Robotics

MEEC

Orientation Estimation and Trajectory Analysis Using Sensor Data

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Group 9

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2 Introduction

This laboratory assignment focuses on two core objectives: understanding the estimation of orientation using data from a rate-gyro sensor and an accelerometer, and demonstrating the application of an industrial-grade serial manipulator. By addressing these objectives, students will gain practical experience in processing sensor data, applying mathematical models, and reconstructing trajectories in both Cartesian and orientation spaces. The work begins with the analysis of sensor data, provided in unique datasets for each group, containing approximately 20 seconds of measurements. The initial 5 seconds represent a static phase where the sensor remains stationary, providing a baseline for filtering and processing. These datasets include accelerometer readings (in milli-g) and rate-gyro readings (in degrees per second), formatted to facilitate computational analysis. The assignment is divided into multiple tasks, starting with data visualization, filtering, and theoretical trajectory reconstruction equations. Further tasks involve the graphical representation of reconstructed trajectories and their interpretation. In the later stages, the focus shifts to the use of the Scorbot VII manipulator. Here, students will derive the robot's direct kinematics equations and assess whether the reconstructed trajectories can be executed by the manipulator. By combining theoretical concepts with practical implementation, this lab offers an in-depth understanding of sensor data processing and robotic manipulator operations.

3 Tasks

3.1 Task 1

Task 1 involves visualizing sensor data by plotting the components of the accelerometer and rate-gyro measurements along their respective axes. This step is critical for understanding the behavior of the sensor during the data collection process and identifying any initial patterns or anomalies in the raw data. The dataset provided contains time-stamped measurements from an accelerometer and a rate-gyro, with values distributed across three axes: x, y, and z. The accelerometer data (measured in milli-g) reflects linear acceleration along each axis, while the rate-gyro data (measured in degrees per second) provides angular velocity along the same axes. The first few seconds of the dataset capture the sensor in a static configuration, offering a baseline for comparison against later movements. Through the visualization of these components in a combined plot, we aim to:

- Distinguish the data trends for each axis.
- Identify any irregularities or noise in the data.
- Provide a foundation for further processing in subsequent tasks.
- This initial analysis will serve as the starting point for understanding the sensor's performance and the nature of the motion captured.

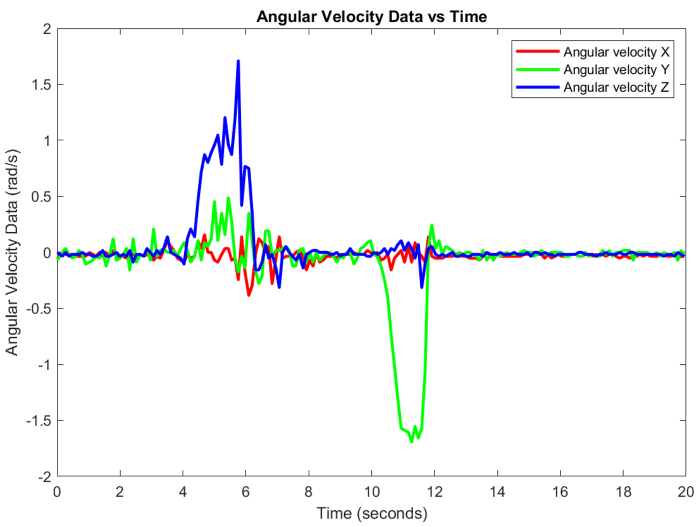

Figure 1: Angular velocity vs time.

