Theoretical Framework for Variable-Speed CNC Spindle Vibration Monitoring in Metal-Cutting CNC Machines

About This Document

This white paper presents a theoretical and mathematical framework for building a robust, adaptive machine learning system to monitor spindle health in CNC environments characterized by variable speeds, mechanical loads, tools, and materials.

Traditional threshold-based condition monitoring systems often fail under these conditions, leading to high false-positive rates or missed failures. This framework blends physics-based equations of motion with residual-driven machine learning, providing a dynamic, contextualized baseline for vibration and temperature behavior.

Rather than attempting to detect faults from raw sensor data, this framework focuses on modeling the expected vibration and temperature behavior based on known physics and contextual variables like tool geometry, RPM, and material type. By predicting and subtracting the expected physical response, we isolate the unexpected component: a context-eliminated residual signal that can be reliably used for anomaly detection.

The content is intended for engineers, data scientists, and researchers working in the fields of predictive maintenance, industrial reliability, and manufacturing analytics. While originally designed with CNC metal-cutting operations in mind, the principles are broadly applicable to other variable-speed rotating machinery.

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Date: July 2025

1 Introduction

Variable speed and ever-changing process conditions (new tools, different depths of cut, feeds, and materials) make static vibration limits nearly useless for CNC spindles. The workable alternative is to treat vibration and load as *functions of operating state*. If we can predict what "healthy" looks like for a given state (speed, torque, tool, material, etc.), then any deviation from that predicted behavior becomes our health indicator.

At a high level:

- Model expected vibration/load features from physics + machine state.
- Compute residuals between measured and expected features.
- Resolve and normalize those residuals (with uncertainty) to detect, trend, and diagnose actual deviations caused by defects and failure modes.

2 Data Needed from CNC/PLC (and Optional Sensors)

Collect time-aligned signals and context from the controller/drive:

- Spindle speed (commanded and actual RPM)
- Spindle motor current or % load (convertible to torque τ and power P)

- Axis feed rates and servo loads (X/Y/Z currents or % load)
- Programmed feed per rev/tooth, depth of cut (a_p), cutting speed (v_c)
- Tool ID and geometry: diameter, flute count Z, overhang, mass/inertia, balance class
- Work material identifier and cutting coefficients (e.g. specific cutting force K_c)
- Process state tags: idle, rapid, air cut, roughing, finishing, tool change
- Optional: accelerometers on spindle housing, acoustic emission, spindle/bearing temperature, coolant state

Every sample (or window) should be tagged with this context so features can be normalized or interpreted correctly.

3 Governing Mathematics

3.1 Rotational & Tooth Frequencies

Fundamental rotating frequency:

$$f_{\text{rot}} = \frac{N}{60}$$

Tooth-pass frequency:

$$f_{\text{tooth}} = \frac{N}{60} Z$$

where N = spindle speed [RPM], Z = number of cutting edges.

3.2 Power / Torque Relations

Instantaneous mechanical power:

$$P = \tau\,\omega$$

Cutting power estimate (turning form, kW):

$$P_c \approx \frac{a_p\, f\, v_c\, K_c}{60 \times 10^3\, \eta}$$

a_p depth of cut [mm], f feed per rev [mm/rev], v_c cutting speed [m/min], K_c specific cutting force [MPa], η efficiency.

Measured vs. predicted:

$$P_{\text{meas}}(t) = \tau(t)\,\omega(t)$$
$$P_{\text{pred}}(t) = \text{from equation above}$$

Rising ratio $P_{\text{meas}}/P_{\text{pred}}$ signals wear or friction.

3.3 Imbalance / Inertia

Centrifugal force:

$$F = m\,r\,\omega^2$$

3.4 Bearing / Gear Fault Orders

Standard defect frequencies (BPFO, BPFI, BSF, FTF) scale with f_{rot}. Gear mesh frequencies scale with shaft speed and tooth counts.

3.5 Tool Engagement Dynamics

Tool geometry and engagement strategy introduce specific dynamic loading characteristics beyond what is captured by flute count and RPM alone. For example, roughing tools with chip-breakers produce interrupted cuts; drills and taps engage axially rather than radially; and finishing tools experience different harmonics due to lighter contact and constant surface speed.

To account for tool orientation, let θ represent the cutting engagement angle between the tool axis and the surface normal. This affects the directionality and magnitude of applied cutting forces.

Effective cutting force vector:

$$\vec{F}_{\text{cut}} = K_c \cdot a_p \cdot f \begin{bmatrix} \cos\theta \\ \sin\theta \end{bmatrix}$$

Where:

- K_c is the specific cutting force for the material [MPa],
- a_p is the depth of cut [mm],
- f is the feed per rev [mm/rev],
- θ is the tool engagement angle [radians].

