西安交通大学
XI'AN JIAOTONG UNIVERSITY

Systems Engineering Institute
Ministry of Education Key Lab for Intelligent Networks and Network Security

动态规划问题举例
Examples in DP

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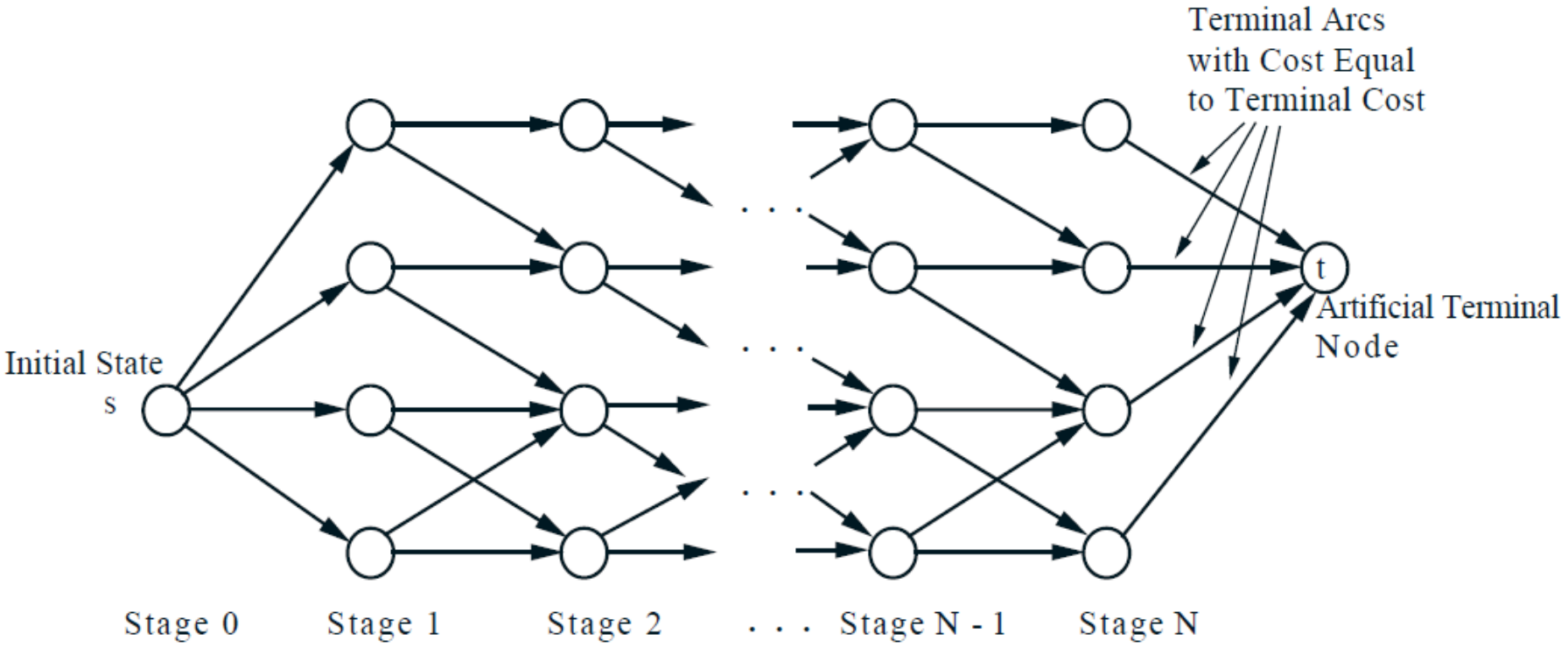
系统工程研究所
吴江

Outline

- 确定性定期多阶段决策问题
- 确定性不定期多阶段决策问题

状态转移图


Terminal Arcs
with Cost Equal
to Terminal Cost
t
Artificial Terminal
Node
Initial State
s
Stage 0
Stage 1
Stage 2
. . .
Stage N - 1
Stage N

