Project Report

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1. Introduction

1.1 Background

Pizza and bread are widespread convenience foods in the world. In the pizza and breadmaking industry, the leavening step is fundamental to creating a dough that conforms to the textural requirements of its class. The overall quality depends chiefly on the dough whose properties are undoubtedly affected by the leavening process, in addition to the flour type and preparation procedure. Leavening brings about changes in the cellular structure of the dough, and hence the textural properties of the bread by expanding gas cell sizes in the dough with carbon dioxide.

A biological leavening agent, yeast, is one of the most used leavening methods. Yeast is a single-celled microorganism related to mushrooms. As the dough is mixed and kneaded, millions of air bubbles are trapped and dispersed throughout the dough. Meanwhile, the yeast in the dough metabolizes the starches and sugars in the flour, turning them into alcohol and carbon dioxide gas. This gas inflates the network of air bubbles, causing the bread to rise. During rising, the yeast divides and multiplies, producing more carbon dioxide. Yeast fermentation is directly affected by the change in temperature because the rate of chemical reactions is affected by temperature.

The goal of this report is to conduct a full-structured experiment design, experiment conduction, data collection, data analysis, establishing a statistical model as well as performing hypothesis testing for the relationships and interpret the results.

1.2 Objective

1) To design, implement an experiment and collect data.

2) To construct a statistical model and analyze relationships as well as perform hypothesis testing.

3) To interpret the results and summarize the findings.

2. Method and results

2.1 Question of interest

Leavened doughs are preparations that include an extensive range of recipes, from bread to pizza and pastries. The common point of all these doughs is that their expansion in cooking is due to the gases produced from yeasts' fermentation. I would like to know

"How does the amount of yeast and temperature affect the rise of dough?"

2.2 Designs of experiment

This experiment has two variables, the amount of yeast and the rising temperature. And the response is the rise of dough. There are 4 levels of Yeast and 4 levels of Temperature. The levels of Yeast are 0, 1%, 2%, and 3% of flour weight. The levels of temperature are 20 °C, 25°C, 30°C, 35°C. And each treatment combination has 4 replicates. I would like to know both the main effects of yeast and temperature and the interaction effects. So, I designed a two-factor factorial experiment.

The two-factor factorial analysis uses the following effects model:

$$y_{ijk} = \mu + \alpha_i + \beta_j + (\alpha\beta)_{ij} + e_{ijk} \qquad \begin{cases} i = 1, 2, 3, 4 \\ j = 1, 2, 3, 4 \\ k = 1, 2, 3, 4 \end{cases}$$

Where μ is the overall mean effect, $\alpha_i$is the effect of the *i*th level of the row factor Yeast, β_j is the effect of the *j*th level of column factor temperature. $(\alpha\beta)_{ij}$ is the effect of the interaction between α_i and β_j, e_{ijk} is a random error component.

2.3 Experiment procedure and data collection

In order to avoid nuisance variation as much as possible, I kept water percentage, flour type, yeast type, and fermentation time the same. And I made all the dough samples of the same weight and shape, and they were kept in identical containers during fermentation.

Four doughs were prepared by hand mixing and kneading in four identical bowls for 2 min at room temperature, 180g unbleached all-purpose flour, 135g water, 0g, 1.8g, 3.6g, 5.4g yeast respectively. Then each of the four doughs was equally divided into 4 samples of about 78g each and incubated in identical plastic containers with lids. Closed containers keep the dough skin moisture. Moisture keeps the skin of the dough supple and soft, promoting a better rise. A thermometer was used to show the environment temperature. I adjusted the thermostat to make room temperature 20°C. 16 dough samples were put in the 20 °C environments to ferment for 2.5h. I marked the initial height of doughs and measured the rise of each dough after fermentation (Figure 1). And then I repeated these steps and put dough samples in 25°C, 30°C, and 35°C environments respectively to ferment for the same amount of time and collect data. 20°C and 25°C are found in room environments. 30°C and 35°C are found in an oven. All data are listed below in the table (Table 1).

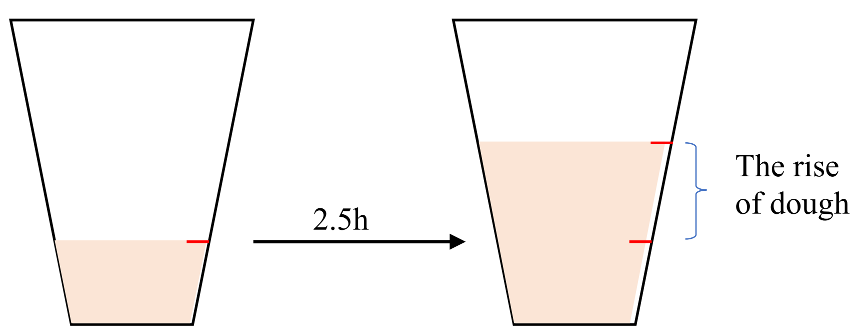


Figure 1. Measurement of the rise of dough samples

Table 1. The rise of doughs (in *mm*)

Amount of yeast (%)	Temperature (℃)							
	20		25		30		35	
0	1	0	0	0	2	0	2	1
	0	0	0	1	0	0	0	0
1	22	16	37	38	35	39	37	35
	21	20	36	33	32	31	36	35
2	22	29	38	36	40	38	39	43
	28	26	39	35	37	39	41	42
3	36	27	37	34	44	38	45	42
	26	26	40	36	36	33	46	41

Note. The amount of yeast is notated as the percentage of flour weight.

