Homework 4.3

① $f(x) = 2x + \frac{6}{x}$

$f'(x) = 2 - \frac{6}{x^2} = 0;\ \pm\sqrt{3}$

$$\begin{array}{ccccccc} + & & - & & - & & + \\ \hline & -\sqrt{3} & & 0 & & \sqrt{3} & \end{array}$$

inc: $x \in (-\infty, -\sqrt{3}) \cup (\sqrt{3}, \infty)$

dec: $x \in (-\sqrt{3}, 0) \cup (0, \sqrt{3})$

local min: $\sqrt{3}$
local max: $-\sqrt{3}$

② $f(x) = 6x + 5\sin x$

$f'(x) = 6 + 5\cos x = 0$

never 80 always ↑

increase: $x \in \mathbb{R}$

decrease: $x \in \varnothing$

no criticals

③ $f(x) = \frac{ax^b}{\ln(x)}$; $f(\frac{1}{7}) = 1$ ↑ local min

$f'(x) = \frac{abx^{b-1}\ln(x) - ax^b \cdot 1/x}{\ln(x)^2} = 0$

$b\ln(x) = 1$; $b = 1/\ln(1/7)$ then, so

$\frac{a(1/7)^b}{\ln(1/7)} = 1$; $a = \frac{\ln(1/7)}{(1/7)^b}$

④ analyzing the graphs and edit like that.

+⑤ +⑥

⑦ $f(x) = 8x^4 + 78x^3 - 30x^2 + 8$

$f'(x) = 32x^3 + 234x^2 - 60x$

$f''(x) = 96x^2 + 468x - 60 = 0$

$12(8x-1)(x+5) = 0$

inflection point $= 1/8, -5$

⑧ $f(x) = x^3 - 6x^2 + 16$

$f'(x) = 3x^2 - 12x = 0$

$x(3x-12)$; $x = 0$ & 4

$$\begin{array}{ccccc} + & & - & & + \\ \hline & 0 & & 4 & \end{array}$$

local min: 4; local max: 0

$f''(x) = 6x - 12$; $\boxed{x = 2}$

⑨ $f(x) = \frac{e^x}{7+e^x}$; $f'(x) = \frac{7e^x}{(7+e^x)^2} = 0$ $\neq 0$

increasing at $(-\infty, \infty)$

decreasing at NA

NA local min and max:

approching HA some

$7e^x(7+e^x)^2 - 7e^x(2)(7+e^x)e^x = 0$

$7 \cdot 49e^x - 7e^{3x} = 0$; $e^x(343 - 7e^{2x}) = 0$

$e^x = 7 \therefore \ln(7) = x$ inflection point

$f''(x) = \frac{e^x(343 - 7e^{2x})}{(7+e^x)^4} = 0$

$$\begin{array}{ccc} + & & - \\ \hline & \ln(7) & \end{array}$$

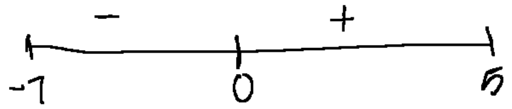
concave ↑: $(-\infty, \ln(7))$
↓: $(\ln 7, \infty)$

Homework 4-3 Cont:

(10) $f(x) = x\sqrt{x^2+9} = \sqrt{x^4+9x^2}$: $[-7,5]$

$f'(x) = \frac{1}{2}(x^4+9x^2)^{-1/2}(4x^3+18x) = 0$

$2x(2x^2+9) = 0$; $x = 0$



$f(0) = 0$; $f(-7) = -53.3$; $f(5) = 29.14$

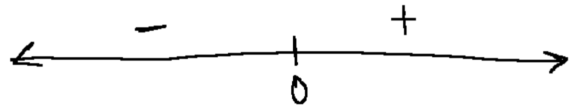
global max: 5, min: −7

$$f''(x) = \frac{18x(x^2+9)^{1/2} - (2x^3+9x)(x^2+9)^{-1/2}}{(x^2+9)} = 0$$

$18x(x^2+9)^{1/2} = (2x^3+9x)(x^2+9)^{-1/2}$

$18x^3 + 162x = 2x^3 + 9x$

$16x^3 + 153x = 0$; $x = 0$; that it

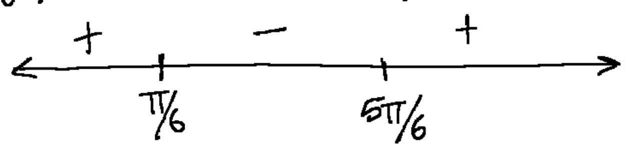


concave up: $(0,\infty)$; down: $(-\infty,0)$

w/ int: $(0,5)$ and $(-7,0)$

(11) $f(x) = \cos x + \frac{1}{2}x$; $f'(x) = \frac{1}{2} - \sin(x)$; $[0,2\pi]$

$\sin^{-1}(\frac{1}{2}) = x$; $x = \pi/6$ and $5\pi/6$



$f''(x) = -\cos(x)$; $f''(\pi/6) = -\frac{\sqrt{3}}{2}$; $f''(5\pi/6) = \frac{\sqrt{3}}{2}$

local max: $\pi/6$ and local min: $5\pi/6$