CS170–Spring 2022 — Homework 0

CurMack

4. In Between Functions

(Find an algorithm that is slower than polynormial and faster than exponential.)
Let:

$$f(n) = 2^{\log n \cdot \log n} = 2^{(\log n)^2}$$

(a) Since $\alpha^n = (2^{\log \alpha})^n = 2^{n \cdot \log \alpha}$ and $\log^2 n = \mathcal{O}(n)$, therefore $f(n) = \mathcal{O}(\alpha^n)$ for all $\alpha > 0$.

(b) Since $n^c = (2^{\log n})^c = 2^{c \log n}$ and $\log^2 n = \Omega(\log n)$, therefore $f(n) = \Omega(n^c)$ for all $c > 0$.

5. Asympotic Bound Practice

Use the L'Hospital rule:

$$\begin{aligned}
\lim_{x \to \infty} \frac{\log x}{x^\epsilon} &= \lim_{x \to \infty} \frac{\frac{\mathrm{d}}{\mathrm{d}x} \log x}{\frac{\mathrm{d}}{\mathrm{d}x} x^\epsilon} \\
&= \lim_{x \to \infty} \frac{\frac{1}{x}}{\epsilon x^{\epsilon - 1}} \\
&= \lim_{x \to \infty} \frac{1}{\epsilon x^\epsilon} \\
&= 0
\end{aligned}$$

Therefore:

$$\log x \in \mathcal{O}(x^\epsilon)$$

for any $\epsilon > 0$.