3. Discussion

3.1 Statistical modeling

In this study, I used the following effects model:

$$y_{ijk} = \mu + \alpha_i + \beta_j + (\alpha\beta)_{ij} + e_{ijk} \qquad \begin{cases} i = 1, 2, 3, 4 \\ j = 1, 2, 3, 4 \\ k = 1, 2, 3, 4 \end{cases}$$

Where μ is the overall mean effect, $\alpha_i$is the effect of the *i*th level of the row factor Yeast, β_j is the effect of the *j*th level of column factor temperature. $(\alpha\beta)_{ij}$ is the effect of the interaction between α_i and β_j, e_{ijk} is a random error component. The following hold:

$$\sum_{i=1}^{a} \alpha_i = 0, \qquad \sum_{j=1}^{b} \beta_j = 0, \qquad \sum_{i=1}^{a} (\alpha\beta)_{ij} = \sum_{j=1}^{b} (\alpha\beta)_{ij} = 0$$

3.2 Hypothesis testing

In this two-factor factorial experiment, both row and column treatments, the amount of yeast and temperature, are of equal interest. Specifically, I am interested in testing hypotheses about the equality of row treatment effects,

$$H_o: \alpha_1 = \alpha_2 = \alpha_3 = \alpha_4$$

$$H_a: \alpha_i \neq 0 \text{ for some } i$$

And the equality of column treatment effects,

$$H_o: \beta_1 = \beta_2 = \beta_3 = \beta_4$$

$$H_a: \beta_j \neq 0 \text{ for some } j$$

I am also interested in determining whether row and column treatments interact. Thus, I also will test

$$H_o: (\alpha\beta)_{ij} = 0 \text{ for all } i, j$$

$$H_o: (\alpha\beta)_{ij} \neq 0 \text{ for some } i, j$$

I uploaded data into R and fitted a two-factor factorial model and the ANOVA test results are shown in Table 2.

Table 2. Analysis of Variance for the rise of dough data

Source of Variation	**Sum of Squares**	**Degree of Freedom**	**Mean Square**	**F_0**	***P*-Value**
Yeast	14276.3	3	4758.8	743.4	< 2.2e-16
Temperature	1223.2	3	407.7	63.7	< 2.2e-16
Interaction	473.9	9	52.7	8.2	2.8e-7
Error	307.3	48	6.4		
Total	16280.7	63			

Because p-values are small, I concluded that there is a significant interaction between the amount of yeast and temperature. And the main effects of yeast and temperature are also significant.

3.3 Multiple Comparisons and Visualizing Effects

The ANOVA table indicates the row and column means differ. I use Tukey's HSD test to make comparisons between the individual row or column means to discover the specific differences. However, in my experiment, the interaction is significant. The comparisons between the means of one factor may be obscured by the interaction. I am interested in detecting differences among the means of the four levels of yeast and I fix factor temperature at a specific level of 20°C and apply Tukey's test to the means of yeast factor at this level. I assume that the best estimate of the error variance is the MS_E from ANOVA table (table 5.2), utilizing the assumption that the experimental error is the same over all treatment combinations.

The four rise averages at 20°C:

$$\bar{y}_{11.} = 0.25 \text{ (0\% yeast)}$$

$$\bar{y}_{21.} = 19.77 \text{ (1\% yeast)}$$

$$\bar{y}_{31.} = 26.25 \text{ (2\% yeast)}$$

$$\bar{y}_{41.} = 28.75 \text{ (3\% yeast)}$$

And

$$T_{0.05} = q_{0.05}(a, f)\sqrt{\frac{MS_E}{n}} = 4.76$$

$T_{.05}$=4.76 and the pairwise comparisons yield

2 vs. 1: 19.77-0.25 =19.52 > $T_{0.05}$=4.76

3 vs. 1: 26.25-0.25 = 26.00 > $T_{0.05}$=4.76

4 vs. 1: 28.75-0.25 = 28.50 > $T_{0.05}$=4.76

3 vs. 2: 26.25-19.77 = 6.48 > $T_{0.05}$=4.76

4 vs. 2: 28.75-19.77 = 8.98 > $T_{0.05}$=4.76

4 vs. 3: 28.75-26.25 = 2.5 < $T_{0.05}$=4.76

The analysis indicates that at 20°C, the mean rise of dough is not significantly different for yeast at levels 3(2%) and 4(3%). Other comparisons are significant.

I constructed graphs of the average rise at each treatment combination to interpret the results of this experiment. The graphs are shown in Figure 2 and Figure 3. Figures 2 and 3 show that because the interaction is significant, these relationships I got from Turkey's test above do not hold for the other levels of the factor. For example, at 25°C, the mean responses are not significantly different for 1%, 2%, and 3% yeast. I did all Tukey's tests in R and added the results in the appendix.

