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Course :- AE-686A (Helicopter Theory)

HOMEWORK-03

## [Question-01]

* **Steps of calculation**

① → Rotor solidity : $\sigma = \dfrac{N_b \cdot c \cdot (R - R_{cut})}{\pi R^2}$

where, $R_{cut} = 20\% R$

→ Hover inflow : $\lambda_h = \sqrt{\dfrac{C_{T_h}}{2}}$

→ Divide blade into $n=10$ segments.

$$dy = \frac{(1-0.2)}{n} = 0.08$$

② → For No twist (Case-02 : $\theta_{tw} = 0°$)

$$\theta_0 = \frac{6 \cdot C_{T_h}}{\sigma \cdot C_{l_\alpha}} + \frac{3}{2}\sqrt{\frac{C_{T_h}}{2}}$$

(Hovering flight)

[from BET for axial flight]

→ Ideal twist (Case-01 & 04 : $\theta_{tip} = \theta \cdot 2$)

$\theta_{75} = \theta_0$ (Reference pitch angle is taken at 75 % R)

$$\theta_{tip} = \frac{4\, C_{T_h}}{\sigma \cdot C_{l_\alpha}} + \sqrt{\frac{C_{T_h}}{2}}$$

→ Ideal taper (Case-04 : $c_{tip} = 0.5 \times c$)

$$c_{tip} = (0.5)c$$

$$\sigma_{tip} = \frac{N_b \cdot c_{tip} \cdot (R - R_{cut})}{\pi R^2}$$

③ * Gaussian Quadrature definition:

$$\int_a^b f(t)\cdot dt = \sum_{i=1}^{m} W_i \cdot f(t_i)\cdot\left(\frac{b-a}{2}\right)$$

$$t_i = \left(\frac{b-a}{2}\right)x_i + \left(\frac{b+a}{2}\right)$$

$$\therefore \int_a^b f(t)\cdot dt = \int_a^b \{W_1\cdot f(t_1) + W_2\cdot f(t_2) + \dots + W_6\cdot f(t_6)\}$$

$W = [W_1, W_2, \dots, W_6]$

$\text{Node} = [x_1, x_2, \dots, x_6]$

→ ~~...~~ Limits of Gaussian quadrature,

$a = 0.2 + dy\,(i-1)$

$b = 0.2 + dy \times i$

- Blade is divided into 10 segments. from $0.2 \to 1$

$$\int_{0.2}^{1} = \int_{0.2}^{0.28} + \int_{0.28}^{0.36} + \int \dots + \dots + \int_{0.92}^{1}$$

and integration is performed for each of 10 segments using 6 point Gaussian quadrature.
(i.e. integration is performed 10 times using for loop in code)

- For gaussian quadrature the variables are changed to ~~t(i)~~ $t_i$ from $x_i$ (i.e Nodes), which are given.

→ case-02 :- $\theta_{tw} = 0°$

$\therefore \theta_{tw2} = 0°$

$\theta_2 = \theta_{75} + (\theta_{tw2})(t - 0.75)$

case-03 :- $\theta_{tw} = -15°$

$\theta_{tw3} = -15°$

$\theta_3 = \theta_{75} + (\theta_{tw3})(t - 0.75)$

case-04 :- Ideal twist & Ideal taper

$$\sigma_4 = \frac{N_b\left(\frac{C\text{-}tip}{t}\right)(R - R_{cut})}{\pi R^2}$$

~~Case~~ Calculation of $C_T$, $C_{P_i}$, $C_{P_p}$

(i.e Thrust coefficient, Induced power coefficient, Profile power coefficient)

* Case 1 :- Ideal twist ($\theta_{tip} = \theta \cdot r$)

→ $\theta_{tip1} = \theta_{tip} = \frac{4 C_{T_h}}{\sigma \cdot C_{l_\alpha}} + \sqrt{\frac{C_{T_h}}{2}}$

→ $\lambda 1 = \frac{\sigma \cdot C_{l_\alpha}}{16} \left[ \sqrt{1 + \frac{32}{\sigma C_{l_\alpha}} \cdot \theta_{tip1}} - 1 \right]$

- induced inflow is constant.

