

E.A.6.7 (HC-VIP)

1.1 Modellazione

Dati i parametri $I, V, G = (I, \text{bus})$ t.c.

- $\text{casa} \in I$
- $V \subseteq I / \{\text{casa}\}$
- $\text{bus} \subseteq I \times I$
- $|V| \leq \frac{|I|}{2}$

Si definiscono le seguenti variabili:

- $T = |I|$ è il numero di *bus* da prendere in un percorso che parte da *casa*, visita tutti gli indirizzi esattamente una volta e ritorna a *casa*
- servono $|I| - 1$ *bus* per visitare tutti gli indirizzi esattamente una volta più 1 *bus* per tornare a casa a fine giornata
- $\mathcal{T} = \{1, \dots, T\}$
- $\mathcal{I} = \{1, \dots, |I|\}$ l'insieme di identificatori per cui esiste una funzione id biettiva t.c.

$$\text{id} : \mathcal{I} \rightarrow I \text{ e } \text{id}(1) = \text{casa} \quad (1)$$

- $\mathcal{V} = \{i \mid i \in \mathcal{I} \wedge \exists v \in V \wedge \text{id}(i) = v\}$
- $X = \{X_{i,j}^t \mid (i,j) \in \text{bus} \wedge t \in \mathcal{T}\}$ l'insieme di variabili dove
 - $X_{i,j}^t$ è vera se l'arco $(i,j) \in \text{bus}$ è stato percorso al t -esimo passo

$$\begin{aligned} \phi = & \phi_{\text{almeno_un_arco_per_passo}} \wedge \\ & \phi_{\text{al_più_un_arco_per_passo}} \wedge \\ & \phi_{\text{almeno_un_arco_per_indirizzo}} \wedge \\ & \phi_{\text{al_più_un_arco_per_indirizzo}} \wedge \\ & \phi_{\text{clienti_VIP_nella_prima_metà}} \wedge \\ & \phi_{\text{partenza_da_casa}} \wedge \\ & \phi_{\text{arrivo_a_casa}} \wedge \\ & \phi_{\text{percorso_valido}} \end{aligned} \quad (2)$$

$$\begin{aligned} \phi_{\text{almeno_un_arco_per_passo}} &= \bigwedge_{t \in \mathcal{T}} \bigvee_{(i,j) \in \text{bus}} X_{i,j}^t \\ \phi_{\text{al_più_un_arco_per_passo}} &= \bigwedge_{\substack{t \in \mathcal{T} \\ (i_1,j_1), (i_2,j_2) \in \text{bus} \\ (i_1,j_1) < (i_2,j_2)}} X_{i_1,j_1}^t \rightarrow \neg X_{i_2,j_2}^t \end{aligned} \quad (3)$$

$$\begin{aligned}
\phi_{\text{almeno_un_arco_per_indirizzo}} &= \bigwedge_{j \in \mathcal{I}} \bigvee_{\substack{t \in \mathcal{T} \\ (i,j) \in \text{bus}}} X_{i,j}^t \\
\phi_{\text{al_più_un_arco_per_indirizzo}} &= \bigwedge_{\substack{t_1, t_2 \in \mathcal{T} \\ (i_1, j), (i_2, j) \in \text{bus} \\ t_1 \leq t_2 \\ i_1 < i_2}} X_{i_1, j}^{t_1} \rightarrow \neg X_{i_2, j}^{t_2} \\
\phi_{\text{clienti_VIP_nella_prima_metà}} &= \bigwedge_{v \in \mathcal{V}} \bigvee_{\substack{t \in \mathcal{T} \\ t \leq \lceil \frac{T}{2} \rceil \\ (i, v) \in \text{bus}}} X_{i, v}^t \\
\phi_{\text{partenza_da_casa}} &= \bigvee_{\substack{i \in \mathcal{I} \setminus \{1\} \\ (1, i) \in \text{bus}}} X_{1, i}^1 \\
\phi_{\text{arrivo_a_casa}} &= \bigvee_{\substack{i \in \mathcal{I} \setminus \{1\} \\ (i, 1) \in \text{bus}}} X_{i, 1}^T \\
\phi_{\text{percorso_valido}} &= \bigwedge_{\substack{t \in \mathcal{T} \setminus \{T\} \\ (i, j) \in \text{bus}}} X_{i, j}^t \rightarrow \bigvee_{(j, k) \in \text{bus}} X_{j, k}^{t+1}
\end{aligned} \tag{3}$$

