Software Engineering

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Contents

1 Software models

Software projects require **design choices** that often can't be driven by experience or reasoning alone. That's why a **model** of the project is needed to compare different solutions. In this course, to describe software systems, we use **discrete time Markov chains**.

1.1 The "Amazon Prime Video" article

If you were tasked with designing the software architecture for **Amazon Prime Video** *(a live streaming service for Amazon)*, how would you go about it? What if you had the **non-functional requirement** to keep the costs minimal? Would you use distributed services?

More often than not, monolith applications are considered **more costly** than the counterpart due to an inefficient usage of resources. But, in a recent article, a Senior SDE at Prime Video describes how they *"**reduced the cost** of the audio/video monitoring infrastructure by **90%**"* [1] by using a monolith application instead of distributed microservices.

While there isn't always definitive answer, one way to go about this kind choice is building a model of the system to compare the solutions. In the case of Prime Video, *"the audio/video monitoring service consists of three major components:"* [1]

- the **media converter** converts input audio/video streams
- the **defect detectors** analyze frames and audio buffers in real-time
- the **orchestrator** controls the flow in the service

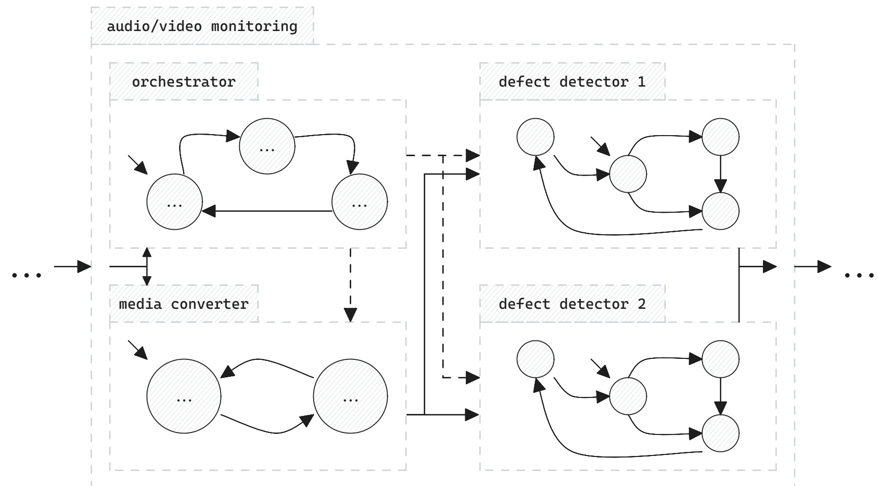


Figure 1: Model of the audio/video monitoring system

The system can be **simulated** by modeling its components as **connected probabilistic stateful automatons**.

1.2 Formal notation

1.2.1 Markov chains

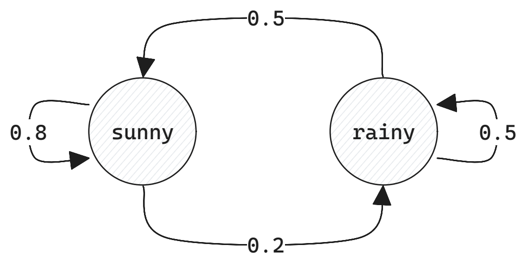
A Markov Chain is defined by a set of **states** S and the **transition probability** $p : S \times S \to [0, 1]$ s.t.

$$\forall s \in S \quad \sum_{s' \in S} p(s'|s) = 1 \tag{1}$$

A simple model of the weather could be $S = \{\text{sunny}, \text{rainy}\}$

$p =$

	sunny	rainy
sunny	0.8	0.2
rainy	0.5	0.5



Unfortunately for us, Markov chains aren't enough: to describe complex systems (e.g. a server) we need the concepts of **input**, **output** and **time**.

1.2.2 DTMC (Discrete Time Markov Chains)

A DTMC M is a tuple (U, X, Y, p, g) s.t.

