Mathematical and Computer Modeling of Biological Processes

Practice 5

Consider the SIR model:

$$\begin{aligned}\frac{dS}{dt} &= A - \beta SI + \gamma R - \mu S,\\ \frac{dI}{dt} &= \beta SI - \nu I - \mu I,\\ \frac{dR}{dt} &= \nu I - \gamma R - \mu R\end{aligned} \tag{5.1}$$

with the following initial conditions:

$$I(t)\big|_{t=0} = 0.2N, \quad R(t)\big|_{t=0} = 0, \tag{5.2}$$

where N is the size of a population.

Summing all the equations in (5.1) yields us a single equation for $N(t)$:

$$\frac{dN}{dt} = A - \mu N. \tag{5.3}$$

The initial condition for (5.3) is

$$N(t)\big|_{t=0} = N_0, \tag{5.4}$$

where N_0 is the initial size of the population.

Tasks

1. Find an analytical solution to model (5.3) with the initial condition (5.4). Estimate $N(t)$ as $t \to \infty$.
2. Solve model (5.1) with the initial condition (5.2) numerically. Draw the graphs for $S(t)$, $I(t)$ and $R(t)$. Consider the cases when the disease-free equilibrium (DFE) is stable and unstable.
3. Estimate the expected secondary infection R_0.

Note: all the model coefficients have been described in Lecture 5. Select the appropriate values for the given coefficients.

(6 points)