1.2 Istanziamento

1.2.1 Variabili

- $I = \{casa, i_1, i_2, i_3, i_4, i_5\}$
- $V = \{i_1\}$
- $bus = \{(casa, i_1), (casa, i_2), (i_1, i_3), (i_1, i_4), (i_2, casa), (i_3, casa), (i_3, i_4), (i_4, i_1), (i_4, i_2)\}$
- $\mathcal{T} = \{1, 2, 3, 4, 5\}$
- $\mathcal{J} = \{1, 2, 3, 4, 5\}$
- $\mathcal{V} = \{2\}$
- $X = \{$

$$\begin{aligned}
&X_{1,2}^1 X_{1,3}^1 X_{2,4}^1 X_{2,5}^1 X_{3,1}^1 \\
&X_{3,5}^1 X_{4,1}^1 X_{4,5}^1 X_{5,2}^1 X_{5,3}^1 \\
&X_{1,2}^2 X_{1,3}^2 X_{2,4}^2 X_{2,5}^2 X_{3,1}^2 \\
&X_{3,5}^2 X_{4,1}^2 X_{4,5}^2 X_{5,2}^2 X_{5,3}^2 \\
&X_{1,2}^3 X_{1,3}^3 X_{2,4}^3 X_{2,5}^3 X_{3,1}^3 \\
&X_{3,5}^3 X_{4,1}^3 X_{4,5}^3 X_{5,2}^3 X_{5,3}^3 \\
&X_{1,2}^4 X_{1,3}^4 X_{2,4}^4 X_{2,5}^4 X_{3,1}^4 \\
&X_{3,5}^4 X_{4,1}^4 X_{4,5}^4 X_{5,2}^4 X_{5,3}^4 \\
&X_{1,2}^5 X_{1,3}^5 X_{2,4}^5 X_{2,5}^5 X_{3,1}^5 \\
&X_{3,5}^5 X_{4,1}^5 X_{4,5}^5 X_{5,2}^5 X_{5,3}^5
\end{aligned}$$
 $\}$

1.2.2 Vincoli

La codifica che ho generato ha 293 clausole anche per un problema così piccolo... non vale la pena elencarle tutte, metto un vincolo d'esempio.

$$\begin{aligned}
\phi_{\text{al più un arco per passo}} = \{ \\
&(X_{1,2}^1 \rightarrow \neg X_{1,3}^1) \wedge (X_{1,2}^1 \rightarrow \neg X_{2,4}^1) \wedge (X_{1,2}^1 \rightarrow \neg X_{2,5}^1) \wedge (X_{1,2}^1 \rightarrow \neg X_{3,1}^1) \wedge (X_{1,2}^1 \rightarrow \neg X_{3,5}^1) \wedge \\
&(X_{1,2}^1 \rightarrow \neg X_{4,1}^1) \wedge (X_{1,2}^1 \rightarrow \neg X_{4,5}^1) \wedge (X_{1,2}^1 \rightarrow \neg X_{5,2}^1) \wedge (X_{1,2}^1 \rightarrow \neg X_{5,3}^1) \wedge (X_{1,3}^1 \rightarrow \neg X_{2,4}^1) \wedge \\
&(X_{1,3}^1 \rightarrow \neg X_{2,5}^1) \wedge (X_{1,3}^1 \rightarrow \neg X_{3,1}^1) \wedge (X_{1,3}^1 \rightarrow \neg X_{3,5}^1) \wedge (X_{1,3}^1 \rightarrow \neg X_{4,1}^1) \wedge (X_{1,3}^1 \rightarrow \neg X_{4,5}^1) \wedge \\
&(X_{1,3}^1 \rightarrow \neg X_{5,2}^1) \wedge (X_{1,3}^1 \rightarrow \neg X_{5,3}^1) \wedge (X_{2,4}^1 \rightarrow \neg X_{2,5}^1) \wedge (X_{2,4}^1 \rightarrow \neg X_{3,1}^1) \wedge (X_{2,4}^1 \rightarrow \neg X_{3,5}^1) \wedge \\
&(X_{2,4}^1 \rightarrow \neg X_{4,1}^1) \wedge (X_{2,4}^1 \rightarrow \neg X_{4,5}^1) \wedge (X_{2,4}^1 \rightarrow \neg X_{5,2}^1) \wedge (X_{2,4}^1 \rightarrow \neg X_{5,3}^1) \wedge (X_{2,5}^1 \rightarrow \neg X_{3,1}^1) \wedge \\
&(X_{2,5}^1 \rightarrow \neg X_{3,5}^1) \wedge (X_{2,5}^1 \rightarrow \neg X_{4,1}^1) \wedge (X_{2,5}^1 \rightarrow \neg X_{4,5}^1) \wedge (X_{2,5}^1 \rightarrow \neg X_{5,2}^1) \wedge (X_{2,5}^1 \rightarrow \neg X_{5,3}^1) \wedge
\end{aligned}$$