基本递推方程

$$f_k(x_k) = \min_{u_k}[G(x_k, u_k, k) + f_{k+1}(x_{k+1})]$$

投资分配问题(纯离散问题)

- 某公司计划用40万元投资项目A, B, C. 下表给出了不同投资规模下的预期利润. 试制定最优投资计划

A			B				C			
1	2	3	1	2	3	4	1	2	3	4
20	30	40	10	20	30	40	10	20	30	40
1.8	2.8	3.2	1.2	1.9	2.5	3	0.8	1.6	2.4	3.1

建模

阶段? → 投资顺序

状态? → 剩余金额

决策? → 投资额

转移方程? → $x_{k+1}=x_k-u_k$

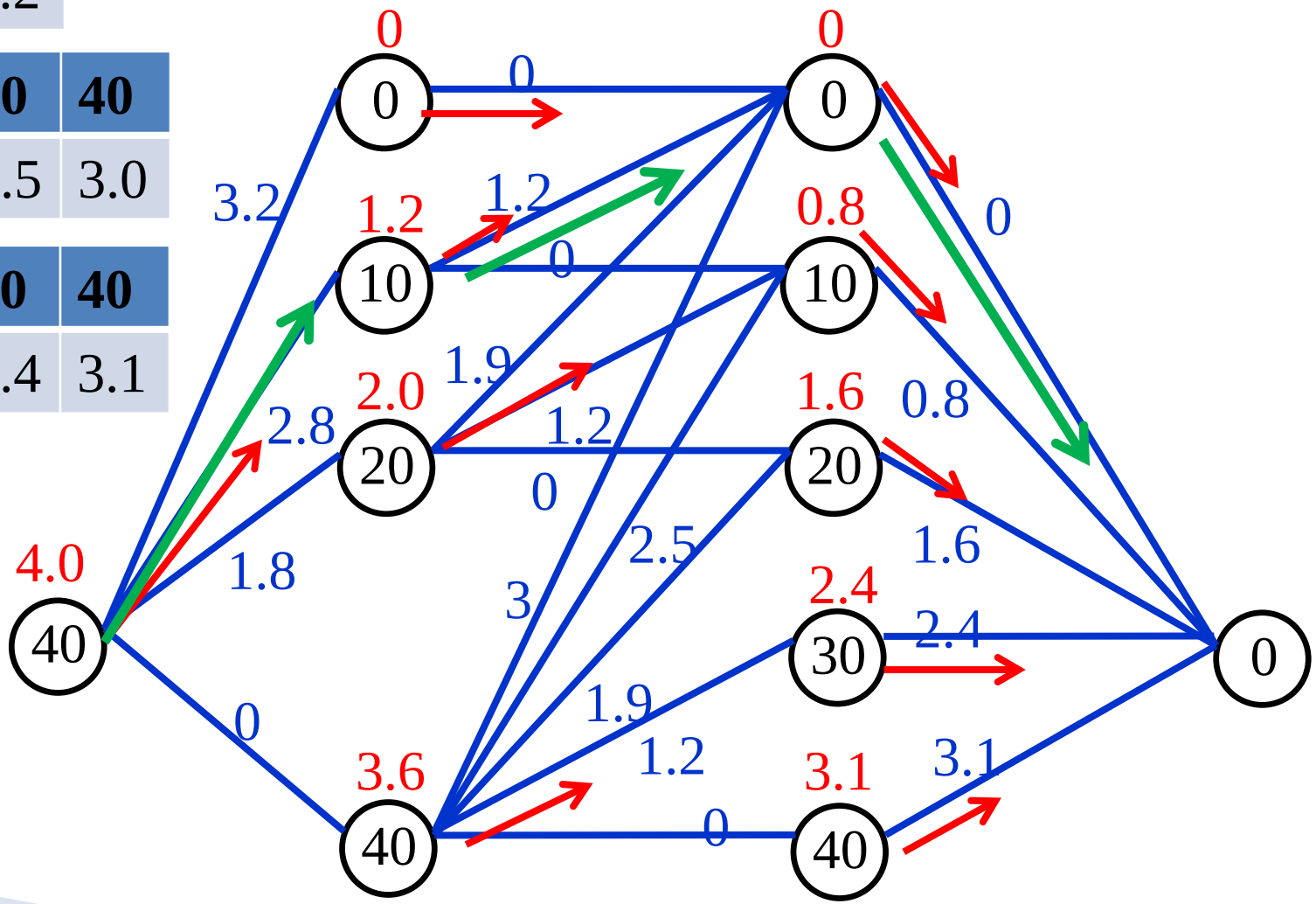
20	30	40
1.8	2.8	3.2

10	20	30	40
1.2	1.9	2.5	3.0

10	20	30	40
0.8	1.6	2.4	3.1

最优投资方案：

A投30万，B投10万，C投0万元



确定性定期多阶段决策问题

例2：(旅行商问题，Traveling Salesman Problem，TSP)

有 $n+1$ 个城市，记为$v_0, v_1, \ldots, v_n$，一个推销员从 v_0 出发，遍访$v_1, \ldots, v_n$ 各恰好一次后再返回 v_0，已知从 v_i 到 v_j 的旅费(或路程长度、耗时等) 为 $d_{i,j}$，求最优路线安排。

解：怎样划分阶段？按自然时序，划分为$n+1$个阶段

怎样定义状态？ 状态：每个阶段/时刻系统所处的状况、态势

