Homework 4-1

① $f(x) = x - x^2$; $f' = 1 - 2x$

$1 - 2x = 0$; $\boxed{x = 1/2}$

maximizes the number by its square

② $x - y = 46$: xy = mini

$f(x) = y(y+46) = y^2 + 46y$

$f' = 2y + 46$; $y = -23$

$x + 23 = 46$; then $x = 23$

$x, y = 23, -23$

③ $x - y^2 = 0$; $\sqrt{x^2 + (y-3)^2}$ = mini

$f(x) = \sqrt{y^4 + (y-3)^2}$ then

$f' = \frac{1}{2}(y^4 + (y-3)^2)^{-1/2}(4y^3 + 2(y-3))$

$f' = \dfrac{2y^3 + y - 3}{\sqrt{y^4 + (y-3)^2}} = 0$; $2y^3 + y - 3 = 0$ at 1

then $y = 1$ & $x = (1)^2 = 1$

min $d = \sqrt{(1)^2 + (1-3)^2} = \sqrt{5}$

④ $\theta = \tan^{-1}\left(\frac{12}{x}\right) - \tan^{-1}\left(\frac{3}{x}\right)$

$$f' = \frac{\frac{-12}{\cancel{x^2}}}{\frac{x^2+144}{\cancel{x^2}}} - \frac{\frac{-3}{\cancel{x^2}}}{\frac{x^2+9}{\cancel{x^2}}}$$

$3x^2 + 3(144) = 12x^2 + 12(9)$

$9x^2 = 324$; $\boxed{x = \pm 6}$

maximized when $x_0 = 6$

⑤ $\frac{7}{x} = \frac{y}{(x+3)}$; x = foot of ladder from fence, y = height at top of build

$y = \frac{7(x+3)}{x}$; Similar $\triangle^{les}$

$L(x)^2$ = length of ladder $= a^2 + b^2$

$L^2 = (x+3)^2 + y^2 = (x+3)^2 + \left[\frac{7x+21}{x}\right]^2$

$2L\,dL = 2(x+3) + 2\left(\frac{7x+21}{x}\right)\left(\frac{-21}{x^2}\right) = 0$

then $x \approx 5.278$; $y \approx 10.979$

$L(x,y) = \sqrt{(x+3)^2 + y^2}$: $L(x,y) = 13.75$ ft

⑥ Springfield $d = \sqrt{(x-0)^2 + (0-5)^2} = \sqrt{x^2 + 25}$

Shelbyville $d = \sqrt{(x-0)^2 + (0+5)^2} = \sqrt{x^2 + 25}$

Centerville $= 11 - x$

Total: $11 - x + 2\sqrt{x^2 + 25}$

$T' = -1 + 2\left(\frac{1}{2}\right)(x^2+25)^{-1/2}(2x) = \dfrac{2x}{\sqrt{x^2+25}} - 1 = 0$

$4x^2 = x^2 + 25$; $x = \frac{5}{\sqrt{3}}$;

$T'' = \left[2\sqrt{x^2+25} - \left(\frac{1}{2}\right)(x^2+25)^{-1/2}(4x)\right] \div x^2 + 25$

$T''\left(\frac{5}{\sqrt{3}}\right) \approx 0.2598$ ✓

$T'\left(\frac{5}{\sqrt{3}}\right) \approx 19.6603$ + the smallest branch length

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Homework 4-7:

⑦ $R(x) = 190 - 5(r-28) = 330 - 5r$

Income $= r \times R(x)$

Inc (max) $= 330r - 5r^2$

$dI = 330 - 5r = 0 ; r = 33 \frac{\$}{day}$

Maximized $=$ In$(33) = \$5445$

⑧ $P = \sqrt{x^2+16} ; P^2 = a^2+b^2$

$C = 1.8\sqrt{x^2+16} + (10-x) ;$

$\frac{dC}{dx} = \frac{1.8x}{\sqrt{x^2+16}} - 1 = 0 ;$ min

$\Rightarrow 1.8^2 x^2 = x^2 + 16 ; x = \sqrt{\frac{16}{2.24}}$

min ≈ 2.673

⑨ $4\cos\theta = d_1 ; d_2 = r\theta ; d_2 = 4\theta$

time $=$ d/speed ; $time_1 = 4\cos\theta/3$

$time_2 = 2\theta/6 = 2\theta/3$

total $t(\theta) = 4\cos\theta/3 + 2\theta/3$

$t' = \frac{-4}{3}\sin(\theta) + \frac{2}{3} = 0 ; \theta = \pi/6$

$t(\frac{\pi}{6}) = \frac{4\cos}{3}(\pi/6) + \frac{\pi}{9} = \frac{2\sqrt{3}}{3} + \frac{\pi}{9}$

$t(\frac{\pi}{2}) = \frac{4\cos(\pi/2)}{3} + \frac{\pi}{3} = \boxed{\frac{\pi}{3}}$

⑩ $6w + 2L = 510$

$L = 255 - 3w$

$A(w) = l \cdot w ; w(255 - 3w)$

$A' = 255 - 6w = 0$ then

$w = 42.5$

largest $A(w) = 42.5(255 - 3(42.5))$

$A = 5418.75 \text{ ft}^2$

⑪ $r^2 = x^2 + y^2 ; V = \pi r^2 h$

then in our question:

$V = \pi y^2 2x : y^2 = r^2 - x^2$

$V = 2\pi x(r^2 - x^2) = 2\pi x r^2 - 2\pi x^3$

$dV = 2\pi r^2 - 6\pi x^2 = 0 ; \quad r^2 = 3x^2$

then $x = \frac{r}{\sqrt{3}} ; y = \sqrt{r^2 - (\frac{r}{\sqrt{3}})^2}$

$y = \sqrt{\frac{3r^2 - r^2}{3}} ; \quad y = r\sqrt{\frac{2}{3}} ; \quad V = \pi r^2 \frac{2}{3}(\frac{2r}{\sqrt{3}})$

$V = \frac{4\pi r^3}{3\sqrt{3}}$ ∀ largest

$V(r) = \frac{4\pi r^3}{3\sqrt{3}}$ ∀ $r = 5 \approx 302.3$ unit^3

⑫ $30 = x + 2y + 2(\frac{x}{2})\frac{\pi}{2} ; x + 2y + \frac{x\pi}{2} = 30$

$A(\text{min}) = xy + \frac{\pi}{2}(\frac{x}{2})^2 ; \quad xy + \frac{\pi x^2}{8}$

$y = 15 - \frac{x}{2} + \frac{x\pi}{4} ; A(x) = x(15 - \frac{x}{2} - \frac{x\pi}{4}) + \frac{\pi x^2}{8}$

$A = 15x - \frac{x^2}{2} - \frac{x^2\pi}{8} ; A' = 15 - x - \frac{x\pi}{4} = 0$

largest $A = 63.01 \qquad 15 = x + \frac{x\pi}{4} ; \quad x = 15/_{1+\pi/4} ; 8.4$

$$2y = \frac{47}{2} - \frac{x}{2} - \frac{x\pi}{4}$$

$$x + 2y + 2\left(\frac{x}{2}\right)\frac{\pi}{2} = 47$$

$$x + 2y + \frac{x\pi}{2} = 47$$

$$xy + \frac{\pi}{2}\left(\frac{x}{2}\right)^2 \rightarrow xy + \frac{\pi x^2}{8}$$