TRANSMISSION DESIGN FOR A FORMULA STUDENT ELECTRIC VEHICLE

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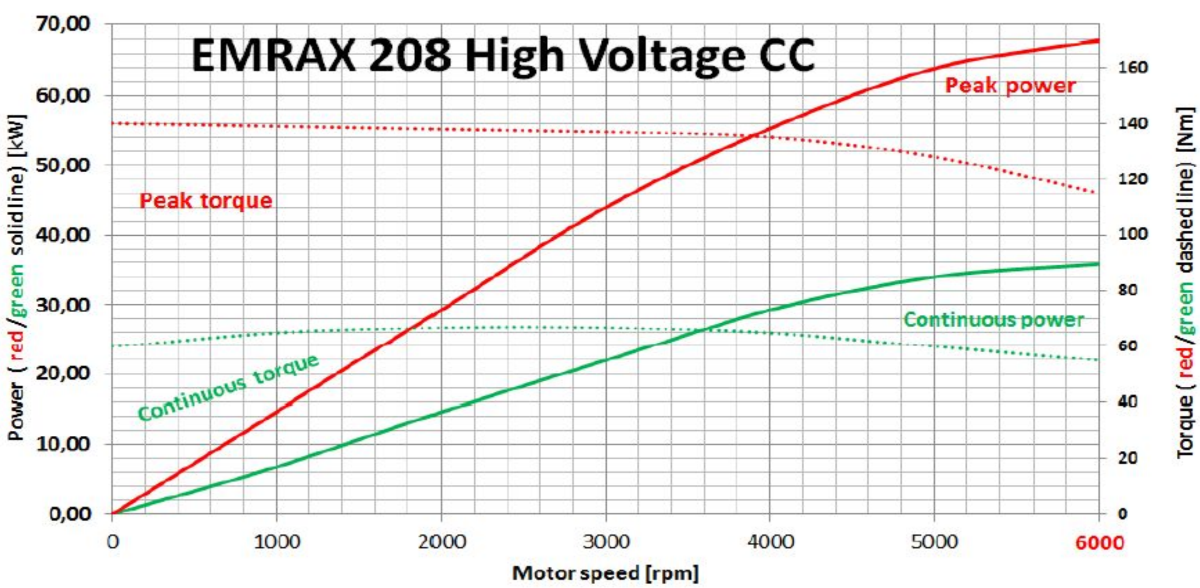
CONTENTS

1. Abstract

The transmission of a car plays a vital role in delivering the motor power and torque to the wheels. This paper presents a comprehensive study of the drivetrain design of the team's maiden EV, starting with the determination of an optimum gear ratio, top speed, maximum longitudinal acceleration, power requirements as well as the design and simulation of a custom planetary gearbox and involute-splined electric motor shaft. Since a single gear reduction will be used in the EV, determining the optimum gear ratio is an important aspect so that in order to prevent the wheel slip, the motor torque that actually reaches the wheels does not exceed the traction limit. To do so, both analytical and numerical analyses were performed with the help of vehicle data and motor specifications alongwith the concepts of vehicle dynamics on the basis of which, the appropriate motor for use in the EV is EMRAX 208. The calculations have been performed in context to the acceleration event (straight line with a length of 75 m from starting line to finish line) where the car will accelerate at its fullest potential and the peak motor torque will be utilized. The power consumed by the car during maximum acceleration and top speed has also been determined. To incorporate the calculated gear ratio of 6:1 in the car, a planetary gearbox will be coupled with a sprocket-chain drive. The design and simulation of the gearbox have been comprehensively discussed. Also, a custom motor shaft, whose design and simulation have been discussed as well, will be manufactured in order to transfer the motor torque to the gearbox. Lastly, it is important to note that the analytical calculations have been performed with the view of minimizing the number of assumptions made and keeping the figures as realistic as possible. Although every model is an abstraction of reality in some way, we are confident that the figures presented are at least in the ballpark.

2. General Vehicle Data

GENERAL CAR DATA	
curb weight(kg)	304
curb weight of K3(kg)	226
engine weight(kg)	40
weight of motor+controller+battery pack(kg)	88
drivetrain weight(kg)	15
aero package weight(kg)	15
mass of driver(kg)	60
gross weight(m)(car + driver)(kg)	364
wheelbase(m)	1.72
height of CG from ground(h)(m)	0.24
horizontal distance of CG from front contact patch center(p)(m)	0.95
horizontal distance of CG from rear contact patch center(q)(m)	0.77
density of air(kg/m^3)	1.23
frontal area of K3(A)(m^2)	0.9
drag coefficient(c_d)	0.8
rolling friction coefficient(C_rr)	0.01
kinetic friction coefficient	1.5
tire radius(m)	0.229
peak motor torque(Nm) (**EMRAX 208**)	150



The determination of the transmission ratio is driven by three main sources: the achievable acceleration time, the top speed of the vehicle and an upper bound on the amount of usable motor power. Firstly, we need to determine the duration of time from standstill position of the car for which the peak motor torque will be maintained and the maximum longitudinal acceleration that the car will have during the same duration.

The peak motor torque can be maintained upto a max. duration of 30 seconds. Hence, as far as the acceleration run is concerned which usually gets completed in less than 5 seconds, we were free to use the peak torque during the entire run but due to the bound of the motor's maximum RPM of 6000 and a power cap of 80 kW, the use of peak torque got limited to 2 seconds. We could have instead decreased the amount of peak torque and increased the time duration for which it is available in order to achieve more acceleration and speed but again, the bounds on motor RPM and power placed a limit on these assumptions.

To determine the max. acceleration during the first 2 seconds, the following calculations were carried out:

$$\frac{\tau \times gr}{radius} = \underbrace{C_{rr} mg + \frac{1}{2} \rho A C_d v^2 + m \frac{dv}{dt}}_{\text{Total tractive effort}}$$

Plugging in the respective values,

$$\frac{dv}{dt} = 9.069 - 0.00123 v^2$$

$$\Rightarrow \frac{v_{n+1} - v_n}{\partial t} = 9.069 - 0.00123 v_n^2$$

$$\Rightarrow \boxed{v_{n+1} = v_n + \partial t \left(9.069 - 0.00123 v_n^2\right)}$$

v_n = velocity at n^{th} second

v_{n+1} = " " $(n+1)^{th}$ second.

∂t = time step (seconds).

$$\boxed{a_{n+1} = \frac{v_{n+1} - v_n}{\partial t} \ (m/s^2)}$$

A spreadsheet was created to determine these values and the results obtained are shown below in the picture:

The maximum acceleration for 2 seconds was thus taken as 9.5 m/s^2 for the calculations (assumed constant) and the speed at the end of this time as 20 m/s.

General data	
motor peak torque(Nm)	150
gear ratio	5.7
mass of car(with driver)(kg)	364
rolling friction coefficient(C_rr)	0.01
tire radius(m)	0.229
density of air(kg/m^3)	1.23
frontal area of K3(A)(m^2)	0.9
drag coefficient(c_d)	0.8
(torque*gear ratio)/tire radius	3733.624454
C_rr*mg	35.672
0.5*rho*A*c_d	0.4428
(J10-J11)/mass	10.15921004
J12/mass	0.001216483516
time step(delta t)(sec.)	0.1

Time(sec.)	Velocity(m/s)	Acceleration(m/s^2)
0	0	0
0.1	1.015921004	10.15921004
0.2	2.031716455	10.15795451
0.3	3.04713531	10.15418855
0.4	4.061926805	10.14791495
0.5	5.075840702	10.13913897
0.6	6.088627539	10.12786836
0.7	7.10003887	10.11411331
0.8	8.109827513	10.09788643
0.9	9.11774779	10.07920277
1	10.12355576	10.05807972
1.1	11.12700947	10.03453705
1.2	12.12786915	10.0085968
1.3	13.12589748	9.980283305
1.4	14.14019632	10.14298838
1.5	15.13179432	9.915980057
1.6	16.11986136	9.880670349
1.7	17.10417204	9.843106882
1.8	18.08450449	9.8033245
1.9	19.06064049	9.761359953
2	20	9.393595105
average acceleration(m/s^2)		9.5

Distance travelled in the first 2 seconds = 19 m.

Speed of the car at the end of 2 seconds = 20 m/s or 72 kmph.

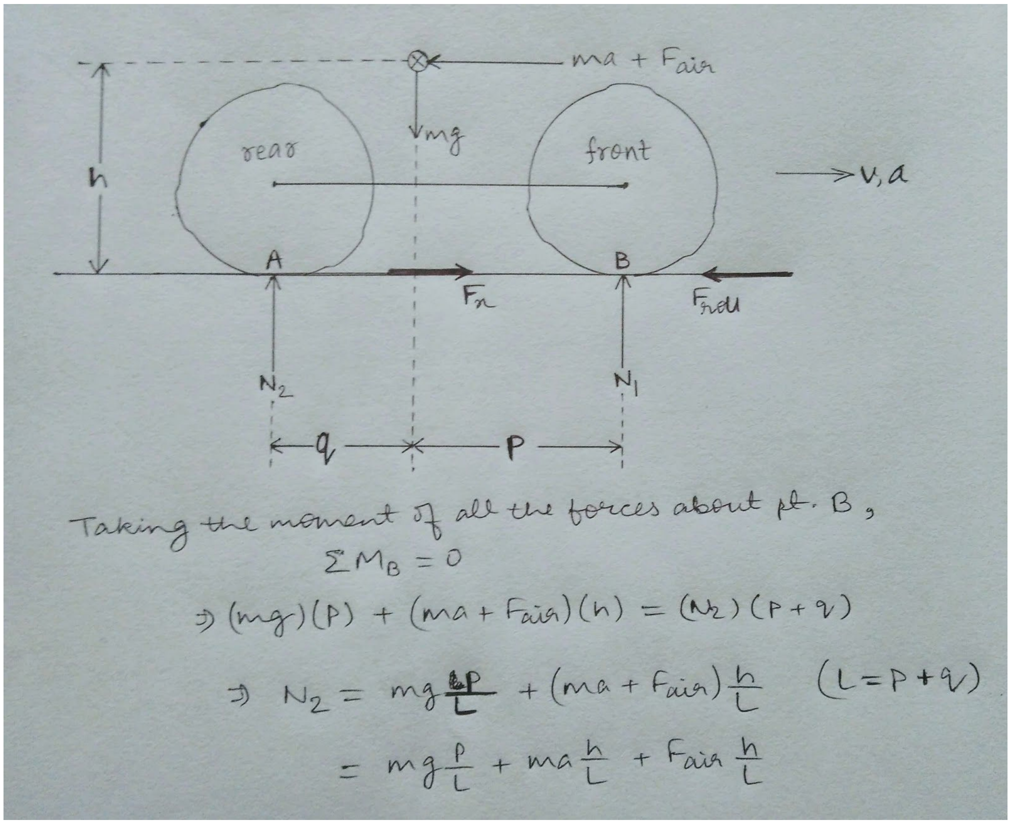
Remaining distance to be traversed = 75 - 19 = 56 m.

Top speed attained at the end of the run = 100 kmph (assumed).

New acceleration for the remaining distance= 3.3 m/s^2 (assumed constant)

Time required to cover the remaining distance = 2.34 seconds.

Hence, total time taken to complete the acceleration run = 4.34 seconds.



3. Calculations to determine the optimum gear ratio

Using the calculations in the figure above, we determined the total load at the rear wheels (N_2) during maximum acceleration. Plugging in the respective values in the final equation, this load came out to be about 250.28 kg. The effect of air drag has been neglected since, at such low speed of the car (20 m/s), the term F_{air}*h/L came out to be very small relative to the first two terms.

Now, considering a static weight distribution of 55% (200.2 kg) and 45% (163.8 kg) at the rear and front side of the car respectively, during maximum acceleration, about 14% (50.08 kg) of the total weight of the car is transferred to the rear wheels.

Friction coefficient = 1.5

Traction available at the rear wheels = 1.5 * 250.28 * 9.8 ~ 3679 N

Tire radius = 0.229 m (using 10-inch wheels)

Total torque at the rear wheels = 3679 * 0.229 ~ 843 Nm

Peak motor torque = 150 Nm

Therefore, required gear ratio = 843 / 150 ~ 5.6

Speed of the car at the end of 2 seconds = 20 m/s

Wheel RPM = (20 * 60) / (2 * π * 0.229) = 834 RPM

Corresponding motor RPM = 834 * 5.6 = 4670 RPM (close to the motor RPM of 4000 upto which peak torque is maintained)

Owing to this gear ratio of 5.6:1, we will finally consider a 6:1 gear ratio since mechanical losses will effectively lower the overall torque output from the gearbox to some extent. We will be implementing a single-stage planetary gearbox coupled with a sprocket-chain drive in the EV, the overall gear ratio being split between the two components.

4. Calculations to determine the power consumed by the vehicle

Power consumption in two situations has been calculated: at max. acceleration and at top speed to ensure that power consumption, in either case, doesn't exceed the specified limit of 80 kW as per the rulebook. The various resistive forces acting on the vehicle have been taken into account.

Case 1: Power consumption at max. acceleration

Speed of the car at the end of 2 seconds = 20 m/s

Corresponding acceleration = 9.5 m/s^2

Rolling friction coefficient = 0.01

Acceleration resistance = 364 * 9.5 = 3458 N

Resistance due to air drag = 0.5 * 1.23 * 0.9 * 0.8 * 20^2 = 177 N

Rolling resistance = 0.01 * 364 * 9.8 = 36 N

Total resistance on the car = 3458 N (neglecting minor effects of air drag and rolling resistance)

Therefore, power consumed= (3458 * 20) / 1000 = 69 kW

Case 2: Power consumption at top speed

Maximum speed of the car at the end of the run (assumed) = 100 kmph

Corresponding acceleration = 3.3 m/s^2

Rolling friction coefficient = 0.01

Acceleration resistance = 364 * 3.3 = 1208 N

Resistance due to air drag = 0.5 * 1.23 * 0.9 * 0.8 * 27.78^2 = 341.67 N

Rolling resistance = 0.01 * 364 * 9.8 = 36 N

Total resistance on the car = 1585 N

Therefore, maximum power consumed= (1585 * 27.78) / 1000 = 44 kW

Wheel RPM at top speed = (27.78 * 60) / (2 * π * 0.229) = 1158 RPM

Corresponding motor RPM = 1158 * 6 = 6506 RPM (close to the maximum motor RPM of 6000).

5. Planetary gearbox design

5.1 Transmission ratio

The transmission ratio is driven by three main sources: the achievable acceleration time, the top speed of the vehicle and an upper bound on the amount of usable motor power. Since the vehicle performs at its fullest potential (acceleration and top speed) during the acceleration event, the transmission ratio is determined on the basis of the maximum torque that needs to be sent to the rear wheels keeping in mind that the high ratio so obtained doesn't limit the vehicle's top speed. The vehicle acceleration is particularly important in the architecture trade analysis as it has a very high return on investment for points at the competition, especially in the acceleration event. Setting a vehicle top speed also plays a significant role in the vehicle design and depends on the maximum speed of the motor.

5.2 Drivetrain Architecture

There are three main techniques used in the drivetrains of common transportation devices. These are the chain drive, belt drive, and gear drive. Each device transfers the torque from the output shaft of the propulsion mechanism and transfers it to a second shaft either increasing or decreasing the torque and speed.

5.2.1 Sprocket-Chain Drive

Formula SAE combustion heavily utilizes the chain drive architecture due to the common implementation of motorcycle engines, predesigned for chain sprockets. The chain drive system is very simple and much more inexpensive as compared to gears. However, if a large gear reduction is necessary, as with electric motors, the secondary sprocket size increases dramatically. The chain drive system also suffers from chain extension requiring constant maintenance for slack in the chain. To attain the gear ratio of 6:1, the front sprocket will be too small leading to a very small wrap angle and area of contact between the chain and sprocket. The sprocket needs to cope with a large magnitude of motor torque (~140 Nm) so this design is not feasible.

5.2.2 Belt Drive

A belt drive system experiences nearly all of the same negative effects of the chain drive with slight decreases in the amount of backlash and impact loading. The operating noise of the belt drive is also lower than that of the chain driven system.

5.2.3 Planetary Gearbox

Planetary gearboxes (also called epicyclic gearing) involve one or more outer gears known as planet gears rotating around a central sun gear. One of the primary advantages offered by planetary gearboxes is very high torque density, a consequence of having multiple gear meshes transmitting torque. For equivalent power rating, planetary gear sets are smaller, lighter and have lower inertia than spur gear pairs. It is also possible to get fairly high reduction ratios in a single stage, certainly as high as 6:1 as required for this application.

