E.B.1.2 (FOL: Studenti ansiosi, inferenza)

$\mathcal{P}$ = { Studente/1, Corso/1, *HaStudiato*/2, Ansioso/1, *Supera*/2}

$$
\begin{aligned}
&\forall\ X\ \forall\ C \\
&\quad (\\
&\qquad \text{Studente}(X) \land \\
&\qquad \text{Corso}(C) \land \\
&\qquad (\text{Ansioso}(X) \lor \neg \textit{HaStudiato}(X, C)) \\
&\quad) \rightarrow \\
&\qquad \neg \textit{Supera}(X, C)
\end{aligned}
$$

Di seguito la riduzione a CNF della formula sopra

$$
\begin{aligned}
&(\text{Studente}(X) \land \text{Corso}(C) \land (\text{Ansioso}(X) \lor \neg \textit{HaStudiato}(X, C))) \rightarrow \neg \textit{Supera}(X, C) = \\
&\qquad \{A \rightarrow B \equiv \neg A \lor B\} \\
&\neg(\text{Studente}(X) \land \text{Corso}(C) \land (\text{Ansioso}(X) \lor \neg \textit{HaStudiato}(X, C)) \lor \neg \textit{Supera}(X, C) = \\
&\qquad \{\text{De Morgan}\} \\
&\neg \text{Studente}(X) \lor \neg \text{Corso}(C) \lor \neg(\text{Ansioso}(X) \lor \neg \textit{HaStudiato}(X, C)) \lor \neg \textit{Supera}(X, C) = \\
&\qquad \{\text{De Morgan}\} \\
&\neg \text{Studente}(X) \lor \neg \text{Corso}(C) \lor (\neg \text{Ansioso}(X) \land \textit{HaStudiato}(X, C)) \lor \neg \textit{Supera}(X, C) = \\
&\qquad \{\text{Associatività di } \lor\} \\
&\neg \text{Studente}(X) \lor \neg \text{Corso}(C) \lor \neg \textit{Supera}(X, C) \lor (\neg \text{Ansioso}(X) \land \textit{HaStudiato}(X, C)) = \\
&\qquad \{\text{Distributività di } \lor\} \\
&\quad (\neg \text{Studente}(X) \lor \neg \text{Corso}(C) \lor \neg \textit{Supera}(X, C) \lor \neg \text{Ansioso}(X)) \land \\
&(\neg \text{Studente}(X) \lor \neg \text{Corso}(C) \lor \neg \textit{Supera}(X, C) \lor \textit{HaStudiato}(X, C))
\end{aligned}
$$

1.1 Inferenza A

1.1.1 KB di partenza

Data la knowledge-base di partenza KB t.c.

$$
\begin{aligned}
&\mathsf{KB} = \{\\
&\quad \exists\ \mathsf{X}\ \mathsf{Studente}(\mathsf{X}) \land \mathsf{Ansioso}(\mathsf{X}),\\
&\quad \exists\ \mathsf{X}\ \exists\ \mathsf{C}\ \mathsf{Studente}(\mathsf{X}) \land \mathsf{Corso}(\mathsf{C}) \land \mathit{HaStudiato}(\mathsf{X}, \mathsf{C}),\\
&\quad \forall\ \mathsf{X}\ \forall\ \mathsf{C}\\
&\qquad (\\
&\qquad\quad \mathsf{Studente}(\mathsf{X}) \land\\
&\qquad\quad \mathsf{Corso}(\mathsf{C}) \land\\
&\qquad\quad (\mathsf{Ansioso}(\mathsf{X}) \lor \neg \mathit{HaStudiato}(\mathsf{X}, \mathsf{C}))\\
&\qquad) \rightarrow\\
&\qquad\quad \neg \mathit{Supera}(\mathsf{X}, \mathsf{C})\\
&\}
\end{aligned}
$$

È vero che $\mathsf{KB} \models (\forall\ \mathsf{X}\ \forall\ \mathsf{C}\ \mathsf{Studente}(\mathsf{X}) \land \mathsf{Corso}(\mathsf{C}) \rightarrow \neg \mathit{Supera}(\mathsf{X}, \mathsf{C}))$?

