OLG Economic Policy (Part 3): Pension Systems

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Outline

1. **Signpost**
2. **Pensions**
   - Fully Funded System
   - Pay-as-you-go (PAYG) System
   - Constant Pensions and PAYG
3. **Accumulation**

Overview

- Previously, we considered long-run steady state optimum and competitive equilibria.
- Then, we considered dynamic equilibria, and, dynamic Pareto-optimal allocations.
- We also studied the steady-state limit of optimal allocations: modified golden rule
- Now, two redistributive policy settings:
  1. Decentralization of Pareto allocation if lump sum taxes available: Second Welfare Theorem
  2. Lump-sum transfers and pensions; effect on capital accumulation:
     - Unfunded pensions: PAYG social security
     - Fully funded social security

Pensions

Questions:

- Effect of pension system on capital accumulation?
- Efficiency of competitive equilibrium under redistributive pension systems?

Two main pension systems:

- Fully Funded (FF)
- Pay-as-you-go (PAYG)

Fully Funded System I

Assume:

- Lump-sum taxes a_t levied from young at t, so that:

  $$c_t^y + s_t = w_t - a_t,$$

  is the young's budget constraint.
- Forced saving: a_t invested; returned with interest to $t+1$ old:

  $$c_{t+1}^o = R_{t+1}^e (s_t + a_t),$$

  is the old's budget constraint, expected for $t+1$.
- Perfect capital markets ...

Fully Funded System II

Given (a_t, k_t), and hence $w_t = w(k_t), R^e_{t+1}(k_t; a_t)$ ...

The best-response saving's rule of the date-t young, $\tilde{s}$, is such that saving at (a_t, k_t) is:

$$s_t = \text{arg}\max_{\tilde{s}} \left\{ U(w_t - a_t - \tilde{s}) + \beta U\left[R^e_{t+1}(\tilde{s} + a_t) \right] \right\}.$$

Now, capital market clearing at the end of time-t requires:

$$K_{t+1} = N_t(s_t + a_t).$$

Fully Funded System III

The solution $s_t = \tilde{s}(w_t, R^e_{t+1}, a_t)$ is characterized by the FONC:

$$\frac{U'(w_t - a_t - s_t)}{\beta U'\left[R^e_{t+1}(s_t + a_t)\right]} = R^e_{t+1}.$$

The optimal savings function has the form:

$$\begin{aligned} s_t &= \tilde{s}(w_t, R^e_{t+1}, a_t) \\ &= s(w_t, R^e_{t+1}) - a_t, \end{aligned}$$

where $s(\cdot)$ plays the same role as the optimal savings function in the model without government.

... we have studied properties of $\tilde{s}(w_t, R^e_{t+1}, a_t)$ earlier ....

Fully Funded System IV

$$
\begin{aligned}
s_t &= \tilde{s}(w_t, R^e_{t+1}, a_t) \\
&= s(w_t, R^e_{t+1}) - a_t,
\end{aligned}
$$

Observations:

- At given k_t ...
- Any increase in the contribution to the pension system is exactly offset ...
- by a decrease of the same quantity in private saving ...
- as long as expectations, and therefore, in equilibrium, R^e_{t+1} unchanged.
- Then the fully funded pension system is neutral with respect to (c^y_t, c^o_{t+1}) for all $t \geq 0$.

Fully Funded System V

Result: ... in equilibrium ...

- Since private and government-forced savings command the same relative price, R_{t+1}, they are perfect substitutes.
- FF pension system merely crowds out private saving
- with perfect capital markets, $w_t - a_t$ in equilibrium, need not be be positive, so that $s_t < 0$ is possible ...
- since $c_t^y > 0$ in equilibrium, a negative s_t implies agents are able to borrow against their pension rights — i.e. claims against future wealth.
- Total per-worker investment $a_t + s_t$ is unchanged $\Rightarrow k_{t+1}$ unchanged.

Fully Funded System VI

Proposition

If capital markets are perfect, then the Fully Funded Pension System is allocation and welfare neutral. That is, it affects neither capital accumulation nor lifetime consumption profiles.

Corollary

A recursive competitive equilibrium (RCE) under the Fully Funded Pension System (FF-RCE) is equivalent to a RCE with no transfer system.

Fully Funded System VII

More remarks:

- This FFPS neutrality result was due to Paul Samuelson (1975).
- This result breaks down if:
    - Young agents are borrowing constrained – so that capital markets are imperfect (or more generally, incomplete):
        - E.g. exogenously, agents cannot borrow so that $s_t \geq 0$;
        - Agents can borrow, but there exists limited commitment to contractual obligations in lending (Azariadis and Lambertini, 2000; Kehoe-Levine (1993) problem;
        - Asymmetric information is lending contracts.

    and/or
    - Agent heterogeneity and intra-generational transfers exist: e.g. distorting political-economic redistribution.