On the basis of the aforementioned information, a single-stage planetary gearbox was decided to be implemented. However, the drivetrain packaging needs to be taken into consideration. The typical power flow would be from motor to gearbox and then to the rear axle. But there arises an issue as to how to connect the gearbox to the rear axle. Using a shaft would be infeasible due to inefficiencies and the need of direct connection to the differential, which would require the opening of the differential.

To alleviate such a situation, it was decided that the gearbox would be connected to a sprocket-chain drive which in turn would transmit power to the rear axle. The overall gear ratio of 6:1 would be divided between the two components, with the aim of minimizing the mass of the system alongwith obtaining a sufficient factor of safety for the gears and sprockets. After a careful analysis, it was decided to design the planetary gearset to obtain a ratio of 4:1 and the sprocket-chain drive to obtain a ratio of 1.5:1.

5.3 Gear Parameters

We decided to use three planet gears to achieve the desired ratio of 4:1. The input from the motor will be given to the sun gear and the output will be provided by the planet carrier, the ring gear being fixed. This configuration of the PGT, being one of the many other possible configurations, provides the highest gear ratio. To determine the number of gear teeth to be used, we needed to satisfy two requirements:

1. N_r = 2*N_p + N_s (N_r- no. of teeth of ring gear and so on).
2. (N_r)/3 and (N_s)/3 should be even integers.

Parameter	Value	Unit
Module	3	mm/tooth
Pressure angle	20	degrees
Number of teeth(sun/planet/ring)	18 / 18 / 54	teeth
Pitch Diameter(sun/planet/ring)	54 / 54 / 162	mm
Face Width(sun/planet/ring)	40.64 / 33.02 / 40.64	mm
Addendum	3	mm
Dedendum	3.75	mm
Whole Depth	6.75	mm
Gear ratio	4:1	---

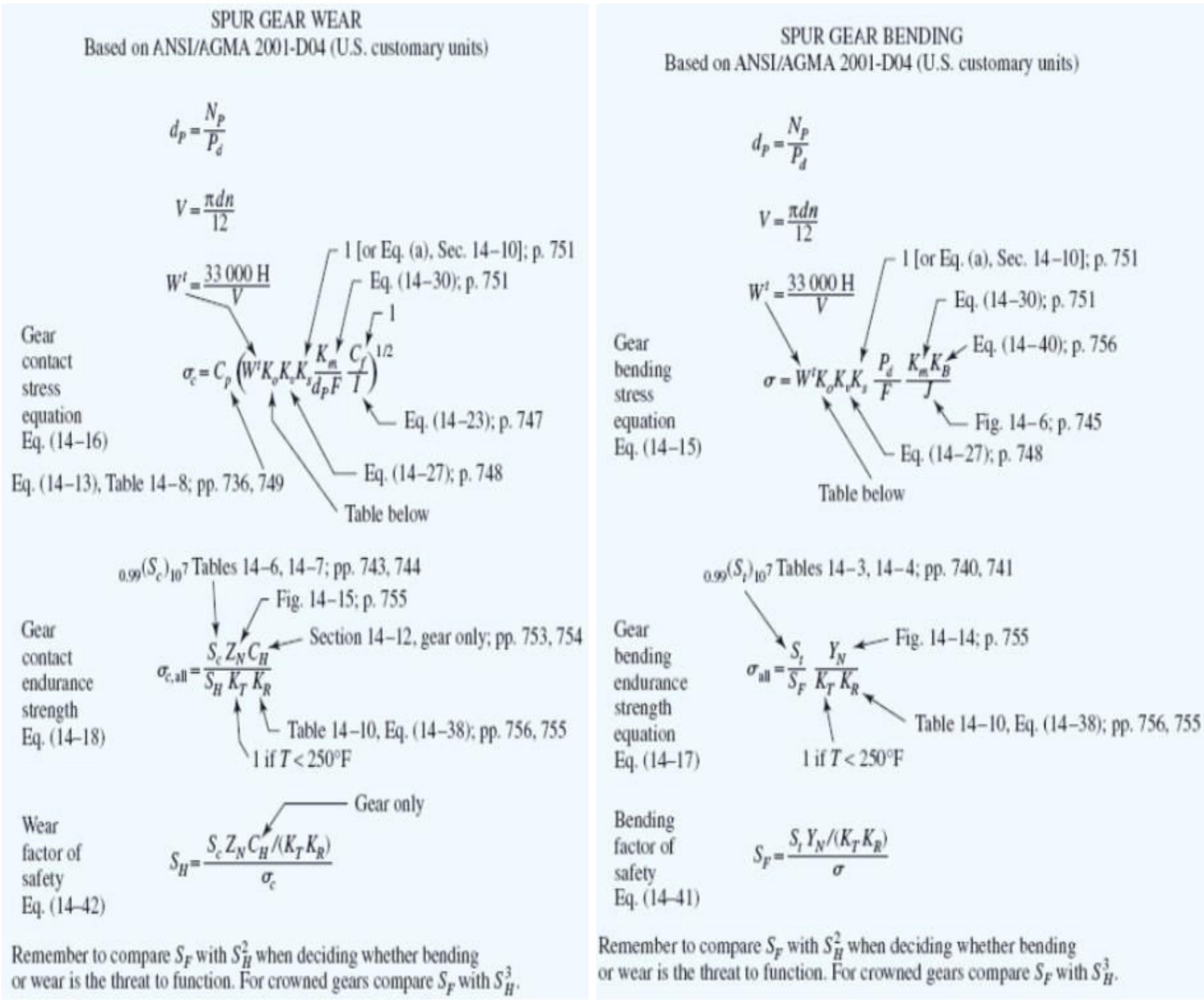
Many different geometries were considered and iterations run on the KISSsoft software, but in the end, a module of 3 provided the best balance of gear ratio and gear sizes. Once this was set, the other requirements fixed the basic parameters shown in the table above.

5.4 Spreadsheet Analysis of Gear Tooth Bending and Wear

During their operation, gear teeth are constantly under bending and contact stresses, too much of which can cause the tooth to wear excessively, deform, or even break off entirely. To ensure that the gear teeth wouldn't fail over their intended lifetime, we calculated the contact and bending stresses they would be under and the strength they'd have against these stresses.

To do these calculations, we used equations from Chapter 14 of Shigley's Mechanical Engineering Design, 10th Edition. These equations were formatted for use between two external spur gears, so the calculations were restricted to the sun and planet gears. Because the sun gear is the smallest and weakest gear, we're confident that despite the fact the ring gear was not analyzed here, we are still covering the worst-case scenario in terms of gear-tooth stresses.

The process of equations that we used is shown below:



Due to the iterative nature of the calculations, we created a spreadsheet to calculate all of the correction factors and other relevant variables necessary to determine our safety factors for gear tooth bending and wear.

We began by specifying the following spreadsheet inputs:

Pitch diameter
Number of teeth
Module
Face Width
Angular Velocity
Pitch Line Velocity

Input Power
Transmitted Load

Next, we calculated all of the following correction factors:

1. *Overload factor (K_o)*

This factor is intended to make allowance for all externally applied loads in excess of the nominal tangential loads. Examples include variations in torque from the mean value due to the firing of pistons, or in our case the torque under numerous sudden acceleration and braking events. We estimated a value for our system using the table given below. Assuming the worst-case scenario, we specified an unlikely overload factor of 1.35 to ensure as big a buffer as possible from failure.

Table of Overload Factors, K_o

	Driven Machine		
Power source	Uniform	Moderate shock	Heavy shock
Uniform	1.00	1.25	1.75
Light shock	1.25	1.50	2.00
Medium shock	1.50	1.75	2.25

2. *Dynamic factor (K_V)*

This factor is used to account for inaccuracies in the manufacturing and meshing of gear teeth in action, including transmission error. Transmission error is defined as the departure from the uniform angular velocity of the gear pair, and can be caused by vibration of the tooth during meshing, wear of the contacting portions of the teeth, tooth friction, and other factors. AGMA defines a set of quality numbers Q which define the tolerances for gears of various sizes manufactured to a specified accuracy in an attempt to account for these effects. Quality numbers from 8 to 12 are of precision quality. Because our gears were being custom-made, we defined our gears to be of quality 12 (the highest quality the equation allows). With the quality number set, the dynamic factor is calculated with the following equation:

$$K_v = \begin{cases} \left(\dfrac{A+\sqrt{V}}{A}\right)^B & V \text{ in ft/min} \\ \left(\dfrac{A+\sqrt{200V}}{A}\right)^B & V \text{ in m/s} \end{cases}$$

where

$$A = 50 + 56(1-B)$$
$$B = 0.25(12-Q_v)^{2/3}$$

3. *Size factor (K_S)*

This factor reflects the nonuniformity of material properties due to size. They can be calculated using the Lewis Form Factor values (Y_p, Y_g) that vary based on the number of teeth. We obtained these values from the table shown below, which is defined for a normal pressure angle of 20° and full-depth teeth:

Number of Teeth	Y	Number of Teeth	Y
12	0.245	28	0.353
13	0.261	30	0.359
14	0.277	34	0.371
15	0.290	38	0.384
16	0.296	43	0.397
17	0.303	50	0.409
18	0.309	60	0.422
19	0.314	75	0.435
20	0.322	100	0.447
21	0.328	150	0.460
22	0.331	300	0.472
24	0.337	400	0.480
26	0.346	Rack	0.485

Once these values were obtained, the Size factor was calculated from the following equation:

$$K_s = \frac{1}{k_b} = 1.192 \left(\frac{F\sqrt{Y}}{P} \right)^{0.0535}$$

4. *Load Distribution factor (K_m)*

This factor modifies the calculated stress equations to reflect the non-uniform distribution of the load across the line of contact. Ideally, the gears in a system will be centered between two bearings, located at the zero slope place when the load is applied. Although the factor is difficult to apply to our planetary and we certainly could have just set it to 1, we felt that calculating a value larger than unity couldn't hurt if we felt we had the capability to do so. This factor is a combination of the following smaller factors:

4.1 Lead Correction factor (C_{mc})

This factor varies based on whether there is any crowning in the tooth profile, with a value of 1 for uncrowned teeth and 0.8 for crowned teeth. Because we did not specify crowning in our gears, we set this value to 1.

4.2 Pinion Proportion factor (C_{pf})

This factor varies based on the face width and pitch diameter of the pinion, with different equations based on different ranges of face width shown below:

$$C_{pf} = \begin{cases} \dfrac{F}{10d} - 0.025 & F \le 1 \text{ in} \\ \dfrac{F}{10d} - 0.0375 + 0.0125F & 1 < F \le 17 \text{ in} \\ \dfrac{F}{10d} - 0.1109 + 0.0207F - 0.000\,228F^2 & 17 < F \le 40 \text{ in} \end{cases}$$

4.3 Pinion Proportion modifier (C_{pm})

This factor compensates for any uncentered placement of the pinion between its bearings.

4.4 Mesh Alignment factor (C_{ma})

This factor accounts for the how well the gears align and mesh based on the condition and types of gears used. Because a larger face width brings on a larger area for misalignment, the value of this factor is based on a quadratic function based on the face width as follows:

$$C_{ma} = A + BF + CF^2$$

Condition	A	B	C
Open gearing	0.247	0.0167	$-0.765(10^{-4})$
Commercial, enclosed units	0.127	0.0158	$-0.930(10^{-4})$
Precision, enclosed units	0.0675	0.0128	$-0.926(10^{-4})$
Extraprecision enclosed gear units	0.00360	0.0102	$-0.822(10^{-4})$

Because our gears will be enclosed and are being custom-made, we specified the condition of precision enclosed gear units when calculating our mesh alignment factor.

4.5 Mesh Alignment Correction factor (C_e)

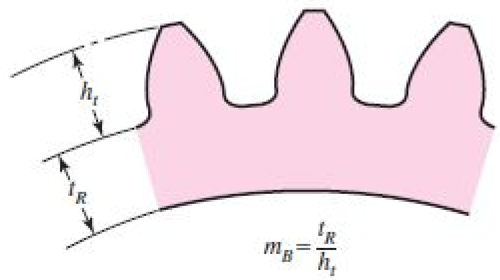
This factor is used to account for any adjustments made to the gearing at assembly to reduce the problems the load distribution factor accounts for. Therefore, if the gearing is not adjusted at the assembly, the value of this factor should be specified as unity. Otherwise, a value of 0.8 is recommended. Because we would have full access to the system at its assembly to spot errors and adjust, we set this value to 0.9.

With all of these smaller factors specified, the Load Distribution Factor was calculated using the following equation:

$$K_m = C_{mf} = 1 + C_{mc}(C_{pf}C_{pm} + C_{ma}C_e)$$

5. *Rim Thickness factor (K_b)*

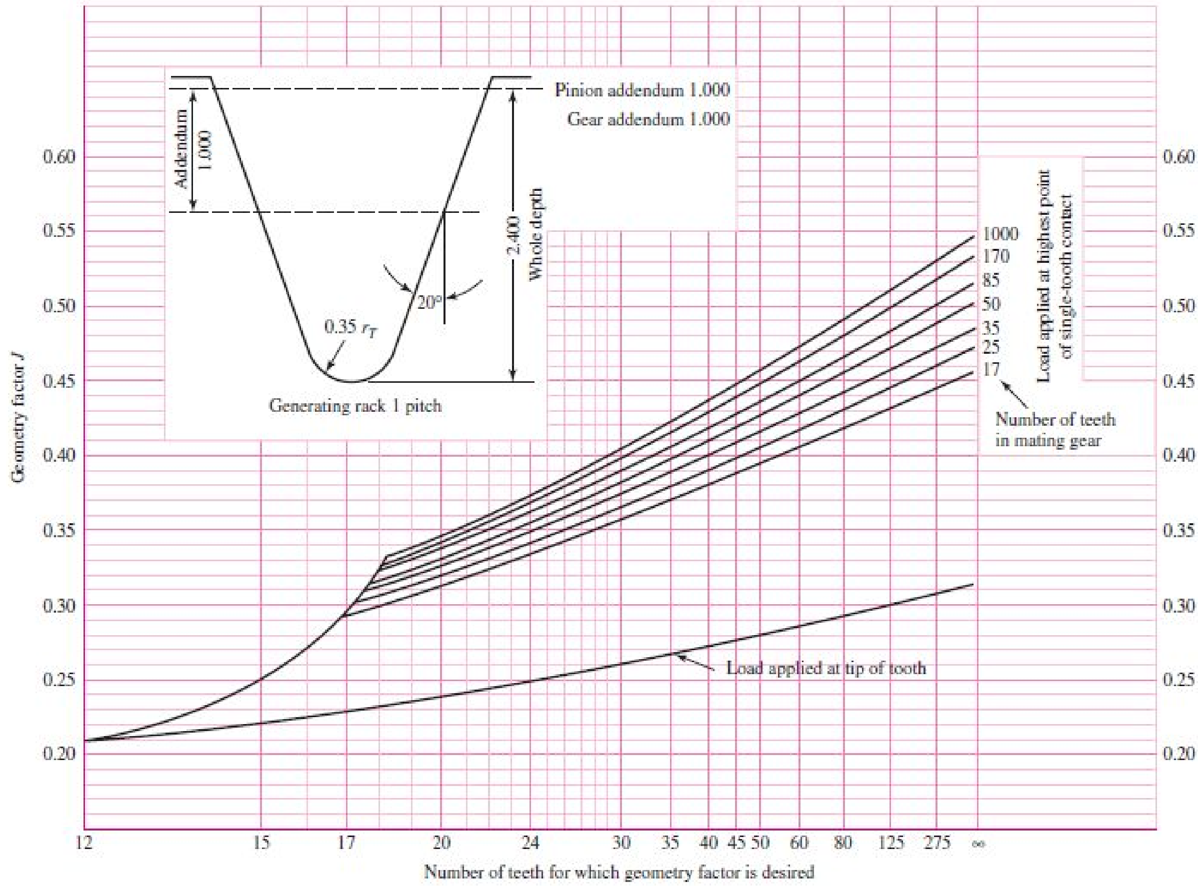
This factor is used when the rim thickness is not sufficient to provide full support for the tooth root, which raises concerns that bending fatigue failure may occur in the rim rather than at the tooth fillet. The factor is a function of the backup ratio m_b.



If the backup ratio is greater than or equal to 1.2, the rim-thickness factor can be set to 1 because the rim thickness is deemed sufficient enough to support the tooth root. Because our backup ratio meets this criterion, we set our rim thickness factor to unity.

