Capability

Improving **process capability** is one of the three necessary requirements of a successful Statistical Process Control (SPC) strategy for quality improvement.

The other two being: obtaining stable processes and reducing variability.

Objectives

- Define Process Capability.
- Describe how to determine if a process is capable.
- Calculate process capability Indices: C_p, C_{pk}, C_{pm}, C_{npk}.
- Build appropriate confidence intervals for process capability indices.

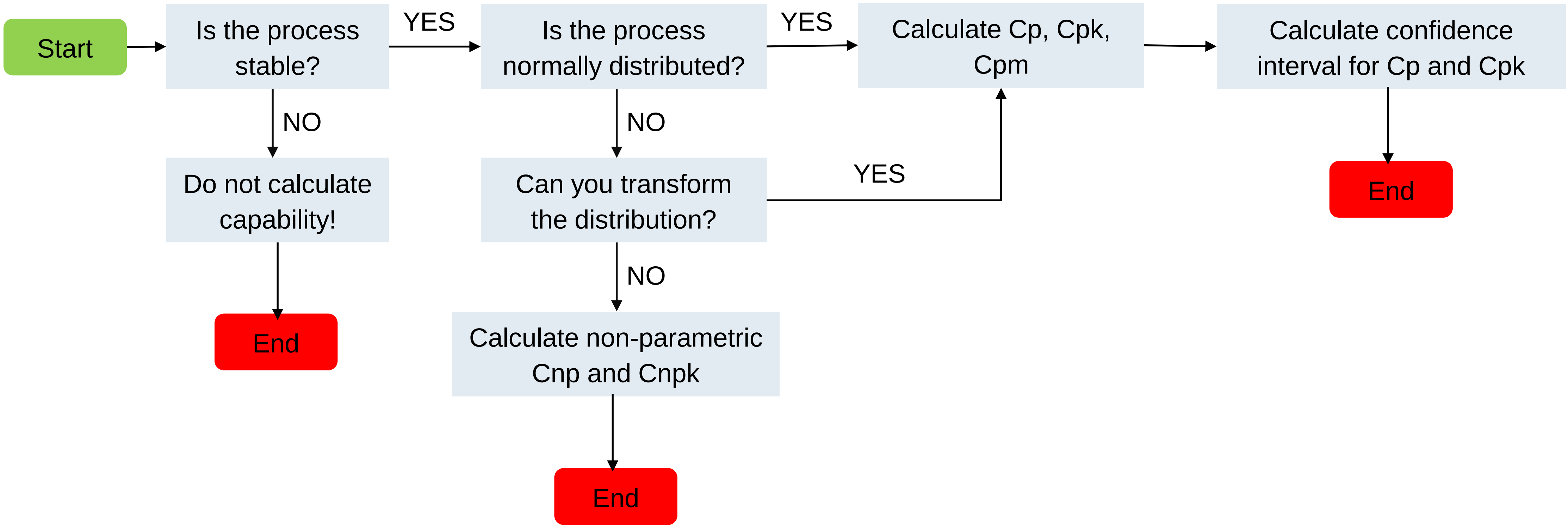
What is Process Capability?

- **Process Capability** is the ability of a process to generate a product that meets engineering and customer specifications.
- **Capability indices** are used to measure process capability and are calculated by comparing the width of the process specification against the width of the process distribution.
- A **capable process** is one where the distributions of the process output measurements are ***centered on target***, and a ***very high*** percentage of the measurements fall within the specification limits.

Capability Indices can be used to:

- Track the relative improvement of a process over time.
- Estimate the percentage of defects or non-conforming product.
- Compare the capability of several processes, each with different units of measurement and different specifications.
- Identify and prioritize processes in need of improvement.
- Be part of the acceptance criteria for transferring a process from development to manufacturing.
- Be part of the qualification criteria for assessing suppliers.

Flowchart for Capability determination



Control limits versus Specification limits

- Control limits are obtained from measurements of a process in statistical control. Control limits are selected so that nearly all measurements or summary statistics of the measurements are within them (usually $+3\sigma$ and -3σ).
- Specification limits are determined by the customer or derived from customer requirements.

