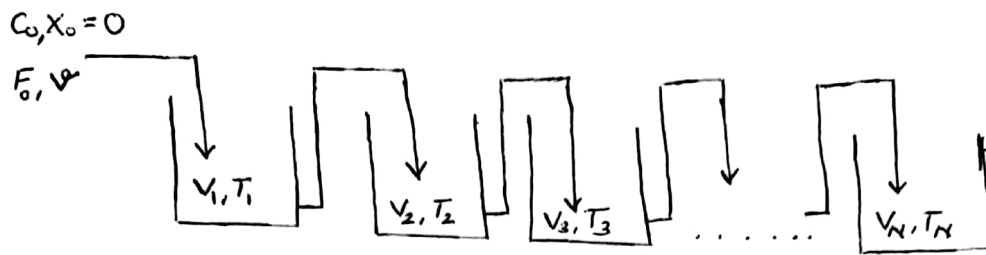


CHEMICAL REACTION ENGINEERING
QUIZ

1.



In first order reaction (where $E=0$), for the i^{th} reactor

$$Z_i = \frac{C_{i-1} V_i}{F_{i-1}} = \frac{C_0 V_i}{F_0} = \frac{V_i}{V} = \frac{C_0 (X_i - X_{i-1})}{-r_{A_i}}$$

as $E=0$ we have $\frac{C_i}{C_0} = (1 - x_i)$

$$\text{Thus } Z_i = \frac{C_0 \left(\left(1 - \frac{C_i}{C_0}\right) - \left(1 - \frac{C_{i-1}}{C_0}\right) \right)}{K C_i} \quad [\because -r_{A_i} = K C_i \text{ in } 1^{\text{st}} \text{ order}]$$

Now as volume of all the reactors are same, so $Z_i = \frac{V_i}{V} = \frac{V_{\text{reactor}}}{V}$ is same for all reactors.

$$\frac{C_0}{C_N} = \frac{1}{1 - X_N} = \frac{C_0}{C_1} \times \frac{C_1}{C_2} \times \frac{C_2}{C_3} \times \dots \times \frac{C_{N-1}}{C_N} = (1 + K Z_i)^N$$

$$\Rightarrow Z_i = \frac{1}{K} \left[\left(\frac{C_0}{C_N} \right)^{\frac{1}{N}} - 1 \right]$$

where N is the no. of reactors.

$$\text{Hence } Z_{N \text{ reactor}} = N Z_i = \frac{N}{K} \left[\left(\frac{C_0}{C_N} \right)^{\frac{1}{N}} - 1 \right] \quad \text{--- (1)}$$

if $N \rightarrow \infty$ then ~~eqn (1) reduces to~~

$$Z_{\infty \text{ reactors}} = \frac{1}{K} \ln \left(\frac{C_0}{C} \right); \text{ which is } Z_{\text{Plug Flow}}$$

So we can say that when no. of MFR in series approaches to ∞ then space time (total) of MFRs approaches to space time of single PFR

2.

For Batch reactor :-

Let the batch reactor be filled with a macrofluid containing reactant 'A'. Since each aggregate / packet of macrofluid acts as its own little batch reactor, conversion is same in all aggregates and is in fact identical to what would be obtained with a microfluid, thus for batch operation degree of ~~freedom~~ segregation doesn't effect conversion.

For plug flow reactor :-

Since plug flow can be visualized as a flow of small batch reactors passing in succession through the vessel, macro and microfluids act alike. So degree of segregation doesn't effect conversion.

For Mixed flow reactor :-

Microfluid :-

In MFR the reactant concentration everywhere drops to the low volume value prevailing the reactor. No clump of molecules retain its high initial concⁿ of 'A'. We may characterize this by saying that each molecule loses ~~its~~ its identity and has no determinable past history. In other words by examining its neighbors we can't tell whether a molecule is new comer or an old timer in the reactor. For this system

$$X_A = \frac{(-r_A)V}{F_{A_0}}$$

as $E=0$ we have $\frac{C_A}{C_{A_0}} = (1-X_A)$

$$\Rightarrow \frac{C_A}{C_{A_0}} = 1 - \frac{(-r_A)\bar{t}}{C_{A_0}}$$

where \bar{t} is the mean residence time of fluid in reactor.

Macrofluid:- When a macrofluid enters a MFR, the reactant concⁿ in an aggregate does not drop immediately to a low value but decreases in the same way as it would be in batch reactor. Thus a molecule in a macrofluid doesn't lose its identity. Its age can be estimated by examining its neighboring molecules.

The performance eqn for a macrofluid in a MFR is given by

$$1 - \bar{X}_A = \frac{\bar{C}_A}{C_{A_0}} = \int_0^{\infty} \left(\frac{C_A}{C_{A_0}} \right)_{\text{Batch}} E dt$$

$$\text{where } E dt = \frac{V}{V} e^{-t/\bar{t}} dt = \frac{e^{-t/\bar{t}}}{\bar{t}} dt$$

$$\text{thus } 1 - \bar{X}_A = \frac{\bar{C}_A}{C_{A_0}} = \int_0^{\infty} \left(\frac{C_A}{C_{A_0}} \right)_{\text{Batch}} \frac{e^{-t/\bar{t}}}{\bar{t}} dt$$