→ Integration for thrust coefficient :

$$C_T = \frac{1}{2} \sigma \int_0^1 C_{l_\alpha} \left( \theta_{tip} - \lambda \right) r \cdot dr$$

Use Gaussian quadrature to ~~calc~~ perform integration over each segments.

$$CT\_1 = \Sigma\, W_i \cdot (f\_ct\_1) \cdot \left(\frac{b-a}{2}\right)$$

where, $f\_ct\_1 = C_T$

→ Integration for induced power coefficient :

$$C_{P_i} = C_T \cdot \lambda = \frac{1}{2} \sigma \cdot C_{l_\alpha} \int_0^1 (\theta_{tip} \lambda - \lambda^2) r\, dr$$

Use Gaussian quadrature for integration over each segments.

$$C_{P_i} = \Sigma (W_i) (f\_cp\_1) \left(\frac{b-a}{2}\right)$$

where, $f\_cp\_1 = C_{P_i} = C_T \cdot \lambda$

→ Integration for profile power coefficient

$$C_{P_p} = \frac{1}{2} \sigma \cdot C_{d0} \int_0^1 r^3 \cdot dr$$

Gaussian quadrature: $C_{P_p} = (W_i)(f\_cpp1) \left(\frac{b-a}{2}\right)$

where, $f\_cp\_p\_1 = C_{P_p}$

* **Case 2 :-** No twist ($\theta_{tw} = 0°$)

→ Pitch angle: $\theta 2 = \theta 75 + (\theta_{tw}2)(t - 0.75)$

$\theta tip2 = (\theta 2)(t)$

→ Inflow: $\lambda 2 = \frac{\sigma C_{l\alpha}}{16} \left\{ \sqrt{1 + \frac{32}{\sigma \cdot C_{l\alpha}} \cdot \theta tip2} - 1 \right\}$

[Not constant as $\theta tip2$ varies.]

→ ~~Then~~ perform integration for $C_T$, $C_{Pi}$ & $C_{P_p}$ ~~for~~ using Gaussian quadrature as discussed in case 01.

$$C_T = \frac{1}{2}\sigma \int_0^1 C_{l\alpha}(\theta tip2 - \lambda)\, r\, dr$$

$$C_{Pi} = \frac{1}{2}\sigma \int_0^1 C_{l\alpha}(\theta tip2 \cdot \lambda - \lambda^2)\, r \cdot dr$$

$$C_{P_p} = \frac{1}{2}\sigma \int_0^1 C_{d_0} \cdot r^3 \cdot dr$$

* **Case 3 :-** Linear twist ($\theta_{tw} = -15°$)

→ Pitch angle: $\theta 3 = \theta 75 + (\theta_{tw}3)(t - 0.75)$

$\theta tip3 = (\theta 3)(t)$

→ Inflow: $\lambda 3 = \frac{\sigma C_{l\alpha}}{16} \left\{ \sqrt{1 + \frac{32}{\sigma C_{l\alpha}} \cdot \theta tip3} - 1 \right\}$

Similarly, perform integration ~~of~~ for $C_T$, $C_{Pi}$ & $C_{P_p}$ using Gaussian quadrature as discussed before.

* **Case 4 :-** Ideal twist & Ideal taper ($C_{tip} = 0.5 \cdot c$).

→ Pitch angle: $\theta tip4 = \theta tip$

→ Inflow: $\lambda 4 = \frac{\sigma C_{l\alpha}}{16} \left\{ \sqrt{1 + \frac{32}{\sigma C_{l\alpha}} \cdot \theta tip4} - 1 \right\}$

Perform integration for $C_T$, $C_{Pi}$ & $C_{P_p}$ using Gaussian quadrature as shown in case 1

## 5. * Collectants :

→ Collect values for $C_T$, $C_{pi}$, $C_{pp}$ for plotting

$$\lambda_{mean} = \frac{sum(\lambda)}{6}$$

i.e. average values of $\lambda$ (from 6 points of each segments)

→ $C_{T mean}$ = average of $C_T$ (from 6 points)

→ $C_{pi}1$ mean = $C_{pi}1$

→ $C_{pp}1$ mean = $C_{pp}1$

→ $C_p1$ mean = $C_{pi}1 + C_{pp}1$

Repeat above procedure for all 4 cases.

## 6. * Results :

→ Total value of $C_T$ is calculated by summing $C_T$ values of each segment.

$$C_T\text{-}1\text{-}total = sum(C_T - 1)$$

→ Similarly, total value of $C_p$ is calculated by

$$C_p\text{-}1\text{-}total = C_{pi}1 + C_{pp}1$$

→ Induced inflow for hover,

$$\underset{(hover)}{\lambda 1} = \sqrt{\frac{C_T\text{-}1\text{-}total}{2}}$$

→ Reference pitch,

$$\theta_{\text{-}75\text{-}1} = \frac{6 \times C_T\text{-}1\text{-}total}{\sigma \cdot C_{l\alpha}} + \frac{3}{2}\sqrt{\frac{C_T\text{-}1\text{-}total}{2}}$$

→ Pitch angle at tip,

$$\theta_{tip} = \frac{4(C_T\text{-}1\text{-}total)}{\sigma \cdot C_{l\alpha}} + \lambda 1_{hover}$$

→ $\theta = \frac{\theta_{tip}}{r}$

⇒ Repeat above steps to calculate $C_{pi}$, $C_{pp}$, $C_T$, $\theta$, $\lambda$ for all four cases.