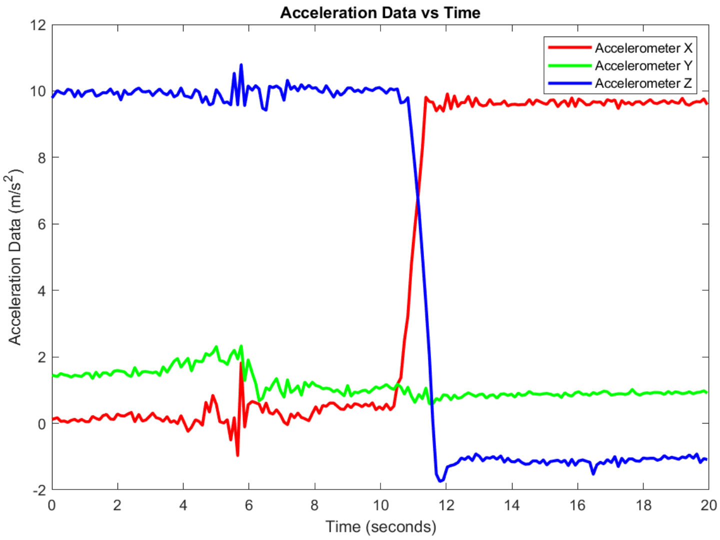

Figure 2: Acceleration vs time

3.2 Task 2

Preprocessing sensor data is a vital step to ensure accurate and meaningful analysis. Raw data, collected from accelerometers and rate-gyros, often contain noise and occasional outliers caused by sensor imperfections, environmental factors, or abrupt movements. These irregularities can obscure the true motion dynamics and negatively affect subsequent analyses. In this case, a median filter was applied to clean the data effectively. This filtering technique is well-suited for handling outliers and reducing noise, as it replaces each data point with the median of its neighbors within a defined window. Unlike other filtering methods, the median filter preserves sharp transitions and edges in the data while removing unwanted spikes, making it ideal for dynamic motion data. The results achieved with the median filter demonstrated significant improvements, yielding smooth and reliable datasets without distortions. This preprocessing ensures that the data is well-prepared for reconstructing accurate trajectories and performing further analysis with confidence in the validity of the underlying patterns.

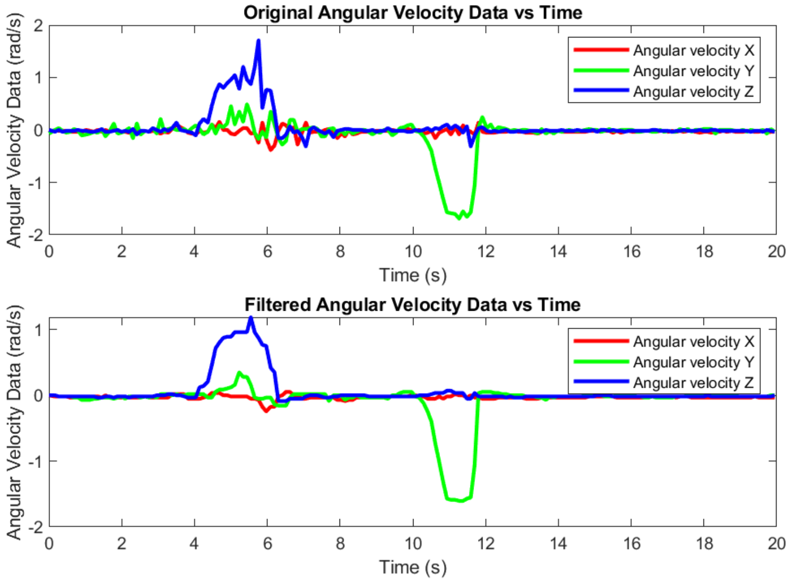

Figure 3: Filtered angular velocity vs original.

