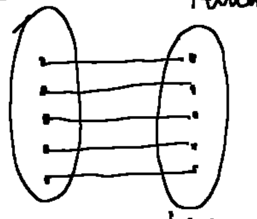
1.5 Inverse functions and logarithms:

Ex1: x = age of rainbow trout
$f(x)$ = length

x	$f(x)$
0	0
1	15
2	20
3	25
4	27

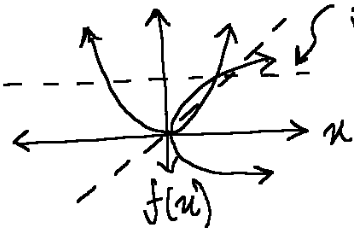


$D \to R = f(x)$; then $R \to D = f^{-1}(x)$

A function $f: D \to R$ is called one-one iff $f(x_1) \neq f(x_2)$ whenever $x_1 \neq x_2$ if true then has an inverse function $f^{-1}: R \to D$ then

$$y = f(x) \iff f^{-1}(y) = x$$

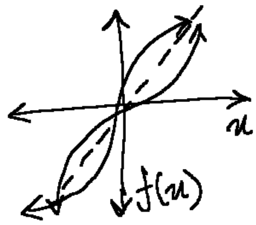
Ex2: is $f(x) = x^2$ one to one?



f doesn't pass the horizontal line test

$x_1 = 2$; $x_2 = -2$ then $f(x_1) = f(x_2)$ yet $x_1 \neq x_2$

Ex3: is $f(x) = x^3$ one to one?



passes the HLT but check algebraic

$f(x) = y$; $x^3 = y$; $x = \sqrt[3]{y} = f^{-1}(x)$
no ambiguity

* Note: Obtain $f^{-1}(x)$ graph by reflecting over $y = x$.

Some formulas:

$f^{-1}(f(x)) = x$ and $f(f^{-1}(x)) = x$ and $f^{-1}(x)^{-1} = f^{-1}(x)$

Continued in the next page w/ more examples.

Ex4: $f(x) = \left(2 + \frac{1}{x}\right)^3$

$D: \{x \in \mathbb{R} \mid x \neq 0\}$

$\left(2 + \frac{1}{x}\right)^3 = y$

$f^{-1}(x) = \dfrac{1}{\sqrt[3]{x} - 2}$

$x \neq 0$ & $y \neq 8$

$D: \{x \in \mathbb{R} \mid x \neq 8\}$

Note the Range of the new function is the Domain of the old function.

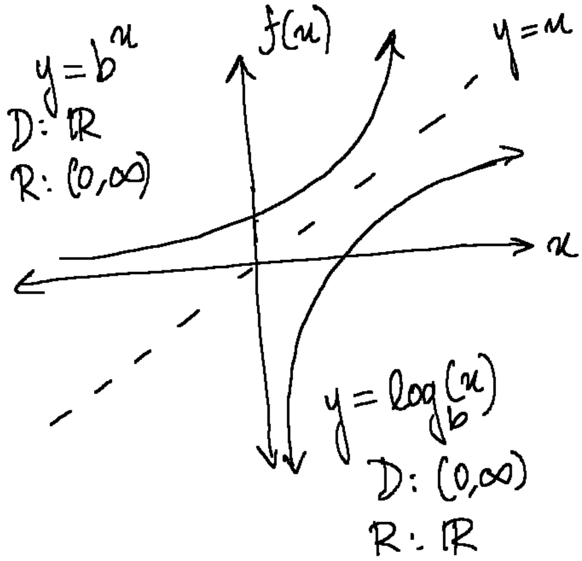
$D: \{x \in \mathbb{R} \mid x \neq 0\}$ & $R: \{y \in \mathbb{R} \mid x \neq 8\}$

$D: \{x \in \mathbb{R} \mid x \neq 8\}$ & $R: \{y \in \mathbb{R} \mid y \neq 0\}$

1.5 logarithms:

$f(x) = b^x \iff f^{-1}(x) = \log_b(x)$

if $b = e$, then we write $f^{-1}(x) = \log_e(x)$ or $\ln(x)$



Ex5:

$\log_2 512 = 9$

$\log_2 (1/512) = -9$

$\log_2 (512 \cdot 512) = 18$

$\log_8 512 = 3$

$\dfrac{\log_2 512}{\log_2 8} = \dfrac{9}{3}$

Rules of log:

$\log_b xy = \log_b x + \log_b y$

$\log_b (x \div y) = \log_b x - \log_b y$

$\log_b x^r = r \log_b x$

$\log_b 1 = 0$; $\boxed{x^0 = 1}$

Change of base formula:

$$\log_b x = \frac{\log_a(x)}{\log_a(b)} = \frac{\ln(x)}{\ln(b)}$$

Computation of $\log_b$

Ex6: Slide Rule

Based on the $\log 2 + \log 3 = \log(2\cdot 3) = \log 6$
with a bunch of rulers lol.

