

Homework 4-2:

① $f(x) = x^3 - 3x^2 + 2x - 5$

continuous
differentiable

$$f(0) = f(2) = -5$$

$$f'(x) = 3x^2 - 6x + 2 = 0$$

$$x^2 - 2x + \frac{2}{3} = 0$$

$$x = 1 \pm \sqrt{\frac{1}{3}}$$

② Creps amount
differentiable

③ $f(x) = x\sqrt{x+5} = x(x+5)^{1/2}$

$$f'(x) = (x+5)^{1/2} + x(\frac{1}{2})(x+5)^{-1/2}$$

$$f'(x) = \sqrt{x+5} + \frac{x}{2\sqrt{x+5}}$$

MVT: $f(b) - f(a) \div b - a; [-5, 0]$

$$\frac{f(-5) - f(0)}{-5 - 0} = \boxed{0} \nearrow$$

$$f'(x) = \frac{3x+10}{2\sqrt{x+5}} = 0; x = -\frac{10}{3}$$

④ $f(x) = 2x^3 - 15x^2 - 36x + 8$

$$f'(x) = 6x^2 - 30x - 36$$

$$\frac{f(-4) - f(10)}{-4 - 10} = \frac{-216}{-14} = \boxed{26}$$

$$f'(x) = 26 = 6x^2 - 30x - 36$$

$$f'(x) = 6x^2 - 30x - 62$$

$$x = \frac{15 \pm \sqrt{597}}{6}$$

⑤ $f(x) = \frac{1}{x}; f'(x) = -\frac{1}{x^2}; [3, 11]$

$$\frac{f(3) - f(11)}{3 - 11} = \frac{-1}{22}$$

$$f'(x) = -\frac{1}{x^2} = \frac{1}{22} = x = \pm\sqrt{22}$$

but only $\sqrt{22}$ in int

⑥ $\frac{f(4) - f(0)}{4 - 0} = \text{Avg} = f'(c) = 8$

$$f(4) - 5 = 32$$

$$f(4) = 27; \text{ possible in } \textcircled{27}$$

⑦ $f(x) = 3 \sin x; |f'(x)| \leq 3$

then by MVT: $|f(a) - f(b)| \leq 3|a - b|$
 \forall all a and b

⑧ $[4, 6]; -4 \leq f'(x) \leq 3; \text{ estimate } f(6) - f(4)$

$$f'(x) \leq \frac{f(6) - f(4)}{6 - 4} \leq f'(x) = (-8, 6)$$

⑨ Points: $(0, 20); (1/6, 35)$

$$\frac{f(1/6) - f(0)}{1/6 - 0} = \frac{35 - 20}{1/6} = 90 \frac{\text{mi}}{\text{hr}^2}$$