Homework 2-5

① look at graph and find discontinuity

$\lim_{x \to a} f(x) = f(a)$ ie $-3, \pm 2, 4$

② Same question:

$x \in [-2,-1) \cup (-1,0) \cup (0,1) \cup (1,4]$

③

$$f(x) = \begin{cases} -2x & x<1 \\ 1 & x=1 \\ 2x & x>1 \end{cases}$$

$\lim_{x \to 1^-} f(x) = -2$; $\lim_{x \to 1^+} f(x) = 2$

$\lim_{x \to 1} f(x) = DNE$ & $f(1)=1$

then limit discontinuous

④ $f(x) = \dfrac{(x+2)\cancel{(x-2)}}{(x-4)\cancel{(x-2)}}$

discontinuous at 2 and 4

2 is whole & 4 is VA

$x \in (-\infty,2) \cup (2,4) \cup (4,\infty)$

⑤ $f(x) = \sqrt{x-5}$

analyze the graph to get

$x \in [5,\infty)$

⑥ $f(x) = \dfrac{x^2+4}{(2-x)(2+x)}$

$\lim_{x \to -2^-} f(x) = -\infty$ & $\lim_{x \to -2^+} f(x) = \infty$

$\lim_{x \to 2^-} f(x) = \infty$ & $\lim_{x \to 2^+} f(x) = -\infty$

these are the discontinuous parts of the eq.

⑦ $f(x) = \dfrac{4x-4}{x^2(x^2-10x+25)}$

$\Rightarrow \dfrac{4(x-1)}{x^2(x-5)^2}$ where point at 0 and 5

$\lim_{x \to 0^-} f(x) = -\infty$ & $\lim_{x \to 0^+} f(x) = -\infty$

$\lim_{x \to 5^-} f(x) = \infty$ & $\lim_{x \to 5^+} f(x) = \infty$

these are the discontinuous parts of the eq.

⑧ $f(x) = \begin{cases} x^2+12x+40 & x<-6 \\ -x^2-12x-32 & x>-6 \end{cases}$

what should $x=-6$ be so that it is continuous

$\lim_{x \to -6^+} f(x) = 4$ & $\lim_{x \to -6^-} f(x) = 4$

then $f(-6) = 4$ to be cont.

⑨ Same as above: $f(x) = \dfrac{2x^2+5x-33}{x-3}$

$f(x) = \dfrac{(2x+11)(x-3)}{(x-3)}$

Continued

(9) $f(x) = 2x + 11$

$\lim_{x \to 3} f(x) = 6 + 11 = 17$

then to make it continuous $f(3) = 17$

(10) $f(x) = \begin{cases} x^2 - c^2 & x < 8 \\ cx + 80 & x \geq 8 \end{cases}$

$64 - c^2 = 8c + 80$ then

$c^2 + 8c + 80 - 64 = 0$

$c^2 + 8c + 16 = (c+4)^2 = 0$

$c = -4$ to be continuous

(11) $f(x) = \begin{cases} 2x & x < 1 \\ cx^2 + d & 1 \leq x < 2 \\ 7x & x \geq 2 \end{cases}$

Two equations:

$2 = c + d$ & $14 = 4c + d$

$d = 2 - c$; $14 = 4c + 2 - c$

$d = 2 - c$; $12 = 3c$; $c = 4$; $d = -2$

to be continuous

(12) $f(x) = \begin{cases} x^2 - 1 & x \leq c \\ 6x - 10 & x > c \end{cases}$

$x^2 - 1 = 6x - 10$; $x^2 - 6x + 9$

then $(x-3)^2$, then $c = 3$

(13) $\lim_{x \to 1} e^{x^2 - 5x + 4} = e^0 = 1$

Used the method of substitution.

(14) $\lim_{x \to \pi} \sin(6x + \sin(7x))$

$\lim_{x \to \pi} \sin(6x + 0) = \sin(6\pi) = 0$

Used the substitution

(15) $\lim_{x \to 5^-} \sin^{-1}\left(\frac{x}{5}\right) = \sin^{-1}(1)$

valid point then $\frac{\pi}{2}$

(16) $\lim_{x \to 1} [2f(x) + f(x)g(x)] = 20$

$2f(1) + f(1)g(1) = 20$; $g(1) = 3$

$2f(1) + 3f(1) = 20$; $f(1) = 4$

(17) Using the IVT, there must be a root between $x \in (-1, 0)$