E.A.5.3 (Edge Colouring)

1.1 Modellazione

1.1.1 Variabili e domini

Dato un grafo $G = (V, E)$ per ogni arco $\big(v_i, v_j\big) \in E$ si definisce una variabile, per cui:

- $X = \big\{C_{i,j} \mid \big(v_i, v_j\big) \in E\big)\}$, dove $C_{i,j}$ indica il colore dell'arco $\big(v_i, v_j\big)$
- $D = \big\{D_{i,j} \mid \big(v_i, v_j\big) \in E \land D_{i,j} = \{1, 2, 3\}\big\}$

1.1.2 Vincoli

Si ha un triancolo per ogni tripla di vertici connessi

$$\begin{aligned} &\forall i, j, k \\ &\quad \big(\big(v_i, v_j\big) \in E \land (v_i, v_k) \in E \land \big(v_j, v_k\big) \in E\big) \implies \\ &\qquad \big(C_{i,j} = C_{i,k} \implies C_{i,j} \neq C_{j,k}\big) \end{aligned} \tag{1}$$

Se due degli archi del triangolo hanno lo stesso colore, allora il terzo deve essere diverso (per non avere archi tutti dello stesso colore). Nel grafo non diretto il verso non conta, quindi $C_{i,j} = C_{j,i}$.

1.2 Istanziazione

1.2.1 Variabili e domini

Dati A = 1, B = 2, C = 3, D = 4, E = 5, G1 = 6, G2 = 7, H = 8, I = 9, J = 10, S = 11; le variabili sono:

$$\begin{gathered} C_{1,5}, C_{5,1}, C_{1,8}, C_{8,1}, C_{1,9}, C_{9,1}, C_{1,11}, C_{11,1}, \\ C_{2,3}, C_{3,2}, C_{2,7}, C_{7,2}, C_{2,9}, C_{9,2}, C_{2,10}, C_{10,2}, \\ C_{2,11}, C_{11,2}, C_{3,4}, C_{4,3}, C_{3,7}, C_{7,3}, C_{3,11}, C_{11,3}, \\ C_{4,5}, C_{5,4}, C_{4,11}, C_{11,4}, C_{5,6}, C_{6,5}, C_{5,8}, C_{8,5}, \\ C_{6,8}, C_{8,6}, C_{7,10}, C_{10,7}, C_{8,9}, C_{9,8}, : [1, 3] \end{gathered} \tag{2}$$

1.2.2 Vincoli

Variabili che rappresentano lo stesso arco devono avere lo stesso colore

- $\langle\big\{C_{1,5}, C_{5,1}\big\}, C_{1,5} = C_{5,1}\rangle$
- $\langle\big\{C_{1,8}, C_{8,1}\big\}, C_{1,8} = C_{8,1}\rangle$
- $\langle\big\{C_{1,9}, C_{9,1}\big\}, C_{1,9} = C_{9,1}\rangle$
- $\langle\big\{C_{1,11}, C_{11,1}\big\}, C_{1,11} = C_{11,1}\rangle$
- $\langle\big\{C_{2,3}, C_{3,2}\big\}, C_{2,3} = C_{3,2}\rangle$
- $\langle\big\{C_{2,7}, C_{7,2}\big\}, C_{2,7} = C_{7,2}\rangle$
- $\langle\big\{C_{2,9}, C_{9,2}\big\}, C_{2,9} = C_{9,2}\rangle$

- $\langle \{C_{2,10}, C_{10,2}\}, C_{2,10} = C_{10,2} \rangle$
- $\langle \{C_{2,11}, C_{11,2}\}, C_{2,11} = C_{11,2} \rangle$
- $\langle \{C_{3,4}, C_{4,3}\}, C_{3,4} = C_{4,3} \rangle$
- $\langle \{C_{3,7}, C_{7,3}\}, C_{3,7} = C_{7,3} \rangle$
- $\langle \{C_{3,11}, C_{11,3}\}, C_{3,11} = C_{11,3} \rangle$
- $\langle \{C_{4,5}, C_{5,4}\}, C_{4,5} = C_{5,4} \rangle$
- $\langle \{C_{4,11}, C_{11,4}\}, C_{4,11} = C_{11,4} \rangle$
- $\langle \{C_{5,6}, C_{6,5}\}, C_{5,6} = C_{6,5} \rangle$
- $\langle \{C_{5,8}, C_{8,5}\}, C_{5,8} = C_{8,5} \rangle$
- $\langle \{C_{6,8}, C_{8,6}\}, C_{6,8} = C_{8,6} \rangle$
- $\langle \{C_{7,10}, C_{10,7}\}, C_{7,10} = C_{10,7} \rangle$
- $\langle \{C_{8,9}, C_{9,8}\}, C_{8,9} = C_{9,8} \rangle$

