1 E.A.5.2

1.1 n-Queens

Il problema n-Queens si potrebbe codificare come problema CSP nel seguente modo. Siano (X, D, C) t.c.

```
• X = \{Q_1, ..., Q_n\}
• D = \{D_1, ..., D_n \mid D_i = \{1, ..., n\}\}
```

- vincoli (due regine non possono stare sulla stessa riga e non si possono attaccare in diagonale):
 - $\begin{array}{l} \bullet \ \ \text{alldiff}(Q_1,...,Q_n) \\ \bullet \ \left\langle \left\{Q_i,Q_j\right\},\Delta\right\rangle \text{ t.c.} \\ -\ i\neq j \land \Delta = \left\{ \left(r_i,r_j\right) \mid r_i,r_j=1,...,n \land |i-j| \neq |r_i-r_j| \right\} \end{array}$

1.2 5-Queens

```
 \begin{split} \bullet & \ X = \{Q_1, Q_2, Q_3, Q_4, Q_5\} \\ \bullet & \ D = \{D_1, D_2, D_3, D_4, D_5 \mid D_i = \{1, 2, 3, 4, 5\}\} \\ \bullet & \ \text{vincoli:} \\ \bullet & \ \text{alldiff}(Q_1, Q_2, Q_3, Q_4, Q_5) \\ \bullet & \ \text{per le diagonali vedere l'ultima pagina...} \\ \end{aligned}
```

1.3 MiniZinc code

```
include "alldifferent.mzn";
int: n = 8; /* l'editor va in crash a n = 21 */
array[1..n] of var 1..n: queens;
constraint alldifferent(queens);
constraint forall(i in 1..n, j in (i + 1)..n)(
   abs(queens[j] - queens[i]) ≠ j - i
);
```

Piccola nota: effettivamente il vincolo descritto in MiniZinc non è identico alla definizione in notazione matematica, andrebbe riscritto così:

```
include "alldifferent.mzn";
int: n = 8; /* l'editor va in crash a n = 21 */
array[1..n] of var 1..n: queens;
constraint alldifferent(queens);
constraint forall(i in 1..n, j in 1..n)(
   i ≠ j → abs(queens[j] - queens[i]) ≠ abs(j - i)
);
```

Con n=20 l'esecuzione del primo codice dura circa 821ms, mentre l'esecuzione per la seconda versione dura circa 1s e 516ms.

