E.A.6.9 (Cards)

1.1 Modellazione

Dati i paramentri n, k siano
- $\mathcal{N} = \{1, ..., n\}$
- $\mathcal{K} = \{1, ..., k\}$
- $\mathcal{P} = \{1, ..., n \cdot k\}$
- $\mathrm{LP} = \left\{X_{n,k,p} \mid n \in \mathcal{N} \land k \in \mathcal{K} \land p \in \mathcal{P}\right\}$ l'insieme di lettere proposizionali t.c.
  - $X_{n,k,p}$ è vera se la k-esima variabile di valore n è in posizione p

Il problema si può modellare con una serie di vincoli

$$\phi = \phi_{\text{ALO\_pos}} \land \phi_{\text{AMO\_pos}} \land \phi_{\text{dist\_1}} \land \phi_{\text{dist\_2}} \land \phi_{\text{alldiff}} \tag{1}$$

(ALO) Ogni carta ha almeno una posizione.

$$\phi_{\text{ALO\_pos}} = \bigwedge_{\substack{n \in \mathcal{N} \\ k \in \mathcal{K}}} \bigvee_{p \in \mathcal{P}} X_{n,k,p} \tag{2}$$

(AMO) Ogni carta ha al più una posizione.

$$\phi_{\text{AMO\_pos}} = \bigwedge_{\substack{n \in \mathcal{N} \\ k \in \mathcal{K} \\ p_1, p_2 \in \mathcal{P} \\ p_1 < p_2}} X_{n,k,p_1} \to \neg X_{n,k,p_2} \tag{3}$$

Ogni carta contrassegnata dal numero n deve essere in posizione tale da avere esattamente n carte che la dividono dalla precedente carta contrassegnata dal numero n (se esiste).

$$\phi_{\text{dist\_1}} = \bigwedge_{\substack{n \in \mathcal{N} \\ k \in \mathcal{K} \setminus \{K\} \\ p \in \mathcal{P} \\ p+n+1 \in \mathcal{P}}} X_{n,k,p} \to X_{n,k+1,p+n+1} \tag{4}$$

Bisogna restringere le posizioni possibili per una carta in modo che la carta successiva (dello lo stesso valore) possa essere posizionata.

$$\phi_{\text{dist\_2}} = \bigwedge_{\substack{n \in \mathcal{N} \\ k \in \mathcal{K} \setminus \{K\} \\ p \in \mathcal{P} \\ p+n+1 \notin \mathcal{P}}} \neg X_{n,k,p} \tag{5}$$

(alldifferent) Tutte le carte devono avere una posizione diversa

$$\phi_{\text{alldiff}} = \bigwedge_{\substack{n_1, n_2 \in \mathcal{N} \\ k_1, k_2 \in \mathcal{N} \\ p \in \mathcal{P} \\ (n_1, k_1) < (n_2, k_2)}} X_{n_1,k_1,p} \to \neg X_{n_2,k_2,p} \tag{6}$$

1.2 Istanziazione

1.2.1 Parametri e variabili

$(n, k) = (4, 2)$

$$\begin{aligned}\text{LP} = \{ \\ & X_{1,1,1}, X_{1,1,2}, X_{1,1,3}, X_{1,1,4}, X_{1,1,5}, X_{1,1,6}, X_{1,1,7}, X_{1,1,8} \\ & X_{1,2,1}, X_{1,2,2}, X_{1,2,3}, X_{1,2,4}, X_{1,2,5}, X_{1,2,6}, X_{1,2,7}, X_{1,2,8} \\ & X_{2,1,1}, X_{2,1,2}, X_{2,1,3}, X_{2,1,4}, X_{2,1,5}, X_{2,1,6}, X_{2,1,7}, X_{2,1,8} \\ & X_{2,2,1}, X_{2,2,2}, X_{2,2,3}, X_{2,2,4}, X_{2,2,5}, X_{2,2,6}, X_{2,2,7}, X_{2,2,8} \\ & X_{3,1,1}, X_{3,1,2}, X_{3,1,3}, X_{3,1,4}, X_{3,1,5}, X_{3,1,6}, X_{3,1,7}, X_{3,1,8} \\ & X_{3,2,1}, X_{3,2,2}, X_{3,2,3}, X_{3,2,4}, X_{3,2,5}, X_{3,2,6}, X_{3,2,7}, X_{3,2,8} \\ & X_{4,1,1}, X_{4,1,2}, X_{4,1,3}, X_{4,1,4}, X_{4,1,5}, X_{4,1,6}, X_{4,1,7}, X_{4,1,8} \\ & X_{4,2,1}, X_{4,2,2}, X_{4,2,3}, X_{4,2,4}, X_{4,2,5}, X_{4,2,6}, X_{4,2,7}, X_{4,2,8} \\ \} \end{aligned} \tag{7}$$

1.2.2 Vincoli

(ALO) Ogni carta ha almeno una posizione.