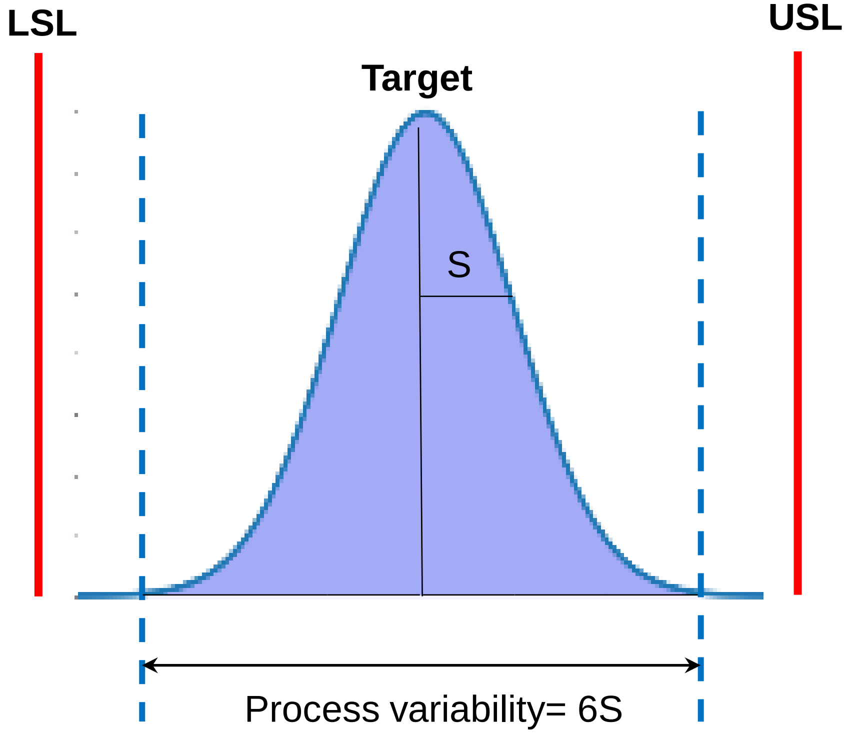
Control limits are not Specification limits

Process potential Capability Index Cp

- **Process potential** is the capability of a process to maintain ***process variability,*** within a specified allowed tolerance width, defined as **USL-LSL**, where **USL** and **LSL** are upper and lower specification limits, respectively.

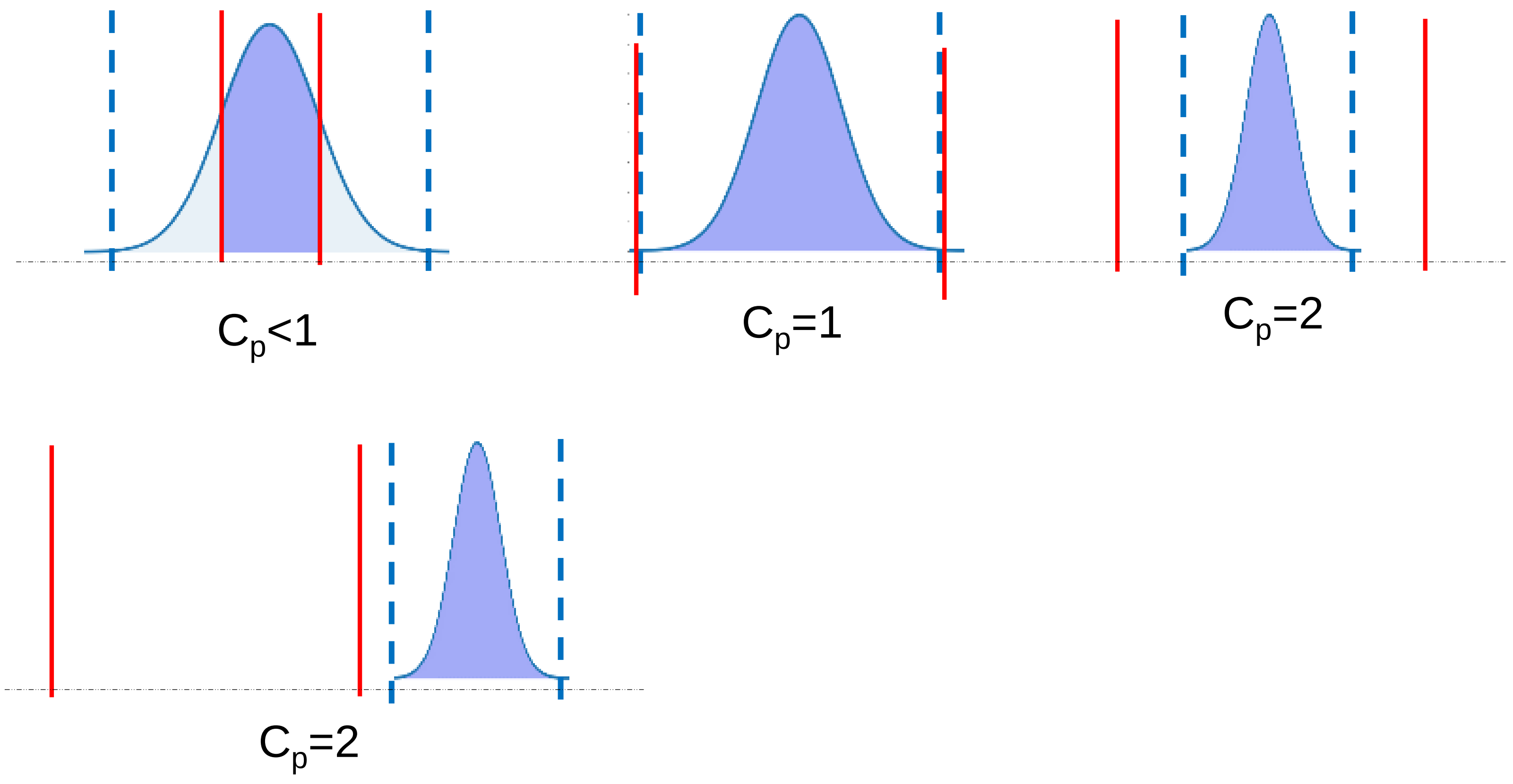
- **The process potential index, Cp**, is the ratio of the allowable process variability divided by the actual variability of the process.