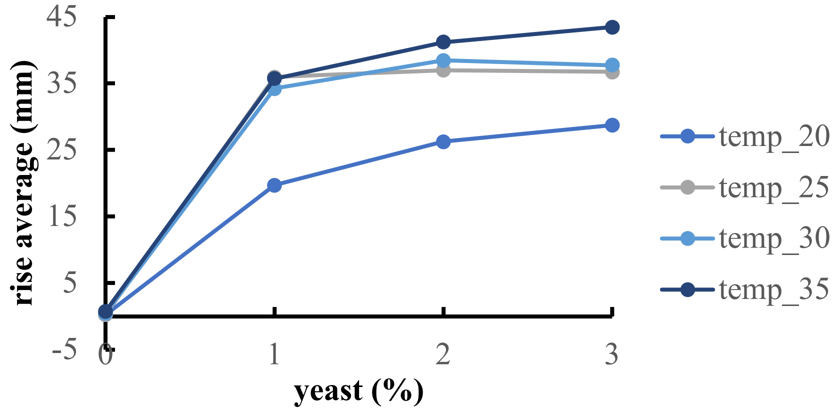


Figure 2. Temperature-yeast plot

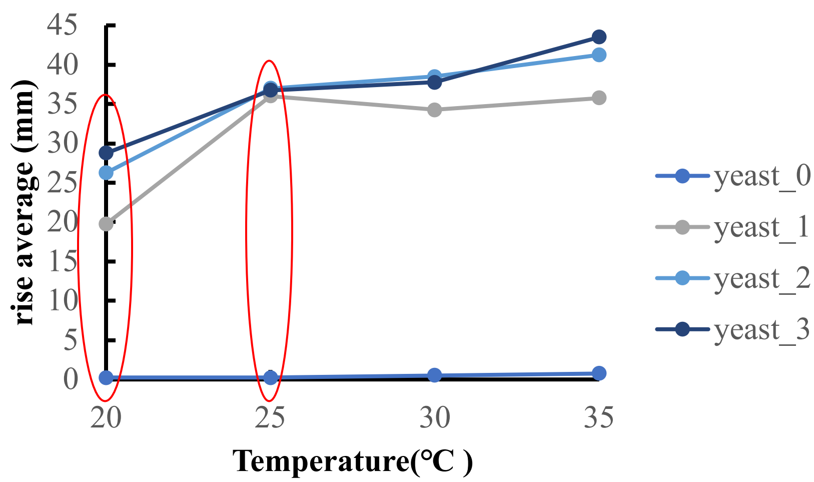
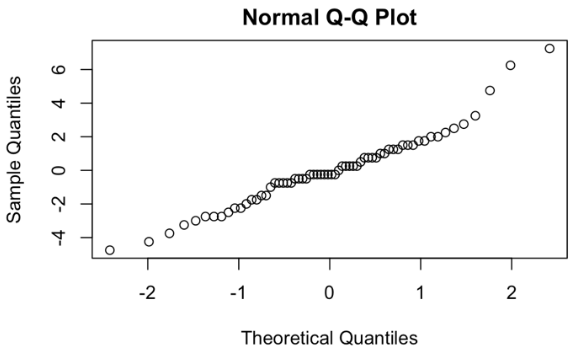


Figure 3. Yeast-temperature plot

3.4 Model Adequacy Checking

In this study, I use residual analysis to check model adequacy. Figure 4 shows the normal probability plot of residuals. The graph does not show wired patterns and the points are in a relatively straight line. The normality assumption has not been violated. Figure 5 shows a sequence of residuals, and it is a random looking, jagged with no big patterns. The residuals are independent. Figure 6 shows the residuals vs predicted data. The plot shows that there is some mild tendency for the variance of the residuals as the rise of dough increases. Figures 7 and 8 plot the residuals versus the amount of yeast and temperature, respectively. Both plots indicate mild inequality of variance.



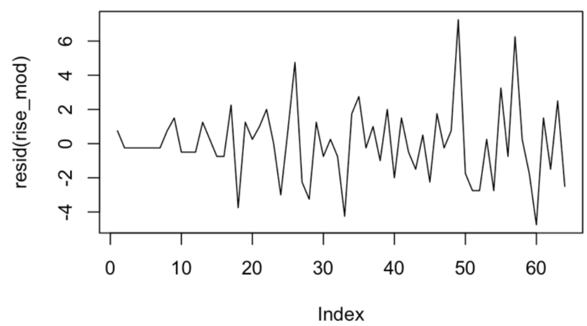


Figure 4. Normal probability plot of the residuals

Figure 5. A sequence of residuals

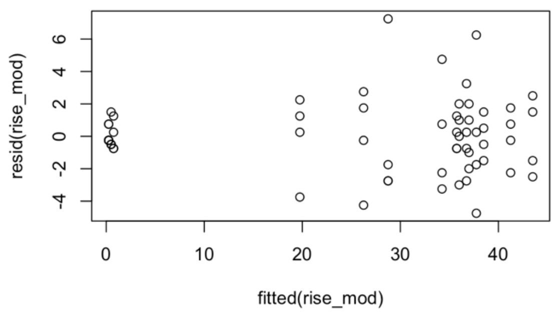
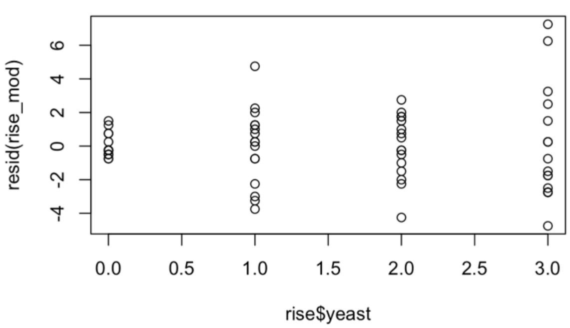


Figure 6. The residuals vs predicted data



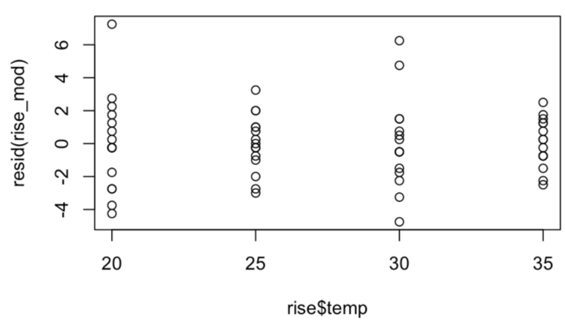


Figure 7. Residuals vs yeast

Figure 8. Residuals vs temperature

4. Conclusion

The model satisfies the normality, indecency, and constant variance assumptions.

The statistical analysis of the data clearly indicates yeast percentage, temperature, and their interaction significantly affect the rise of dough. I also notice that, as shown in Figure 2, at 20℃ a lower temperature, the increase of yeast percentage affects the rise of dough more effectively than that at other higher temperatures. This is because when the temperature is low, the yeast is less active, and the rise of dough is more sensitive to yeast amount. At a higher temperature, when the percentage of yeast is larger than 1%, the increase of yeast amount will not effectively increase the rise of dough. This is because when temperature is high, the yeast is active, and less yeast is needed

5. Future improvement

Because of the limited experiment time and space, I conducted an experiment with a comparatively large factor interval. In future work, I will fine the levels of factors. For example, the levels of yeast will be 0, 0.5%,1%, 1.5%, 2%, 2.5% and 3%.

