Aalto University
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Structured Prediction of Network Response

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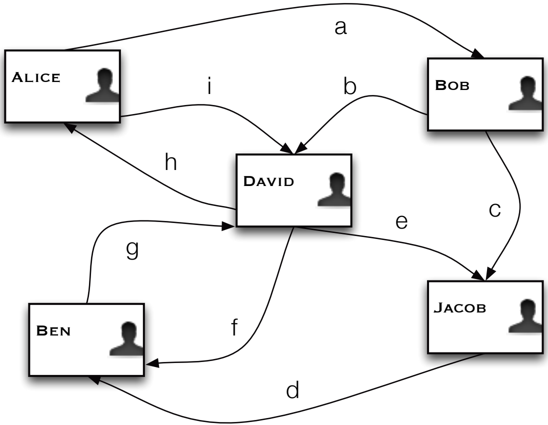
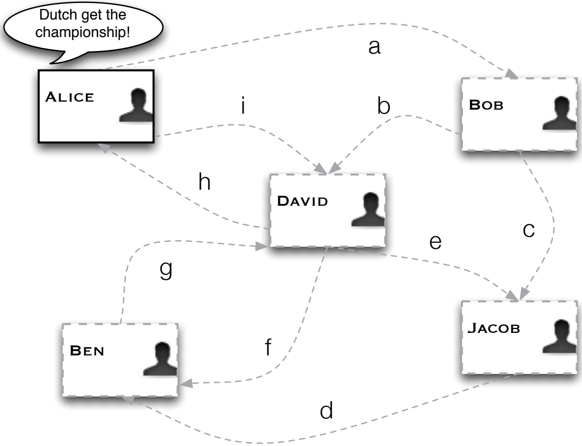
Example: Predicting Network Response

A twitter (follower-ship) network consists of five users.



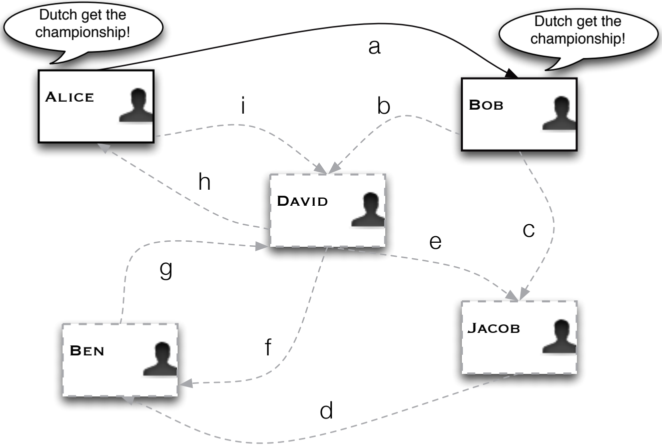
Example: Predicting Network Response

Alice tweets a message after World Cup final.



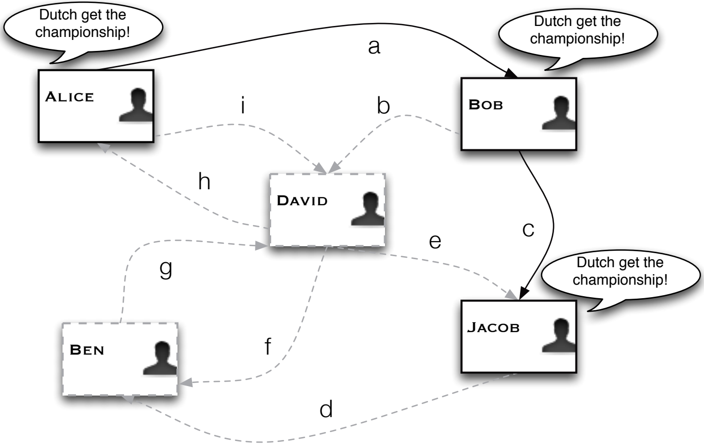
Example: Predicting Network Response

Bob sees the message and retweets the message from Alice.



