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A Network-Based Model for the Integrated Weekly Aircraft Maintenance Routing and Fleet Assignment Problem

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Most studies in airline operations planning research are focused on the optimization problems that deal with a daily flight schedule, which is considered to be the same for every day in the week. While the weekly schedule is more realistic and practical, it increases the complexity of the optimization problems drastically. In this paper, we present a novel weekly rotation-tour network representation for the *weekly aircraft maintenance routing problem* (WAMRP). Based on this representation, we propose a new network-based mixed-integer linear programming (LP) formulation for the WAMRP; namely, *weekly rotation-tour network model* (WRTNM). The main advantage of this formulation is that the size of WRTNM only increases linearly with the size of the weekly schedule, and it provides a very tight LP relaxation. In addition, because of the tight LP relaxation, we develop a diving heuristic to solve WRTNM efficiently and effectively. To assess the performance of WRTNM, we tested the WRTNM using eight real-life test cases. The computational results show that the proposed model is very compact and scalable, and is able to find the optimal solutions to the schedule with 5,700 flights and 330 aircraft, approximately the size of the world's largest airlines fleet, within five minutes. We also propose an integrated model to solve the WAMRP with the weekly fleet assignment problem simultaneously. We tested the integrated model on nine self-constructed test cases. The computational results show that the integrated model generates near-optimal solutions to the schedules with 1,700 flights, 8 fleets with 110 aircraft, and approximately a medium-sized airline, in a reasonable time.

Key words: transportation: air; maintenance; networks: scheduling; programming: integer; optimization

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1. Introduction

In the last decade, airlines' profit margin has been continuously pressured by their growing exposure to a high-cost low-fare environment. The increasing cost in capital, labor, and fuel, and the elevated competition from budget carriers have tied the airlines' profitability to the current economic downturn. In 2009, five out of nine major passenger airlines in the United States (i.e., US Airways, Continental, United, Delta, and American) suffered from severe net losses, where American airlines alone lost about \$1.5 billion. As a whole, these nine airlines collectively lost about \$3.4 billion (McCartney 2010). How to efficiently use their expensive resources and generate the maximum profit is always a grand challenge in the airline planning operations. This challenge has motivated numerous research studies from both industry and academia. The majority of those studies, however, are focused largely on *daily schedule* problems, in

which the schedule and the profitability of the flights are considered to be the same everyday in the week. For the U.S. domestic airlines, the schedules are normally the same during weekdays and slightly different during weekends due to the demand variation. For international airlines, on the other hand, the *daily schedule* assumption is no longer valid. For example, Lufthansa airlines only offer three flights per week from Frankfurt to Anchorage. Apparently, planning problems based on the *weekly schedule* are much more realistic and of interest to such international airlines. Even for U.S. domestic airlines, considering the *weekly schedule* during the planning stage may increase their revenues. However, a very few studies in the airline schedule planning literature deal with the *weekly schedule* problems. One reason is because the weekly problems are much harder, and their problem complexity increases drastically with the number of flights to be scheduled. Specifically, the models and solution

methodologies for the daily schedule problems can be used to solve the weekly problems by extending the planning duration from a single day to an entire week; however, the models and computational complexities will increase exponentially. To overcome this computational obstacle, there have been a few mathematical models and decomposition techniques developed for the weekly schedule problems (Desaulniers et al. 1997a; Barnhart et al. 1998; Barnhart and Shenoi 1998; Ioachim et al. 1999; Sriram and Haghani 2003; Belanger et al. 2006; Grönkvist 2006; Haouari et al. 2011; Weide, Ryan, and Ehrgott 2010). Nevertheless, these models and techniques usually cannot obtain the optimal solutions for large real-life problems in a reasonable time. In this paper, we focus our research on the weekly aircraft maintenance routing problem (WAMRP).

The aircraft maintenance routing problem (AMRP) is to determine the flight routes for every aircraft such that the maintenance requirements are satisfied. The sizes of the traditional models for the AMRP usually increase exponentially with the number of flights in the schedule, and these models are commonly solved iteratively using column-generation approaches (Desaulniers et al. 1997b; Barnhart et al. 1998; Cordeau et al. 2001; Elf and Kaibel 2003; Mercier, Cordeau, and Soumis 2005; Mercier and Soumis 2007) or row generation approaches (Clarke et al. 1997; Boland, Clarke, and Nemhauser 2000). Liang et al. (2011) proposed a compact rotation-tour network model for the daily AMRP, where the size of the model only increases linearly with the number of flights in the schedule. However, this model cannot be directly extended to the weekly schedule because the weekly time-space network does not preserve the critical properties to model the maintenance requirement.

Thus, in this paper, we propose a novel network-based model for the WAMRP, which is inspired by the model proposed in Liang et al. (2011). The advantage of the proposed model is twofold. First, the size of the proposed mathematical model only increases linearly with the number of flights to be scheduled. Therefore, this compact and scalable model can be solved directly by most commercial mathematical programming packages. Second, we notice from our computational experience that the proposed time-space network flow-based model provides very tight linear programming (LP) relaxation bounds, which can help to find good integer solutions efficiently. In addition, we further extend the proposed model to solve the integration of the WAMRP with the weekly fleet assignment problem (WFAP). Because of the tight LP relaxation bounds of the integrated model, we propose a diving heuristic to efficiently solve the integrated model.

The remainder of the paper is organized as follows. In §2, we give a background on airline schedule planning, especially on the fleet assignment problem (FAP) and the AMRP, and present the widely used time-space network structure. In §3, we mathematically present the new rotation-tour network model of WAMRP. Section 4 presents the mathematical model to integrate the WFAP and WAMRP, and the solution methodology. Section 5 presents the computational results of WAMRP and integrated problem. Finally, we conclude our work and discuss some future studies in §6.

2. Background

In this section, we first introduce major optimization problems that often arise in airline planning operations. Then, we discuss in detail the AMRP and its integration with other planning operations. Toward the end of the section, we present the traditional time-space network representation of the airline network and its extension to the AMRP.

2.1. Airline Planning Operations

Generally, airline planning operations involve four major optimization problems: flight schedule design, fleet assignment, aircraft maintenance routing, and crew scheduling. The *flight schedule design problem* is to decide which flights should be offered based on traffic forecasts, airline network analysis, and profitability analysis. After a flight schedule is obtained, a variety of aircraft fleets are assigned to individual flights based on passenger demands, revenues, and operating cost, so that the total profit is maximized. This operation is often referred to as the FAP. Given an assigned aircraft fleet, the AMRP is to determine the rotation of individual aircraft, so that adequate maintenance opportunities are provided to each and every aircraft. The *crew scheduling problem* is to determine the best set of crew pairings (referred to as the crew pairing problem (CPP)) to cover all the aircraft fleets and to construct personalized monthly schedules (referred to as the rostering problem) for crew members.

Traditionally, airlines solve these planning operations separately and sequentially because of their sizes and intractability of the integrated problems. Nevertheless, with the evolution of the computational capability, researchers have been able to develop better solution approaches by integrating multiple planning operations and solving them simultaneously. For example, it has been shown that a substantial saving on the crew cost can be obtained by solving the CPP with the FAP and/or the AMRP together (Cordeau et al. 2001; Klabjan et al. 2002; Cohn and Barnhart 2003; Mercier, Cordeau, and Soumis 2005; Mercier and Soumis 2007; Sandhu and Klabjan 2007; Weide,

Ryan, and Ehrgott 2010). In Yan and Tseng (2002), Lohatepanont and Barnhart (2004), and Serali, Bae, and Haouari (2010), it has been shown that significant benefits can be achieved by solving the integration of the schedule design problem with the FAP simultaneously. In this paper, we are especially interested in the WAMRP and its integration with the WFAP. In the following subsections, we provide a detailed literature review on the AMRP and the recent advances of its integration with the FAP.

2.2. AMRP

Given a set of flights and a fleet of aircraft, the AMRP is to determine the flight routes for every aircraft such that the maintenance requirements, which are set by the Federal Aviation Administration (FAA) and individual airline companies, are satisfied. The aircraft maintenances considered in AMRP are called daily check, which is needed every two to four days. It includes a walk around inspection, a checkup on lights, emergency equipment, servicing engine oil, etc., and repairs if needed. It normally lasts from one to three hours for a checkup, and several hours for a repair. To not affect the aircraft use during the day, a daily check is usually performed at night (Talluri 1998; Gopalan and Talluri 1998; Sriram and Haghani 2003). Only certain airports, called maintenance stations, are capable of performing the maintenance operation. A feasible solution to the AMRP normally contains a generic aircraft route during a rolling time horizon. This generic solution does not assign individual aircraft to flights explicitly. However, in the operational stage, this solution can serve as a reference for assigning individual aircraft.

