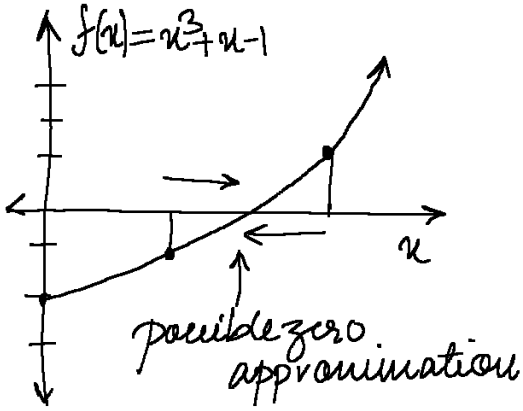
4.8 Newtonic Method:

Ex1: $f(x) = x^3 + x - 1 = 0$

$f(0) = -1$: neg ; $f(1) = 1$: pos



Bisection Method + root:

f root in $[0,1]$ mid = $f(.5)$: neg

f root in $[.5,1]$ mid = $f(.75)$: pos

f root in $[.5,.75]$ mid .....

Ex2: $f(x) = x^3 + x - 1 = 0$

Newtonic Method: $f' = 3x^2 + 1$

Initial guess: $x_1 = 1$

$L(x) = f'(1)(x-1) + f(1)$

$= 4(x-1) + 1 = 0$

$x = 0.75$

Second guess: $x_2 = 0.75$

$L(x) = f'(.75)(x-.75) + f(.75)$

$= 0.75 - f(.75)/f'(.75)$

$x = 0.68$

n^{th} guess: $x_n = x_n$

$L(x_n) = f'(x_n)(x - x_n) + f(x_n) = 0$

$= f'(x_n)(x - x_n) = -f(x_n)$

then next guess $= x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$

Newtonic Iteration:

x_1 = initial guess

$x_{n+1} = x_n - (f(x_n) \div f'(x_n))$

Then $x = \lim_{n \to \infty} x_n$ is a zero of f (then it exists)

Note: Newtonic iteration may fail to converge if the initial guess x_1 is chosen too far away from the actual root.