6. *Bending-Strength geometry factor (J)*

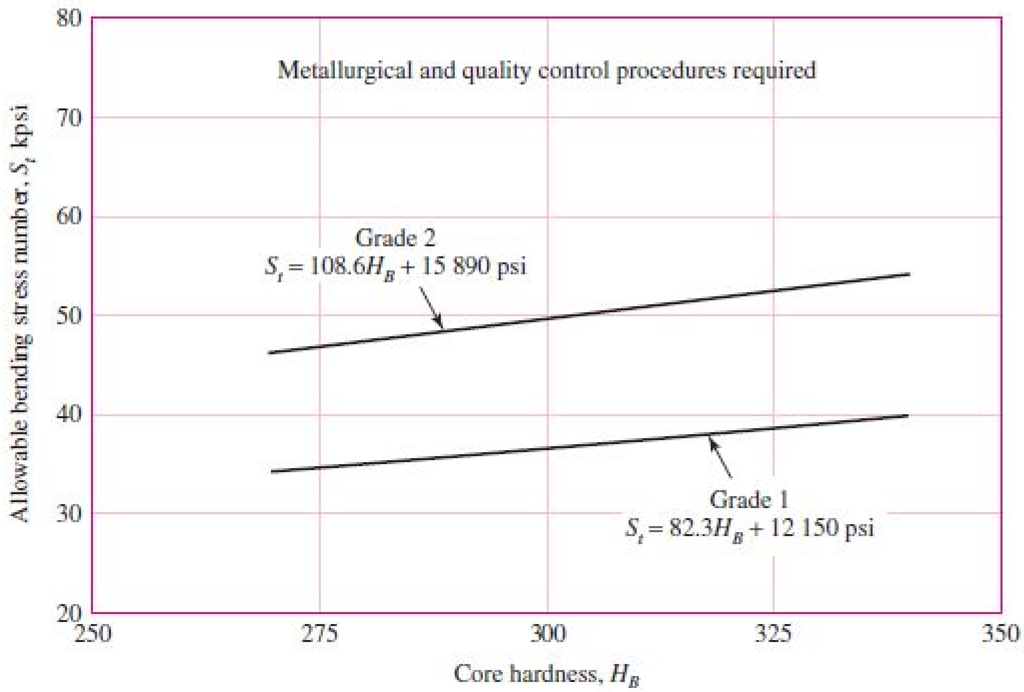
This factor depends on the ratio of the numbers of teeth between the meshing gears, shown in the graph below:



To use the graph, first find the number of teeth on the gear you want to find the factor for on the x-axis. Then trace your way vertically to the black line that represents the number of teeth of the mating gear. Simply trace horizontally left from the point of intersection to determine the geometry factor for that gear. Then do the same for the other gear in the pair.

7. *Allowable Bending strength (S_t)*

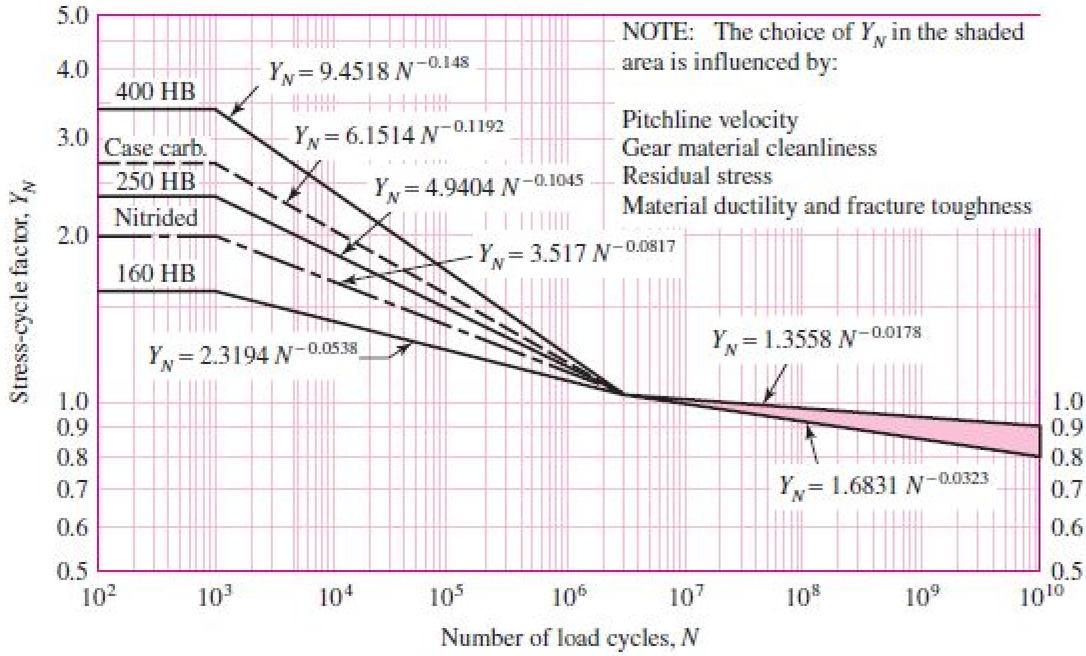
This factor dictates how much bending stress our gear teeth can handle and is to what we will compare our calculated stresses in order to determine whether the teeth will fail in bending. To put it another way, this is our tooth bending “strength”, and it depends on the metallurgical quality and Brinell hardness of the material used in our gears as shown in the graph below:



8. Bending Stress Cycle factor (Y_n)

This factor is used to modify the gear strength for lifes other than 10^7 cycles (where Yn is given a value of unity). Because our gears are spinning at different speeds, they will have a different number of lifetime cycles, so this factor must be calculated for both the pinion and gear. Because different materials and treatments can vary lifetime reliability, different equations for the bending stress cycle factor are required for each type. We used the following graph to determine the right equation for our system:

Nitriding and carburizing are two options available to us for the heat treatment of the gears, both suitable for low-carbon steels like AISI 4340. The benefit of nitriding over carburizing is that the former is done after the gears have been machined, quenched and tempered to bring in the required toughness. It is done at a temperature below the transformation temp. of steel (austenite to martensite) so eventually very little or no distortion in gear size is observed. In contrast, carburizing is done after machining the gears and at a temp. above the steel's transformation temp. which causes distortion in its size and a slight increase in volume, after which it is quenched and tempered. Since our gears need to be very precisioned, distortion in size is not acceptable. Hence, we will go for nitriding of the gears which costs relatively lesser than carburizing on the cost front too.



9. *Pitting Resistance geometry factor (I)*

This is a rather self-explanatory factor that specifies both gears resistance to pitting based on the pressure angle, load sharing ratio, and the speed ratio of the gears using the following equation:

$$I = \begin{cases} \dfrac{\cos\phi_t \sin\phi_t}{2m_N}\dfrac{m_G}{m_G+1} & \text{external gears} \\ \dfrac{\cos\phi_t \sin\phi_t}{2m_N}\dfrac{m_G}{m_G-1} & \text{internal gears} \end{cases}$$

Since the sun and planet are external gears, we will use the first equation. The load sharing ratio (m_N) is equal to the face width divided by the total length of the lines in contact. Lucky for us, for spur gears, the load sharing ratio is equal to unity, so we don't need to go into specifics on how this is found. m_G is the speed ratio of the gears, simply defined as $m_G = N_G/N_P$. It should be noted that this equation is intended for use in external gears only.

10. *Surface Condition factor (C_f)*

This factor depends on the surface finish, residual stress, and other related properties of the material used. Standard surface conditions for gear teeth haven't yet been established, but if the surface condition is known to be poor, AGMA recommends a value greater than 1. Because our gears are being custom-made, our manufacturers will provide a proper surface finish, so we set this value to unity.

11. Elastic Coefficient (C_p)

This is used in the equation for contact stress and is based on the ductility of the materials used in the pinion and gear. More specifically, it depends on their elastic moduli and Poisson's ratios as follows:

$$C_p = \left[\frac{1}{\pi \left(\frac{1 - \nu_P^2}{E_P} + \frac{1 - \nu_G^2}{E_G} \right)} \right]^{1/2}$$

12. Allowable Contact stress (S_c)

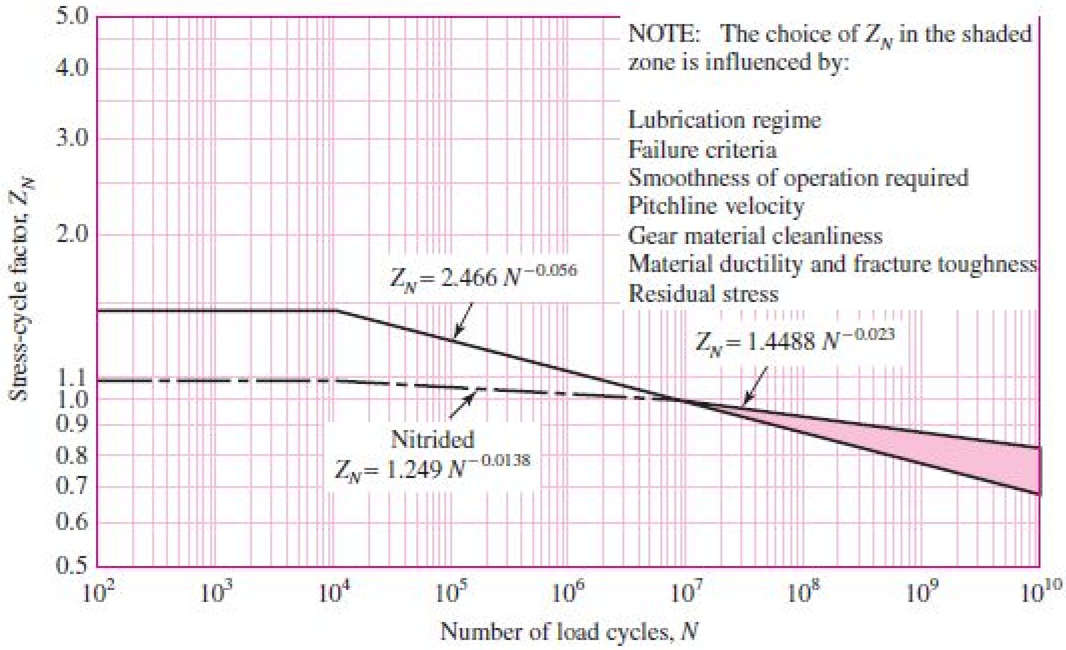
This factor dictates how much contact stress our gear teeth can handle and is to what we will compare our calculated stresses in order to determine whether the teeth will fail in wear. To put it another way, this is our tooth bending “strength”, and it depends on the same properties as the allowable bending stress, as shown below:

Material Designation	Heat Treatment	Minimum Surface Hardness[1]	Allowable Contact Stress Number,[2] S_c, psi		
			Grade 1	Grade 2	Grade 3
Steel[3]	Through hardened[4]	See Fig. 14–5	See Fig. 14–5	See Fig. 14–5	—
	Flame[5] or induction hardened[5]	50 HRC	170 000	190 000	—
		54 HRC	175 000	195 000	—
	Carburized and hardened[5]	See Table 9*	180 000	225 000	275 000
	Nitrided[5] (through hardened steels)	83.5 HR15N	150 000	163 000	175 000
		84.5 HR15N	155 000	168 000	180 000
2.5% chrome (no aluminum)	Nitrided[5]	87.5 HR15N	155 000	172 000	189 000
Nitralloy 135M	Nitrided[5]	90.0 HR15N	170 000	183 000	195 000
Nitralloy N	Nitrided[5]	90.0 HR15N	172 000	188 000	205 000
2.5% chrome (no aluminum)	Nitrided[5]	90.0 HR15N	176 000	196 000	216 000

The material designation is through-hardened nitrided steel (Grade 1).

13. Pitting Stress Cycle factor (Z_N)

This factor works using the same premise as the bending stress cycle factor, but for contact stress instead. In a similar process, as followed above, the graph below is used to determine the right equation. Just as before, we chose to use the nitrided line for our calculation.



14. Hardness Ratio factor (C_h)

This factor is used only for the gear in the pair, or in our case for the planet. Because the pinion has a smaller number of teeth than the gear, it's subjected to more cycles of contact stress over time. By making the pinion harder than the gear, a uniform surface strength can be achieved. This factor is meant to account for this difference in hardness when calculating the contact stresses on the gear. In our case we did not specify different hardnesses for the sun or planet gears (i.e. our hardness ratio = 1), so Ch = 1 for both the pinion and the gear.

15. Reliability factor (K_r)

This factor is used to account for any specified reliability other than 99%. It's used as an additional safety factor to ensure that you are completely safe using your system for its intended purpose. We specified a reliability factor of 1.

Reliability	K_R (Y_Z)
0.9999	1.50
0.999	1.25
0.99	1.00
0.90	0.85
0.50	0.70

16. Temperature factor (K_t)

This factor is given a value greater than 1 when the system is operated in temperatures greater than 250°F. Because our planetary is not expected to see any temperatures that high, we set this factor to 1.

5.5 Spreadsheet results

With all of these properties and factors determined, we were able to calculate bending/contact stresses, bending/contact strength, and bending/wear safety factors for our sun and planet gears. Eventually, the lowest factor of safety was 1.45 for the planet gear tooth wear. Other factors of safeties obtained were as follows:

	Sun Gear	**Planet Gear**
Bending FoS	3.22	2.69
Wear FoS	1.58	1.45

Further, with the help of a simple calculation, a contact ratio of 1.53 was obtained, which was very appropriate.

After carrying out the analysis based on AGMA standards and obtaining satisfactory results, the next requirement was to validate these results by carrying out certain hand calculations of various stresses that the gear teeth will have to sustain during the gearbox operation and further validate these results by ANSYS simulation.

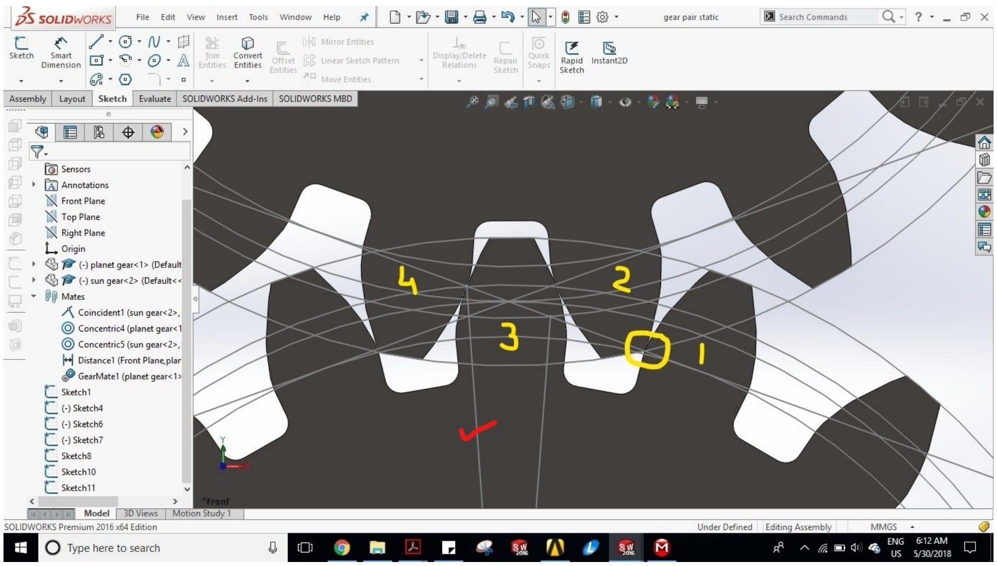
5.6 Calculation of stresses on gear teeth and Factor of Safety

A gear tooth can be assumed to be a rectangular beam of uniform cross-section supported at the gear root. This tooth is acted upon by bending and shear stresses as a result of the transmitted load acting on it along the line of action (LOA). Using the concept of bending of beams, we calculated the various stresses, namely, Principal, Shear, and von Mises. Further, the calculations required the concept of HPSTC and LPSTC, the basic underlying principle of which is the line of contact when two teeth of two separate gears mesh with each other. These two contact points are used to evaluate the bending and contact stresses on the gear tooth.

5.6.1 Calculation of HPSTC and LPSTC diameters

When considering a single tooth of the drive gear, its contact with the tooth/teeth of the driven gear begins and ends at the points of intersection of the addendum circle with the line of action(LOA). Using this fact, base circles, addendum circles and the line of action (tangential to both base circles) were sketched in the CAD itself and a tooth of the driven gear(2) was brought into contact with a tooth of the drive gear(1) at the intersection point of the driven gear's addendum circle and the LOA as shown below (yellow circle).

For a given gear, the 4 circles in the order of increasing diameters (or inward to outward) are the base circle, pitch circle, HPSTC circle, and addendum circle.