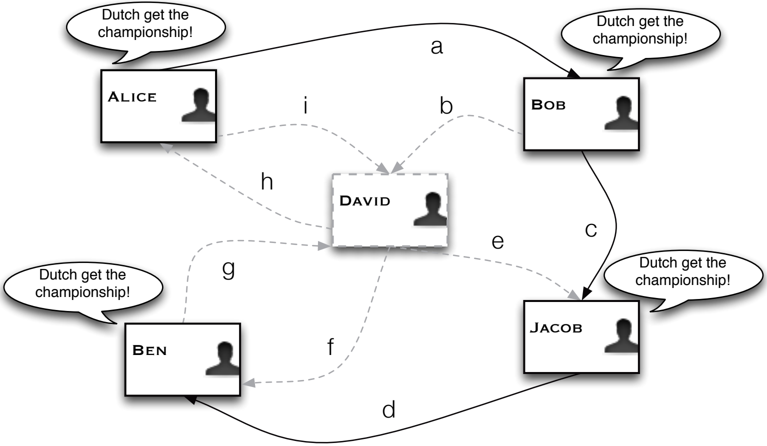
Example: Predicting Network Response

Jacob retweets the message from Bob.



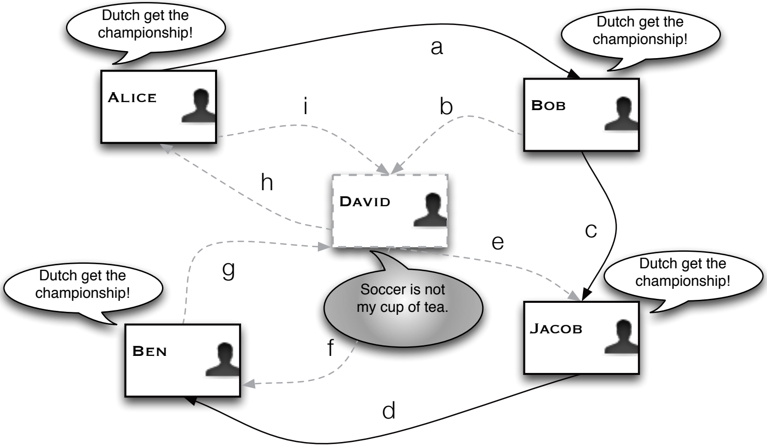
Example: Predicting Network Response

Ben retweets the message from Jacob.



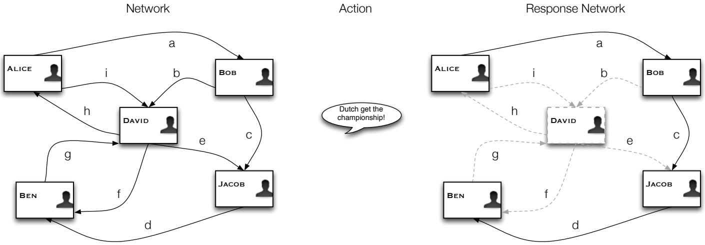
Example: Predicting Network Response

David does not like soccer.



Network Response Problem

- Definition:
  - Given a complex network G, and an action **a** performed on the network.
  - Task: predict the subnetwork that responses to the action.
    - Which nodes $v \in V_{\mathbf{a}}$ perform the action? $V_{\mathbf{a}} = \{Alice, Bob, Jacob, Ben\}$
    - Which directed edges $e \in E_{\mathbf{a}}$ relay the action from one node to its neighbors? $E_{\mathbf{a}} = \{a, c, d\}$

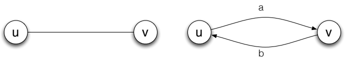


Contributions

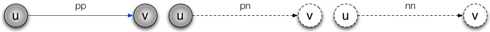
- Context-sensitive prediction. The influence from node u to node v not only depends on their connections but also depends on the action $\mathbf{a}$ under consideration.
- Structure output learning. We model the problem as predicting for each action (e.g. a tweet) a response network (*directed acyclic graph*).
- Efficient inference algorithms to discover the response network:
  - Semidefinite programming relaxation of integer quadratic programming with approximation guarantee.
  - Greedy algorithm as a scalable approach.
- Empirically performance is shown to be better than the state-of-the-art.

Model

- Model is defined on directed network.
  - Any undirected network can be seen as special case by replacing undirected edges with two directed ones.

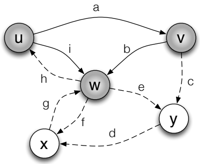


- Notation of edge labels:



- Feature maps:
  - *Input feature*: Encode $\mathbf{a}$ as $\phi(\mathbf{a})$ (e.g. bag-of-word of a tweet).
  - *Output feature*: Encode $G_{\mathbf{a}}$ as $\psi(G_{\mathbf{a}})$ (e.g. a set of edges and their labels)

$$\begin{aligned}\psi(G_{\mathbf{a}}) &= \{a_{pp}, a_{pn}, a_{nn}, b_{pp}, b_{pn}, b_{nn}, c_{pp}, c_{pn}, c_{nn}, \cdots\} \\ &= \{1, \quad 0, \quad 0, \quad 1, \quad 0, \quad 0, \quad 0, \quad 1, \quad 0, \cdots\}\end{aligned}$$