Two common objectives considered in the AMRP include a *short connect* penalty cost and a *through revenue* between connecting flights. A short connect happens when the *turn time* between two connecting flights is less than the *minimum sit time*. The turn time is the time difference between the second flight departure time and the first flight arrival time, and the minimum sit time is the minimum time for a crew to change aircraft between two connecting flights (also including the time to unload an aircraft after its arrival at the gate, and the time of preparation for the next departure). A short connect is undesirable because it may lead to an infeasible schedule or a high cost for the CPP. A through revenue between two connecting flights is measured by the number of passengers who stay on the same aircraft between two connecting flights. It is worth mentioning that some airlines consider the AMRP as a pure feasibility problem because the cost of AMRP is relatively small compared to other planning operations such as FAP and CPP.

The solution methodology for the AMRP can be categorized into three approaches. The most commonly

used approach in the literature, which was first proposed in Desaulniers et al. (1997b) and Barnhart et al. (1998), is to model a sequence of aircraft rotation as connecting flight strings and find the optimal routing by solving a set-partitioning problem. This model has been further modified and extended in Cordeau et al. (2001), Elf and Kaibel (2003), Mercier, Cordeau, and Soumis (2005), Mercier and Soumis (2007). In this approach, the decision variables represent the maintenance feasible flight sequences between two maintenance stations with a maintenance at the end. Various column-generation and branch-and-price solution approaches were developed to solve this type of models. The second approach models the AMRP as an Euler tour problem or asymmetric traveling salesman problem with side constraints (Clarke et al. 1997; Gopalan and Talluri 1998; Talluri 1998; Boland, Clarke, and Nemhauser 2000). The last and most recent approach models the AMRP as a network flow problem (Liang et al. 2011). The network model has been shown to be very compact and scalable as it is able to solve the real-life daily AMRP in a reasonable time.

2.3. Integration of the FAP and AMRP

The integration of the FAP and AMRP has continuously attracted many researchers to develop effective models and efficient solution approaches. The traditional FAP usually does not consider the feasibility of the aircraft maintenance schedule. Therefore, when solving the FAP followed by the AMRP sequentially, the solution to the FAP might not be maintenance feasible for the following AMRP. Barnhart et al. (1998) are the first to solve the integrated FAP and AMRP using a set-partitioning-based model, as mentioned previously. Ioachim et al. (1999) solve the weekly integrated problem with schedule synchronization constraints, which require the departure times for flights with the same identifier (flight number). Recently, Haouari, Aissaoui, and Mansour (2009) propose a multicommodity network flow model for the integrated problem. Haouari et al. (2011) propose assignment- and set-partitioning-based models to solve the same problem addressed in Haouari, Aissaoui, and Mansour (2009). A Benders' decomposition method is proposed to solve the assignment-based model, and a column-generation method is proposed to solve the set-partitioning model. In Mansour et al. (2011), the same group solves the integrated FAP and AMRP with the options of chartered aircraft and flight retiming. A set-partitioning-based formulation is proposed to model the problem, and a column-generation-based heuristic is developed to solve the model. Note that in Haouari, Aissaoui, and Mansour (2009), Haouari et al. (2011), and Mansour et al. (2011), the authors do not consider the maintenance constraints because of the special airline operational pattern. In Papadakos (2009),

the author proposes an integrated model to solve the FAP, AMRP, and CPP simultaneously. The integrated model extends the flight string model to handle the CPP. The integrated model is solved by an enhanced Benders' decomposition method combined with column generation.

Note that most of the above papers for the integrated problem do not consider maintenance requirements explicitly in their model except Barnhart et al. (1998) and Papadakos (2009). Considering maintenance requirements explicitly in the integrated problems poses significant difficulties in both modeling and solution approaches. For example, in Papadakos (2009), it needs more than 27 hours to solve the integrated FAP, AMRP, and CPP for a schedule of 705 flights with 167 aircraft of six fleets.

2.4. Time-Space Network

The time-space network is widely used to model the airline planning operations, including the FAP, AMRP, CPP, etc. The time-space network first appears in Hane et al. (1995) to solve the FAP. In a daily time-space network (shown in Figure 1(a)), a time line represents a station, which consists of a series of event nodes representing flight departures and/or arrivals at the station. To allow connection between flights, a minimum ground time is added on the actual flight arrival time for computing the time of an arrival node. In the time-space network, there are three types of arcs: ground, flight, and overnight arcs. Ground arcs represent one or more aircraft staying at the same station for a period of time. Flight arcs represent flights

between airports. Overnight arcs ensure the continuity of the aircraft routing from the current planning period to the next. With ground arcs and overnight arcs, it is possible to preserve the aircraft balance and allow all possible connections between the arrival flights and the following departure flights at a station.

Two preprocessing methods; namely, *node aggregation* and *island isolation*, proposed in Hane et al. (1995), can be used to reduce the size of time-space network (shown in Figure 1(b)). Node aggregation allows the combination of consecutive arrival nodes and subsequent consecutive departure nodes and the elimination of the unnecessary ground arcs. Island isolation can eliminate a ground arc if it is not necessary to have aircraft on the ground arc during the specific period.

Liang et al. (2011) propose a modified time-space network; namely, rotation-tour network, to model the daily AMRP. They duplicate the daily time-space network for D times, where D is the maximal days allowed between two consecutive maintenances. They also remove all the overnight arcs in the traditional time space, and create a set of time-reversible maintenance arcs to represent maintenance opportunities. The new maintenance arcs start at the end of every day at a maintenance station and end at the beginning of the same maintenance station time line. It is obvious that any flight sequences in the rotation-tour network cannot violate the D -day maintenance constraints. In Figure 1(c), we show a two-day rotation-tour network for the daily AMRP.

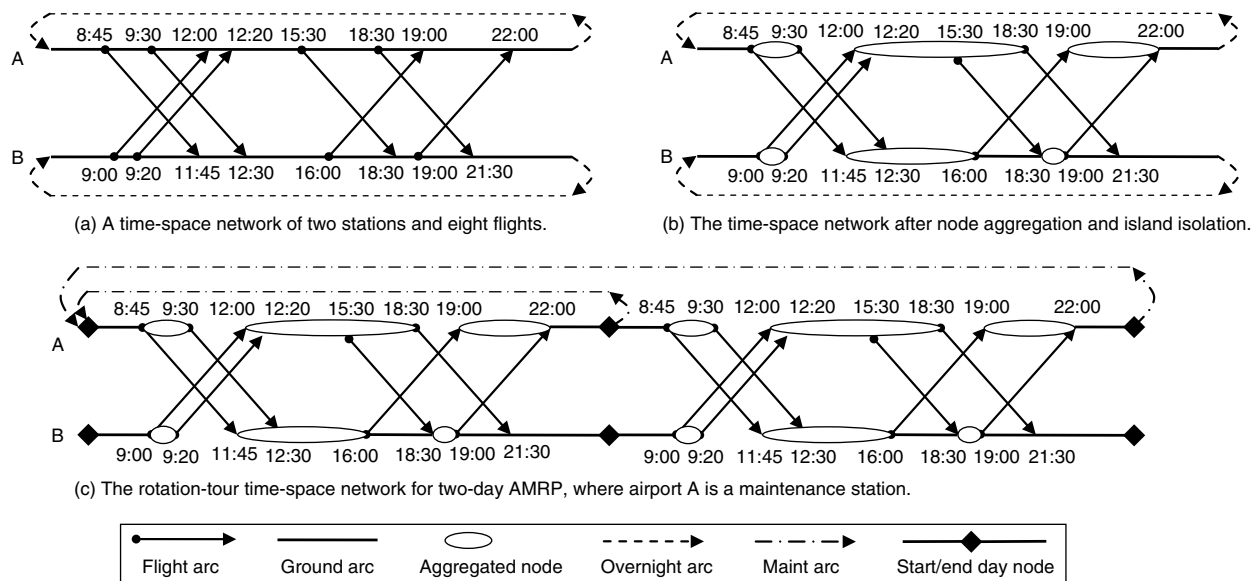


Figure 1 Time-Space Network with Three Stations and Eight Flights for FAP and Rotation-Tour Network for Two-Day AMRP

Notes. The time progresses horizontally from left to right. In A, there are eight flights, and hence 16 nodes in the time-space network. In B, we show the network after node aggregation and island isolation, where only six ground arcs and seven nodes are necessary.