1.1.2 CNF

$$
\begin{aligned}
&\alpha =\\
&(\forall\ \mathsf{X}\ \forall\ \mathsf{C}\ \mathsf{Studente}(\mathsf{X}) \land \mathsf{Corso}(\mathsf{C}) \rightarrow \neg \mathit{Supera}(\mathsf{X}, \mathsf{C})) =\\
&\qquad \{\text{Semplificazione}\}\\
&(\mathsf{Studente}(\mathsf{X}) \land \mathsf{Corso}(\mathsf{C})) \rightarrow \neg \mathit{Supera}(\mathsf{X}, \mathsf{C}) =\\
&\qquad \{\mathsf{A} \rightarrow \mathsf{B} \equiv \neg \mathsf{A} \lor \mathsf{B}\}\\
&\neg(\mathsf{Studente}(\mathsf{X}) \land \mathsf{Corso}(\mathsf{C})) \lor \neg \mathit{Supera}(\mathsf{X}, \mathsf{C}) =\\
&\qquad \{\text{De Morgan}\}\\
&\neg \mathsf{Studente}(\mathsf{X}) \lor \neg \mathsf{Corso}(\mathsf{C}) \lor \neg \mathit{Supera}(\mathsf{X}, \mathsf{C})\\
&\quad \{\text{La negazione si ottiene con De Morgan}\}\\
&\neg\alpha = \mathsf{Studente}(\mathsf{X}) \land \mathsf{Corso}(\mathsf{C}) \land \mathit{Supera}(\mathsf{X}, \mathsf{C})
\end{aligned}
$$

$$
\begin{aligned}
&\mathsf{KB}_{\mathsf{CNF}} = \{\\
&\quad \mathsf{Studente}(S_1),\\
&\quad \mathsf{Ansioso}(S_1),\\
&\\
&\quad \mathsf{Studente}(S_2),\\
&\quad \mathsf{Corso}(C_1),\\
&\quad \mathit{HaStudiato}(S_2, C_1),\\
&\\
&\quad (\neg \mathsf{Studente}(\mathsf{X}) \lor \neg \mathsf{Corso}(\mathsf{C}) \lor \neg \mathit{Supera}(\mathsf{X}, \mathsf{C}) \lor \neg \mathsf{Ansioso}(\mathsf{X})),\\
&\quad (\neg \mathsf{Studente}(\mathsf{X}) \lor \neg \mathsf{Corso}(\mathsf{C}) \lor \neg \mathit{Supera}(\mathsf{X}, \mathsf{C}) \lor \mathit{HaStudiato}(\mathsf{X}, \mathsf{C}))\\
&\}
\end{aligned}
$$

Si applica l'algoritmo di risoluzione su

$$\mathtt{KB}_{\mathtt{CNF}} \land \mathtt{Studente(Y)} \land \mathtt{Corso(D)} \land \mathit{Supera}\mathtt{(Y,\ D)}$$

1. $(\neg\mathtt{Studente(X)} \lor \neg\mathtt{Corso(C)} \lor \neg\mathit{Supera}\mathtt{(X,\ C)} \lor \mathit{HaStudiato}\mathtt{(X,\ C)}) \land (\mathtt{Studente}(S_2))$
   $\vDash (\neg\mathtt{Corso(C)} \lor \neg\mathit{Supera}\mathtt{(}S_2\mathtt{,\ C)} \lor \mathit{HaStudiato}\mathtt{(}S_2\mathtt{,\ C)})$
2. $(\neg\mathtt{Corso(C)} \lor \neg\mathit{Supera}\mathtt{(}S_2\mathtt{,\ C)} \lor \mathit{HaStudiato}\mathtt{(}S_2\mathtt{,\ C)}) \land (\mathit{HaStudiato}\mathtt{(}S_2\mathtt{,\ }C_1\mathtt{)})$
   $\vDash (\neg\mathtt{Corso(}C_1\mathtt{)} \lor \neg\mathit{Supera}\mathtt{(}S_2\mathtt{,\ }C_1\mathtt{)})$
3. $(\neg\mathtt{Corso(}C_1\mathtt{)} \lor \neg\mathit{Supera}\mathtt{(}S_2\mathtt{,\ }C_1\mathtt{)}) \land (\mathtt{Corso(}C_1\mathtt{)})$
   $\vDash \neg\mathit{Supera}\mathtt{(}S_2\mathtt{,\ }C_1\mathtt{)}$
4. $(\neg\mathit{Supera}\mathtt{(}S_2\mathtt{,\ }C_1\mathtt{)}) \land (\mathit{Supera}\mathtt{(Y,\ D)})$
   $\vDash ()$

Avendo trovato la clausola vuota, si ha che $\mathtt{KB}_{\mathtt{CNF}} \land \neg\alpha$ non è soddisfacibile, quindi $\mathtt{KB}_{\mathtt{CNF}} \vDash \alpha$

1.2 Inferenza B

1.2.1 KB di partenza

Data la knowledge-base di partenza KB t.c.

