Orbital Motion of EZ Aquarii Triple Star System

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1 Introduction

This project encompasses the motion of the EZ Aquarii triple star system and their related orbital motions interpreted as a hierarchical three-body problem. Stars Aquarii A and Aquarii C (referred as star A and star C from here on out) form a spectroscopic binary [1] with a 3.8 day orbit, whereas their companion star Aquarii B (referred to as star B from here on out) orbits the spectroscopic binary with an ~ 823 day period. The A-C binary orbits each other at nearly circular orbits, whereas star B orbits the AC system with an eccentricity of 0.439 ± 0.001. All three stars are MV-type red dwarfs, where A has a mass of 0.1187 M_{Sun} , B with 0.1145 M_{Sun} and C with 0.0930 M_{Sun}. The AC binary orbits each other with a constant distance of 0.03 AU while B orbits the binary with an average separation from the AC binary's COM at 1.22 AU [2]. The orbits of all three of the stars are along their own plane within space, confining their motion to 2-dimensions.

2 Methodology

A hierarchical three body approach signifies a subsystem - the AC binary - within the Aquarii system as a singular body. The entire system will consist of star B orbiting the AC COM, treating this system almost as 2 2-body problems. Ultimately the inertia of star B will affect the motion of the COM of the AC binary, yet won't disturb the AC orbits themselves around their COM. On the other hand, the entire AC system will contribute to the orbital motion and movement of star B, ultimately creating an eccentric orbit along the binary's axis. The stars all orbit along their own plane, creating a simpler calculation of their motions.

Initial velocities and positions can be defined from the relation of known semimajor axes between the systems. Due to differences in mass, one can use the mass-velocity relation to compute the constant velocities of star's A and C about the center of mass of the system. .

$$v_i = \pm \frac{m_j}{M_{AC}} * \frac{2\Pi a_{AC}}{T_{AC}}$$

This relationship shows that the ratio of the masses of the objects are inversely proportional to their velocity relationship. The velocity of the objects can be calculated from the known period of the system (~3.78 D) but since masses are unequal it's multiplied by the ratio of the other objects' mass divided by the total mass. The stars will have their velocities in the opposite directions where the gravitational acceleration between them will keep their distance between one another constant.

To find the velocity of star B, the vis-visa equation will be used as a function of the distance between star B to the center of mass of the A-C system as the orbit of B is elliptical around A and C.

$$v(r) = \sqrt{G(M_{AC} + m_{B})\left(\frac{2}{r} - \frac{1}{a}\right)}$$

Here, M_{AC} is the mass of the AC system, M_B is the mass of star B, a is chosen initially as the average separation between AC to B, and r is the position along the elliptical orbit between B's COM and the AC center of mass. This velocity will become smaller as r increases, meaning that at apoapsis the velocity will be lowest for star B whereas at periapsis it will be highest.

Initial positions for A and C are defined as their constant separation (0.03AU) while for B it is initially defined as its average separation (1.22AU). The distances of A from C and B from the AC COM will initially be along the x-axis, whereas the initial velocities will be along the y-axis. B's position will ultimately vary between periapsis and apoapsis which can ultimately be checked from the following equations:

$$r_{peri} = a(1 - e)$$
$$r_{apo} = a(1 + e)$$

Where e is star B's eccentricity and a is the average separation between B and the AC system.

Each body enacts a gravitational force against one another, but since a hierarchical approach is being taken star B will be affected by the combined mass and COM of the AC system. For the gravitational force each body will enact on another for the two separate systems, and an acceleration will be calculated from the formula

$$a_{ij} = \sum_{j \neq i} \frac{-GM_j}{r_{ij}^2}$$

I and J cannot equal one another as one's own body cannot cause an acceleration on its own center of mass. In this sense however, only star B and the AC system as a whole will be accelerating, as stars A and C orbit each other at nearly circular orbits allowing for a near constant velocity throughout their binary orbit.

Once all of the initial parameters and the acceleration function for both of the 2 2-body systems is defined, the Runge-Kutta 4 integration method will be used to evaluate and update positions and velocities in a 3 dimensional space based on the defined acceleration function, and ultimately will calculate and solve the equations of motion for the three objects.

All of the calculations were calculated using coding programs in C and Python - C to calculate data point values and save them to a text file for the positions and velocities of each object as a function of time. The data point values would then be read into python scripts to ultimately produce movies (GIFs) of the orbital motions in 2 and 3 dimensional visuals. Matplotlib would be the primary library used to plot and create the animations of the orbits.