PAYG System I

- Consider per-period balanced-budget PAYG system ...
- And, sequence of lump-sum transfers $\{a_t\}_{t\in\mathbb{N}}$ such that

$$z_t = (1+n)a_t \geq 0$$

  for all $t \in \mathbb{N}$.
- Equilibrium with such a PAYG system is equivalent to the economy with positive lump-sum transfers.
- We have characterized the latter before ... so ...

PAYG System II

Definition

A recursive competitive equilibrium with perfect foresight under the PAYG social security system (RCE-PAYG), beginning from a known k_0, is a sequence $\{k_t\}_{t\in\mathbb{N}}$, such that for all $t \in \mathbb{N}$,

- $a_t > 0$,
- $k_t > 0$, and
- $G(k_t, k_{t+1}) :=$
  $(1+n)k_{t+1} - \tilde{s}\left(w(k_t) - a_t, (1+n)a_{t+1}, \tilde{f}'(k_{t+1})\right) = 0.$

PAYG System III

Not all transfers are consistent with a well-defined RCE-PAYG:

- A policy $\{a_t | a_t > 0\}_{t\in\mathbb{N}}$ is *sustainable* if the RCE-PAYG it induces exists.
- A policy which, at some point in time, results in a negative income to the workers, $w(k_t) - a_t < 0$, is said to be unsustainable.

Smallest sustainable initial per-worker capital stock: ...

- Denote the greatest lower bound on initial capital per worker, (k_0), such that $\{a_t\}_{t\in\mathbb{N}}$ is sustainable, as $\underline{k}(a)$.

PAYG System IV

The following result states that:

- if savings are always higher than or equal to the investment required to sustain an arbitrary path of capital, say $\{k_t\}_{t\in\mathbb{N}}$, ...
- then there always exists a RCE with a path of capital higher than $\{k_t\}_{t\in\mathbb{N}}$.
- This result will allow us to define precisely a notion of a *smallest sustainable capital* ... later.

PAYG System V

This result (i.e. next Lemma) is obtained as follows:

1. Suppose we have a sequence $\{k_t\}_{t\in\mathbb{N}}$ such that for all $t \geq 0$:

$$(1+n)k_{t+1} \leq \tilde{s}\Big(w(k_t) - a_t, (1+n)a_{t+1}, \tilde{f}'(k_{t+1})\Big)$$
$$\Rightarrow G(k_t, k_{t+1}) \leq 0,$$

   and, $w(k_t) - a_t$.

2. Consider some $k_0' > k_0$.
   - Note: diminishing marginal product of labor implies $w'(k) < 0$. Also: we showed $\tilde{s}_w > 0$ ... so ...
   - ... it can be shown that $G(k_t, k_{t+1})$ is decreasing in k_t, for fixed k_{t+1}. ...

3. Then, we have $G(k_0', k_1) < G(k_0, k_1) \leq 0$.

PAYG System VI

4. Since $\tilde{s}\left(w(k_t) - a_t > 0, (1+n)a_{t+1}, \tilde{f}'(k_{t+1})\right)$ is bounded above by $w(k_t) - a_t$, then $\lim_{k_1 \to +\infty} G(k_0', k_1) = +\infty$.
5. $G(k_0, k_1)$ is continuous in its arguments. Therefore, there exists some $k_1' \geq k_1$, such that $G(k_0', k_1') = 0$.
6. Now reset $k_1' = k_1$, and repeat logic from step 1. Inductively, we would have proved that there exists a sequence $\{k_t'\}_{t \in N}$ satisfying RCE-PAYG conditions:
   - $a_t > 0$,
   - $k_t > 0$, and
   - $G(k_t, k_{t+1}) = 0$.

   and that it attains a higher capital path than $\{k_t\}_{t \in \mathbb{N}}$.

PAYG System VII

Lemma

Let $\{k_t\}_{t\in\mathbb{N}}$ *be a sequence such that* $G(k_t, k_{t+1}) \leq 0$ *and* $w(k_t) - a_t > 0$, *for all* $t \geq 0$. *There exists a RCE-PAYG sequence* $\{k'_t\}_{t\in N}$ *such that* $k'_t \geq k_t$ *for all* $t \geq 0$.

