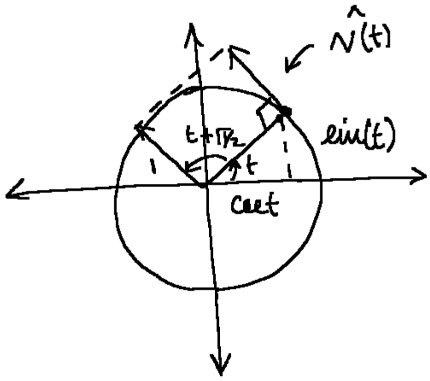
3.3 Derivative of Trig functions:



Position Vector: $\begin{bmatrix} x(t) \\ y(t) \end{bmatrix} = \begin{bmatrix} \cos(t) \\ \sin(t) \end{bmatrix}$ $\Big\rangle \frac{d}{dx}$

Velocity Vector: $\begin{bmatrix} x'(t) \\ y'(t) \end{bmatrix} = \begin{bmatrix} -\sin(t) \\ \cos(t) \end{bmatrix}$

Because: $x't = \cos(t+\pi/2) = -\sin(t)$

$y't = \sin(t+\pi/2) = \cos(t)$

Definition: look TB & precise definition

The derivative of $\frac{d}{dx}(\sin\theta) = \cos\theta$ and $\frac{d}{dx}(\tan\theta) = \sec^2\theta$ & derivative and derivative of $\frac{d}{dx}(\cos\theta) = -\sin\theta$

Ex2: $f(x) = x^3\cos(x)$

$f'(x) = 3x^2\cos(x) - x^3\sin(x)$

$\frac{dy}{dx} = 3x^2\cos x - x^3\sin(x)$

Ex3: $\lim_{x\to 0} \frac{\sin x}{x} = 1$?

$\lim_{x\to 0} \frac{f(x)-f(a)}{x-a} = f'(a)$: $a=0$, $f(x)=\sin(x)$

then $f' = \cos(0) = 1$

Ex3: $f(x) = \tan x$

$\frac{dy}{dx} = \frac{\sin x}{\cos x} = \frac{\cos^2 x + \sin^2 x}{\cos^2 x}$

then $\tan x = \sec^2 x$

Ex5: $f(x) = \sin^2\theta + \cos^2\theta$

where $f(x) = 1$ then

$\frac{dy}{dx}$ of $1 = 0$ & $\sin^2\theta + \cos^2\theta$

Ex6: $f(t) = 5\sin(t)$ & displace

v: $f'(t) = 5\cos(t)$ and

$\vec{a}$: $f''(t) = -5\sin(t)$ neg $\because$

Newton laws:

Acceleration ~ Velocity

~ -displacement.

Ex7: $f^7(t) = \sin(t) = ?$

$f^7(t) = -\cos(t)$

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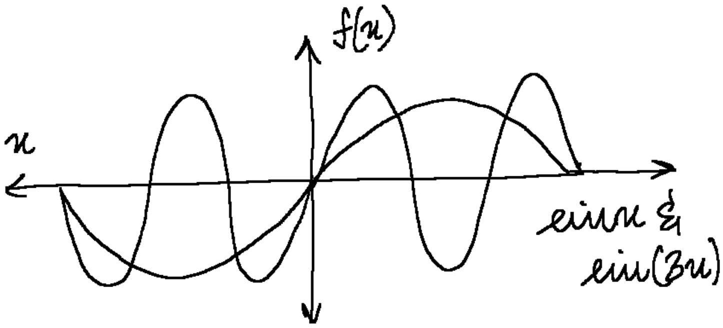
3.4 Chain Rule:

Ex1: $f(x) = \sin(3x)$

$$f'(x) = \lim_{x\to 0} \frac{\sin(3x) - \sin(3\cdot 0)}{x - 0} \Rightarrow \lim_{x\to 0} \frac{\sin(3x)\,3}{x\cdot 3} \Rightarrow 3\lim_{x\to 0} \frac{\sin(3x)}{3x}$$

then $u = 3x$ and $f'(u) = 1$ then $f'(0) = 3$

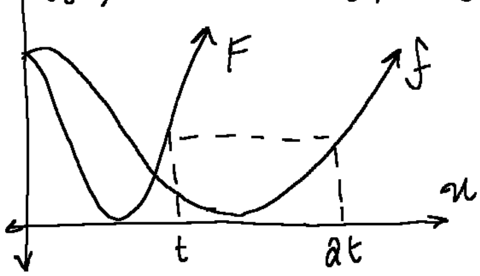


multiply the factor on the outside as the outside graph is squeezed on the inside