## 7 *Plotting of values

(a-i) Variation of pitch angle with $r$ ($\theta$ v/s $r$)

(a-ii) Numerical & Analytical solution for No twist case.

(b) Variation of AOA v/s $r$

$$\alpha = \theta - \phi = \theta - \frac{\lambda}{r}$$

(c) Inflow ($\lambda$) v/s $r$

(d) (i) $C_T$ v/s $r$

(ii) $C_Q$ v/s $r$

(iii) $C_{Q_i}$ v/s $r$

(iv) $C_{Q_p}$ v/s $r$

# Observations from graph:

(1) pitch angle v/s $r$

case-(i): Ideal twist (Hyperbolic variation)

$$\theta = \frac{\theta_{tip}}{r}$$

case-(ii): No twist ($\theta_{tw} = 0°$)

- Twist is zero, hence $\theta$ remains constant $\theta = \theta_0 = \theta_{75}$

case-(iii): Linear twist ($\theta_{tw} = -15°$)

$$\theta = \theta_{75} + \theta_{tw}(r - 0.75)$$

- $\theta$ variation is linear (decreases with $r$)

case-(iv): Ideal twist & Ideal taper.

~~$\theta$ variation is higher~~ values

- $\theta$ values are higher as compared to simple ideal/hyperbolic twist because blade chord decreases with $r$ and more pitch / Angle of attack is required to generate same $C_T$.

(2) Numerical solution v/s Analytical solution (for ideal twist)

→ Closed form exact formula:

$$\theta_{tip} = \theta r = \frac{4C_T}{\sigma \cdot C_{l\alpha}} + \sqrt{\frac{C_T}{2}} \quad \text{[constant]}$$

(3) Variation of AOA v/s r.

→ From BET, $\alpha = \theta - \phi = \theta - \frac{\lambda}{r}$

where, $\phi = \frac{\lambda}{r} = \tan^{-1}\left(\frac{u_p}{u_T}\right)$

Generally small for hover case.

hence, plot is similiar to $\theta$ v/s r.

(4) Variation of $\lambda$ v/s r.

→ For ideal twist $\lambda$ is uniform.

$\boxed{\text{Question-02}}$

No blade twist case ($\theta_{tw} = 0°$)

step 1 :- Find $\theta$ as discussed in question-01

$$\sigma = \frac{N_b \cdot c \; (R - R_{cut})}{\pi R^2}$$

$$\lambda_h = \sqrt{\frac{C_{T_h}}{2}}$$

$$dy = \frac{(1 - 0.2)}{n}$$

$$\theta_0 = \frac{6 \cdot C_{T_h}}{\sigma \cdot C_{l\alpha}} + \frac{3}{2}\sqrt{\frac{C_{T_h}}{2}}$$

$$\theta_{tw} = 0°$$

$$\theta = \theta_0 + \theta_{tw}(t - 0.75)$$

$$\theta_{tip} = \theta \cdot t$$

where, t is calculated from Gaussian qudrature.

Step 2 : Solve for $\lambda$ with $F=1$ (No tip loss)

$$\lambda(r) = \frac{\sigma \cdot C_{l\alpha}}{16 \cdot F}\left\{\sqrt{1+\frac{32 \cdot F \cdot \theta r}{\sigma \cdot C_{l\alpha}}}-1\right\} \quad (1)$$

Step 3 : Calculate F using $\lambda(r)$ of step-2.

$$f(r) = \frac{N_b}{2}\left(\frac{1-r}{\lambda(r)}\right)$$

$$F(r) = \frac{2}{\pi} \cdot \cos^{-1}\left[\exp(-f)\right]$$

Step 4: Recalculate $\lambda(r)$ using step-2.

Step 5: Recalculate $F(r)$ using step-3.

Step 6:- Iterate till convergence. (Ex. error $= 10^{-6}$)

# Observations from graph/plots:

(1) Reference pitch angle v/s r

→ The pitch angle for tip-loss condition is higher to ~~co~~ compensate lost $C_T$ due to tip-loss.

$$\theta_{\text{with tip loss}} > \theta_{\text{without tip loss}}$$

(2) Inflow v/s r

→ The inflow is increased ~~k~~ near tip for tip-loss case. ~~and $C_T$ is increased~~

(3) Thrust distribution $dC_T$ v/s r

→ Thrust is decreased near tip due to tip loss while ~~for~~ ~~t~~ thrust is more for no tip-loss case, near tip.

(4) Torque distribution $dC_P$ or $dC_Q$ v/s r

→ Torque drops near tip for tip loss case.