$$\begin{aligned}\phi_{\text{ALO\_pos}} = \\ & (X_{1,1,1} \lor X_{1,1,2} \lor X_{1,1,3} \lor X_{1,1,4} \lor X_{1,1,5} \lor X_{1,1,6} \lor X_{1,1,7} \lor X_{1,1,8}) \land \\ & (X_{1,2,1} \lor X_{1,2,2} \lor X_{1,2,3} \lor X_{1,2,4} \lor X_{1,2,5} \lor X_{1,2,6} \lor X_{1,2,7} \lor X_{1,2,8}) \land \\ & (X_{2,1,1} \lor X_{2,1,2} \lor X_{2,1,3} \lor X_{2,1,4} \lor X_{2,1,5} \lor X_{2,1,6} \lor X_{2,1,7} \lor X_{2,1,8}) \land \\ & (X_{2,2,1} \lor X_{2,2,2} \lor X_{2,2,3} \lor X_{2,2,4} \lor X_{2,2,5} \lor X_{2,2,6} \lor X_{2,2,7} \lor X_{2,2,8}) \land \\ & (X_{3,1,1} \lor X_{3,1,2} \lor X_{3,1,3} \lor X_{3,1,4} \lor X_{3,1,5} \lor X_{3,1,6} \lor X_{3,1,7} \lor X_{3,1,8}) \land \\ & (X_{3,2,1} \lor X_{3,2,2} \lor X_{3,2,3} \lor X_{3,2,4} \lor X_{3,2,5} \lor X_{3,2,6} \lor X_{3,2,7} \lor X_{3,2,8}) \land \\ & (X_{4,1,1} \lor X_{4,1,2} \lor X_{4,1,3} \lor X_{4,1,4} \lor X_{4,1,5} \lor X_{4,1,6} \lor X_{4,1,7} \lor X_{4,1,8}) \land \\ & (X_{4,2,1} \lor X_{4,2,2} \lor X_{4,2,3} \lor X_{4,2,4} \lor X_{4,2,5} \lor X_{4,2,6} \lor X_{4,2,7} \lor X_{4,2,8}) \end{aligned}$$

(AMO) Ogni carta ha al più una posizione.

$$\begin{aligned}\phi_{\text{AMO\_pos}} = \\ & (X_{1,1,1} \to \neg X_{1,1,2}) \land (X_{1,1,1} \to \neg X_{1,1,3}) \land (X_{1,1,1} \to \neg X_{1,1,4}) \land (X_{1,1,1} \to \neg X_{1,1,5}) \land (X_{1,1,1} \to \neg X_{1,1,6}) \land \\ & (X_{1,1,1} \to \neg X_{1,1,7}) \land (X_{1,1,1} \to \neg X_{1,1,8}) \land (X_{1,1,2} \to \neg X_{1,1,3}) \land (X_{1,1,2} \to \neg X_{1,1,4}) \land (X_{1,1,2} \to \neg X_{1,1,5}) \land \\ & (X_{1,1,2} \to \neg X_{1,1,6}) \land (X_{1,1,2} \to \neg X_{1,1,7}) \land (X_{1,1,2} \to \neg X_{1,1,8}) \land (X_{1,1,3} \to \neg X_{1,1,4}) \land (X_{1,1,3} \to \neg X_{1,1,5}) \land \\ & (X_{1,1,3} \to \neg X_{1,1,6}) \land (X_{1,1,3} \to \neg X_{1,1,7}) \land (X_{1,1,3} \to \neg X_{1,1,8}) \land (X_{1,1,4} \to \neg X_{1,1,5}) \land (X_{1,1,4} \to \neg X_{1,1,6}) \land \\ & (X_{1,1,4} \to \neg X_{1,1,7}) \land (X_{1,1,4} \to \neg X_{1,1,8}) \land (X_{1,1,5} \to \neg X_{1,1,6}) \land (X_{1,1,5} \to \neg X_{1,1,7}) \land (X_{1,1,5} \to \neg X_{1,1,8}) \land \end{aligned}$$