Structure Output Prediction Model

- Compatibility score for $(\mathbf{a}, G_{\mathbf{a}})$: $F(\mathbf{a}, G_{\mathbf{a}}, \mathbf{w}) = \langle \mathbf{w}, \varphi(\mathbf{a}, G_{\mathbf{a}}) \rangle$
  - $\mathbf{w}$ is the feature weight to be learned.
  - $\varphi(\mathbf{a}, G_{\mathbf{a}}) = \phi(\mathbf{a}) \otimes \psi(G_{\mathbf{a}})$ is joint feature map.
  - Intuition: given an action $\mathbf{a}$, the score of correct response graph $(\mathbf{a}, G_{\mathbf{a}})$ should be higher than any incorrect response graph $(\mathbf{a}, G'_{\mathbf{a}})$.

$$F(\mathbf{a}, G_{\mathbf{a}}, \mathbf{w}) > F(\mathbf{a}, G'_{\mathbf{a}}, \mathbf{w}), \quad \forall G'_{\mathbf{a}} \in \mathcal{H}(G)$$

- $\mathbf{w}$ is learned by solving structured output learning problem

$$\begin{aligned}
\min_{\mathbf{w},\xi} \quad & \frac{1}{2}||\mathbf{w}||_2^2 + C\sum_{i=1}^{m}\xi_i \\
\textbf{s.t.} \quad & F(a_i, G_{\mathbf{a}_i}; \mathbf{w}) > \operatorname*{\textbf{argmax}}_{G'_{\mathbf{a}_i} \in \mathcal{H}(G)} (F(a_i, G'_{\mathbf{a}_i}, \mathbf{w}) \\
& + \ell_G(G_{\mathbf{a}_i}, G'_{\mathbf{a}_i})) - \xi_i, \xi_i \geq 0, \forall i \in \{1, \cdots, m\},
\end{aligned}$$

Inference Problem

- To solve the optimization, we have to solve similar inference problem appeared both in training and in prediction.
- In prediction phase:
  - Given the feature weight **w** and the complex network G.
  - To find out a network $H^* = (V_H, E_H)$ that gives the maximal compatibility score for a given action **a**

$$
\begin{aligned}
H^*(a) &= \operatorname*{\mathbf{argmax}}_{H \in \mathcal{H}(G)} F(\mathbf{a}, H; \mathbf{w}) \\
&= \operatorname*{\mathbf{argmax}}_{H \in \mathcal{H}(G)} \langle \mathbf{w}, \phi(a) \otimes \psi(H) \rangle \\
&= \operatorname*{\mathbf{argmax}}_{H \in \mathcal{H}(G)} \sum_{e \in E^H} s_{\mathbf{y}_e}(e, a, \mathbf{w})
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NP-hardness

$$H^*(a) = \underset{H \in \mathcal{H}(G)}{\mathbf{argmax}} \sum_{e \in E^H} s_{\mathbf{y}_e}(e, a, \mathbf{w}) \quad (1)$$

Lemma

Finding the graph that maximizes Eq. (1) is an $\mathcal{NP}$-hard problem.

Proof.

Reduction from MAX-CUT problem. □

Approximate Inference through SDP relaxation

- SDP inference:
  - We formulate the inference problem as *integer quadratic programming* (IQP).
    - Introduce for each node $u \in V$ a binary variable $x_u \in \{-1, +1\}$.
    - Introduce a special variable $x_0 \in \{-1, +1\}$ to distinguish activated node.

$$
\begin{aligned}
\textbf{max} \ \frac{1}{4} \sum_{(u,v)\in E} & [s_{pn}(u,v)(1 + x_0 x_u - x_0 x_v - x_u x_v) \\
& + s_{nn}(u,v)(1 - x_0 x_u - x_0 x_v + x_u x_v) \\
& + s_{pp}(u,v)(1 + x_0 x_u + x_0 x_v + x_u x_v)] \\
\textbf{s.t.} \quad & x_0, x_u, x_v \in \{-1, +1\}, \ \text{for all } u, v \in V,
\end{aligned}
$$

  - IQP is solved by *semidefinite programming relaxation* (SDP).
  - Optimization guarantee $E[Z] \geq (\alpha - \epsilon) Z_R$ with $\alpha > 0.796$, Z is objective achieved by SDP, Z_R is objective of IQP.