$$C_p = \frac{USL - LSL}{6s}$$



S= Process standard deviation

Various levels of Potential Capability



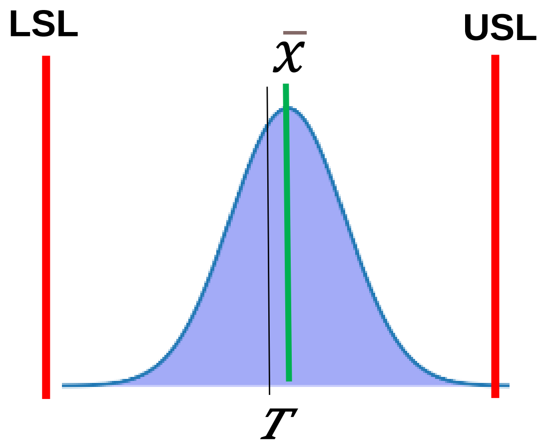
- $C_p<1$ are not capable processes.
- $C_p=1$ are just potentially capable processes.
- $C_p\geq 2$ are very good potentially capable processes.
- A process with $C_p>>1$ can have non-conforming product if it is not centered within USL and LSL.

Different Potential Capability values: Having potential capability greater that one does not necessarily mean all production is within specifications.

Process performance Capability Index C_{pk} (Real Capability Index)

- **Process performance** is the capability of a process to maintain ***process variability,*** within a specified allowed tolerance width, defined as **USL-LSL**, and centered on **Target**.

- **The real capability index, Cpk**, considers the deviation of the process mean from the process target.



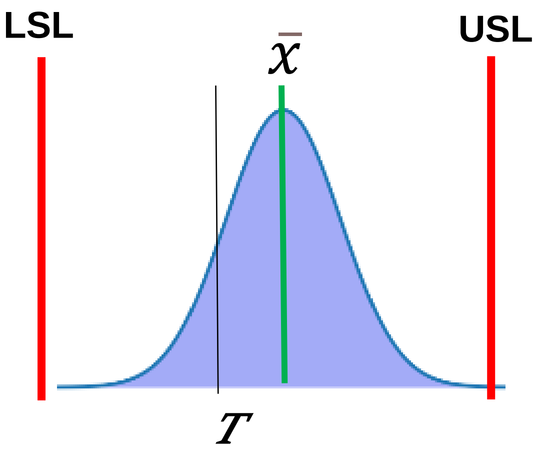
$$C_{pk} = C_p(1 - k)$$

$$k = \frac{2|T - \bar{x}|}{USL - LSL}$$

$$T = \frac{USL + LSL}{2}$$

$\bar{x}$ = process mean

Index C_{pk} for two sided specifications where Target is not at midpoint of specs