$$(X_{1,1,6} \to \neg X_{1,1,7}) \wedge (X_{1,1,6} \to \neg X_{1,1,8}) \wedge (X_{1,1,7} \to \neg X_{1,1,8}) \wedge (X_{1,2,1} \to \neg X_{1,2,2}) \wedge (X_{1,2,1} \to \neg X_{1,2,3}) \wedge$$
$$(X_{1,2,1} \to \neg X_{1,2,4}) \wedge (X_{1,2,1} \to \neg X_{1,2,5}) \wedge (X_{1,2,1} \to \neg X_{1,2,6}) \wedge (X_{1,2,1} \to \neg X_{1,2,7}) \wedge (X_{1,2,1} \to \neg X_{1,2,8}) \wedge$$
$$(X_{1,2,2} \to \neg X_{1,2,3}) \wedge (X_{1,2,2} \to \neg X_{1,2,4}) \wedge (X_{1,2,2} \to \neg X_{1,2,5}) \wedge (X_{1,2,2} \to \neg X_{1,2,6}) \wedge (X_{1,2,2} \to \neg X_{1,2,7}) \wedge$$
$$(X_{1,2,2} \to \neg X_{1,2,8}) \wedge (X_{1,2,3} \to \neg X_{1,2,4}) \wedge (X_{1,2,3} \to \neg X_{1,2,5}) \wedge (X_{1,2,3} \to \neg X_{1,2,6}) \wedge (X_{1,2,3} \to \neg X_{1,2,7}) \wedge$$
$$(X_{1,2,3} \to \neg X_{1,2,8}) \wedge (X_{1,2,4} \to \neg X_{1,2,5}) \wedge (X_{1,2,4} \to \neg X_{1,2,6}) \wedge (X_{1,2,4} \to \neg X_{1,2,7}) \wedge (X_{1,2,4} \to \neg X_{1,2,8}) \wedge$$
$$(X_{1,2,5} \to \neg X_{1,2,6}) \wedge (X_{1,2,5} \to \neg X_{1,2,7}) \wedge (X_{1,2,5} \to \neg X_{1,2,8}) \wedge (X_{1,2,6} \to \neg X_{1,2,7}) \wedge (X_{1,2,6} \to \neg X_{1,2,8}) \wedge$$
$$(X_{1,2,7} \to \neg X_{1,2,8}) \wedge (X_{2,1,1} \to \neg X_{2,1,2}) \wedge (X_{2,1,1} \to \neg X_{2,1,3}) \wedge (X_{2,1,1} \to \neg X_{2,1,4}) \wedge (X_{2,1,1} \to \neg X_{2,1,5}) \wedge$$
$$(X_{2,1,1} \to \neg X_{2,1,6}) \wedge (X_{2,1,1} \to \neg X_{2,1,7}) \wedge (X_{2,1,1} \to \neg X_{2,1,8}) \wedge (X_{2,1,2} \to \neg X_{2,1,3}) \wedge (X_{2,1,2} \to \neg X_{2,1,4}) \wedge$$
$$(X_{2,1,2} \to \neg X_{2,1,5}) \wedge (X_{2,1,2} \to \neg X_{2,1,6}) \wedge (X_{2,1,2} \to \neg X_{2,1,7}) \wedge (X_{2,1,2} \to \neg X_{2,1,8}) \wedge (X_{2,1,3} \to \neg X_{2,1,4}) \wedge$$
$$(X_{2,1,3} \to \neg X_{2,1,5}) \wedge (X_{2,1,3} \to \neg X_{2,1,6}) \wedge (X_{2,1,3} \to \neg X_{2,1,7}) \wedge (X_{2,1,3} \to \neg X_{2,1,8}) \wedge (X_{2,1,4} \to \neg X_{2,1,5}) \wedge$$
$$(X_{2,1,4} \to \neg X_{2,1,6}) \wedge (X_{2,1,4} \to \neg X_{2,1,7}) \wedge (X_{2,1,4} \to \neg X_{2,1,8}) \wedge (X_{2,1,5} \to \neg X_{2,1,6}) \wedge (X_{2,1,5} \to \neg X_{2,1,7}) \wedge$$
$$(X_{2,1,5} \to \neg X_{2,1,8}) \wedge (X_{2,1,6} \to \neg X_{2,1,7}) \wedge (X_{2,1,6} \to \neg X_{2,1,8}) \wedge (X_{2,1,7} \to \neg X_{2,1,8}) \wedge (X_{2,2,1} \to \neg X_{2,2,2}) \wedge$$
$$(X_{2,2,1} \to \neg X_{2,2,3}) \wedge (X_{2,2,1} \to \neg X_{2,2,4}) \wedge (X_{2,2,1} \to \neg X_{2,2,5}) \wedge (X_{2,2,1} \to \neg X_{2,2,6}) \wedge (X_{2,2,1} \to \neg X_{2,2,7}) \wedge$$
$$(X_{2,2,1} \to \neg X_{2,2,8}) \wedge (X_{2,2,2} \to \neg X_{2,2,3}) \wedge (X_{2,2,2} \to \neg X_{2,2,4}) \wedge (X_{2,2,2} \to \neg X_{2,2,5}) \wedge (X_{2,2,2} \to \neg X_{2,2,6}) \wedge$$
$$(X_{2,2,2} \to \neg X_{2,2,7}) \wedge (X_{2,2,2} \to \neg X_{2,2,8}) \wedge (X_{2,2,3} \to \neg X_{2,2,4}) \wedge (X_{2,2,3} \to \neg X_{2,2,5}) \wedge (X_{2,2,3} \to \neg X_{2,2,6}) \wedge$$
$$(X_{2,2,3} \to \neg X_{2,2,7}) \wedge (X_{2,2,3} \to \neg X_{2,2,8}) \wedge (X_{2,2,4} \to \neg X_{2,2,5}) \wedge (X_{2,2,4} \to \neg X_{2,2,6}) \wedge (X_{2,2,4} \to \neg X_{2,2,7}) \wedge$$
$$(X_{2,2,4} \to \neg X_{2,2,8}) \wedge (X_{2,2,5} \to \neg X_{2,2,6}) \wedge (X_{2,2,5} \to \neg X_{2,2,7}) \wedge (X_{2,2,5} \to \neg X_{2,2,8}) \wedge (X_{2,2,6} \to \neg X_{2,2,7}) \wedge$$
$$(X_{2,2,6} \to \neg X_{2,2,8}) \wedge (X_{2,2,7} \to \neg X_{2,2,8}) \wedge (X_{3,1,1} \to \neg X_{3,1,2}) \wedge (X_{3,1,1} \to \neg X_{3,1,3}) \wedge (X_{3,1,1} \to \neg X_{3,1,4}) \wedge$$
$$(X_{3,1,1} \to \neg X_{3,1,5}) \wedge (X_{3,1,1} \to \neg X_{3,1,6}) \wedge (X_{3,1,1} \to \neg X_{3,1,7}) \wedge (X_{3,1,1} \to \neg X_{3,1,8}) \wedge (X_{3,1,2} \to \neg X_{3,1,3}) \wedge$$
$$(X_{3,1,2} \to \neg X_{3,1,4}) \wedge (X_{3,1,2} \to \neg X_{3,1,5}) \wedge (X_{3,1,2} \to \neg X_{3,1,6}) \wedge (X_{3,1,2} \to \neg X_{3,1,7}) \wedge (X_{3,1,2} \to \neg X_{3,1,8}) \wedge$$
$$(X_{3,1,3} \to \neg X_{3,1,4}) \wedge (X_{3,1,3} \to \neg X_{3,1,5}) \wedge (X_{3,1,3} \to \neg X_{3,1,6}) \wedge (X_{3,1,3} \to \neg X_{3,1,7}) \wedge (X_{3,1,3} \to \neg X_{3,1,8}) \wedge$$
$$(X_{3,1,4} \to \neg X_{3,1,5}) \wedge (X_{3,1,4} \to \neg X_{3,1,6}) \wedge (X_{3,1,4} \to \neg X_{3,1,7}) \wedge (X_{3,1,4} \to \neg X_{3,1,8}) \wedge (X_{3,1,5} \to \neg X_{3,1,6}) \wedge$$
$$(X_{3,1,5} \to \neg X_{3,1,7}) \wedge (X_{3,1,5} \to \neg X_{3,1,8}) \wedge (X_{3,1,6} \to \neg X_{3,1,7}) \wedge (X_{3,1,6} \to \neg X_{3,1,8}) \wedge (X_{3,1,7} \to \neg X_{3,1,8}) \wedge$$
$$(X_{3,2,1} \to \neg X_{3,2,2}) \wedge (X_{3,2,1} \to \neg X_{3,2,3}) \wedge (X_{3,2,1} \to \neg X_{3,2,4}) \wedge (X_{3,2,1} \to \neg X_{3,2,5}) \wedge (X_{3,2,1} \to \neg X_{3,2,6}) \wedge$$
$$(X_{3,2,1} \to \neg X_{3,2,7}) \wedge (X_{3,2,1} \to \neg X_{3,2,8}) \wedge (X_{3,2,2} \to \neg X_{3,2,3}) \wedge (X_{3,2,2} \to \neg X_{3,2,4}) \wedge (X_{3,2,2} \to \neg X_{3,2,5}) \wedge$$
$$(X_{3,2,2} \to \neg X_{3,2,6}) \wedge (X_{3,2,2} \to \neg X_{3,2,7}) \wedge (X_{3,2,2} \to \neg X_{3,2,8}) \wedge (X_{3,2,3} \to \neg X_{3,2,4}) \wedge (X_{3,2,3} \to \neg X_{3,2,5}) \wedge$$
$$(X_{3,2,3} \to \neg X_{3,2,6}) \wedge (X_{3,2,3} \to \neg X_{3,2,7}) \wedge (X_{3,2,3} \to \neg X_{3,2,8}) \wedge (X_{3,2,4} \to \neg X_{3,2,5}) \wedge (X_{3,2,4} \to \neg X_{3,2,6}) \wedge$$
$$(X_{3,2,4} \to \neg X_{3,2,7}) \wedge (X_{3,2,4} \to \neg X_{3,2,8}) \wedge (X_{3,2,5} \to \neg X_{3,2,6}) \wedge (X_{3,2,5} \to \neg X_{3,2,7}) \wedge (X_{3,2,5} \to \neg X_{3,2,8}) \wedge$$
$$(X_{3,2,6} \to \neg X_{3,2,7}) \wedge (X_{3,2,6} \to \neg X_{3,2,8}) \wedge (X_{3,2,7} \to \neg X_{3,2,8}) \wedge (X_{4,1,1} \to \neg X_{4,1,2}) \wedge (X_{4,1,1} \to \neg X_{4,1,3}) \wedge$$
$$(X_{4,1,1} \to \neg X_{4,1,4}) \wedge (X_{4,1,1} \to \neg X_{4,1,5}) \wedge (X_{4,1,1} \to \neg X_{4,1,6}) \wedge (X_{4,1,1} \to \neg X_{4,1,7}) \wedge (X_{4,1,1} \to \neg X_{4,1,8}) \wedge$$