[illegible]

$$\begin{aligned}
& (X_{4,1}^4 \rightarrow \neg X_{5,2}^4) \wedge (X_{4,1}^4 \rightarrow \neg X_{5,3}^4) \wedge (X_{4,5}^4 \rightarrow \neg X_{5,2}^4) \wedge (X_{4,5}^4 \rightarrow \neg X_{5,3}^4) \wedge (X_{5,2}^4 \rightarrow \neg X_{5,3}^4) \wedge \\
& (X_{1,2}^5 \rightarrow \neg X_{1,3}^5) \wedge (X_{1,2}^5 \rightarrow \neg X_{2,4}^5) \wedge (X_{1,2}^5 \rightarrow \neg X_{2,5}^5) \wedge (X_{1,2}^5 \rightarrow \neg X_{3,1}^5) \wedge (X_{1,2}^5 \rightarrow \neg X_{3,5}^5) \wedge \\
& (X_{1,2}^5 \rightarrow \neg X_{4,1}^5) \wedge (X_{1,2}^5 \rightarrow \neg X_{4,5}^5) \wedge (X_{1,2}^5 \rightarrow \neg X_{5,2}^5) \wedge (X_{1,2}^5 \rightarrow \neg X_{5,3}^5) \wedge (X_{1,3}^5 \rightarrow \neg X_{2,4}^5) \wedge \\
& (X_{1,3}^5 \rightarrow \neg X_{2,5}^5) \wedge (X_{1,3}^5 \rightarrow \neg X_{3,1}^5) \wedge (X_{1,3}^5 \rightarrow \neg X_{3,5}^5) \wedge (X_{1,3}^5 \rightarrow \neg X_{4,1}^5) \wedge (X_{1,3}^5 \rightarrow \neg X_{4,5}^5) \wedge \\
& (X_{1,3}^5 \rightarrow \neg X_{5,2}^5) \wedge (X_{1,3}^5 \rightarrow \neg X_{5,3}^5) \wedge (X_{2,4}^5 \rightarrow \neg X_{2,5}^5) \wedge (X_{2,4}^5 \rightarrow \neg X_{3,1}^5) \wedge (X_{2,4}^5 \rightarrow \neg X_{3,5}^5) \wedge \\
& (X_{2,4}^5 \rightarrow \neg X_{4,1}^5) \wedge (X_{2,4}^5 \rightarrow \neg X_{4,5}^5) \wedge (X_{2,4}^5 \rightarrow \neg X_{5,2}^5) \wedge (X_{2,4}^5 \rightarrow \neg X_{5,3}^5) \wedge (X_{2,4}^5 \rightarrow \neg X_{5,3}^5) \wedge \\
& (X_{2,5}^5 \rightarrow \neg X_{3,5}^5) \wedge (X_{2,5}^5 \rightarrow \neg X_{4,1}^5) \wedge (X_{2,5}^5 \rightarrow \neg X_{4,5}^5) \wedge (X_{2,5}^5 \rightarrow \neg X_{5,2}^5) \wedge (X_{2,5}^5 \rightarrow \neg X_{5,3}^5) \wedge \\
& (X_{3,1}^5 \rightarrow \neg X_{3,5}^5) \wedge (X_{3,1}^5 \rightarrow \neg X_{4,1}^5) \wedge (X_{3,1}^5 \rightarrow \neg X_{4,5}^5) \wedge (X_{3,1}^5 \rightarrow \neg X_{5,2}^5) \wedge (X_{3,1}^5 \rightarrow \neg X_{5,3}^5) \wedge \\
& (X_{3,5}^5 \rightarrow \neg X_{4,1}^5) \wedge (X_{3,5}^5 \rightarrow \neg X_{4,5}^5) \wedge (X_{3,5}^5 \rightarrow \neg X_{5,2}^5) \wedge (X_{3,5}^5 \rightarrow \neg X_{5,3}^5) \wedge (X_{4,1}^5 \rightarrow \neg X_{4,5}^5) \wedge \\
& (X_{4,1}^5 \rightarrow \neg X_{5,2}^5) \wedge (X_{4,1}^5 \rightarrow \neg X_{5,3}^5) \wedge (X_{4,5}^5 \rightarrow \neg X_{5,2}^5) \wedge (X_{4,5}^5 \rightarrow \neg X_{5,3}^5) \wedge (X_{5,2}^5 \rightarrow \neg X_{5,3}^5) \wedge \\
& \}
\end{aligned}$$

