3.9 Related Rates:

Ex1: $V = \frac{4}{3}\pi r^3$

V(t) = Volume at t

R(t) = Radius at t

$V(t) = \frac{4}{3}\pi r(t)^3$

$\frac{dV}{dt} = 4\pi r(t)^2 \frac{dr}{dt}$

$3 = 4\pi r(2)^2 \frac{dr}{dt}$ then

$\frac{dr}{dt} = \frac{3}{16\pi}$ units

Ex2: $x^2 + y^2 = z^2$

$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 2z\frac{dz}{dt}$

$x\frac{dx}{dt} + y\frac{dy}{dt} = z\frac{dz}{dt}$

$x = 8;\ y = 4;\ x' = 20;\ y' = -40$

$z = \sqrt{3^2 + 4^2} = 5$

$\frac{dz}{dt} = \frac{3}{5}(20) + \frac{4}{5}(-40) = -44$

Ex3: $x^3 + 2y^3 - 5xy = 0$

$(x,y) = (2,1);\ x' = 4$

$3x^2x' + 6y^2 y' - 5x'y - 5xy'$

$3(2)^2(4) + 6(1)^2 y' - 5(4) - 5(2)y'$

$y'(6-10) = 20 - 48$

$\frac{dy}{dt} = \frac{20-48}{6-10} = \frac{-28}{-4} = 7$

$y' = 7$ m/s speed

Ex4: $x^2 + y^2 = z^2$; $y = 6$ always

$z = 10m;\ \frac{dz}{dt} = -4 m/s;\ \frac{dx}{dt} = ?;\ y = 6$

$2x\frac{dx}{dt} + 0 = 2z\frac{dz}{dt}$; 6 constant always

$x = \sqrt{100-36} = 8;\ \frac{dx}{dt} = \frac{-40}{8} = \boxed{-5 \frac{m}{s}}$

3.10 linear approximations

Ex1: $f(x) = \sqrt{x}$; tangent at 4

$\frac{dy}{dx} = \frac{1}{2\sqrt{x}};\ \frac{1}{2(2)} = 1/4$

Points are x,y $\Rightarrow$ (4,2)

$2 = 1/4(4) + b;\ y(x) = 1/4 x + 1$

Ex2: Approximate $\sqrt{4.1}$

Use the tangent line because 4 is close the 4.1 then $f(4.1) = \sqrt{4.1}$

then $f(4.1) = 2.025$

$\sqrt{4.1} = 0.0248 \approx 2.025$; close

linear approximation of f at a:

$$y = \underbrace{f'(a)}_{\text{sensitivity}}(x-a) + f(a)$$

Ex3: Approximate $(0.98)^3$

$f(x) = x^3;\ f(1) = 1;\ f'(x) = 3x^2$

$f'(1) = 3;\ y = 3(x-1) + 1$

$y = 3x - 2;\ y = 3(0.98) - 2 \approx 0.94$

Ex4: Approximate $\sin(1)$

$\frac{\pi}{3} \approx 1$; $\sin\left(\frac{\pi}{3}\right) = \frac{\sqrt{3}}{2}$; $\cos\left(\frac{\pi}{3}\right) = 1/2$

$f(x) = \sin x$; $f'(x) = \cos x$

$f(\pi/3) = \frac{\sqrt{3}}{2}$; $f'(\pi/3) = 1/2$; $f(1) \approx ?$

$L(x) = \frac{1}{2}\left(x - \frac{\pi}{3}\right) + \frac{\sqrt{3}}{2}$ then

$L(1) \approx \sin(1)$ then true