f(\tau)d\tau = \frac{F(s)}{s}+\frac{\int_{-\infty}^{0^-} f(\tau)d\tau}{s}$ $f^{(\cdots)}$ 表示…次求导, $f^{(-\cdots)}$ 表示…次积分.

S域微分: $-tf(t) \to \frac{dF(s)}{ds}$ 初/终值: $f(0^+)=\lim_{t\to 0^+}f(t)=\lim_{s\to\infty}sF(s)$, $\lim_{t\to\infty}f(t)=\lim_{s\to 0}sF(s)$.

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②表 $\delta(t) \to 1$

$\delta(t-t_0) \to e^{-st_0}$ $\delta^{(n)}t \to s^n$

$u(t) \to \frac{1}{s} \Leftrightarrow -u(-t)$ $u(t)-u(t-T) \to \frac{1-e^{-sT}}{s}$. 部分分式分解: 挨个代入回检验(逐一检验!)

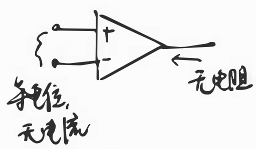
$\frac{t^n u(t)}{n!}=u^{(-n)}(t) \to \frac{1}{s^{n+1}} \Leftrightarrow -u^{(-n)}(-t)$. $Q=\omega_0 L/R=\frac{1}{\omega_0 RC}$, $BW=R/L$. 标度放缩:

以下均为单边.
$\left\{\begin{array}{l}A=K_mA_0\\ \omega=K_f\omega_0\end{array}\right. \Rightarrow$ $R'=K_mR_0$, $L'=\frac{K_m}{K_f}L_0$, $C=\frac{1}{K_mK_f}C_0$

e^{-at} ~~(t≥0)~~ $\to \frac{1}{s+a}$ $\frac{t^n}{n!}e^{at} \to \frac{1}{(s-a)^{n+1}}$ 双边指数 $e^{-b|t|} \to \frac{-2b}{s^2-b^2}$

$\sin\omega t \to \frac{\omega}{s^2+\omega^2}$ $\cos\omega t \to \frac{s}{s^2+\omega^2}$ $e^{-at}\sin\omega t \to \frac{\omega}{(s+a)^2+\omega^2}$ $e^{-at}\cos\omega t \to \frac{s+a}{(s+a)^2+\omega^2}$ 运放: 虚短 $\left\{\begin{array}{l}R_{out}=0\\ u_同=u_反\end{array}\right.$

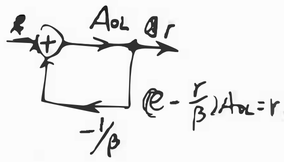
$t\sin\omega t \to \frac{2s\omega}{(s^2+\omega^2)^2}$ $t\cos\omega t \to \frac{s^2-\omega^2}{(s^2+\omega^2)^2}$ $\sinh(at) \to \frac{a}{s^2-a^2}$ $\cosh(at) \to \frac{s}{s^2-a^2}$ 虚断 $i_同=i_反=0$



$i(t)=U(t)/R \to i(s)=U(s)/R$

$u(t)=L\frac{di}{dt} \to U(s)=sLI(s)-LI(0)$, $I(s)=\frac{U(s)}{sL}+\frac{I(0)}{s}$ } 注意放能时源的方向!

$i(t)=C\,du/dt \to U(s)=\frac{1}{sC}i(s)+\frac{u(0^-)}{s}$, $i(s)=sCU(s)-Cu(0^-)$



独立源: 阶跃函数 u/s i/s
+线性

傅里叶 $\omega_k=k\omega_0$, 基波频率 $=k\cdot\frac{2\pi}{T_1}$

系数: $F_i=\frac{\langle f(t),e^{j\omega_k t}\rangle}{\langle e^{j\omega_k t},e^{j\omega_k t}\rangle}$ (正交分解 G-S)

$F_{直流}=\frac{\langle f(t),1\rangle}{\langle 1,1\rangle}=\frac{1}{T}\int_{t_0}^{t_0+T}f(t)dt$

变换: $\mathcal{F}:=\frac{1}{2\pi}\int_{-\infty}^{\infty}f(t)e^{-j\omega t}dt$, $\mathcal{F}^{-1}=\frac{1}{2\pi}\int_{-\infty}^{\infty}F(\omega)e^{j\omega t}d\omega$

