Emergence of Neuronal synchronisation in coupled areas.

CNS Seminar- SS24
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Contents

1. Introduction
2. Neuronal Synchronisation
3. The model
4. Results

Introduction

Synchronisation in nature

- Dynamical systems often exhibit collective behaviour, which is often termed as *emergence*.
- Emergence is often found in multitude of systems which possess, dynamical, chemical and biological processes.
- One such great examples of emergent phenomena is **Synchronisation**

The Kuramoto Model and Order parameter

- Different mathematical frameworks has been developed for describing synchronisation. One such model which describes which describes synchrony between a set of phase oscillators is the Kuramoto model (Kuramoto 1984).
- The set of randomized phase oscillators θ_i are coupled through a sinusoidal interaction $\sin(\theta_i - \theta_j)$

$$\dot{\theta}_i = \omega_i + \frac{k}{N}\sum_{j=1}^{N}\sin(\theta_j - \theta_i), \quad i = 1, \ldots, N \tag{1}$$

- with k being the coupling strength. It is known that when k exceeds a threshold k_c, the entire system of oscillators spontaneously locks to a common frequency: all the oscillators synchronize.
- An order parameter can also be defined for quantifying 'synchronous' behaviour

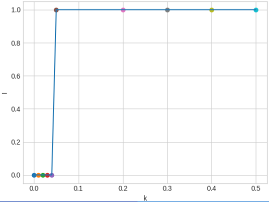
$$R \equiv re^{i\psi} = \frac{1}{N}\sum_{j=1}^{N} e^{i\theta_j}. \tag{2}$$

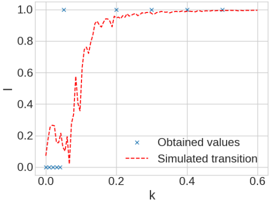
No coupling

With nonzero coupling

Experimental realisations of the model

(Prasad et al. 2024)





Neuronal Synchronisation

The idea

- Neural synchrony has been a topic of great interest, with being observed and found in multiple experimental setups (for a comprehensive list see Protachevichz et al. 2021)

  - Recently synchronisation in excitatory and inhibitory synapses has been observed in computational models.

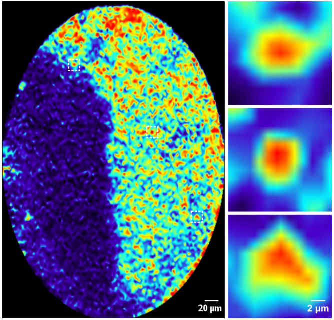


Figure: Source: The Scientist, 2020

- This work (Protachevicz et al. 2021) tries to investigate the influence of inhibitory and excitatory connections from one brain area to an another brain area and then emergence of synchronous behaviour.

The model

Nature of cortex and mathematical framework I

- The cortex mainly constitutes of excitatory pyramidal neurons and inhibitory interneurons.
- Excitatory neurons have a relatively low firing rate than inhibitory ones, and in the mammalian cortex they show regular spike (RS) and fast spike (FS) intervals respectively.

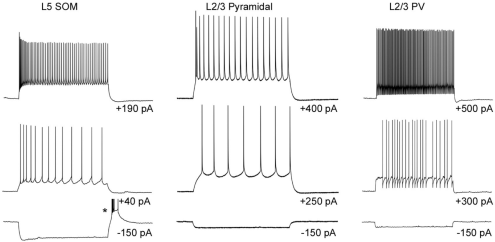


Figure: Neske et al. 2015

- Excitatory neurons shows adaptation in their firings, while inhibitory ones shows neglible adaptation.

Nature of cortex and mathematical framework II

- The properties of the cortical neurons as mentioned before can be modelled by a **Adaptive Exponential Leaky Integrate and Fire - AELIF** model.

AELIF model

$$C_m \frac{dV}{dt} = -g_L(V - E_L) + g_L \Delta_T \exp\left(\frac{V - V_T}{\Delta_T}\right) - w + I$$

$$\tau_w \frac{dw}{dt} = a(V - E_L) - w$$

Spike generation

$$\text{if } V \geq V_{spike} \text{ then } \begin{cases} V \to V_{reset} \\ w \to w + b \end{cases}$$