$$(X_{4,1,2} \to \neg X_{4,1,3}) \land (X_{4,1,2} \to \neg X_{4,1,4}) \land (X_{4,1,2} \to \neg X_{4,1,5}) \land (X_{4,1,2} \to \neg X_{4,1,6}) \land (X_{4,1,2} \to \neg X_{4,1,7}) \land$$
$$(X_{4,1,2} \to \neg X_{4,1,8}) \land (X_{4,1,3} \to \neg X_{4,1,4}) \land (X_{4,1,3} \to \neg X_{4,1,5}) \land (X_{4,1,3} \to \neg X_{4,1,6}) \land (X_{4,1,3} \to \neg X_{4,1,7}) \land$$
$$(X_{4,1,3} \to \neg X_{4,1,8}) \land (X_{4,1,4} \to \neg X_{4,1,5}) \land (X_{4,1,4} \to \neg X_{4,1,6}) \land (X_{4,1,4} \to \neg X_{4,1,7}) \land (X_{4,1,4} \to \neg X_{4,1,8}) \land$$
$$(X_{4,1,5} \to \neg X_{4,1,6}) \land (X_{4,1,5} \to \neg X_{4,1,7}) \land (X_{4,1,5} \to \neg X_{4,1,8}) \land (X_{4,1,6} \to \neg X_{4,1,7}) \land (X_{4,1,6} \to \neg X_{4,1,8}) \land$$
$$(X_{4,1,7} \to \neg X_{4,1,8}) \land (X_{4,2,1} \to \neg X_{4,2,2}) \land (X_{4,2,1} \to \neg X_{4,2,3}) \land (X_{4,2,1} \to \neg X_{4,2,4}) \land (X_{4,2,1} \to \neg X_{4,2,5}) \land$$
$$(X_{4,2,1} \to \neg X_{4,2,6}) \land (X_{4,2,1} \to \neg X_{4,2,7}) \land (X_{4,2,1} \to \neg X_{4,2,8}) \land (X_{4,2,2} \to \neg X_{4,2,3}) \land (X_{4,2,2} \to \neg X_{4,2,4}) \land$$
$$(X_{4,2,2} \to \neg X_{4,2,5}) \land (X_{4,2,2} \to \neg X_{4,2,6}) \land (X_{4,2,2} \to \neg X_{4,2,7}) \land (X_{4,2,2} \to \neg X_{4,2,8}) \land (X_{4,2,3} \to \neg X_{4,2,4}) \land$$
$$(X_{4,2,3} \to \neg X_{4,2,5}) \land (X_{4,2,3} \to \neg X_{4,2,6}) \land (X_{4,2,3} \to \neg X_{4,2,7}) \land (X_{4,2,3} \to \neg X_{4,2,8}) \land (X_{4,2,4} \to \neg X_{4,2,5}) \land$$
$$(X_{4,2,4} \to \neg X_{4,2,6}) \land (X_{4,2,4} \to \neg X_{4,2,7}) \land (X_{4,2,4} \to \neg X_{4,2,8}) \land (X_{4,2,5} \to \neg X_{4,2,6}) \land (X_{4,2,5} \to \neg X_{4,2,7}) \land$$
$$(X_{4,2,5} \to \neg X_{4,2,8}) \land (X_{4,2,6} \to \neg X_{4,2,7}) \land (X_{4,2,6} \to \neg X_{4,2,8}) \land (X_{4,2,7} \to \neg X_{4,2,8})$$