状态(v_i, V)：v_i为当前时刻所在城市，V为尚未经过的城市集合(V中不包含 v_0) 思考：状态数目？$O(2^n)$

无后效性？

决策$(v_i, V) \to (v_j, V \setminus \{v_j\}), v_j \in V$ 决策费用为 $d_{i,j}$

思考：画状态转移图？ 应利用基本方程求解！

确定性定期多阶段决策问题

例2：(旅行商问题，Traveling Salesman Problem，TSP)

状态 (v_i, V)　　决策 $(v_i, V) \to \left(v_j, V \setminus \{v_j\}\right), v_j \in V$

怎样列基本方程？基本方程是关于cost-to-go的递推方程。

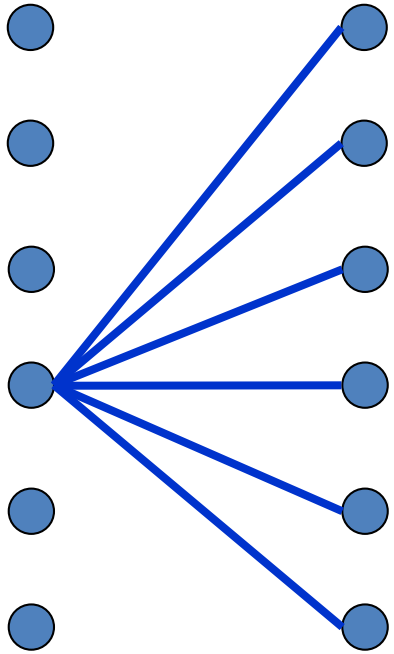
$f(v_i, V) = ?$ 从v_i出发，遍访V中所有城市各恰好一次，再回到 v_0的最短路程长度

状态转移图上求解过程的启示……

边界条件？

$$\begin{cases} f(v_i, \phi) = d_{i,0} \quad , \quad \forall v_i \neq v_0 \\ f(v_i, V) = \min\limits_{v_j \in V} \left\{ d_{i,j} + f\left(v_j, V \setminus \{v_j\}\right) \right\} \end{cases}$$

求 $f\left(v_0, \{v_1, v_2, \cdots, v_n\}\right) = ?$



确定性定期多阶段决策问题

例2：(旅行商问题，Traveling Salesman Problem，TSP)

