4.3 Derivative effect the shape of Graph

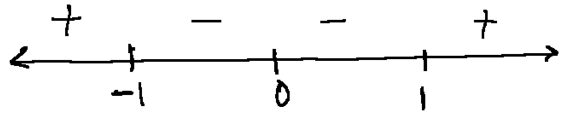
Increasing/decreasing test:

if $f'(x)>0$ on an interval, then increasing
if $f'(x)<0$ on an interval, then decreasing } on an interval

Ex 1: $f(x)=3x^5-5x^3+3$

$15x^4-15x^2=0$; $x^2(15x^2-15)=0$

$x=0$ m2; $x=\pm 1$

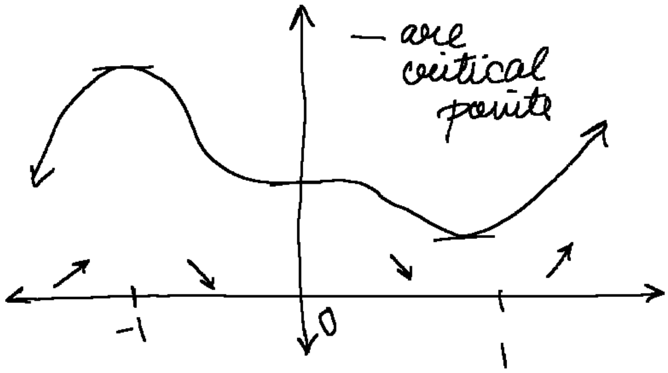


D of f: $(-\infty,-1)\cup(-1,0)\cup(0,1)\cup(1,\infty)$

increasing at: $(-\infty,-1)\cup(1,\infty)$

increasing at: $(-\infty,-1]\cup[1,\infty)$

decreasing at: $[-1,1]$



First derivative test:

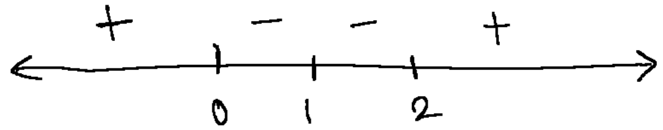
Suppose that c is a critical number of f.

→ If f' change from pos to neg at c, then c is a local max

→ If f' change from neg to pos at c, then c is a local min.

Ex 2: $f(x)=\dfrac{x^2}{x-1}$; $f'(x)=\dfrac{2x(x-1)-x^2}{(x-1)^2}=0$

$\dfrac{x^2-2x}{(x-1)^2}=0$; $x=0$ & 2 critical points



D of f: $(-\infty,0)\cup(0,1)\cup(1,2)\cup(2,\infty)$

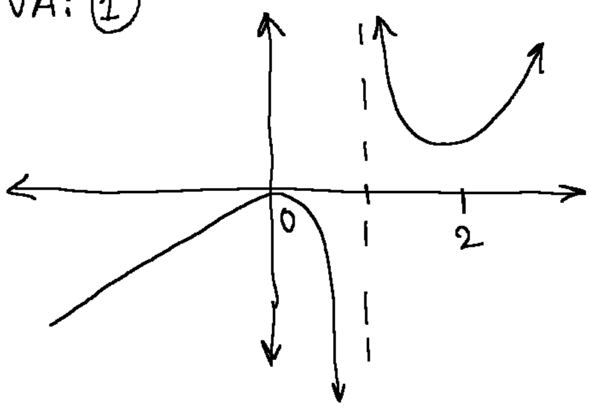
local min: at 2

local max: at 0

increase: $(-\infty,0]\cup[2,\infty)$

decrease: $[0,2]$

VA: 1



What does f'' say about f?

Definition:

Concave up: The graph of f lies above all its tangent lines

Concave down: The graph of f lies below all its tangent line

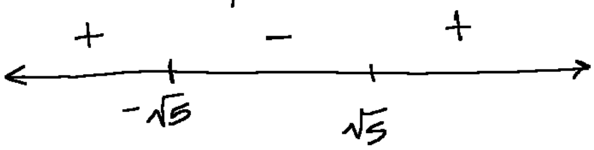
If f change from concave up to down at c then c is called the inflection point: $f''(c)=0$

Concavity Test:

→ if $f''(x)>0$ on an interval, then f is concave up on that int.

→ if $f''(x)<0$ on an interval, then f is concave down

$f''(c)=0$ then c is inflection point



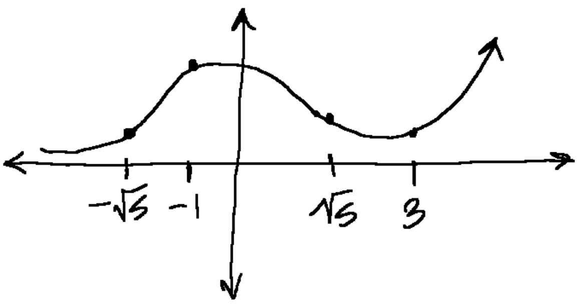
Ex3: $f(x)=(x^2-4x+1)e^x$

$f'(x)=(2x-4)e^x+(x^2-4x+1)e^x$

$f'(x)=e^x(x^2-2x-3)$; $x=-1,3$

$f''(x)=(2x-2)e^x+(x^2-2x-3)e^x$

$f''(x)=e^x(x^2-5)$; $x=\pm\sqrt{5}$



Second derivative test:

→ if $f'(c)=0$ and $f''(c)>0$, then f has a local min at c

→ if $f'(c)=0$ and $f''(c)<0$, then f has a local max at c

if $f'(c)=0$ and $f''(c)=0$ then inconclusive