This formulation enables direction-sensitive vibration prediction and allows the model to differentiate between axial and radial loading events based on tool behavior.

3.6 Composite Mechanical Load Estimation

The effective mechanical load on the spindle-bearing assembly depends not just on torque or RPM individually, but on a complex interaction of tool geometry, material, feedrate, and orientation.

Define a composite mechanical load scalar L as:

$$L = \alpha \cdot \tau + \beta \cdot \|\vec{F}_{\text{cut}}\| + \gamma \cdot \left(\frac{P_{\text{meas}}}{P_{\text{pred}}}\right)$$

Where:

- τ is the instantaneous spindle torque [Nm],
- $\vec{F}_{\text{cut}}$ is the cutting force vector from Section 3.5,
- P_{meas} is the measured spindle power [kW],
- P_{pred} is the predicted power based on cutting equations,
- α, β, and γ are tunable weight coefficients to balance the influence of each term.

This scalar L can be used as a unified load index for vibration and thermal model inputs, enabling more precise residual calculation under varied machining conditions.

4 Sensor Inputs & Signal Conditioning

The primary real-time inputs to this framework are:

- **Tri-axial Vibration (acceleration)**: sampled at high frequency (typically 10–50 kHz),
- **Spindle Temperature**: sampled at 1–10 Hz,
- **Power Consumption and Torque**: derived or measured values from drive telemetry.

4.1 Signal Preprocessing

Raw vibration signals are preprocessed through a chain of operations:

1. **Bandpass Filtering** to remove high-frequency noise and isolate relevant resonance bands.

2. **Resampling or RPM-normalized Windowing**: signal windows are dynamically adjusted to match spindle rotations, enabling phase-coherent analysis across variable speeds.
3. **Time-Domain Feature Extraction**: RMS, peak, crest factor, kurtosis, and zero-crossing rate are computed per window.
4. **Spectral Decomposition**: via FFT, STFT, or CWT (continuous wavelet transform), yielding frequency-domain signatures.

4.2 Feature Alignment and Normalization

Once time-frequency representations are computed, features are aligned with contextual spindle parameters (RPM, tool ID, feedrate). This contextual metadata is appended to each feature vector.

All features are min-max normalized and log-scaled if necessary to match the scale of the physics-informed expected output $\hat{y}$.

4.3 Adaptive Feature Strategy

To handle high-variability machining (tool swaps, RPM shifts, interrupted cuts), the system dynamically selects:

- Fixed-length time windows (in milliseconds) for stable cuts,
- RPM-synchronized rotation windows (in revolutions) for harmonically aligned analysis.

This hybrid windowing enables robust comparison between measured and predicted values, regardless of cutting conditions.

5 Healthy Behavior Model (HBM)

With clean, context-rich features extracted from the sensor signals, we must now construct a modeling architecture capable of predicting the expected healthy behavior. This Healthy Behavior Model acts as digital twins of the spindle's mechanical health and provide the baseline against which residual deviations can be measured.

The goal of the HBM is not simply to model observed sensor values, but to estimate the expected behavior of the system under known operating conditions. By doing so, we explicitly eliminate the contribution of those known variables (RPM, tool, material, etc.) to the measured vibration and temperature. The resulting residual can then be interpreted as a deviation from ideal behavior, with all contextual variation accounted for.

Map operating state $\mathbf{x}$ to expected features $\hat{\mathbf{y}}$:

$$\hat{\mathbf{y}} = g(\mathbf{x}), \quad \mathbf{x} = [N, T, f, a_p, Z, K_c, \theta, M, \omega_{\text{cut}}, \text{coolant}, \text{tool\_class}, \ldots]$$

The machine learning components of this framework are not black-box predictors but physics-informed estimators designed to account for contextual variability. Three major classes of model architectures are recommended based on the operating complexity and available data fidelity:

5.1 Supervised Regression Models (Baseline Modeling)

For environments with rich labeled datasets, use supervised learning algorithms to estimate baseline spindle behavior under healthy conditions.

- **Gradient Boosting Machines (GBM)** such as XGBoost or LightGBM can handle non-linear interactions between input features (RPM, tool ID, angle, load) and expected vibration/temperature outputs.

- **Multivariate Polynomial Regression** may be appropriate when physical relationships are well-defined and nonlinear but relatively low-dimensional.

These models form the baseline estimator $\hat{y}_{\text{vib}}$ in the residual equation:

$$r_{\text{vib}} = y_{\text{meas}} - \hat{y}_{\text{vib}} \approx \text{fault signature}$$

5.2 Physics-Informed Neural Networks (PINNs)

When partial physics knowledge is available (e.g., torque curves, resonance frequencies, tool-material damping coefficients), integrate that knowledge as constraints or embedded functions inside neural network loss functions.

