E.B.3.2.2.1 (I Rossi, Inferenza)

NOTA: uso una notazione più simile a quella del libro, l'ho trovata un po' più comoda.

1.1 Inferenza incompleta ma educativa

$$\mathbf{P}(F = true \mid L = true, A = false) =$$

$$\alpha \sum_c \sum_i \mathbf{P}(F = true)\mathbf{P}(L = true \mid F = true)\mathbf{P}(A = false \mid C = c)\mathbf{P}(C = c \mid I = i, F = true) =$$

$$\{\text{spostamento dei termini fuori dalle sommatorie}\}$$

$$\alpha\mathbf{P}(F = true)\mathbf{P}(L = true \mid F = true) \sum_c \mathbf{P}(A = false \mid C = c) \sum_i \mathbf{P}(C = c \mid I = i, F = true) =$$

$$\{\text{eliminazione delle variabili}\}$$

$$\alpha\ \mathbf{f}_1 \times \mathbf{f}_2 \times \sum_c \mathbf{f}_3(C) \times \sum_i \mathbf{f}_4(C, I) =$$

$$\{\text{calcolo dei fattori}\}$$

$$\mathbf{f}_5(C) = \mathbf{f}_4(C, i) + \mathbf{f}_4(C, \neg i) = \begin{pmatrix} .99 \\ .01 \end{pmatrix} + \begin{pmatrix} .9 \\ .1 \end{pmatrix} = \begin{pmatrix} 1.89 \\ .11 \end{pmatrix}$$

$$\mathbf{f}_6 = \mathbf{f}_3(c) \times \mathbf{f}_5(c) + \mathbf{f}_3(\neg c) \times \mathbf{f}_5(\neg c) = .3 \times 1.89 + .99 \times .11 = .6759$$

$$\text{result} = \alpha\ \mathbf{f}_1 \times \mathbf{f}_2 \times \mathbf{f}_6 = \alpha\ .15 \times .6 \times .6759 = \alpha\ .06083$$

1.2 Inferenza completa

$$\mathbf{P}(F \mid L = \mathit{true}, A = \mathit{false}) =$$

$$\alpha \sum_c \sum_i \mathbf{P}(F)\mathbf{P}(L = \mathit{true} \mid F)\mathbf{P}(A = \mathit{false} \mid C = c)\mathbf{P}(C = c \mid I = i, F) =$$

$$\{\text{spostamento dei termini fuori dalle sommatorie}\}$$

$$\alpha\mathbf{P}(F)\mathbf{P}(L = \mathit{true} \mid F) \sum_c \mathbf{P}(A = \mathit{false} \mid C = c) \sum_i \mathbf{P}(C = c \mid I = i, F) =$$

$$\{\text{eliminazione delle variabili}\}$$

$$\alpha\ \mathbf{f}_1(F) \times \mathbf{f}_2(F) \times \sum_c \mathbf{f}_3(C) \times \sum_i \mathbf{f}_4(C, I, F) =$$

$$\{\text{calcolo dei fattori}\}$$

$$\mathbf{f}_5(C, F) = \mathbf{f}_4(C, i, F) + \mathbf{f}_4(C, \neg i, F) = \begin{pmatrix} .99 & .97 \\ .01 & .03 \end{pmatrix} + \begin{pmatrix} .9 & .3 \\ .1 & .7 \end{pmatrix} = \begin{pmatrix} 1.89 & 1 \\ .11 & .73 \end{pmatrix}$$

$$\mathbf{f}_6(F) = \mathbf{f}_3(c) \times \mathbf{f}_5(c, F) + \mathbf{f}_3(\neg c) \times \mathbf{f}_5(\neg c, F) = .3 \times \begin{pmatrix} 1.89 \\ 1 \end{pmatrix} + .99 \times \begin{pmatrix} .11 \\ .73 \end{pmatrix} = \begin{pmatrix} .567 \\ .3 \end{pmatrix} + \begin{pmatrix} .1088 \\ .7227 \end{pmatrix} = \begin{pmatrix} .6758 \\ 1.0227 \end{pmatrix}$$

$$\text{result} = \alpha\ \mathbf{f}_1(F) \times \mathbf{f}_2(F) \times \mathbf{f}_6(F) = \alpha \begin{pmatrix} .15 \\ .85 \end{pmatrix} \times \begin{pmatrix} .6 \\ .05 \end{pmatrix} \times \begin{pmatrix} .6758 \\ 1.0227 \end{pmatrix} = \alpha \begin{pmatrix} .09 \\ .0425 \end{pmatrix} \times \begin{pmatrix} .6758 \\ 1.0227 \end{pmatrix} =$$

$$\alpha \begin{pmatrix} .060822 \\ .04346475 \end{pmatrix} = 9.588945863208894 \begin{pmatrix} .060822 \\ .04346475 \end{pmatrix} = \begin{pmatrix} 0.5832188652920913 \\ 0.4167811347079088 \end{pmatrix}$$

I rossi hanno una probabilità del 58.3% di stare fuori casa.