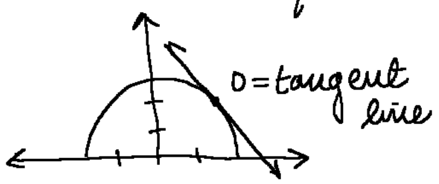
3.5 Implicit Differentiation:

Ex1: $x^2+y^2=5^2$ at point $(3,4)$
slope at ↗



$y^2=25-x^2$; $y=\sqrt{25-x^2}$

$\frac{dy}{dx}=\frac{1}{2}(25-x^2)^{-1/2}\cdot 2x=\frac{-x}{\sqrt{25-x^2}}$

$\frac{dy}{dx}\Big|_3=\frac{-3}{\sqrt{16}}=\boxed{\frac{-3}{4}}$

Implicit differentiation:

$x^2+y^2=5^2$

$2x+2y\frac{dy}{dx}=0$

$\frac{dy}{dx}2y=-2x$; $\frac{dy}{dx}=\frac{-2x}{2y}$

$\frac{dy}{dx}=\boxed{\frac{-x}{y}}$

∀ points $(3,4)$ slope of tangent is $\boxed{-3/4}$

Ex2: $x^3+2y^3-5xy=0$: $(2,1)$

$3x^2+6y^2\cdot\frac{dy}{dx}-\left(5y+5x\frac{dy}{dx}\right)=0$

$3x^2+6y^2\frac{dy}{dx}-5y+5x\frac{dy}{dx}=0$

$\frac{dy}{dx}=(5y-3x^2)\div(5x+6y^2)$

$\frac{dy}{dx}=(5-3(4))\div(10+6)=\boxed{\frac{-7}{16}}$

Ex3: $x^4+y^4=17$; $(2,1)$

$4x^3+4y^3\cdot\frac{dy}{dx}=0$; $\frac{dy}{dx}=\frac{-4x^3}{4y^3}=\frac{-x^3}{y^3}$

$\frac{d^2y}{dx^2}=\frac{-3x^2(y^3)-3y^2(-x^3)\frac{dy}{dx}}{y^6}$

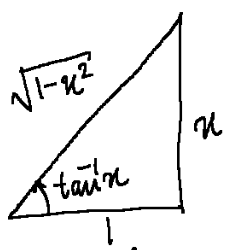
$\frac{dy}{dx}\Big|_{(2,1)}=\frac{-8}{1}=-8$ & $\frac{d^2y}{dx^2}=\boxed{180}$

Ex4: $\tan y=x$; $y=\tan^{-1}(x)$

$\sec^2 y\frac{dy}{dx}=1$

$\frac{dy}{dx}=\cos^2(y)$; $y=\tan^{-1}(x)$

$\frac{dy}{dx}=\cos^2(\tan^{-1}(x))$



then $\frac{dy}{dx}=$

$\left(\frac{1}{\sqrt{1-x^2}}\right)^2=\frac{1}{1+x^2}$

Simplifying: $\frac{1}{1+x^2}$

Practice time:

$\frac{d}{dx}\tan^{-1}(x)=\frac{1}{1+x^2}$

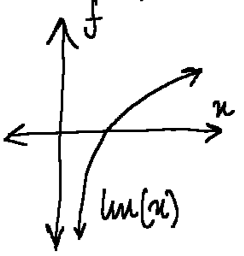
$\frac{d}{dx}(\sin^{-1}(x)=\frac{1}{\sqrt{1+x^2}}$

$\frac{d}{dx}(\cos^{-1}(x))=\frac{-1}{\sqrt{1+x^2}}$

3.6 Derivative of log functions

Ex1: $f(x) = \ln x$; $e^y = x$

implicit differentiation



$$e^y \frac{dy}{dx} = 1$$

$$\frac{dy}{dx} = \frac{1}{e^y} \text{ : but}$$

$$e^y = x ; \frac{dy}{dx} = \frac{1}{x}$$

Thm Definition:

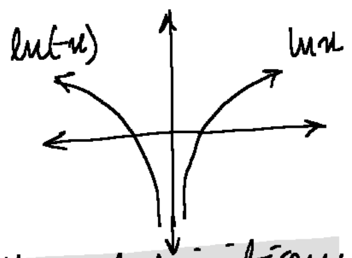
$$\frac{d}{dx}(\ln x) = \frac{1}{x}$$

due to the implicit diff and the something else (geometric interpretation)

Ex2: $$\frac{d}{dx}(\log_2 x) = \frac{d}{dx}\frac{\ln x}{\ln 2} = \frac{1}{x \ln 2}$$

Thm def: $$\frac{d}{dx}(\log_b x) = \frac{1}{x \ln(b)}$$

Ex3: $f(x) = \ln|x|$



$$\frac{d}{dx}(\ln x) = \boxed{\frac{1}{x}}$$

$$\frac{d}{dx}(\ln -x) = \frac{-1}{-x} = \boxed{\frac{1}{x}}$$

thm definition: $\frac{d}{dx}(\ln|x|) = \frac{1}{x} ; x \neq 0$

The idea of the antiderivative:

$$\frac{d}{dx}\left(\frac{x^{n+1}}{n+1}\right) = \frac{(n+1)x^n}{(n+1)} = x^n$$

logarithmic derivation:

Ex5: $f(x) = \ln|\cos x|$

$$f'(x) = -\sin x / \cos x = \boxed{-\tan x}$$

Ex6: $f(x) = \ln|x^n|$

$$f'(x) = \frac{nx^{n-1}}{x^n} = \boxed{\frac{n}{x}} \text{ or } n \ln x = \boxed{\frac{n}{x}}$$

Ex7: $f(x) = \ln|f(x)|$

$$f'(x) = \boxed{f'(x) \div f(x)}$$

Ex8: $f(x) = x^7\sqrt{x+1} \div \sin x$

$$\ln(f(x)) = 7\ln x + \frac{1}{2}\ln(x+1) - \ln \sin x$$

$$\frac{f'(x)}{f(x)} = \frac{7}{x} + \frac{1}{2x+2} - \frac{\cos x}{\sin x}\left(\frac{x^7\sqrt{x+1}}{\sin x}\right)$$

Ex9: $fx = (x+1)(x+2)^5(x+3)^7(x+4)^9$

$$\ln fx = \ln(x+1) + 5\ln(x+2) + 7\ln(x+3) + 9\ln(x+4)$$

$$\frac{dy}{dx} = \frac{1}{x+1} + \frac{5}{x+2} + \frac{7}{x+3} + \frac{9}{x+4}(fx)$$

The number as limit e

$$\lim_{x\to0^+} \ln\left((1+x)^{1/x}\right) = \lim_{x\to0^+} \frac{1}{x}\ln(1+x)$$

$$\lim_{x\to0^+} \frac{\ln(1+x)-\ln(1+0)}{x-0} ; \frac{d}{dx} = \frac{1}{1+x} ; f'(0)=1$$

then by def

$$\lim_{x\to0^+} (1+x)^{1/x} = \lim_{x\to0^+} e^{\ln(1+x)^{1/x}} = e^1 = e$$

$$\lim_{x\to0^+} (1+x)^{1/x} = e$$

Substitute $n = \frac{1}{x}$

$$\lim_{1/x\to0^+} \left(1+\frac{1}{n}\right)^n = e \text{ or}$$

$$\lim_{n\to\infty} \left(1+\frac{1}{n}\right)^n = e$$

$$\lim_{n\to\infty} \left(1-\frac{1}{n}\right)^n = \frac{1}{e}$$

Definition of the e as limit

Ex12: Account balance: \$1

interest rate: 0.01

Account balance in 100 yrs

$$\$1(1.01)^{100} \approx 2.704\cdots$$

$$\Rightarrow \$1\left(1+\frac{1}{100}\right)^{100} \Rightarrow \boxed{e}$$

Ex13: Cat catches mouse: 1/1000 Prop

Propability mouse survive:

$$\left(1-\frac{1}{1000}\right)^{1000} \approx 0.367\cdots \boxed{\frac{1}{e}}$$

Continuously happening