Both the yeast and temperature can be represented as numeric variables, a polynomial model can be fitted and compared with the factorial model.

There are many factors that affect the rise of dough. In this experiment, I focused on yeast and temperature. For the future work, I will add other factors such as flour type, yeast type, water percentage.

Appendix

```
rise<-read.csv("rise.csv")
rise$yeast<-as.factor(rise$yeast)
rise$temp<-as.factor(rise$temp)
str(rise)
```

```
## 'data.frame':    64 obs. of  3 variables:
##  $ yeast: Factor w/ 4 levels "0","1","2","3": 1 1 1 1 1 1 1 1 1 1 ...
##  $ temp : Factor w/ 4 levels "20","25","30",..: 1 1 1 1 2 2 2 2 3 3 ...
##  $ rise : int  1 0 0 0 0 0 0 1 2 0 ...
```

```
rise_mod<-lm(rise~yeast*temp,data=rise)
anova(rise_mod)
```

```
## Analysis of Variance Table
##
## Response: rise
##            Df  Sum Sq Mean Sq  F value    Pr(>F)
## yeast       3 14276.3  4758.8 743.4361 < 2.2e-16 ***
## temp        3  1223.2   407.7  63.6965 < 2.2e-16 ***
## yeast:temp  9   473.9    52.7   8.2259 2.851e-07 ***
## Residuals  48   307.3     6.4
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

```
library(DescTools)
PostHocTest(aov(rise_mod),method="hsd")$'yeast:temp'
```

```
##                    diff       lwr.ci     upr.ci         pval
## 1:20-0:20  1.950000e+01  13.03689473  25.963105 0.000000e+00
## 2:20-0:20  2.600000e+01  19.53689473  32.463105 0.000000e+00
## 3:20-0:20  2.850000e+01  22.03689473  34.963105 0.000000e+00
## 0:25-0:20  1.121325e-14  -6.46310527   6.463105 1.000000e+00
## 1:25-0:20  3.575000e+01  29.28689473  42.213105 0.000000e+00
## 2:25-0:20  3.675000e+01  30.28689473  43.213105 0.000000e+00
## 3:25-0:20  3.650000e+01  30.03689473  42.963105 0.000000e+00
## 0:30-0:20  2.500000e-01  -6.21310527   6.713105 1.000000e+00
## 1:30-0:20  3.400000e+01  27.53689473  40.463105 0.000000e+00
## 2:30-0:20  3.825000e+01  31.78689473  44.713105 0.000000e+00
## 3:30-0:20  3.750000e+01  31.03689473  43.963105 0.000000e+00
## 0:35-0:20  5.000000e-01  -5.96310527   6.963105 1.000000e+00
## 1:35-0:20  3.550000e+01  29.03689473  41.963105 0.000000e+00
## 2:35-0:20  4.100000e+01  34.53689473  47.463105 0.000000e+00
## 3:35-0:20  4.325000e+01  36.78689473  49.713105 0.000000e+00
## 2:20-1:20  6.500000e+00   0.03689473  12.963105 4.739071e-02
```

```
## 3:20-1:20  9.000000e+00   2.53689473  15.463105 7.173303e-04
## 0:25-1:20 -1.950000e+01 -25.96310527 -13.036895 0.000000e+00
## 1:25-1:20  1.625000e+01   9.78689473  22.713105 6.229222e-10
## 2:25-1:20  1.725000e+01  10.78689473  23.713105 9.390755e-11
## 3:25-1:20  1.700000e+01  10.53689473  23.463105 1.509570e-10
## 0:30-1:20 -1.925000e+01 -25.71310527 -12.786895 0.000000e+00
## 1:30-1:20  1.450000e+01   8.03689473  20.963105 1.767174e-08
## 2:30-1:20  1.875000e+01  12.28689473  25.213105 3.452350e-12
## 3:30-1:20  1.800000e+01  11.53689473  24.463105 2.159384e-11
## 0:35-1:20 -1.900000e+01 -25.46310527 -12.536895 1.179501e-12
## 1:35-1:20  1.600000e+01   9.53689473  22.463105 1.000295e-09
## 2:35-1:20  2.150000e+01  15.03689473  27.963105 0.000000e+00
## 3:35-1:20  2.375000e+01  17.28689473  30.213105 0.000000e+00
## 3:20-2:20  2.500000e+00  -3.96310527   8.963105 9.879993e-01
## 0:25-2:20 -2.600000e+01 -32.46310527 -19.536895 0.000000e+00
## 1:25-2:20  9.750000e+00   3.28689473  16.213105 1.775498e-04
## 2:25-2:20  1.075000e+01   4.28689473  17.213105 2.625829e-05
## 3:25-2:20  1.050000e+01   4.03689473  16.963105 4.250984e-05
## 0:30-2:20 -2.575000e+01 -32.21310527 -19.286895 0.000000e+00
## 1:30-2:20  8.000000e+00   1.53689473  14.463105 4.269167e-03
## 2:30-2:20  1.225000e+01   5.78689473  18.713105 1.415702e-06
## 3:30-2:20  1.150000e+01   5.03689473  17.963105 6.124488e-06
## 0:35-2:20 -2.550000e+01 -31.96310527 -19.036895 0.000000e+00
## 1:35-2:20  9.500000e+00   3.03689473  15.963105 2.840189e-04
## 2:35-2:20  1.500000e+01   8.53689473  21.463105 6.745146e-09
## 3:35-2:20  1.725000e+01  10.78689473  23.713105 9.390755e-11
## 0:25-3:20 -2.850000e+01 -34.96310527 -22.036895 0.000000e+00
## 1:25-3:20  7.250000e+00   0.78689473  13.713105 1.495584e-02
## 2:25-3:20  8.250000e+00   1.78689473  14.713105 2.761804e-03
## 3:25-3:20  8.000000e+00   1.53689473  14.463105 4.269167e-03
## 0:30-3:20 -2.825000e+01 -34.71310527 -21.786895 0.000000e+00
## 1:30-3:20  5.500000e+00  -0.96310527  11.963105 1.780287e-01
## 2:30-3:20  9.750000e+00   3.28689473  16.213105 1.775498e-04
## 3:30-3:20  9.000000e+00   2.53689473  15.463105 7.173303e-04
## 0:35-3:20 -2.800000e+01 -34.46310527 -21.536895 0.000000e+00
## 1:35-3:20  7.000000e+00   0.53689473  13.463105 2.224607e-02
## 2:35-3:20  1.250000e+01   6.03689473  18.963105 8.682146e-07
## 3:35-3:20  1.475000e+01   8.28689473  21.213105 1.091012e-08
## 1:25-0:25  3.575000e+01  29.28689473  42.213105 0.000000e+00
## 2:25-0:25  3.675000e+01  30.28689473  43.213105 0.000000e+00
## 3:25-0:25  3.650000e+01  30.03689473  42.963105 0.000000e+00
## 0:30-0:25  2.500000e-01  -6.21310527   6.713105 1.000000e+00
## 1:30-0:25  3.400000e+01  27.53689473  40.463105 0.000000e+00
## 2:30-0:25  3.825000e+01  31.78689473  44.713105 0.000000e+00
## 3:30-0:25  3.750000e+01  31.03689473  43.963105 0.000000e+00
## 0:35-0:25  5.000000e-01  -5.96310527   6.963105 1.000000e+00
## 1:35-0:25  3.550000e+01  29.03689473  41.963105 0.000000e+00
## 2:35-0:25  4.100000e+01  34.53689473  47.463105 0.000000e+00
## 3:35-0:25  4.325000e+01  36.78689473  49.713105 0.000000e+00
## 2:25-1:25  1.000000e+00  -5.46310527   7.463105 9.999998e-01
## 3:25-1:25  7.500000e-01  -5.71310527   7.213105 1.000000e+00
## 0:30-1:25 -3.550000e+01 -41.96310527 -29.036895 0.000000e+00
## 1:30-1:25 -1.750000e+00  -8.21310527   4.713105 9.997399e-01
## 2:30-1:25  2.500000e+00  -3.96310527   8.963105 9.879993e-01
```