Ogni carta contrassegnata dal numero n deve essere in posizione tale da avere esattamente n carte che la dividono dalla precedente carta contrassegnata dal numero n (se esiste)

$$\phi_{\text{dist\_1}} =$$
$$(X_{1,1,1} \to X_{1,2,3}) \land (X_{1,1,2} \to X_{1,2,4}) \land (X_{1,1,3} \to X_{1,2,5}) \land$$
$$(X_{1,1,4} \to X_{1,2,6}) \land (X_{1,1,5} \to X_{1,2,7}) \land (X_{1,1,6} \to X_{1,2,8}) \land$$
$$(X_{2,1,1} \to X_{2,2,4}) \land (X_{2,1,2} \to X_{2,2,5}) \land (X_{2,1,3} \to X_{2,2,6}) \land$$
$$(X_{2,1,4} \to X_{2,2,7}) \land (X_{2,1,5} \to X_{2,2,8}) \land (X_{3,1,1} \to X_{3,2,5}) \land$$
$$(X_{3,1,2} \to X_{3,2,6}) \land (X_{3,1,3} \to X_{3,2,7}) \land (X_{3,1,4} \to X_{3,2,8}) \land$$
$$(X_{4,1,1} \to X_{4,2,6}) \land (X_{4,1,2} \to X_{4,2,7}) \land (X_{4,1,3} \to X_{4,2,8})$$

Bisogna restringere le posizioni possibili per una carta in modo che la carta successiva (con lo stesso valore) possa essere posizionata.

$$\phi_{\text{dist\_2}} =$$
$$\neg X_{1,1,7} \land \neg X_{1,1,8} \land \neg X_{2,1,6} \land \neg X_{2,1,7} \land \neg X_{2,1,8} \land \neg X_{3,1,5} \land \neg X_{3,1,6} \land$$
$$\neg X_{3,1,7} \land \neg X_{3,1,8} \land \neg X_{4,1,4} \land \neg X_{4,1,5} \land \neg X_{4,1,6} \land \neg X_{4,1,7} \land \neg X_{4,1,8}$$

(alldifferent) Tutte le carte devono avere una posizione diversa

$$\phi_{\text{alldiff}} =$$
$$(X_{1,1,1} \to \neg X_{1,2,1}) \land (X_{1,1,2} \to \neg X_{1,2,2}) \land (X_{1,1,3} \to \neg X_{1,2,3}) \land (X_{1,1,4} \to \neg X_{1,2,4}) \land (X_{1,1,5} \to \neg X_{1,2,5}) \land$$
$$(X_{1,1,6} \to \neg X_{1,2,6}) \land (X_{1,1,7} \to \neg X_{1,2,7}) \land (X_{1,1,8} \to \neg X_{1,2,8}) \land (X_{1,1,1} \to \neg X_{2,1,1}) \land (X_{1,1,2} \to \neg X_{2,1,2}) \land$$
$$(X_{1,1,3} \to \neg X_{2,1,3}) \land (X_{1,1,4} \to \neg X_{2,1,4}) \land (X_{1,1,5} \to \neg X_{2,1,5}) \land (X_{1,1,6} \to \neg X_{2,1,6}) \land (X_{1,1,7} \to \neg X_{2,1,7}) \land$$
$$(X_{1,1,8} \to \neg X_{2,1,8}) \land (X_{1,1,1} \to \neg X_{2,2,1}) \land (X_{1,1,2} \to \neg X_{2,2,2}) \land (X_{1,1,3} \to \neg X_{2,2,3}) \land (X_{1,1,4} \to \neg X_{2,2,4}) \land$$