for 1st order reaction

$$\left(\frac{C_A}{C_{A_0}} \right)_{\text{Batch}} = e^{-kt}$$

$$\rightarrow 1 - \bar{X}_A = \frac{\bar{C}_A}{C_{A_0}} = \frac{1}{\bar{t}} \int_0^{\infty} e^{-kt} e^{-t/\bar{t}} dt$$

$$\rightarrow \frac{\bar{C}_A}{C_{A_0}} = \frac{1}{1 + k\bar{t}}$$

for microfluid

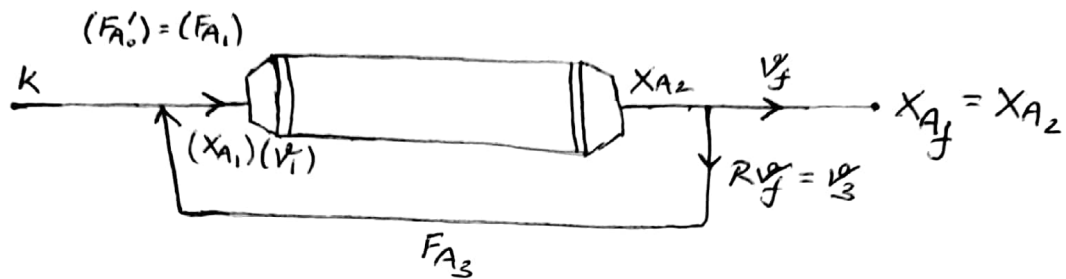
$$-r_A = kC_A \quad [\because 1^{\text{st}} \text{ order}]$$

$$\Rightarrow \frac{C_A}{C_{A_0}} = 1 - \frac{(-r_A)\bar{t}}{C_{A_0}} = 1 - \frac{kC_A\bar{t}}{C_{A_0}}$$

$$\text{or } \frac{C_A}{C_{A_0}} = \frac{1}{1 + k\bar{t}}$$

as the equations for macro and micro fluids are same thus we conclude that the degree of segregation has no effect on conversion for 1st order reaction.

3.



Across the reactor plug flow gives

$$\frac{V}{F_{A_0}'} = \int_{x_{A_1}}^{x_{A_2} = x_{A_f}} \frac{dx_A}{-r_A}$$

where F_{A_0}' would be the feed rate of 'A' if the stream entering the reactor (fresh feed plus recycle) ~~where~~ were unconverted

$$\begin{aligned} \rightarrow F_{A_0}' &= \text{fresh feed} + \text{Recycle stream} \\ &= F_{A_0} + R F_{A_0} \\ &= (1+R) F_{A_0} \end{aligned}$$

$$\begin{aligned} R &= \frac{\text{volume recycled}}{\text{volume leaving the reactor}} \\ R &\in [0, \infty) \end{aligned}$$

$$x_{A_1} = \frac{1 - C_{A_1}/C_{A_0}}{1 + E_A C_{A_1}/C_{A_0}} \quad \text{--- (1)}$$

as $E=0$ we have $x_{A_1} = (1 - C_{A_1}/C_{A_0})$

$$C_{A_1} = \frac{F_{A_1}}{v_1} = \frac{F_{A_0} + F_{A_3}}{v_0 + R v_f} = \frac{F_{A_0} + R F_{A_0} (1 - x_{A_f})}{v_0 + R v_f}$$

$$\text{thus } C_{A_1} = C_{A_0} \left(\frac{1+R - R x_{A_f}}{1+R} \right) \quad \text{--- (2)} \quad \left\{ \text{where } \frac{F_{A_0}}{v_0} = C_{A_0} \right\}$$

for eqn ① & ②

$$X_{A_i} = \frac{R}{R+1} \cdot X_{A_f}$$

$$\rightarrow \frac{V}{F_{A_0}} = \int_{X_{A_i} = \frac{R}{R+1} X_{A_f}}^{X_{A_f}} \frac{dX_A}{-r_A}$$

$$\rightarrow \frac{V}{F_{A_0}} = (R+1) \int_{\frac{R}{R+1} X_{A_f}}^{X_{A_f}} \frac{dX_A}{-r_A}$$

$$Z = \frac{C_{A_0} V}{F_{A_0}} = -(R+1) \int_{\frac{C_{A_0} + R C_{A_f}}{R+1}}^{C_{A_f}} \frac{dC_A}{-r_A} \quad \left\{ \begin{array}{l} \text{as } E_A = 0 \text{ and} \\ dC_A = -C_A \cdot dX_A \end{array} \right\}$$

and also $-r_A = k C_A$ [\because Ist order reaction]

$$\rightarrow Z = -(R+1) \int_{\frac{C_{A_0} + R C_{A_f}}{R+1}}^{C_{A_f}} \frac{dC_A}{k C_A} \quad \text{--- ③}$$

$$\rightarrow kZ = -(R+1) \ln \left(\frac{C_{A_f}(R+1)}{C_{A_0} + R C_{A_f}} \right)$$

$$\rightarrow \frac{kZ}{R+1} = \ln \left[\frac{C_{A_0} + R C_{A_f}}{(R+1) C_{A_f}} \right]$$

Putting $R = 0$ in eqn ③ $\Rightarrow Z = \frac{1}{k} \ln \left(\frac{C_{A_0}}{C_{A_f}} \right)$ (Plug Flow)

Putting $R = \infty$ in eqn ③ $\Rightarrow Z = \frac{1}{k} \int_{X_{A_0}}^{X_{A_f}} \frac{dX_A}{1 - X_A}$ (MFR)