状态 (v_i, V)　　决策 $(v_i, V) \to (v_j, V \setminus \{v_j\}), v_j \in V$

注意：非对称TSP

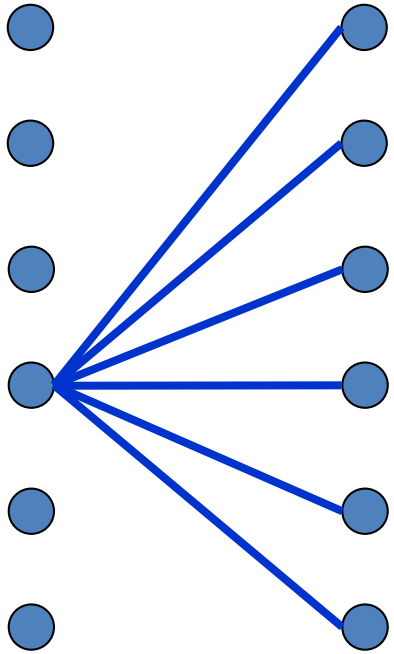
实例

$$D = \begin{array}{c} \begin{matrix} v_0 & v_1 & v_2 & v_3 \end{matrix} \\ \begin{bmatrix} 0 & 8 & 5 & 6 \\ 6 & 0 & 8 & 5 \\ 7 & 9 & 0 & 5 \\ 9 & 7 & 8 & 0 \end{bmatrix} \end{array} \begin{matrix} v_0 \\ v_1 \\ v_2 \\ v_3 \end{matrix}$$

最优解：$v_0 \to v_2 \to v_3 \to v_1 \to v_0$

P170~171

计算复杂性分析



$$\begin{cases} f(v_i, \phi) = d_{i,0}, & \forall v_i \neq v_0 \\ f(v_i, V) = \min\limits_{v_j \in V} \{ d_{i,j} + f(v_j, V \setminus \{v_j\}) \} \end{cases}$$

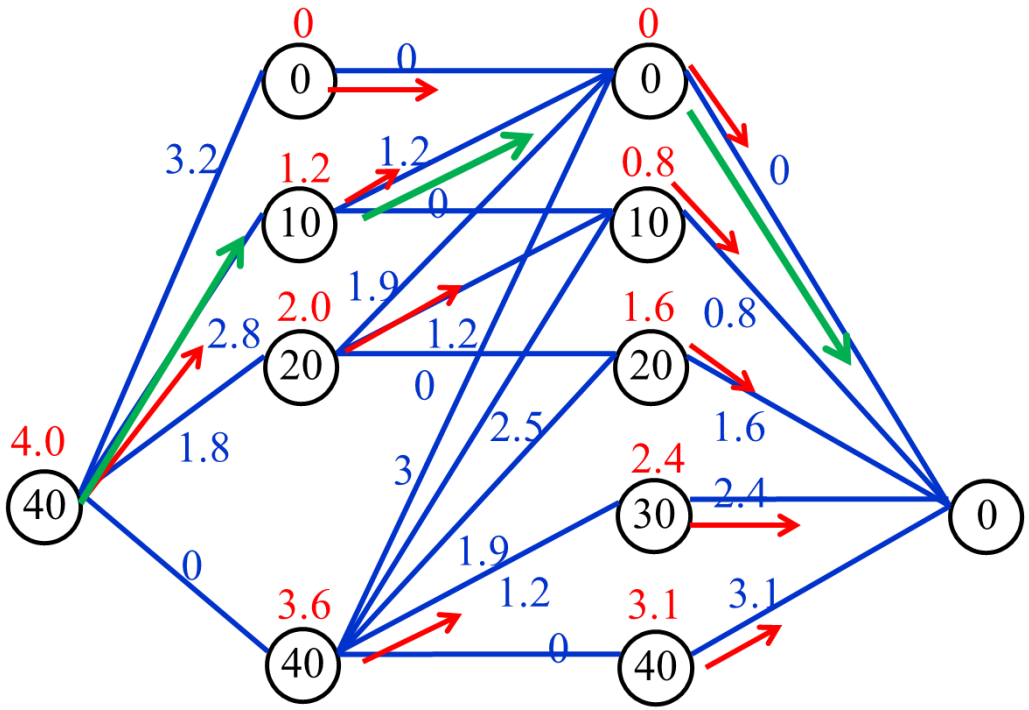
求 $f(v_0, \{v_1, v_2, \cdots, v_n\}) = ?$

