Homework-4:

(1) $\lim_{x\to 0} \frac{\sin x}{x} = \frac{\cos(x)}{1} = 1$

$\lim_{x\to 0} \frac{e^x - 1 - x}{x^2} = \frac{e^x - 1}{2x} = \frac{e^x}{2} = \boxed{\frac{1}{2}}$

$\lim_{x\to 0} \frac{e^x - e^{-x}}{\sin x} = \frac{e^x + e^{-x}}{\cos x} = \boxed{\frac{2}{1}}$

$\lim_{x\to -2} \frac{x^2+5x+6}{x-2} = !\frac{0}{0} = 0$

$\lim_{x\to 2} \frac{x^2+5x+6}{x-2} = !\frac{0}{0} = \boxed{\text{DNE}}$

(2) $\lim_{x\to \infty} \frac{(\ln x)^2}{2^x} =$ Numerator grows slower so $\boxed{0}$

(3) $\lim_{x\to 1} \frac{x^4-3x^3+x^2-x+2}{x^3-2x+1} = \frac{4x^3-9x^2+2x-1}{3x^2-2}$

then $\frac{4-9+2-1}{3-2} = \frac{-4}{1}$

$\lim_{x\to 0} \frac{\cos(x)-1}{x^2} = \frac{-\sin(x)}{2x} = \frac{-\cos(x)}{2}$

then $\frac{-1}{2} =$ limit

$\lim_{x\to 0} \frac{e^x - 2^x}{x} = \frac{e^x - 2^x \ln 2}{1} = \frac{e^0 - 2^0 \ln(2)}{1}$

(4) $\lim_{x\to 0} \frac{6^x - 10^x}{x} = 6^x \ln(6) - 10^x \ln(10)$

$\Rightarrow \ln(6) - \ln(10)$

(5) $\lim_{p\to 0} \frac{\ln(\frac{1}{2}x^p + \frac{1}{2}y^p)}{p} \Rightarrow \frac{\frac{1}{2}x^p \ln(x) + \frac{1}{2}y^p \ln(y)}{\frac{1}{2}x^p + \frac{1}{2}y^p}$

then $\frac{1}{2}\ln(x) + \frac{1}{2}\ln(y)$

(5b) $\lim_{p\to 0} (\frac{1}{2}x^p + \frac{1}{2}y^p)^{1/p} = \frac{\ln(\frac{1}{2}x^p + \frac{1}{2}y^p)}{p}$

then e^(5) answer

(6) $\lim_{x\to 0} \frac{\sin x - \tan x}{x^2 \sin x} = \frac{\cos x - \sec^2 x}{2x \sin x + x^2 \cos x}$

$\Rightarrow \frac{-\sin x - 2(\sec x)\sec x \tan x}{2\sin x + 2x\cos x + 2x\cos x - x^2 \sin x}$

$\Rightarrow \frac{-\cos x - 2\sec^4 x + 4\sec^2 x \tan^2 x}{2\cos x + 4\cos x \mp 4x\sin x - 2x\sin x + x^2\cos x}$

simplify: $\frac{-3}{6} = \boxed{\frac{-1}{2}}$

(7) $f(x) = \frac{\ln x}{1+(\ln x)^2}$; $\lim_{x\to 0^+} \frac{\ln x}{1+(\ln x)^2} = \frac{1/x}{1+2\ln x \cdot 1/x}$

$\lim_{x\to 0^+} \frac{1}{x+2\ln x} \Rightarrow \boxed{0}$

$\lim_{x\to \infty} \frac{1}{x+2\ln x} \Rightarrow \boxed{0}$

(8) $\lim_{x\to 0} \frac{x^2 \sin(1/x)}{\tan x}$; squeeze th.

