
Relational algebra and relational calculus

- Relational algebra and relational calculus are formal languages associated with the relational model.
- Informally, relational algebra is a (high-level) procedural language and relational calculus a non-procedural language.
- However, formally both are equivalent to one another.
- A language that produces a relation that can be derived using relational calculus is relationally complete.

Two mathematical Query Languages form the basis for “real” languages (e.g. SQL), and for implementation:

Relational Algebra: More **operational**, very useful for representing execution plans.

- Relational algebra operations work on one or more relations to define another relation without changing the original relations.
- Both operands and results are relations, so output from one operation can become input to another operation.
- Allows expressions to be nested, just as in arithmetic. This property is called closure.
- Five basic operations in relational algebra: Selection, Projection, Cartesian product, Union, and Set Difference.
- Basic operations:
 - **Selection** (σ) Selects a subset of rows from relation. Works on a single relation R and defines a relation that contains only those tuples (rows) of R that satisfy the specified condition (*predicate*). p is called the **selection predicate**, r can be the name of a table, or another query

- **Projection** (π) Deletes unwanted columns from relation. Works on a single relation R and defines a relation that contains a vertical subset of R, extracting the values of specified attributes and eliminating duplicates.

- **Cross-product** (\times) Allows us to combine two relations. • Defines a relation that is the concatenation of every tuple of relation R with every tuple of relation S.
- For sets A, B, the **Cartesian product**, or **cross product**, of A and B is denoted by $A \times B$ and equals $\{(a, b) \mid a \in A, b \in B\}$
- Elements of $A \times B$ are ordered pairs. For $(a, b), (c, d) \in A \times B$, $(a, b) = (c, d)$ if and only if $a = c$ and $b = d$

Properties:

1. If A, B are finite, it follows from the rule of product that $|A \times B| = |A||B|$
2. Although we generally will not have $A \times B = B \times A$, we will have $|A \times B| = |B \times A|$

- **Set-difference** ($-$) Tuples in reln. 1, but not in reln. 2. • Defines a relation consisting of the tuples that are in relation R, but not in S.
- R and S must be union-compatible.

- **Union** (\cup) Tuples in reln. 1 and in reln. 2. • $R \cup S$
- Union of two relations R and S defines a relation that contains all the tuples of R, or S, or both R and S, duplicate tuples being eliminated.
- R and S must be union-compatible.

- If R and S have I and J tuples, respectively, union is obtained by concatenating them into one relation with a maximum of $(I + J)$ tuples.

- Additional operations:
- Intersection, *join*, division, renaming: Not essential, but (very!) useful.
- Since each operation returns a relation, *operations can be composed!* (Algebra is “closed”.)
- These perform most of the data retrieval operations needed.
- Also have Join, Intersection, and Division operations, which can be expressed in terms of 5 basic operations

Join Operations

- Join is a derivative of Cartesian product.
- Equivalent to performing a Selection, using join predicate as selection formula, over Cartesian product of the two operand relations.
- One of the most difficult operations to implement efficiently in an RDBMS and one reason why RDBMSs have intrinsic performance problems.

Theta join (q-join)

- Defines a relation that contains tuples satisfying the predicate F from the Cartesian product of R and S.
- The predicate F is of the form $R.a_i \text{ q } S.b_j$ where q may be one of the comparison operators ($<$, \neq , $>$, \geq , $=$, π).

Natural join

- An Equijoin of the two relations R and S over all common attributes x. One occurrence of each common attribute is eliminated from the result.

Outer join

- To display rows in the result that do not have matching values in the join column, use Outer join.
- (Left) outer join is join in which tuples from R that do not have matching values in common columns of S are also included in result relation.

Semijoin

- Defines a relation that contains the tuples of R that participate in the join of R with S.

Division

- $R \div S$
- Defines a relation over the attributes C that consists of set of tuples from R that match combination of every tuple in S.
- Expressed using basic operations:

$$T_1 \leftarrow P_C(R)$$

$$T_2 \leftarrow P_C((S \times T_1) - R)$$

$$T \leftarrow T_1 - T_2$$

- Idea:* For A/B , compute all x values that are not ‘disqualified’ by some y value in B.
- x value is *disqualified* if by attaching y value from B, we obtain an xy tuple that is not in A.

Relational Calculus: Lets users describe what they want, rather than how to compute it.
(Non-operational, *declarative*.)

Relations

Let $A = \{0,1,2\}$, $B = \{1,2,3\}$. $A \times B = \{(0,1), (0,2), (0,3), (1,1), (1,2), (1,3), (2,1), (2,2), (2,3)\}$

Let say an element x in A is related to an element y in B iff x is less than y . $x R y$: x is related to y

$\backslash 0 R 1, 0 R 2, 0 R 3, 1 R 2, 1 R 3, 2 R 3$

\backslash The set of all ordered pair in $A \times B$ where elements are related $\{(0,1), (0,2), (0,3), (1,2), (1,3), (2,3)\}$

• For sets A, B , a (binary) relation R from A to B is a subset of $A \times B$. Any subset of $A \times A$ is called a (binary) relation on A

• Given an ordered pair (a, b) in $A \times B$, x is related to y by R ($x R y$) iff (x, y) is in R

• In general, for finite sets A, B with $|A| = m$ and $|B| = n$, there are 2^{mn} relations from A to B , including the empty relation as well as the relation $A \times B$ itself

Let $A = \{2, 3, 4\}$, $B = \{4, 5\}$. Then

$A \times B = \{(2, 4), (2, 5), (3, 4), (3, 5), (4, 4), (4, 5)\}$.

The following are some of the relations from A to B .

i. Δ

ii. $\{(2, 4)\}$

iii. $\{(2, 4), (2, 5)\}$

iv. $\{(2, 4), (3, 4), (4, 4)\}$

v. $\{(2, 4), (3, 4), (4, 5)\}$

vi. $A \times B$

Since $|A \times B| = 6$, there are 2^6 possible relations from A to B (for there are 2^6 possible subsets of $A \times B$)

Aggregate Operations

• Applies aggregate function list, AL , to R to define a relation over the aggregate list.

• AL contains one or more ($\langle \text{aggregate_function} \rangle, \langle \text{attribute} \rangle$) pairs.

• Main aggregate functions are: COUNT, SUM, AVG, MIN, and MAX.

Grouping Operation

• Groups tuples of R by grouping attributes, GA , and then applies aggregate function list, AL , to define a new relation.

• AL contains one or more ($\langle \text{aggregate_function} \rangle, \langle \text{attribute} \rangle$) pairs.

• Resulting relation contains the grouping attributes, GA , along with results of each of the aggregate functions.