性质: ① $\mathcal{F}(\mathcal{F}(f(t)))=2\pi f(-\omega)$ (对称性) ②线性

③ $\mathcal{F}(f(at))=\frac{1}{|a|}F(\frac{\omega}{a})$ ④ $\mathcal{F}(f(t-t_0))=F(\omega)e^{-j\omega t_0}$
$\mathcal{F}(f(t)e^{j\omega_0 t})=F(\omega-\omega_0)$

⑤ $\mathcal{F}(\frac{d^n f(t)}{dt^n})=(j\omega)^nF(\omega)$
$\mathcal{F}(\frac{d^{-1}f(t)}{dn})=\mathcal{F}\int_{-\infty}^{t}f(\tau)d\tau=\frac{F(\omega)}{j\omega}+\pi F(0)\delta(\omega)$

⑥ $f_1(t)*f_2(t) \to F_1(\omega)F_2(\omega)$ / $f_1(t)f_2(t)=\frac{1}{2\pi}F_1(\omega)*F_2(\omega)$

⑦周期信号 $\sum f_0(t)u(t-kT) \to F_n=\frac{1}{T_1}F_0(\omega)|_{\omega=n\omega_1}$, ω_1为基波频率.

$e^{-at}u(t) \to \frac{1}{a+j\omega}$

$e^{-|a|t} \to \frac{2a}{a^2+\omega^2}$

$u(t+\frac{\tau}{2})-u(t-\frac{\tau}{2}) \to \tau\,\mathrm{sinc}\frac{\omega\tau}{2}$ (零点 $2\pi/\omega_c$)

$e^{-(\frac{t}{\tau})^2} \to \sqrt{\pi}\tau e^{-(\frac{\omega\tau}{2})^2}$

$\cos\frac{\pi t}{\tau}(u(t+\frac{\tau}{2})-u(t-\frac{\tau}{2})) \Rightarrow \frac{2\tau}{\pi}\cdot\frac{\cos\frac{\omega\tau}{2}}{1-(\frac{\omega\tau}{\pi})^2}$

$(1+\cos\frac{2\pi t}{\tau})(u(t+\frac{\tau}{2})-u(t-\frac{\tau}{2})) \Rightarrow \frac{\tau}{2}\frac{\mathrm{sinc}\frac{\omega\tau}{2}}{1-(\frac{\omega\tau}{2\pi})^2}$

$(1-2\frac{|t|}{\tau})(u(t+\frac{\tau}{2})-u(t-\frac{\tau}{2})) \to \frac{\tau}{2}\mathrm{sinc}^2\frac{\omega\tau}{4}$

$(\frac{t}{a}+1)(u(t+a)-u(t)) \to \frac{1}{a\omega^2}(1+j\omega a-e^{j\omega a})$

$te^{-at}u(t) \to \frac{1}{(a+j\omega)^2}$

$\cos\omega_0 t\,(u(t+\frac{\tau}{2})-u(t-\frac{\tau}{2}))=\frac{\tau}{2}(\mathrm{sinc}\frac{(\omega+\omega_0)\tau}{2}+\mathrm{sinc}\frac{(\omega-\omega_0)\tau}{2})$

$\delta(t) \to 1$

$u(t) \to \frac{1}{j\omega}+\pi\delta(\omega)$

$1 \to 2\pi\delta$

$\sum\delta(t-nT_1) \to \omega_1\sum\delta(\omega-n\omega_1)$

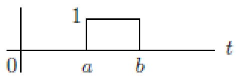
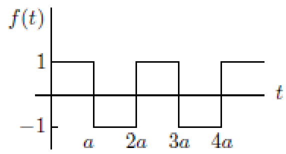
$e^{-at}\cos\omega_0 t\,u(t) \to \frac{a+j\omega}{(a+j\omega)^2+\omega_0^2}$

$$X[k]=\frac{1}{N}\sum_{n=0}^{N-1}x(nT_s)e^{-j\frac{2\pi}{NT_s}knT_s}=\frac{1}{N}\sum_{n=0}^{N-1}x[n]e^{-j\frac{2\pi}{N}kn}$$