1.3 Codifica SATCodec (a questo giro in Rust)

```
use std::collections::BTreeSet;

use computer_braining::framework::sat_codec::*;
use serde::Serialize;

#[derive(PartialEq, Eq, PartialOrd, Ord, Hash, Debug, Serialize)]
struct X(usize, usize, usize);

fn main() {
    use Literal::Neg;

    let addresses_cardinality = 5;

    let addresses = 1..=addresses_cardinality;
    let steps = 1..=addresses_cardinality;
    let vips = BTreeSet::from([2]);

    #[rustfmt::skip]
    // Already sorted
    let buses: Vec<(usize, usize)> = vec![
        (1, 2), (1, 3), (2, 4), (2, 5), (3, 1), (3, 5), (4, 1),
        (4, 5), (5, 2), (5, 3)
    ];

    let mut encoder = Encoder::new();

    // Almeno un arco per passo
    for t in steps.clone() {
        let mut c = encoder.clause_builder();
        for &(i, j) in buses.iter() {
            c.add(X(t, i, j));
        }
        encoder = c.end()
    }

    // Al più un arco per passo
    for t in steps.clone() {
        for (index, &(i1, j1)) in buses.iter().enumerate() {
            for &(i2, j2) in buses.iter().skip(index + 2) {
                let mut c = encoder.clause_builder();
                c.add(Neg(X(t, i1, j1)));
                c.add(Neg(X(t, i2, j2)));
                encoder = c.end();
            }
        }
    }

    // Almeno un arco per indirizzo
    for j in addresses.clone() {
        let mut c = encoder.clause_builder();
        for t in steps.clone() {
            c.add(X(t, j, 2));
        }
    }
}
```

```

        for &(i, k) in buses.iter() {
            if k == j {
                c.add(X(t, i, j));
            }
        }
    }
    encoder = c.end();
}

// Al più un arco per indirizzo
for t1 in steps.clone() {
    for t2 in t1 + 1..*steps.end() {
        for j in addresses.clone() {
            for i1 in addresses.clone() {
                for i2 in i1 + 1..*addresses.end() {
                    if buses.contains(&(i1, j)) &&
buses.contains(&(i2, j)) {
                        let mut c = encoder.clone_builder();
                        c.add(Neg(X(t1, i1, j)));
                        c.add(Neg(X(t2, i2, j)));
                        encoder = c.end();
                    }
                }
            }
        }
    }
}

// Clienti VIP nella prima metà
for &v in vips.iter() {
    let mut c = encoder.clone_builder();
    for t in 1..steps.end().div_ceil(2) {
        for i in addresses.clone() {
            if buses.contains(&(i, v)) {
                c.add(X(t, i, v));
            }
        }
    }
    encoder = c.end();
}

// Partenza da casa
let mut c = encoder.clone_builder();
for i in 2..*addresses.end() {
    if buses.contains(&(1, i)) {
        c.add(X(1, 1, i));
    }
}
encoder = c.end();

// Arrivo a casa
let mut c = encoder.clone_builder();
for i in 2..*addresses.end() {

```

```

        if buses.contains(&(i, 1)) {
            c.add(X(*steps.end(), i, 1));
        }
    }
    encoder = c.end();

    // Percorso valido
    for t in *steps.start() .. *steps.end() - 1 {
        for &(i, j) in buses.iter() {
            let mut c = encoder.clone_builder();
            c.add(Neg(X(t, i, j)));
            for k in addresses.clone() {
                if buses.contains(&(j, k)) {
                    c.add(X(t + 1, j, k))
                }
            }
            encoder = c.end();
        }
    }

    encoder.end();
}

```

1.3.1 ...

Devo ancora implementare il decodificatore. La decodifica l'ho fatta a mano a questo giro, e negli esempi che ho provato funziona perfettamente.