Indefinite integrals & net change

Definition (indefinite integral):

if F is the anti-derivative of f, then we can write

$$\int f(x)\,dx = F(x)+C$$

Ex1: $\int_2^3 x^2\,dx = \frac{x^3}{3}\Big]_2^3$

$\frac{3^3}{3} - \frac{2^3}{3} \Rightarrow$ definite integral

$\int x^2\,dx = \frac{x^3}{3}+C \Rightarrow$ indef

Ex2: $\int (-x+5\sin x)-3\,dx$

$-\frac{x^2}{2} - 5\cos x - 3x + C$

indefinite integral answer

$\Rightarrow -\frac{x^2}{2} - 5\cos x - 3x + C$

Rules:

$\int f+g\,dx = \int f\,dx + \int g\,dx$

$\int f-g\,dx = \int f\,dx - \int g\,dx$

$\int cf\,dx = c\int f(x)\,dx$

$\int k\,dx = kx + C$ [anti-deriv]

$\int x^b\,dx = \frac{x^{b+1}}{b+1} + C;\ b \neq -1$

$\int 1/x\,dx = \ln|x| + C$

$\int e^x\,dx = e^x + C$

$\int \sin x = -\cos x + C;\ \int \cos x = \sin x + C$

Net Change th.

$$\int_a^b F'(x)\,dx = \int_a^b \frac{dF}{dx}\,dx = F(b)-F(a) = \Delta F$$

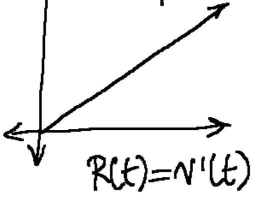
Just a different notation of FTC 2

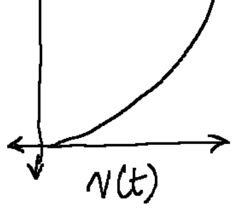
Ex4: Filling a pool:

Volume of pool: 1000

flow rate: $t = R(t)$

seconds to fill pool?





Net change: $\int_0^T \frac{dV}{dt}\,dt = V(T) - V(0) = V(t)$

$V(T) = \int_0^T \frac{dV}{dt}\,dt = \int_0^T R(t)\,dt = \int_0^T t\,dt = \frac{t^2}{2}\Big]_0^T$

$V(T) = 1000;\ T^2 = 2000;\ T = \sqrt{2000}$

At 44.7 seconds after starting, the pool fills up successfully.

Substitution Rule:

Chain Rule: a theoretical approach:

if F is the antiderivative of f:

$$\frac{d}{dx}\left(F(g(x))\right) = F'(g(x))\,g'(x) = f(g(x))\,g'(x)$$

work in reverse!

$$\int f(g)\,g'(x)\,dx = F(g(x)) + C$$

Ex 1:

$$\frac{d}{dx}\sqrt{\sin x} = \frac{\cos x}{2\sqrt{\sin x}}$$

$$\int \frac{\cos x}{2\sqrt{\sin x}}\,dx = \sqrt{\sin x} + C$$

Ex 2:

$$\int 2x e^{x^2}\,dx = \int g'(x)\,f(g)\,dx$$

$u = x^2$; $du = 2x$

$\int e^u\,du = e^u$ then $e^{x^2} = F(x)$

Substitution Rule, indeterminate integral

if $u = g(x)$ then,

$$\int f(g(x))\,g'(x)\,dx = \int f(u)\,du$$

Ex 3:

$$\int \sin(2x)\,dx$$

$u = 2x$; $du = 2\,dx$

$\frac{1}{2}\int \sin(u)\,du \Rightarrow \frac{-1}{2}\cos(u) + C$

$F(x) = \frac{-1}{2}\cos(2x) + C$

Ex 4:

$$\int \frac{\ln x}{x}\,dx$$

$u = \ln x$; $du = \frac{1}{x}\,dx$

$\int u\,du \Rightarrow \frac{u^2}{2} + C$ then

$F(x) = \frac{\ln^2 x}{2} + C$

Ex 5:

$$\int_0^1 \sin(2x)\,dx$$

$u = 2x$; $du = 2\,dx$

$\frac{1}{2}\int \sin(u)\,du \Rightarrow \frac{-1}{2}\cos(2x)\Big]_0^1$

$\frac{-1}{2}\cos(2) + \frac{1}{2}\cos(0)$