- U, X and Y aren't empty *(otherwise stuff doesn't work)*
- U is the set of **input values**
- X is the set of **states**
- Y is the set of **output values**
- $p : X \times X \times U \to [0, 1]$ is the **transition probability**
- $g : X \to Y$ is the **output function**

The same constrain in Equation 1 holds for the DTMC, with an important difference: the **probability depends on the input value**. This means that **for each input value**, the sum of the probabilities to transition for **that input value** must be 1.

$$\forall x \in X \ \ \forall u \in U \quad \sum_{x' \in X} p(x'|x, u) = 1 \tag{2}$$

Let M be a DTMC, let t be a time **instant** and d a time **interval**

$$\begin{aligned} X(0) &= x_0 \\ X(t+d) &= \begin{cases} x_0 & \text{with probability} \ \ p(x_0 \mid X(t), U(t)) \\ x_1 & \text{with probability} \ \ p(x_1 \mid X(t), U(t)) \\ \dots \end{cases} \end{aligned} \tag{3}$$

(TODO: I don't like it…) We denote with $U(t)$ the **input value** of M at time t (the same goes for $X(t)$ and $Y(t)$), yada yada…

1.2.2.1 An example of DTMC

Let's consider the development process of a team. We can define a DTMC $M = (U, X, Y, p, g)$ s.t.

- $U = \{()\}$, as it doesn't have any input
- $X = \{0, 1, 2, 3\}$
- $Y = \text{Cost} \times \text{Duration (in months)}$

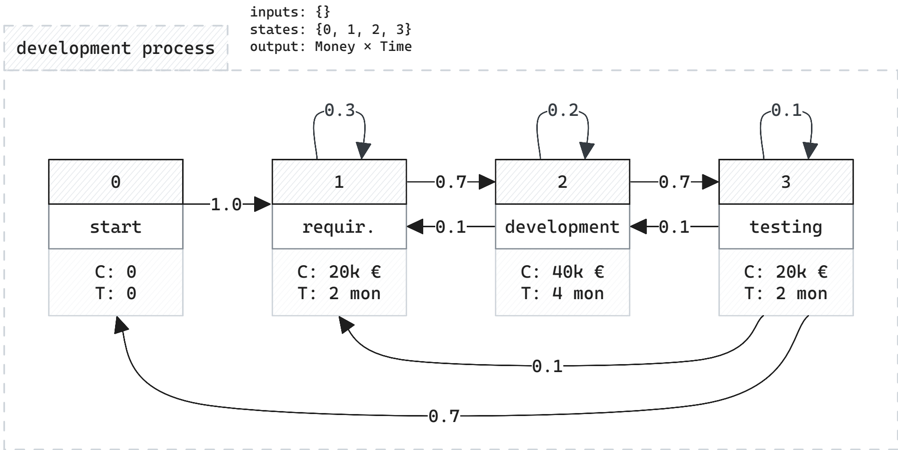


Figure 2: the model of a team's development process

$$g(x) = \begin{cases} (0, 0) & \text{if} \quad x = 0 \\ (20000, 2) & \text{if} \quad x = 1 \\ (40000, 4) & \text{if} \quad x = 2 \\ (20000, 2) & \text{if} \quad x = 3 \end{cases} \tag{4}$$

1.2.3 Network of Markov Chains

TODO...

1.3 Tips and tricks

1.3.1 Calculate average incrementally

Given a set of values $V = \{v_1, ..., v_n\}$ s.t. $|V| = n$ the average ε_n is s.t.

$$\varepsilon_n = \frac{\sum_{i=0}^{n} v_i}{n} \tag{5}$$

The problem with this method is that, by adding up all the values before the division, the **numerator** could easily **overflow**, as the biggest integer we can represent precisely with singe-precision floating-point number is 16777216 [2].