AELIF spike dynamics - Constant step

AELIF spike dynamics - Ramping step current

Connection Matrix and Parameter Values adapted

Parameter	Description	Value
N	AEIFs in each area	1,000 neurons
Areas	Number of areas	2
A	Area number	1 or 2
N_T	Total number of neurons	2,000 neurons
C_m	Capacitance membrane	200 pF
g_L	Leak conductance	12 nS
E_L	Leak reversal potential	–70 mV
I	Constant input current	270 pA
Δ_T	Slope factor	2 mV
V_T	Threshold potential	–50 mV
τ_w	Adaptation time constant	300 ms
V_r	Reset potential	–58 mV

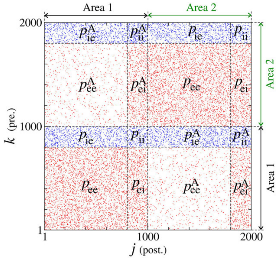


Figure: Probability connection matrix

Parameter	Description	Value
M_{ij}	Adjacent matrix elements	0 or 1
τ_s	Synaptic time constant	2.728 ms
t_{fin}	Final time to analyses	100 s
t_{ini}	Initial time to analyses	20 s
a_j	Subthreshold adaptation	[1.9, 2.1] nS •
		0 nS ⋆
b_j	Triggered adaptation	70 pA •
		0 pA ⋆
V_{REV}	Synaptic reversal potential	V_{REV}^{exc} = 0 mV •
		V_{REV}^{inh} = -80 mV ⋆
g_s	Synaptic conductances	g_{ee}, g_{ei}, g_{ei}^A, g_{ee}^A •
		g_{ii}, g_{ie}, g_{ii}^A, g_{ie}^A ⋆
g_{ee}	Excitatory to excitatory ⊙	0.5 nS •
g_{ei}	Excitatory to inhibitory ⊙	1 or 2 nS •
g_{ii}	Inhibitory to inhibitory ⊙	2 nS ⋆
g_{ie}	Inhibitory to excitatory ⊙	1.5 nS ⋆
g_{ee}^A	Excitatory to excitatory ⊕	[0,3] nS •
g_{ei}^A	Excitatory to inhibitory ⊕	[0,6] nS •
g_{ii}^A	Inhibitory to inhibitory ⊕	[0,4] nS ⋆
g_{ie}^A	Inhibitory to excitatory ⊕	[0,4] nS ⋆
d_j	Time delay	d_{exc} = 1.5 ms •
		d_{inh} = 0.8 ms ⋆

The connections inside the area are identified by ⊙ and between the areas by ⊕.

Figure: Adapted parameter values

Definition of the Network

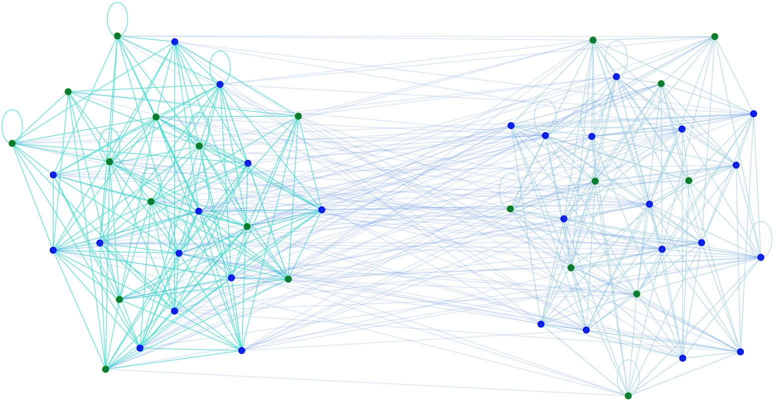


Figure: Model of two neuronal areas A (right) and B (left)

Modified model

Dynamics of each neuron given a node j

$$C_m \frac{dV_j}{dt} = g_L(V_j - E_L) + g_L \Delta T \exp\left(\frac{V_j - V_T}{\Delta T}\right) - w_j + I + I_j^{\text{chem}} \tag{3}$$