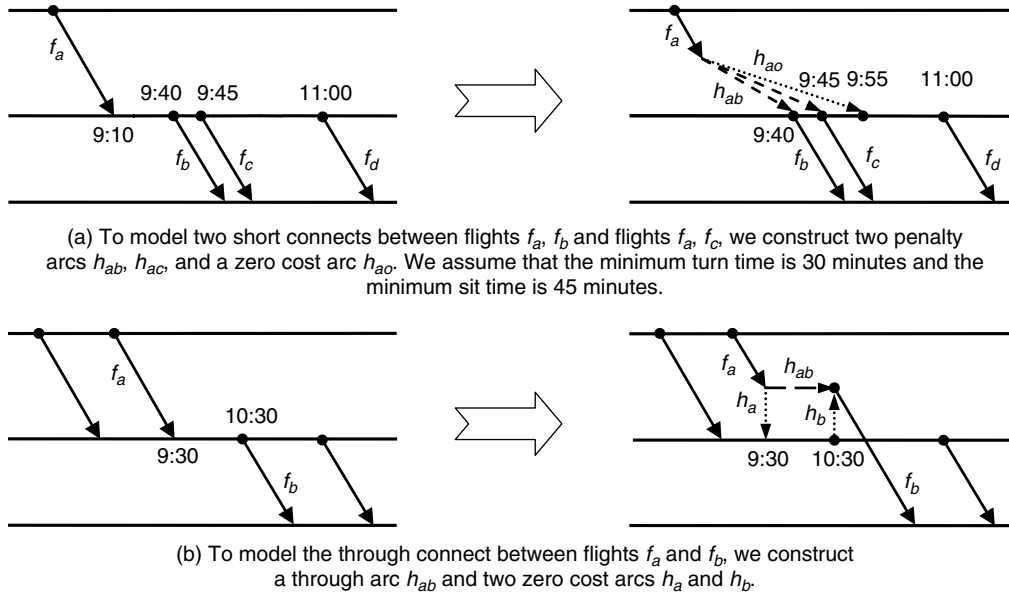


Figure 2 Construction of Penalty Arcs for Short Connects and Through Arcs for Through Revenue Connects

Notes. For illustration purposes, we assume that the end time of a flight arc is equal to the arrival time of the flight in this figure to avoid confusion. However, in a general time-space network, the end time of a flight arc is normally set to the arrival time of the flight plus the minimum turn time.

Liang et al. (2011) also propose two sets of additional arcs to represent the undesired short connects and profitable connects with through revenue in the time-space network. In Figure 2(a), we show how to model the short connects in the time-space network. For any short connect between an arrival flight and a departure flight, we construct a penalty arc to represent the short connect. Considering the example shown in Figure 2(a), we have two short connects between flights f_a , f_b and flights f_a , f_c . Instead of connecting the arrival flight f_a with the ground arc directly, we construct penalty arcs h_{ab} and h_{ac} to connect the arrival flight with the ground arc. The end time of a penalty arc h_{ab} (or h_{ac}) is the departure time of the short connect flight f_b (or f_c). To allow non-penalty connections between the arrival flight f_a and the later departure flights, we also create a zero cost arc h_{a0} connecting the arrival flight f_a with the ground arc. The end time of the zero cost arc h_{a0} equals the arrival time of the flight f_a plus the minimum sit time.

In Figure 2(b), we show how to model the through revenue connects. For any through connect, we build a through arc connecting the arrival flight with the departure flight. Consider the example shown in Figure 2(b), we have a through connect between flights f_a and f_b . Instead of connecting f_a and f_b with the ground arc, we create a through arc h_{ab} connecting flights f_a and f_b . The profit of through arc h_{ab} is the number of passengers with the itinerary f_a followed by f_b . To allow nonthrough connects between f_a/f_b and other flights, we create a zero cost arc h_a/h_b to connect the f_a/f_b and the ground arc.

3. Weekly Rotation-Tour Time-Space Network Model

Given a weekly schedule containing F flights, the WAMRP is to find a generic cyclic aircraft route so that the overall penalty cost or (negative) through revenue is minimized. The number of aircraft required to construct a solution route has to be less than or equal to the fleet size K . Additionally, an aircraft needs to undergo a maintenance every D days or less, and maintenances can only be performed at one of the maintenance stations denoted by M . There is a maintenance capacity Q_{mp} on p day of the week at maintenance station m , where $p \in P = \{1, 2, \dots, 7\}$ and $m \in M$. Note the maintenance capacity of a station might vary from day to day in a week because the available man-hours might change slightly between weekdays and weekends at some stations.

In this section, we first present the weekly rotation-tour network to represent the WAMRP. We then introduce the new weekly rotation-tour network model (WRTNM) to formulate the WAMRP.

3.1. Construction of Weekly Rotation-Tour Network (WRTN)

The WRTN for the WAMRP can be constructed as follows (as shown in Figure 3). We first construct seven D -day time-space networks as there are seven days in a week. Each D -day time-space network starts at day p and ends at day $p + D - 1$, where $p \in \{1, 2, \dots, 7\}$. If $p + D - 1$ is greater than 7, we divide it by 7 and take the remainder to ensure that the day index is smaller than or equal to 7. Without loss of generality, we assume that all the day indices calculated

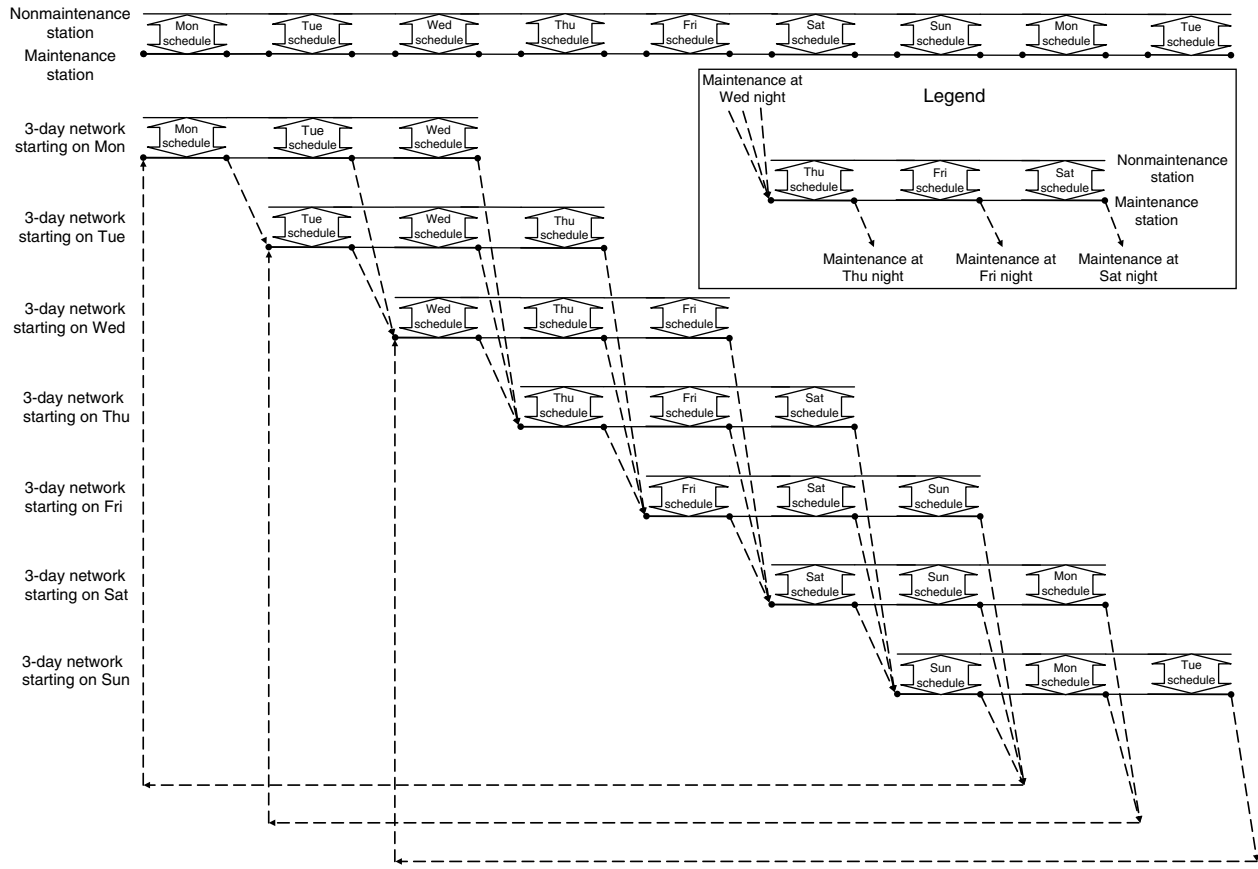


Figure 3 WRTN for the WAMRP

Notes. The maximum days allowed between two consecutive maintenances is three. Only the maintenance arcs, which connect seven D -day time-space networks, are shown in the figure explicitly; whereas the arcs within every D -day time-space network are represented by two-sided bold arrows implicitly.

