E.A.5.2 (n-Queens)

1.1 Modello

Il problema n-Queens si potrebbe codificare come problema CSP nel seguente modo. Siano (X, D, C) t.c.

- $X = \{Q_1, ..., Q_n\}$
- $D = \{D_1, ..., D_n \mid D_i = \{1, ..., n\}\}$
- vincoli (due regine non possono stare sulla stessa riga e non si possono attaccare in diagonale):
  - alldiff$(Q_1, ..., Q_n)$
  - $\langle \{Q_i, Q_j\}, \Delta \rangle$ t.c.
    - $i \neq j \wedge \Delta = \{(r_i, r_j) \mid r_i, r_j = 1, ..., n \wedge |i - j| \neq |r_i - r_j|\}$

1.2 5-Queens

- $X = \{Q_1, Q_2, Q_3, Q_4, Q_5\}$
- $D = \{D_1, D_2, D_3, D_4, D_5 \mid D_i = \{1, 2, 3, 4, 5\}\}$
- vincoli:
  - alldiff$(Q_1, Q_2, Q_3, Q_4, Q_5)$
  - per le diagonali vedere l'ultima pagina...

1.3 Codifica MiniZinc

```
include "alldifferent.mzn";

int: n = 8; /* l'editor va in crash a n = 21 */
array[1..n] of var 1..n: queens;
constraint alldifferent(queens);
constraint forall(i in 1..n, j in (i + 1)..n)(
  abs(queens[j] - queens[i]) ≠ j - i
);
```

Piccola nota: effettivamente il vincolo descritto in MiniZinc non è identico alla definizione in notazione matematica, andrebbe riscritto così:

```
include "alldifferent.mzn";

int: n = 8; /* l'editor va in crash a n = 21 */
array[1..n] of var 1..n: queens;
constraint alldifferent(queens);
constraint forall(i in 1..n, j in 1..n)(
  i ≠ j → abs(queens[j] - queens[i]) ≠ abs(j - i)
);
```

Con $n = 20$ l'esecuzione del primo codice dura circa 821ms, mentre l'esecuzione per la seconda versione dura circa 1s e 516ms.