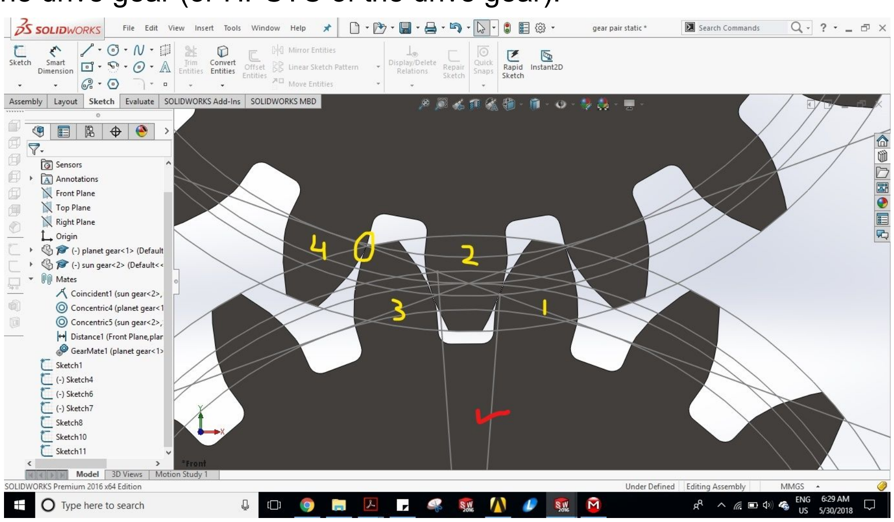


When this was done, the contact of the next pair of teeth(3 and 4) marks the location of HPSTC of the drive gear (or LPSTC of the driven gear).

Next, a line was sketched from the drive gear's center to the HPSTC and hence, the HPSTC diameter was measured.

HPSTC diameter = 74.08 mm

To measure the LPSTC diameter, a similar approach is used except that we are concerned with the point of intersection of the drive gear's addendum circle with the LOA (yellow circle)and thereafter, the contact of the previous pair of teeth(1 and 2) marks the location of LPSTC of the drive gear (or HPSTC of the drive gear).



Next, a line was sketched from the drive gear's center to the LPSTC and hence, the LPSTC diameter was measured.

LPSTC diameter = 70.30 mm

Next, to calculate the bending stress due to the load applied at the HPSTC, we need to know the tooth thickness. Since the tooth thickness is non-uniform due to the involute profile, it was approximated nearly at the tooth root where it is minimum and where the bending stress will be maximum.

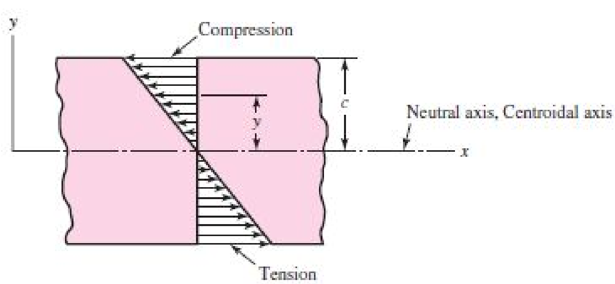
Tooth thickness = 4.76 mm or 0.187 inch.

Also, we require the distance of the point of application of the transmitted load from the tooth root. This distance is approximated as:

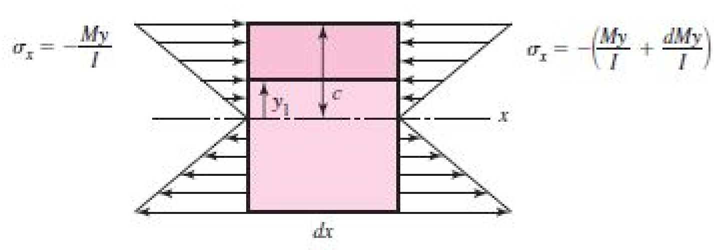
HPSTC circle radius - pitch circle radius + dedendum = 5.313 mm or 0.209 in.

5.6.2 Principal, shear, and von Mises stresses on gear teeth

We approximate a gear tooth as a rectangular beam fixed to the tooth root with a transmitted load applied at a certain distance from the root (perpendicular to the beam's longitudinal axis, i.e., along -ve y-axis). Consider the free end of the tooth pointing towards the positive x-axis. The stresses acting on the beam will be bending stress (σ_X) and transverse shear stress (τ_{YX}), the former being maximum at the tooth's outer surface and zero at the neutral axis while the latter being maximum at the neutral axis and zero at the outer surface.



Bending stress variation



Transverse shear stress variation

$|(\sigma_X)_{MAX}| = (6 * W * L) / (F * t^2)$ (outer surface) and $|(\tau_{YX})_{MAX}| = (3 * V) / (2 * A)$ (neutral axis) where,

W = load on gear tooth along the LOA= 395.45 lbf.

L = 0.209 in.

F = face width = 1.6 in.

t = tooth thickness = 0.187 in.

V = shear force at the HPSTC cross section = W = 395.45 lbf.

A = transverse cross-sectional area at HPSTC = F * t = 0.30 in^2

Now, for a gear tooth under consideration, we have two critical elements: the neutral axis and the outer tooth surface where the stress analysis is to be done on a transverse plane normal to the tooth longitudinal axis (x-axis) and passing through the tooth root.

1. *Neutral axis:*

At this axis, the bending stress is zero while the transverse shear stress is maximum, i.e., the neutral plane is a plane of pure shear ($\sigma_X = \sigma_Y = 0$ while $\tau_{YX} = 3V/2A$). Recall that we have considered the free end of the gear tooth to be pointing towards the +ve x-axis and the load along the -ve y-axis.

Calculating the principal normal stresses and arranging in descending order, we get:

$$\sigma_1 = \tau_{YX}$$
$$\sigma_2 = 0 \qquad (\sigma_1 > \sigma_2 > \sigma_3)$$
$$\sigma_3 = -\tau_{YX}$$

Hence, the *max. principal stress* at the neutral axis is $\sigma_1 = \tau_{YX}$ = 1977.25 lbf/in^2 or 13.53 MPa.

Now, evaluating the principal shear stresses based on Mohr's circle, we have:

$$\tau_{1/2} = (\sigma_1 - \sigma_2) / 2$$
$$\tau_{2/3} = (\sigma_2 - \sigma_3) / 2$$
$$\tau_{1/3} = (\sigma_1 - \sigma_3) / 2$$

We find that the max. of the three is always $\tau_{1/3}$ when σ_1 , σ_2 and σ_3 are arranged in decreasing order ($\sigma_1 \geq \sigma_2 \geq \sigma_3$).

Hence, the *max. shear stress* at the neutral axis is $\tau_{1/3} = (\sigma_1 - \sigma_3) / 2 = \tau_{YX}$ = 13.53 MPa.

Now, using the formula for the *von Mises stress*, we have:

$$\sigma' = \tau_{YX}\sqrt{3} = 23.43 \text{ MPa.}$$

2. *Tooth outer surface:*

At this surface, the bending stress is maximum (near the tooth) while the transverse shear stress is zero.

$$(\sigma_X = (6 * W * L) / (F * t^2) \ , \sigma_Y = 0 \ , \tau_{XY} = 0)$$

Note: The neutral surface is along the xz-plane and here we have assumed that the tooth outer surface considered in the calculations is above the neutral surface towards the +ve y-axis. So the bending stress will be tensile here (+ve sign).

Calculating the principal normal stresses and arranging in descending order, we get:

$$\sigma_1 = \sigma_X$$
$$\sigma_2 = 0 \qquad (\sigma_1 > \sigma_2 = \sigma_3)$$
$$\sigma_3 = 0$$

Hence, the *max. principal stress* is $\sigma_1 = \sigma_X$ = 8863.11 lbf/in^2 or 60.65 MPa.

Now, calculating the principal shear stresses as above, we have:
Max. shear stress $\tau_{1/3} = (\sigma_1 - \sigma_3) / 2 = \sigma_X / 2$ = 30.33 MPa.

Now, using the formula for the *von Mises stress*, we have:

$$\sigma' = \sigma_X = 60.65 \text{ MPa.}$$

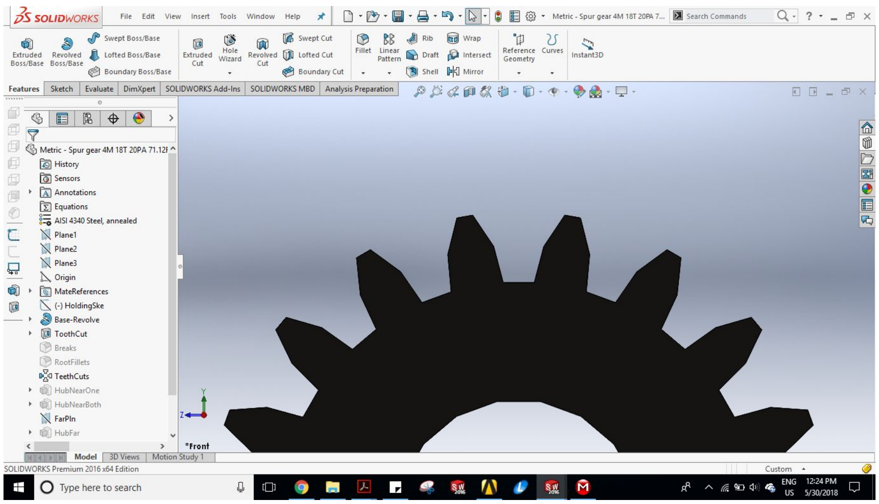
5.6.3 Calculation of Factor of Safety

The yield theory used to calculate the Factor of Safety is Distortion-Energy theory which is applicable for ductile and isotropic materials. The material chosen for the gears is AISI 4340 Steel which is widely used to manufacture transmission gears and shafts. It can easily be heat-treated via annealing and finally nitrided to provide a surface hardness in order to prevent failure from wear stresses. Also, even after annealing, it can be easily machined to provide a good surface finish that gives an extra safety factor against wear.

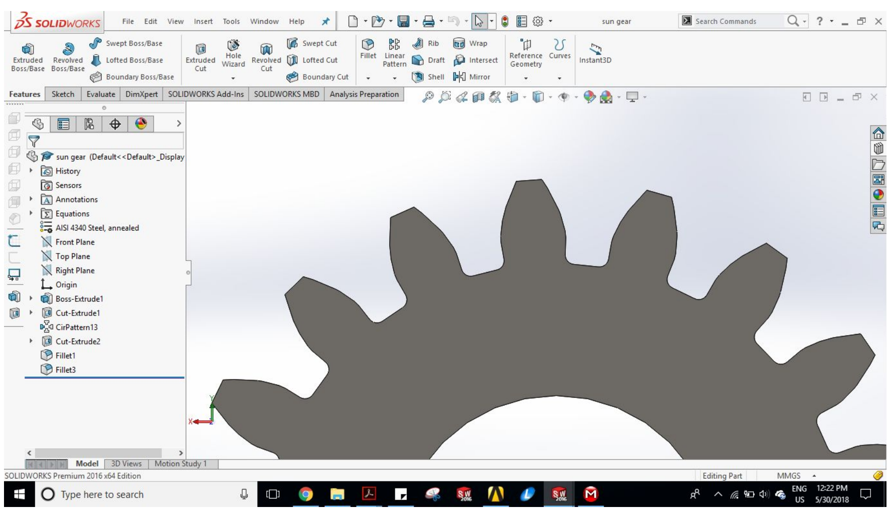
Yield strength of annealed AISI 4340 Steel ~ 470 MPa (assumed same in tension and compression)
Maximum von Mises stress obtained above = 60.65 MPa
Using the Distortion-Energy theory,
Factor of Safety = 470 / 60.65 = 7.75

5.7 CAD of gears

The gears were imported from the SOLIDWORKS toolbox and modified as per the module, no. of teeth, etc. but the gears so obtained did not exhibit an involute tooth profile. Rather, the tooth profile looked linear with flat faces. This made it difficult to establish proper contact between gear teeth. Hence, an effort was made to make gears from scratch using equations for the involute profile and many others. These equations were then set in SOLIDWORKS and the gears were made having a proper involute profile. These equations are so set that we can generate any spur gear just by changing few values.



Imported from SOLIDWORKS toolbox (linear tooth profile)

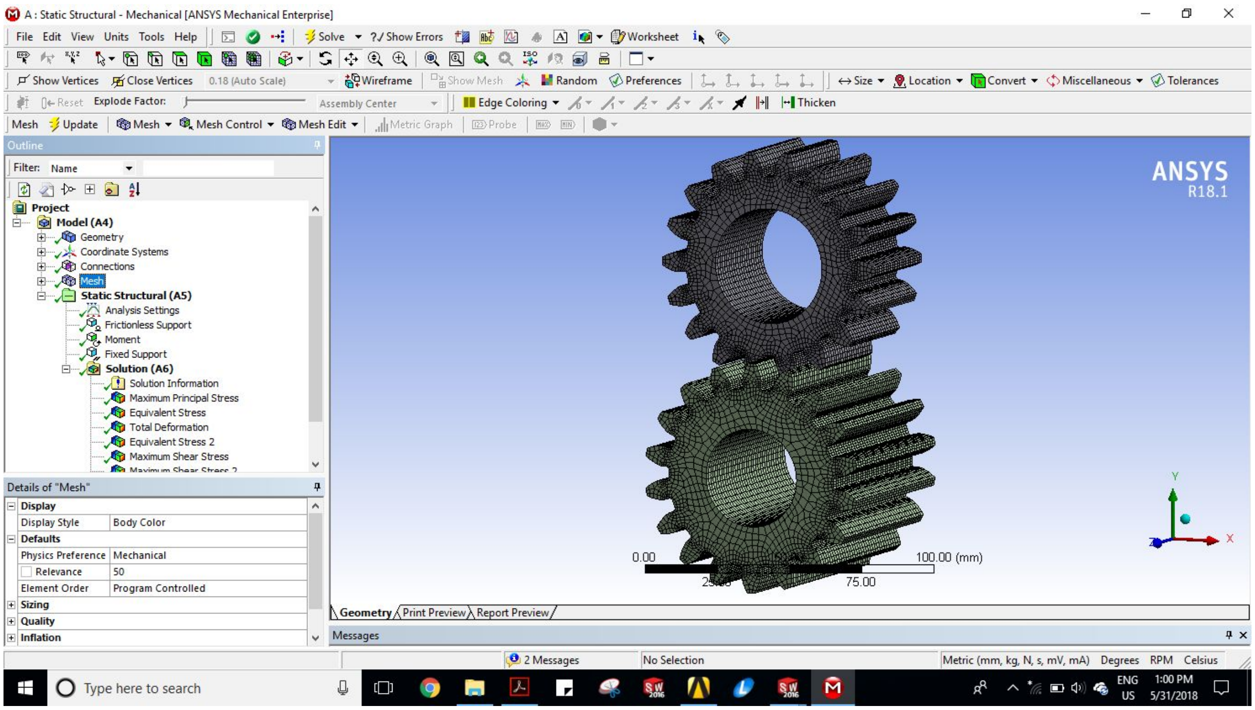


Made from scratch using equations (involute tooth profile)

5.8 ANSYS Static Structural Analysis

Though the analysis of the entire planetary gearset is required, doing so consumed a lot of time due to the need of using multiple load steps and also, errors in establishing proper contact between the sun and planet gears was encountered. A simpler solution was to analyze only a pair of sun and planet gear with the sun gear given one-third (50 Nm)(since there are 3 planet gears) of the total torque (150 Nm) that is actually input by the motor. This reduced the simulation time significantly by applying the torque all at once and also, better contact was established between the gears.