Ex7: Decay of ^{241}Pu = 5% per year

Find half life:

x: # years

fraction left after x many years Same as

$$f(x) = (0.95)^x = 1/2$$

Solution 1:

$$x = \log_{.95}(1/2)$$

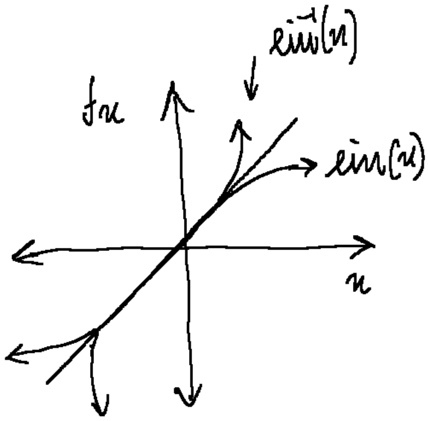
then $\dfrac{\ln(0.5)}{\ln(.95)} \approx 13.5$ years

Solution 2:

$$\ln(0.95)^x = \ln(1/2)$$

$$x = \frac{\ln(0.5)}{\ln(.95)} \approx 13.5 \text{ years}$$

Inverse Trigonometric Functions:



$f(x) = \sin x$

D: $x \in [-\pi/2, \pi/2]$

R: $y \in [-1, 1]$

$f^{-1}(x) = \arcsin(x)$

R: $x \in [-1, 1]$

D: $y \in [-\pi/2, \pi/2]$

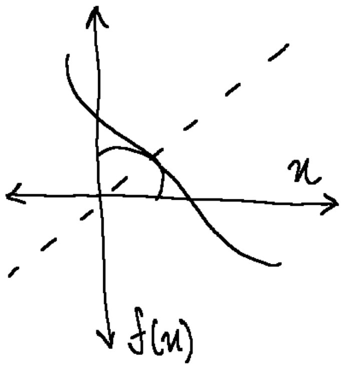
Ex8:

$$\sin^{-1}\left(\frac{\sqrt{2}}{2}\right) = x = \frac{\pi}{4}$$

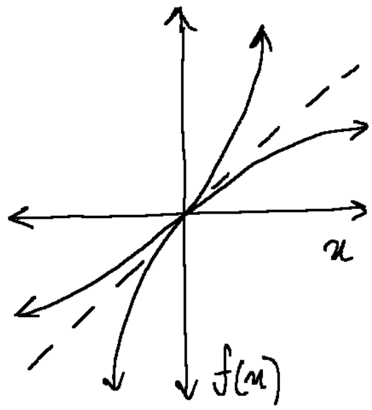
$$\sin(x) = \frac{\sqrt{2}}{2}$$

$$\theta = \pi/4$$

Draw a unit circle or use the Pythagoras theorem to figure out the other value

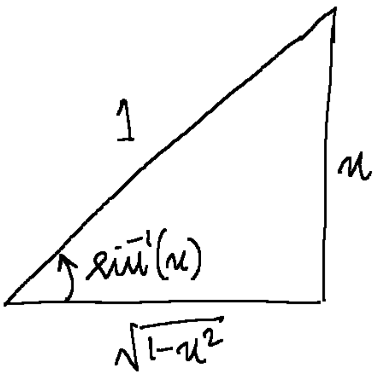


$\cos^{-1}(x) = \arccos(x)$
where Domain
Restricted to
$D: x \in [0, 2\pi]$



$\tan^{-1} = \arctan x$
where D is
restricted
$D: x \in [\frac{-\pi}{2}, \frac{\pi}{2}]$

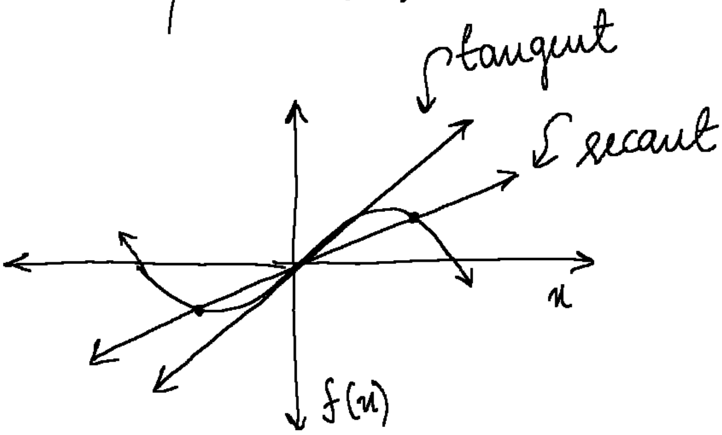
Ex 9: $\tan(\sin^{-1}(x))$



$1^2 = x^2 + b^2$ where
$b = \sqrt{1-x^2}$
then $\tan\theta = \frac{x}{\sqrt{1-x^2}}$
then $\tan(\sin^{-1}(x))$

2.1 Tangent & the velocity problems:

Ex: The tangent problem:
$f(x) = \sin(x)$
Find the slope of tangent line at point: $(0,0)$



Slope of secant through $(0,0)$ and $(x, f(x))$

$$\frac{f(x) - f(0)}{x - 0} = \frac{\sin(x)}{x}$$

get the value closer and close to 0 to get the tanget line.

x	0	1	0.1
$\frac{\sin x}{x}$	0.45	0.84	0.998

the slope = 1