in the remainder of the paper are smaller or equal to 7 by using the modular operation, as mentioned previously. We denote the D -day time-space network starting at day p of the week, where $p \in \{1, 2, \dots, 7\}$, as S_p . For example, network S_6 , where $D = 3$, is the time-space network containing Saturday, Sunday, and Monday.

We define N as a set of nodes in the seven D -day time-space networks, where node $n \in N$ represents a flight departure or arrival at a station. There are three types of arcs in any S_p network: flight, ground, and connection arcs. The flight and ground arcs are constructed in the same way as in the traditional time-space network. The connection arcs are constructed in the same way as in Liang et al. (2011), as previously discussed in §2.4 and Figure 2. It is noted that we need to construct D flight arcs, each in one of S_p network, where $p \in \{i, \dots, i + D - 1\}$ for every flight f departing on day $i + D - 1$. For example, we need to construct three flight arcs for a flight departing on Saturday in S_4 , S_5 , and S_6 networks, respectively.

We also create a set of capacitated maintenance arcs connecting seven S_p networks. Specifically, a maintenance arc starts at the end of day i at maintenance

station m in S_p network, where $i \in \{p, \dots, p + D - 1\}$, and ends at the beginning time line of maintenance station m in network S_{i+1} . For example, as shown in Figure 3, for network S_2 (here $D = 3$) spanning on Tuesday, Wednesday, and Thursday, we create three maintenance arcs leaving this time-space network at the end of Tuesday, Wednesday, and Thursday. The maintenance arc, which starts at the end of Tuesday, terminates at the beginning of network S_3 ; and the maintenance arc, which starts at the end of Wednesday, terminates at the beginning of network S_4 .

By constructing the WRTN, it is guaranteed that no aircraft rotations violate the maximum D -day maintenance constraints. This is because without going through any maintenance arcs, an aircraft only stays in a single S_p network in WRTN, which lasts for D days. To build a flight route lasting more than D days, a maintenance arc has to be inserted in the route. In other words, any flow in the weekly rotation network must traverse a maintenance arc after $d \in \{1, \dots, D\}$ days to respect the maintenance requirements. Also, it is noticed that an aircraft can fly at most D days after a maintenance, because in the WRTN, an aircraft will enter a new S_p network after

going through any maintenance arcs. For any maintenance station of a S_p network, there are D outgoing maintenance arcs, each at the end of day $i \in \{p, \dots, p + D - 1\}$. Therefore, an aircraft can perform a maintenance after $d \in \{1, \dots, D\}$ days of flying, and no maintenance opportunities are ignored in the network.

Finally, it is worth mentioning that for the daily rotation network (as shown in Figure 1(c)), a flow might cover the same flight multiple times. Whereas it will not be the case for the weekly problem, because D , the maximum days allowed between maintenance, is less than planning the duration of seven days in practice.

3.2. Mathematical Modeling for WAMRP

To facilitate the discussion of our model, we first list all the notations as follows.

Sets, Elements, and Constants

- D : the maximum days between two consecutive maintenances.
- p : $p \in \{1, 2, \dots, 7\}$ represent the day of week.
- S_p : the D -day time-space network starts on day p .
- N : the set of nodes (events) in the WRTN, indexed by n .
- N_p : the set of nodes in the S_p network, where $p \in \{1, 2, \dots, 7\}$.
- F : the set of flights in the weekly schedule, indexed by f .
- F_p : the set of flight arcs in the S_p network. It is worth mentioning that F_p are the arcs in the network, whereas F are the flights in reality. Therefore $\sum_{p \in \{1, \dots, 7\}} |F_p| = D \times |F|$ because each flight appears D times in the weekly network.
- H : the set of connect arcs in the WRTN (either for penalty connects or for through revenue connects), indexed by h .
- H_p : the set of connect arcs in the S_p network.
- c_h : the cost of connect arc h , where $h \in H$.
- M : the set of maintenance stations, indexed by m .
- G : the set of maintenance arcs in the WRTN, indexed by g .
- g_{mpd} : the maintenance arc at station m at the end of day p after d days flying, where $m \in M$, $p \in \{1, \dots, 7\}$, and $d \in D$. Notice that index (mpd) uniquely define a maintenance arc in the WRTN.
- Q_{mp} : the maintenance capacity at maintenance station m on day p .
- L : the set of ground arcs in the WRTN, indexed by l .
- l_n^+ : the ground arc before node n .
- l_n^- : the ground arc after node n .
- K : the size of the aircraft fleet.
- O : the arbitrary count time for counting the number of aircraft.

- F_O : the set of flight arcs passing the count time O in WRTN.
- H_O : the set of connect arcs passing the count time O in WRTN.
- L_O : the set of ground arcs passing the count time O in WRTN.
- G_O : the set of maintenance arcs passing the count time O in WRTN.

Indication Parameters

- α_{fnp}^+ : the binary indicator such that $\alpha_{fnp}^+ = 1$ if flight f in the S_p network starts at node n , and 0 otherwise.
- α_{fnp}^- : the binary indicator such that $\alpha_{fnp}^- = 1$ if flight f in the S_p network ends at node n , and 0 otherwise.
- γ_{hpn}^+ : the binary indicator such that $\gamma_{hpn}^+ = 1$ if arc h in the S_p network starts at node n , and 0 otherwise.
- γ_{hpn}^- : the binary indicator such that $\gamma_{hpn}^- = 1$ if arc h in the S_p network ends at node n , and 0 otherwise.
- β_{mpdn}^+ : the binary indicator such that $\beta_{mpdn}^+ = 1$ if maintenance arc g_{mpd} starts at node n , and 0 otherwise.
- β_{mpdn}^- : the binary indicator such that $\beta_{mpdn}^- = 1$ if maintenance arc g_{mpd} ends at node n , and 0 otherwise.

Variables

- x_{fp} : the binary variable such that $x_{fp} = 1$ if flight f is flown in the S_p network, and 0 otherwise.
- y_{hp} : the binary variable such that $y_{hp} = 1$ if connect arc h is in the S_p network, and 0 otherwise.
- z_{mpd} : the integer variable representing the number of aircraft on maintenance arc g_{mpd} in the weekly network.
- w_l : the integer variable representing the number of aircraft on ground arc l .

Given the above notations, a WRTNM for the WAMRP is presented as follows.