```
## 3:30-1:25  1.750000e+00  -4.71310527   8.213105 9.997399e-01
## 0:35-1:25 -3.525000e+01 -41.71310527 -28.786895 0.000000e+00
## 1:35-1:25 -2.500000e-01  -6.71310527   6.213105 1.000000e+00
## 2:35-1:25  5.250000e+00  -1.21310527  11.713105 2.361602e-01
## 3:35-1:25  7.500000e+00   1.03689473  13.963105 9.943767e-03
## 3:25-2:25 -2.500000e-01  -6.71310527   6.213105 1.000000e+00
## 0:30-2:25 -3.650000e+01 -42.96310527 -30.036895 0.000000e+00
## 1:30-2:25 -2.750000e+00  -9.21310527   3.713105 9.717965e-01
## 2:30-2:25  1.500000e+00  -4.96310527   7.963105 9.999609e-01
## 3:30-2:25  7.500000e-01  -5.71310527   7.213105 1.000000e+00
## 0:35-2:25 -3.625000e+01 -42.71310527 -29.786895 0.000000e+00
## 1:35-2:25 -1.250000e+00  -7.71310527   5.213105 9.999963e-01
## 2:35-2:25  4.250000e+00  -2.21310527  10.713105 5.734220e-01
## 3:35-2:25  6.500000e+00   0.03689473  12.963105 4.739071e-02
## 0:30-3:25 -3.625000e+01 -42.71310527 -29.786895 0.000000e+00
## 1:30-3:25 -2.500000e+00  -8.96310527   3.963105 9.879993e-01
## 2:30-3:25  1.750000e+00  -4.71310527   8.213105 9.997399e-01
## 3:30-3:25  1.000000e+00  -5.46310527   7.463105 9.999998e-01
## 0:35-3:25 -3.600000e+01 -42.46310527 -29.536895 0.000000e+00
## 1:35-3:25 -1.000000e+00  -7.46310527   5.463105 9.999998e-01
## 2:35-3:25  4.500000e+00  -1.96310527  10.963105 4.779064e-01
## 3:35-3:25  6.750000e+00   0.28689473  13.213105 3.268814e-02
## 1:30-0:30  3.375000e+01  27.28689473  40.213105 0.000000e+00
## 2:30-0:30  3.800000e+01  31.53689473  44.463105 0.000000e+00
## 3:30-0:30  3.725000e+01  30.78689473  43.713105 0.000000e+00
## 0:35-0:30  2.500000e-01  -6.21310527   6.713105 1.000000e+00
## 1:35-0:30  3.525000e+01  28.78689473  41.713105 0.000000e+00
## 2:35-0:30  4.075000e+01  34.28689473  47.213105 0.000000e+00
## 3:35-0:30  4.300000e+01  36.53689473  49.463105 0.000000e+00
## 2:30-1:30  4.250000e+00  -2.21310527  10.713105 5.734220e-01
## 3:30-1:30  3.500000e+00  -2.96310527   9.963105 8.360310e-01
## 0:35-1:30 -3.350000e+01 -39.96310527 -27.036895 0.000000e+00
## 1:35-1:30  1.500000e+00  -4.96310527   7.963105 9.999609e-01
## 2:35-1:30  7.000000e+00   0.53689473  13.463105 2.224607e-02
## 3:35-1:30  9.250000e+00   2.78689473  15.713105 4.524388e-04
## 3:30-2:30 -7.500000e-01  -7.21310527   5.713105 1.000000e+00
## 0:35-2:30 -3.775000e+01 -44.21310527 -31.286895 0.000000e+00
## 1:35-2:30 -2.750000e+00  -9.21310527   3.713105 9.717965e-01
## 2:35-2:30  2.750000e+00  -3.71310527   9.213105 9.717965e-01
## 3:35-2:30  5.000000e+00  -1.46310527  11.463105 3.062468e-01
## 0:35-3:30 -3.700000e+01 -43.46310527 -30.536895 0.000000e+00
## 1:35-3:30 -2.000000e+00  -8.46310527   4.463105 9.987904e-01
## 2:35-3:30  3.500000e+00  -2.96310527   9.963105 8.360310e-01
## 3:35-3:30  5.750000e+00  -0.71310527  12.213105 1.314218e-01
## 1:35-0:35  3.500000e+01  28.53689473  41.463105 0.000000e+00
## 2:35-0:35  4.050000e+01  34.03689473  46.963105 0.000000e+00
## 3:35-0:35  4.275000e+01  36.28689473  49.213105 0.000000e+00
## 2:35-1:35  5.500000e+00  -0.96310527  11.963105 1.780287e-01
## 3:35-1:35  7.750000e+00   1.28689473  14.213105 6.545207e-03
## 3:35-2:35  2.250000e+00  -4.21310527   8.713105 9.957413e-01
```

