1.2 Mathematical Models: A catalog of Functions

8/29/23

Mathematical Modelling:

IRL problem → Math Model → Conclusion → Predictions

Ex1:) Tide level in Half Moon Bay

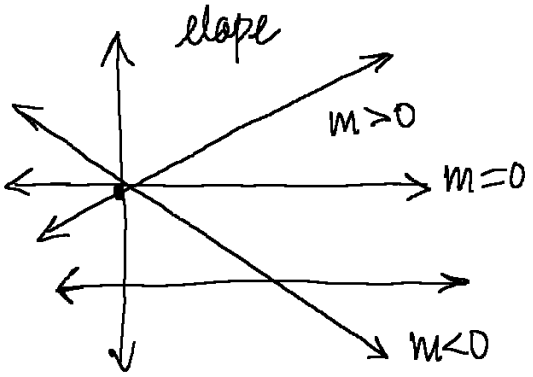
[assumptions are made off of sin(x) functions etc]

chances of undersampling are high ↑

Linear Models:

$f(x) = mx + b$ ← y-int

slope ↗



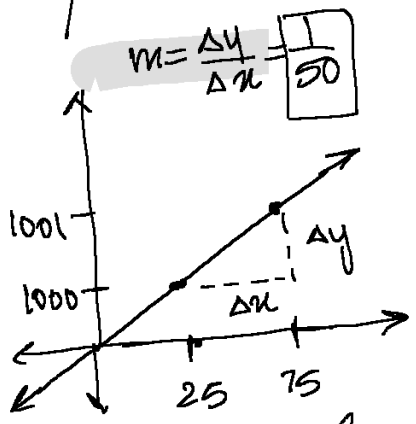
Ex:) Aluminum Rod

model: length depends linearly on temperature

x = temp(°C)	y = length(mm)
25	1000
75	1001

$m = \frac{\Delta y}{\Delta x} = \frac{1}{50}$

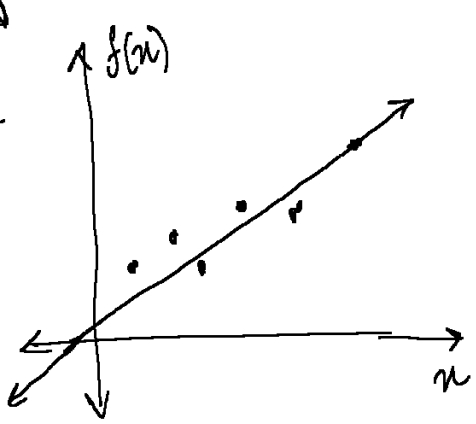
$f(x) = \frac{1}{50}x + 999.5$



Melting etc to be considered.

Note: Preferably more than 2 data points to have accuracy.

→ linear Regression model to get the line of best fit



Polynomials:

degree, coefficient

$$f(x) = a_n x^n + a_{n-1} x^{n-1} + \cdots + a_2 x^2 + a_1 x + a_0$$

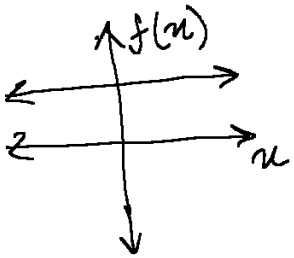
linear function

quadratic function

degree 0: $n=0$

$f(x) = a_0$

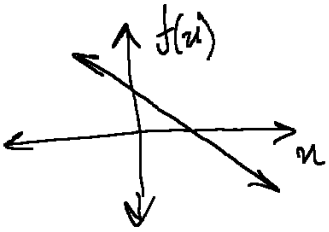
Ex) $f(x) = 7$



degree 1: $n=1$

$f(x) = a_1 x + a_0$

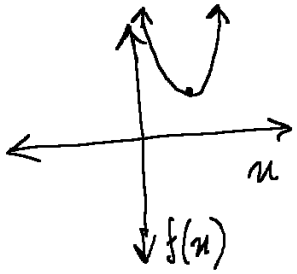
Ex): $f(x) = -3x + 5$



degree 2: $n=2$

$f(x) = a_2 x^2 + a_1 x + a_0$

Ex: $f(x) = (x-1)^2 - 1$



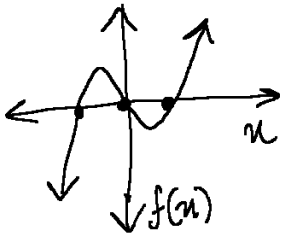
cubic func

degree 3: $n=3$

$f(x) = a_3 x^3 + a_2 x^2 + a_1 x + a_0$

Ex): $x(x-1)(x+1)$

$\Rightarrow x^3 - x$



. . . . and so on for the polynomial functions

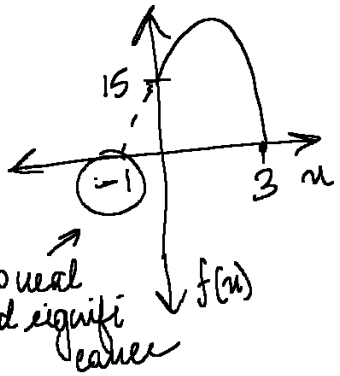
Ex: Height of a ball at time x:

$f(x) = -5x^2 + 10x + 15$

When does the ball hit $y=0$?

$$x_{y_2} = \frac{-10 \pm \sqrt{10^2 - 80(15)}}{-10} = (-1, 3)$$

Due to mathematical model



Ex: Solve: $x^3 - 7x + 6 = 0$

Find one solution: $x=1$ [hopefully given the question]