Property	Section	Periodic Signal	Fourier Series Coefficients
		$\left.\begin{matrix}x(t)\\y(t)\end{matrix}\right\}$ Periodic with period T and fundamental frequency $\omega_0 = 2\pi/T$	$\begin{matrix}a_k\\b_k\end{matrix}$
Linearity	3.5.1	$Ax(t)+By(t)$	Aa_k+Bb_k
Time Shifting	3.5.2	$x(t-t_0)$	$a_k e^{-jk\omega_0 t_0} = a_k e^{-jk(2\pi/T)t_0}$
Frequency Shifting		$e^{jM\omega_0 t}x(t) = e^{jM(2\pi/T)t}x(t)$	a_{k-M}
Conjugation	3.5.6	$x^*(t)$	a^*_{-k}
Time Reversal	3.5.3	$x(-t)$	a_{-k}
Time Scaling	3.5.4	$x(\alpha t), \alpha>0$ (periodic with period T/α)	a_k
Periodic Convolution		$\int_T x(\tau)y(t-\tau)d\tau$	Ta_kb_k
Multiplication	3.5.5	$x(t)y(t)$	$\sum_{l=-\infty}^{+\infty} a_l b_{k-l}$
Differentiation		$\frac{dx(t)}{dt}$	$jk\omega_0 a_k = jk\frac{2\pi}{T}a_k$
Integration		$\int_{-\infty}^{t} x(t)\,dt$ (finite valued and periodic only if $a_0 = 0$)	$\left(\frac{1}{jk\omega_0}\right)a_k = \left(\frac{1}{jk(2\pi/T)}\right)a_k$
Conjugate Symmetry for Real Signals	3.5.6	$x(t)$ real	$\begin{cases}a_k = a^*_{-k}\\ \Re e\{a_k\} = \Re e\{a_{-k}\}\\ \Im m\{a_k\} = -\Im m\{a_{-k}\}\\ \lvert a_k\rvert = \lvert a_{-k}\rvert\\ \sphericalangle a_k = -\sphericalangle a_{-k}\end{cases}$
Real and Even Signals	3.5.6	$x(t)$ real and even	a_k real and even
Real and Odd Signals	3.5.6	$x(t)$ real and odd	a_k purely imaginary and odd
Even-Odd Decomposition of Real Signals		$\begin{cases}x_e(t) = \mathcal{E}v\{x(t)\} & [x(t) \text{ real}]\\ x_o(t) = \mathcal{O}d\{x(t)\} & [x(t) \text{ real}]\end{cases}$	$\begin{matrix}\Re e\{a_k\}\\ j\Im m\{a_k\}\end{matrix}$

Parseval's Relation for Periodic Signals

$$\frac{1}{T}\int_T |x(t)|^2 dt = \sum_{k=-\infty}^{+\infty} |a_k|^2$$

$L18$	$e^{-at}(1-at)$	$\frac{p}{(p+a)^2}$
$L20$	$\frac{1}{t}\sin at\cos bt,$	$\frac{1}{2}\left(\arctan\frac{a+b}{p}+\arctan\frac{a-b}{p}\right)$
$L21$	$\frac{e^{-at}-e^{-bt}}{t}$	$\ln\frac{p+b}{p+a}$
$L22$	$1-\operatorname{erf}\left(\frac{a}{2\sqrt{t}}\right), \quad a>0$ (See Chapter 11, Section 9)	$\frac{1}{p}e^{-a\sqrt{p}}$
$L23$	$J_0(at)$ (See Chapter 12, Section 12)	$(p^2+a^2)^{-1/2}$
$L24$	$u(t-a) = \begin{cases}1, & t>a>0\\ 0, & t<a\end{cases}$ (unit step, or Heaviside function)	$\frac{1}{p}e^{-pa}$
$L25$	$f(t) = u(t-a)-u(t-b)$ 	$\frac{e^{-ap}-e^{-bp}}{p}$
$L26$	$f(t)$ 	$\frac{1}{p}\tanh\left(\frac{1}{2}ap\right)$

