8.8 Exponential growth and decay

Ex1: $f(t) = Ce^{kt}$

$f' = Cke^{kt} = kf(t)$

(global relation)

OR

Ex2: $y(t) = Ce^{kt}$

$\frac{dy}{dt} = ky$

(local relation)

Theorem:

All the solution of the differential equations such that $f'(t) = kf(t)$ are in the form $f(t) = Ce^{kt}$

Proof: assuming $f(t) > 0$

$f'(t) = kf(t)$

$f'(t) \div f(t) = k$

$e^{\ln f = kt + b} = Ce^{kt}$

Ex3: $f'(t) = 5f(t)$; $f(2) = 3$

$f(t) = Ce^{5t}$; $3 = Ce^{10}$; $f(t) = \frac{3}{e^{10}}e^{5t}$

using the theorem stated above

General Solutions

$y = Ce^{kt}$ and $C \& k > 0$ = growth and $C \& k < 0$ exponential decay.

Ex3: $y(t) = \#$ cells at time

$y(0) = 2000$

$\frac{dy}{dt} = 0.01y$; local relation

$y(t) = Ce^{0.01t}$; $y(t) = 2000e^{0.01t}$

$2 = e^{0.01t}$; $100\ln(2) = t$

when there are 4000 cells.

Ex4: $y(t) = \#\ Pu^{241}$ atoms

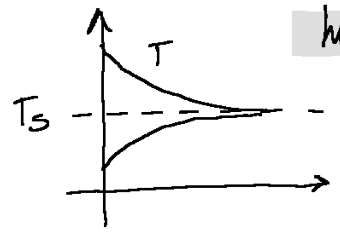
$\frac{dy}{dt} = -0.048y$: local relation

$y(t) = Ce^{-0.048t}$; $\frac{1}{2} = e^{-0.048t}$

$\frac{\ln(\frac{1}{2})}{-0.048} = t$: ie when Pu^{241} is at is half life.

Newtons law of Cooling

Intuition



$T(t)$ = temperature at t

T_S = temp at surrounding

The rate of cooling is proportional to the temperature difference b/w the object and its surroundings.

$$\frac{dT}{dt} = -k(T - T_S)$$

After integration you get: $T(t) = Ce^{-kt} + T_S$ global Rule

Ex5 Temperature:

$T_0 = 80°C$

room $T = 16°C$

$T(10) = 20°C$

then T after 15 mins

$T(t) = Ce^{-kt} + 16°C$

$80 = C + 16 ; C = 64$

$T(t) = 64e^{-kt} + 16°C$

$20°C = 64e^{-k(10)} + 16°C$

$\ln(1/16) = -k(10)$

$k = \ln(16)$ then calc

$T(1/4) = 64e^{-k(1/4)} + 16 = 48$

use calculator please (mental math)

Continuously Compounded interest:

Current balance: \$100

interest rate : 20%

annually: $\$120 = \$100(1+0.20) = \$120$

bi-annual: \$ $= \$100(1+0.2/2)^2$

quarterly: \$ $= \$100(1+0.2/4)^4$

continuous: $\lim_{n\to\infty} 100\left(1+\frac{0.2}{n}\right)^n = 100e^{0.2}$

A_0 = principle or initial

r = interest rate

n = compounded times

$A_0\left(1+\frac{r}{n}\right)^t : \lim_{n\to\infty} f(x) = A_0 e^{rt}$

compounded continuously