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Transformate Laplace, z şi z modificate

$f(t)$ $\blacktriangleleft f[t] \blacktriangleright$	$f(s)=\mathbf{L}[f(t)]$	$f(z) = \mathbf{Z}\{f[t]\} = Z\{f(s)\}$	$f_\vartheta(z) = \mathbf{Z}\{f[t,\vartheta]\} = Z_\vartheta\{f(s)\}$
(1)	(2)	(3)	(4)
$\delta(t)$	1	5	x
$\blacktriangleleft \delta[t] \blacktriangleright$	x	1	x
$1\ ,\ \sigma(t)$	$\frac{1}{s}$	$\frac{z}{z-1}$	$\frac{z}{z-1}$
$\blacktriangleleft (-1)^t \blacktriangleright$	x	$\frac{z}{z+1}$	x
t	$\frac{1}{s^2}$	$\frac{hz}{(z-1)^2}$	$\frac{hz\cdot[\vartheta z+(1-\vartheta)]}{(z-1)^2}$
$\frac{1}{2}\cdot t^2$	$\frac{1}{s^3}$	$\frac{h^2z(z+1)}{2(z-1)^3}$	$\frac{h^2z[\vartheta^2z^2+(1+2\vartheta-2\vartheta^2)z+(1-\vartheta)^2]}{2\cdot(z-1)^3}$
$\frac{1}{3!}\cdot t^3$	$\frac{1}{s^4}$	$\frac{h^3z(z^2+4z+1)}{3!\cdot(z-1)^4}$	$\frac{h^3z\,[\vartheta^3z^3+(1+3\vartheta+3\vartheta^2-3\vartheta^3)z^2+(4-6\vartheta^2+3\vartheta^3)z+(1-\vartheta)^3]}{3!\cdot(z-1)^4}$
$\frac{1}{n!}\cdot t^n$	$\frac{1}{s^{n+1}}$	$\frac{1}{n!}\cdot\lim\limits_{a\to 0}\frac{\partial^n}{\partial a^n}\left\{\frac{z}{z-e^{-ah}}\right\}$	$\frac{1}{n!}\cdot\lim\limits_{a\to 0}\frac{\partial^n}{\partial a^n}\left\{\frac{ze^{a\vartheta h}}{z-e^{-ah}}\right\}$
$a^t\ ,\ a>0$	$\frac{1}{s-\ln a}$	$\frac{z}{z-a^h}$	$\frac{z\,a^{\vartheta h}}{z-a^h}$
$\blacktriangleleft (-a)^t, a>0 \blacktriangleright$	x	$\frac{z}{z+a}$	x
$\frac{a^t}{t!}\ ,\ a>0$	x	$e^{\frac{a}{z}}$	x
e^{-at}	$\frac{1}{s+a}$	$\frac{z}{z-e^{-ah}}$	$\frac{z\cdot e^{-a\vartheta h}}{z-e^{-ah}}$
$\delta(t)-a\cdot e^{-at}$	$\frac{s}{s+a}$	x	x
$t\cdot e^{-at}$	$\frac{1}{(s+a)^2}$	$\frac{hze^{-ah}}{(z-e^{-ah})^2}$	$\frac{hze^{-a\vartheta h}\cdot[\vartheta z+(1-\vartheta)\cdot e^{-ah}]}{(z-e^{-ah})^2}$
$(1-at)\cdot e^{-at}$	$\frac{s}{(s+a)^2}$	$\frac{z\cdot[z-(1+ah)e^{-ah}]}{(z-e^{-ah})^2}$	$\frac{ze^{-a\vartheta h}}{(z-e^{-ah})^2}\cdot[(1-a\vartheta h)z-(1+ah-a\vartheta h)e^{-ah}]$
t^2e^{-at}	$\frac{2}{(s+a)^3}$	$\frac{h^2ze^{-ah}(z+e^{-ah})}{(z-e^{-ah})^3}$	$\frac{h^2ze^{-a\vartheta h}}{(z-e^{-ah})^3}[\vartheta^2z^2+(1+2\vartheta-2\vartheta^2)e^{-ah}z+(1-\vartheta)^2e^{-2ah}]$
$t^n e^{at}$	$\frac{n!}{(s+a)^{n+1}}$	$\frac{\partial^n}{\partial a^n}\left\{\frac{z}{z-e^{-ah}}\right\}$	$(-1)^n\cdot\frac{\partial^n}{\partial a^n}\left\{\frac{ze^{-a\vartheta h}}{z-e^{-ah}}\right\}$