动态规划 v.s. 非线性(混合整数)规划

- 确定性定期多阶段决策问题基本上都可以转化为非线性(混合整数)规划问题.
- 非线性(混合整数)规划问题转化为DP:
  - 最优化原理
  - 无后效性
  - 子问题的重叠性
- DP求解的原因
  - 全局解v.s.局部解
  - 中间信息
  - 求解效率

基本递推方程

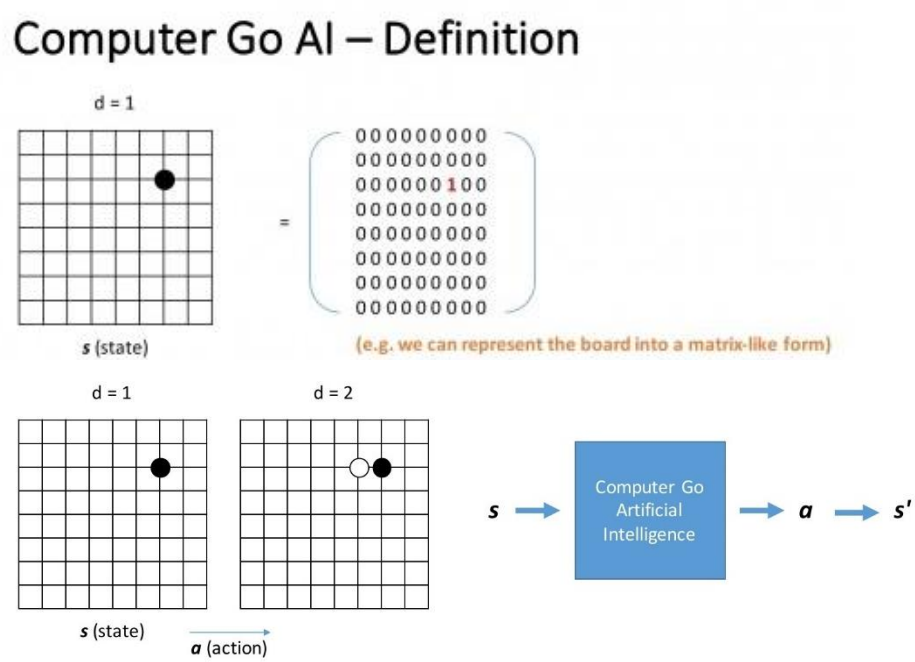
$$f_k(x_k) = \min_{u_k}[G(x_k, u_k, k) + f_{k+1}(x_{k+1})]$$



$$\pi^*(s) = \arg_a \min[r(s,a) + V^*(\delta(s,a))]$$

Mastering the Game of Go with Deep Neural Networks and Tree Search

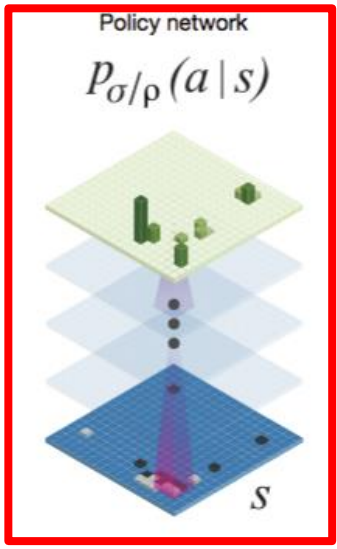
(Nature 529, 484–489, 28 January 2016)



$$\pi^*(s) = \arg_a \max[r(s,a) + V^*(\delta(s,a))]$$

$|S|=3^{361}$ $|A_k|=361-2(k-1)$ $|\Omega|=361*359*357*.....$

Reducing "act candidates"



