3.1 Derivative of Polynomials & Exponents:

3.2: Power Rule and Quotient Rule:

$f(x) = 1$; $\frac{dy}{dx} = 0$ & $f(x) = x$; $\frac{dy}{dx} = 1$ & $f(x) = x^2$; $\frac{dy}{dx} = 2x$ & $f(x) = x^3$; $\frac{dy}{dx} = 3x^2$

Power Rule:

$$\frac{d}{dx} x^n = n x^{n-1}$$ [* Read TB + derivation]

This formula is also true if n is not a positive number.

Ex 6: $f(x) = x^{1/2}$

$\frac{1}{2} x^{-1/2} = \frac{1}{2\sqrt{x}}$

$\frac{dy}{dx} = \boxed{\frac{1}{2\sqrt{x}}}$

Ex 7: $f(x) = 1/x$

$x^{-1} = -x^{-2}$

$\frac{dy}{dx} = \boxed{\frac{-1}{x^2}}$

Ex 8: $f(x) = 1/x^5$

$x^{-5} = -5x^{-6}$

$\frac{dy}{dx} = \boxed{\frac{-5}{x^6}}$

Ex 9: $f(x) = 1/x^6$

$x^{-6} = -6x^{-7}$

$\frac{dy}{dx} = \boxed{\frac{-6}{x^7}}$

New derivative from known derivative:

$$\left.\begin{aligned} \frac{d}{dx}(cf(x)) &= c\left(\frac{d}{dx} f(x)\right) \\ \frac{d}{dx}(f+g) &= \frac{df}{dx} + \frac{dg}{dx} \\ \frac{d}{dx}(f-g) &= \frac{df}{dx} - \frac{dg}{dx} \end{aligned}\right\}$$

share the same rule as the limit laws

Ex 9: $f(x) = 7x + 5x^{1/3} - 3$

$\frac{dy}{dx} = 7 + \frac{5}{3} x^{-2/3}$; $\frac{dy}{dx} = 7 + \frac{5}{3x^{2/3}}$

$$\frac{dy}{dx} = 7 + \frac{5}{3x^{2/3}}$$

Ex 10: $f(x) = \frac{\sqrt{x} - 1}{x} = \frac{1}{\sqrt{x}} - \frac{1}{x}$

$\frac{dy}{dx} = x^{-1/2} - x^{-1} = \frac{-1}{2} x^{-3/2} + x^{-2}$

$$\frac{dy}{dx} = \frac{-1}{2x^{3/2}} + \frac{1}{x^2}$$

Ex: $f(x) = x^5$

what is $\frac{d^5y}{dx^5}$? Taylor polynomials

$f_5(x^5) = 5!$

$f'(x) = 5x^4;\ f''(x) = 5\cdot 4x^3$

$f'''(x) = 5\cdot 4\cdot 3x^2 \ \ldots\ f^5(x) = 5! = 120$

Ex: 12 $f(x) = x(x+1) = x^2+x$

then what is $f'(x)$

$$\frac{dy}{dx} = 2x+1$$

* Maybe use chain Rule in the future *

Derivative of Exponential Function

Ex: $f(x) = 3^x$

$$f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h} \Rightarrow \lim_{h\to 0} \frac{3^{x+h}-3^x}{h} \Rightarrow 3^x \lim_{h\to 0} \frac{3^h-1}{h}$$

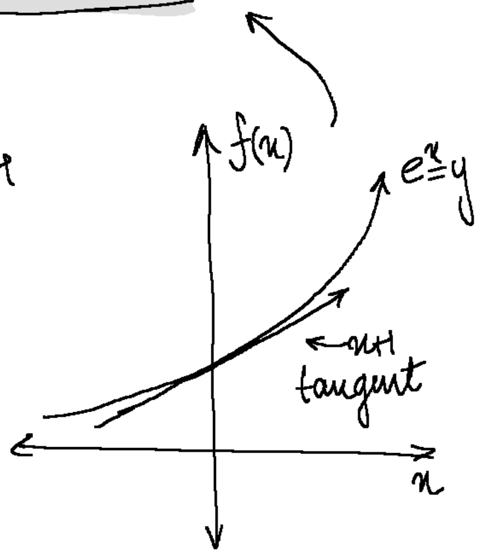
then true ∀ every b:

$$\frac{d}{dx}(b^x) = \left(\lim_{h\to 0} \frac{b^h-1}{h}\right)\cdot b^x \text{ where } \lim_{h\to 0} \frac{b^h-1}{h} = \ln(b)$$

