Optimization and Linear Algebra for Machine Learning (MO431)

Tiago Trocoli (226078)

March 2019

1 Homework

For an usual inner product, the eigenvectors associated to distinct eigenvalues of a symmetric real matrix are orthogonal to each other.

Solution:

If A is a symmetric matrix then it has distinct eigenvectors, let name two of them $\mathbf{v}_1$ and $\mathbf{v}_2$ having λ and μ as their eigenvalues, respectively, so

$$\lambda < \mathbf{v}_1, \mathbf{v}_2 >=< \lambda\mathbf{v}_1, \mathbf{v}_2 >=< A\mathbf{v}_1, \mathbf{v}_2 >,$$

since the inner product is the usual one, therefore

$$< A\mathbf{v}_1, \mathbf{v}_2 >=< \mathbf{v}_1, A^T\mathbf{v}_2 >,$$

then

$$< \lambda\mathbf{v}_1, \mathbf{v}_2 >=< \mathbf{v}_1, \mu\mathbf{v}_2 >\Rightarrow (\lambda - \mu) < \mathbf{v}_1, \mathbf{v}_2 >= 0,$$

but $\lambda - \mu \neq 0$, thus

$$< \mathbf{v}_1, \mathbf{v}_2 >= 0 \Rightarrow \mathbf{v}_1 \perp \mathbf{v}_2. \quad \square$$