4.1 Maximum and Minimum Values

Definition:

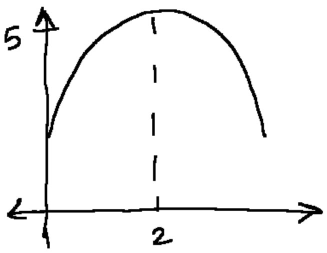
Suppose that c is a number in the function $f(c)$ is called:

→ absolute max of f: if $f(c) \geq f(x)$ $\forall$ all $\mathbb{R}$ in D

→ absolute min of f: if $f(c) \leq f(x)$ $\forall$ all R in D

Extremum: is a min and max

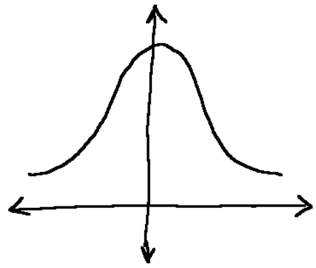
Ex1: $f(x) = 5-(x-2)^2$



abs max: 5
at $x=2$

abs min: None
at $x=$ DNE

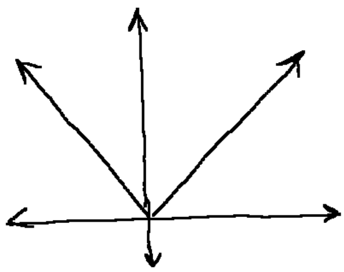
Ex2: $f(x) = 1/1+x^2$



abs max: 1
at $x=0$

abs min: None
at $x=$ DNE

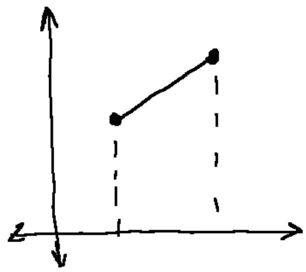
Ex3: $f(x)=|x|$



abs min: 0
at $x=0$

abs max: None
at $x=$ DNE

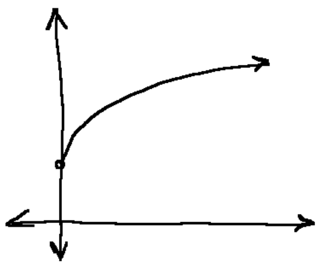
Ex4: $f(x)=x+1$: $D \, \forall x \in [1,3]$ if $(1,3)$ ↓ DNE $\forall$ all!



abs min: 2
at $x=1$

abs max: 4
at $x=3$

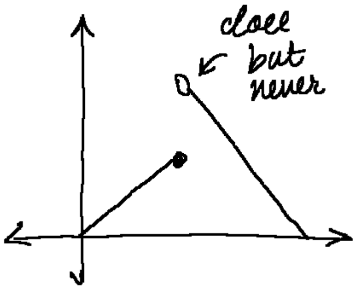
Ex5: $f(x)=\sqrt{x}+1$



abs min: 0
at $x=0$

abs max: None
at $x=$ DNE

Ex6: $f(x) = \begin{cases} x & x \leq 1 \\ 3-x & x > 1 \end{cases}$



abs max: DNE
at $x=$ DNE

abs min: None
at $x=$ DNE

Extreme Value theorem:

if f is continuous on a closed interval of the form $[a,b]$, then f must have an absolute max and min.

Definition

suppose that c is a number in the D of the function where c is not an endpoint then:

→ local max: $f(c) \geq f(x)$ ∀ all $x \approx c$ in D of f

→ local min: $f(c) \leq f(x)$ ∀ all $x \approx c$ in D of f

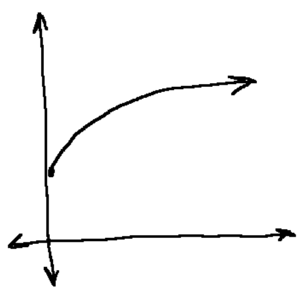
Convention: Endpoints are ! local extremums.

Fermat's theorem

if f has a local maximum or minimum at c, then

$f'(c) = 0$ OR f is ! differentiable at c

Ex 7: $f(x) = \sqrt{x+1}$



local max: None
at x = DNE
local min: None
at x = DNE

Ex 8: $f(x) = x^3$ something

Definition:

A number c in the interior of the domain f is called the critical number if $f'(c) = 0$ or is ! differentiable at c.

Fermat's theorem Restated:

If f has a local extremum at c (in the interior of the domain) then c is a critical # of f.