Current Board

```
00 000 0000
00 000 1000
0-100 1-1100
01 001-1000
00 00-10000
00 000 0000
0-1000 0000
00 000 0000
```

Prediction Model

Next Action

```
000000000
000000000
000000000
000000000
000001000
000000000
000000000
000000000
```

s ***f*: *s* → *a*** ***a***

Expert Moves Imitator Model (w/ CNN)

30,000,000 < s , a >

Updated Model ver 1.3 **VS** **Updated Model ver 1.7**

30,000,000 < s , a >

Current Board

```
00 000 0000
00 000 1000
0-100 1-1100
01 001-1000
00 00-10000
00 000 0000
0-1000 0000
00 000 0000
```

Deep Learning (13 Layer CNN)

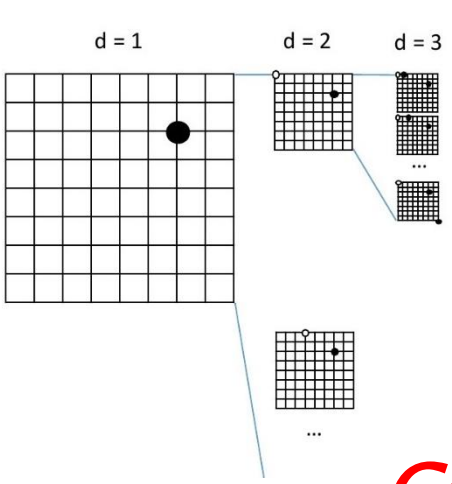
```
000000  000
000000  000
000000  000
000000.2 0.100
000000.4 0.200
000000.1  000
000000  000
000000  000
```

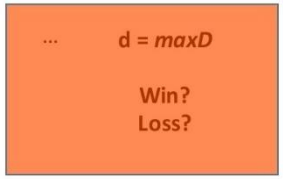
Next Action

```
000000000
000000000
000000000
000000000
000001000
000000000
000000000
000000000
```

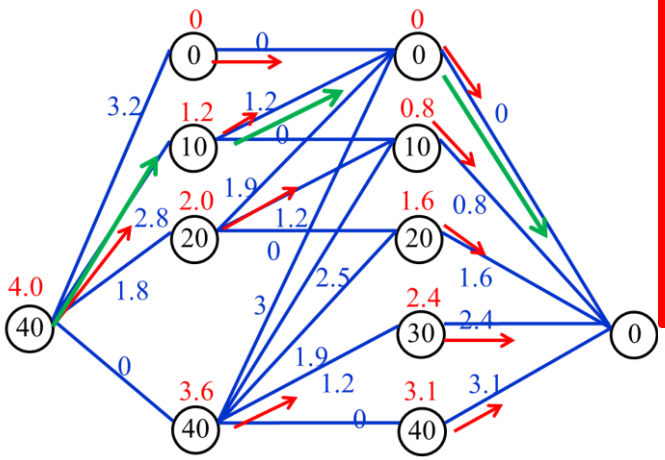
s ***g*: *s* → p(*a*|*s*)** **p(*a*|*s*)** **argmax** ***a***

