Taxes, Transfers and RCE in OLG

The Big Picture. This week we reinforce what we have studied thus far:

1. Relation to competitive equilibrium; failure of first welfare theorem
2. Competitive equilibrium with transfer: second welfare theorem

These are groundwork which pave the way for analyses on the role of government (redistributive) policies in the OLG environment.

Please prepare your answers before attending tutorials.

Assesssable Homework 1. Consider a partial order (preferences) over the consumption set $\mathbb{R}_{++} := \{c \in \mathbb{R} | c > 0\}$, represented by the iso-elastic family of utility functions $U(\cdot; \theta)$:

$$U(c;\theta) = \frac{c^{1-\theta}}{1-\theta} - \frac{1}{1-\theta}, \qquad \theta > 0.$$

1. One version of this utility function (an all-time favorite in macro applications), is the special case of $\lim_{\theta \to 1} U(c;\theta) = \ln(c)$. Verify that this is true.
2. If U is interpreted as a per-period utility function, then, verify that these functions exhibit the property of a "constant intertemporal elasticity of substitution", i.e. $\sigma(c) = U'(c)/[U''(c)c] = \sigma$. What is the expression for the constant σ here?

Assesssable Homework 2. Consider agents with lifetime preference representations:

$$U(c_t) + \beta U(d_{t+1}),$$

where $U(x) = \ln(x)$, for $x > 0$, $\beta \in (0,1)$, c_t is young-age consumption, and d_t is consumption of the date-t old. Each young agent is endowed with 1 unit of time. Define total resources at time-t, given state k_t, as

$$\tilde{f}(k_t) = k_t^{\alpha} + (1-\delta)k_t,$$

where $\alpha \in (0,1)$ and $\delta \in (0,1]$. Assume initial per-worker capital stock k_0 is given. The population of young agents, of measure N_t, grow at a constant rate $n > -1$. Assume N_0 is given. All agents are prices takers—i.e. they take the real wage w_t and gross rental rate R_t as given. Denote R^e_{t+1} as agents' subjective expectation of next period capital return, where the expectation is formed at each date t. Old agents own the capital stock at time t.

There is lump-sum tax/transfer, $a_t = \tau_t w_t \in \mathbb{R}$, where $\tau_t < 1$ for all $t \in \mathbb{N}$. The transfer to (tax on) the old is $z_t \in \mathbb{R}$. The transfer system is such that, for each period, the budget for the system is balanced.

1. Now characterize a (perfect-foresight) competitive equilibrium and lump sum taxes/transfers. To do so, follow these steps:
   (a) Set up the consumer's decision problem for every generation $t \geq 0$. Derive exactly the optimal decision rule(s) for the consumer, given (w_t, R^e_{t+1}).

(b) Set up the firm's decision problem. [Hint: denote k_t^d and N_t^d respectively, as the firm's demand of per-worker capital and labor services at time t.] Derive exactly the firm's best response functions, given w_t and R_t.

(c) Using the final goods market clearing condition and consumers' budget constraint, verify that the capital market clears. Derive what this condition is.

(d) Now state, precisely, what a *perfect-foresight recursive competitive equilibrium with transfers* is, in this example economy.