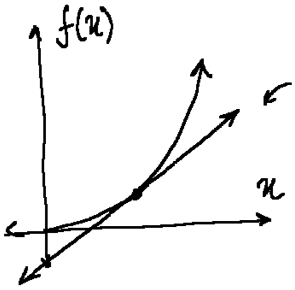
2.7 Derivative and Rate of Change:

The tangent and velocity problem

Ex1:



tangent line $m=10$

$f(x) = 5x^2$ at $x=1$

$$\lim_{x \to 1} \frac{f(x)-f(1)}{x-1} = \frac{5x^2-1}{x-1} = \boxed{10}$$

Simplifies to it : derivative

Ex2: Dropping a ball : $f(t) = 5t^2$

$$\lim_{h \to 0} \frac{f(1+h)-f(1)}{h} \Rightarrow 10+h \Rightarrow 10$$

where the 10 is instantaneous velocity at that graph

* Note the instantaneous velocity at a point is the derivative of a function

Definition:

The derivative of a function f at a number a is defined as

$$f'(a) = \lim_{h \to 0} \frac{f(a+h)-f(a)}{h} = \lim_{x \to a} \frac{f(x)-f(a)}{x-a} = \frac{dy}{dx}\bigg|_{x=a}$$

Ex3: $f(x) = x^3$

$$\lim_{x \to a} \frac{\cancel{(x-a)}(x^2+xa+a^2)}{\cancel{(x-a)}} = 3a^2$$

[Power Rule]

Ex4: Rate of change:

$V(t) = \sqrt{t} \longrightarrow \frac{dV}{dt} =$ gallons/sec

$$\frac{dV}{dt} = \frac{1}{2\sqrt{t}} ; \quad \frac{dV}{dt}\bigg|_{t=3} = \frac{1}{2\sqrt{3}}$$

Instantaneous Rate of Change of a quantity Y (in time)

$$f' = \lim_{\Delta t \to 0} \frac{\Delta y}{\Delta t} = \frac{dy}{dt} ;$$ ← Average vs instantaneous!

If the dependence between Y and t is expressed in terms of function $Y: f(t)$

Means the entire process of derivation

Ex5: Rate of change (general)

$D(v) = 10v^2$

$1\,\text{gal} \rightarrow D(2) - D(1) = 30\text{ mi} \rightarrow 30\frac{\text{mi}}{\text{ga}}$

$0.1\,\text{gal} \rightarrow D(1.1) - D(1) = 2.1\text{mi} \rightarrow 21$ "

$\Delta t\,\text{gal} \rightarrow D(1+\Delta t) - D(1)$ $\quad \frac{dD}{dV}$

then

$$\lim_{\Delta V \to 0} \frac{D(1+\Delta t) - D(1)}{\Delta V} = 20v\frac{\text{mi}}{\text{ga}}$$

Ex6: Rate of Change (general) BAC

$V(r) = \pi r^2$

new $V(5+\Delta r) = 10\pi\Delta r + \pi\Delta r^2$

Average Rate of Change:

$$\frac{\Delta V}{\Delta r} = \frac{10\pi\Delta r + \pi\Delta r^2}{\Delta r} = 10\pi + \pi\Delta r$$

then $\left.\frac{dV}{dr}\right|_{r=5} = 10\pi$; $A \approx \frac{\Delta V}{\Delta r}$

Instantaneous Rate of change of quantity Y on x

$$\lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x} = \frac{dY}{dx} = f'$$

if the dependence between Y and t is expressed in terms of a function $y = f(x)$

2.8 Derivative as a function:

$f(x) = x^2$: $f'(a) = 2a$; $f'(x) = 2x$

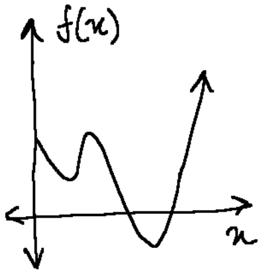
Definition

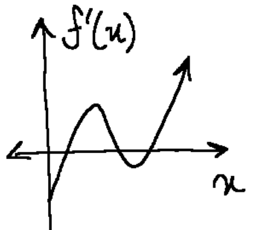
$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \lim_{x' \to x} \frac{f(x') - f(x)}{x' - x} = \frac{dy}{dx}$$

f is called differentiable at x if the limit exists and is not $\pm\infty$ or forms any sharp angle.

Try depending on the derivative rules to find any derivative

Ex5:



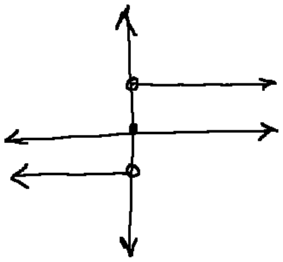


Change a function to its derivative func!

Theorem:

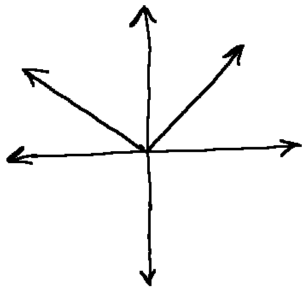
If f is differentiable at a, then it is also continuous at a

Ex6: $f(x) = \begin{cases} -1 & x<0 \\ 0 & x=0 \\ 1 & x>0 \end{cases}$



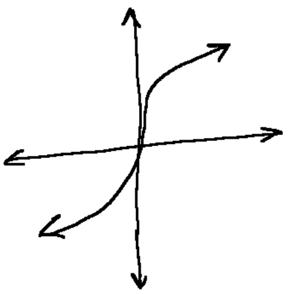
! differentiable ∵
not cont
but also
graph $dy = \infty$

Ex7: $f(x) = |x|$



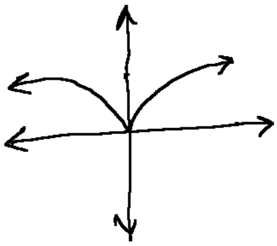
! differentiable ∵
$\lim_{x \to 0^+} f(x) = 1$
$\lim_{x \to 0^-} f(x) = -1$

Ex8: $f(x) = x^{1/3}$



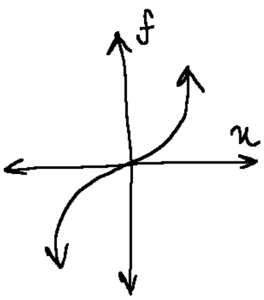
! differentiable ∵
$\lim_{x \to 0^+} x^{-2/3} = \infty$
tangent to y-axis
$= \infty$

$f(x) = $ IDK

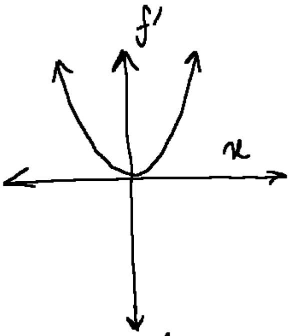


! differentiable ∵
cusps are
also sharp
corners.

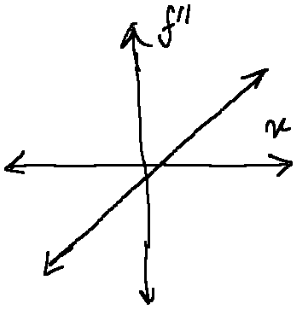
Higher Derivatives:



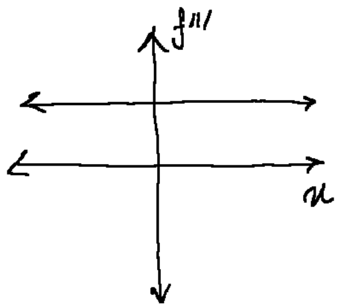
Position



Velocity



Acceleration



jerk