$L5$	$t^k, \ k>-1$	$\frac{k!}{p^{k+1}}$ or $\frac{\Gamma(k+1)}{p^{k+1}}$
$L6$	$t^k e^{-at}, \ k>-1$	$\frac{k!}{(p+a)^{k+1}}$ or $\frac{\Gamma(k+1)}{(p+a)^{k+1}}$
$L7$	$\frac{e^{-at}-e^{-bt}}{b-a}$	$\frac{1}{(p+a)(p+b)}$
$L8$	$\frac{ae^{-at}-be^{-bt}}{a-b}$	$\frac{p}{(p+a)(p+b)}$
$L9$	$\sinh at$	$\frac{a}{p^2-a^2}$
$L10$	$\cosh at$	$\frac{p}{p^2-a^2}$
$L11$	$t\sin at$	$\frac{2ap}{(p^2+a^2)^2}$
$L12$	$t\cos at$	$\frac{p^2-a^2}{(p^2+a^2)^2}$
$L13$	$e^{-at}\sin bt$	$\frac{b}{(p+a)^2+b^2}$
$L14$	$e^{-at}\cos bt$	$\frac{p+a}{(p+a)^2+b^2}$
$L15$	$1-\cos at$	$\frac{a^2}{p(p^2+a^2)}$
$L16$	$at-\sin at$	$\frac{a^3}{p^2(p^2+a^2)}$
$L17$	$\sin at - at\cos at$	$\frac{2a^3}{(p^2+a^2)^2}$

Property	Periodic Signal	Fourier Series Coefficients
	$\left.\begin{matrix}x[n]\\y[n]\end{matrix}\right\}$ Periodic with period N and fundamental frequency $\omega_0 = 2\pi/N$	$\left.\begin{matrix}a_k\\b_k\end{matrix}\right\}$ Periodic with period N
Linearity	$Ax[n]+By[n]$	Aa_k+Bb_k
Time Shifting	$x[n-n_0]$	$a_k e^{-jk(2\pi/N)n_0}$
Frequency Shifting	$e^{jM(2\pi/N)n}x[n]$	a_{k-M}
Conjugation	$x^*[n]$	a^*_{-k}
Time Reversal	$x[-n]$	a_{-k}
Time Scaling	$x_{(m)}[n] = \begin{cases}x[n/m], & \text{if } n \text{ is a multiple of } m\\ 0, & \text{if } n \text{ is not a multiple of } m\end{cases}$ (periodic with period mN)	$\frac{1}{m}a_k$ $\begin{pmatrix}\text{viewed as periodic}\\ \text{with period } mN\end{pmatrix}$
Periodic Convolution	$\sum_{r=\langle N\rangle} x[r]y[n-r]$	Na_kb_k
Multiplication	$x[n]y[n]$	$\sum_{l=\langle N\rangle} a_l b_{k-l}$
First Difference	$x[n]-x[n-1]$	$(1-e^{-jk(2\pi/N)})a_k$
Running Sum	$\sum_{k=-\infty}^{n} x[k]$ $\begin{pmatrix}\text{finite valued and periodic only}\\ \text{if } a_0 = 0\end{pmatrix}$	$\left(\frac{1}{(1-e^{-jk(2\pi/N)})}\right)a_k$
Conjugate Symmetry for Real Signals	$x[n]$ real	$\begin{cases}a_k = a^*_{-k}\\ \Re e\{a_k\} = \Re e\{a_{-k}\}\\ \Im m\{a_k\} = -\Im m\{a_{-k}\}\\ \lvert a_k\rvert = \lvert a_{-k}\rvert\\ \sphericalangle a_k = -\sphericalangle a_{-k}\end{cases}$
Real and Even Signals	$x[n]$ real and even	a_k real and even
Real and Odd Signals	$x[n]$ real and odd	a_k purely imaginary and odd
Even-Odd Decomposition of Real Signals	$\begin{cases}x_e[n] = \mathcal{E}v\{x[n]\} & [x[n] \text{ real}]\\ x_o[n] = \mathcal{O}d\{x[n]\} & [x[n] \text{ real}]\end{cases}$	$\begin{matrix}\Re e\{a_k\}\\ j\Im m\{a_k\}\end{matrix}$

Parseval's Relation for Periodic Signals

$$\frac{1}{N}\sum_{n=\langle N\rangle} |x[n]|^2 = \sum_{k=\langle N\rangle} |a_k|^2$$

无论N如何变化，DFT的谐波频率始终在区间$[0,2\pi]$之内：

$$0 \le \omega_k = \frac{2\pi}{N}k < 2\pi$$

当$N\to\infty$，DFT对于区间$[0,2\pi]$的划分越来越细致，ω_k从离散的值最终变成整个实数区间。（相当于往区间$[0,2\pi]$中不断插值），或者说DFT是DTFT在$\omega=\frac{2\pi}{N}k$处的采样。

DTFT的定义为：

$$X(\omega) = \sum_{n=-\infty}^{\infty} x[n]e^{-j\omega n}, 0 \le \omega < 2\pi$$

DTFT是$N\to\infty$时DFT的极限。