3 Data and Results

The resultant orbital motions of the system and its stellar components is periodic throughout each ~ 823 day period for star B. Data points were calculated in intervals of 1000 second timesteps, where every 1000 individual sets of position and velocity data were skipped. 30 frames are plotted per second in each of the GIF's. Circular motion is displayed below in a GIF involving stars A and C:

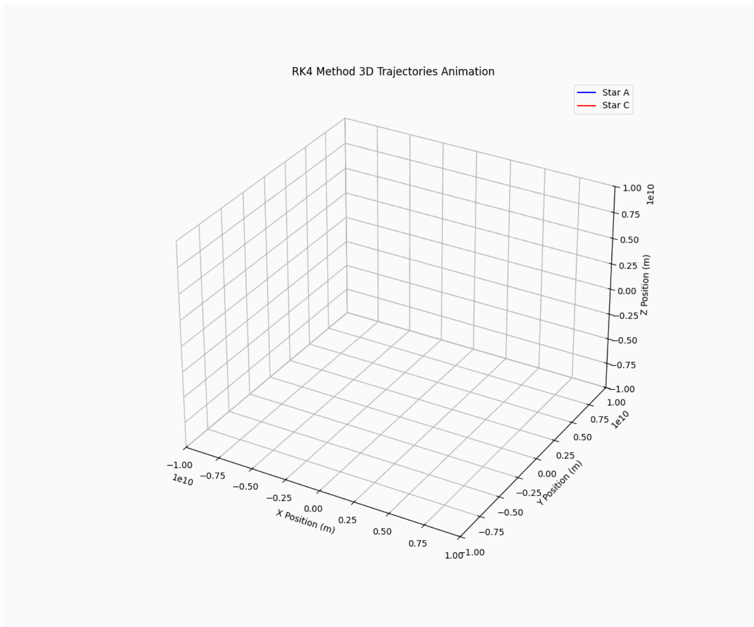

Figure 1: Motion of stars A and C relative to one another orbiting at constant velocity.

Figure 1 displays relative distances between stars A and C stay constant with total separation of 0.03 AU from each COM, with star C at a distance ~ 0.01655 AU from the AC COM and ~ 0.01345 AU between star A and the AC COM due to their mass imbalance. Their constant total velocities of $3.58 * 10^4$ m/s for star A and $4.56 * 10^4$ m/s for star C are shown from this constant separation.

The movie below shows the entire 3 body motion of the system given a kick in the y-axis of the entire 3 body systems' center of mass due to initial velocity parameters along the y axis for the objects.

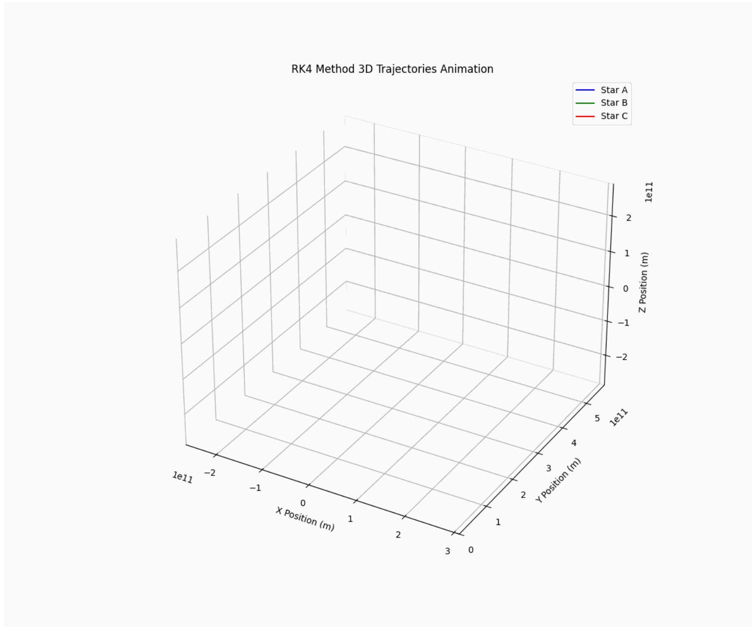

Figure 2: Entire 3 body motion of the EZ_Aquarii triple star system.

Star B's motion is ultimately confined within the boundaries of distances of 0.683 AU at periapsis from the AC COM to 1.756 at apoapsis. We can see that the system ultimately continues to move along the y axis due to this initial kick, where practically there isn't any force acting on the objects, but was added to display the motion of the system for example if it were all moving along through a galaxy parallel with the y-axis for visual effect. An overhead view of this along the xy plane is shown below.