Ex2: Position of car 1: $f(t) = (t+1)^2$

car 2: $f(t) = f(2t)$

$F'(t) = 2f'(2t)$



Chain Rule (Simple)

if $F(x) = f(cx)$, then

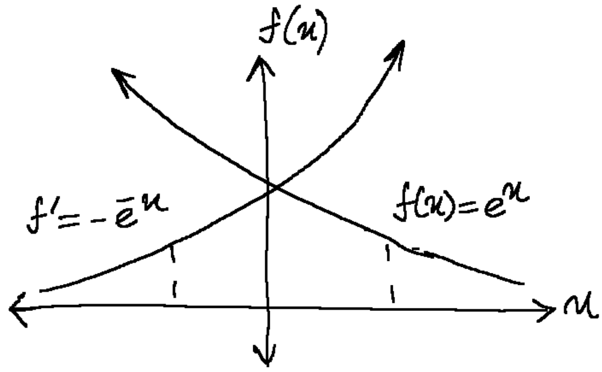
$$F'(x) = cf'(cx)$$

Reflects back on the position of the cars.

Ex3: $f(x) = e^x$

$F(x) = f(-x) = f(-1x) = e^{-x}$

$F'(x) = (-1)f'(-x) = -e^{-x}$



Ex4: $h(t)$ = height at t (m)

$T(h)$ = temp at height (°C)

$h(60) = 200\,m$ & $T(200) = 10°C$

$\frac{dh}{dt}(60) = 5\frac{m}{s}$ $\quad \frac{dT}{dt}(200) = -0.3\frac{°C}{m}$

$\frac{°C}{s}$

$$\frac{d}{dt}(T(h(t))) = \frac{\Delta T}{\Delta t} = \frac{\Delta T}{\Delta h}\cdot\frac{\Delta h}{\Delta t} = \frac{dT}{dt} = 5(-0.3)$$

Chain Rule (Full Version):

if $F(x) = f(g(x))$ then

$$F'(x) = f'(g(x)) \cdot g'(x)$$

In other words, if $y = f(u)$ and $u = g(x)$ then $y = F(x)$

$$F'(x) = \frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx} \text{ where } \frac{dy}{du} = f' \text{ \& } \frac{du}{dx} = g'$$

Ex6: $f(t) = \tan(x^3)$

$3x^2 \sec^2(x^3) = f'(x)$

Ex7: $f(t) = (1-x^2)^{1/2}$

$f'(t) = \frac{1}{2}(1-x^2)^{-1/2} \cdot (-2x) = \frac{-x}{\sqrt{1-x^2}}$

Power Rule Combined w/ chain Rule:

$$\frac{d}{dx}(g(x))^n = n(g(x))^{n-1} \cdot g'(x) : \text{Same pattern}$$

Ex8: $f(t) = (x^2+1)^3$

$3(x^2+1)^2(2x) = 6x(x^2+1)^2$

Ex9: $f(t) = (1+e^x)^{-1}$

$f'(t) = -1(1+e^x)^{-2} e^x = \frac{-e^x}{(1+e^x)^2}$

Ex10: $f(t) = 2^x$

$\frac{d}{dx} 2^x = \frac{d}{dx} = e^{\ln(2)x} = \ln(2) \cdot 2^x$

Exponential Func:

$f(t) = b^x$; $f'(t) = b^x \ln(b)$

Ex11: $\frac{d}{dx}(\sin(\cos x^2)) = \cos(\cos x^2) \cdot \frac{d}{dx}(\cos(x^2))$

$\to \cos(\cos x^2)\, 2x \sin(x^2)$

Chain Rule + 3 func:

$$\frac{d}{dx} f(g(h(x))) = f'(gh(x)) \cdot \frac{d}{dx}(g(hx))$$

$$\Rightarrow f'(gh) \cdot g'(h)\, h'(x)$$

Chain Rule ∀ 3 functions:

if $y = f(u)$; $u = g(x)$; $x = h(t)$ then $y = f(g(h))$

$$\frac{dy}{dt} = \frac{dy}{du} \cdot \frac{du}{\cancel{dx}} \cdot \frac{dx}{dt} = \frac{dy}{dt}$$