There is a way to calculate ε_{n+1} given ε_n

$$\begin{aligned}
\varepsilon_{n+1} = \frac{\sum_{i=0}^{n+1} v_i}{n+1} = \frac{\left(\sum_{i=0}^{n} v_i\right) + v_{n+1}}{n+1} = \frac{\sum_{i=0}^{n} v_i}{n+1} + \frac{v_{n+1}}{n+1} = \\
= \frac{\left(\sum_{i=0}^{n} v_i\right) n}{(n+1)n} + \frac{v_{n+1}}{n+1} = \frac{\sum_{i=0}^{n} v_i}{n} \cdot \frac{n}{n+1} + \frac{v_{n+1}}{n+1} = \frac{\varepsilon_n \cdot n + v_{n+1}}{n+1}
\end{aligned} \tag{6}$$

1.3.2 Euler's method for differential equations

Got it from here baby [3]

2 C++

This section will cover the basics needed for the exam.

2.1 How to use #include <random>

The <random> library offers useful tools to build our models. It makes Markov chains and probabilistic models really easy to implement.

2.1.1 random() vs random_device vs default_random_engine

In C++ there are many ways to **generate random numbers**, I'm gonna keep it short and sweet: don't use random(), use std::random_device to generate the **seed** and from then on just some random engine like std::default_random_engine.

```
#include <iostream>
#include <random>

int main() {
    std::cout << random() ① << std::endl;

    std::random_device random_device; ②
    std::cout << random_device() ③ << std::endl;

    std::default_random_engine r_engine(random_device() ④ );
    std::cout << r_engine() ⑤ << std::endl;
}
```

Listing 1: examples/random.cpp

I'll explain: random() ① doesn't work very well (TODO: link to the article*), you're encouraged to use the generators C++ offers... but each works in a different way, and you have to choose the best one depending on your needs. std::random_device ② is used to generate the **seed** ④ because it uses device randomness (if available, TODO: link to docs / article *), but it's really slow. That **seed** is used to to instantiate one of the other engines ⑤, like ::default_random_engine (TODO: link with different engines and types). TODO BONUS: only r_eng used with distr

Note:
If you see ③ and ⑤, random_device and r_engine aren't functions, they are instances, but you can call the () operator on them because C++ has operator overloading, and it allows you to call custom operators on instances... the same goes for std::cout and <<

2.1.2 Distributions

Just the capability of generating random numbers isn't enough, we often need to manipulate those numbers to fit our needs. Luckly, `C++` covers basically all of them… for example, this is how easy it is to simulate a transition matrix in Figure 2:

```cpp
#include <iostream>
#include <random>

const size_t HORIZON = 15;

int main() {
    std::random_device random_device;
    std::default_random_engine random_engine(random_device());

    std::discrete_distribution<size_t> transition_matrix[] = {
        {0, 1},
        {0, .3, .7},
        {0, .2, .2, .6},
        {0, .1, .2, .1, .6},
        {1},
    };

    size_t state = 0;
    for (size_t time = 0; time < HORIZON; time++) {
        state = transition_matrix[state](random_engine);
        std::cout << state << std::endl;
    }
}
```

Listing 2: `examples/transition_matrix.cpp`

2.1.2.1 `std::uniform_int_distribution`

2.1.2.2 `std::uniform_real_distribution`

2.1.2.3 `std::bernoulli_distribution`

2.1.2.4 `std::poisson_distribution`

2.1.2.5 `std::geometric_distribution`

2.1.2.6 `std::discrete_distribution`

2.2 Dynamic structures

2.2.1 `std::vector<T>()` instead of `malloc()`

2.2.2 `std::deque<T>()`

2.2.3 Sets

2.2.4 Maps

2.3 I/O

2.3.1 #include <iostream>

2.3.2 Files

3 Exercises

Each exercise has 4 digits `xxxx` that are the same as the ones in the `software` folder in the course material.

3.1 `[1000]` First examples

Now we have to put together our **formal definitions** and our `C++` knowledge to build some simple DTMCs and networks.

3.1.1 `[1100]` A simple Markov chain

Let's begin our modeling journey by implementing a DTMC M s.t.

