Homework 3-6:

① $f(x) = x\ln x - x$

$f'(x) = \ln x + 1 - 1 = \ln x$

② $f(x) = \ln(x^8(x+7)^9(x^2+8)^4)$

$f'(x) = 8x^7(x+7)^9(x^2+8)^4 +$

$9x^8(x+7)^8(x^2+8)^4 + 8x^9(x^2+8)^3(x+7)^9 \div$

$(x^8(x+7)^9(x^2+8)^4)$; divided by all

③ $f(x) = \ln\left(\sqrt{\frac{2x-6}{3x+7}}\right)$

$f(x) = \frac{1}{2}\ln(2x-6) - \frac{1}{2}\ln(3x+7)$

$f'(x) = \frac{2}{2(2x-6)} - \frac{3}{2(3x+1)}$

④ $y = \ln(x^2+y^2)$; $(-\sqrt{e-4}, 2)P$

$\frac{dy}{dx} = \frac{2x+2yy'}{x^2+y^2}$; $x^2y' + y^2y' - 2yy' = 2x$

$\frac{dy}{dx} = \frac{2x}{x^2+y^2-2y}$; given P

$y' = \frac{-2}{\sqrt{e^2-4}} \approx -1.0864$

⑤ $f(x) = 2\cos(5\ln x)$

$f'(x) = -2\sin(5\ln x) \cdot \frac{5}{x} \;\forall\; f'(1)$

$f'(1) = -2\sin(0) \cdot \frac{5}{1} = 0$

⑥ $f(x) = 2\log_9(x)$

$f'(x) = \frac{2}{x\ln(9)} \;\forall\; f'(3) = \frac{2}{3\ln(9)}$

⑦ $y = \sqrt{x}e^{x^4}(x^2+1)^4$

$\ln(y) = \frac{1}{2}\ln(x) + x^4\ln(e) + 4\ln(x^2+1)$

$\ln(y) = \frac{1}{2}\ln x + x^4 + 4\ln(x^2+1)$

$\frac{1}{y}\frac{dy}{dx} = \frac{1}{2x} + 4x^3 + \frac{8x}{x^2+1}$ then

$\frac{dy}{dx} = \sqrt{x}e^{x^4}(x^2+1)^4\left(1/2x + 4x^3 + 8x/x^2+1\right)$

⑧ $f(x) = x^5(x-6)^5 \div (x^2+6)^2$

$\ln(y) = 5\ln(x) + 5\ln(x-6) - 2\ln(x^2+6)$

$\frac{1}{y}\frac{dy}{dx} = \frac{5}{x} + \frac{5}{x-6} - \frac{4x}{x^2-6}$ then

$\frac{dy}{dx} = \frac{x^5(x-6)^5}{(x^2-6)^2}\left(\frac{5}{x} + \frac{5}{x-6} - \frac{4x}{x^2+6}\right)$