$1-e^{-at}$	$\frac{a}{s(s+a)}$	$\frac{(1-e^{-ah})z}{(z-1)(z-e^{-ah})}$	$\frac{(1-e^{-a\vartheta h})z^2+(e^{-a\vartheta h}-e^{-ah})z}{(z-1)(z-e^{-ah})}$

$at-1+e^{-at}$	$\frac{a^2}{s^2(s+a)}$	$\frac{(ah-1+e^{-ah})z^2}{(z-1)^2(z-e^{-ah})}+$ $+\frac{(1-ahe^{-ah}-e^{-ah})z}{(z-1)^2(z-e^{-ah})}$	$\frac{z}{(z-1)^2(z-e^{-ah})}\cdot\{[a\vartheta h-1+e^{-a\vartheta h})\cdot z^2+$ $+[ah(1-\vartheta-\vartheta e^{-ah})+1-2e^{-a\vartheta h}+e^{-ah}]\cdot z+$ $+\left[e^{-a\vartheta h}-ahe^{-ah}(1-\vartheta)-e^{-ah}\right]$
$e^{-at}-e^{-bt}$	$\frac{b-a}{(s+a)\cdot(s+b)}$	$\frac{z(e^{-ah}-e^{-bh})}{(z-e^{-ah})(z-e^{-bh})}$	$\frac{(e^{-a\vartheta h}-e^{-b\vartheta h})z^2+(e^{-(a+b\vartheta)h}-e^{-(b+a\vartheta)h})z}{(z-e^{-ah})(z-e^{-bh})}$
$a(1-e^{-bt})-$ $-b(1-e^{-at})$	$\frac{ab(a-b)}{s(s+a)(s+b)}$	$\frac{z}{(z-1)(z-e^{-ah})(z-e^{-bh})}\cdot$ $\cdot\{(a-b-ae^{-bh}+be^{-ah})\cdot z+$ $[(a-b)\cdot e^{-(a+b)h}-ae^{-ah}+be^{-bh}]$	$\frac{(a-b)z}{z-1}+\frac{bze^{a\vartheta h}}{z-e^{-ah}}-\frac{aze^{-b\vartheta h}}{z-e^{-bh}}$
$ab(a-b)t+$ $(b^2-a^2)-$ $-b^2e^{-at}+$ $+a^2e^{-bt}$	$\frac{a^2b^2(a-b)}{s^2(s+a)(s+b)}$	$\frac{ab(a-b)hz}{(z-1)^2}+\frac{(b^2-a^2)z}{z-1}-$ $-\frac{b^2z}{z-e^{-ah}}+\frac{a^2z}{z-e^{-bh}}$	$\frac{ab(a-b)hz}{(z-1)^2}+\frac{[ab(a-b)\vartheta h+b^2-a^2]\cdot z}{z-1}-$ $-\frac{b^2e^{-a\vartheta h}z}{z-e^{-ah}}+\frac{a^2e^{-b\vartheta h}z}{z-e^{-bh}}$
$\sin\omega_0 t$	$\frac{\omega_0}{s^2+\omega_0^2}$	$\frac{z\sin\omega_0 h}{z^2-2z\cos\omega_0 h+1}$	$\frac{z^2\sin\vartheta\omega_0 h+z\sin(1-\vartheta)\omega_0 h}{z^2-2z\cos\omega_0 h+1}$
$\cos\omega_0 t$	$\frac{s}{s^2+\omega_0^2}$	$\frac{z(z-\cos\omega_0 h)}{z^2-2z\cos\omega_0 h+1}$ *caz special*: $\omega_0 h=\pi$ $\mathbf{Z}\left\{(-1)^t\right\}=\frac{z}{z+1}$	$\frac{z^2\cos\vartheta\omega_0 h-z\cos(1-\vartheta)\omega_0 h}{z^2-2z\cos\omega_0 h+1}$ *caz special*: $\omega_0 h=\pi$ $\frac{z\cos\vartheta\pi}{z+1}$
$\frac{\cos at-\cos bt}{b^2-a^2}$	$\frac{s}{(s^2+a^2)\cdot(s^2+b^2)}$	$\frac{1}{b^2-a^2}\cdot$ $\cdot\left[\frac{z(z-\cos ah)}{z^2-2z\cos ah+1}-\right.$ $\left.-\frac{z(z-\cos bh)}{z^2-2z\cos bh+1}\right]$	$\frac{1}{b^2-a^2}\cdot\left[\frac{z^2\cos\vartheta ah-z\cos(1-\vartheta)ah}{z^2-2z\cos ah+1}-\right.$ $\left.-\frac{z^2\cos\vartheta bh-z\cos(1-\vartheta)bh}{z^2-2z\cos bh+1}\right]$