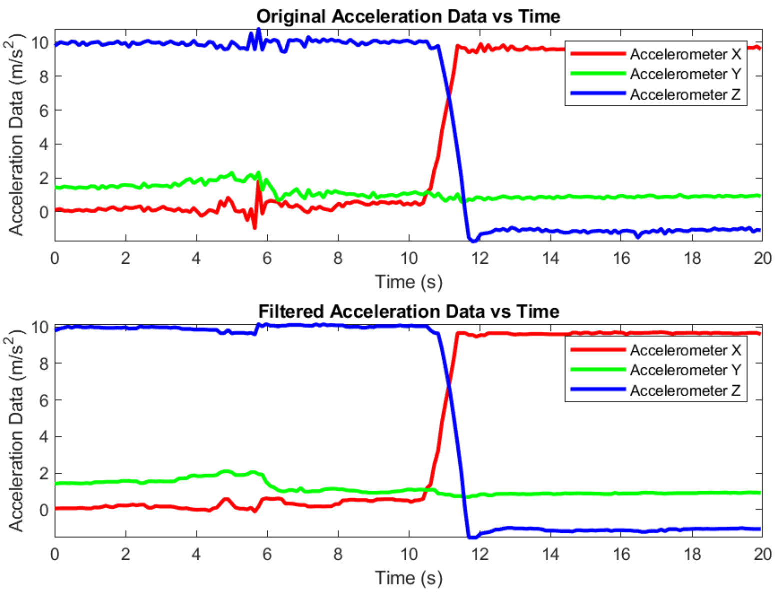

Figure 4: Filtered acceleration vs original

3.3 Task 3

3.4 Task 4

To reconstruct the sensor orientation in Euler angles (α, β, γ) using rate-gyro data, we analyze the relationship between angular velocities $(\omega_x, \omega_y, \omega_z)$ and the derivatives of the Euler angles. The **ZYX Euler angle convention** was selected because it provides an intuitive sequence for controlling robotic manipulators, such as the Scorbot VII, allowing separate adjustments for direction, height, and tool orientation.

3.4.1 Theory and Mathematical Formulation

In this convention, the angles represent rotations applied in a specific order:

- **Yaw** (γ) — rotation about the global Z-axis. This initial rotation aligns the manipulator to face the target, orienting its base towards the goal.
- **Pitch** (β) — rotation about the new Y-axis after the yaw rotation. This step adjusts the vertical alignment of the manipulator relative to the goal.
- **Roll** (α) — rotation about the new X-axis following the yaw and pitch rotations. This final adjustment ensures the gripper or tool is properly oriented for interaction with the target.

By properly parameterizing these rotations, we ensure predictable and precise control of the manipulator's movements, essential for tasks involving accurate positioning and end-effector orientation.

Parameterization To reconstruct the orientation accurately, we define the following parameters:

- α, β, γ: Euler angles representing the sensor's orientation in 3D space.
- $\omega_x, \omega_y, \omega_z$: Angular velocities provided by the rate-gyro, indicating rotational speeds around the sensor's local axes. These values are taken from the dataset provided in the file.
- Δt: Time interval between consecutive measurements, essential for discrete numerical integration.
- $\mathbf{J}(\alpha, \beta, \gamma)$: Jacobian matrix that relates the angular velocities to the changes in the Euler angles.

The task is to relate the angular velocities $(\omega_x, \omega_y, \omega_z)$ to the time derivatives of the Euler angles $(\dot{\alpha}, \dot{\beta}, \dot{\gamma})$, using a transformation matrix.

The relationship between angular velocities and the derivatives of the Euler angles is defined by the Jacobian matrix, shown in Equation (1):

$$\begin{bmatrix} \dot{\alpha} \\ \dot{\beta} \\ \dot{\gamma} \end{bmatrix} = \mathbf{J}^{-1}(\alpha, \beta, \gamma) \cdot \begin{bmatrix} \omega_x \\ \omega_y \\ \omega_z \end{bmatrix} \tag{1}$$

Where $\mathbf{J}(\alpha, \beta, \gamma)$ is the Jacobian matrix that links the angular velocities to the changes in Euler angles. For the ZYX convention, the Jacobian matrix is given by Equation (2):

$$\mathbf{J}(\alpha, \beta) = \begin{bmatrix} 1 & \sin(\alpha)\tan(\beta) & \cos(\alpha)\tan(\beta) \\ 0 & \cos(\alpha) & -\sin(\alpha) \\ 0 & \frac{\sin(\alpha)}{\cos(\beta)} & \frac{\cos(\alpha)}{\cos(\beta)} \end{bmatrix} \tag{2}$$

Each term in the Jacobian matrix reflects the dependency between rotations and angular velocities. For instance, the appearance of trigonometric functions, such as $\sin(\alpha)$ or $\cos(\beta)$, results from the sequential nature of the ZYX rotations and their cumulative impact on the orientation.

