2.4 The Precise definition of a limit:

$\lim_{x \to a} f(x) = L$ means $f(x) = L$ if $x \approx a$

Precision can be as good as we want if x is chosen sufficiently close to x; $x \neq a$

$|f(x) - L|$ can be made as small as we want as long as $|x-a|$ is sufficiently to 0 but $\neq 0$

Precision of $f(x) \approx L$: $|f(x) - L|$

Closeness of $x \approx a$: $|x-a|$

Definition

We say that $\lim_{x \to a} f(x) = L$ if:

For every $\varepsilon > 0$

there is a # $\delta > 0$ such that

if $0 < \underbrace{|x-a|}_{\text{closeness}} < \delta$, then $\underbrace{|f(x) - L|}_{\text{Precision}} < \varepsilon$

Ex 1: $\lim_{x \to 1} f(x) = 12$; $f(x) = 10x + 2$; $f(1) = 12$

then $0 < |x-1| < \delta$, then $|f(x) - 12| < \varepsilon$

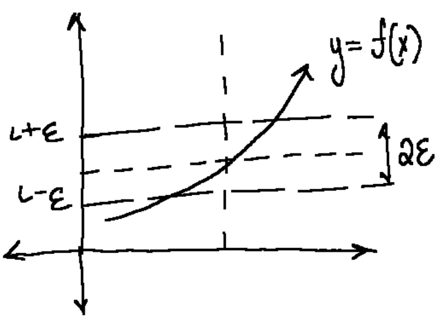
then $x = 0.01$, smallest value

Ex 2: $\lim_{x \to 0} f(x) = 0$; $f(x) = \sin\left(\frac{\pi}{x}\right)$;

then $0 < |x-0| < \delta$, then $|f(x)| < \varepsilon$

then $x = 0.08$. smallest Value

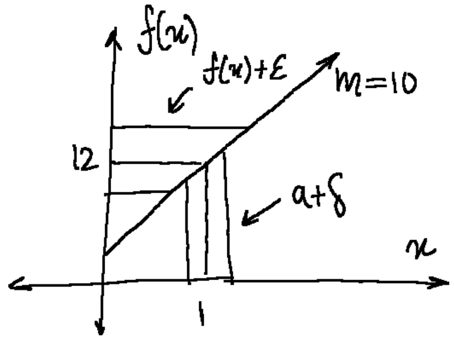
Ex 3:



please look at a video explaining the same concept

Ex3: $\lim_{x\to 1} f(x) = 12$; $f(x) = 10x + 12$

How to choose δ ∀ given ε?



what we want is, the vertical lines should always be within the horizontal lines:

largest: $|f(x)-12| < \varepsilon$

then $|10x+10| \Rightarrow |10(x-10)| \Rightarrow |x-1| < \frac{\varepsilon}{10} = \delta$

Proof that $\lim_{x\to 1} f(x) = 12$:

Given $\varepsilon > 0$, let $\delta = \frac{\varepsilon}{10} > 0$. Then:

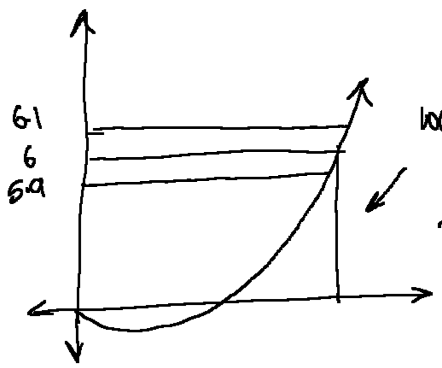
if $0 < |x-1| < \delta$, then $|f(x) - 12| \ldots 10|x-1| < 10\frac{\varepsilon}{10} = \varepsilon$

Ex4: $\lim_{x\to 0} f(x) \neq 0$: $f(x) = \sin(\frac{\pi}{x})$

How to choose ε?

Ex5: $f(x) = x^2 - x$; Find the largest $\delta > 0$ such that:

$0 < |x-3| < \delta$ then $|f(x) - 6| < 0.1 = \varepsilon$



want this to be b/w that range

$x^2 - x = f(x) = 5.9$

$x^2 - x = f(x) = 6.1$

$x_1 = 2.979\ldots$ & $x_2 = 3.019$

$\boxed{\text{Domain}}$

if $2.979 < x < 3.019$, then $5.9 < f(x) < 6.1$ or $|f(x) - 6| < 0.1$

$|3 - 2.979| \approx 0.21$

$|3 - 3.019| \approx 0.19 = \delta$ ✓ smallest distance

Ex6: Real world intuition:

6 metal square should be 10cm width
to get $1000cm^3 = 1l$ with vol tolerance of $\pm 1 cm^3$

x = width $\quad f(10) = 1000$

$f(x)$ = volume = x^3

Want:

$$|f(x) - 1000| < 1$$

$$999 < f(x) < 1001$$

$$9.9966 < x < 10.0033$$

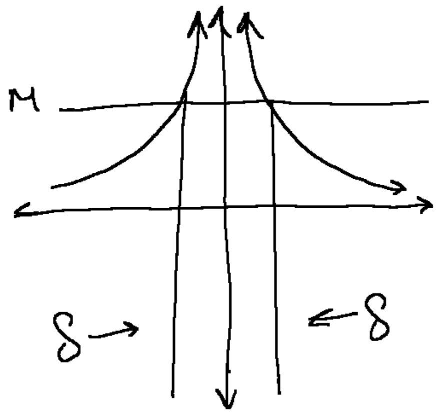
Then we want the supplier to provide a with an accuracy of

$10 {}^{+0.003322\ldots}_{-0.003344\ldots}$ smallest

10 ± 0.003

Definition:

We say that $\lim_{x \to a} f(x) = \infty$ if:
For every where there is a # $\delta > 0$ such that
$0 < |x - a| < \delta$, then $f(x) > M$



want to do the same drit