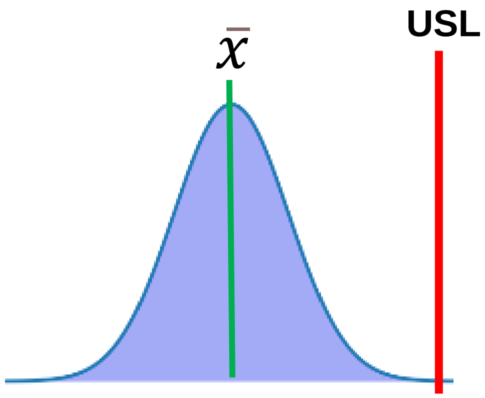


$$C_{pk} = min\left(C_{pl}, C_{pu}\right)$$

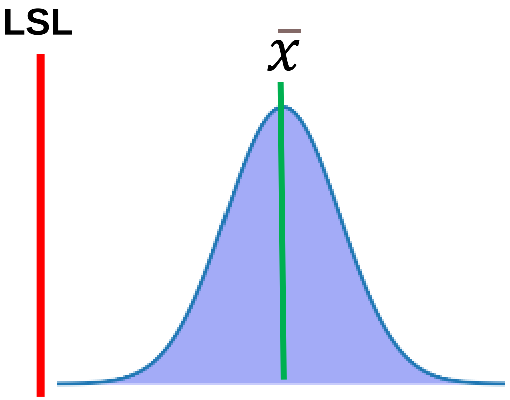
$$C_{pl} = \frac{T - LSL}{3s}\left[1 - \frac{|T - \bar{x}|}{T - LSL}\right]$$

$$C_{pu} = \frac{USL - T}{3s}\left[1 - \frac{|T - \bar{x}|}{USL - T}\right]$$

Index C_{pk} for one sided specifications



$$C_{pk} = \frac{USL - \bar{x}}{3s}$$



$$C_{pk} = \frac{\bar{x} - LSL}{3s}$$

Assumptions for C_p and C_{pk}

- The process is stable.
- The distribution of measurements is normal.
- There is a single source of variation. Or at least one dominant source of variation.

If data does not come from a normal distribution, there are three alternatives to consider:

1. Transform the data and calculate capability indices on the transformed data.
2. Use capability indices based on percentiles. A non-parametric approach.
3. Fit an alternative distribution to the data. Consult your Master Black Belt.

C_{np}, and C_{npk}: Non-parametric Capability indices

$$C_{np} = \frac{USL - LSL}{P_{0.995} - P_{0.005}}$$

$$C_{npk} = min\left(\frac{USL - P_{0.5}}{P_{0.995} - P_{0.5}}, \frac{P_{0.5} - LSL}{P_{0.5} - P_{0.005}}\right)$$

P_{α} = α percentile of the data distribution. α϶(0,1)

Confidence intervals for C_p and C_{pk}

$$\underbrace{CI_L = C_p\sqrt{\frac{\chi^2_{(\alpha/2,n-1)}}{n-1}} \qquad CI_U = C_p\sqrt{\frac{\chi^2_{(1-\alpha/2,n-1)}}{n-1}}}_{C_p}$$

$$CI = (CI_L, CI_U)$$

@ 1-α probability

$$\underbrace{CI_U = 1 + Z_{1-\alpha/2}\sqrt{\frac{1}{9nC_{pk}^2} + \frac{1}{2n}} \qquad CI_L = 1 - Z_{1-\alpha/2}\sqrt{\frac{1}{9nC_{pk}^2} + \frac{1}{2n}}}_{C_{pk}}$$

$\chi^2_{(\alpha/2,n-1)}$ and $\chi^2_{(1-\alpha/2,n-1)}$ are the α/2 and 1-α/2 percentiles of a Chi-Square distribution with n-1 degrees of freedom.

$Z_{1-\alpha/2}$ is the1-α/2 percentile of a standard normal probability distribution.

Some notes on Capability indices:

- Avoid using only C_{pk} as a measure of capability for two-sided specification processes.
- A capability index is as meaningful as the specifications on which it is calculated.
- Capability indices should not be used on unstable processes.
- If the specifications for a process output change, then recalculate your capability indices.
- Use enough runs or measurements to compute indices.
- Interpret C_p or C_{pk} in terms of defect rates (ppm values) only if the assumption of a normal population has been verified for that process.
- C_p and C_{pk} are not for attribute (non-continuous) data (ex. number of defects, counts, etc.).
- Do not average the C_p or $C_{pk's}$ across several processes.