WRTNM

$$\min \sum_{p \in \{1, \dots, 7\}} \sum_{h \in H_p} c_h y_{hp} \quad (1)$$

$$\text{s.t.} \sum_{p: f \in F_p} x_{fp} = 1 \quad \forall f \in F, \quad (2)$$

$$\begin{aligned} & \sum_{f \in F} \sum_{p: f \in F_p} \alpha_{fnp}^+ x_{fp} + \sum_{m \in M} \sum_{d \in D} \sum_{p \in \{1, \dots, 7\}} \beta_{mpdn}^+ z_{mpd} \\ & + \sum_{h \in H} \sum_{p: h \in H_p} \gamma_{hpn}^+ y_{hp} + w_{l_n^+} \\ & = \sum_{f \in F} \sum_{p: f \in F_p} \alpha_{fnp}^- x_{fp} + \sum_{m \in M} \sum_{d \in D} \sum_{p \in \{1, \dots, 7\}} \beta_{mpdn}^- z_{mpd} \\ & + \sum_{h \in H} \sum_{p: h \in H_p} \gamma_{hpn}^- y_{hp} + w_{l_n^-} \quad \forall n \in N, \end{aligned} \quad (3)$$

$$\sum_{d \in D} z_{mpd} \leq Q_{mp} \quad \forall m \in M, p \in \{1, \dots, 7\}, \quad (4)$$

$$\sum_{f_p \in F_O} x_{fp} + \sum_{g_{mpd} \in G_O} z_{mpd} + \sum_{l \in L_O} w_l \leq K \quad (5)$$

$$x_{fp}, y_{hp} \in \{0, 1\} \quad \forall p \in \{1, \dots, 7\}, \\ \forall f \in F_p, \forall h \in H_p, \quad (6)$$

$$z_{mpd} \in \{0, 1, \dots, Q_{mp}\} \quad \forall m \in M, \\ \forall p \in \{1, \dots, 7\}, \forall d \in D, \quad (7)$$

$$w_l \in Z^+ \quad \forall l \in L. \quad (8)$$

The objective function (1) is to minimize the total penalty cost. The assignment constraints (2) ensure that each flight is covered once in the rotation solution. The flow balance constraints (3) ensure that the number of inbound aircraft is equal to the number of outbound aircraft at each node. The capacity constraints (4) ensure that the number of aircraft being maintained at each station is not greater than the station's capacity. The fleet size constraint (5) ensures that the total number of aircraft used is not greater than the size of the fleet. Constraints (6)–(8) are the binary integrality constraints for variables. It is interesting to note that the integrality constraints (7)–(8) can be relaxed because of flow balance constraints (3) and binary constraints (6). The total number of variables and constraints in WRTNM is $O(D|F|)$, and the number of nonzero entries in the problem matrix is $O(D|F|^2)$.

Note that the solution to WRTNM does not provide details of routing sequences. Instead, the solution provides a set of connecting flight, ground, connection, and maintenance arcs. We can use a general Eulerian tour algorithm (Chartrand and Oellermann 1993) to construct generic routing sequences in polynomial time. To construct an Eulerian tour, we first convert the set of connecting arcs in the solution into a simple digraph. In particular, we replace a solution arc with value π by π parallel arcs, each with capacity one. Then, we use the Euler tour algorithm to find a rotation, which covers all the arcs in the graph. We can also extract the individual flight strings from each S_p network by traveling the flight, ground, and connection arcs in that S_p network.

3.3. Operational Issues

In this section, we demonstrate that the proposed model can be easily extended to take into account several operational issues. We also discuss the limitations of WRTNM and provide alternative solution methods.

Red-Eye Flights. In the WRTN, we can model the red-eye flights in a similar way as in Liang et al. (2011). Assume that a red-eye flight departs on day d and arrives on day $d + 1$. Different from normal flights that are duplicated D times in WRTN, we need to construct $D - 1$ duplicated arcs for a red-eye flight,

each in a S_p network, where $p \in \{d - D + 1, \dots, d\}$. For example, we need to construct two arcs for a red-eye flight that departs on Thursday and arrives on Friday when $D = 3$: one arc in S_3 network (containing Wednesday, Thursday, and Friday) and the other arc in S_4 network (containing Thursday, Friday, and Saturday). Because we assume that the maintenances are only performed at night, it is obvious that the maximum number of red-eye flights an aircraft can fly without a maintenance is $D - 1$. Hence the maintenance feasibility of the schedule with red-eye flights is guaranteed by WRTN.

Short Maintenance Duration. It is important to note that when constructing WRTN, the *end of day* at a station is defined as the last departure/arrival event at the station, and the *beginning of a day* is defined as the first arrival/departure event at the station. The night duration between the end of a day and the beginning of the next day is normally longer than the required maintenance duration. However, in some extreme situation, the night duration could be shorter than the required maintenance duration.

For example, as shown in Figure 4(a), flight B lands at 24:00 midnight, and flight C departs at 3:00 in the morning next day at a maintenance station. In such situation, the duration at night is three hours, which is less than the four-hour maintenance duration. To handle this, we can construct multiple maintenance arcs to represent maintenances with different starting and ending times. For example, as shown in Figure 4(b), we can construct two types of maintenance arcs, one starts from 23:00 on day d and ends at 3:00 the next day; and the other starts from 24:00 midnight and ends at 4:00 the next day.

Limitations of WRTNM. The proposed model is not suitable when maintenance can be performed during the daytime. Although for most domestic fleets and many international fleets, maintenances are performed at night, for certain fleets, e.g., fleets to service Europe–North America flights, maintenance can be performed during the daytime. To handle such a case, we suggest that the string model (Barnhart et al. 1998) should be used because it has no restriction on when maintenance should be performed.

Also, it is worth mentioning that the proposed model only considers the maximum number of days between two consecutive maintenances, which is a simplified version of maintenance constraints set by the FAA and the airlines. In particular, the proposed model does not incorporate some operational constraints such as maximum flying time and the total number of takeoffs between two successive maintenances. Although the solutions obtained by WRTNM satisfy these operational constraints in our computational study, WRTNM does not guarantee that the solutions are always feasible to these constraints.

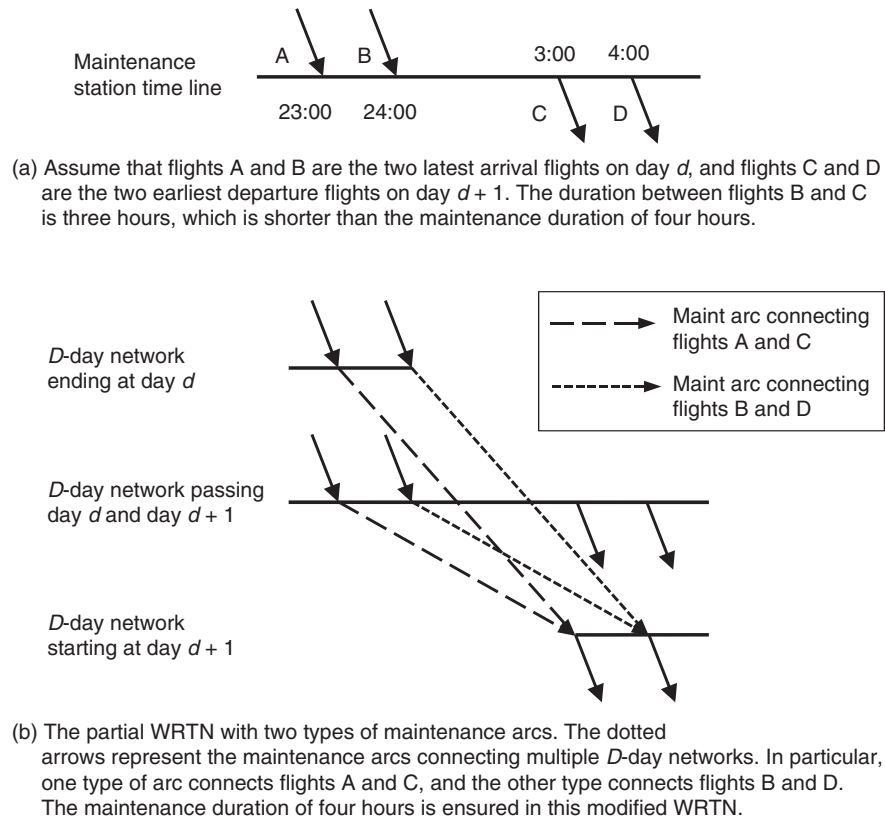


Figure 4 We Construct Multiple Maintenance Arcs When the Duration between “the End of the Previous Day” and “the Beginning of the Next Day” Is Shorter than the Required Maintenance Duration

To the best of our knowledge, there are two models in the literature that guarantee the feasibility to all the operational maintenance constraints. The first one is the string model (Barnhart et al. 1998) because we can always generate flight strings satisfying all the operational constraints with constrained shortest path algorithm. Recently, Haouari, Shao, and Sherali (2013) proposed a quadratic mixed-integer model to solve the daily AMRP with the consideration of all the operational maintenance constraints (maximum number of days, maximum flying time, and total number of takeoffs between two successive maintenances), and the linearized model can solve very large test cases in a reasonable time.

