Mathematical and Computer Modeling of Biological Processes

Practice 7

Consider the predator-prey model:

$$\begin{aligned}
\frac{dx}{dt} &= r_1 x\left(1-\frac{x}{k_1}\right) - b_1 xy + \beta_1 xz, \\
\frac{dy}{dt} &= r_2 y\left(1-\frac{y}{k_2}\right) - b_2 xy + \beta_2 yz, \\
\frac{dz}{dt} &= r_3 z\left(1-\frac{z}{k_3}\right) - \gamma_1 xz - \gamma_2 yz,
\end{aligned} \tag{7.1}$$

where x and y are competing predators, and z is the prey.

Initial conditions are given by

$$x(t)\big|_{t=0} = x_0, \quad y(t)\big|_{t=0} = y_0, \quad z(t)\big|_{t=0} = z_0, \tag{7.2}$$

where $x_0 < y_0 < z_0$.

Tasks

1. Solve model (7.1) with initial conditions (7.2) using the Runge-Kutta method.
2. Select the model parameters to obtain the following cases:
   a) $x \to 0$, $y \to 0$ and $z \to 0$;
   b) equilibrium state of the system, i.e. all the species coexist;
   c) $x \to 0$, y and z remain stable (coexist).
3. Draw the graphs for all the given cases.

Note: all the model coefficients have been described in Lecture 7. Select the appropriate values for the given coefficients.

(4 points)