Non esistono archi tutti dello stesso colore

- $\langle \{C_{0,4}, C_{0,7}, C_{4,7}\}, C_{0,4} = C_{0,7} \implies C_{0,4} \neq C_{4,7}) \rangle$
- $\langle \{C_{0,7}, C_{0,4}, C_{7,4}\}, C_{0,7} = C_{0,4} \implies C_{0,7} \neq C_{7,4}) \rangle$
- $\langle \{C_{0,7}, C_{0,8}, C_{7,8}\}, C_{0,7} = C_{0,8} \implies C_{0,7} \neq C_{7,8}) \rangle$
- $\langle \{C_{0,8}, C_{0,7}, C_{8,7}\}, C_{0,8} = C_{0,7} \implies C_{0,8} \neq C_{8,7}) \rangle$
- $\langle \{C_{1,2}, C_{1,6}, C_{2,6}\}, C_{1,2} = C_{1,6} \implies C_{1,2} \neq C_{2,6}) \rangle$
- $\langle \{C_{1,2}, C_{1,10}, C_{2,10}\}, C_{1,2} = C_{1,10} \implies C_{1,2} \neq C_{2,10}) \rangle$
- $\langle \{C_{1,6}, C_{1,2}, C_{6,2}\}, C_{1,6} = C_{1,2} \implies C_{1,6} \neq C_{6,2}) \rangle$
- $\langle \{C_{1,6}, C_{1,9}, C_{6,9}\}, C_{1,6} = C_{1,9} \implies C_{1,6} \neq C_{6,9}) \rangle$
- $\langle \{C_{1,9}, C_{1,6}, C_{9,6}\}, C_{1,9} = C_{1,6} \implies C_{1,9} \neq C_{9,6}) \rangle$
- $\langle \{C_{1,10}, C_{1,2}, C_{10,2}\}, C_{1,10} = C_{1,2} \implies C_{1,10} \neq C_{10,2}) \rangle$
- $\langle \{C_{2,1}, C_{2,6}, C_{1,6}\}, C_{2,1} = C_{2,6} \implies C_{2,1} \neq C_{1,6}) \rangle$
- $\langle \{C_{2,1}, C_{2,10}, C_{1,10}\}, C_{2,1} = C_{2,10} \implies C_{2,1} \neq C_{1,10}) \rangle$
- $\langle \{C_{2,3}, C_{2,10}, C_{3,10}\}, C_{2,3} = C_{2,10} \implies C_{2,3} \neq C_{3,10}) \rangle$
- $\langle \{C_{2,6}, C_{2,1}, C_{6,1}\}, C_{2,6} = C_{2,1} \implies C_{2,6} \neq C_{6,1}) \rangle$
- $\langle \{C_{2,10}, C_{2,1}, C_{10,1}\}, C_{2,10} = C_{2,1} \implies C_{2,10} \neq C_{10,1}) \rangle$
- $\langle \{C_{2,10}, C_{2,3}, C_{10,3}\}, C_{2,10} = C_{2,3} \implies C_{2,10} \neq C_{10,3}) \rangle$
- $\langle \{C_{3,2}, C_{3,10}, C_{2,10}\}, C_{3,2} = C_{3,10} \implies C_{3,2} \neq C_{2,10}) \rangle$
- $\langle \{C_{3,10}, C_{3,2}, C_{10,2}\}, C_{3,10} = C_{3,2} \implies C_{3,10} \neq C_{10,2}) \rangle$
- $\langle \{C_{4,0}, C_{4,7}, C_{0,7}\}, C_{4,0} = C_{4,7} \implies C_{4,0} \neq C_{0,7}) \rangle$
- $\langle \{C_{4,5}, C_{4,7}, C_{5,7}\}, C_{4,5} = C_{4,7} \implies C_{4,5} \neq C_{5,7}) \rangle$
- $\langle \{C_{4,7}, C_{4,0}, C_{7,0}\}, C_{4,7} = C_{4,0} \implies C_{4,7} \neq C_{7,0}) \rangle$
- $\langle \{C_{4,7}, C_{4,5}, C_{7,5}\}, C_{4,7} = C_{4,5} \implies C_{4,7} \neq C_{7,5}) \rangle$
- $\langle \{C_{5,4}, C_{5,7}, C_{4,7}\}, C_{5,4} = C_{5,7} \implies C_{5,4} \neq C_{4,7}) \rangle$
- $\langle \{C_{5,7}, C_{5,4}, C_{7,4}\}, C_{5,7} = C_{5,4} \implies C_{5,7} \neq C_{7,4}) \rangle$
- $\langle \{C_{6,1}, C_{6,2}, C_{1,2}\}, C_{6,1} = C_{6,2} \implies C_{6,1} \neq C_{1,2}) \rangle$
- $\langle \{C_{6,1}, C_{6,9}, C_{1,9}\}, C_{6,1} = C_{6,9} \implies C_{6,1} \neq C_{1,9}) \rangle$
- $\langle \{C_{6,2}, C_{6,1}, C_{2,1}\}, C_{6,2} = C_{6,1} \implies C_{6,2} \neq C_{2,1}) \rangle$
- $\langle \{C_{6,9}, C_{6,1}, C_{9,1}\}, C_{6,9} = C_{6,1} \implies C_{6,9} \neq C_{9,1}) \rangle$
- $\langle \{C_{7,0}, C_{7,4}, C_{0,4}\}, C_{7,0} = C_{7,4} \implies C_{7,0} \neq C_{0,4}) \rangle$
- $\langle \{C_{7,0}, C_{7,8}, C_{0,8}\}, C_{7,0} = C_{7,8} \implies C_{7,0} \neq C_{0,8}) \rangle$