3.4. Diving Heuristic

Based on our preliminary computational study, the integrality gaps between optimal LP relaxations and integer solutions of the WRTNM are less than 0.1%. This result matches the observations from most time-space network-based models for airline planning problems (Hane et al. 1995; Barnhart et al. 1998; Sherali, Bish, and Zhu 2006). Also, the number of cuts added by CPLEX at the root node of the branch-and-bound tree (ILOG 2006) is very small compared to the problem size, whereas these cuts improve the

objective value insignificantly. The strong LP relaxation of WRTNM encourages us to apply a simple diving heuristic (Ball 2011) to simplify the problem and obtain good solutions in a timely fashion.

Here, we propose an iterative heuristic to fix the variables based on their values in the LP solution. Given a fractional LP solution, we first fix all the variables whose values are equal to 1. Because of the constraints in Equation (2), we fix the corresponding nonselected variables to 0. Then, we resolve the simplified mixed-integer program (MIP) using CPLEX solver. If no integer solution is obtained within a limited time, we then solve the LP relaxation of the model and fix the variables whose value are greater than $1 - \sigma$, where σ is the stepsize of the diving heuristic. It is important to note that by rounding up multiple fractional variables, the problem might become infeasible. When infeasibility is detected, we reduce the value of σ by half and the algorithm continues. The detailed algorithm is shown in Figure 5.

4. Integrated WFAP with WAMRP

It has been pointed out in §2.2 that the results of FAP do not guarantee the feasibility of the AMRP, because the maintenance requirements are not considered in FAP. Therefore it is necessary to study the integrated

Diving Heuristic

Input: WRTNM problem matrix;
Heuristic step length σ ;
Predefined IP solution time limit τ ;

```

0  $E = 0$ ;
1 Solve the LP relaxation of WRTNM;
2 WHILE current LP solution is fractional
3   Select a set of variables  $\hat{X}$  such that  $x_{fp} \geq 1 - E \times \sigma$ 
   for all  $x_{fp} \in \hat{X}$ ;
4    $E = E + 1$ ;
5   Set the lower bound of  $x_{fp}$  to 1,  $\forall x_{fp} \in \hat{X}$ ;
6   Set the upper bound of  $x_{f\bar{p}}$  to 0,  $\forall x_{f\bar{p}}$  such that
    $x_{fp} \in \hat{X}, \bar{p} \neq p$ ;
7   Solve the IP of restricted WRTNM;
8   If a feasible solution of restricted WRTNM is obtained
   within time  $\tau$ ;
9     If an optimal solution of restricted WRTNM
   is obtained;
10      Select the best integer solutions available,
      algorithm ends;
11   Else If a feasible but not optimal solution
   is obtained;
12     Record the solution if it is better than the current
     best solution;
13   End If
14 End If
15 Solve the LP relaxation of the restricted WRTNM;
16 If the restricted WRTNM is infeasible;
17   Restore lower bounds for  $x_{fp} \in \hat{X}$  and upper bounds
   for  $x_{f\bar{p}}$  such that  $x_{fp} \in \hat{X}, \bar{p} \neq p$ ;
18    $\sigma = \sigma/2$ ;
19    $E = E - 1$ ;
20 End If
21 End While
22 Select the best integer solutions available,
   algorithm ends.
```

Figure 5 Pseudocode of the Diving Heuristic

WFAP with WAMRP. The network representation of WRTNM can be extended to the integrated WFAP and WAMRP. The objective of WFAP is to assign the flights to aircraft of several fleets so that the total profit is maximized. To model the integrated WFAP with WAMRP, we introduce a new set of available fleets I , which increases the dimensionality of variables with fleet index. In other words, we create a WRTN for each of the fleet $i \in I$. To facilitate the discussion, we define the following additional notations.

Sets, Elements, and Constants

- I : the set of fleets, indexed by i .
- D^i : the maximum days between maintenances for fleet i .
- S_p^i : the D -day time-space network starts on day p for fleet i .
- N^i : the set of nodes (events) in the WRTN for fleet i , indexed by n .
- N_p^i : the set of nodes in the S_p^i network, where $p \in \{1, 2, \dots, 7\}$.
- F_p^i : the set of flight arcs in the S_p^i network.

- r_f^i : the revenue of assigning flight f to fleet i .
- M^i : the set of maintenance stations for fleet i , indexed by m .
- G^i : the set of maintenance arcs in the WRTN for fleet i , indexed by g .
- g_{mpd}^i : as the maintenance arc at station m at the end of day p after d days flying for fleet i .
- J : the set of aircraft families, indexed by j . Each aircraft family contains one or more fleets, which are usually serviced using the same maintenance facility.
- I_j : the set of fleets contained in aircraft family $j \in J$.
- Q_{mp}^j : the maintenance capacity at station m on day p for aircraft family j .
- L^i : the set of ground arcs in the WRTN for fleet i , indexed by l .
- K^i : the size of the aircraft fleet i .
- F_O^i : the set of flight arcs passing the count time O in the WRTN for fleet i .
- L_O^i : the set of ground arcs passing the count time O in the WRTN for fleet i .
- G_O^i : the set of maintenance arcs passing the count time O in the WRTN for fleet i .

Indication Parameters

- α_{fnp}^{i+} : is 1 if flight f in the S_p^i network starts at node n for fleet i , and 0 otherwise.
- α_{fnp}^{i-} : is 1 if flight f in the S_p^i network ends at node n for fleet i , and 0 otherwise.
- β_{mpdn}^{i+} : is 1 if maintenance arc g_{mpd}^i starts at node n for fleet i , and 0 otherwise.
- β_{mpdn}^{i-} : is 1 if maintenance arc g_{mpd}^i ends at node n for fleet i , and 0 otherwise.

Variables

- x_{fp}^i : is a binary variable such that $x_{fp}^i = 1$ if flight f is flown in the S_p^i network, and 0 otherwise.
- z_{mpd}^i : is an integer variable representing the number of aircraft on maintenance arc g_{mpd}^i in the WRTN for fleet i .
- w_l^i : is an integer variable representing the number of aircraft on ground arc l for fleet i .

The MIP formulation of WFAP and WAMRP using WRTNM is given by

$$\max \sum_{i \in I} \sum_{f \in F} \sum_{p: f \in F_p^i} r_f^i x_{fp}^i \quad (9)$$

$$\text{s.t.} \sum_{i \in I} \sum_{p: f \in F_p^i} x_{fp}^i = 1 \quad \forall f \in F, \quad (10)$$

$$\begin{aligned} & \sum_{f \in F} \sum_{p: f \in F_p^i} \alpha_{fnp}^{i+} x_{fp}^i + \sum_{m \in M^i} \sum_{d \in D^i} \sum_{p \in \{1, \dots, 7\}} \beta_{mpdn}^{i+} z_{mpd}^i + w_{ln}^i \\ &= \sum_{f \in F} \sum_{p: f \in F_p^i} \alpha_{fnp}^{i-} x_{fp}^i \\ &+ \sum_{m \in M^i} \sum_{d \in D^i} \sum_{p \in \{1, \dots, 7\}} \beta_{mpdn}^{i-} z_{mpd}^i + w_{ln}^i \\ &\quad \forall n \in N^i, \forall i \in I, \quad (11) \end{aligned}$$

$$\sum_{i \in I_j} \sum_{d \in D^i} z_{mpd}^i \leq Q_{mp}^j \quad \forall m \in M^i, p \in \{1, \dots, 7\}, \quad \forall j \in J, \quad (12)$$

$$\sum_{f_p \in F_O^i} x_{fp}^i + \sum_{g_{mpd} \in G_O^i} z_{mpd}^i + \sum_{l \in L_O^i} w_l^i \leq K^i \quad \forall i \in I, \quad (13)$$

$$x_{fp}^i \in \{0, 1\} \quad \forall f \in F_p^i, \forall p \in \{1, \dots, 7\}, \quad \forall i \in I, \quad (14)$$

$$z_{mpd}^i \in \{0, 1, \dots, Q_{mp}^i\} \quad \forall m \in M^i, \forall p \in \{1, \dots, 7\}, \quad \forall d \in D^i, \forall i \in I, \quad (15)$$

$$w_l^i \in Z^+ \quad \forall l \in L, \forall i \in I. \quad (16)$$

The objective function (9) is to maximize the total profit of all the flights. Here, we do not consider the cost of WAMRP but only the profit r_f^i of the flights, because the cost of WAMRP is negligible compared to the cost of WFAP. However, the connection and maintenance costs can be included in the integrated model easily as in WRTNM. Constraints (10) ensure that each flight is assigned to a single fleet. Constraints (11) are the flow balance constraints, which ensure the flow balance within the WRTN of any fleet. Constraints (12) ensure the number of maintenance performed on day p , at maintenance station m , for fleet i does not exceed the maintenance capacity. Constraints (13) are the plan count constraints for any fleet i . Because the size of the integrated model is compact, we use commercial solvers to solve the integrated model directly. If no satisfied solution is obtained in a reasonable time, we use the diving heuristic presented in §3.4 to obtain the solution in a timely fashion.

