WAVELET ANALYSIS OF E.C.G. SIGNALS

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ABSTRACT

The complete separation of relevant waves from the E.C.G. signals prior to interpretation is still an open problem because of the noisy nature of the input data. A preliminary attempt is proposed in this paper based on wavelet analysis. Decomposition procedures are reviewed and partial noiseless recomposition is explicited. Some results in low signal to noise ratio are provided.

INTRODUCTION

The wavelets analysis represents a decomposition on a set of functions obtained after several translations and dilations of the so called "analysing wavelet": $\{\psi_{a,b}(t) = \frac{1}{\sqrt{|a|}}\psi(\frac{t-b}{a}); (a,b) \in \mathbb{R}^* \times \mathbb{R}\}$.

This set is generally redundant and does not always constitute a base of $L^2(\mathbb{R})$. These decomposition algorithms are computationally expansive. Different authors [1], [2] have proposed wavelets orthonormal base and Mallat [2] has defined an efficient algorithm requiring less computations.

To be monitored the ECG signal must be decomposed in significant components characterizing cardiac behaviour ; these components (P, QRS, T waves) are often masked by noises (50 hz, baseline drift, muscular noise, artifacts,...). To face these draw backs, we look for decomposition algorithms which work locally in time and frequency domain.

METHOD DESCRIPTION

We consider an orthonormal base of wavelets $(\psi_{m,n})$ $m,n \in \mathbb{Z}$, $a = 2^{-m}$, $b = n.a$. Let V_m be the associated multiresolution analysis; V_m, $m \in Z$ is an increasing serie of closed subspaces of $L^2(\mathbb{R})$ satisfying :

i) $V_{-\infty} \subset \ldots\ldots V_{n-1} \subset V_n \subset \ldots\ldots\ldots \subset V_\infty$

ii) $\bigcap_{m \in Z} V_m = \{0\}$; $\overline{\bigcup_{m \in Z} V_m} = L^2(\mathbb{R})$

where $\bar{E}$ denotes the closure of E

iii) $f(.) \in V_m \Leftrightarrow f(2.) \in V_{m+1}$

iv) $\exists\, \varphi \in V_o$: for any $m \in Z$ the collection, $(\varphi_{m,n})$, $n \in Z$, is a base of V_m with

$$\varphi_{m,n}(x) = \sqrt{2^m}\ .\ \varphi(2^m.x - n) \quad .$$

$<X,Y> = \int_{-\infty}^{+\infty} X(t)\bar{Y}(t)dt$, denotes the scalar product of X by Y in $L^2(\mathbb{R})$ where $\bar{Y}$ is the conjugate of Y.

We consider in the following paragraph the operator A_m such that :

$$A_m \ : \ f \in L^2(\mathbb{R}) \rightarrow A_m f \in V_m$$

It corresponds to an orthogonal projection onto the subspace V_m, according to the scalar product previously defined. $A_m f$ is an approximation of f. Since $V_m \subset V_{m+1}$, the approximation $A_{m+1}f$ is better than the approximation $A_m f$:

$$\|A_m f - f\| \geqslant \|A_{m+1} f - f\|$$

Consider the space spanned by the family $(\psi_{m-1,n})$ denoted W_{n-1} . This vectorial space is orthogonal to V_{m-1} and V_m is the direct sum of V_{m-1} and W_{m-1} :

$$V_m = W_{m-1} \oplus V_{m-1}$$

since $V_{-1} \subset V_o$, we have :

$$\varphi_{-1,o}(x) = \frac{1}{\sqrt{2}}\varphi\left(\frac{x}{2}\right) = \sum_n h_n\ \varphi(x-n) = \sum_n h_n\ \varphi_{o,n}(x)$$

We have $\psi(x) = \sqrt{2}\sum_n g_n\ \varphi(2x-n)$ where

$g_n = (-1)^n h_{1-n}$ [1] [2]. Within the framework of multiresolution analysis $(V_m)_{m \in Z}$, let f(x) be a signal belonging to V_o :

$$f(x) = \sum_n C_{o,n}\ \varphi(x - n)$$

where the serie $(C_{o,n})_{n \in Z}$ can be considered as a discrete signal associated to f(x) through a decomposition onto the $(\varphi_{o,n})_{n \in Z}$. The following relations can then be written :

- $V_o = V_{-1} \oplus W_{-1}$
- $A_{-1}f \in V_{-1}$, $\quad A_{-1}f = \sum_n C_{-1,n}\ \varphi_{-1,n}$
- $D_{-1}f \in W_{-1}$, $\quad D_{-1}f = \sum_n d_{-1,n}\ \psi_{-1,n}$
- $\varphi(\frac{x}{2}) = \sqrt{2}\ \sum_n h_n\ \varphi(x - n)$,

$\psi(\frac{x}{2}) = \sqrt{2}\sum_n g_n \varphi(x - n)$ with $g_n = (-1)^n\ h_{1-n}$

from which, we can compute [1], [2] the equalities :

$$C_{-1,n} = \sum_k C_{o,k}\ .\ h_{k-2n}$$

$$d_{-1,n} = \sum_k C_{o,k}\ .\ g_{k-2n}$$

The series $(C_{-1,n})$ and $(d_{-1,n})$ are then obtained through convolution of the serie $(C_{o,k})$ with the series $(\tilde{h}_k) = (h_{-k})$ and $(\tilde{g}_k) = (g_{-k})$ respectively, by subsampling these last ones at a ratio 2. It is easy to see that the procedure can be extended by recurrence and that more generally :

$$\left\{ \begin{array}{l} C_{j-1,n} = \sum_k C_{j,k} \cdot h_{k-2n} \\ \\ d_{j-1,n} = \sum_k C_{j,k} \cdot g_{k-2n} \end{array} \right. \qquad (1)$$

$$\left\{ \begin{array}{l} A_j f = A_{j-1} f + D_{j-1} f \\ \\ V_j = V_{j-1} \oplus W_{j-1} \end{array} \right. \qquad (2)$$

