

4.7 Optimization Problems:

Ex1: Circumference: $2x+2y=12$

$$y=6-x; A(x)=xy$$

$$\text{domain: } x \in [0,6] \text{ or } (0,6)$$

$$A(x)=6x-x^2; A'=6-2x=0$$

$$\text{critics at } \textcircled{3}$$

$$\text{Abs max: } f(0); f(3); f(6)$$

which are $[0,6]$ then

Max Area: $\text{9cm} \times \text{width} = 3\text{cm}$
and height = 3cm

Strategy:

1. Identify and name the independent variable
2. Express situation as a func
3. Identify the domain
4. Express the mathematical goal
5. Find the abs min/max
6. Interpret your result.

Ex2: Speed: $l=4\frac{\text{m}}{\text{s}}; w=1\frac{\text{m}}{\text{s}}; t=\frac{d}{s}$

$$t(s) = \frac{y}{4} + \sqrt{(x-8)^2 + 36}; x: 8-x \text{ \& } y=6$$

$$\text{domain: } x \in [0,\infty) \text{ or } [0,8]$$

$$t'(s) = \frac{1}{4} + \frac{1}{2}((x-8)^2 + 36)^{-1/2}(2x-16) = 0$$

$$\frac{1}{4} = \frac{8-x}{\sqrt{(x-8)^2 + 36}}; (x-8)^2 + 36 = 16(8-x)^2$$

$$(8-x)^2 = \frac{36}{15}; x = 8 \pm \sqrt{\frac{36}{15}}; x \approx 6.45$$

Optimal running time is 7.74s w/ an
opt. distance of 8.45m.

Finding abs extrema + half

$$\text{OI: } (a,b); (-\infty,b); (a,\infty); (-\infty,\infty)$$

$$\text{CI: } [-\infty,b]; [a,\infty]; [a,b]; (a,b) \dots$$

1. Compute all critics of f
2. Compute value of f of closed endp...
3. Compute value of f of open endp...
4. Combine Results.
if largest value is found w/
open interval, then DNE

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Ex3: Speed: $l = 2\frac{m}{s}$; $w = 1\frac{m}{s}$; $t = \frac{d}{s}$
 $t(x) = \frac{x}{2} + \sqrt{(x-2)^2 + 36}$; $x = x-2$; $y = 6$
 domain: $[0, \infty)$

$t'(x) = \frac{1}{2} + \frac{x-2}{\sqrt{(x-2)^2 + 36}} = 0$; simplify

! domain $\begin{cases} x = 2 - \sqrt{12} = -1.4 \\ f(2 - 2\sqrt{3}) \approx 6.19 \end{cases}$

then: $f(0) = 6.32$; $f(\infty) = \infty$
 then optimal time within
 given interval is $x=0$ at 6.32
 so start swimming
 immediately.

Ex4: $f(x) = \sqrt{x^2 + \left(\frac{9m}{x-6}\right)^2}$; $x \in (6, \infty)$

$f' = 2\sqrt{x^2 + \left(\frac{9m}{x-6}\right)^2}^{-1} \frac{d}{dx} \left(x^2 + \left(\frac{9m}{x-6}\right)^2 \right)$

simplify to get
 critical points of $6 + 3\sqrt[3]{18} \approx 13.86$

$f(13.86) \approx 21.07$ then

abs max = 21.07 at 13.86

Interpret: largest pipe that fits
 is 21.07m with the
 specified conditions.