

Homework 4.5:

① Random True and false question: logical

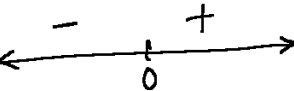
② $f(x) = 7x^3 + 7x$

$f'(x) = 21x^2 + 7$; then None & criticals

increasing: $(-\infty, \infty)$

decrease: $x \in \emptyset$

local min & max: None

$f''(x) = 42x$; 

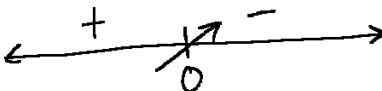
concave up: $(0, \infty)$

concave down: $(-\infty, 0)$

inflection: (1)

③ $f(x) = e^{-25x^2}$

$f'(x) = e^{-25x^2}(-5x) = 0$



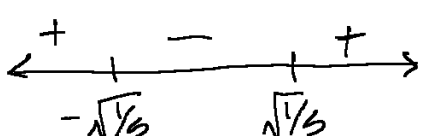
increase: $(-\infty, 0)$

decrease: $(0, \infty)$

local max: 5; min: None

$f''(x) = -5e^{-25x^2} + 25x^2 e^{-25x^2} = 0$

$f''(x) = (25x^2 - 5) = \pm\sqrt{1/5}$

 inflection point

concave up: $(-\infty, \sqrt{1/5}) \cup (\sqrt{1/5}, \infty)$

concave down: $(-\sqrt{1/5}, \sqrt{1/5})$

④ $f(x) = 5x(x^2+4)^{-1}$; $f'(x) = 5(x^2+4)^{-1} - 5x(x^2+4)^{-2}(2x)$

$f'(x) = (-5x^2-20)/(x^2+4)^2$; CP = None

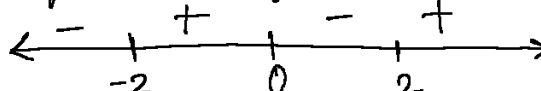
increase: none w/ VA: (± 2)

decrease: $(-\infty, -2) \cup (-2, 2) \cup (2, \infty)$

local min & max: None

$f''(x) = \frac{-10x(x^2+4)^{-2} - (20x^2+80x)(x^2+4)^{-3}}{(x^2+4)^4}$

simplify w/ inflection = 0 & ± 2 DNE



concave up: $(-2, 0) \cup (2, \infty)$

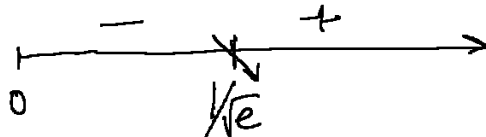
down: $(-\infty, -2) \cup (0, 2)$

$\lim_{x \rightarrow \infty} \frac{5x}{x^2+4} = 0 = \text{horizontal asymptote}$

$x^2+4=0$; (± 2) Vertical Asymptote

⑤ $f(x) = 3x^2 \ln(x)$; $f' = 6x \ln(x) + 3x$

$6x \ln(x) + 3x = 0$; $x = e^{-1/2} \neq 0$ PNE

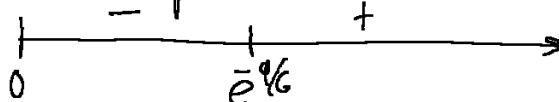


increase: $(1/e, \infty)$; dec: $(0, 1/e)$

local min: $1/e$; max: None

$f''(x) = 6 \ln(x) + 9$; $f''(x) = \frac{-9}{6} = \ln(x)$

inflection P = $e^{-9/6}$



concave up: $(e^{-9/6}, \infty)$; down: $(0, e^{-9/6})$

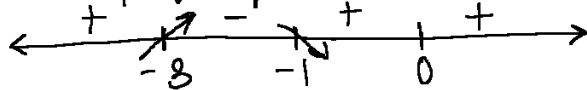
HW 4-6: (continued):

⑥ $f(x) = x^{1/3}(x+3)^{2/3}$

$$f'(x) = \frac{1}{3}x^{-2/3}(x+3)^{2/3} + x^{1/3}\frac{2}{3}(x+3)^{-1/3}$$

$$\frac{(x+3)^{2/3}}{3x^{2/3}} = -\frac{2x^{1/3}}{3(x+3)^{1/3}}$$

simplify equation: $x = -1$



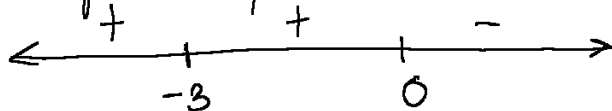
increasing: $(-\infty, -3) \cup (-1, \infty)$

decreasing: $(-3, -1)$

local max: -3 & local min: -1

$$f''(x) = \frac{-2(x+3)^{2/3}}{9x^{5/3}} - \frac{2x^{1/3}}{9(x+3)^{4/3}} + \frac{4}{9x^{2/3}(x+3)^{3/3}}$$

inflection point: 0 (sign change)



concave up then: $(-\infty, -3) \cup (-3, 0)$

concave down: $(0, \infty)$