The relations (1) and (2) (see [1]) point out the recurrent scheme for decomposition. Here, we describe how to recompose the signal step by step.

$$\sum_n C_{j,n}\, \varphi_{j,n} = \sum_n C_{j-1,n}\, \varphi_{j-1,n} + \sum_n d_{j-1,n}\, \psi_{j-1,n}$$

We can obtain the serie $(C_{j,n})$ from the series $(C_{j-1,n})$ and $(d_{j-1,n})$ by making use of the orthonormality of $(\varphi_{j,n})$ [3] :

$$C_{j,n} = \sum_m C_{j-1,m} \cdot h_{n-2m} + \sum_m d_{j-1,m} \cdot g_{n-2m}$$

This relation makes appear the convolution product between the series $(C_{j-1,m})$, $(d_{j-1,m})$, oversampled in a ratio one to two, and the series (h_m), (g_m) respectively [1]. (h_n) and (g_n) can be interpreted as the impulse responses of a low pass and a high pass filter respectively (cut-off frequency = 1/4).

IMPLEMENTATION AND APPLICATION

For the application under consideration, we have to our disposal a discrete representation of an analogical signal s(t) . The analysis is then performed on $(s_k)_{k\in Z} = (s(k))_{k\in Z}$ where the sampling rate is normalized to unit. The following remark is of concern : if the support of $\hat{s}$ is included within $\left[-\frac{1}{2}, \frac{1}{2}\right]$, the sequence of samples (s_k) can be assimilated to the serie $(C_{o,k})$ resulting from the decomposition of s(t) onto V_o :

$$C_{o,k} = \langle s, \varphi_{o,k} \rangle$$

$$(A_o s)(t) = \sum_k C_{o,k}\, \varphi(t - k)$$

Indeed

$$C_{o,k} = \int s(t)\, \varphi(t - k) dt$$
$$= (s(t) * \varphi(-t))\ (k)$$

and considering that $\hat{\varphi}(f)$ is the response of a band-pass filter with support $\left[-\frac{1}{2}, \frac{1}{2}\right]$, which is approximately true, it can tnen be stated that :

$$\hat{s}(f)\ .\ \hat{\varphi}(-f) \simeq \hat{s}(f)$$

and

$$C_{o,k} = (s(t) * \varphi(-t))(k) \simeq s_k$$

So, a sampling satisfying the Shannon's condition provides the serie $(C_{o,k})$. The result of the E.C.G. signal decomposition with the algorithm described in 3, is depicted on figure 1. We can see that the P,QRS,T waves do not tend to be localized in distinct subspaces. But from figure 2, the baseline drift can be satisfactory cancelled. To do this, the E.C.G. signal is reconstructed after eliminating the approximation of low levels. These results have been confirmed by multiple tests. The wavelets analysis, equivalent to a filter bank cannot separate efficiently the P,QRS,T waves, due to the overlaps of their frequency supports. As the operator A_{-n} corresponds to a low-pass filter with a cut-off frequency $(1/2)^{n+1}$, the baseline drift can be suppressed.

CONCLUSION

We have presented here an orthonormal wavelets analysis, equivalent to a filter bank of which cut-off frequencys are related to (2^n) serie. This ratio, equal to 2, can be modified [1], but we do not have any efficient algorithm to our disposal. Furthermore, we can imagine to change the sampling frequency of the digitized E.C.G. signal in a rational ratio, by employing Decimation and Interpolation techniques. The characteristic components of E.C.G. could then be better and decisional framework built.

RERERENCES

[1] DAUBECHIES I., Com.Pure Appl. Math, 41, pp. 909-996, 1988.

[2] MALLAT S.G., IEEE Trans.PAMI,2,7,pp.674-693, 1989

[3] CARRAULT G., and al, AIM report N°A1006-1.1. B-F-R-900223, Feb.1990.

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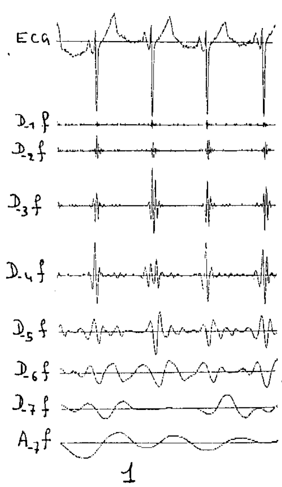

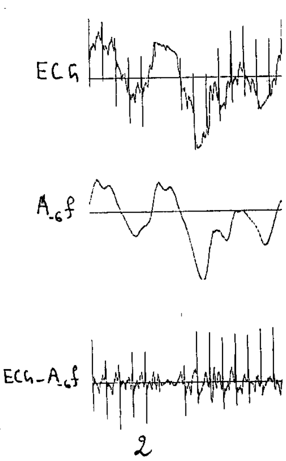

Figure 1

An example of ECG Signal decomposition

Figure 2

Baseline drift cancellation