$$
\begin{aligned}
&(X_{1,1,5} \to \neg X_{2,2,5}) \wedge (X_{1,1,6} \to \neg X_{2,2,6}) \wedge (X_{1,1,7} \to \neg X_{2,2,7}) \wedge (X_{1,1,8} \to \neg X_{2,2,8}) \wedge (X_{1,2,1} \to \neg X_{2,1,1}) \wedge \\
&(X_{1,2,2} \to \neg X_{2,1,2}) \wedge (X_{1,2,3} \to \neg X_{2,1,3}) \wedge (X_{1,2,4} \to \neg X_{2,1,4}) \wedge (X_{1,2,5} \to \neg X_{2,1,5}) \wedge (X_{1,2,6} \to \neg X_{2,1,6}) \wedge \\
&(X_{1,2,7} \to \neg X_{2,1,7}) \wedge (X_{1,2,8} \to \neg X_{2,1,8}) \wedge (X_{1,2,1} \to \neg X_{2,2,1}) \wedge (X_{1,2,2} \to \neg X_{2,2,2}) \wedge (X_{1,2,3} \to \neg X_{2,2,3}) \wedge \\
&(X_{1,2,4} \to \neg X_{2,2,4}) \wedge (X_{1,2,5} \to \neg X_{2,2,5}) \wedge (X_{1,2,6} \to \neg X_{2,2,6}) \wedge (X_{1,2,7} \to \neg X_{2,2,7}) \wedge (X_{1,2,8} \to \neg X_{2,2,8}) \wedge \\
&(X_{1,1,1} \to \neg X_{3,1,1}) \wedge (X_{1,1,2} \to \neg X_{3,1,2}) \wedge (X_{1,1,3} \to \neg X_{3,1,3}) \wedge (X_{1,1,4} \to \neg X_{3,1,4}) \wedge (X_{1,1,5} \to \neg X_{3,1,5}) \wedge \\
&(X_{1,1,6} \to \neg X_{3,1,6}) \wedge (X_{1,1,7} \to \neg X_{3,1,7}) \wedge (X_{1,1,8} \to \neg X_{3,1,8}) \wedge (X_{1,1,1} \to \neg X_{3,2,1}) \wedge (X_{1,1,2} \to \neg X_{3,2,2}) \wedge \\
&(X_{1,1,3} \to \neg X_{3,2,3}) \wedge (X_{1,1,4} \to \neg X_{3,2,4}) \wedge (X_{1,1,5} \to \neg X_{3,2,5}) \wedge (X_{1,1,6} \to \neg X_{3,2,6}) \wedge (X_{1,1,7} \to \neg X_{3,2,7}) \wedge \\
&(X_{1,1,8} \to \neg X_{3,2,8}) \wedge (X_{1,2,1} \to \neg X_{3,1,1}) \wedge (X_{1,2,2} \to \neg X_{3,1,2}) \wedge (X_{1,2,3} \to \neg X_{3,1,3}) \wedge (X_{1,2,4} \to \neg X_{3,1,4}) \wedge \\
&(X_{1,2,5} \to \neg X_{3,1,5}) \wedge (X_{1,2,6} \to \neg X_{3,1,6}) \wedge (X_{1,2,7} \to \neg X_{3,1,7}) \wedge (X_{1,2,8} \to \neg X_{3,1,8}) \wedge (X_{1,2,1} \to \neg X_{3,2,1}) \wedge \\
&(X_{1,2,2} \to \neg X_{3,2,2}) \wedge (X_{1,2,3} \to \neg X_{3,2,3}) \wedge (X_{1,2,4} \to \neg X_{3,2,4}) \wedge (X_{1,2,5} \to \neg X_{3,2,5}) \wedge (X_{1,2,6} \to \neg X_{3,2,6}) \wedge \\
&(X_{1,2,7} \to \neg X_{3,2,7}) \wedge (X_{1,2,8} \to \neg X_{3,2,8}) \wedge (X_{1,1,1} \to \neg X_{4,1,1}) \wedge (X_{1,1,2} \to \neg X_{4,1,2}) \wedge (X_{1,1,3} \to \neg X_{4,1,3}) \wedge \\
&(X_{1,1,4} \to \neg X_{4,1,4}) \wedge (X_{1,1,5} \to \neg X_{4,1,5}) \wedge (X_{1,1,6} \to \neg X_{4,1,6}) \wedge (X_{1,1,7} \to \neg X_{4,1,7}) \wedge (X_{1,1,8} \to \neg X_{4,1,8}) \wedge \\
&(X_{1,1,1} \to \neg X_{4,2,1}) \wedge (X_{1,1,2} \to \neg X_{4,2,2}) \wedge (X_{1,1,3} \to \neg X_{4,2,3}) \wedge (X_{1,1,4} \to \neg X_{4,2,4}) \wedge (X_{1,1,5} \to \neg X_{4,2,5}) \wedge \\
&(X_{1,1,6} \to \neg X_{4,2,6}) \wedge (X_{1,1,7} \to \neg X_{4,2,7}) \wedge (X_{1,1,8} \to \neg X_{4,2,8}) \wedge (X_{1,2,1} \to \neg X_{4,1,1}) \wedge (X_{1,2,2} \to \neg X_{4,1,2}) \wedge \\
&(X_{1,2,3} \to \neg X_{4,1,3}) \wedge (X_{1,2,4} \to \neg X_{4,1,4}) \wedge (X_{1,2,5} \to \neg X_{4,1,5}) \wedge (X_{1,2,6} \to \neg X_{4,1,6}) \wedge (X_{1,2,7} \to \neg X_{4,1,7}) \wedge \\
&(X_{1,2,8} \to \neg X_{4,1,8}) \wedge (X_{1,2,1} \to \neg X_{4,2,1}) \wedge (X_{1,2,2} \to \neg X_{4,2,2}) \wedge (X_{1,2,3} \to \neg X_{4,2,3}) \wedge (X_{1,2,4} \to \neg X_{4,2,4}) \wedge \\
&(X_{1,2,5} \to \neg X_{4,2,5}) \wedge (X_{1,2,6} \to \neg X_{4,2,6}) \wedge (X_{1,2,7} \to \neg X_{4,2,7}) \wedge (X_{1,2,8} \to \neg X_{4,2,8}) \wedge (X_{2,1,1} \to \neg X_{2,2,1}) \wedge \\
&(X_{2,1,2} \to \neg X_{2,2,2}) \wedge (X_{2,1,3} \to \neg X_{2,2,3}) \wedge (X_{2,1,4} \to \neg X_{2,2,4}) \wedge (X_{2,1,5} \to \neg X_{2,2,5}) \wedge (X_{2,1,6} \to \neg X_{2,2,6}) \wedge \\
&(X_{2,1,7} \to \neg X_{2,2,7}) \wedge (X_{2,1,8} \to \neg X_{2,2,8}) \wedge (X_{2,1,1} \to \neg X_{3,1,1}) \wedge (X_{2,1,2} \to \neg X_{3,1,2}) \wedge (X_{2,1,3} \to \neg X_{3,1,3}) \wedge \\
&(X_{2,1,4} \to \neg X_{3,1,4}) \wedge (X_{2,1,5} \to \neg X_{3,1,5}) \wedge (X_{2,1,6} \to \neg X_{3,1,6}) \wedge (X_{2,1,7} \to \neg X_{3,1,7}) \wedge (X_{2,1,8} \to \neg X_{3,1,8}) \wedge \\
&(X_{2,1,1} \to \neg X_{3,2,1}) \wedge (X_{2,1,2} \to \neg X_{3,2,2}) \wedge (X_{2,1,3} \to \neg X_{3,2,3}) \wedge (X_{2,1,4} \to \neg X_{3,2,4}) \wedge (X_{2,1,5} \to \neg X_{3,2,5}) \wedge \\
&(X_{2,1,6} \to \neg X_{3,2,6}) \wedge (X_{2,1,7} \to \neg X_{3,2,7}) \wedge (X_{2,1,8} \to \neg X_{3,2,8}) \wedge (X_{2,2,1} \to \neg X_{3,1,1}) \wedge (X_{2,2,2} \to \neg X_{3,1,2}) \wedge \\
&(X_{2,2,3} \to \neg X_{3,1,3}) \wedge (X_{2,2,4} \to \neg X_{3,1,4}) \wedge (X_{2,2,5} \to \neg X_{3,1,5}) \wedge (X_{2,2,6} \to \neg X_{3,1,6}) \wedge (X_{2,2,7} \to \neg X_{3,1,7}) \wedge \\
&(X_{2,2,8} \to \neg X_{3,1,8}) \wedge (X_{2,2,1} \to \neg X_{3,2,1}) \wedge (X_{2,2,2} \to \neg X_{3,2,2}) \wedge (X_{2,2,3} \to \neg X_{3,2,3}) \wedge (X_{2,2,4} \to \neg X_{3,2,4}) \wedge \\
&(X_{2,2,5} \to \neg X_{3,2,5}) \wedge (X_{2,2,6} \to \neg X_{3,2,6}) \wedge (X_{2,2,7} \to \neg X_{3,2,7}) \wedge (X_{2,2,8} \to \neg X_{3,2,8}) \wedge (X_{2,1,1} \to \neg X_{4,1,1}) \wedge \\
&(X_{2,1,2} \to \neg X_{4,1,2}) \wedge (X_{2,1,3} \to \neg X_{4,1,3}) \wedge (X_{2,1,4} \to \neg X_{4,1,4}) \wedge (X_{2,1,5} \to \neg X_{4,1,5}) \wedge (X_{2,1,6} \to \neg X_{4,1,6}) \wedge \\
&(X_{2,1,7} \to \neg X_{4,1,7}) \wedge (X_{2,1,8} \to \neg X_{4,1,8}) \wedge (X_{2,1,1} \to \neg X_{4,2,1}) \wedge (X_{2,1,2} \to \neg X_{4,2,2}) \wedge (X_{2,1,3} \to \neg X_{4,2,3}) \wedge \\
&(X_{2,1,4} \to \neg X_{4,2,4}) \wedge (X_{2,1,5} \to \neg X_{4,2,5}) \wedge (X_{2,1,6} \to \neg X_{4,2,6}) \wedge (X_{2,1,7} \to \neg X_{4,2,7}) \wedge (X_{2,1,8} \to \neg X_{4,2,8}) \wedge \\
&(X_{2,2,1} \to \neg X_{4,1,1}) \wedge (X_{2,2,2} \to \neg X_{4,1,2}) \wedge (X_{2,2,3} \to \neg X_{4,1,3}) \wedge (X_{2,2,4} \to \neg X_{4,1,4}) \wedge (X_{2,2,5} \to \neg X_{4,1,5}) \wedge \\
&(X_{2,2,6} \to \neg X_{4,1,6}) \wedge (X_{2,2,7} \to \neg X_{4,1,7}) \wedge (X_{2,2,8} \to \neg X_{4,1,8}) \wedge (X_{2,2,1} \to \neg X_{4,2,1}) \wedge (X_{2,2,2} \to \neg X_{4,2,2}) \wedge
\end{aligned}
$$