- $\langle \{C_{7,4}, C_{7,0}, C_{4,0}\}, C_{7,4} = C_{7,0} \implies C_{7,4} \neq C_{4,0}) \rangle$
- $\langle \{C_{7,4}, C_{7,5}, C_{4,5}\}, C_{7,4} = C_{7,5} \implies C_{7,4} \neq C_{4,5}) \rangle$
- $\langle \{C_{7,5}, C_{7,4}, C_{5,4}\}, C_{7,5} = C_{7,4} \implies C_{7,5} \neq C_{5,4}) \rangle$
- $\langle \{C_{7,8}, C_{7,0}, C_{8,0}\}, C_{7,8} = C_{7,0} \implies C_{7,8} \neq C_{8,0}) \rangle$
- $\langle \{C_{8,0}, C_{8,7}, C_{0,7}\}, C_{8,0} = C_{8,7} \implies C_{8,0} \neq C_{0,7}) \rangle$
- $\langle \{C_{8,7}, C_{8,0}, C_{7,0}\}, C_{8,7} = C_{8,0} \implies C_{8,7} \neq C_{7,0}) \rangle$
- $\langle \{C_{9,1}, C_{9,6}, C_{1,6}\}, C_{9,1} = C_{9,6} \implies C_{9,1} \neq C_{1,6}) \rangle$
- $\langle \{C_{9,6}, C_{9,1}, C_{6,1}\}, C_{9,6} = C_{9,1} \implies C_{9,6} \neq C_{6,1}) \rangle$
- $\langle \{C_{10,1}, C_{10,2}, C_{1,2}\}, C_{10,1} = C_{10,2} \implies C_{10,1} \neq C_{1,2}) \rangle$
- $\langle \{C_{10,2}, C_{10,1}, C_{2,1}\}, C_{10,2} = C_{10,1} \implies C_{10,2} \neq C_{2,1}) \rangle$
- $\langle \{C_{10,2}, C_{10,3}, C_{2,3}\}, C_{10,2} = C_{10,3} \implies C_{10,2} \neq C_{2,3}) \rangle$
- $\langle \{C_{10,3}, C_{10,2}, C_{3,2}\}, C_{10,3} = C_{10,2} \implies C_{10,3} \neq C_{3,2}) \rangle$

1.3 Codifica in MiniZinc

```
set of int: V = 1..11;
array [V, V] of var 1..3: C;

any: E = [|
    /* ... matrice di adiacenza del grafo, dove E[i, j]
indica che c'è un arco da v_i a v_j */
|];

constraint  forall(i in V, j in V, k in V)(
    (E[k, i] /\ E[k, j] /\ E[i, j]) → (C[k, i] == C[k, j] →
C[k, i] ≠ C[i, j])
);

constraint  forall(i in V, j in V)(
    C[i, j] == C[j, i]
);
```

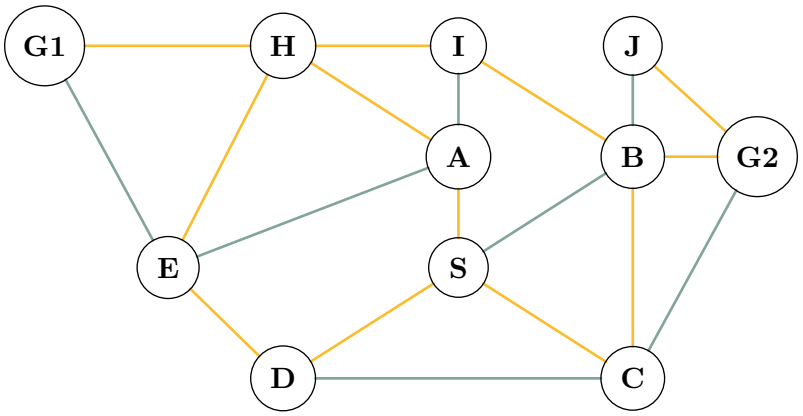

Figura 1: soluzione generata da `MiniZinc`