```
PostHocTest(aov(rise_mod),method="hsd")$yeast
```

```
##          diff     lwr.ci     upr.ci          pval
```

```
## 1-0 31.0000 28.619403 33.380597 0.000000e+00
## 2-0 35.3125 32.931903 37.693097 0.000000e+00
## 3-0 36.2500 33.869403 38.630597 0.000000e+00
## 2-1  4.3125  1.931903  6.693097 8.517170e-05
## 3-1  5.2500  2.869403  7.630597 2.334909e-06
## 3-2  0.9375 -1.443097  3.318097 7.222288e-01
```

```
PostHocTest(aov(rise_mod),method="hsd")$temp
```

```
##          diff     lwr.ci    upr.ci         pval
## 25-20  8.7500  6.3694025 11.130597 1.929568e-13
## 30-20  9.0000  6.6194025 11.380597 0.000000e+00
## 35-20 11.5625  9.1819025 13.943097 0.000000e+00
## 30-25  0.2500 -2.1305975  2.630597 9.922800e-01
## 35-25  2.8125  0.4319025  5.193097 1.465775e-02
## 35-30  2.5625  0.1819025  4.943097 3.038598e-02
```

```
rise20<-rise[rise$temp=="20",]
aggregate(rise20$rise,list(rise20$yeast),FUN=mean)
```

```
##   Group.1     x
## 1       0  0.25
## 2       1 19.75
## 3       2 26.25
## 4       3 28.75
```

```
rise25<-rise[rise$temp=="25",]
aggregate(rise25$rise,list(rise25$yeast),FUN=mean)
```

```
##   Group.1     x
## 1       0  0.25
## 2       1 36.00
## 3       2 37.00
## 4       3 36.75
```

```
rise30<-rise[rise$temp=="30",]
aggregate(rise30$rise,list(rise30$yeast),FUN=mean)
```

```
##   Group.1     x
## 1       0  0.50
## 2       1 34.25
## 3       2 38.50
## 4       3 37.75
```

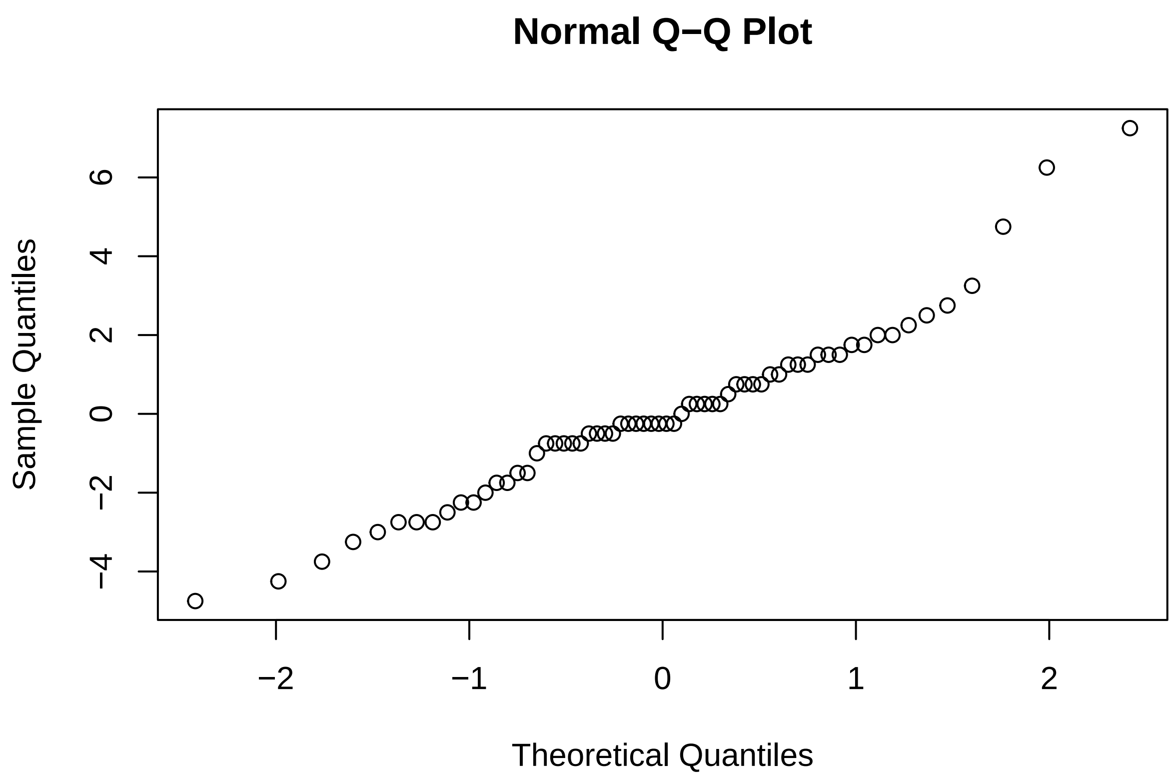
```
rise35<-rise[rise$temp=="35",]
aggregate(rise35$rise,list(rise35$yeast),FUN=mean)
```

