CS170– Spring 2022— Homework 5

CurMack

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Knapsack with repetition

Define:

$$K(w) = \text{maximum value achievable with a knapsack of capacity } w.$$

express this in terms of smaller subproblems:

$$k(w) = \max_{i:w_i \le w} \{K(w - w_i) + v_i\}$$

where as usual our convention is that the maximum over an empty set is 0.
So the algorithm:

$K(0) \leftarrow 0$
for $w = 1 : W$ **do**
    $K(w) \leftarrow \max\{K(w - w_i) + v_i : w_i \le w\}$
return $K(W)$

Runtime: $\mathcal{O}(nW)$

Knapsack without repetition

Definr:

$$K(w, j) = \text{maximum value achievable using a knapsack of capacity } w \text{ and items } 1, \ldots j.$$

The answer we seek is $K(W, n)$.
express a subproblem $K(w, j)$ in terms of smaller subproblems:

$$K(w, j) = \max\{K(w - w_j, j - 1) + v_j, K(w, j - 1)\}.$$

So the algorithm:

Initialize all $K(0, j) \leftarrow 0$ and all $K(w, 0) \leftarrow 0$
for $j = 1 : n$ **do**
    for $w = 1 : W$ **do**
        if $w_j > w$ **then**
            $K(w, j) \leftarrow K(w, j - 1)$
        else
            $K(w, j) \leftarrow \max\{K(w, j - 1), K(w - w_j, j - 1) + v_j\}$
return $K(W, n)$

Runtime: $\mathcal{O}(nW)$

2 Copper Pipes

Main Idea: Knapsack with repetition:

Algorithm 1: `cutPipe`$(price, n)$

$val(0) \leftarrow 0$
for $w = 1 : n$ **do**
  $max\_val \leftarrow 0$
  for $i = 1 : w$ **do**
    $max\_val \leftarrow \max\{val(w-i) + price(i), max\_val\}$
  $val(i) \leftarrow max\_val$
return $val(n)$

Runtime: $\mathcal{O}(n^2)$

3 Egg Drop

(a) $f(1,k) = 1$: drop the egg from the single floor.
$f(0,k) = 0$: there is only one possible value for l.
$f(n,1) = n$: drop it from floor 1 and going up, until it breaks.
$f(n,0) = \infty$: the problem is unsolvable if we have no eggs to drop.

(b) the recurrence relation is:

$$f(n,k) = \min_{x \in \{1 \dots n\}} \max\{f(x-1, k-1), f(n-x, k)\}$$

Suppose we drop at floor x:

- breaks: we know $l \in [0, x-1]$, and we have $k-1$ eggs $\Rightarrow f(x-1, k-1)$
- doesn't break: now $l \in [x, n]$, k eggs $\Rightarrow f(n-x, k)$

And we pick the beat x