

E.A.6.8 (Graph Colouring with Red Self-Loops)

1.1 Modellazione

Dati i parametri $G = (V, E)$ siano

- $\mathcal{C} = \{R, B, C\}$
- $X = \{X_v^c \mid v \in V \wedge c \in \mathcal{C}\}$ l'insieme di variabili dove
 - X_v^c è vera se il nodo v ha colore c

Il problema presenta 4 vincoli

$$\phi = \phi_1 \wedge \phi_2 \wedge \phi_3 \wedge \phi_4$$

(ALO) Ogni nodo ha almeno un colore.

$$\phi_1 = \bigwedge_{v \in V} \bigvee_{c \in \mathcal{C}} x_v^c \quad (1)$$

(AMO) Ogni nodo ha al più un colore.

$$\phi_2 = \bigwedge_{\substack{v \in V \\ c_1, c_2 \in \mathcal{C} \\ c_1 < c_2}} X_v^{c_1} \rightarrow \neg X_v^{c_2} \quad (2)$$

1. Non esistono nodi collegati da un arco colorati con lo stesso colore.

$$\phi_3 = \bigwedge_{\substack{(u,v) \in E \\ c \in \mathcal{C} \\ u < v}} X_u^c \rightarrow \neg X_v^c \quad (3)$$

2. Ogni nodo $v \in V$ che ha un cappio (un arco da v a v) è colorato con il colore R .

$$\phi_4 = \bigwedge_{(v,v) \in E} X_v^R \quad (4)$$

1.2 Istanziamento

1.2.1 Parametri e variabili

$V = \{A, B, C, D, E, G1, G2, H, I, J, S\}$

$E = \{$

(A, E), (E, A), (A, H), (H, A), (A, I), (I, A), (A, S), (S, A), (B, C), (C, B),
(B, G2), (G2, B), (B, I), (I, B), (B, J), (J, B), (B, S), (S, B), (C, D), (D, C),
(C, G2), (G2, C), (C, S), (S, C), (D, E), (E, D), (D, S), (S, D), (E, G1), (G1, E),
(E, H), (H, E), (G1, H), (H, G1), (G2, J), (J, G2), (H, I), (I, H), (J, J)

$\}$

$$\begin{aligned}
X = \{ & \\
& X_A^R, X_A^B, X_A^C, X_B^R, X_B^B, X_B^C, X_C^R, X_C^B, X_C^C, \\
& X_D^R, X_D^B, X_D^C, X_E^R, X_E^B, X_E^C, X_{G1}^R, X_{G1}^B, X_{G1}^C, \\
& X_{G2}^R, X_{G2}^B, X_{G2}^C, X_H^R, X_H^B, X_H^C, X_I^R, X_I^B, X_I^C, \\
& X_J^R, X_J^B, X_J^C, X_S^R, X_S^B, X_S^C \\
& \}
\end{aligned}$$

1.2.2 Vincoli

(ALO) Ogni nodo ha almeno un colore.

$$\begin{aligned}
\phi_1 = & \\
& (X_A^R \vee X_A^B \vee X_A^C) \wedge (X_B^R \vee X_B^B \vee X_B^C) \wedge (X_C^R \vee X_C^B \vee X_C^C) \wedge \\
& (X_D^R \vee X_D^B \vee X_D^C) \wedge (X_E^R \vee X_E^B \vee X_E^C) \wedge (X_{G1}^R \vee X_{G1}^B \vee X_{G1}^C) \wedge \quad (5) \\
& (X_{G2}^R \vee X_{G2}^B \vee X_{G2}^C) \wedge (X_H^R \vee X_H^B \vee X_H^C) \wedge (X_I^R \vee X_I^B \vee X_I^C) \wedge \\
& (X_J^R \vee X_J^B \vee X_J^C) \wedge (X_S^R \vee X_S^B \vee X_S^C)
\end{aligned}$$

(AMO) Ogni nodo ha al più un colore.