```
##   Group.1     x
## 1       0  0.75
## 2       1 35.75
## 3       2 41.25
## 4       3 43.50
```

```r
qtukey(0.95,4,48)*sqrt(6.4/4)
```

```
## [1] 4.760808
```

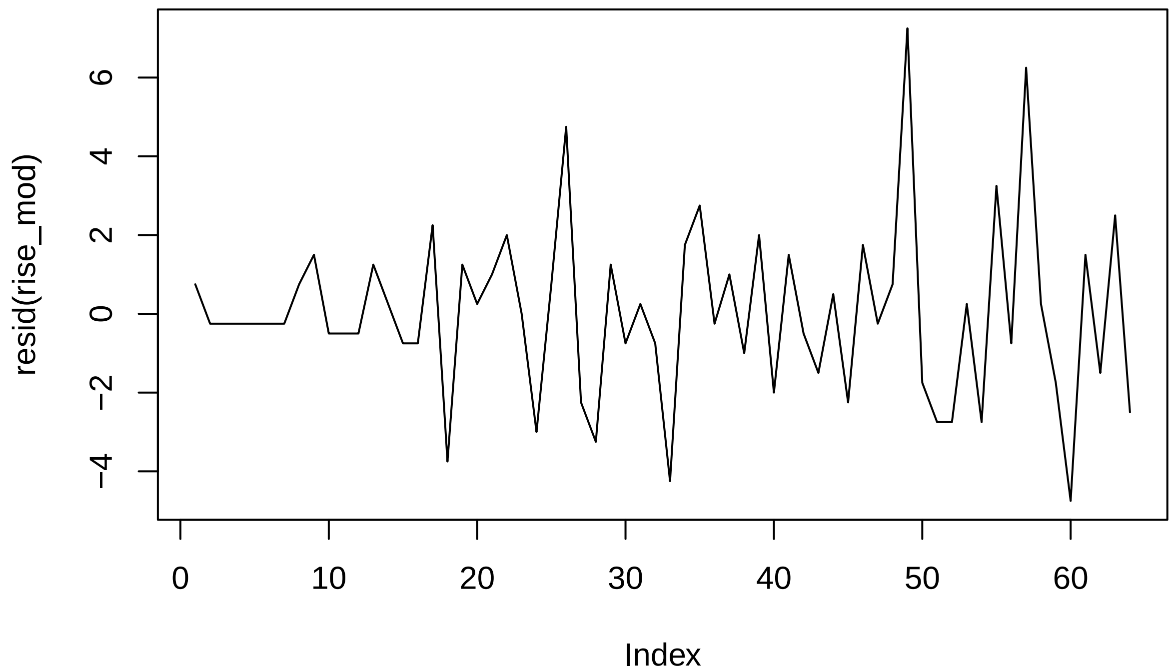
```r
#Normality assumption
qqnorm(resid(rise_mod))
```



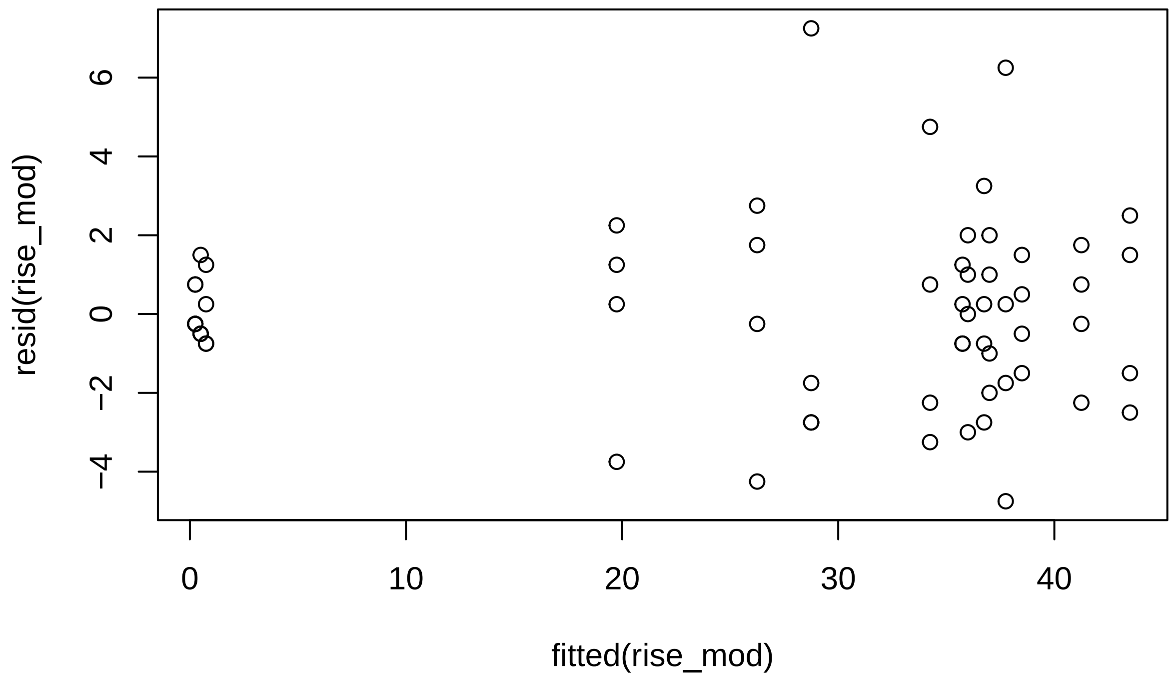