5.8.1 Mesh



5.8.2 Boundary conditions

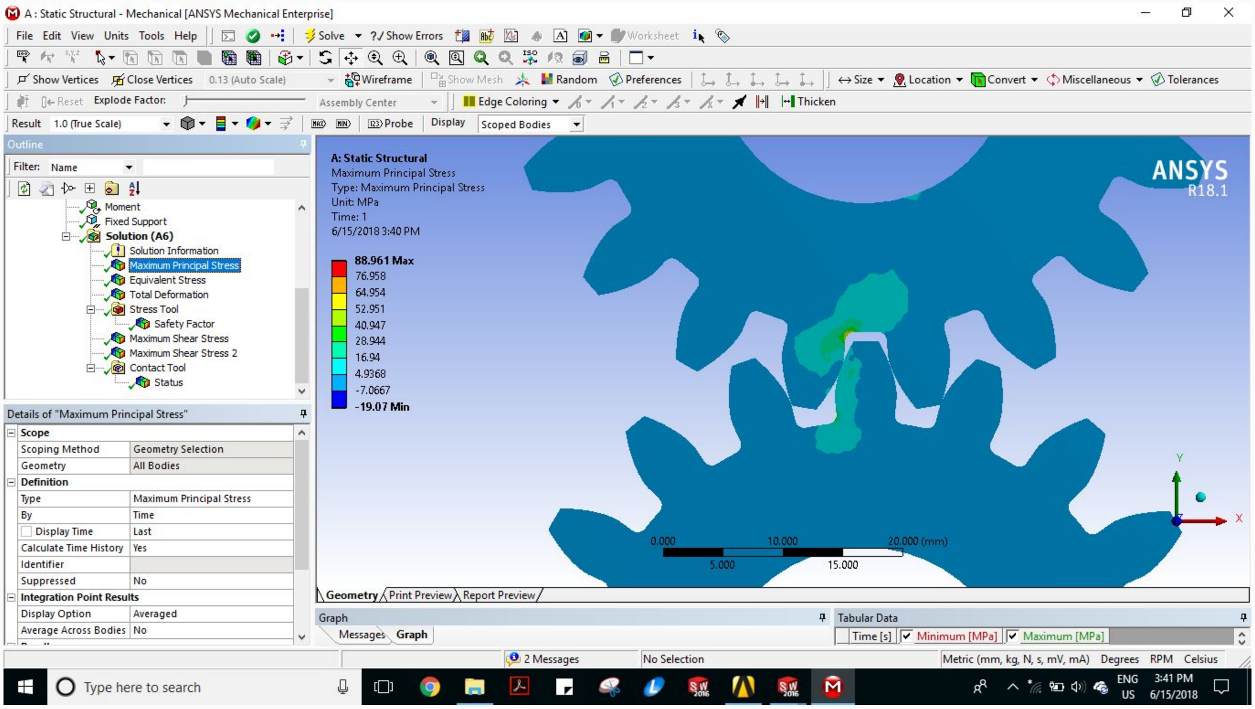
1. *Frictionless support* - the inner circular face of the sun gear
2. *Fixed support* - the inner circular face of the planet gear
3. *Moment* - provided to the inner circular face of the sun gear
4. In the analysis settings, *large deflection was turned “on”* because the inclusion of large deflection means that ANSYS accounts for changes in stiffness due to changes in the shape of the parts being simulated. It captures the non-linear effects as the shape of the tooth changes upon deflection since the general Hooke’s law does not hold in such situations.

5.8.3 Analysis settings

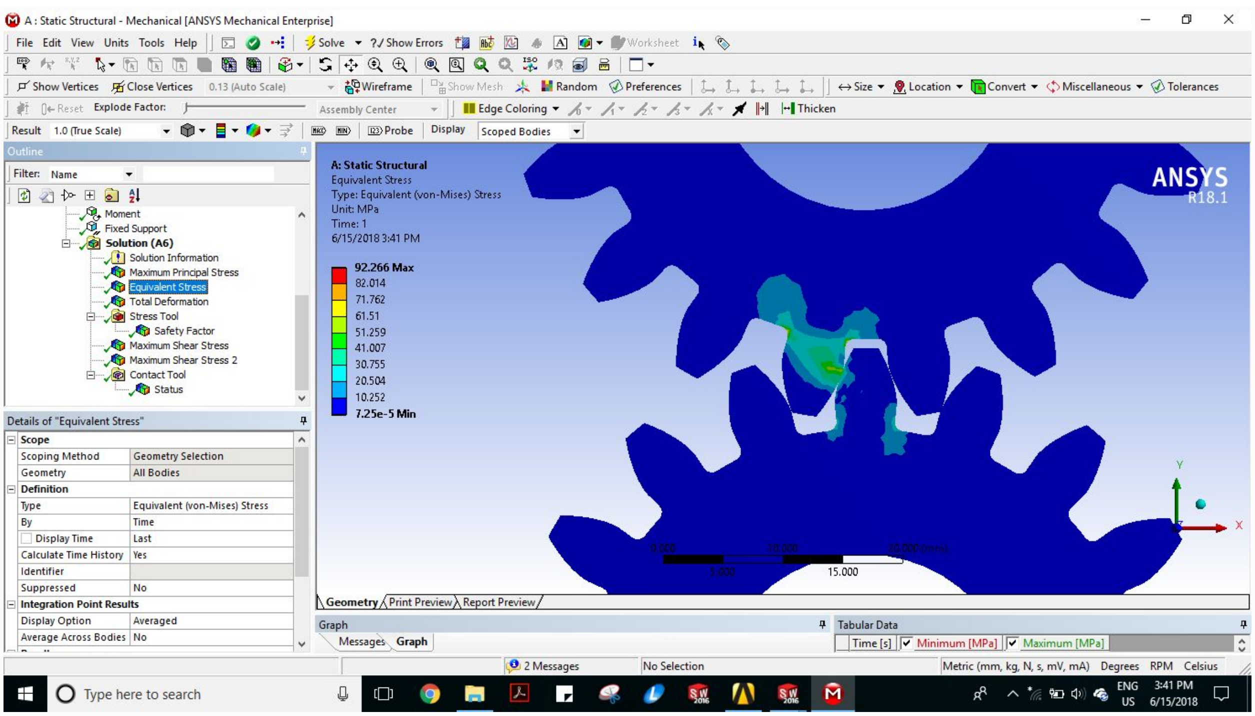
1. A *frictional contact* was made between teeth 3 and 4. The friction coefficient was varied from 0 to 0.3 but very little or no changes in the stress values were observed. Higher values cannot be used since the gear tooth are meant to be smooth enough to prevent their wear upon contact. Hence, the values given in this doc are for mew = 0.

2. In the definition settings, the behaviour was set to *“Asymmetric”* which means that the contact surfaces (of planet gear) are constrained from penetrating the target surfaces (of sun gear). Care should be taken to decide which body will be the contact element and which will be the target element when using asymmetric behaviour. “Symmetric” behaviour could also be used which constrains both contact and target elements from penetrating each other. While this seems a better setting, it is computationally time-taking and interpreting the results is more difficult since the results are reported on both surfaces which are not the “true” values of stresses. It is easy and straightforward to deduce results using asymmetric behaviour.
3. Surfaces in contact should not interpenetrate, rather they should slide over each other when there is a chance of penetration. In the advanced settings, the formulation was set to *“Augmented Lagrange”* which adds additional controls to automatically reduce the penetration of surfaces and this setting is recommended when using frictionless or frictional contacts.
4. ANSYS provides a contact offset value that can be entered manually or can be automatically calculated to provide an *“Adjust to Touch”* configuration. To avoid any chance of gap between the two teeth, the “Adjust to Touch” option was used to close the gap and establish an initial contact between the teeth surfaces.

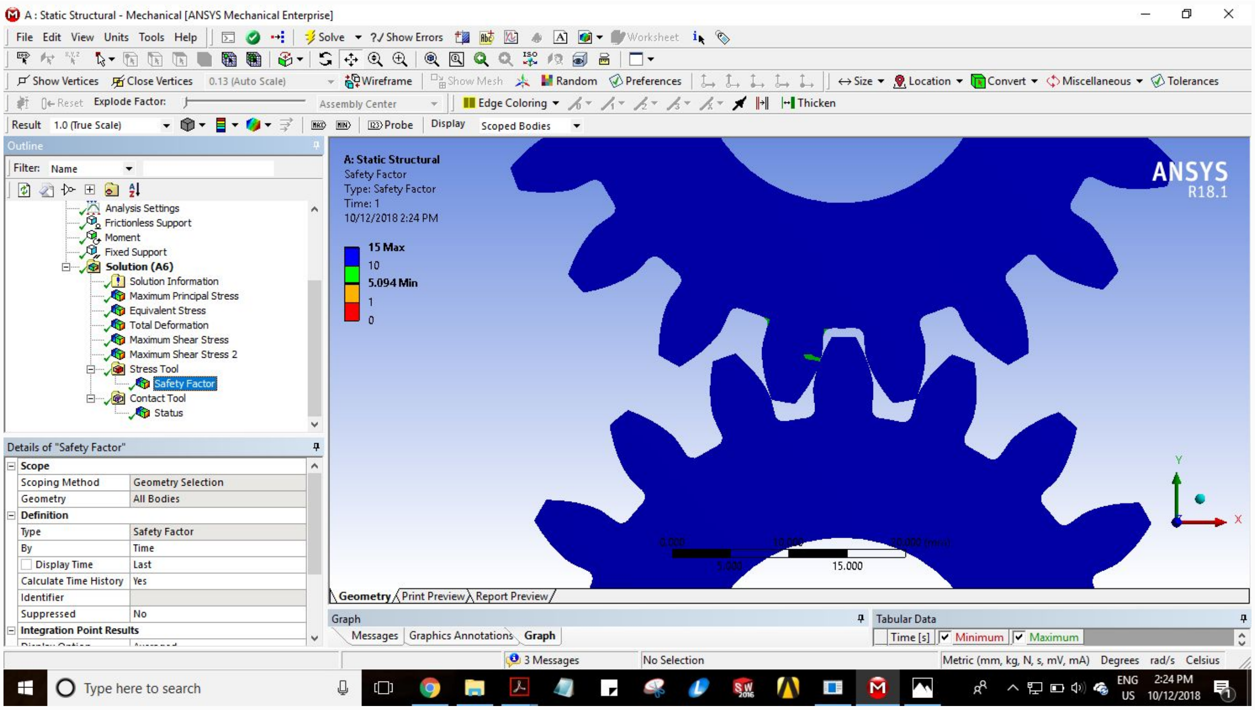
5.8.4 Results



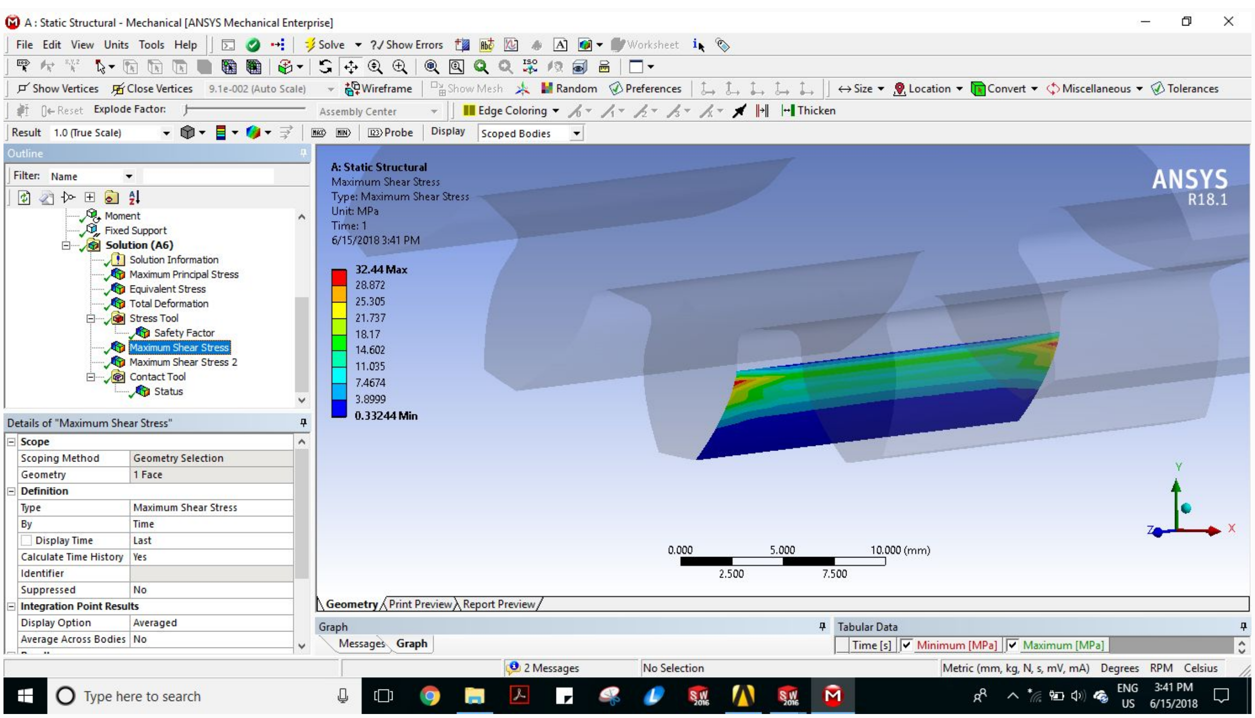
Max. Principal stress = 88.96 MPa



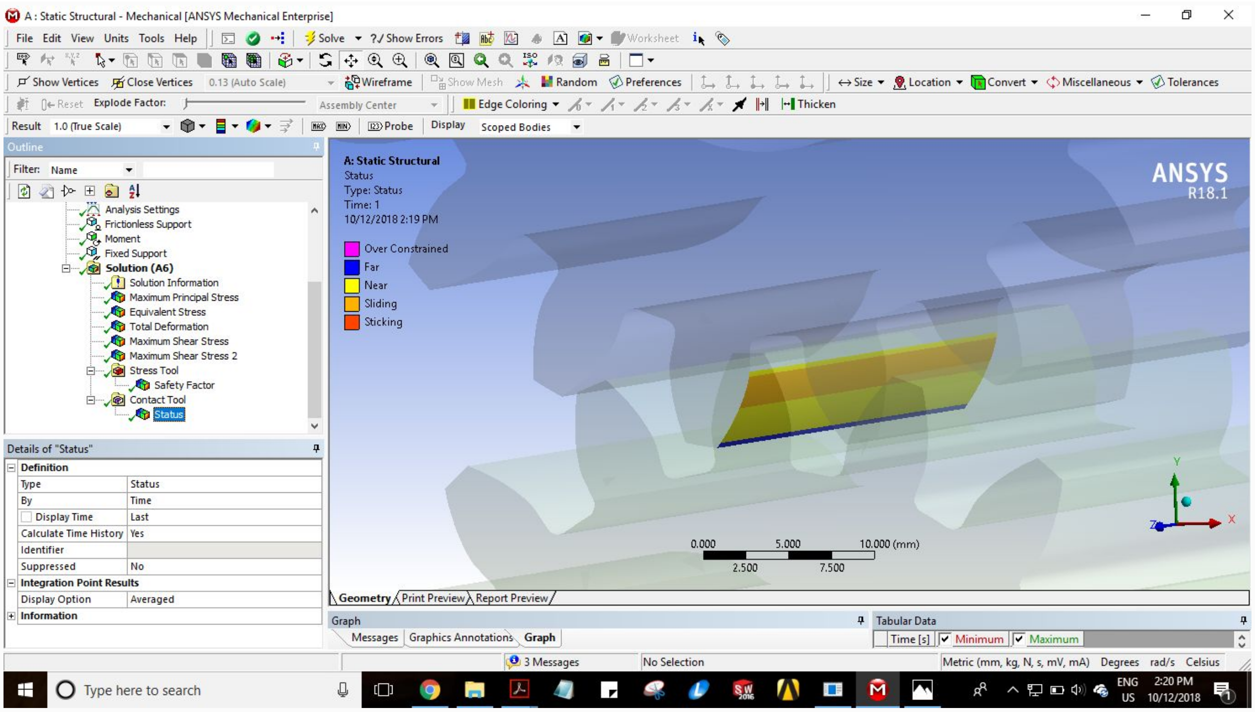
von Mises stress = 92.27 MPa



FoS = 5.10



Max. shear stress = 32.44 MPa



Stress profile on gear tooth (sliding+near+far). The region of sliding is in proximity to HPSTC.

	AGMA	*Hand calculations (analytical)*
Max. bending stress	80.10 MPa	60.65 MPa

	Hand calculations (analytical)	*ANSYS (numerical)*
Max. principal stress	60.65 MPa	88.96 MPa
von-Mises stress	60.65 MPa	92.27 MPa
Max. shear stress	30.33 MPa	32.44 MPa
Factor of Safety	7.75	5.10

5.9 Conclusion and further scope

As obvious from the results obtained from the foregoing analysis, it appears that the values of stresses obtained from hand calculations and ANSYS simulation are not close to each other, the simulation values being more. However, it must be noted that for simplicity, the hand calculations have been performed assuming the gear tooth to be a rectangular beam of uniform cross-section (which actually isn't since the tooth thickness is non-uniform) and then applying the concept of beam theory to obtain the results. Hence, this seems to account for the difference in results. On the other hand, establishing a proper contact between gear teeth in ANSYS was an important part of this analysis since the contact behaviour dictates the precision of the results. With the help of various ANSYS settings, proper contact was established and the results obtained. Nonetheless, since the values obtained from the simulation are more than the hand calculated values, it shouldn't be much of a concern since shedding mass off the gears by making proper lightening holes should almost match-up the stress results, simultaneously reducing the mass of the gears. Care must be taken regarding tolerances and fillet radii so as to minimize the manufacturing difficulties.

