Homework 3-5:

① $f: 5\sin x + 2\cos y - 5\sin x\cos y + x = 7\pi$

$f': 5\cos x - 2\sin y\frac{dy}{dx} - 5\cos x\cos y + 5\sin x\sin y\frac{dy}{dx} + 1 = 0$

$5\sin x\sin y\frac{dy}{dx} - 2\sin y\frac{dy}{dx} = 5\cos x\cos y - 5\cos x - 1$

$\frac{dy}{dx} = \frac{5\cos x\cos y - 5\cos x - 1}{5\sin x\sin y - 2\sin y} = \frac{4}{-(-2)} = 2$

② $5x(f(x))^5 + x^4 f(x) = 66;\ f(-3) = 1;\ f'(-3) = ?$

$5f^5 + 5x(5f^4)\frac{df}{dx} + 4x^3 f + x^4\frac{df}{dx} = 0$

$\frac{df}{dx} = \frac{-5f^5 - 4x^3 f}{5x(5f^4) + x^4} = \frac{103}{6}$ plug in Vals

③ $x^4 + 2xy + y^2 = 9$ at $(1,2)$

$4x^3 + 2y + 2x\frac{dy}{dx} + 2y\frac{dy}{dx} = 0$

$\frac{dy}{dx} = \frac{-2y - 4x^3}{2x + 2y} = \frac{-4}{3}$

④ $V = \frac{4}{3}\pi r^3;\ \frac{dV}{dt} = 4\pi r^2\frac{dr}{dt}$

$\frac{dr}{dt} = 3;\ r = 3;\ \frac{dV}{dt} = 4\pi(9)(3)$

then $\frac{dV}{dt} = 108\pi$

⑤ $f(x) = 2x^4\tan^{-1}(8x^4)$

$\frac{df}{dx} = 8x^3\tan^{-1}(8x^4) + \frac{2x^4(32x^3)}{1+64x^8}$

$f'(x) = 8x^3\tan^{-1}(8x^4) + 64x^5(1+64x^8)^{-1}$

⑥ $\ln(2y) = 3xy$

$\frac{dy}{dx} = \frac{3y}{1/y - 3x};\ \frac{d^2y}{dx^2} = \frac{3(\frac{1}{y} - 3x)\frac{dy}{dx} - 3y(\frac{-1}{y^2}\frac{dy}{dx} - 3)}{(\frac{1}{y} - 3x)^2}$

$0 = \frac{d^2y}{dx^2} = 9y - 3y\left(\frac{-1}{y^2}\left(\frac{1}{y} - 3x\right) - 3\right);\ \frac{3y}{\frac{1}{y} - 3x} = \frac{1}{2y} = x$

then $\frac{d^2y}{dx^2} = 0$ at $\left(\frac{1}{e^{3/2}}, \frac{e^{3/2}}{2}\right)$ ↗

3-5 Continued

(a) $f(x) = 4x + 10x^{15}$; $c = -14$

$f(-1) = -14$ then $f^{-1}(-14) = -1$

$f'(x) = 4 + 150x^{14}$; $f'(-1) = 154$

then $f^{-1}(c)' = \boxed{1/154}$

(b) $f(x) = x^2 - 11x + 32$; $c = 8$

$x^2 - 11x + 32 = 8$

$x(x-8) - 3(x-8) : (x-8)(x-3)$

$[5.5, \infty)$ then (8)

$f(8) = 8$ then $f^{-1}(8) = 8$

$f'(x) = 2x - 11$; $f'(8) = 5$

then $f^{-1}(c)' = \boxed{1/5}$