1.3 New functions & Old functions

Translating graphs:

$f(x) = \sqrt{x}$

$D: x \in [0, \infty)$; $R: y \in [0, \infty)$

new point shifted by 1

$f(x) = \sqrt{x} + 1$

$D: x \in [0, \infty)$; $R: y \in [1, \infty)$

$y - 1 = \sqrt{x}$

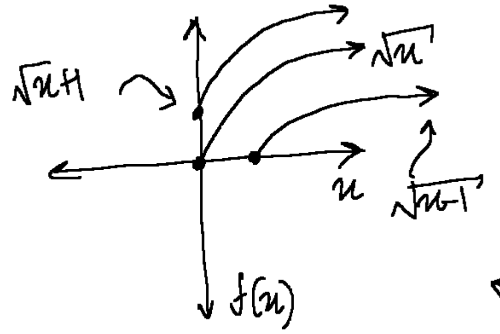
Similar to

$y = \sqrt{x-1}$

new point shifted right

$f(x) = \sqrt{x-1}$

$D: x \in [1, \infty)$; $R: y \in [0, \infty)$

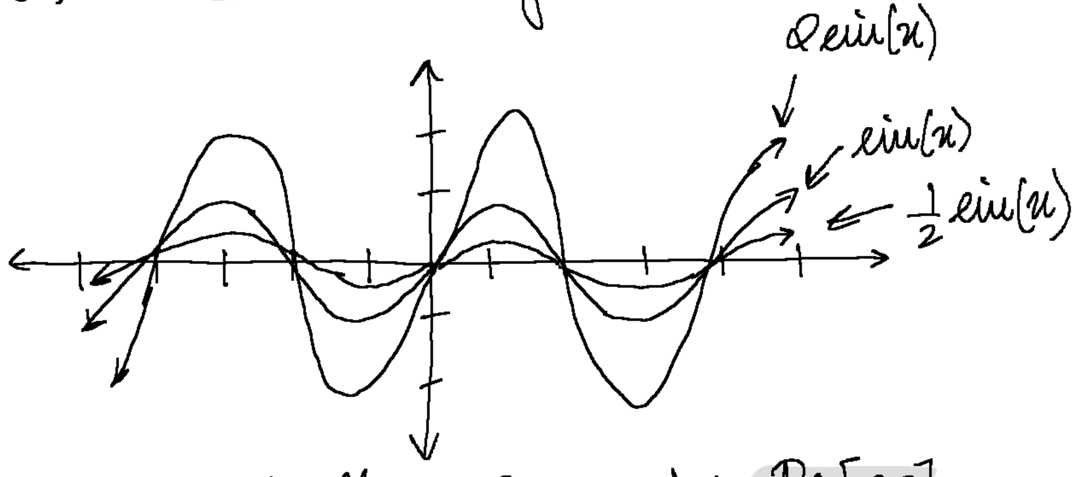


shift up/down: $y = f(x) + c$ where $c > 0$ & $c < 0$

shift right/down: $y = f(x - c)$ where $c > 0$ & $c < 0$

Stretching and Squeezing:

$f(x) = \sin(x)$ where $D: x \in \mathbb{R}$ & $R: y \in [-1, 1]$

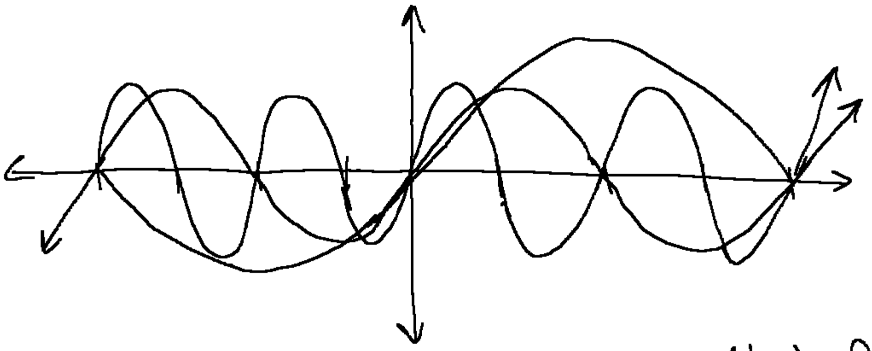


Stretch by 2 vertically $y = 2\sin x$ where $R: [-2, 2]$

Squeeze by 1/2 vertically $y = \frac{1}{2}\sin x$ where $R: [-\frac{1}{2}, \frac{1}{2}]$