PAYG System VIII

- The previous result suggests that we can define "sustainable policy" via defining a "lowest sustainable initial per-worker capital".
- Mathematically, we are just working with properties of the set of real numbers: $k \in \mathbb{R}_+$, and the RCE-PAYG conditions enforce a sequence $\{k_t\}_{t \in \mathbb{N}}$ to have an well-defined infimum (or greatest lower bound) for its initial point.

PAYG System IX

Definition

Given $\{a_t | a_t > 0\}_{t \in \mathbb{N}}$, the lowest sustainable initial per-worker capital stock, $\underline{k}$, is the greatest lower bound of the set of all possible initial capital stocks such that a RCE-PAYG exists:

$$\underline{k} = \begin{cases} \inf\{k_0 \in \mathbb{R}_+ : \exists \{k_t | k_t > 0\}_{t \in \mathbb{N}} \text{ s.t. } G(k_t, k_{t+1}) = 0\} \\ +\infty \text{ otherwise} \end{cases}$$

PAYG System X

Proposition (Existence)

- *For all* $k_0 > \underline{k}$, *there exists a RCE-PAYG beginning from* k_0.
- *For all* $k_0 \in (0, \underline{k})$, *there is no RCE-PAYG from* k_0.

Proof:

- Non-existence follows immediately from the definition of $\underline{k}$.
- Existence follows from the previous Lemma.

PAYG System XI

Remarks:

- So a competitive equilibrium under a PAYG social security system can exist.
- The necessary condition of $w(k_t) > a_t$ for all $t \geq 0$ is not easy to verify, in order to arrive at the last proposition.
- Is there another condition that is independent of the equilibrium outcome at each t? i.e. is there a restriction on initial conditions such that a RCE-PAYG exists?
- The answer is in the affirmative. To do so, we defined notions of:
  - "sustainable" policy
  - Sustainable initial capital stock, $k_0 > \underline{k}$.

  Then we proved existence of RCE-PAYG if initially, $k_0 > \underline{k}$.

PAYG and constant pensions I

For more insight, consider constant policies s.t.:

- $(z_t, a_t) = (z, a)$ for all $t \in \mathbb{N}$.
- Balanced budget: $z = (1 + n)a > 0$.

What is the effect of constant policy a on $\underline{k} := \underline{k}(a)$?

PAYG and constant pensions II

Proposition (Property of $\underline{k}(a)$)

The lowest sustainable initial per-worker capital $\underline{k}(a)$ is non-decreasing with respect to constant policy a.

Proof (outline):

- Use definition of $\underline{k}$.
- Property of $\tilde{s}$ such that $\partial G(k_t, k_{t+1}; a)/\partial a > 0$.
- Apply previous Lemma again.

PAYG and constant pensions III

We can now use $\underline{k}(a)$ to prove that there exists a maximal sustainable transfer policy, beyond which there is no RCE-PAYG ...

Proposition (Maximal sustainable policy)

There is a threshold $0 \leq \overline{a} < +\infty$,

$$\overline{a} = \sup\{a \geq 0 : \underline{k}(a) \text{ is finite }\}.$$

That is,

- *for all* $a < \overline{a}$, $\underline{k}(a)$ *is finite, and*
- *for all* $a > \overline{a}$, $\underline{k}(a) = +\infty$, *and no RCE-PAYG exists.*

PAYG and constant pensions IV

Proof:

1. First we show $\underline{k}(a) = +\infty$ for $a > \overline{a}$.
    - Note: $\exists \tilde{k}$ (finite) such that for all $k > \tilde{k}$, $w(k) < (1+n)k$, since $w'(k) < 0$.
    - Suppose $\exists$ a RCE-PAYG $\{k_t\}_{t \in \mathbb{N}}$ with transfer $a = w(\tilde{k})$, i.e. transfer of maximum lifetime income, satisfying:

$$(1+n)k_{t+1} = \tilde{s}\left(w(k_t) - a, (1+n)a, \tilde{f}'(k_t)\right).$$

    - For k_t to be a RCE-PAYG outcome, we necessarily have $w(k_t) > a = w(\tilde{k})$.
    - And for all $k_t > \tilde{k}$, $w(k_t) < (1+n)k_t$ by definition of $\tilde{k}$. Thus we have:

$$(1+n)k_{t+1} < w(k_t) - a < w(k_t) < (1+n)k_t \Rightarrow k_{t+1} < k_t.$$

PAYG and constant pensions V

- The sequence $\{k_t\}$ is decreasing and has a limit $k_\infty \geq \tilde{k}$, since for all t, $k_t \geq \tilde{k}$. The limit k_∞ satisifies $w(k_\infty) \leq (1+n)k_\infty$, and

$$\begin{aligned}(1+n)k_\infty &= \tilde{s}\left(w(k_\infty) - a, (1+n)a, \tilde{f}'(k_\infty)\right) \\ &< w(k_\infty) - a < w(k_\infty) \leq (1+n)k_\infty - a.\end{aligned}$$

- That is we have concluded that $(1+n)k_\infty < (1+n)k_\infty - a$! A contradiction. Thus, $\underline{k}(a) = +\infty$ if $a = w(\tilde{k})$ and trivially, if $a > w(\tilde{k})$. As a result it must be that $\overline{a} < w(\tilde{k})$.
- By the definition of $\overline{a}$, we have for $a > \overline{a}$, $\underline{k}(a) = +\infty$.