$$\begin{aligned}
\phi_2 = & \\
& (\neg X_A^B \vee \neg X_A^R) \wedge (\neg X_A^B \vee \neg X_A^C) \wedge (\neg X_A^C \vee \neg X_A^R) \wedge \\
& (\neg X_B^B \vee \neg X_B^R) \wedge (\neg X_B^B \vee \neg X_B^C) \wedge (\neg X_B^C \vee \neg X_B^R) \wedge \\
& (\neg X_C^B \vee \neg X_C^R) \wedge (\neg X_C^B \vee \neg X_C^C) \wedge (\neg X_C^C \vee \neg X_C^R) \wedge \\
& (\neg X_D^B \vee \neg X_D^R) \wedge (\neg X_D^B \vee \neg X_D^C) \wedge (\neg X_D^C \vee \neg X_D^R) \wedge \\
& (\neg X_E^B \vee \neg X_E^R) \wedge (\neg X_E^B \vee \neg X_E^C) \wedge (\neg X_E^C \vee \neg X_E^R) \wedge \\
& (\neg X_{G1}^B \vee \neg X_{G1}^R) \wedge (\neg X_{G1}^B \vee \neg X_{G1}^C) \wedge (\neg X_{G1}^C \vee \neg X_{G1}^R) \wedge \quad (6) \\
& (\neg X_{G2}^B \vee \neg X_{G2}^R) \wedge (\neg X_{G2}^B \vee \neg X_{G2}^C) \wedge (\neg X_{G2}^C \vee \neg X_{G2}^R) \wedge \\
& (\neg X_H^B \vee \neg X_H^R) \wedge (\neg X_H^B \vee \neg X_H^C) \wedge (\neg X_H^C \vee \neg X_H^R) \wedge \\
& (\neg X_I^B \vee \neg X_I^R) \wedge (\neg X_I^B \vee \neg X_I^C) \wedge (\neg X_I^C \vee \neg X_I^R) \wedge \\
& (\neg X_J^B \vee \neg X_J^R) \wedge (\neg X_J^B \vee \neg X_J^C) \wedge (\neg X_J^C \vee \neg X_J^R) \wedge \\
& (\neg X_S^B \vee \neg X_S^R) \wedge (\neg X_S^B \vee \neg X_S^C) \wedge (\neg X_S^C \vee \neg X_S^R)
\end{aligned}$$

1. Non esistono nodi collegati da un arco colorati con lo stesso colore.

$$\begin{aligned}
\phi_3 = & \\
& (\neg X_A^R \vee \neg X_E^R) \wedge (\neg X_A^B \vee \neg X_E^B) \wedge (\neg X_A^C \vee \neg X_E^C) \wedge \quad (7) \\
& (\neg X_A^R \vee \neg X_H^R) \wedge (\neg X_A^B \vee \neg X_H^B) \wedge (\neg X_A^C \vee \neg X_H^C) \wedge
\end{aligned}$$

$$\begin{aligned}
& (\neg X_A^R \vee \neg X_I^R) \wedge (\neg X_A^B \vee \neg X_I^B) \wedge (\neg X_A^C \vee \neg X_I^C) \wedge \\
& (\neg X_A^R \vee \neg X_S^R) \wedge (\neg X_A^B \vee \neg X_S^B) \wedge (\neg X_A^C \vee \neg X_S^C) \wedge \\
& (\neg X_B^R \vee \neg X_C^R) \wedge (\neg X_B^B \vee \neg X_C^B) \wedge (\neg X_B^C \vee \neg X_C^C) \wedge \\
& (\neg X_B^R \vee \neg X_{G2}^R) \wedge (\neg X_B^B \vee \neg X_{G2}^B) \wedge (\neg X_B^C \vee \neg X_{G2}^C) \wedge \\
& (\neg X_B^R \vee \neg X_I^R) \wedge (\neg X_B^B \vee \neg X_I^B) \wedge (\neg X_B^C \vee \neg X_I^C) \wedge \\
& (\neg X_B^R \vee \neg X_J^R) \wedge (\neg X_B^B \vee \neg X_J^B) \wedge (\neg X_B^C \vee \neg X_J^C) \wedge \\
& (\neg X_B^R \vee \neg X_S^R) \wedge (\neg X_B^B \vee \neg X_S^B) \wedge (\neg X_B^C \vee \neg X_S^C) \wedge \\
& (\neg X_C^R \vee \neg X_D^R) \wedge (\neg X_C^B \vee \neg X_D^B) \wedge (\neg X_C^C \vee \neg X_D^C) \wedge \\
& (\neg X_C^R \vee \neg X_{G2}^R) \wedge (\neg X_C^B \vee \neg X_{G2}^B) \wedge (\neg X_C^C \vee \neg X_{G2}^C) \wedge \\
& (\neg X_C^R \vee \neg X_S^R) \wedge (\neg X_C^B \vee \neg X_S^B) \wedge (\neg X_C^C \vee \neg X_S^C) \wedge \\
& (\neg X_D^R \vee \neg X_E^R) \wedge (\neg X_D^B \vee \neg X_E^B) \wedge (\neg X_D^C \vee \neg X_E^C) \wedge \\
& (\neg X_D^R \vee \neg X_S^R) \wedge (\neg X_D^B \vee \neg X_S^B) \wedge (\neg X_D^C \vee \neg X_S^C) \wedge \\
& (\neg X_E^R \vee \neg X_{G1}^R) \wedge (\neg X_E^B \vee \neg X_{G1}^B) \wedge (\neg X_E^C \vee \neg X_{G1}^C) \wedge \\
& (\neg X_E^R \vee \neg X_H^R) \wedge (\neg X_E^B \vee \neg X_H^B) \wedge (\neg X_E^C \vee \neg X_H^C) \wedge \\
& (\neg X_{G1}^R \vee \neg X_H^R) \wedge (\neg X_{G1}^B \vee \neg X_H^B) \wedge (\neg X_{G1}^C \vee \neg X_H^C) \wedge \\
& (\neg X_{G2}^R \vee \neg X_J^R) \wedge (\neg X_{G2}^B \vee \neg X_J^B) \wedge (\neg X_{G2}^C \vee \neg X_J^C) \wedge \\
& (\neg X_H^R \vee \neg X_I^R) \wedge (\neg X_H^B \vee \neg X_I^B) \wedge (\neg X_H^C \vee \neg X_I^C)
\end{aligned} \tag{7}$$