Then $x^3 - 7x + 6 / x - 1$

$$x-1 \overline{)\, x^3+0x^2-7x+6}$$

x^2+x-6

x^3-x^2 ↓

$-x^2-7x$

$-x^2+1x$ ↓

$-6x+6$

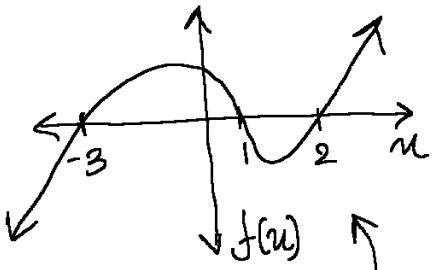
$-6x+6$

0

polynomial division

Polynomial division converts into quadratic function. $(x+2)(x-3)$

$x^3-7x+6=(x-1)(x^2-x-6)$

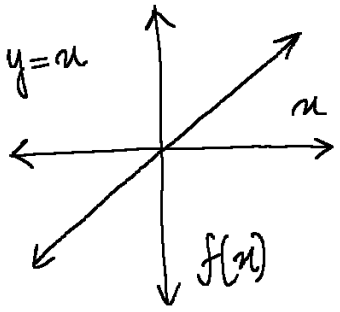


$f(x)=(x-1)(x+3)(x-2)$

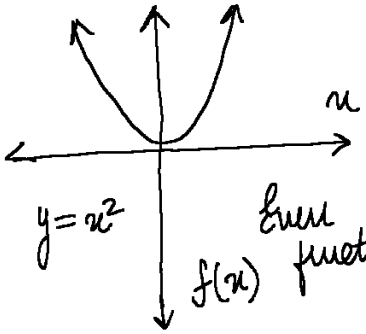
Power Functions: Exponent

$f(x)=x^a$

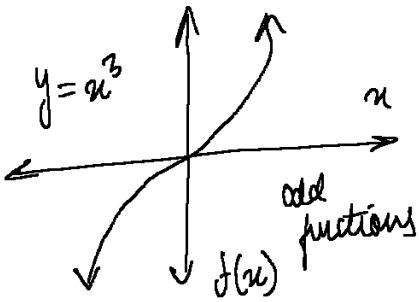
case $a=n$



case $=2$



case $=3$

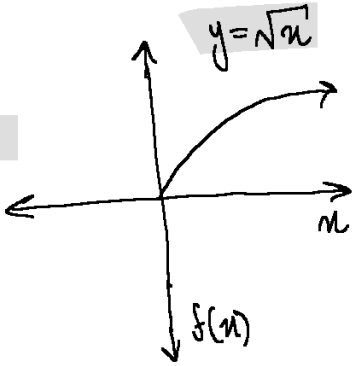


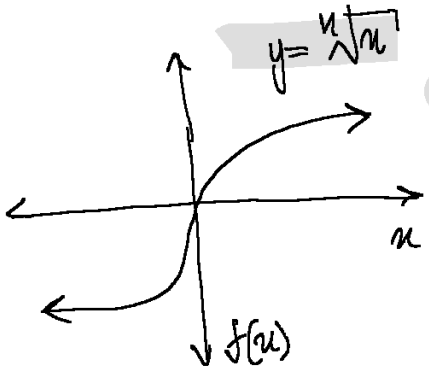
Power Rule:

$x^a \cdot x^b = x^{a+b}$; $x^a \div x^b = x^{a-b}$; $(x^a)^b = x^{a \cdot b}$; $x^0=1$

case $a=\frac{1}{n}$ where $f(x)=x^{1/n}=\sqrt[n]{x}$

if $n=$ even





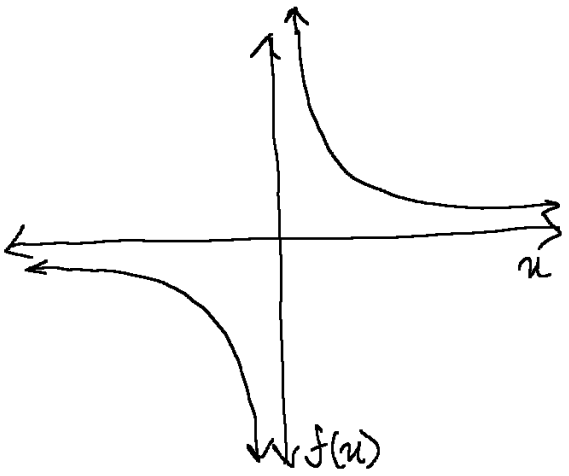
if $n=$ odd

Case $a=-1$: $f(x)=x^{-1}=\frac{1}{x}$

Domain: $\mathbb{R}\to\{0\}$

$x\in(-\infty,0)\cup(0,\infty)$

Models proportionality in the functions



Rational Functions

$f(x)=\frac{P(x)}{q(x)}$ ↔ polynomials where $q(x)$ domain is not equal to 0

Ex 6)

$f(x)=\frac{x^3+2x+10}{x^2-1}$ $\quad x^2-1\neq 0$, $x\neq\pm 1$

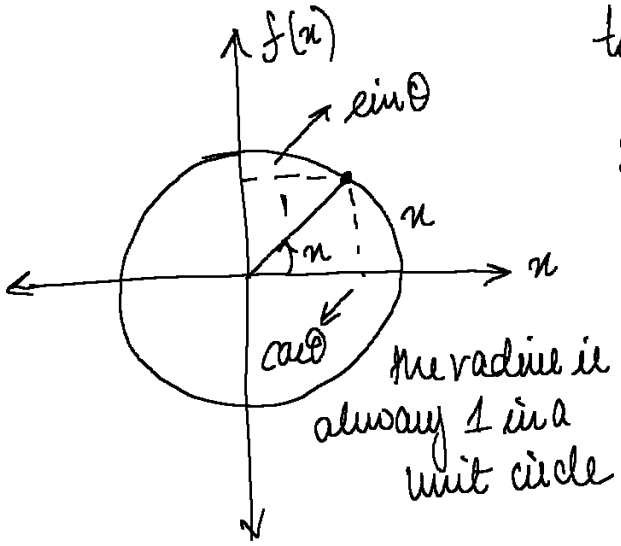
where the domain is $D: \mathbb{R}\setminus\{\pm 1\}$

$(-\infty,-1)\cup(-1,1)\cup(1,\infty)$

Algebraic Functions Anything consisting of the operations like $+,-,\div,\times,\sqrt[n]{}$ etc.

