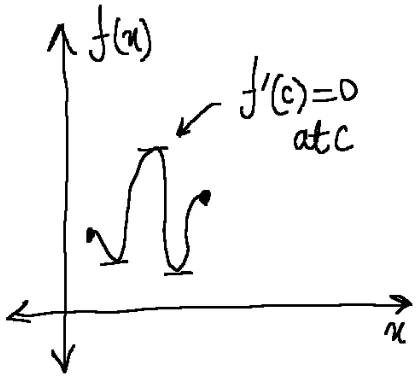
4.2 Mean Value theorem:

Rolle's theorem

let f function that satisfies the assumpts:

1. f is continuous $[a,b]$
2. f is differentiable (a,b)
3. $f(a)=f(b)$: has same height

There is a $c \in (a,b)$ where $f'(c)=0$



Proof: EVT:

→ there is $c_{min} \in [a,b]$ such that $f(c_m)$ is abs min val

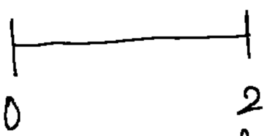
→ there is $c_{max} \in [a,b]$ such that $f(c_m)$ is abs max val

then c_{min} or c_{max} : $f'(c)=0$ given the min or max value

if they are endpoints, then $f(c)=f(c) \Rightarrow$ abs min; max

Ex1: $f(t)=$ Bart Train

$f(0)=12$ and $f(2)=12$



Roll's th. $\Rightarrow f'(c)=0$

$\forall$ some $c \in [0,2]$

Ex2: $f(t)=x^2-x+1$

$f(-1)=3$ & $f(2)=3$

Rolls th. $\Rightarrow f'(c)=0$

$\forall$ some $c \in [-1,2]$

Ex3: $f(x)=x^3-3x^2+6x+7$

at least 1 root : IVT

at most 1 root : Rolle th.

$f'(c)=0=3x^2-6x+6$

$\Rightarrow x^2-2x+2 \neq 0$

no solution; then only 1 sol $\forall f(x)$

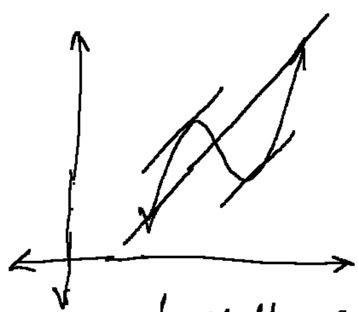
Mean Value th.

let f function that satisfies the:

1. f is continuous $[a,b]$
2. f is differentiable (a,b)

then there is some $c \in (a,b)$ such that

$$f'(c) = \frac{f(b)-f(a)}{b-a}$$



have the same slopes.

$h(x) = f(x) - L(x)$

$L(x) = \frac{f(b)-f(a)}{b-a}(x-a) + f(a)$ $\Big\}$ $L'(x) = \frac{f(b)-f(a)}{b-a}$, $L(a) = f(a)$, $L(b) = f(b)$

h is continuous and differentiable w/

$$h(a) = f(a) - L(a) = 0 \quad \& \quad h(b) = f(b) - L(b) = 0$$

Ex4 $f(t) = BART$

$f(1) = 12$ & $f(3) = 52$

$f'(c) = \frac{52-12}{3-1} = 20$

then average vel

by MVT: $t \in (1,3)$ train has $\vec{v}$ 20m/s

Ex5: $[2,7]$: $f(2) = 1$ and

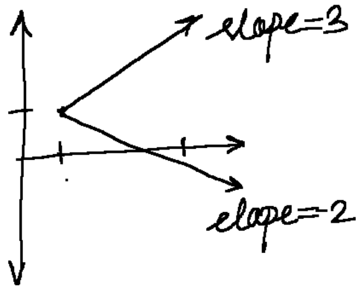
$-2 \le f'(x) \le 3$; $x \in (2,7)$

Then $\le f(7) \le 1 + 3 \cdot 5 = 16$

$1 + (-2) \cdot 5 = -9$

There is c by MVT: $f'(c)$

$= \frac{f(7)-1}{5} = \frac{16-1}{5} = 3$



Theorem:

if $f'(x) = 0$ $\forall$ all $x \in (a,b)$, & f is continuous on $[a,b]$, then f is constant, so $f'(x) = C$ $\forall$ some # C

Same graphical ideas of mean value theorem

Corollary:

If $f'(x) = g'(x)$ $\forall$ all $x \in (a,b)$ then $f(x) = g(x) + c$ $\forall$ some constant c

Proof: $H(x) = f(x) - g(x)$

$h'(x) = f'(x) - g'(x) = 0$

$\Rightarrow h(x) = C$

$f(x) = g(x) + C$

Ex6: $\sin^{-1}(x) = \frac{\pi}{2} - \cos^{-1}(x)$

$f(x) = \sin^{-1}(x) = 1/\sqrt{1-x^2}$

$g(x) = -\cos^{-1}(x) = 1/\sqrt{1-x^2}$

$f'(x) = g'(x)$

Corollary: $f(x) = g(x) + C$

$\sin^{-1}(x) = -\cos^{-1}(x) + C$

Plug $x = 0$ then

$0 = \sin^{-1}(0) = -\cos^{-1}(0) + C = -\frac{\pi}{2} + C$

$\boxed{C = \pi/2}$