2. Ogni nodo $v \in V$ che ha un cappio (un arco da v a v) è colorato con il colore R .

$$\phi_4 = X_J^R \quad (8)$$

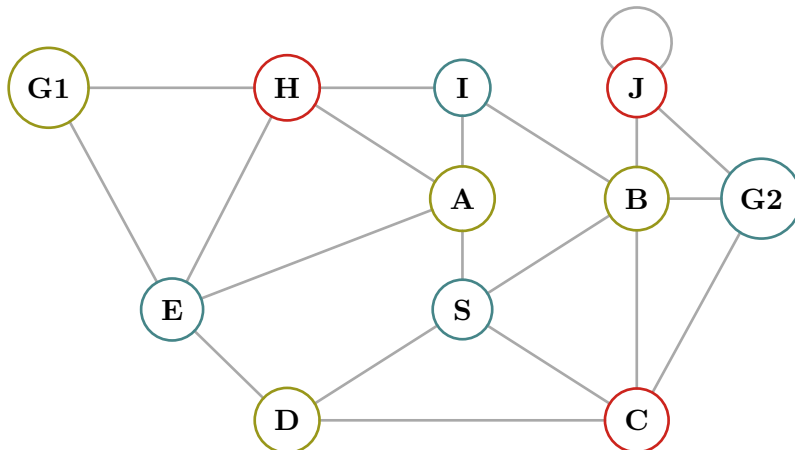


Figura 1: soluzione generata da picosat

1.3 Codifica

```
use computer_braining::framework::sat_codec::*;
use serde::Serialize;

#[derive(Clone, Copy, Hash, PartialEq, Eq, PartialOrd, Ord,
Serialize, Debug)]
enum Color {
    R,
    B,
    C,
}

type Node = &'static str;

#[derive(Hash, PartialEq, Eq, PartialOrd, Ord, Serialize, Debug)]
struct X(Node, Color);

fn main() {
    use Color::*;
    use Literal::Neg;

    #[rustfmt::skip]
    let nodes = [
        "A", "B", "C", "D", "E", "G1", "G2", "H", "I", "J", "S"
    ];

    #[rustfmt::skip]
    let edges = [
        ("A", "E"), ("A", "H"), ("A", "I"), ("A", "S"),
        ("B", "C"), ("B", "G2"), ("B", "I"), ("B", "J"),
        ("B", "S"), ("C", "D"), ("C", "G2"), ("C", "S"),
        ("D", "E"), ("D", "S"), ("E", "G1"), ("E", "H"),
        ("G1", "H"), ("G2", "J"), ("H", "I"), ("J", "J")
    ];

    let colors = [R, B, C];

    let mut encoder = Encoder::new();

    // ALO
    for v in nodes {
        let mut c = encoder.clause_builder();
        for color in colors {
            c.add(X(v, color));
        }
        encoder = c.end();
    }

    // AMO
    for v in nodes {
        for (i_1, &color_1) in colors.iter().enumerate() {
            for &color_2 in colors.iter().skip(i_1 + 1) {
```

```

        let mut c = encoder clause_builder();
        c.add(Neg(X(v, color_1)));
        c.add(Neg(X(v, color_2)));
        encoder = c.end();
    }
}

// 1. + 2.
for (u, v) in edges {
    if u == v {
        let mut c = encoder clause_builder();
        c.add(X(v, R));
        encoder = c.end();
    } else {
        for color in colors {
            let mut c = encoder clause_builder();
            c.add(Neg(X(u, color)));
            c.add(Neg(X(v, color)));
            encoder = c.end();
        }
    }
}

encoder.end();
}

```