GREEDY Inference

- GREEDY inference:
  - We have shown that the inference problem can be expressed equivalently as nodes and their scores

$$H^*(\mathbf{a}) = \underset{H \in \mathcal{H}(G)}{\mathbf{argmax}} \sum_{v_i \in V_p^H} F_m(v_i).$$

  - The greedy algorithm iteratively maximizes the equation by adding one node into activited node set V_p^H in each iteration.
  - The time complexity for greedy inference algorithm is $\Theta(|E| \log |V|)$.
  - We are able to run GREEDY algorithm on network with upto 2000 nodes.

Experiment: Context-sensitive Prediction

- Experiment settings:
  - We assume action is known (e.g. bag-of-word of a tweet).
  - Task is to predict the response network given an action.
  - *Predicted Subgraph Coverage* (PSC) is the relative size of correctly predicted subgraph in terms of node labels.)
- Result:

Dataset	Node Accuracy			Node F_1 Score			Edge Acc		*PSC*		
	SVM	MMCRF	SPIN	SVM	MMCRF	SPIN	SVM	SPIN	SVM	MMCRF	SPIN
memeS	**73.4**	68.0	*72.2*	39.0	*39.8*	**47.1**	**62.7**	45.6	23.4	*25.3*	**33.6**
memeM	**82.1**	79.0	*81.5*	29.1	*30.1*	**38.0**	61.1	**68.8**	18.6	*18.8*	**28.3**
memeL	**89.9**	88.3	*89.8*	26.7	*27.1*	**35.0**	45.5	**80.0**	17.7	*18.9*	**27.6**
M700	91.9	**94.1**	*92.1*	*13.8*	7.3	**14.2**	26.3	**93.0**	*29.4*	23.9	**34.4**
M1k	94.1	**95.8**	*94.2*	**10.9**	3.5	*9.3*	26.6	**94.7**	*33.7*	16.6	**35.2**
M2k	*96.8*	**97.6**	96.7	**6.2**	1.4	*3.4*	25.3	**97.6**	**34.6**	9.6	*14.7*
L700	89.7	**92.4**	*89.7*	*16.2*	9.4	**17.3**	26.5	**90.4**	*9.5*	6.7	**12.5**
L1k	*92.4*	**94.4**	91.5	*12.4*	6.4	**13.9**	26.4	**92.3**	*6.1*	4.4	**8.4**
L2k	*92.5*	**94.5**	91.9	*12.3*	5.4	**12.7**	26.5	**93.2**	*6.0*	2.9	**7.2**
Geom.	85.5	*86.4*	**86.6**	*19.8*	12.6	**20.3**	32.6	**79.7**	*18.9*	14.2	**21.7**

Experiment: Context-free Prediction

- Experiment settings:
  - We assume action is unknown.
  - Task is to predict directed edges from a cascade of actions and compare the results against other influence network prediction methods.
  - The measure of success is *Precision@K*, where we ask for top-*K* percent edge predictions and compute the precision.
- Result:

Dataset	**Model**	**T** (10^3**s**)	Precision @ *K*					
			10%	20%	30%	40%	50%	60%
memeS	SPIN	*5.50*	**82.9**	**81.0**	**76.0**	**74.0**	**74.0**	**70.0**
	ICM-EM	**0.01**	60.3	63.5	65.1	62.0	62.0	61.5
	NETRATE	5.83	*76.2*	*73.8*	*70.4*	*68.7*	*68.7*	*66.8*
memeM	SPIN	*5.52*	**82.7**	**72.1**	**70.5**	**69.2**	**69.2**	**67.9**
	ICM-EM	**0.02**	56.3	55.3	56.8	57.4	57.4	56.3
	NETRATE	13.93	*61.2*	*64.6*	*62.9*	*62.5*	*62.5*	*62.4*
memeL	SPIN	*4.75*	**82.2**	**73.6**	**69.1**	**66.7**	**66.7**	**65.9**
	ICM-EM	**0.01**	52.1	55.7	54.2	56.5	56.5	56.7
	NETRATE	12.63	*56.5*	*57.8*	*60.0*	*59.3*	*59.3*	*59.4*

Conclusion

- We have developed a structured output learning approach for network response prediction problem.
- To outperform competing methods, our model takes the advantage of two sources of prior knowledge:
  - The context given by the action descriptions.
  - The structure of an underlying network (e.g. followership in Twitter, friendship in Facebook).
- The inference problem is $\mathcal{NP}$-hard, and is in practice tackled by two algorithms:
  - SDP relaxation with approximation guarantee.
  - a fast GREEDY heuristics.

- Poster No. 565 tonight.
- Thank you !