$$
\begin{aligned}
&\mathtt{KB} = \{\\
&\quad \exists\ \mathtt{X}\ \mathtt{Studente(X)} \land \neg \mathtt{Ansioso(X)},\\
&\quad \forall\ \mathtt{X}\ \exists\ \mathtt{C}\ \mathtt{Studente(X)} \rightarrow (\mathtt{Corso(C)} \land \mathit{HaStudiato}(\mathtt{X}, \mathtt{C})),\\
&\quad \forall\ \mathtt{X}\ \forall\ \mathtt{C}\\
&\qquad (\\
&\qquad\quad \mathtt{Studente(X)} \land\\
&\qquad\quad \mathtt{Corso(C)} \land\\
&\qquad\quad (\mathtt{Ansioso(X)} \lor \neg \mathit{HaStudiato}(\mathtt{X}, \mathtt{C}))\\
&\qquad) \rightarrow\\
&\qquad\quad \neg \mathit{Supera}(\mathtt{X}, \mathtt{C})\\
&\}
\end{aligned}
$$

È vero che $\mathtt{KB} \vDash (\forall\ \mathtt{X}\ \exists\ \mathtt{C}\ \mathtt{Studente(X)} \rightarrow (\mathtt{Corso(C)} \land \mathit{Supera}(\mathtt{X}, \mathtt{C})))$?

1.2.2 CNF

$$
\begin{gathered}
\forall\ \mathtt{X}\ \exists\ \mathtt{C}\ \mathtt{Studente(X)} \rightarrow (\mathtt{Corso(C)} \land \mathit{HaStudiato}(\mathtt{X}, \mathtt{C})) =\\
\{\text{Semplificazione}\}\\
\mathtt{Studente(X)} \rightarrow (\mathtt{Corso}(C_1(\mathtt{X})) \land \mathit{HaStudiato}(\mathtt{X}, C_1(\mathtt{X}))) =\\
\{\mathtt{A} \rightarrow \mathtt{B} \equiv \neg \mathtt{A} \lor \mathtt{B}\}\\
\neg \mathtt{Studente(X)} \lor (\mathtt{Corso}(C_1(\mathtt{X})) \land \mathit{HaStudiato}(\mathtt{X}, C_1(\mathtt{X}))) =\\
\{\text{Distributività di } \lor\}\\
(\neg \mathtt{Studente(X)} \lor \mathtt{Corso}(C_1(\mathtt{X}))) \land\\
(\neg \mathtt{Studente(X)} \lor \mathit{HaStudiato}(\mathtt{X}, C_1(\mathtt{X})))
\end{gathered}
$$

$$
\begin{gathered}
\alpha =\\
\forall\ \mathtt{X}\ \exists\ \mathtt{C}\ \mathtt{Studente(X)} \rightarrow (\mathtt{Corso(C)} \land \mathit{Supera}(\mathtt{X}, \mathtt{C})) =\\
\{\text{Semplificazione}\}\\
\mathtt{Studente(X)} \rightarrow (\mathtt{Corso}(C_2(\mathtt{X})) \land \mathit{Supera}(\mathtt{X}, C_2(\mathtt{X}))) =\\
\{\mathtt{A} \rightarrow \mathtt{B} \equiv \neg \mathtt{A} \lor \mathtt{B}\}\\
\neg \mathtt{Studente(X)} \lor (\mathtt{Corso}(C_2(\mathtt{X})) \land \mathit{Supera}(\mathtt{X}, C_2(\mathtt{X}))) =\\
\{\text{Distributività di } \lor\}\\
(\neg \mathtt{Studente(X)} \lor \mathtt{Corso}(C_2(\mathtt{X}))) \land\\
(\neg \mathtt{Studente(X)} \lor \mathit{Supera}(\mathtt{X}, C_2(\mathtt{X})))\\
\{\text{La negazione si ottiene con De Morgan}\}\\
\neg\alpha =\\
\neg(\neg \mathtt{Studente(X)} \lor \mathtt{Corso}(C_2(\mathtt{X}))) \lor\\
\neg(\neg \mathtt{Studente(X)} \lor \mathit{Supera}(\mathtt{X}, C_2(\mathtt{X}))) =\\
\{\text{De Morgan}\}\\
(\mathtt{Studente(X)} \land \neg \mathtt{Corso}(C_2(\mathtt{X}))) \lor\\
(\mathtt{Studente(X)} \land \neg \mathit{Supera}(\mathtt{X}, C_2(\mathtt{X}))) =\\
\{\text{Distributività}\}
\end{gathered}
$$