6. Motor Shaft Design

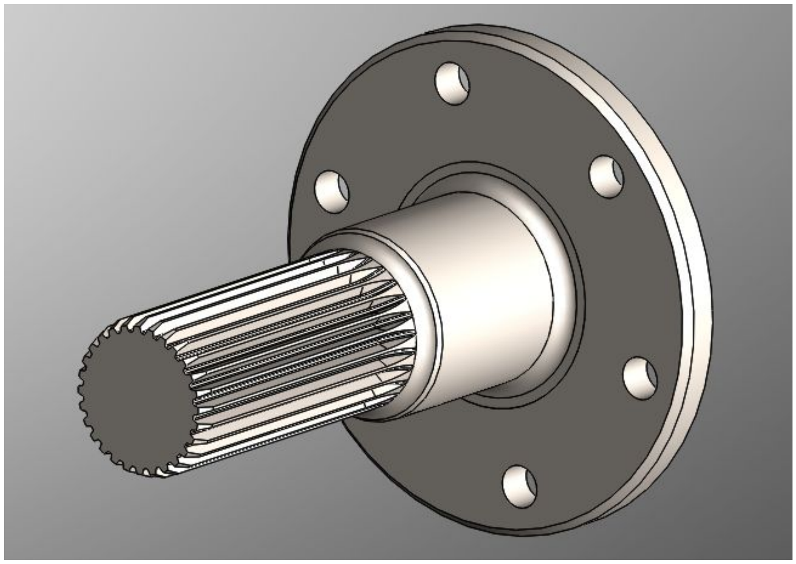
6.1 Need of a custom designed shaft

Owing to the configuration of the planetary gearset to be used in the car, i.e., input to be given from the motor to the sun gear and output to be delivered by the planet carrier, we need a shaft extending out from the motor via which the torque will be transmitted to the sun gear. Since we will be using an EMRAX motor, readily made shafts are available on demand along with the motor. They are of two types:

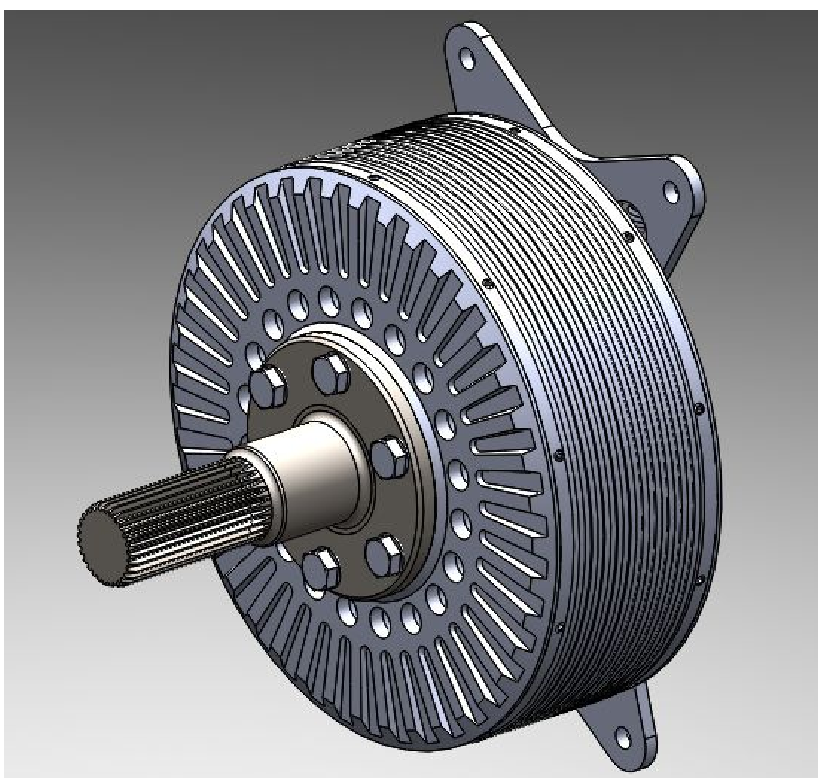
1. Flanged shaft with inner splines (FSI)- delivers output from the front side of the motor
2. Extended shaft with outer splines (ESO)- delivers output from the rear side of the motor

Since the mounting points of the motor, including the bracket, are provided on its rear side, the ESO doesn't seem the right choice for the required shaft. Also, since the FSI has inner splines, the outer surface being smooth, it can not mate with the inner splines of the sun gear. A possible solution thought out was that we could bolt the ESO in place of the FSI's position so that we could then have outer splines of the shaft available to be mated with the sun gear's inner splines. But the length of the grooves made in the ESO was insufficient for the sun gear to be mounted on it. As a remedy to this problem, we could have cut and increased the length of the grooves, but this would tamper the shaft and would result in the loss of its structural strength. Purchasing an expensive shaft and then modifying it this way is a risky task. Plus, the diameter of the bolts needed to mount the two shafts are different and also the distance between diametrically opposite bolting holes is different.

Thus, the only way out was to design our own custom shaft with outer splines as per the requirements and then either mount it on the motor directly in place of the FSI or insert it into the FSI itself (as suggested by the EMRAX technical support team). Directly mounting the custom shaft in place of the FSI would be a better option for three reasons: firstly, this would rule out the need to purchase a shaft (FSI) along with the motor and save us money which would otherwise be used to manufacture the custom shaft; secondly, this would minimize the parts count from two to one; thirdly, the extra work to fit the custom shaft into the FSI won't be required. The following analysis was carried out to design the shaft.



CAD Model of the Custom Motor Shaft (~ 1kg)



Custom shaft bolted at the ESO's position with six M8 bolts

6.2 Loads on the shaft and material selection

We evaluated our motor shaft to determine whether the shaft would fail in fatigue. The term “fatigue” is used because this type of failure generally occurs after a lengthy period of repeated stress or strain cycling. When considering a shaft driven by a motor, the applied stress may be axial (tension-compression), flexural (bending) or torsional (twisting) in nature. Axial stresses are not so prominent and are neglected in our analysis. Only bending and torsional stresses are considered.

For the material of the shaft, three options were considered: AISI 4340, AISI 4140 and AISI 4130 Steels. The main objective was to get the best strength to weight ratio of the three as well as a sufficiently high endurance limit. But since a motor shaft is a critical design element that sustains repeated torsional as well as flexural stresses, the main factor that needs to be given importance is the endurance limit which is the stress level below which a specimen can withstand cyclic stresses indefinitely without exhibiting fatigue failure. Upon analytical calculations performed, the following results were obtained for the same geometry of the shaft:

Material of the shaft	**Fatigue FoS**	**Yield FoS**	**Endurance Limit**
AISI 4340 Steel	4.18	5.03	189 MPa
AISI 4140 Steel	3.75	4.44	172 MPa
AISI 4130 Steel	3.34	4.99	153 MPa

Though AISI 4130 made shaft weighs the least and offers a good enough strength to weight ratio of the three, its endurance limit is low. A minor loss of weight doesn’t matter as much as the strength gain of the shaft is concerned. Hence, AISI 4340 Steel stands out as the best candidate for the shaft material.

The table below presents important material properties and geometric dimensions for shaft strength analysis.

Material properties (Annealed AISI 4340 Steel)	**Value**	**Geometric dimensions**	**Value**
Ultimate strength (S_{ut})	745 MPa	Shaft length	109.14 mm
Yield strength (S_y)	470 MPa	Major diameter (D)	36 mm
Elastic modulus (E)	200 GPa	Minor diameter (d)	30 mm
Modulus of Rigidity (G)	80 GPa	Fillet radius (r)	3 mm
Number of Cycles (N)	10^7	Minimum area (A)	706.86 mm^2
		Moment arm (L)	75.63 mm

6.3 Spreadsheet analysis of shaft stresses, geometry, and FoS for fatigue and yield

The analysis began with finding the endurance strength of the shaft (S_e), determined by the equation below:

$$S_e = k_a k_b k_c k_d k_e k_f S'_e$$

An explanation for the chosen values for each factor in this equation is provided below:

1. *Endurance Limit (S'_e)*

This is the endurance strength of a similar specimen under completely reversible loading, and is defined by the following equation:

$$S'_e = \begin{cases} 0.5 S_{ut} & S_{ut} \leq 200 \text{ kpsi (1400 MPa)} \\ 100 \text{ kpsi} & S_{ut} > 200 \text{ kpsi} \\ 700 \text{ MPa} & S_{ut} > 1400 \text{ MPa} \end{cases}$$

Because S_{ut} in our case is <1400 MPa, the first of the three equations is used to find the endurance limit.

2. *Surface Factor (k_a)*

This factor is based on the quality of the surface finish and ultimate tensile strength of the material. Using the equation and table below, we assumed a machined surface finish in our calculations.

$$k_a = a S_{ut}^b$$

Surface Finish	Factor a S_{ut}, kpsi	Factor a S_{ut}, MPa	Exponent b
Ground	1.34	1.58	−0.085
Machined or cold-drawn	2.70	4.51	−0.265
Hot-rolled	14.4	57.7	−0.718
As-forged	39.9	272.	−0.995

3. *Size Factor (k_b)*

This factor is determined by the minimum diameter of the shaft being analyzed using the following equations:

$$k_b = \begin{cases} (d/0.3)^{-0.107} = 0.879d^{-0.107} & 0.11 \le d \le 2 \text{ in} \\ 0.91d^{-0.157} & 2 < d \le 10 \text{ in} \\ (d/7.62)^{-0.107} = 1.24d^{-0.107} & 2.79 \le d \le 51 \text{ mm} \\ 1.51d^{-0.157} & 51 < d \le 254 \text{ mm} \end{cases}$$

Because d= 30 mm, we used the third equation.

4. *Loading Factor (k_c)*

This factor depends on the loading scenario of the specimen being analyzed (axial, bending, torsion). Because we analyzed the shaft under a combined loading scenario (torque and bending), this factor was given a value of 1.

5. *Temperature Factor (k_d)*

For our analysis, we are assuming that the shaft will operate at room temperature environments and hence, this factor is prescribed a value of 1.

6. *Reliability Factor (k_e)*

A reliability of 99.9 % was chosen and a K_e value of 0.753 was set based on the following table.

Reliability, %	Transformation Variate z_a	Reliability Factor k_e
50	0	1.000
90	1.288	0.897
95	1.645	0.868
99	2.326	0.814
99.9	3.091	0.753
99.99	3.719	0.702
99.999	4.265	0.659
99.9999	4.753	0.620

7. *Miscellaneous-Effects Factor (k_f)*

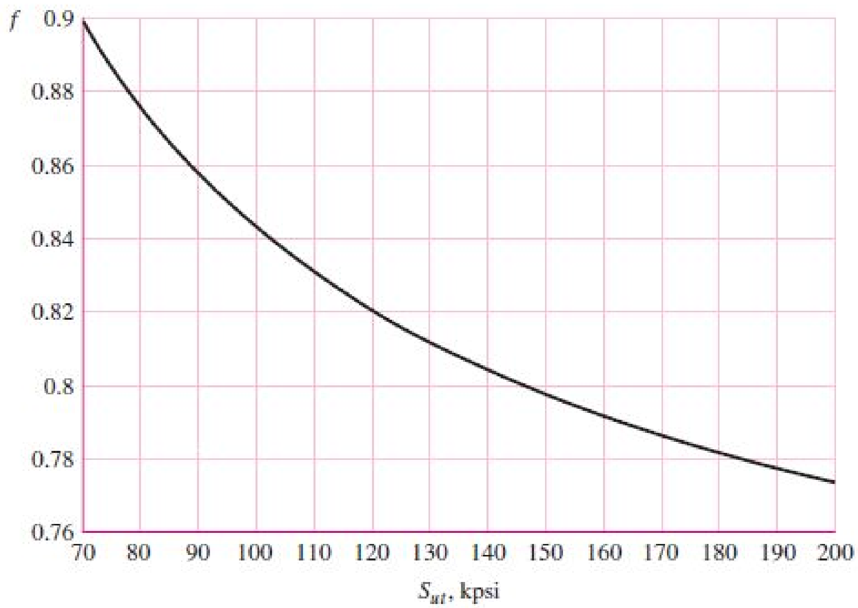
Assigned a value of 1.

With our S_e calculated, we then calculated the fatigue strength (S_f) using the equations below. The fatigue strength fraction (f) is found using the graph shown below:

$$S_f = a\ N^b$$

$$a = \frac{(f\,S_{ut})^2}{S_e}$$

$$b = -\frac{1}{3}\log\left(\frac{f\,S_{ut}}{S_e}\right)$$

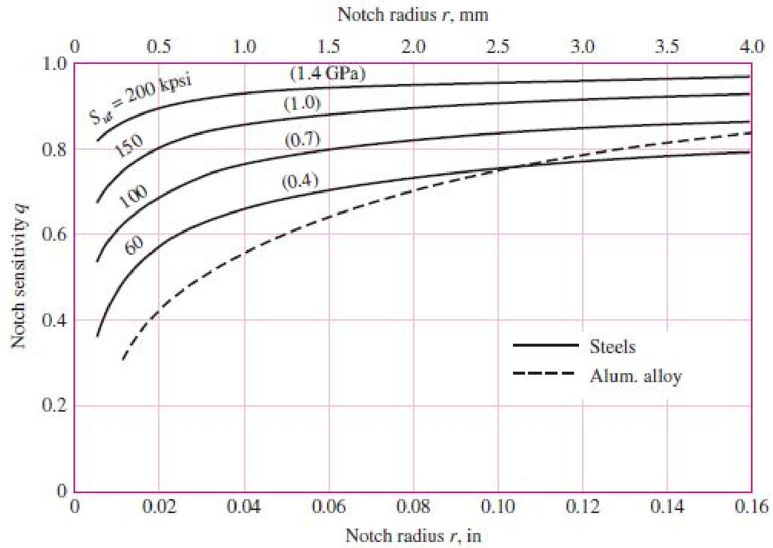


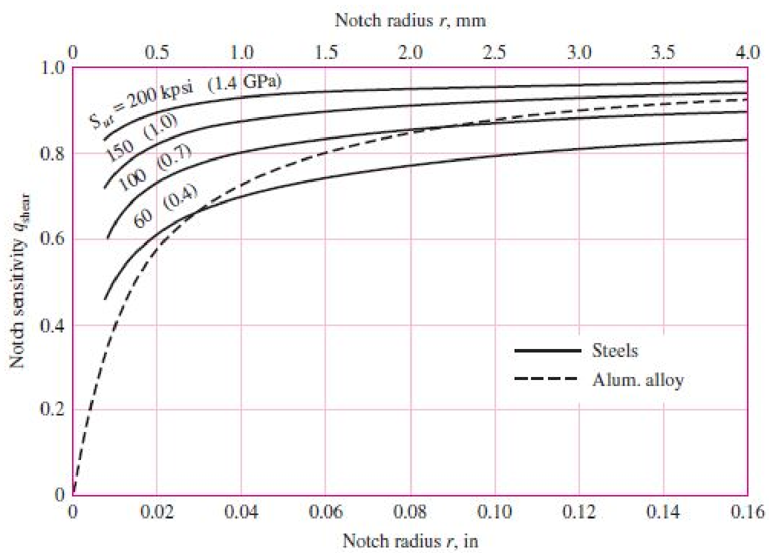
Corresponding to S_{ut}= 745 MPa (108 kpsi), f was assigned a value of 0.83.

To calculate the mean and alternating stresses, the fatigue stress concentration factors (K_f and K_{fs}) must be found using the stress concentration factors (K_t and K_{ts}) and the notch sensitivity (q and q_{shear}):

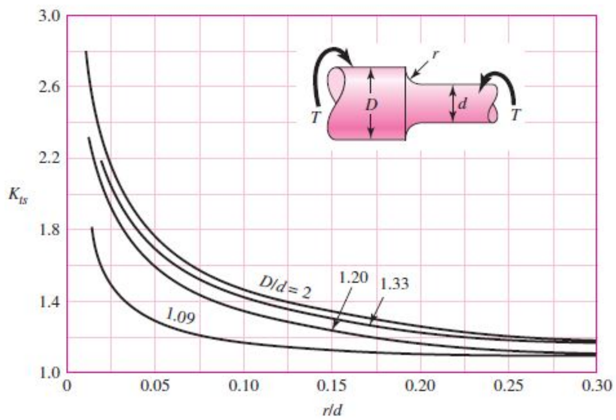
$$K_f = 1 + q(K_t - 1) \qquad K_{fs} = 1 + q_{\text{shear}}(K_{ts} - 1)$$

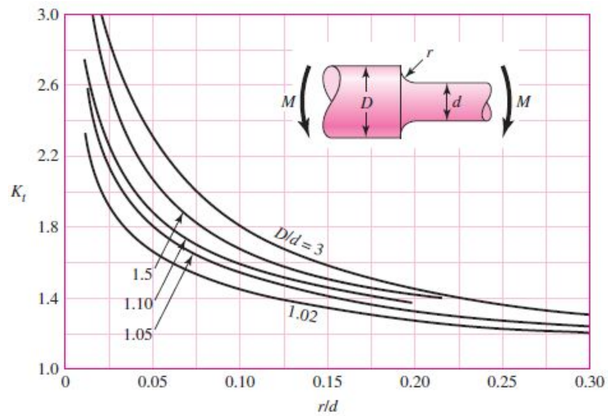
The Notch Sensitivity (q and q_{shear}) defines how sensitive the specimen is to notches in fatigue loading. It was determined using the graph below using the notch radius and ultimate tensile strength.