Remember: Euler's # is defined to be a # with the slope of tangent line to the graph $e^x = \boxed{1}$

$$\text{then: } 1 = \frac{dy}{dx}\Big|_{x=0} e^x \overset{\uparrow}{=} \left(\lim_{h\to 0} \frac{e^h-1}{h}\right)\cdot e^0 = 1$$

$$\text{then } \lim_{h\to 0} \frac{e^h-1}{h} = 1 \text{ where } \boxed{\frac{d}{dx} e^x = e^x}$$



Ex14: $f(x) = x^2+7e^x$

$$\frac{dy}{dx} = 2x+7e^x$$

Ex15: $f(x) = e^{x+1} = e^x\cdot e$

$$\frac{dy}{dx} = e^x\cdot e = e^{x+1}$$

3.2 The product and the quotient rule:

Recall: $(fg)' \neq f'g'$

I $f(x) = 2+3x$; $f(0) = 2$; $f'(0) = 3$

$g(x) = 5+7x$; $f(0) = 5$; $f'(0) = 7$

$fg(x) = (2+3x)(5+7x)$

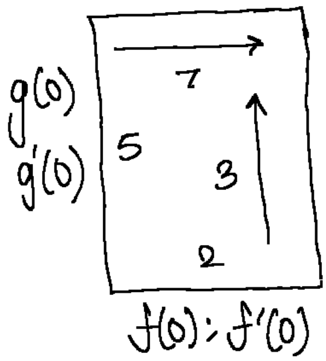
$10 + 29x + 21x^2 = fg(x)$

$(fg)'(0) = 42x + 29$

$42(0) + [2\cdot 7 + 3\cdot 5] = \boxed{29}$

Same as: $f(0)g'(0) + f'(0)g(0)$

II Rectangle w/ lengths f & g



$A(t) = f(t)\cdot g(t)$

$\Delta t = 0.0001$

$$\frac{\Delta A}{\Delta t} = \frac{f\cdot g'\Delta t + g' f \Delta t}{\Delta t}$$

$A'(t) = f\cdot g' + g' f$

Product Rule:

$$\frac{d}{dx}(fg) = fg' + f'g \text{ or } f\frac{dg}{dt} + \frac{df}{dt}g$$

Ex3: $f(x) = x^2e^x$

$$\frac{dy}{dx} = 2xe^x + x^2e^x$$

Ex4: $f(x) = e^{2x} = e^xe^x$

$$\frac{dy}{dx} = e^xe^x + e^xe^x = 2e^{2x}$$

Ex5: $f(x) = e^x(7x^4+x^3)$

$$\frac{dy}{dx} = e^x(7x^4+x^3) + e^x(28x^3+3x^2)$$

The quotient Rule:

Ex7: $\left(\frac{1}{g}\right)' = ? = \frac{-g'}{g^2}$

$1 = \frac{1}{g}\cdot g$ then we get

then: $\frac{d}{dx}(1) = \frac{d}{dx}\left(\frac{1}{g}\right)\cdot g = \frac{d}{dx}\left(\frac{1}{g}\right)\cdot g + \frac{1}{g}\frac{dg}{dx}$

Ex8: $\frac{d}{dx}\frac{1}{x^n} = \frac{-nx^{n-1}}{x^{2n}}$

$\Rightarrow -nx^{-n-1}$

Ex9: $\frac{d}{dx}e^{-x} = \frac{1}{e^x} = -e^{-x}$

Quotient Rule: (simplified Version)

$$\frac{d}{dx}\left(\frac{1}{g(x)}\right) = -\frac{g'(x)}{g^2(x)}$$

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Ex10: $\left(\frac{f}{g}\right)' = \left(f \cdot \frac{1}{g}\right)' + f\left(\frac{1}{g}\right)'$ but thats already given

$(f \div g)' = \frac{f'g - g'f}{g^2}$ which is the quotient rule