5. Computational Results

In this section, we report empirical results of the proposed models. All test problems were solved using an Intel I7-2640 M 2.80 GHz laptop with 8 GB of memory running on Windows 7 platform. Computational times reported in this section were obtained from the laptop's internal timing calculations. All the mathematical modeling and algorithms were implemented in C++ language. All LP and MIP problems were solved using a CPLEX callable library version 12.1. We set the CPLEX relative MIP gap tolerance to 0.01%.

5.1. Computational Experience for WAMRP

The test instances used to benchmark WRTNM in this study are constructed based on the real-life operational aircraft schedule from a major U.S. airline, which is publicly accessible on the airline website. In particular, we construct a total of eight test cases, in which the first six test cases are constructed from six different fleets, respectively, and the last two are constructed by combining flights from multiple fleets. To construct the first six test cases, we first select six

representable fleets, and extract the corresponding airline flights within a particular week of the published schedule. Because what we obtained is an operational schedule, it does not guarantee the cyclic constraints geographically, i.e., the number of incoming flights to an airport is not equal to the number of outgoing flights in a particular week. To ensure the feasibility of WAMRP, we develop an Eulerian tour algorithm (Chartrand and Oellermann 1993) to obtain the maximal set of flights, which are maintenance feasible. We obtain the fleet size information from the airline website. Because we have no information on maintenance stations, we assume that an airport is a maintenance station if the number of departure/arrival flights in that airport is greater than a threshold. We minimize the total number of maintenance stations by increasing this threshold and maintaining the feasibility of the schedule at the same time. For example, for Boeing 757–200 flights, we assume that an airport is a maintenance station if the pairs of in/out flights at the airport are greater than 30.

We combine the schedules of two fleets AIR-320 and 737–800 to create a larger test case SIM-001, and combine the schedules of three fleets 757–200, AIR-320, and 737–800 to create the largest test case SIM-002 just for testing purposes. It is interesting to note that the last test case SIM-002 is about the size of the world's largest fleet, Southwest Airline Boeing 737–700 (350 aircraft). For all the test cases, we assume that the maximum number of days between two consecutive maintenances is 4, which is a realistic estimation of the airline regulation. The detailed information of eight test cases are shown in Table 1.

Table 2 presents the solution information of WRTNM with the objective of minimizing the total penalty cost of short connects. *S-Connect* records the number of all possible short connects in the schedule. In our computation, we assume that the minimum turn time for aircraft is 30 minutes and minimum sit time for crew is 45 minutes. We set the short connect cost to be the short time ($\text{short time} = \min(\text{minimum sit time} - \text{real connection time}, 0)$) of the connect. *Cols* and *Cols-P* represent the number of variables before and after preprocessing. Similarly,

Table 1 Characteristics of Eight Test Cases

Test cases	Flights	Fleet size	Red eyes	Airports	Maint stations
757-300	348	22	14	18	2
737-500	658	35	0	34	3
CRJ-700	972	55	0	40	6
757-200	1,428	88	54	31	6
AIR-320	2,080	123	38	62	7
737-800	2,240	122	54	84	12
SIM-001	4,342	245	92	107	28
SIM-002	6,072	333	156	109	30

Table 2 Performance Characteristics of WRTNM When Minimizing the Total Penalty Cost from Short Connects

Test cases	S-connect	Cols	Cols-P	Rows	Rows-P	NonO	NonO-P	LP	IP	Gap (%)	IP time (sec)	Cuts	Nodes
757-300	21	5,068	3,220	4,039	2,191	11,722	8,026	0	0*	0	1	7	1
737-500	138	9,574	6,654	7,224	4,304	22,530	16,690	120	120*	0	1	26	1
CRJ-700	463	13,872	10,504	9,259	5,891	33,796	27,060	1,675	1,675*	0	2	17	1
757-200	412	18,777	12,621	12,995	6,755	45,183	32,999	25	25*	0	2	20	1
AIR-320	789	28,581	20,445	19,518	11,382	69,015	52,743	174	174*	0	11	27	1
737-800	656	31,011	20,971	22,061	12,021	74,221	54,141	82	82*	0	3	16	1
SIM-001	1,667	58,329	40,513	38,899	21,083	141,857	106,225	90	90*	0	140	10	1
SIM-002	3,521	81,817	59,045	49,851	27,079	203,204	157,660	64	64*	0	20	3	1

*Denotes the optimal solutions.

Rows with Rows-P, and NonO with NonO-P represent the numbers of constraints, and nonzero entities in the problem matrix before and after preprocessing, respectively. Note that in our computational study, we only use node aggregation as the preprocessing method. LP represents the solution values of LP relaxation of WRTNM. IP represents the integer solution to WRTNM. Gap represents the LP-IP gap of the model, and is computed by $(IP - LP)/IP$. Time records the total solution time of the problem. Cut records the number of cuts added by CPLEX solver when solving the model. Nodes records the number of branch-and-bound nodes CPLEX solver explored to obtain the solution.

We can see from Table 2 that WRTNM produces optimal solutions to all test cases within two minutes. Also, it is important to note that the LP bounds provided by WRTNM are very tight. Zero LP-IP gaps are obtained for all eight test cases. The tight LP bounds again help to find optimal solutions quickly. The number of cuts generated by CPLEX to WRTNM is quite small compared to the problem size. Also, the optimal integer solutions are found at the root nodes of the CPLEX branch-and-bound trees for all test cases. Finally, the preprocessing procedure reduces the problem sizes by about 1/3 for all the test cases.

Table 3 presents the solution information of WRTNM with the objective of maximizing the total profit from through revenue. Similar information is recorded in Table 3 as in Table 2. Particularly, we record the number of possible through connects in the second column of Table 3. Because we do not have real

life through connects information, we assume that a through connect only occurs on the airline hubs for one-stop service with spoke-hub-spoke structure. In particular, we build a through connect between any arrival and departure flights if the connection time among them is between 45 minutes and 3 hours. We randomly generate a through value for any through connect such that the accumulative through value from/to any flight does not exceed the aircraft's capacity.

As we can see from Table 3, we obtain optimal solutions to all eight test cases. For the first seven test cases, we are able to obtain optimal solutions in less than two minutes computational time. Similar with the results shown in Table 2, the LP bounds provided by WRTNM are very tight (within 0.114% optimal). The number of cuts generated by CPLEX is very small compared to the problem size, and they do not improve the LP objective value significantly. Optimal and near-optimal solutions are found at the root nodes of the branch-and-bound trees. For test case SIM-002, the computational times are significantly longer than the other test cases. However, it is still within 15 minutes, which is acceptable for a planning problem.

5.2. Computational Experience for Integrated WFAP and WAMRP

The test cases used for the integrated model are constructed from multiple fleet schedules presented in Table 1. As we can see from the integrated model (9)–(16), the size of the integrated model primarily

Table 3 Performance Characteristics of WRTNM When Maximizing the Total Revenue from Through Connects

Test cases	T-connect	Cols	Cols-P	Rows	Rows-P	NonO	NonO-P	LP	IP	Gap (%)	Time (sec)	Cuts	Nodes
757-300	0	5,064	3,088	4,119	2,143	11,630	7,678	0	0*	0	1	14	1
737-500	1,109	13,890	10,410	7,656	4,176	39,482	32,522	1,717	1,717*	0	1	21	1
CRJ-700	1,496	19,456	14,384	10,711	5,639	55,080	44,936	2,101	2,103*	0.11	7	0	500
757-200	1,380	23,612	15,676	13,995	6,059	64,200	48,328	2,218	2,218*	0	2	39	1
AIR-320	3,585	41,720	30,612	21,582	10,474	120,622	98,406	4,356	4,352*	0.11	78	203	173
737-800	10,167	70,380	58,288	23,573	11,481	231,360	207,176	6,718	6,718*	0	8	0	1
SIM-001	10,647	98,158	74,314	43,055	19,211	299,612	251,924	9,610	9,610*	0	75	12	1
SIM-002	8,243	107,181	74,221	56,375	23,415	305,685	239,765	8,191	8,191*	0	881	6	1

*Denotes the optimal solutions.