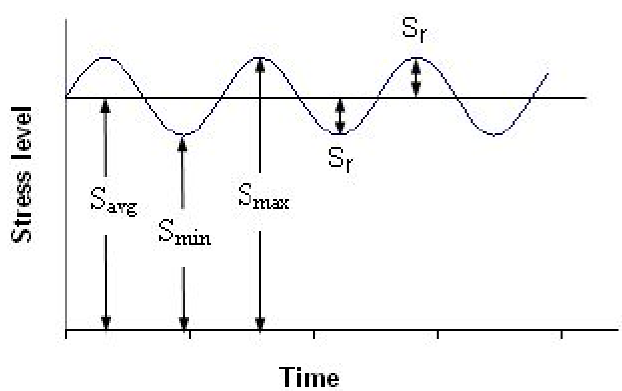
The Stress Concentration Factors (K_t and K_{ts}) were found based on tabulated graphical values for a round bar with shoulder fillet under bending (K_t) and torsional loading (K_{ts}).





On iterating the values of r/d and D/d, we finally arrived at the values of r/d= 0.1 and D/d= 1.2 (the recommended value of D/d is between 1.2 and 1.5, so considering 1.2 gave the best result for the value of D).

Once the strength of the shaft was known, we calculated the von Mises mean and alternating stresses. Since the shaft in our case will be under constant bending and torsion, the bending stress is completely reversed and the torsion is steady. A generalized stress condition can be defined as a combination of purely reversing stress (S_r) superimposed on a steady stress (S_{avg}).



$$\text{Steady stress} = S_{avg} = \frac{S_{max} + S_{min}}{2}$$

$$\text{Reversing stress} = S_r = \frac{S_{max} - S_{min}}{2}$$

Hence in our case, the mean stress (T_m) is only from the shear stress on the shaft due to the motor torque and the alternating stress (M_a) is only from the bending stress on the shaft due to the distributed radial load (on the sun gear due to planet gears), approximated as a point force F. The values of T_a and M_m are zero.

The mean stress was calculated using the following von Mises stress formula which is based on the Distortion-Energy theory for ductile materials:

$$\sigma_m' = (\sigma_m^2 + 3\tau_m^2)^{1/2} = \left[\left(\frac{32K_f M_m}{\pi d^3}\right)^2 + 3\left(\frac{16K_{fs}T_m}{\pi d^3}\right)^2\right]^{1/2}$$

The alternating stress was calculated using the following formula:

$$\sigma_a' = (\sigma_a^2 + 3\tau_a^2)^{1/2} = \left[\left(\frac{32K_f M_a}{\pi d^3}\right)^2 + 3\left(\frac{16K_{fs}T_a}{\pi d^3}\right)^2\right]^{1/2}$$

Next, considering a minimum Factor of Safety of n = 4 to be achieved, we calculated the minor diameter (d) of the shaft using the following formula:

$$d = \left(\frac{16n}{\pi}\left\{\frac{1}{S_e}\left[4(K_f M_a)^2 + 3(K_{fs}T_a)^2\right]^{1/2} + \frac{1}{S_{ut}}\left[4(K_f M_m)^2 + 3(K_{fs}T_m)^2\right]^{1/2}\right\}\right)^{1/3}$$

This value came out to be 30 mm. Using D/d = 1.2, the value of the major diameter D came out to be 36 mm and using r/d = 0.1, the value of the fillet radius came out to be 3 mm.

Finally, the actual fatigue Factor of Safety (n) was calculated using the following modified Goodman formula by substituting the respective values:

$$\frac{1}{n} = \frac{\sigma_a'}{S_e} + \frac{\sigma_m'}{S_{ut}}$$

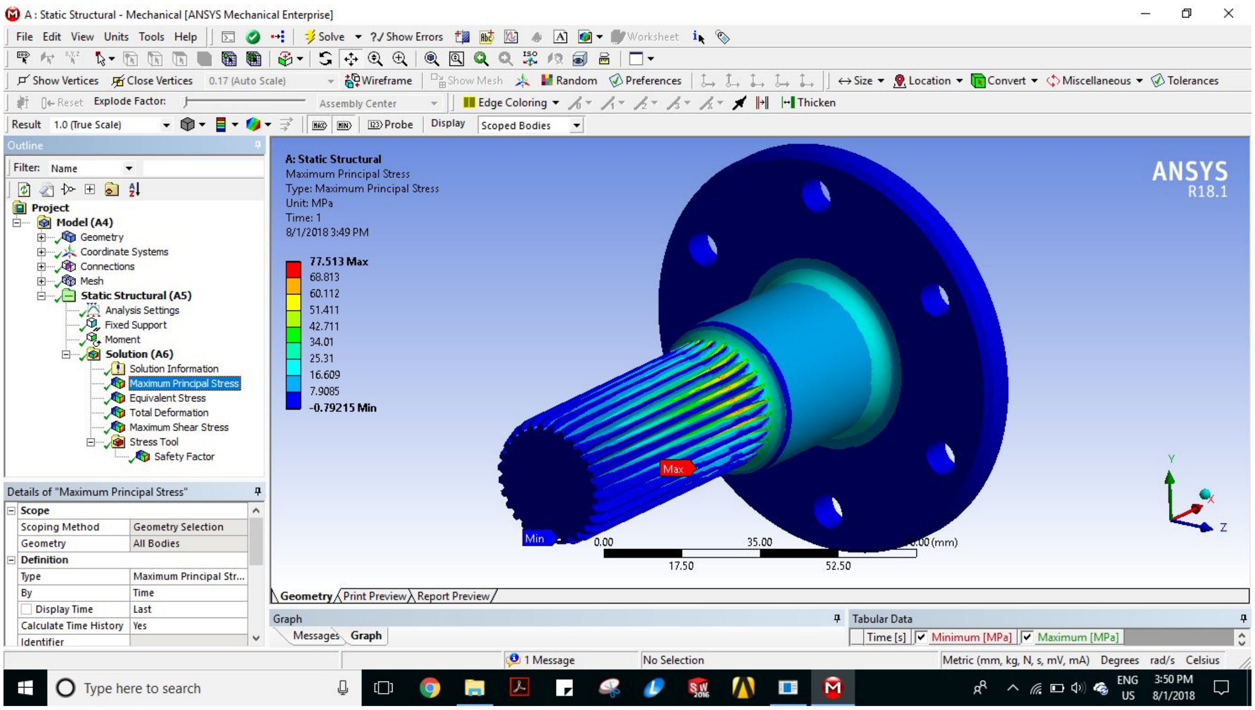
A fatigue Factor of Safety of 4.18 was obtained.

However, the modified Goodman criterion does not guard against yielding, requiring a separate check for yielding. A von Mises maximum stress is calculated for this purpose. This stress is estimated by simply adding the mean and alternating stresses already obtained above and the yield Factor of Safety was calculated using the following formula:

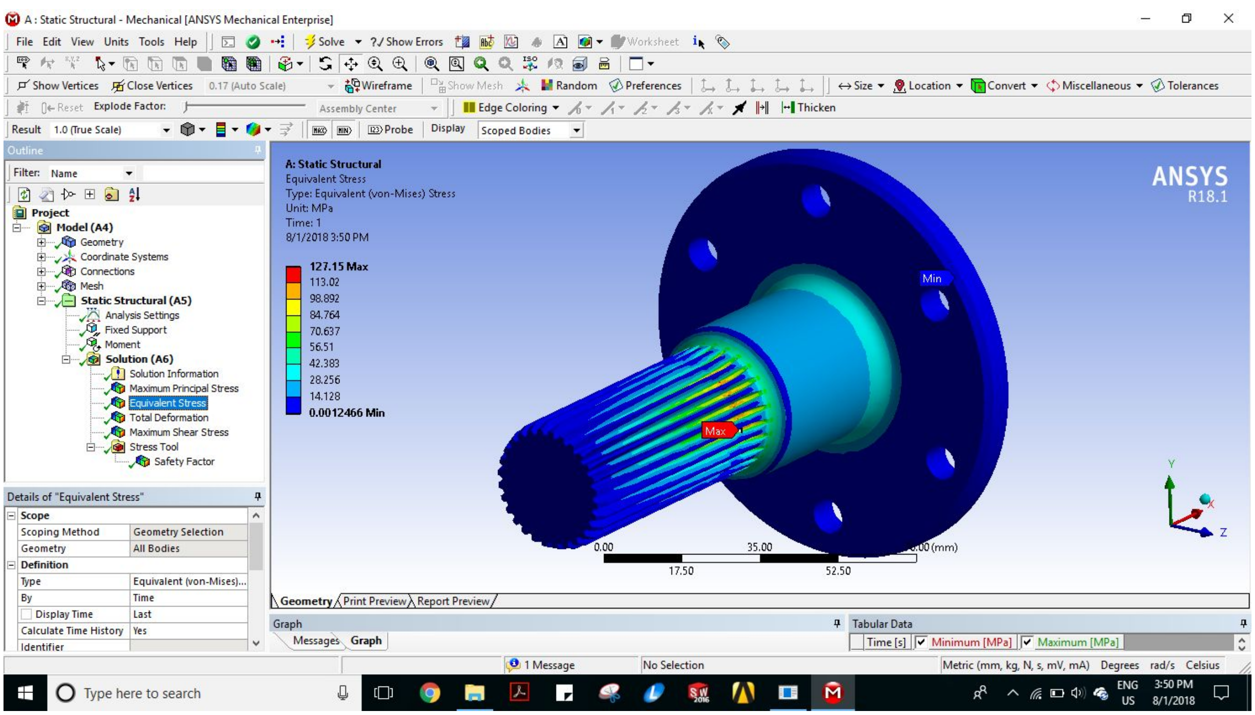
$$n_y = \frac{S_y}{\sigma_{\max}'}$$

A yield Factor of Safety of 5.03 was obtained.