$$\tau_w \frac{dw_j}{dt} = a_j(V_j - E_L) - w_j \tag{4}$$

$$\tau_s \frac{dg_j}{dt} = -g_j \tag{5}$$

Update rules $V_j \geq V_T$

$$V_j \to V_r \tag{6}$$

$$w_j \to w_j + b_j \tag{7}$$

$$g_j \to g_j + g_s \tag{8}$$

Metrics

Measure	Definition
Complex Phase Order Parameter $R(t)$	$R(t) = \left\lvert \frac{1}{N_T} \sum_{j=1}^{N_T} \exp\left(i\psi_j(t)\right) \right\rvert$
Phase of Each Neuron $\psi_j(t)$	$\psi_j(t) = 2\pi m + 2\pi \frac{t - t_{j,m}}{t_{j,m+1} - t_{j,m}}$
Time-Average Order Parameter R	$R = \frac{1}{t_{\text{fin}} - t_{\text{ini}}} \int_{t_{\text{ini}}}^{t_{\text{fin}}} R(t)\, dt$
Order Parameter for Each Area $R_A(t)$	$R_A(t) = \left\lvert \frac{1}{N} \sum_{j=(A-1)N+1}^{AN} \exp\left(i\psi_j(t)\right) \right\rvert$
Mean Order Parameter R_A	$R = \frac{1}{t_{\text{fin}} - t_{\text{ini}}} \int_{t_{\text{ini}}}^{t_{\text{fin}}} R_A(t)\, dt$
Resultant Phase Angle of Each Area $\Phi_A(t)$	$\Phi_A(t) = \arctan\left(\frac{R_A^y(t)}{R_A^x(t)}\right)$
Relative Phase Angle for Each Area $\Phi'_A(t)$	$\Phi'_A(t) = \Phi_A(t) - \Phi_1(t)$
Mean Value of the Relative Phase Angle Φ'_2	$\Phi'_2 = \frac{1}{t_{\text{fin}} - t_{\text{ini}}} \int_{t_{\text{ini}}}^{t_{\text{fin}}} \Phi'_2(t)\, dt$

Table: Synchronization Measures and definitions

Schematic representation of mean order parameter.

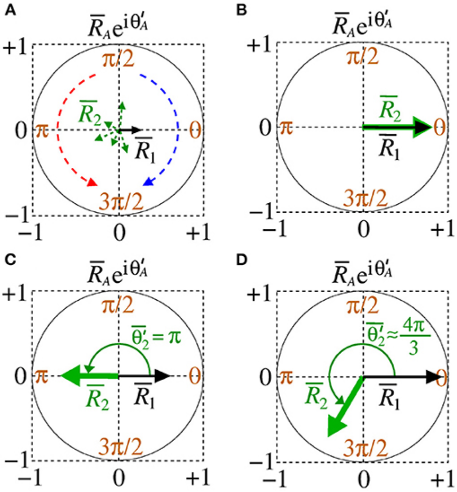
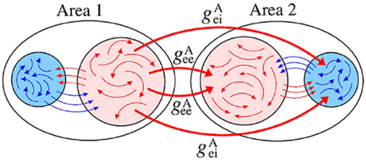


Figure: Qualitative descriptions of phase synchronisation.

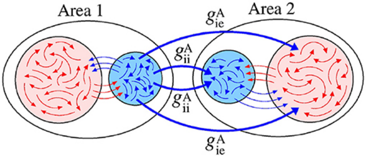
Results

Unidirectional connections

- We consider two cases, where the areas are coupled by the means of excitatory (a) and inhibitory (b) connections.

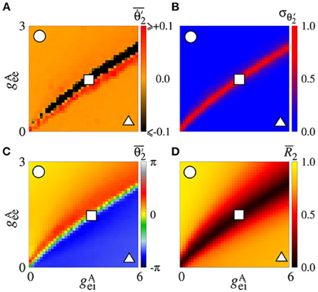


(a) Unidirectional Interaction by the means of Exc. Connections

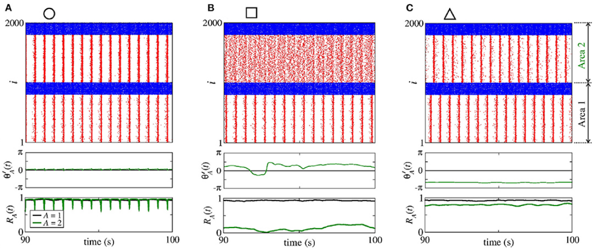


(b) Unidirectional Interaction by the means of Inh. Connections

Synchrony in Unidirectional connections - I



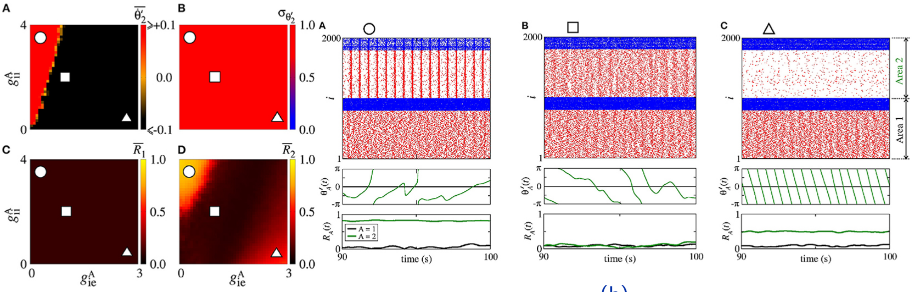
(a)