Horizontal Stretching

D: $x \in \mathbb{R}$; R: $x \in [-1,1]$



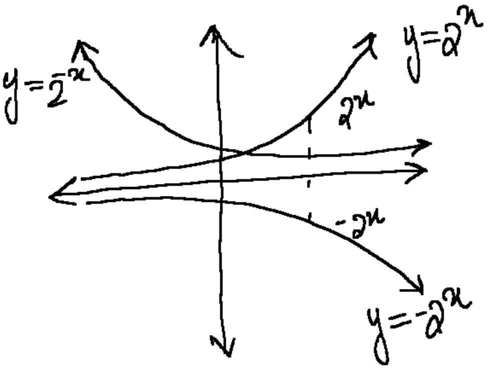
Stretch by 2 horizontally $y = \sin(\frac{1}{2}x)$ } D: $x \in \mathbb{R}$

Squeeze by 2 horizontally $y = \sin(2x)$ } R: $y \in [-1,1]$

Reflecting graphs:

$f(x) = 2^x$

D: $x \in \mathbb{R}$ & R: $y \in (0, \infty)$



Reflected the graph over the x-axis

D: $x \in \mathbb{R}$ but R: $y \in (-\infty, 0)$

then $f(x) = -2^x$

Reflected the graph over the y-axis

$f(x) = 2^{-x}$

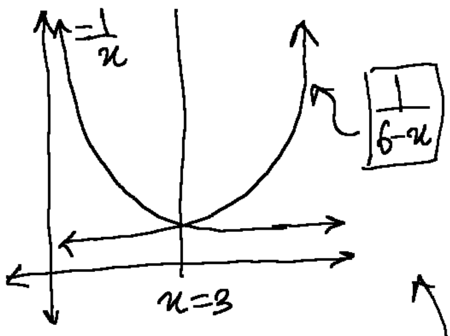
D: $x \in \mathbb{R}$ but R: $(0, \infty)$

Reflect along x axis $y = -f(x)$

Reflect along y-axis $y = f(-x)$

Applying both $-f(-x)$ gives you a rotation of $180°$

Reflect $f(x)=\frac{1}{x}$ over line $x=3$



D: $x\in(-\infty,6)$; R: $y\in(0,\infty)$

shift the graph left by 3 ← 3=axis

$f(x)=\frac{1}{x+3}$

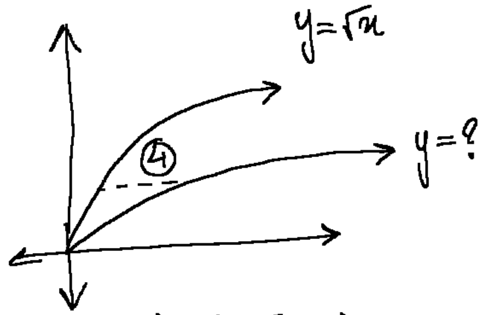
Reflect the y-axis then ← flip

$f(x)=\frac{1}{-x+3}$

shift back by 3 to org. position ← again back

$f(x)=\frac{1}{-(x-3)+3}\to\frac{1}{-x+3+3}=\boxed{\frac{1}{-x+6}}$

Stretching $f(x)=\sqrt{x}$ by 4:



D: $x\in[0,\infty)$; R: $[0,\infty)$

horizontal stretching is

$y=\sqrt{\frac{1}{4}x}$ ∵ $\sqrt{\frac{1}{4}}\sqrt{x}=\boxed{\frac{1}{2}\sqrt{x}}$

is the same as stretching

$y=\frac{1}{2}\sqrt{x}$

Linear is Same stretched or horizontally stretched

Combination of functions

Given two function lead to: $f+g$; $f-g$; fg ; $\frac{f}{g}$

Domain and Range may change:

Ex:

$f(x)=\frac{1}{x-2}$; D: $x\in(-\infty,2)\cup(2,\infty)$

$g(x)=\sqrt{x}$; D: $x\in[0,\infty)$

Case I: $f+g(x) = f(x) + g(x)$

$$\Rightarrow \frac{1}{x-2} + \sqrt{x}$$

$\mathcal{D}: x \in [0,2) \cup (2,\infty)$

common Domain

Case II: $\frac{f}{g}(x) = \frac{f(x)}{g(x)}$

$$\Rightarrow \frac{1}{x-2} \times \frac{1}{\sqrt{x}}$$

$\mathcal{D}: x \in (0,2) \cup (2,\infty)$

exclude zero

Composition functions

$f(g(x))$ $g(f(x))$

Given two function: $f(x)$ & $g(x)$ give $f \circ g(x)$ & $g \circ f(x)$

Domain & Range change!