```r
shapiro.test((resid(rise_mod)))
```

```
##
##  Shapiro-Wilk normality test
##
## data:  (resid(rise_mod))
## W = 0.95835, p-value = 0.03005
```

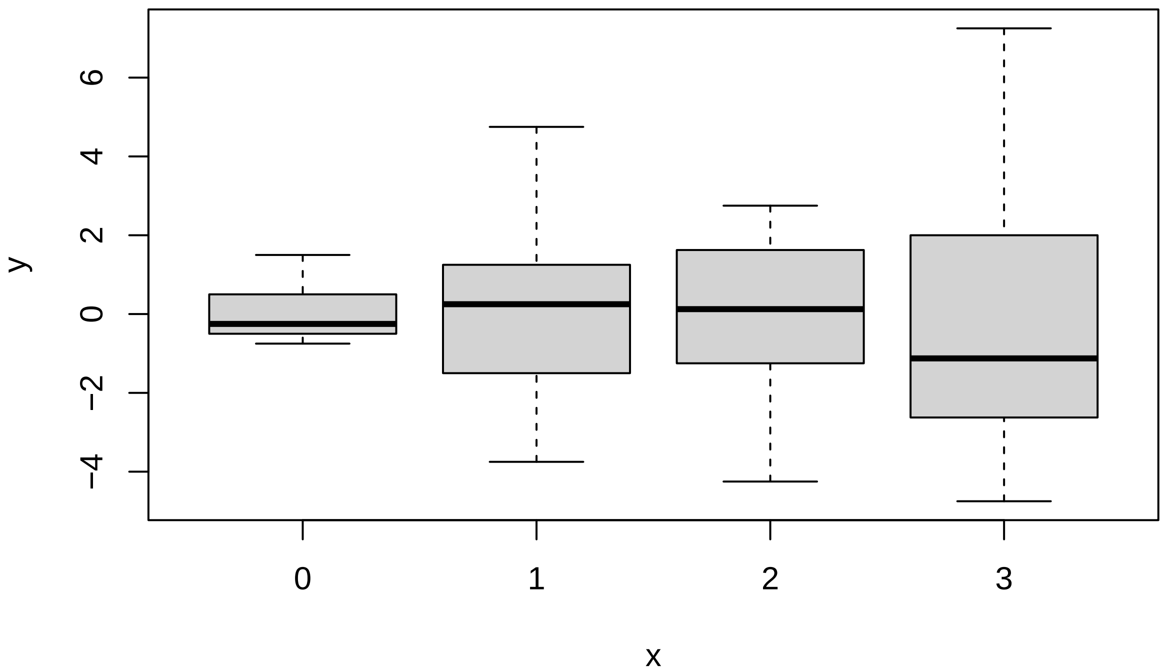
```r
#Independence
plot(resid(rise_mod),type="l")
```



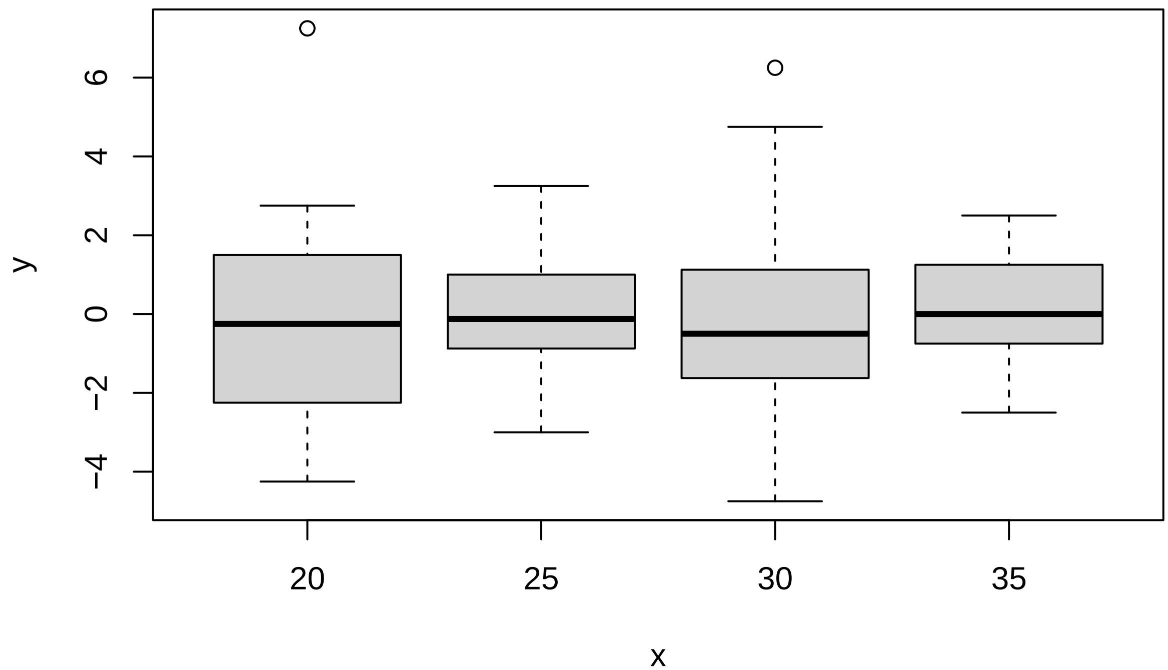

```
#Constant variance
plot(fitted(rise_mod),resid(rise_mod))
```




```
plot(rise$yeast,resid(rise_mod))
```




```
plot(rise$temp,resid(rise_mod))
```



y
6
4
2
0
−2
−4
20
25
30
35
x