- **PINNs** can model real-world conditions while incorporating ODEs or algebraic expressions from classical mechanics directly into the training process.
- This approach is especially useful for variable tool paths, angles, and advanced multi-axis CNC geometries.

5.3 Hybrid Model Architectures

For high-dimensional or high-variance environments, a hybrid system can be employed:

- **Encoder-decoder networks** or **Autoencoders** to learn latent representations of healthy spindle behavior across tool types and load profiles.
- **Residual learning**: train a secondary model (e.g., a CNN or RNN) on residuals to classify or score anomalies.
- **Online Learning**: incorporate mini-batch updates or reinforcement learning for systems that evolve or adapt to new tools/materials in production.

5.4 Justification of Model Selection

These model families were chosen due to:

- The need to generalize across multiple operating conditions with sparse labels.
- The ability to incorporate known physical relationships into loss functions or feature engineering.
- Their success in real-time fault detection across adjacent domains (e.g., rotating equipment monitoring, bearing diagnostics, and electric motor health).

6 Contextual Variable Elimination and Normalization Layer

Once residuals are calculated by isolating abnormal behavior from the expected physics-based output, we require a method to interpret these deviations in a statistically meaningful way. This section presents techniques to aggregate, score, and visualize those residuals for real-time health estimation.

In variable-speed CNC machining, sensor readings vary widely with operating conditions. A cutting tool drilling into stainless steel at 2000 RPM will naturally generate more vibration and heat than the same tool milling aluminum at 1200 RPM. These changes are not indicative of failure—they are expected consequences of physics.

To address this, the proposed framework does not rely on fixed thresholds. Instead, it uses known physical relationships (e.g. torque, cutting force, heat transfer) and real-time context variables (RPM, tool ID, material, etc.) to model the expected vibration and temperature behavior.

By subtracting this prediction from the observed value, the framework isolates the unexpected residual—revealing only the health-relevant signal.

Let the full machine context vector be:

$$x = [N, T, f, a_p, Z, K_c, \theta, M, \omega_{\text{cut}}, \text{coolant}, \text{tool\_class}, \ldots]$$

Where:

- N = spindle RPM
- T = torque/load
- f = feed rate or feed per tooth
- a_p = depth of cut
- Z = number of flutes / cutting edges
- K_c = specific cutting force of the material
- θ = tool cutting angle or approach angle
- M = material identifier / properties
- ω_{cut} = rotational speed of cut engagement (could be relative to surface)

Let $y_{\text{meas}}(t)$ be the actual measured sensor vector (e.g., vibration amplitude, temp). Let $\hat{y}(x)$ be the expected output from the Healthy Behavior Model, given context x.

Then the residual:

$$r(t) = y_{\text{meas}}(t) - \hat{y}(x(t))$$

And the normalized anomaly score (z-score):

$$z_i(t) = \frac{y_i(t) - \hat{y}_i(x)}{\sigma_i(x)}$$

Therefore, a generalized model

$$\hat{y}_{\text{vib}} = h_{\text{phys}}(x) + \epsilon_{\text{model}}$$

where $h_{\text{phys}}(x)$ is the output from the physics model. By subtracting the modeled physical response $\hat{y}_{\text{vib}}$, we isolate unexplained behavior:

$$r_{\text{vib}} = y_{\text{meas}} - \hat{y}_{\text{vib}} \approx \text{fault signature}$$

Given predicted outputs $\hat{\mathbf{y}}$ and measured sensor readings $\mathbf{y}_{\text{meas}}$, we define the residual error vector:

$$\mathbf{e} = \mathbf{y}_{\text{meas}} - \hat{\mathbf{y}}$$

Uncertainty Quantification

While deterministic models provide point estimates of $\hat{\mathbf{y}}$, modeling the uncertainty around these predictions enables more reliable decision-making and confidence-aware health scoring. There are several practical methods for incorporating uncertainty into this framework:

- **Bayesian Regressors:** Approaches such as Gaussian Processes or Bayesian Neural Networks yield predictive distributions $P(\hat{\mathbf{y}} \mid \mathbf{x})$ rather than single values. These allow for explicit quantification of confidence intervals around each predicted signal.
- **Monte Carlo Dropout:** By keeping dropout active during inference and sampling multiple forward passes, one can estimate epistemic uncertainty from the variance across outputs. This method scales well and is straightforward to integrate into existing neural models.