From this matrix, we derive the equations for the time derivatives of the Euler angles, presented in Equations (3), (4), and (5):

$$\dot{\alpha} = \omega_x + \sin(\alpha)\tan(\beta)\cdot\omega_y + \cos(\alpha)\tan(\beta)\cdot\omega_z \tag{3}$$

$$\dot{\beta} = \cos(\alpha)\cdot\omega_y - \sin(\alpha)\cdot\omega_z \tag{4}$$

$$\dot{\gamma} = \frac{\sin(\alpha)}{\cos(\beta)}\cdot\omega_y + \frac{\cos(\alpha)}{\cos(\beta)}\cdot\omega_z \tag{5}$$

Example Calculation To illustrate the use of these formulas with real data, consider an entry from the dataset provided in the file. For the time step corresponding to $t = 692028\,\mu s$, the rate-gyro measurements are:

$$\omega_x = -2.0^\circ/\text{s}, \quad \omega_y = 1.0^\circ/\text{s}, \quad \omega_z = -1.0^\circ/\text{s} \tag{6}$$

We convert them to radians per second:

$$\omega_x \approx -0.0349\,\text{rad/s}, \quad \omega_y \approx 0.0175\,\text{rad/s}, \quad \omega_z \approx -0.0175\,\text{rad/s} \tag{7}$$

For the initial conditions, let's assume the current Euler angles are all zero:

$$\alpha_k = 0.0\,\text{rad}, \quad \beta_k = 0.0\,\text{rad}, \quad \gamma_k = 0.0\,\text{rad}$$

We can calculate the derivatives using Equations (3), (4), and (5):

$$\begin{aligned}
\dot{\alpha}_k &= -0.0349 + \sin(0.0)\cdot\tan(0.0)\cdot 0.0175 + \cos(0.0)\cdot\tan(0.0)\cdot(-0.0175)\\
\dot{\beta}_k &= \cos(0.0)\cdot 0.0175 - \sin(0.0)\cdot(-0.0175) = 0.0175\\
\dot{\gamma}_k &= \frac{\sin(0.0)}{\cos(0.0)}\cdot 0.0175 + \frac{\cos(0.0)}{\cos(0.0)}\cdot(-0.0175) = -0.0175
\end{aligned}$$

After calculating the derivatives, they are:

$$\dot{\alpha}_k \approx -0.0349\,\text{rad/s}, \quad \dot{\beta}_k \approx 0.0175\,\text{rad/s}, \quad \dot{\gamma}_k \approx -0.0175\,\text{rad/s}$$

These derivatives will then be integrated over time to estimate the updated Euler angles.

3.4.2 Numerical Integration

To estimate the Euler angles over time, numerical integration is performed using the angular velocities. The continuous integration formulas are given in Equations (8), (9), and (10):

$$\alpha(t) = \alpha_0 + \int_0^t \dot{\alpha}\, dt \tag{8}$$

$$\beta(t) = \beta_0 + \int_0^t \dot{\beta}\, dt \tag{9}$$

$$\gamma(t) = \gamma_0 + \int_0^t \dot{\gamma}\, dt \tag{10}$$

For discrete time intervals, the equations become:

$$\alpha_{k+1} = \alpha_k + \dot{\alpha}_k \cdot \Delta t \tag{11}$$

$$\beta_{k+1} = \beta_k + \dot{\beta}_k \cdot \Delta t \tag{12}$$

$$\gamma_{k+1} = \gamma_k + \dot{\gamma}_k \cdot \Delta t \tag{13}$$

Here, Δt is the time interval between consecutive measurements.

3.5 Task 5

3.6 Task 6

3.7 Task 7

3.8 Task 8

4 Conclusion

References

Appendices