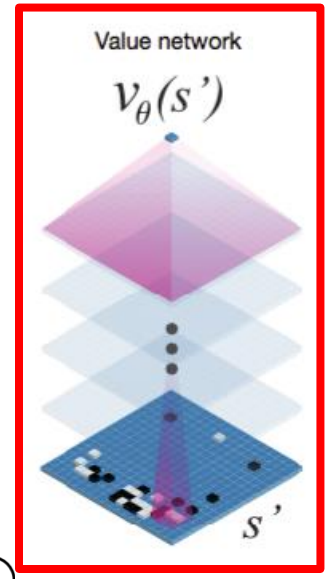
Board Evaluation





Instead of simulating unt

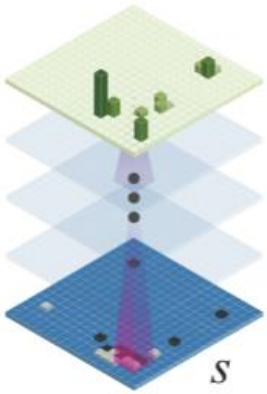




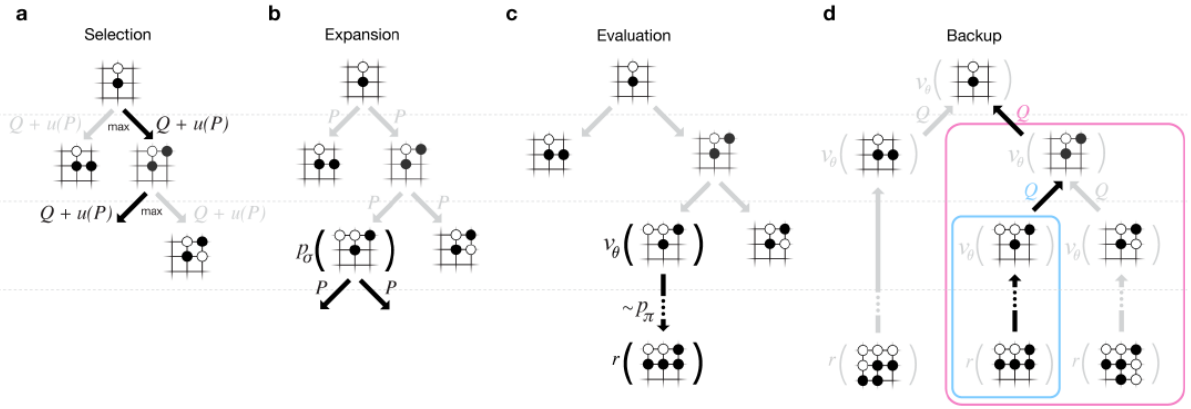
Cost to go?