$$(X_{2,2,3} \to \neg X_{4,2,3}) \wedge (X_{2,2,4} \to \neg X_{4,2,4}) \wedge (X_{2,2,5} \to \neg X_{4,2,5}) \wedge (X_{2,2,6} \to \neg X_{4,2,6}) \wedge (X_{2,2,7} \to \neg X_{4,2,7}) \wedge$$
$$(X_{2,2,8} \to \neg X_{4,2,8}) \wedge (X_{3,1,1} \to \neg X_{3,2,1}) \wedge (X_{3,1,2} \to \neg X_{3,2,2}) \wedge (X_{3,1,3} \to \neg X_{3,2,3}) \wedge (X_{3,1,4} \to \neg X_{3,2,4}) \wedge$$
$$(X_{3,1,5} \to \neg X_{3,2,5}) \wedge (X_{3,1,6} \to \neg X_{3,2,6}) \wedge (X_{3,1,7} \to \neg X_{3,2,7}) \wedge (X_{3,1,8} \to \neg X_{3,2,8}) \wedge (X_{3,1,1} \to \neg X_{4,1,1}) \wedge$$
$$(X_{3,1,2} \to \neg X_{4,1,2}) \wedge (X_{3,1,3} \to \neg X_{4,1,3}) \wedge (X_{3,1,4} \to \neg X_{4,1,4}) \wedge (X_{3,1,5} \to \neg X_{4,1,5}) \wedge (X_{3,1,6} \to \neg X_{4,1,6}) \wedge$$
$$(X_{3,1,7} \to \neg X_{4,1,7}) \wedge (X_{3,1,8} \to \neg X_{4,1,8}) \wedge (X_{3,1,1} \to \neg X_{4,2,1}) \wedge (X_{3,1,2} \to \neg X_{4,2,2}) \wedge (X_{3,1,3} \to \neg X_{4,2,3}) \wedge$$
$$(X_{3,1,4} \to \neg X_{4,2,4}) \wedge (X_{3,1,5} \to \neg X_{4,2,5}) \wedge (X_{3,1,6} \to \neg X_{4,2,6}) \wedge (X_{3,1,7} \to \neg X_{4,2,7}) \wedge (X_{3,1,8} \to \neg X_{4,2,8}) \wedge$$
$$(X_{3,2,1} \to \neg X_{4,1,1}) \wedge (X_{3,2,2} \to \neg X_{4,1,2}) \wedge (X_{3,2,3} \to \neg X_{4,1,3}) \wedge (X_{3,2,4} \to \neg X_{4,1,4}) \wedge (X_{3,2,5} \to \neg X_{4,1,5}) \wedge$$
$$(X_{3,2,6} \to \neg X_{4,1,6}) \wedge (X_{3,2,7} \to \neg X_{4,1,7}) \wedge (X_{3,2,8} \to \neg X_{4,1,8}) \wedge (X_{3,2,1} \to \neg X_{4,2,1}) \wedge (X_{3,2,2} \to \neg X_{4,2,2}) \wedge$$
$$(X_{3,2,3} \to \neg X_{4,2,3}) \wedge (X_{3,2,4} \to \neg X_{4,2,4}) \wedge (X_{3,2,5} \to \neg X_{4,2,5}) \wedge (X_{3,2,6} \to \neg X_{4,2,6}) \wedge (X_{3,2,7} \to \neg X_{4,2,7}) \wedge$$
$$(X_{3,2,8} \to \neg X_{4,2,8}) \wedge (X_{4,1,1} \to \neg X_{4,2,1}) \wedge (X_{4,1,2} \to \neg X_{4,2,2}) \wedge (X_{4,1,3} \to \neg X_{4,2,3}) \wedge (X_{4,1,4} \to \neg X_{4,2,4}) \wedge$$
$$(X_{4,1,5} \to \neg X_{4,2,5}) \wedge (X_{4,1,6} \to \neg X_{4,2,6}) \wedge (X_{4,1,7} \to \neg X_{4,2,7}) \wedge (X_{4,1,8} \to \neg X_{4,2,8})$$