Case III: $f \circ g(x) = f(g(x))$

$$\Rightarrow \frac{1}{\sqrt{x}-2}$$

$\mathcal{D}: x \in [0,4) \cup (4,\infty)$

Case IV: $g \circ f(x) = g(f(x))$

$$\Rightarrow \sqrt{\frac{1}{x-2}}$$

$\mathcal{D}: x \in (2,\infty)$

We can also compose 3 functions: $g(x)$: $f(x)$: $h(x)$

then $f \circ g \circ h(x) = f(g(h(x)))$

Case V: $f(x) = \sin(x)$

$g(x) = x^2 + 2x$

$h(x) = x+1$

$f \circ g \circ h(x) = f((x+1)^2 + 2(x+1))$

$$\Rightarrow \sin[(x+1)^2 + 2(x+1)]$$

Domain: Same logic.

1.4. Exponential Functions

1.4 Exponential Functions

Exponential functions: $f(x) = b^x$ (b: base when $b>0$; x: exponent)

Ex: $f(x) = 4^x$

x	$f(x)$
-1	$1/4$
0	1
1	4
2	16

$\times 4$

Some of the Exponent Rules:

$b^n = b \cdot b \cdot b \cdots n$ times $\qquad \left(\frac{1}{b}\right)^n = \frac{1}{b} \cdots \frac{1}{b^n}$

$b^0 = 1$; $b^{-n} = \frac{1}{b^n} \qquad b^{1/n} = \sqrt[n]{b}$; $b^{a/c} = \sqrt[c]{b^a}$

Ex: 2^π what does mean to define of π being raised to 2

then defined as $2^3 < 2^\pi < 2^4 \approx 8 < 2^\pi < 16$

keep getting a closer value $2^{3.1} < 2^\pi < 2^{3.2} \approx 8.57\ldots < 2^\pi < 9.18\ldots$

$\vdots$

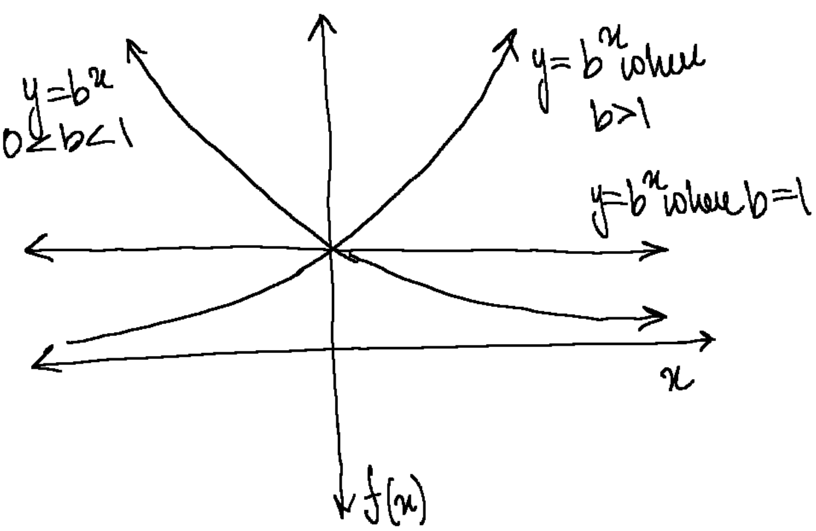
$2^{3.141} < 2^\pi < 2^{3.142} \approx \underline{8.8}213\ldots\ 2^\pi < \underline{8.8}227\ldots$

Therefore we know, $2^\pi \approx 8.82\ldots$

Ex3: $f(x) = \left(\frac{1}{2}\right)^x$

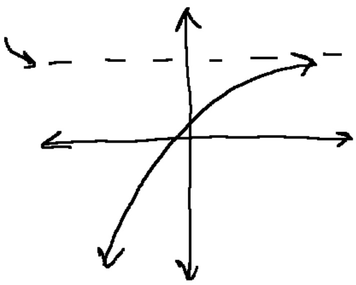
x	$f(x)$
-1	2
0	1
1	$1/2$
2	$1/4$
3	$1/8$

$\times \frac{1}{2}$



Ex4: Graph: $f(x) = 1 - 4^{-x}$

VA



Reflect over the y axis
↓
Reflect over the x axis
↓
Translate one unit up

Applications:

Ex5: Number of bacterium: 100
growth: double every hr
x = hours
$f(x)$ = # of bacteria

Ex6: Decay of ^{241}Pu: 5% per Year
inital mass: 7mg
x = # years
$f(x)$ = mass of Pu

$f(x) = 100 \cdot 2^x \longrightarrow \boxed{f(x) = cb^x} \longleftarrow f(x) = 7 \cdot (0.95)^x$

Euler's Numbers:

$e = 2.7182818\ldots$ irrational function

$f(x) = e^x$ and $f(x) = \ln(x)$