- $U = \{()\}$ s.t. '()' is the **empty tuple**
- $X = [0,1] \times [0,1]$, each state is a pair ③ of **real** ① numbers in $[0,1]$
- $Y = [0,1] \times [0,1]$
- $p : X \times X \times U \to X = \mathcal{U}(0,1) \times \mathcal{U}(0,1)$, the transition probability is a **uniform** distribution ②
- $g : X \to Y : (r_0, r_1) \mapsto (r_0, r_1)$ outputs the current state ④
- $X(0) = (0,0)$ ③

```
/* ... */

using real_t ① = double;
const size_t HORIZON = 10;

int main() {
    std::random_device random_device;
    std::default_random_engine random_engine(random_device());
    std::uniform_real_distribution<real_t> uniform(0, 1); ②

    std::vector<real_t> state(2, 0); ③
    std::ofstream log("log");

    for (size_t time = 0; time <= HORIZON; time++) {
        for (auto &r : state) r = uniform(random_engine); ②
        log << time << ' ';
        for (auto r : state) log << r << ' '; t ④
        log << std::endl;
    }

    log.close();
    return 0;
}
```

Listing 3: `software/1100/main.cpp`

3.1.2 [1200] Connect Markov chains pt.1

In this exercise we build 2 DTMCs M_0, M_1 like the one in the first example Section 3.1.1, with the difference that, and $U_i = [0,1] \times [0,1]$:

- $U_0(t+d) = Y_1(t)$
- $U_1(t+d) = Y_0(t)$

TODO: formula to get input from other stuff and calculate the state..., maybe define

$$\begin{aligned} p : X \times X \times U \to [0,1] \\ (x_0, x_1), (x_0', x_1'), (u_0, u_1) \mapsto ... \end{aligned} \tag{7}$$

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```cpp
const size_t HORIZON = 100;
struct DTMC { real_t state[2]; };

int main() {
    std::vector<DTMC> dtmcs(2, {0, 0});

    for (size_t time = 0; time <= HORIZON; time++) {
        for (size_t r = 0; r < 2; r++) {
            dtmcs[0].state[r] =
                dtmcs[1].state[r] * uniform(random_engine);
            dtmcs[1].state[r] =
                dtmcs[0].state[r] + uniform(random_engine);
        }
    }
}
```

Listing 4: `software/1200/main.cpp`

3.1.3 [1300] Connect Markov Chains pt.2

The same as above, but with a different connection

```cpp
int main() {
    std::vector<DTMC> dtmcs({{1, 1}, {2, 2}});

    for (size_t time = 0; time <= HORIZON; time++) {
        dtmcs[0].state[0] =
            .7 * dtmcs[0].state[0] + .7 * dtmcs[0].state[1];
        dtmcs[0].state[1] =
            -.7 * dtmcs[0].state[0] + .7 * dtmcs[0].state[1];

        dtmcs[1].state[0] =
            dtmcs[1].state[0] + dtmcs[1].state[1];
        dtmcs[1].state[1] =
            -dtmcs[1].state[0] + dtmcs[1].state[1];
```

```
    }
}
```

Listing 5: `software/1300/main.cpp`

3.1.4 `[1400]` Connect Markov Chains pt.3

The same as above, but with a different connection

3.2 `[2000]` Traffic light

In this example we want to model a **traffic light**. The three versions of the system on the drive (`2100`, `2200` and `2300`) do the same thing with a different code structure.

```
const size_t HORIZON = 1000;
enum Light { GREEN = 0, YELLOW = 1, RED = 2 };

int main() {
    auto random_timer_duration =
        std::uniform_int_distribution<>(60, 120);

    Light traffic_light = Light::RED;
    size_t timer = random_timer_duration(random_engine);

    for (size_t time = 0; time <= HORIZON; time++) {
        if (timer > 0) {
            timer--;
            continue;
        }

        traffic_light =
            (traffic_light == RED
                 ? GREEN
                 : (traffic_light == GREEN ? YELLOW : RED));
        timer = random_timer_duration(random_engine);
    }
}
```

Listing 6: `software/2000/main.cpp`

3.3 `[3000]` Control center

3.3.1 `[3100]` No network

3.3.2 `[3200]` Network monitor (no faults)

3.3.3 `[3300]` Network monitor (faults, no repair)

3.3.4 `[3400]` Network monitor (faults, repair)

3.3.5 [3500] Network monitor (faults, repair, correct protocol)

3.4 [4000] Statistics

3.4.1 [4100] Expected value

3.4.2 [4200] Probability

3.5 [5000] Transition matrix

One of the ways to implement a Markov Chain (like in Section 1.2.1) is by using a **transition matrix**. The simplest implemenation can be done by using a `std::discrete_distribution` by using the trick in Listing 2.

3.5.1 [5100] Random transition matrix

In this example we build a **random transition matrix**.