Table 4 Characteristics of Nine Integrated WFAF and WAMRP Test Cases

Test cases	Flights	Aircraft	Fleets	Origin flights
INT1-4	1,006	62	4	757-300, 737-500
INT2-4	1,320	78	4	757-300, CRJ-700
INT3-4	1,636	96	4	737-500, CRJ-700
INT4-4	1,776	110	4	757-200, 757-300
INT5-4	2,086	128	4	737-500, 757-200
INT1-8	1,032	62	8	757-300, 737-500
INT2-8	1,352	78	8	757-300, CRJ-700
INT3-8	1,686	96	8	737-500, CRJ-700
INT4-8	1,780	110	8	757-200, 757-300

depends on both the number of flights in the schedule, and the number of fleets in WFAF. Therefore, we vary these numbers when generating the integrated test cases. Specifically, we construct two sets of test cases, one set with four fleets and the other set with eight fleets. There are five instances in the four-fleet test set and four instances in the eight-fleet test set. We assume that the demand for every flight is uniformly distributed between 80–320. We also assume that the capacities of the fleets in the four-fleet test set are 110, 170, 230, and 290 with 60 seats difference between two adjacent fleets. Similarly, we assume that the fleet capacities for the eight-fleet test set are from 95 to 305 with 30 seats difference between two adjacent fleets. The number of aircraft in each fleet is the same for all test cases. The profit obtained by assigning flight f to fleet i is computed by the following equation:

$$r_f^i = \min\{dmd_f, cap_i\} \times dur_f - 0.1 \max\{0, dmd_f - cap_i\} \times dur_f. \quad (17)$$

Here, dmd_f is the randomly generated demand for flight f , cap_i is the capacity of aircraft fleet i , and dur_f is the flying time of flight i . The profit r_f^i in Equation (17) contains two parts: the first part computes the revenue, and the second part computes the lost sale penalty. For simplicity, we only consider flight-based profit, but not itinerary-based profit.

In Table 4, we present the nine test cases used to test the proposed integrated model. As we can see from Table 4, the numbers of flights vary approximately from 1,000 to 2,000, which are about the sizes of small to medium airlines. We also list the schedules used to construct the test cases in the last column of the Table 4.

In our computational study, we assume that every flight is eligible to be assigned to any fleet. In other words, we construct $|I|$ rotation-tour networks, and each network contains all flights F . In reality, if it is infeasible or unattractive to assign flight f to fleet i (e.g., the flight distance of f might be too long for some small fleet; the demand is too small for some fleet with large capacity), we can exclude flight f in the rotation network for fleet i to reduce the size of the integrated model. We set the maximum computational time to be four hours (14,400 seconds).

In Table 5, we present the solution values of the integrated model using CPLEX directly. As we can see from Table 5, the weekly integrated model can solve all five 4-fleet test cases optimally. The computational time for the 4-fleet test cases increases from 32 seconds to about an hour. However, we notice that the number of fleets affects the computational time greatly. We can only obtain the integral solution for the first two 8-fleet test cases (INT1-8 and INT2-8) within the four-hour computational time, and we cannot prove solution optimality for any 8-fleet test cases. For all five 4-fleet test cases and the first two 8-fleet test cases, we notice that the LP-IP gaps are very small, which match our observation for WRTNM.

To obtain good solutions in a reasonable time, we apply the diving heuristic to the integrated weekly model. In particular, we realized that the number of fleets affects the difficulty of the test cases significantly in our preliminary study. Also, the number of fractional variables in LP relaxation for the eight-fleet test cases is much larger than that for the four-fleet test cases. Therefore we want to fix more variables in the diving heuristic for the eight-fleet test cases. Thus we decided to use 1.0 for the four-fleet test cases and 0.8 for the eight-fleet test cases in the diving heuristic.

Table 5 Performance Characteristics of Integrated WFAF with WRTNM Using CPLEX Directly

Test cases	Cols	Rows	NonO	LP	LP time (sec)	IP	IP time (sec)	Gap (%)	Cuts	Nodes
INT1-4	34,288	20,210	85,704	144,087	14	144,087*	32	0.00	0	6
INT2-4	44,208	25,112	111,256	148,619	13	148,621*	57	0.00	12	48
INT3-4	54,460	30,736	137,084	170,381	19	170,384*	405	0.00	16	4
INT4-4	51,520	25,258	133,356	268,932	27	268,932*	211	0.00	1	1
INT5-4	62,248	31,626	159,948	309,253	40	309,267*	3,467	0.00	3	77
INT1-8	68,576	39,414	171,408	142,344	21	142,380	14,400	0.03	2	27
INT2-8	88,416	48,904	222,512	146,258	29	146,298	14,400	0.03	0	> 300
INT3-8	108,920	59,836	274,168	167,092	61	—	14,400	—	—	—
INT4-8	109,416	50,700	286,272	283,215	77	—	14,400	—	—	—

*Denotes the optimal solutions.

Table 6 Performance Characteristics of Integrated WFAP with WRTNM Using the Diving Heuristic

Test cases	Iteration	Threshold	Var up	Var down	IP	IP time (sec)	Gap (%)	Cuts	Nodes
INT1-4	1	1.0	515	7,713	144,087*	23	0.00	0	3
INT2-4	1	1.0	303	4,541	148,626	815	0.00	0	28
INT3-4	1	1.0	355	5,325	170,384*	58	0.00	0	1
INT4-4	1	1.0	532	7,821	268,932*	1,346	0.00	0	6
INT5-4	1	1.0	307	4,589	309,267*	5,001	0.01	8	86
INT1-8	3	0.8	282	8,710	142,407	7,835	0.04	0	> 200
INT2-8	4	0.8	279	8,617	146,303	4,659	0.03	0	30
INT3-8	3	0.8	303	9,393	167,425	14,400	0.20	2	9
INT4-8	3	0.8	289	8,895	283,640	14,400	0.15	2	0

*Denotes the optimal solutions.

In Table 6, we present the solutions of the integrated model using the diving heuristic. As we can see, we obtain the optimal and near-optimal solutions to all nine test cases. Particularly, the LP-IP gaps are within 0.2% for all test cases. Also, by applying the diving heuristic, the computational times are reduced for six out of nine test cases. Specifically, for the four-fleet test cases, we can obtain optimal and near-optimal solutions within two hours. For the eight-fleet test cases, we are able to obtain very good IP solutions with small LP-IP gaps within four hours. Overall, by fixing 16%–28% of the flights, we can obtain very good IP solutions for the integrated model in a reasonable time.

6. Conclusion

In this paper, we presented a new weekly rotation-tour network representation for the WAMRP. Based on this representation, we proposed a new mixed-integer LP formulation for the WAMRP; namely, WRTNM. We proposed a heuristic to solve WRTNM in a timely fashion. The computational results showed that the proposed model is very compact and scalable, and is able to find the optimal solutions to schedule with 5,700 flights and 330 aircraft in minutes. We also proposed an integrated model to solve the WFAP and WAMRP simultaneously. A diving heuristic was used to solve the integrated model efficiently. The computational results showed that the integrated model generates near-optimal solutions to the schedules with 1,700 flights, and 8 fleets with 110 aircraft, approximately a medium-sized airline, in a reasonable time. The computational results showed that WRTNM and the integrated model provide good LP relaxation bounds for all test cases.

The current WRTNM can only handle simple cost structures. Extending the proposed network representation for more complex requirements and/or cost will be an interesting future research direction. The compact formulation of WRTNM might also be beneficial to the integration of WAMRP with other planning operations such as the schedule design problem and CPP. Evaluating the new integrated

problems with WRTNM might be another interesting future research direction. Finally, the proposed WRTN representation might facilitate researchers for various weekly planning problems in other areas of transportation, scheduling, and networking.

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