(b)

Figure: (a) Metrics plotted by varying g^{A}_{ee} and g^{A}_{ei}. The parameter space $g^{A}_{ee} \in [0, 3]$ and $g^{A}_{ei} \in [0, 6]$ (b) Raster plots describing spike trains in both areas. The sender area is synchronized while the receiver is not. Synchronised sender area can generate phase and shift-phase synchronisation in the receiver area.

Synchrony in Unidirectional connections - II



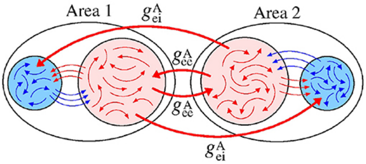
(a)

(b)

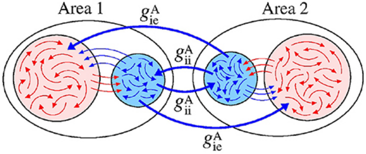
Figure: (a) Metrics plotted by varying g^A_{ii} and g^A_{ie}. The parameter space $g^A_{ii} \in [0, 4]$ and $g^A_{ie} \in [0, 3]$ (b) Raster plots describing spike trains in both areas. The sender area is desynchronized. However, Desynchronised sender area can actuate on the initially desynchronised receiver area generating synchrony or desynchrony or silence the neurons.

Bidirectional connections

- We consider two cases, where the areas are coupled by the means of excitatory (a) and inhibitory (b) connections.

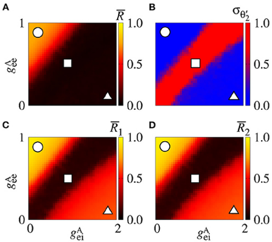


(a) Bidirectional Interaction by the means of Exc. Connections

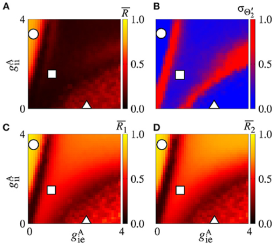


(b) Bidirectional Interaction by the means of Inh. Connections

Synchrony in Bidirectional interactions I



(a) Metrics plotted for excitatory bidirectional connections



(b) Metrics plotted for Inhibitory bidirectional connections

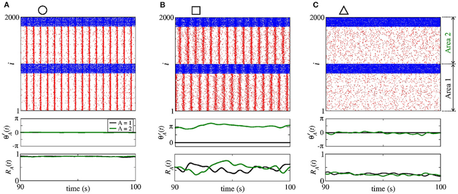


Figure: Raster plotting showing synchrony between two areas by bidirectional inhibitory coupling

Key observations

Unidirectional interactions

- **Excitatory Connections:**
  - Synchronization is influenced by g_{ee}^{A} and g_{ei}^{A} conductances.
  - Specific combinations of g_{ee}^{A} and g_{ei}^{A} result in high levels of synchronization.
- **Inhibitory Connections:**
  - g_{ii}^{A} and g_{ie}^{A} conductances also play a crucial role.
  - Different combinations of g_{ii}^{A} and g_{ie}^{A} lead to varying degrees of synchronization.

Bidirectional interactions

- **Excitatory Connections:**
  - Bidirectional excitatory connections enhance synchronization.
  - Both areas can achieve high synchronization levels and exhibit anti-phase synchronization.
- **Inhibitory Connections:**
  - Bidirectional inhibitory connections result in complex synchronization dynamics.
  - Balanced inhibitory conductances are crucial for stable synchronization.

Conclusions

- The study demonstrates the emergence of different synchronization patterns between two coupled neuronal areas.
- The role of synaptic conductances in synchronization is critical.
- Both magnitude and type of conductance significantly affect synchronization patterns.
- Computational models provide insights into neuronal synchronization dynamics.
- Implications for brain function and disorders, e.g., epilepsy and Parkinson's disease.

Figure: Image Credits : DALL-E

Code for generating animations as well as a minimal implementation of the code used in the paper can be found in my public Github repository @Cup-cake-lover