```
#include <fstream>
#include <random>
#include <vector>

using real_t = double;
const size_t HORIZON = 20, STATES_SIZE = 10;

int main() {
    std::random_device random_device;
    std::default_random_engine random_engine(random_device());
    auto random_state = (1)
        std::uniform_int_distribution<>(0, STATES_SIZE - 1);
    std::uniform_real_distribution<> random_real_0_1(0, 1);

    std::vector<std::discrete_distribution<>>
        transition_matrix(STATES_SIZE); (2)
    std::ofstream log("log.csv");

    for (size_t state = 0; state < STATES_SIZE; state++) {
        std::vector<real_t> weights(STATES_SIZE); (3)
        for (auto &weight : weights)
            weight = random_real_0_1(random_engine);

        transition_matrix[state] = (4)
            std::discrete_distribution<>(weights.begin(),
                                         weights.end());
    }

    size_t state = random_state(random_engine);
    for (size_t time = 0; time <= HORIZON; time++) {
        log << time << " " << state << std::endl;
        state = transition_matrix[state (5) ](random_engine); (6)
    }

    log.close();
    return 0;
}
```

Listing 7: `software/5100/main.cpp`

A **transition matrix** is a `vector<discrete_distribution<>>` ② just like in Listing 2. Why can we do this? First of all, the states are numbered from `0` to `STATES_SIZE - 1`, that's why we can generate a random state ① just by generating a number from `0` to `STATES_SIZE - 1`.

The problem with using a simple `uniform_int_distribution` is that we don't want to choose the next state uniformly, we want to do something like in Figure 3.

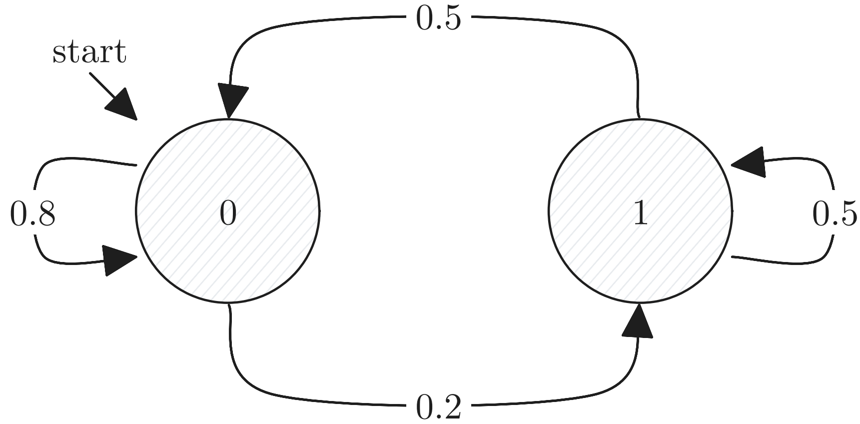

Figure 3: A simple Markov Chain

Luckly for us `std::discrete_distribution<>` does exactly what we want. It takes a list of weights $w_0, w_1, w_2, ..., w_n$ and assigns each index i the probability $p(i) = \frac{\sum_{i=0}^{n} w_i}{w_i}$ (the probability is proportional to the weight, so we have that $\sum_{i=0}^{n} p(i) = 1$ like we would expect in a Markov Chain).

To instantiate the `discrete_distribution` ④, unlike in Listing 2, we need to first calculate the weights ③, as we don't know them in advance.

To randomly generate the next state ⑥ we just have to use the `discrete_distribution` assigned to the current state ⑤.

3.5.2 `[5200]` Software development & error detection

Our next goal is to model the software development process of a team. Each phase takes the team 4 days to complete, and, at the end of each phase the testing team tests the software, and there can be 3 outcomes:

- **no error** is introduced during the phase (we can't actually know it, let's suppose there is an all-knowing "oracle" that can tell us there aren't any errors)
- **no error detected** means that the "oracle" detected an error, but the testing team wasn't able to find it

- **error detected** means that the "oracle" detected an error, and the testing team was able to find it

If we have **no error**, we proceed to the next phase... the same happens if **no error was detected** (because the testing team sucks and didn't find any errors). If we **detect an error** we either reiterate the current phase (with a certain probability, let's suppose 0.8), or we go back to one of the previous phases with equal probability (we do this because, if we find an error, there's a high chance it was introduced in the current phase, and we want to keep the model simple).

In this exercise we take the parameters for each phase (the probability to introduce an error and the probability to not detect an error) from a file.