$$
\begin{gathered}
((\text{Studente}(X) \land \neg\text{Corso}(C_2(X))) \lor \text{Studente}(X)) \land \\
((\text{Studente}(X) \land \neg\text{Corso}(C_2(X))) \lor \neg \textit{Supera}(X, C_2(X))) = \\
\{\text{Distributività}\} \\
(\text{Studente}(X) \lor \text{Studente}(X)) \land \\
(\neg\text{Corso}(C_2(X)) \lor \text{Studente}(X)) \land \\
(\text{Studente}(X) \lor \neg \textit{Supera}(X, C_2(X))) \land \\
(\neg\text{Corso}(C_2(X)) \lor \neg \textit{Supera}(X, C_2(X))) \\
\{\text{Semplificazione + ridenominazione}\} \\
\text{Studente}(Y) \land \\
(\neg\text{Corso}(C_2(Y)) \lor \text{Studente}(Y)) \land \\
(\text{Studente}(Y) \lor \neg \textit{Supera}(Y, C_2(Y))) \land \\
(\neg\text{Corso}(C_2(Y)) \lor \neg \textit{Supera}(Y, C_2(Y)))
\end{gathered}
$$

$$
\begin{aligned}
&\text{KB}_{\text{CNF}} = \{ \\
&\quad \text{Studente}(S_1), \\
&\quad \neg\text{Ansioso}(S_1), \\
&\quad \\
&\quad (\neg\text{Studente}(Z) \lor \text{Corso}(C_1(Z))), \\
&\quad (\neg\text{Studente}(Z) \lor \textit{HaStudiato}(Z, C_1(Z))), \\
&\quad \\
&\quad (\neg\text{Studente}(X) \lor \neg\text{Corso}(C) \lor \neg \textit{Supera}(X, C) \lor \neg\text{Ansioso}(X)), \\
&\quad (\neg\text{Studente}(X) \lor \neg\text{Corso}(C) \lor \neg \textit{Supera}(X, C) \lor \textit{HaStudiato}(X, C)) \\
&\}
\end{aligned}
$$

Si applica l'algoritmo di risoluzione su

$$\text{KB}_{\text{CNF}} \land \neg\alpha$$

1. La ricerca fallisce con il solver, quindi dovrei provare tutte le clausole finché non se ne possono aggiungere altre.

1.3 Prover9 & Mace4

1.3.1 Inferenza A

```
formulas(sos).
    (exists x Studente(x) & Ansioso(x)).
    (all x (exists c Studente(x) & Corso(c) & HaStudiato(x, c))).
    (all x all c ((Studente(x) & Corso(c) & (Ansioso(x) | -HaStudiato(x, c))) -> -Supera(x, c))).
end_of_list.

formulas(goals).
    (all x all c (Studente(x) & Corso(c)) ->  -Supera(x, c)).
end_of_list.
```

Il solver è in grado di dimostrare il goal

```
  1 (exists x Studente(x)) & Ansioso(x) # label(non_clause).  [assumption].
  2 (all x ((exists c Studente(x)) & Corso(c) & HaStudiato(x,c))) # label(non_clause).
  [assumption].
  3 (all x all c (Studente(x) & Corso(c) & (Ansioso(x) | -HaStudiato(x,c)) -> -Supera(x,c))) #
  label(non_clause).  [assumption].
  4 (all x all c (Studente(x) & Corso(c))) -> -Supera(x,c) # label(non_clause) # label(goal).
  [goal].
  5 -Studente(x) | -Corso(y) | -Ansioso(x) | -Supera(x,y).  [clausify(3)].
  7 Studente(x).  [clausify(2)].
  11 Ansioso(x).  [clausify(1)].
  12 -Corso(x) | -Ansioso(y) | -Supera(y,x).  [resolve(5,a,7,a)].
  14 Corso(c).  [clausify(2)].
  18 -Corso(x) | -Supera(y,x).  [resolve(12,b,11,a)].
  20 Supera(c1,c).  [deny(4)].
  22 -Supera(x,c).  [resolve(18,a,14,a)].
  24 $F.  [resolve(22,a,20,a)].
```

1.3.2 Inferenza B

```
formulas(sos).
    (exists x Studente(x) & -Ansioso(x)).
    (all x Studente(x) -> (exists c Corso(c) & HaStudiato(x, c))).
    (all x all c ((Studente(x) & Corso(c) & (Ansioso(x) | -HaStudiato(x, c))) -> -Supera(x, c))).
end_of_list.

formulas(goals).
    (all x (exists c Studente(x) -> (Corso(c) & Supera(x, c)))).
end_of_list.
```

La ricerca fallisce.