$e^{-at}\sin\omega_0 t$	$\frac{\omega_0}{(s+a)^2+\omega_0^2}$	$\frac{ze^{-ah}\sin\omega_0 h}{z^2-2ze^{-ah}\cos\omega_0 h+e^{-2ah}}$ *caz special*: $\omega_0 h=\pi/2$ $\mathbf{Z}\left\{\frac{1-(-1)^t}{2}(-e^{ah})^t\right\}=$ $=\frac{ze^{-ah}}{z^2+e^{-2ah}}$	$\frac{[z\sin\vartheta\omega_0 h+e^{-ah}\sin(1-\vartheta)\omega_0 h]\cdot z\cdot e^{-a\vartheta h}}{z^2-2ze^{-ah}\cos\omega_0 h+e^{-2ah}}$ *caz special*: $\omega_0 h=\pi/2$ $\frac{(z\sin\vartheta\frac{\pi}{2}+e^{-ah}\cos\vartheta\frac{\pi}{2})\cdot z\cdot e^{-a\vartheta h}}{z^2+e^{-2ah}}$
$e^{-at}\cos\omega_0 t$	$\frac{s+a}{(s+a)^2+\omega_0^2}$	$\frac{z(z-e^{-ah}\cos\omega_0 h)}{z^2-2ze^{-ah}\cos\omega_0 h+e^{-2ah}}$ caz special: $\omega_0 h=\pi$ $\mathbf{Z}\left\{\left(-e^{ah}\right)^t\right\}=\frac{z}{z+e^{-ah}}$	$\frac{[z\cos\vartheta\omega_0 h-e^{-ah}\cos(1-\vartheta)\omega_0 h]\cdot z\cdot e^{-a\vartheta h}}{z^2-2ze^{-ah}\cos\omega_0 h+e^{-2ah}}$ caz special: $\omega_0 h=\pi$ $\frac{(z\cos\vartheta\frac{\pi}{2}-e^{-ah}\sin\vartheta\frac{\pi}{2})\cdot z\cdot e^{-a\vartheta h}}{z^2-2ze^{-ah}\cos\omega_0 h+e^{-2ah}}$
$\mathrm{sh}\omega_0 t$	$\frac{\omega_0}{s^2-\omega_0^2}$	$\frac{z\cdot\mathrm{sh}\omega_0 h}{z^2-2z\cdot\mathrm{ch}\omega_0 h+1}$	$\frac{z\cdot[z\cdot\mathrm{sh}\vartheta\omega_0 h+\mathrm{sh}(1-\vartheta)\omega_0 h]}{z^2-2z\cdot\mathrm{ch}\omega_0 h+1}$
$\mathrm{ch}\omega_0 t$	$\frac{s}{s^2-\omega_0^2}$	$\frac{z(z-\mathrm{ch}\omega_0 h)}{z^2-2z\cdot\mathrm{ch}\omega_0 h+1}$	$\frac{z\cdot[z\mathrm{ch}\vartheta\omega_0 h-\mathrm{ch}(1-\vartheta)\omega_0 h]}{z^2-2z\cdot\mathrm{ch}\omega_0 h+1}$
$\sqrt{t}$	$\frac{1}{2s}\cdot\sqrt{\frac{\pi}{s}}$	xx	xx

$\frac{1}{\sqrt{t}}\cdot e^{-at}, t > 0$	$\sqrt{\frac{\pi}{s+a}}$	xx	xx
$\{ C_t^k \}$ $(C_t^k=0,\ t<k)$	x	$\frac{z}{(z-1)^{k+1}}$	x
$\frac{1}{2}+\sum_{k=1,3,..}^{\infty}\frac{2\sin k\pi t}{k\pi}$	$\frac{1}{s(1+e^{-s})}$	xx	xx
$\{ f[0]=0,\ f[t]=(-1)^{t-1}t^{-1}, t\in \mathbf{N}^* \}$	x	$\ln\left(1+\frac{1}{z}\right)$	x
$\{ f[0]=0,\ f[t]=a^{t-1}t^{-1}, t\in \mathbf{N}^* \}$	x	$\frac{1}{a}\cdot\ln\frac{z}{z-a}$	x

În tabel s-au folosit notațiile „x” şi „xx” pentru situațiile în care transformatele respective nu există, respectiv nu prezintă interes.