HW2-3:

① Plug in the value for the limits & as long as
$$\lim_{x\to a^-} f(x) = \lim_{x\to a^+} f(x)$$
then not DNE. Else Ez

② Plug in the value: no simplification

③ $f(x) = \begin{cases} \sqrt{-4-x}+5 & x<-5 \\ 5 & x=5 \\ 3x+21 & x>-5 \end{cases}$

$\lim_{x\to -5^-} f(x) = \sqrt{-4+5}+5 = 6$

$\lim_{x\to -5^+} f(x) = 3(-5)+21 = 6$

$\lim_{x\to -5} f(x) = 6 \because \lim_{x\to 5^-} = \lim_{x\to 5^+}$

④ Plug in the values no simplification

⑤ $\lim_{x\to 1} \frac{x^2+2x-3}{x-1} = \frac{(x+3)\cancel{(x-1)}}{\cancel{(x-1)}}$

$\lim_{x\to 1} x+3 = 4$

⑥ $\lim_{x\to 1} \frac{x^3-1}{x^2-1} = \frac{x^3-1}{(x+1)(x-1)}$

Polynomial Division

$\begin{array}{r|rrrr} 1 & 1 & 0 & 0 & -1 \\ & \downarrow & 1 & 1 & 1 \\ \hline & 1 & 1 & 1 & 0 \end{array}$ then $x^3-1/x-1 \Rightarrow x^2+x+1$

$\lim_{x\to 1} \frac{x^2+x+1}{x+1} = \boxed{\frac{3}{2}}$

⑦ $\lim_{x\to 1} \frac{1/x - 1/3}{x-3} = \frac{3-x/3x}{x-3}$

$\Rightarrow \frac{3-x}{3x}\left(\frac{1}{x-3}\right) = \frac{-1}{3x}$

$\lim_{x\to 3} \frac{-1}{3(3)} = \boxed{\frac{-1}{9}}$

⑧ Plug in the value from the table & evaluate

⑨ $\lim_{x\to -11^-} \frac{|x+11|}{x+11}$ then

$|x+11| = \begin{cases} x+11 & x>-11 \\ -(x+11) & x<-11 \end{cases}$

$\lim_{x\to -11^-} \frac{-\cancel{(x+11)}}{\cancel{x+11}} = -1$

⑩ $\lim_{x\to 3} \frac{15-3x-|x^2-5x|}{|x^2-25|-16}$

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(11) $f(x) = 3x^2+8$

$$\lim_{h\to 0} \frac{f(-4+h)-f(-4)}{h}$$

$$\frac{3(-4+h)^2+8-(3(-4)^2+8)}{h}$$

$\Rightarrow$ Simplies to

$\lim_{h\to 0} 6x$ where $6(-4) = \boxed{-24}$

(12) $$\lim_{y\to 0} \frac{3}{y(y+1)} - \frac{3}{y}$$

$$\Rightarrow \frac{3}{y(y+1)} - \frac{3(y+1)}{y(y+1)}$$

then $\lim_{y\to 0} \frac{-3y}{y(y+1)} = \frac{-3}{y+1}$

$\lim_{y\to 0} f(x) = \boxed{\frac{-3}{1}}$

(13) $$\lim_{x\to 4} \frac{1}{x+4} + \frac{\cancel{x-4}}{(x+4)\cancel{(x-4)}}$$

$$\Rightarrow \frac{1}{x+4} + \frac{1}{x+4} = \frac{2}{x+4}$$

$\lim_{x\to 4} f(x) = \boxed{1/4}$

(14) $$\lim_{x\to 25} \frac{25-t}{5-\sqrt{t}}; \quad 25-t = 5^2-(\sqrt{t})^2$$

$$\Rightarrow \frac{(5+\sqrt{t})\cancel{(5-\sqrt{t})}}{\cancel{(5-\sqrt{t})}}$$

$\lim_{x\to 25} 5+\sqrt{t} = 10$

(16) $$\lim_{h\to 0} \frac{\sqrt{6a+6h}-\sqrt{6a}}{h}$$

$$\frac{(\sqrt{6a+6h}-\sqrt{6a})(\sqrt{6a+6h}+\sqrt{6a})}{h(\sqrt{6a+6h}+\sqrt{6a})}$$

$$\Rightarrow \frac{\cancel{6a}+6\cancel{h}-\cancel{6a}}{\cancel{h}(\sqrt{6a+6h}+\sqrt{6a}}$$

$\lim_{x\to 0} f(x) = \boxed{\frac{6}{2\sqrt{6a}}}$

(17) $\lim_{t\to 0} \left(\frac{7}{t\sqrt{49+t}} - \frac{1}{t}\right)$ where

$$\lim_{t\to 0}\left(\frac{7-\sqrt{49+t}}{t\sqrt{49+t}} \times \frac{7+\sqrt{49+t}}{7+\sqrt{49+t}}\right)$$

$\lim_{t\to 0} \frac{49-(49+t)+0+0}{t\sqrt{49+t}(7+\sqrt{49+t})} = \frac{-1}{\text{denominator w/o } t}$

$$\lim_{t\to 0} f(x) = \frac{-1}{\sqrt{49}(7+\sqrt{49})} \to \frac{-1}{7(14)}$$

(18) $\lim_{x\to 2} f(x) =$ where $8x-18 \le f(x) \le x^2+4x-14$

$\lim_{x\to 2} f(x) = 8(2)-18 = -2$

Using the squeeze th. as it is between $h(x)$ and $g(x)$

(16) $f(x) = \begin{cases} 6x+m & x<2 \\ -6x^2+2m & x\ge 2 \end{cases}$

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15 $f(x) = \begin{cases} 6x+m & x<2 \\ -6x^2+2m & x\geq 2 \end{cases}$

$$\lim_{x\to 2^+} -6x^2+2m = -6(2)^2+2m$$
$$\Rightarrow 2m-24$$

$$\lim_{x\to 2^-} 6x+m = 6(2)+m$$
$$\Rightarrow 12+m$$

Then value $\forall$ m to make $\lim\limits_{x\to 2^-} = \lim\limits_{x\to 2^+}$

$$2m-24 = 12+m$$
$$m = 12+24 = \boxed{32}$$

10 $$\lim_{x\to 3} \frac{15-3x-|x^2-5x|}{|x^2-25|-16}$$

$$\lim_{x\to 3} \frac{15-3x+x^2-5x}{-x^2+25-16}$$

$$\lim_{x\to 3} \frac{\cancel{(x-3)}(x-5)}{-(x+3)\cancel{(x-3)}} = \frac{-(x-5)}{x+3}$$

$$\lim_{x\to 3} \frac{2}{6} = \boxed{1/3}$$