Policy network

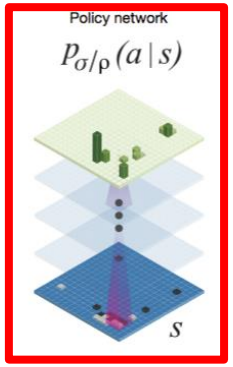
$p_{\sigma/\rho}(a \mid s)$



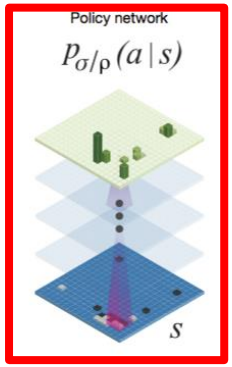
Monte-Carlo tree search



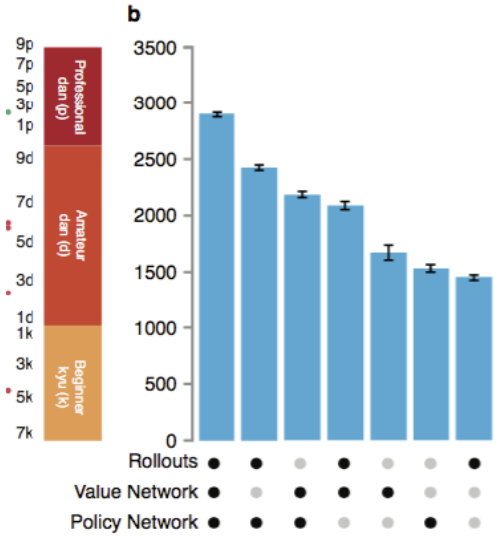
How AlphaGo selected its move

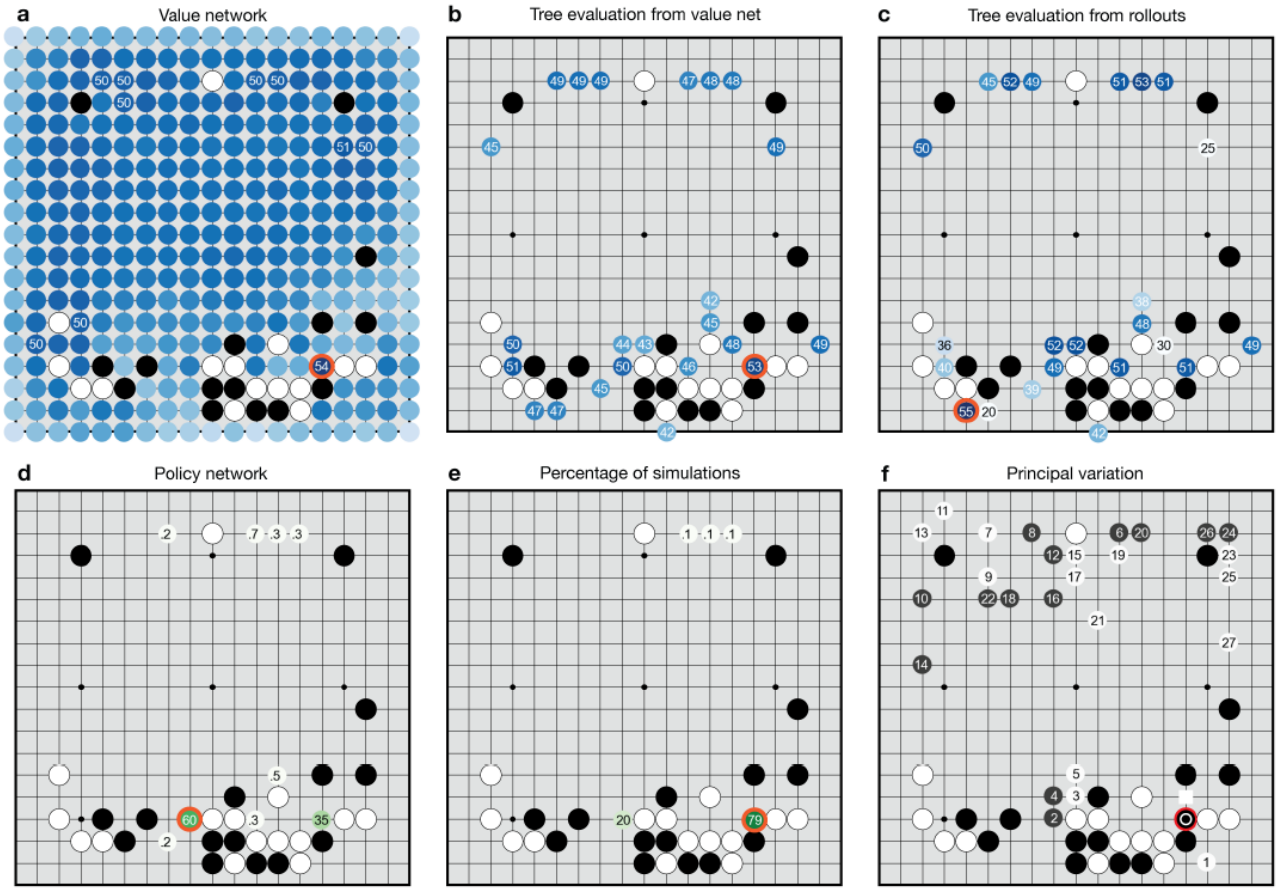


Bread reduction



Depth reduction





Nature 2017: Mastering the game of Go without human knowledge

1. without any human data
2. only stones as input features
3. single neural network
4. without any Monte Carlo rollouts