$-1 \le \sin(\frac{1}{x}) \le 1$

$\frac{-x^2}{\tan x} \le \frac{x^2 \sin(1/x)}{\tan x} \le \frac{x^2}{\tan x}$

$\lim_{x\to 0} \frac{-x^2}{\tan x} \Rightarrow \frac{-2x}{\sec^2 x} = 0$

$\lim_{x\to 0} \frac{x^2}{\tan x} \Rightarrow \frac{2x}{\sec^2 x} = 0$

Homework 4-4

(9) $\lim_{x\to 0^+} x^{4.1}\ln x \Rightarrow \frac{\ln x}{x^{-4.1}}$

$\lim_{x\to 0^+} \frac{1/x}{-4.1x^{-5.1}} \approx 0.284x^{4.1}$

which is $\boxed{0}$

(10) $\lim_{x\to\infty} \frac{\tan^{-1}(x^7)}{x^7} = \frac{\pi/2}{\infty}$

which is 0

$\lim_{x\to 0^+} \sqrt[7]{x}\ln x = \frac{\ln x}{x^{-1/7}}$

$\Rightarrow \frac{1/x}{-\frac{1}{7}x^{-8/7}} = -7x^{1/7} = \boxed{0}$

(11) $\lim_{x\to\infty} 6xe^{1/x} - 6x = \frac{e^{1/x}-1}{1/6x}$

$\Rightarrow \frac{e^{1/x}-1}{(6x)^{-1}} \Rightarrow \frac{e^{1/x}(-1/x^2)}{-(6x)^{-2}(6)} \Rightarrow 6e^{1/x} = 6$

(12) $\lim_{x\to 0}\left(\frac{4}{x} - \frac{4}{\sin x}\right) = \frac{4\sin x - 4x}{x\sin x}$

$\Rightarrow \frac{4\cos x - 4}{\sin x + x\cos x} \Rightarrow \frac{-4\sin x}{2\cos x - x\sin x} = \boxed{0}$

(13) $\lim_{x\to 0} \sqrt{x}\ln x = \frac{\ln x}{x^{-1/2}} = \frac{1/x}{-\frac{1}{2}x^{-3/2}} = -2x^{1/2}$

$\Rightarrow -2(0)^{1/2} = 0$

(13) $\lim_{x\to 0} \ln x + \frac{1}{\sqrt{x}} = \frac{\sqrt{x}\ln x + 1}{\sqrt{x}}$

lim top $= 1$
lim bot $= 0^+$ $= \boxed{\infty}$

(13) $\lim_{x\to 0}\left(\frac{1}{e^x-1} - \frac{1}{x}\right) = \frac{x - e^x - 1}{xe^x - x}$

$\frac{1-e^x}{e^x + xe^x - 1} \Rightarrow \frac{-e^x}{2e^x + xe^x} = \frac{-1}{2}$

(14) $\lim_{x\to\infty} \sqrt{x^2+6x+9} - x = (x+3) - x = 3$

simplify this thing

(15) $\lim_{x\to\infty}\left(\frac{2^x+4^x}{4}\right)^{1/t} \Rightarrow e^{\frac{\ln\left(\frac{2^x+4^x}{4}\right)}{x}}$

$\Rightarrow e^{\frac{1/4(2^x\ln(2)+4^x\ln(4))}{1/4(2^x+4^x)}} \Rightarrow$ simplify this

$\lim_{x\to\infty} e^{\frac{2\ln 2(x-1)}{x}}\left(1/2^x + 1\right)^{1/x} \Rightarrow 4$

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Homework 4-4:

(17) $\lim\limits_{x\to\infty} \frac{P(x)}{e^x} = 0$

$\lim\limits_{x\to\infty} \frac{P(x)}{\ln x} = \text{DNE}$

$\lim\limits_{x\to 0} (1+x)^{1/x} = \frac{\ln(x+1)}{x}$

$\frac{1}{x+1} = 1 = \boxed{e}$

$\lim\limits_{x\to\infty} \left(1+\frac{1}{x}\right)^x = \boxed{e}$ ← def of e

$\lim\limits_{x\to\infty} \left(1+\frac{1}{x}\right)^{2x} = \boxed{e^2}$

$\lim\limits_{x\to 0^+} x^x = \frac{\ln(x)}{1/x} = \frac{1}{\cancel{x}} \cdot \frac{x^2}{-1} = 0$

then $e^0 = \boxed{1}$

$\lim\limits_{x\to 0^+} x^{x^x} = 0$