- **Ensemble Models:** Training multiple independent models or bootstrapped versions allows for empirical variance estimation. Differences in predictions reflect uncertainty due to initialization, training data variability, or structural ambiguity.
- **Uncertainty-Aware Residuals:** When the model outputs a distribution (e.g., with mean $\hat{\mu}$ and covariance $\hat{\Sigma}$), the residual vector $\mathbf{e}$ can be scaled accordingly:

  $$\mathbf{e}_{\text{norm}} = \hat{\Sigma}^{-1/2}(\mathbf{y}_{\text{meas}} - \hat{\mu})$$

  This normalized residual accounts for model uncertainty in each component of the signal, making subsequent anomaly detection and scoring more robust.

7 Health Scoring & Aggregation Residual Decision Layer

The final output of this system is a scalar or vector-valued *health score* representing the current condition of the spindle system. This score is intended to quantify how far the system's real-world signals deviate from their physics-informed, normalized expectations.

7.1 Residual Vector Computation

After all sensor inputs are processed and expected outputs are modeled (via physics, ML, or hybrid approaches), we define the residual vector:

$$\mathbf{r} = \mathbf{x}_{\text{observed}} - \hat{\mathbf{x}}_{\text{expected}}$$

where $\mathbf{x}_{\text{observed}}$ is the measured sensor data vector (e.g., vibration features, thermal readings) and $\hat{\mathbf{x}}_{\text{expected}}$ is the model-predicted healthy-state baseline under current conditions.

7.2 Mahalanobis Distance Scoring

The Mahalanobis distance provides a scale-invariant metric for residual deviation:

$$D_M = \sqrt{(\mathbf{r} - \boldsymbol{\mu}_r)^T \boldsymbol{\Sigma}_r^{-1} (\mathbf{r} - \boldsymbol{\mu}_r)}$$

where $\boldsymbol{\mu}_r$ and $\boldsymbol{\Sigma}_r$ are the mean and covariance of residuals under known healthy operation. This metric is particularly suited to multi-dimensional inputs with correlated features.

7.3 Control Chart Integration

For real-time monitoring, control limits (e.g., based on 3σ thresholds) can be derived from the distribution of D_M over time. Deviations exceeding these bounds are flagged as anomalies. Integration into SPC (Statistical Process Control) dashboards allows operations teams to visualize changes in machine behavior even before failure occurs.

7.4 Unsupervised Anomaly Models (Optional)

In contexts where labeled failure data is scarce, the system can include:

- **Autoencoders**: trained to reconstruct healthy-state features, with reconstruction error used as a proxy for anomaly.
- **Isolation Forests or LOF (Local Outlier Factor)**: trained on residual vectors to detect statistical outliers.

- **PCA-based Monitoring**: where the first k principal components are monitored for drift or orthogonal deviations.

These models operate in parallel with Mahalanobis-based metrics, offering ensemble detection to improve robustness in edge cases (e.g., unusual tooling or cutting states).

7.5 Health Score Output

The final health score $H(t)$ can be a composite scalar:

$$H(t) = w_1 D_M(t) + w_2 \cdot \text{ReconstructionError}(t) + \cdots$$

where weights w_i are empirically determined or dynamically adjusted through adaptive learning. A declining $H(t)$ indicates increasing degradation or unmodeled behavior.

Thresholds for alerting and downtime planning are derived through historical backtesting and integration with maintenance outcomes.

8 Generalization and Application Across Machine Variants

The methodology outlined in this framework is designed for portability across diverse CNC machines, spindles, and tool configurations. Generalization is made possible by parameterizing the model according to the physical characteristics of each system.

Specifically, any machine, tool, or configuration with a known mechanical specification and a validated baseline of "healthy" behavior can serve as the foundation for parameterization. The full context vector $\mathbf{x}$, as defined earlier, captures critical operational variables such as RPM, tool type, engagement angle, and material properties. Provided this input vector is properly scaled or transformed to accommodate new hardware characteristics (e.g., different spindle inertia or damping coefficients), the physics-informed model $h_{\text{phys}}(\mathbf{x})$ remains structurally valid.

To support generalization:

- Baseline parameter sets can be grouped by machine family or spindle class.
- Few-shot learning or online reparameterization can be applied to adapt to new tools with minimal data.
- Transfer learning or multi-task model training can be used to share weights between similar spindles.
- Tooling and material profiles can be stored as lookup tables or submodels, invoked dynamically based on job metadata.

This allows the framework to be deployed flexibly across environments while preserving the rigor of physics-derived behavior and statistical anomaly detection.

Bottom line: By blending know equations in mechanics and physics (order tracking, cutting-power equations, imbalance dynamics) with current machine context and ML analysis models, we can begin to build a trustworthy, machine-agnostic spindle-health monitoring system for CNC metal-cutting machines that operate along a broad spectrum of varied spindle types, tools, speeds, materials, and operation types.