1.3 Codifica

```rust
use crate::encoder::*;
use serde::Serialize;

#[derive(Clone, Copy, Hash, PartialEq, Eq, PartialOrd, Ord,
Serialize, Debug)]
pub struct X(usize, usize, usize);

pub fn encode_instance(card_k: usize, card_n: usize) -> (String,
Vec<X>) {
    use Literal::Neg;

    let mut encoder = EncoderSAT::new();
    let card_p = card_n * card_k;

    // ALO_pos
    for n in 1..=card_n {
        for k in 1..=card_k {
            encoder.add((1..=card_p).map(|p| X(n, k,
p).into()).collect());
        }
    }

    // AMO_pos
    for n in 1..=card_n {
        for k in 1..=card_k {
            for p1 in 1..=card_p {
                for p2 in p1 + 1..=card_p {
                    encoder.add(vec![Neg(X(n, k, p1)), Neg(X(n,
k, p2))])
                }
            }
        }
    }

    // dist_1 + dist_2
    for n in 1..=card_n {
        for k in 1..card_k {
            for p in 1..=card_p {
                if p + n < card_p {
                    encoder.add(vec![Neg(X(n, k, p)), X(n, k + 1,
p + n + 1).into()])
                } else {
                    encoder.add(vec![Neg(X(n, k, p))]);
                }
            }
        }
    }

    // alldifferent
    for n1 in 1..=card_n {
        for n2 in 1..=card_n {
```

```
            for k1 in 1..=card_k {
                for k2 in 1..=card_k {
                    for p in 1..=card_p {
                        if (n1, k1) < (n2, k2) {
                            encoder.add(vec![Neg(X(n1, k1, p)),
Neg(X(n2, k2, p))]);
                        }
                    }
                }
            }
        }
    }

    encoder.end()
}
```

1.4 Statistiche