I vincoli per cui le regine non si possono attaccare in diagonale

```
\{\{Q_1,Q_2\},\{(1,1),(1,3),(1,4),(1,5),(2,2),(2,4),(2,5),(3,1),(3,3),(3,5),(4,1),(4,2),(4,4),(5,1),(5,2),(5,3),(5,5)\}
\{\{Q_1,Q_3\},\{(1,1),(1,2),(1,4),(1,5),(2,1),(2,2),(2,3),(2,5),(3,2),(3,3),(3,4),(4,1),(4,3),(4,4),(4,5),(5,1),(5,2),(5,4),(5,5)\}\}
\langle \{Q_1,Q_4\}, \{(1,1),(1,2),(1,3),(1,5),(2,1),(2,2),(2,3),(2,4),(3,1),(3,2),(3,3),(3,4),(3,5),(4,2),(4,3),(4,4),(4,5),(5,1),(5,3),(5,4),(5,5)\} \rangle
\langle \{Q_1,Q_5\},\{(1,1),(1,2),(1,3),(1,4),(2,1),(2,2),(2,3),(2,4),(2,5),(3,1),(3,2),(3,3),(3,4),(3,5),(4,1),(4,2),(4,3),(4,4),(4,5),(5,2),(5,3),(5,4),(5,5)\} \rangle
\{\{Q_2,Q_1\},\{(1,1),(1,3),(1,4),(1,5),(2,2),(2,4),(2,5),(3,1),(3,3),(3,5),(4,1),(4,2),(4,4),(5,1),(5,2),(5,3),(5,5)\}
\{\{Q_2,Q_3\},\{(1,1),(1,3),(1,4),(1,5),(2,2),(2,4),(2,5),(3,1),(3,3),(3,5),(4,1),(4,2),(4,4),(5,1),(5,2),(5,3),(5,5)\}
\langle \{Q_2,Q_4\}, \{(1,1),(1,2),(1,4),(1,5),(2,1),(2,2),(2,3),(2,5),(3,2),(3,3),(3,4),(4,1),(4,3),(4,4),(4,5),(5,1),(5,2),(5,4),(5,5)\} \rangle
\langle \{Q_2,Q_5\}, \{(1,1),(1,2),(1,3),(1,5),(2,1),(2,2),(2,3),(2,4),(3,1),(3,2),(3,3),(3,4),(3,5),(4,2),(4,3),(4,4),(4,5),(5,1),(5,3),(5,4),(5,5)\} \rangle
\{\{Q_3,Q_1\},\{(1,1),(1,2),(1,4),(1,5),(2,1),(2,2),(2,3),(2,5),(3,2),(3,3),(3,4),(4,1),(4,3),(4,4),(4,5),(5,1),(5,2),(5,4),(5,5)\}
\{\{Q_3,Q_2\},\{(1,1),(1,3),(1,4),(1,5),(2,2),(2,4),(2,5),(3,1),(3,3),(3,5),(4,1),(4,2),(4,4),(5,1),(5,2),(5,3),(5,5)\}
\{\{Q_3,Q_4\},\{(1,1),(1,3),(1,4),(1,5),(2,2),(2,4),(2,5),(3,1),(3,3),(3,5),(4,1),(4,2),(4,4),(5,1),(5,2),(5,3),(5,5)\}
\{\{Q_3,Q_5\},\{(1,1),(1,2),(1,4),(1,5),(2,1),(2,2),(2,3),(2,5),(3,2),(3,3),(3,4),(4,1),(4,3),(4,4),(4,5),(5,1),(5,2),(5,4),(5,5)\}\}
\langle \{Q_4,Q_1\},\{(1,1),(1,2),(1,3),(1,5),(2,1),(2,2),(2,3),(2,4),(3,1),(3,2),(3,3),(3,4),(3,5),(4,2),(4,3),(4,4),(4,5),(5,1),(5,3),(5,4),(5,5)\} \rangle
\langle \{Q_4,Q_2\}, \{(1,1),(1,2),(1,4),(1,5),(2,1),(2,2),(2,3),(2,5),(3,2),(3,3),(3,4),(4,1),(4,3),(4,4),(4,5),(5,1),(5,2),(5,4),(5,5)\} \rangle
\{\{Q_A,Q_3\},\{(1,1),(1,3),(1,4),(1,5),(2,2),(2,4),(2,5),(3,1),(3,3),(3,5),(4,1),(4,2),(4,4),(5,1),(5,2),(5,3),(5,5)\}
\{\{Q_4,Q_5\},\{(1,1),(1,3),(1,4),(1,5),(2,2),(2,4),(2,5),(3,1),(3,3),(3,5),(4,1),(4,2),(4,4),(5,1),(5,2),(5,3),(5,5)\}
\langle \{Q_5,Q_1\},\{(1,1),(1,2),(1,3),(1,4),(2,1),(2,2),(2,3),(2,4),(2,5),(3,1),(3,2),(3,3),(3,4),(3,5),(4,1),(4,2),(4,3),(4,4),(4,5),(5,2),(5,3),(5,4),(5,5)\} \rangle
\langle \{Q_5,Q_2\}, \{(1,1),(1,2),(1,3),(1,5),(2,1),(2,2),(2,3),(2,4),(3,1),(3,2),(3,3),(3,4),(3,5),(4,2),(4,3),(4,4),(4,5),(5,1),(5,3),(5,4),(5,5)\} \rangle
\{\{Q_5,Q_3\},\{(1,1),(1,2),(1,4),(1,5),(2,1),(2,2),(2,3),(2,5),(3,2),(3,3),(3,4),(4,1),(4,3),(4,4),(4,5),(5,1),(5,2),(5,4),(5,5)\}\}
\{\{Q_5,Q_4\},\{(1,1),(1,3),(1,4),(1,5),(2,2),(2,4),(2,5),(3,1),(3,3),(3,5),(4,1),(4,2),(4,4),(5,1),(5,2),(5,3),(5,5)\}
```