$f(x)=\frac{1+x^2-\sqrt{x}}{\sqrt{1+\sqrt{1+x}}}$ ..... they are real numbers

Trigonometric Functions:

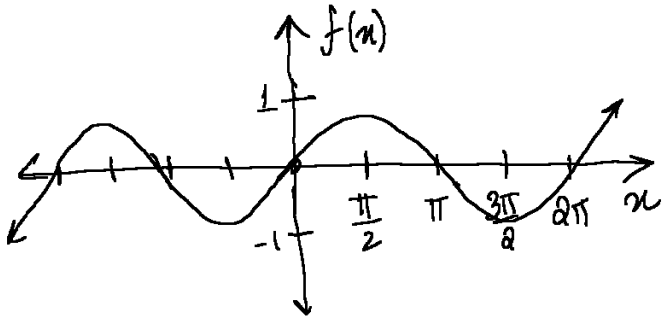


The radius is always 1 in a unit circle

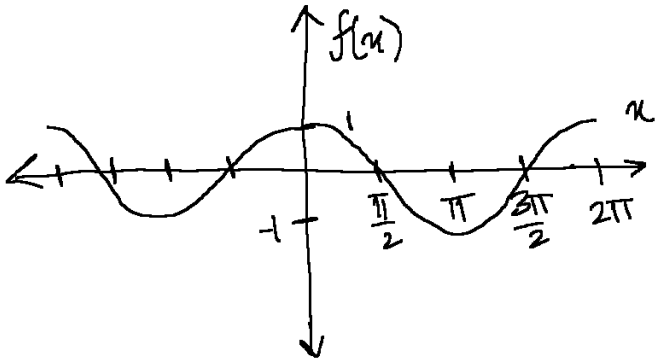
$\tan\theta = \frac{\sin\theta}{\cos\theta}$

Everything vertical is the cosθ while everything horizontal is sinθ.

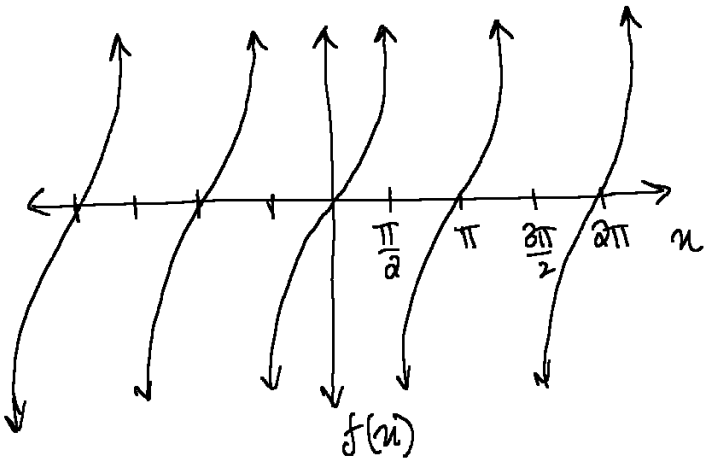
$f(x) = \sin\theta$



$f(x) = \cos\theta$



$f(x) = \tan\theta$



Important Functions:

$\sin(x+\pi) = -\sin(x)$

$\sin(x+2\pi) = \sin(x)$

$\sin(-x) = -\sin(x)$

$\cos(x+\pi) = -\cos(x)$

$\cos(x+2\pi) = \cos(x)$

$\cos(-x) = \cos(x)$

$\cos(x)^2 + \sin(x)^2 = 1$