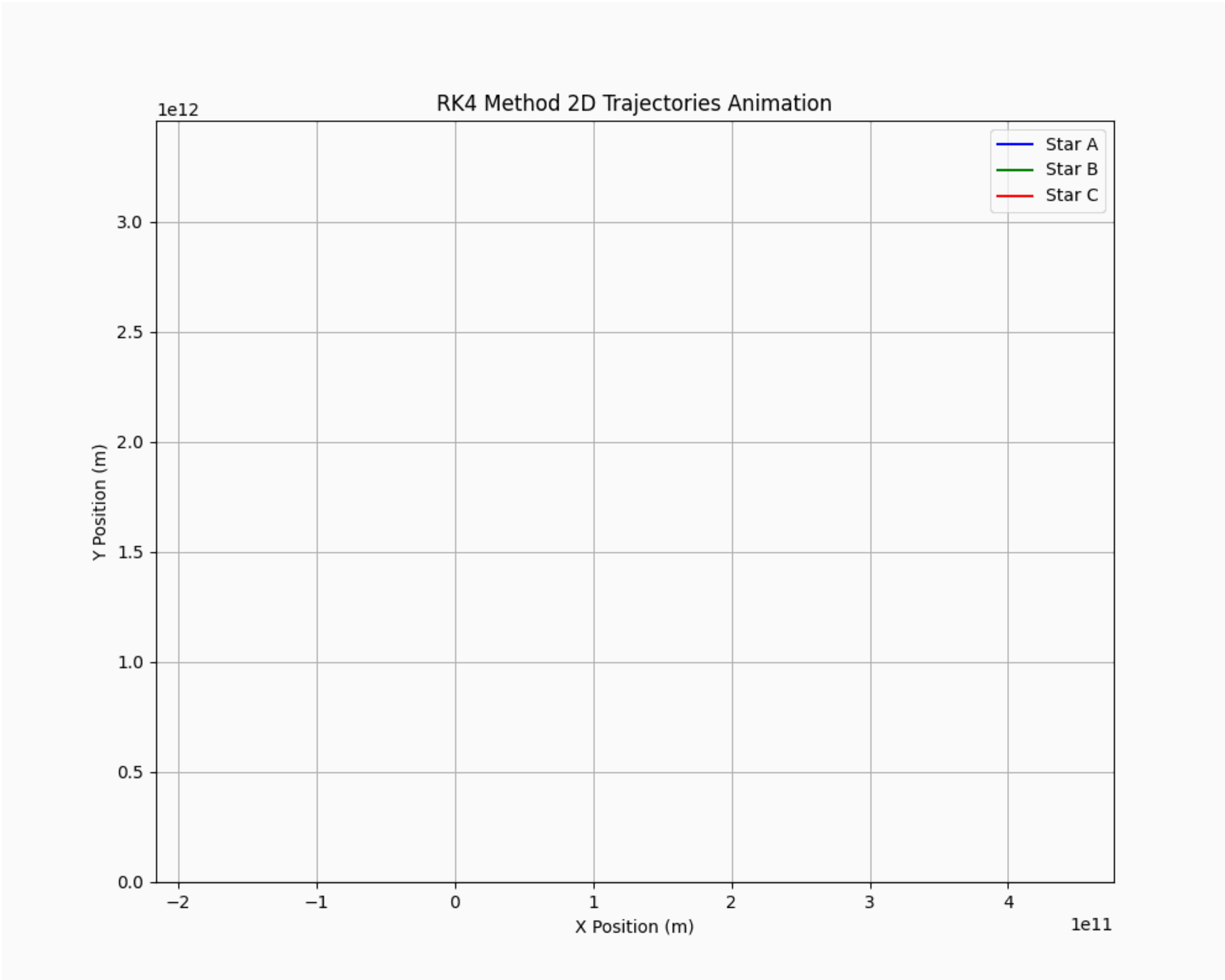

Figure 3: Overhead view of the entire motion of the EZ_Aquarii system viewed onto the xy plane.

We can see in Figure 3 the entire resultant motion doesn't occur at all in the z-direction due to the orbit along a singular plane that is projected onto the xy plane in our simulation. We can see star B move fastest radially when it is seen closest to stars A and C and move slowest when furthest away.

4 Conclusion

The periodic orbital motions of the EZ_Aquarii triple star system were calculated and plotted using a Runge-Kutta 4th order integration method from C and Python code. The overall motion in this simulation is seen as periodic based on the condition of constant separation between the inner A-C binary, leading to simplified, periodic motion throughout the orbitals. In reality, motions may not be this periodic and are more likely chaotic due to other external forces, as well as potential perturbations from star B onto the individual stars A and C themselves, which could lead to chaotic motion. This simulation provides a simplified version of the hierarchical three body problem where separation of stars A to C stays constant.

Ultimately, this provides a good basis for potential orbital motions of other 3-body systems if periodic, which can lead to further understanding of how other systems may interact with one another.

5 References

[1] “EZ Aquarii.” *Wikipedia*, Wikimedia Foundation, 22 Nov. 2024, en.wikipedia.org/wiki/EZ_Aquarii.

[2] *EZ Aquarii 3*, chview.nova.org/solcom/stars/ez-aqr3.htm. Accessed 5 Dec. 2024.

6 Coding Attachments

https://github.com/ricotti/ASTR415-Fall24/tree/main/TERM-PROJECTs/CL_SS_project