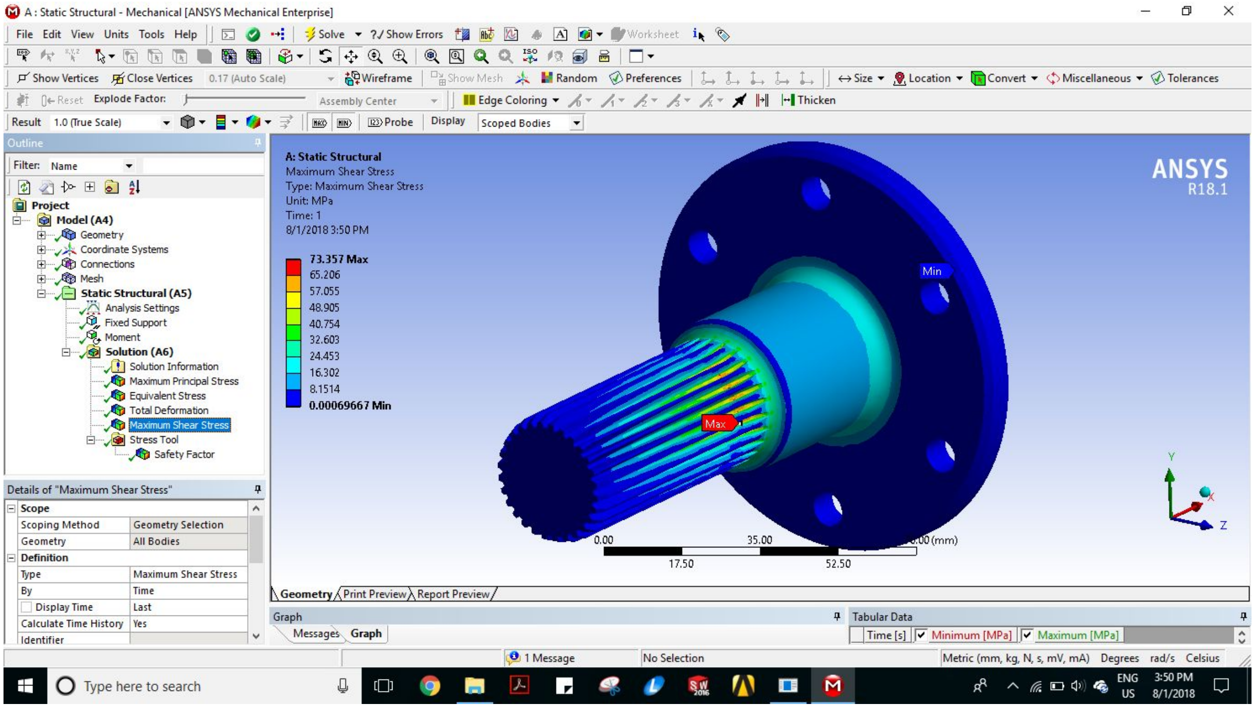
6.4 ANSYS simulation results



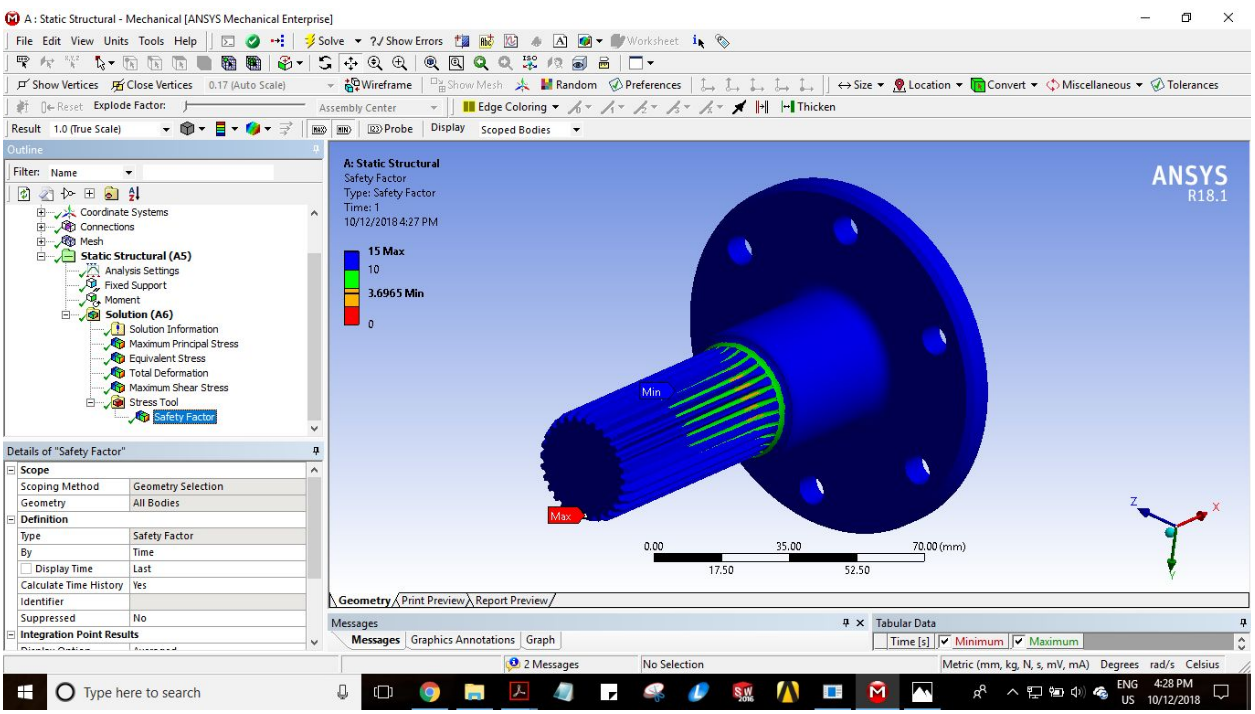
Max. Principal stress = 77.51 MPa



von Mises stress = 127.15 MPa



Max. Shear stress = 73.36 MPa



Factor of Safety = 3.69

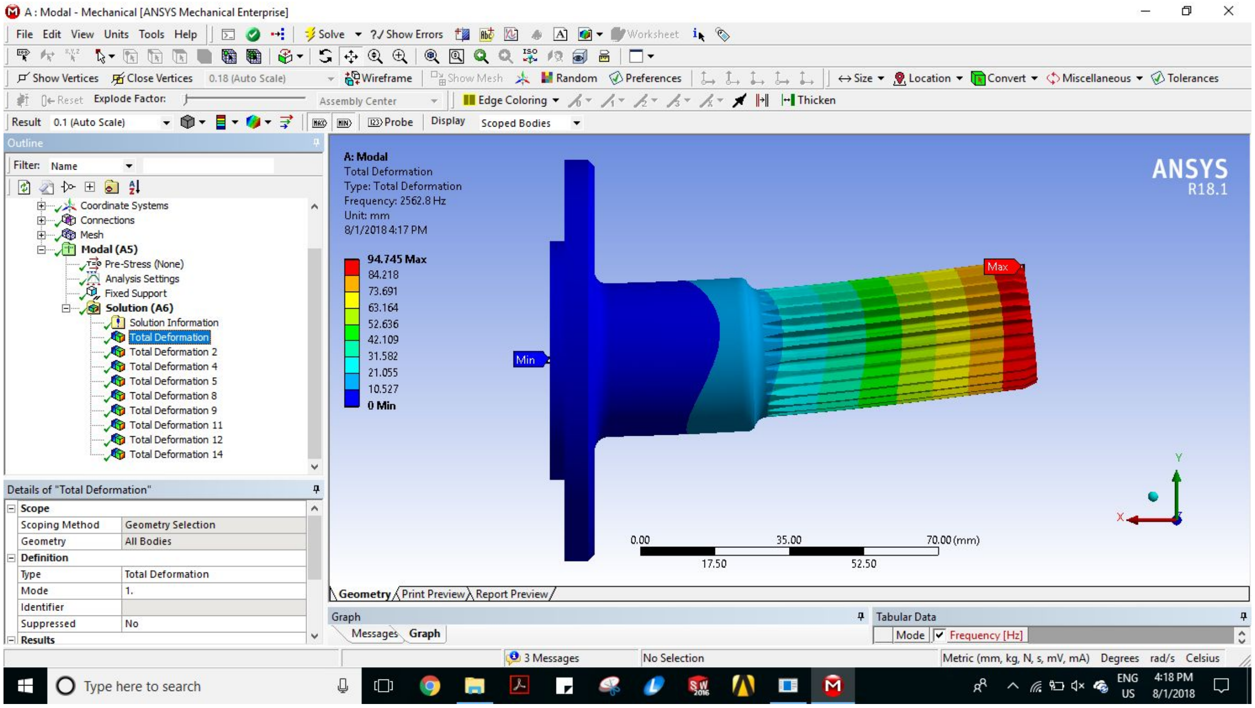
6.5 Shaft MODAL Analysis

As aforementioned, the motor shaft will be under simultaneous actions of bending as well as torsional loading, with the latter being steady in magnitude and direction but the former will alternate between tensile and compressive regimes. In simple terms, if a gear revolves around a shaft (while also rotating about its own axis), then in one complete revolution, if a particular point on the surface of the shaft is considered, that point will be acted upon by tensile stress as well as compressive stress; tensile when the gear is on the portion of the shaft containing the point and compressive when the gear is on the portion of the shaft opposite to that containing the point (here, only the extreme values of the two stresses is being talked about). Hence, it can be imagined that if such a fluctuating state of stress exists, the shaft is prone to vibrate due to the bending load. Also, since the driving motor will operate at varying speeds (hence, frequencies), these speed variations may also give rise to torsional vibrations. Overall, the shaft is prone to both bending and torsional vibrations but because the length of the shaft is quite small (~ 10cm) and since the radial load exerted by the planet gear on the sun gear (hence, on the shaft) is small compared to the tangential load, failure due to vibrations is not that prominent. Had the shaft been long enough, there might have been a difference in the torque input at one end from the motor and the torque output from the other end from the shaft, leading to vibration on a relatively larger scale.

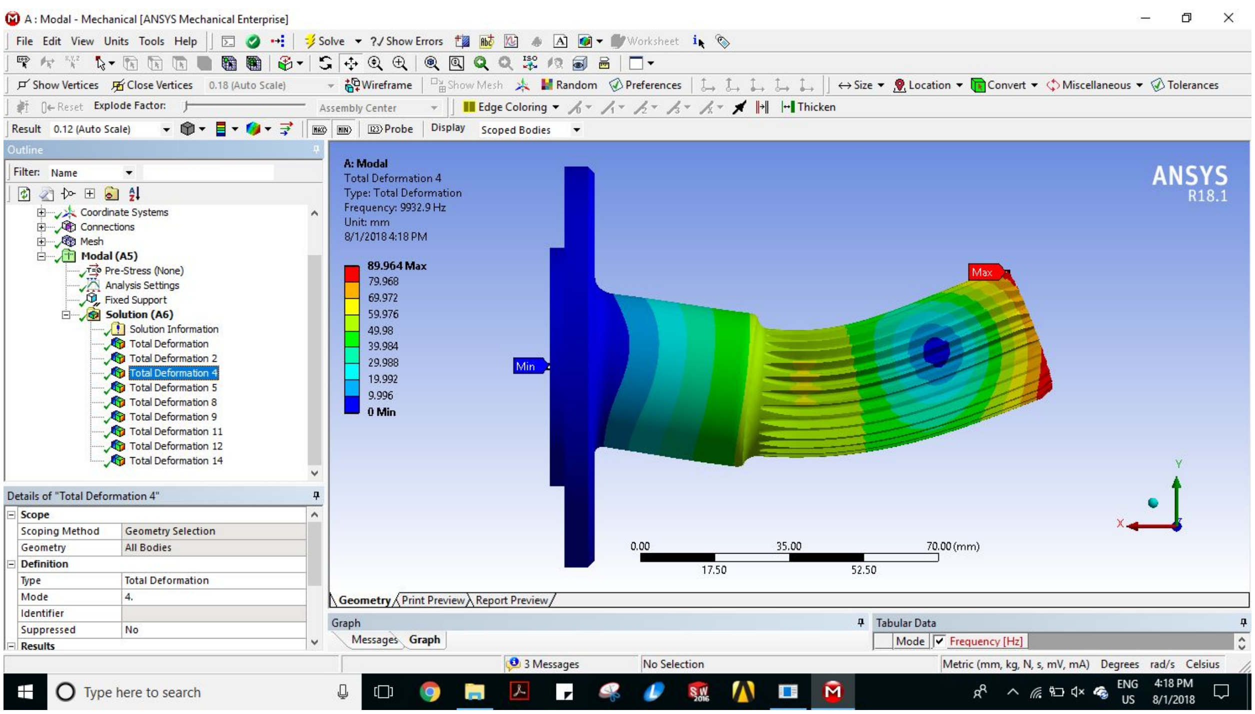
Nonetheless, to account for such vibrations and ensuring that the designed shaft is capable of sustaining them, a Modal analysis of the shaft was carried out in ANSYS, the underlying concept being that if the motor's input frequency to the shaft matches the shaft's natural frequency, the shaft might begin to vibrate and lead to failure. Considering the extreme case when the motor operates at its maximum frequency at 6000 RPM, the shaft's natural frequency must be significantly above it so that it can sustain frequencies even below the maximum limit.

Maximum motor RPM = 6000
Number of poles = 20 (10 pole pairs)
Maximum operating frequency = (6000 x 20) / 120 = 1000 Hz

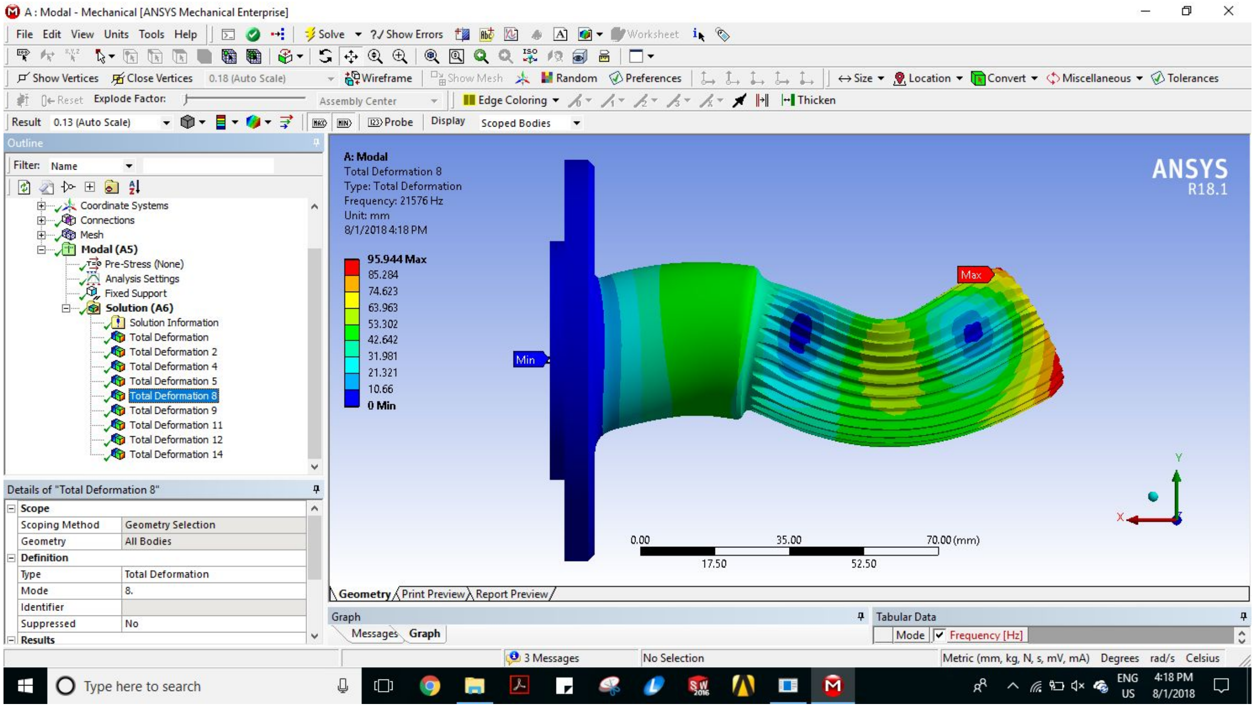
Hence, the shaft's natural frequency must be significantly above 1000 Hz. The following Modal analysis results were obtained in ANSYS.



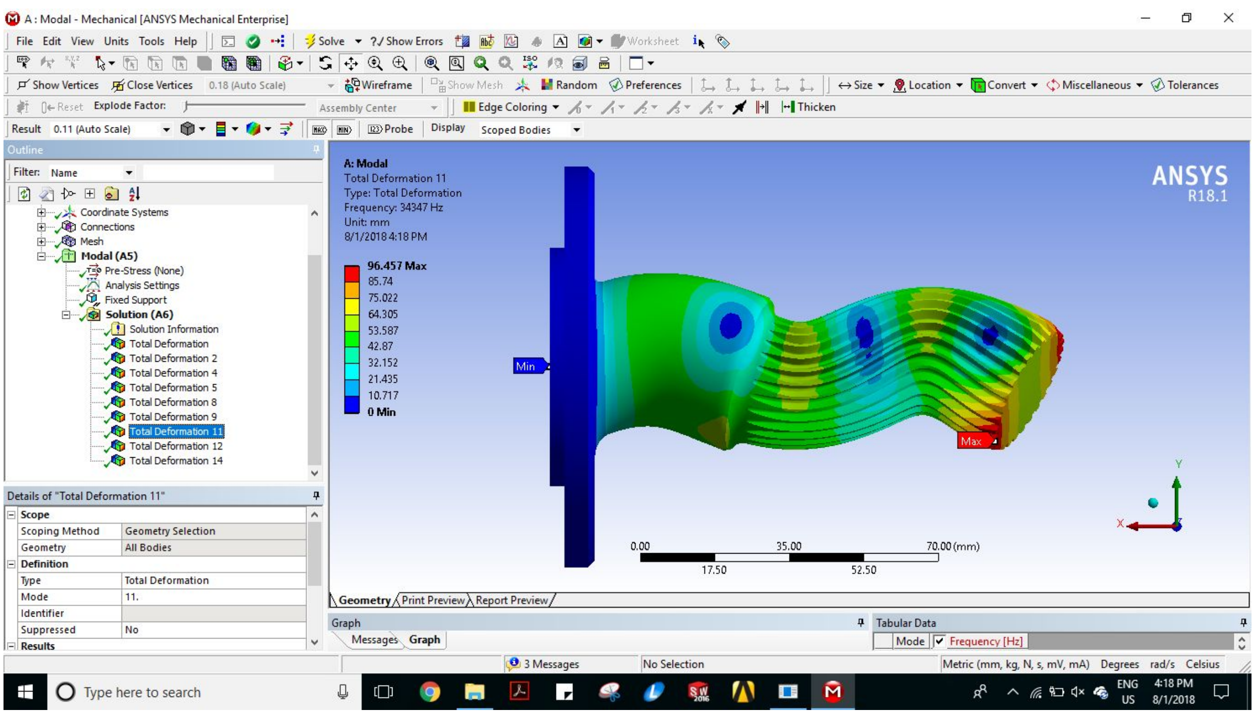
Fundamental frequency = 2563 Hz



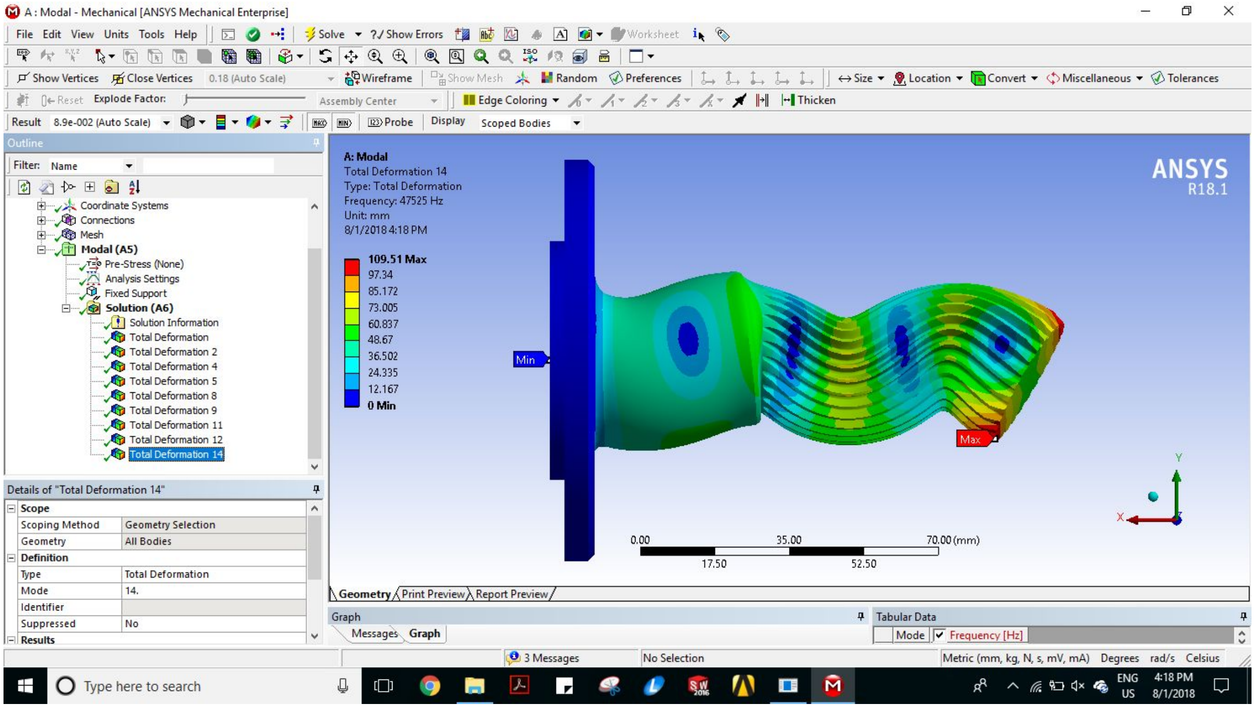
3rd harmonic frequency = 9933 Hz



5th harmonic frequency = 21576 Hz



7th harmonic frequency = 34347 Hz



9th harmonic frequency = 47525 Hz

Clearly, the fundamental frequency (2563 Hz) is well above the limiting frequency of 1000 Hz, thus keeping the shaft safe against vibrations.

CAD assembly of EMRAX 208 motor, motor shaft and planetary gearset (excluding planet carrier):