PAYG and constant pensions VI

2. Now, we show for $a < \overline{a}$, $\underline{k}(a)$ is finite.
   - for $a < \overline{a}$, $\exists a'$ such that $a < a' \leq \overline{a}$ (by definition of the supremum), such that $\underline{k}(a') < +\infty$.
   - By previous proposition, $\underline{k}(a)$ is nondecreasing so that $\underline{k}(a) \leq \underline{k}(a') < \infty$.

$\blacksquare$

PAYG and constant pensions VII

If we layer another assumption on preferences (A4 in de la Croix and Michel) we can also guarantee that the RCE-PAYG is unique.

Assumption (A4)

The intertemporal elasticity of substitution is bounded below by unity:

$$\sigma(c) = \frac{U'(c)}{U''(c) \cdot c} \geq 1.$$

This is sufficient for the following condition to hold:

PAYG and constant pensions VIII

Assumption (H3a)

For all $k, k' > 0$, such that $k \geq \underline{k}(a)$ and $k' \geq \underline{k}(a)$,

$$G(k, k'; a) = 0 \Rightarrow \frac{\partial G(k, k'; a)}{\partial k'} > 0,$$

i.e. the zero of the RCE-PAYG condition is increasing in next period capital.

PAYG and constant pensions IX

Proposition

Given the last condition, when $\underline{k}(a)$ is positive and finite, it is the smallest positive steady state of the dynamics satisfying RCE-PAYG with constant transfer, $G(k_t, k_{t+1}; a) = 0$.

See example with Cobb-Douglas technology and log utility.

PAYG and constant pensions X

Definition

The compatibility set D_p is the set of (k, a) pairs such that there exists a RCE-PAYG with constant transfer a and initial capital k:

$$D_p = \{(k, a) \in \mathbb{R}^2_+ : k > 0 \text{ and } k \geq \underline{k}(a)\}.$$

PAYG and constant pensions XI

Proposition (Unique RCE-PAYG)

Assume H3a,

H1 *for all* $c > 0$, $U'(c) > 0$, $U''(c) < 0$, *and* $\lim_{c \to 0} = +\infty$, *and*

H2 *for all* $k > 0$, $\tilde{f}'(k) > 0$, $\tilde{f}''(k) < 0$.

Then for any $(k_0, a) \in D_p$, *there exists a unique RCE-PAYG* $\{k_t\}_{t \in \mathbb{N}}$ *with constant pension* a *and initial state* k_0, *such that*

$$G(k_t, k_{t+1}; a) = 0 \Leftrightarrow k_{t+1} = g(k_t; a).$$

Capital accumulation and PAYG I

Proposition

Assume H1, H2, and H3a, for all a. For $(k_0, a) \in D_p$, there exists a unique RCE-PAYG $\{k_t\}_{t\in\mathbb{N}}$ beginning from k_0 with constant pension a, and with long run state $\lim_{t\to\infty} k_t = k$.

- *Following a drop in a to $a' < a$, the RCE-PAYG $\{k'_t\}_{t\in\mathbb{N}}$ starting from k_0 is such that $k'_t > k_t$ for all $t \geq 1$. In the long run, provided $k > 0$, we have $k' > k$.*
- *Following a rise in a to $a'' > a$, either:*
  - *Case $(k_0, a) \in D_p$: there exists a RCE-PAYG $\{k''_t\}_{t\in\mathbb{N}}$ starting from k_0 is such that $k''_t < k_t$ for all $t \geq 1$. In the long run, provided $k > 0$, we have $k'' < k$; or*
  - *Case $(k_0, a) \notin D_p$: there is no longer a RCE-PAYG from k_0.*

Capital accumulation and PAYG II

Remarks:

- Introducing PAYG pension lowers capital stock along transition path and along steady state path.
- This is welfare improving only if the OLG economy has an initial (inefficient) over-accumulation problem.
- Else, the introduction of pension benefits only the first generation old, and is welfare reducing for all subsequent generations.