```
#include <...>

using real_t = double;
const size_t HORIZON = 800, PHASES_SIZE = 3;

enum Outcome (1) {
    NO_ERROR = 0,
    NO_ERROR_DETECTED = 1,
    ERROR_DETECTED = 2
};

int main() {
    std::random_device random_device;
    std::default_random_engine urng(random_device());
    std::uniform_real_distribution<> uniform_0_1(0, 1);
    std::vector<std::discrete_distribution<>>
        phases_error_distribution;

    {
        std::ifstream probabilities("probabilities.csv");
        real_t probability_error_introduced,
            probability_error_not_detected;

        while (probabilities >> probability_error_introduced >>
                probability_error_not_detected)
            phases_error_distribution.push_back(
                (2) std::discrete_distribution<>({
                    1 - probability_error_introduced,
                    probability_error_introduced *
                        probability_error_not_detected,
                    probability_error_introduced *
                        (1 - probability_error_not_detected),
                }));
```

```
        probabilities.close();
        assert(phases_error_distribution.size() ==
                PHASES_SIZE);
    }

    real_t probability_repeat_phase = 0.8;

    size_t phase = 0;
    std::vector<size_t> progress(PHASES_SIZE, 0);
    std::vector<Outcome> outcomes(PHASES_SIZE, NO_ERROR);

    for (size_t time = 0; time < HORIZON; time++) {
        progress[phase]++;

        if (progress[phase] == 4) {
            outcomes[phase] = static_cast<Outcome>(
                phases_error_distribution[phase](urng));
            switch (outcomes[phase]) {
            case NO_ERROR:
            case NO_ERROR_DETECTED:
                phase++;
                break;
            case ERROR_DETECTED:
                if (phase > 0 && uniform_0_1(urng) >
                                    probability_repeat_phase)
                    phase = std::uniform_int_distribution<>(
                        0, phase - 1)(urng);
                break;
            }

            if (phase == PHASES_SIZE)
                break;

            progress[phase] = 0;
        }
    }

    return 0;
}
```

Listing 8: `software/5300/main.cpp`

TODO: `class enum` vs `enum`. We can model the outcomes as an `enum` ①... we can use the `discrete_distribution` trick to choose randomly one of the outcomes ②. The other thing we notice is that we take the probabilities to generate an error and to detect it from a file.

3.5.3 `[5300]` Optimizing costs for the development team

If we want we can manipulate the "parameters" in real life: a better experienced team has a lower probability to introduce an error, but a higher cost. What we can do is:

1. randomly generate the parameters (probability to introduce an error and to not detect it)
2. simulate the development process with the random parameters

By repeating this a bunch of times, we can find out which parameters have the best results, a.k.a generate the lowest development times (there are better techinques like simulated annealing, but this one is simple enough for us).

3.5.4 `[5400]` Key performance index

We can repeat the process in exercise `[5300]`, but this time we can assign a parameter a certain cost, and see which parameters optimize cost and time (or something like that? Idk, I should look up the code again).

3.6 `[6000]` Complex systems

3.6.1 `[6100]` Insulin pump

3.6.2 `[6200]` Buffer

3.6.3 `[6300]` Server

4 Exam

4.1 Development team (time & cost)

4.2 Backend load balancing

4.3 Heater simulation

5 CASE library

TODO...

Bibliography

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