I vincoli per cui le regine non si possono attaccare in diagonale

$\langle\{Q_1, Q_2\}, \{(1,1),(1,3),(1,4),(1,5),(2,2),(2,4),(2,5),(3,1),(3,3),(3,5),(4,1),(4,2),(4,4),(5,1),(5,2),(5,3),(5,5)\}\rangle$
$\langle\{Q_1, Q_3\}, \{(1,1),(1,2),(1,4),(1,5),(2,1),(2,2),(2,3),(2,5),(3,2),(3,3),(3,4),(4,1),(4,3),(4,4),(4,5),(5,1),(5,2),(5,4),(5,5)\}\rangle$
$\langle\{Q_1, Q_4\}, \{(1,1),(1,2),(1,3),(1,5),(2,1),(2,2),(2,3),(2,4),(3,1),(3,2),(3,3),(3,4),(3,5),(4,2),(4,3),(4,4),(4,5),(5,1),(5,3),(5,4),(5,5)\}\rangle$
$\langle\{Q_1, Q_5\}, \{(1,1),(1,2),(1,3),(1,4),(2,1),(2,2),(2,3),(2,4),(2,5),(3,1),(3,2),(3,3),(3,4),(3,5),(4,1),(4,2),(4,3),(4,4),(4,5),(5,2),(5,3),(5,4),(5,5)\}\rangle$
$\langle\{Q_2, Q_1\}, \{(1,1),(1,3),(1,4),(1,5),(2,2),(2,4),(2,5),(3,1),(3,3),(3,5),(4,1),(4,2),(4,4),(5,1),(5,2),(5,3),(5,5)\}\rangle$
$\langle\{Q_2, Q_3\}, \{(1,1),(1,3),(1,4),(1,5),(2,2),(2,4),(2,5),(3,1),(3,3),(3,5),(4,1),(4,2),(4,4),(5,1),(5,2),(5,3),(5,5)\}\rangle$
$\langle\{Q_2, Q_4\}, \{(1,1),(1,2),(1,4),(1,5),(2,1),(2,2),(2,3),(2,5),(3,2),(3,3),(3,4),(4,1),(4,3),(4,4),(4,5),(5,1),(5,2),(5,4),(5,5)\}\rangle$
$\langle\{Q_2, Q_5\}, \{(1,1),(1,2),(1,3),(1,5),(2,1),(2,2),(2,3),(2,4),(3,1),(3,2),(3,3),(3,4),(3,5),(4,2),(4,3),(4,4),(4,5),(5,1),(5,3),(5,4),(5,5)\}\rangle$
$\langle\{Q_3, Q_1\}, \{(1,1),(1,2),(1,4),(1,5),(2,1),(2,2),(2,3),(2,5),(3,2),(3,3),(3,4),(4,1),(4,3),(4,4),(4,5),(5,1),(5,2),(5,4),(5,5)\}\rangle$
$\langle\{Q_3, Q_2\}, \{(1,1),(1,3),(1,4),(1,5),(2,2),(2,4),(2,5),(3,1),(3,3),(3,5),(4,1),(4,2),(4,4),(5,1),(5,2),(5,3),(5,5)\}\rangle$
$\langle\{Q_3, Q_4\}, \{(1,1),(1,3),(1,4),(1,5),(2,2),(2,4),(2,5),(3,1),(3,3),(3,5),(4,1),(4,2),(4,4),(5,1),(5,2),(5,3),(5,5)\}\rangle$
$\langle\{Q_3, Q_5\}, \{(1,1),(1,2),(1,4),(1,5),(2,1),(2,2),(2,3),(2,5),(3,2),(3,3),(3,4),(4,1),(4,3),(4,4),(4,5),(5,1),(5,2),(5,4),(5,5)\}\rangle$
$\langle\{Q_4, Q_1\}, \{(1,1),(1,2),(1,3),(1,5),(2,1),(2,2),(2,3),(2,4),(3,1),(3,2),(3,3),(3,4),(3,5),(4,2),(4,3),(4,4),(4,5),(5,1),(5,3),(5,4),(5,5)\}\rangle$
$\langle\{Q_4, Q_2\}, \{(1,1),(1,2),(1,4),(1,5),(2,1),(2,2),(2,3),(2,5),(3,2),(3,3),(3,4),(4,1),(4,3),(4,4),(4,5),(5,1),(5,2),(5,4),(5,5)\}\rangle$
$\langle\{Q_4, Q_3\}, \{(1,1),(1,3),(1,4),(1,5),(2,2),(2,4),(2,5),(3,1),(3,3),(3,5),(4,1),(4,2),(4,4),(5,1),(5,2),(5,3),(5,5)\}\rangle$
$\langle\{Q_4, Q_5\}, \{(1,1),(1,3),(1,4),(1,5),(2,2),(2,4),(2,5),(3,1),(3,3),(3,5),(4,1),(4,2),(4,4),(5,1),(5,2),(5,3),(5,5)\}\rangle$
$\langle\{Q_5, Q_1\}, \{(1,1),(1,2),(1,3),(1,4),(2,1),(2,2),(2,3),(2,4),(2,5),(3,1),(3,2),(3,3),(3,4),(3,5),(4,1),(4,2),(4,3),(4,4),(4,5),(5,2),(5,3),(5,4),(5,5)\}\rangle$
$\langle\{Q_5, Q_2\}, \{(1,1),(1,2),(1,3),(1,5),(2,1),(2,2),(2,3),(2,4),(3,1),(3,2),(3,3),(3,4),(3,5),(4,2),(4,3),(4,4),(4,5),(5,1),(5,3),(5,4),(5,5)\}\rangle$
$\langle\{Q_5, Q_3\}, \{(1,1),(1,2),(1,4),(1,5),(2,1),(2,2),(2,3),(2,5),(3,2),(3,3),(3,4),(4,1),(4,3),(4,4),(4,5),(5,1),(5,2),(5,4),(5,5)\}\rangle$
$\langle\{Q_5, Q_4\}, \{(1,1),(1,3),(1,4),(1,5),(2,2),(2,4),(2,5),(3,1),(3,3),(3,5),(4,1),(4,2),(4,4),(5,1),(5,2),(5,3),(5,5)\}\rangle$