

Case Study: Student Performance Analysis with Linear Algebra

1. Introduction

In this case study, we will use Linear Algebra concepts (vectors, vector operations, matrices, and matrix operations) to analyze student performance data.

Linear Algebra is powerful because it allows us to represent and manipulate data in a structured way. For example:

- Each student's scores can be represented as a vector.
- A group of students' scores across subjects can be represented as a matrix.
- We can then use vector and matrix operations to compute totals, averages, and weighted grades.

2. Problem Setup

We have the scores of 4 students in 3 subjects (Mathematics, English, Science):

Student	Math	English	Science
Student 1	80	70	90
Student 2	60	85	75
Student 3	95	88	92
Student 4	70	60	65

We will represent this dataset as a matrix:

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[[80, 70, 90],  
 [60, 85, 75],  
 [95, 88, 92],  
 [70, 60, 65]]
```

3. Step-by-Step Analysis

3.1 Representing Data as a Matrix

Represent the scores as a 4x3 matrix, where rows = students and columns = subjects.

3.2 Total Scores per Student (Vector Addition)

Compute the total score for each student by summing across the row vector.

3.3 Applying Weights (Scalar Multiplication)

Apply subject weights (e.g., Math is twice as important as others) using scalar multiplication.

3.4 Average Score per Subject (Matrix Operations)

Compute the average score for each subject by calculating the column-wise mean.

3.5 Final Grades using Matrix Multiplication

Use matrix multiplication with a weight vector (e.g., $[0.5, 0.3, 0.2]$) to calculate final grades.

3.6 Comparing Students (Vector Subtraction)

Compare two students' performances by subtracting their score vectors.

4. Key Takeaways

- Vectors represent individual students' scores.
- Vector addition/subtraction helps compare performance.
- Scalar multiplication applies importance/weights to subjects.
- Matrices represent the whole dataset.
- Matrix multiplication is used for weighted grading.

5. Add-on Exercises

1. Create a new 5×4 matrix of scores (5 students, 4 subjects). Compute each student's total score and the average per subject.
2. Suppose one subject (e.g., Science) is 3 times more important than others. Apply scalar multiplication and recompute totals.
3. Define new weights for grading (e.g., 40%, 20%, 30%, 10%). Compute final grades using matrix multiplication.
4. Compare Student 3 and Student 4 using vector subtraction. Who performed better in each subject?