Monetary Economics with OLG

The Big Picture. We took the OLG model apart and considered the role of money. To keep things tractable without losing the main message, we shut down production and considered instead an endowment general equilibrium environment. The monetary economics of OLG here is a metaphor and mathematically, it is similar to the ideas of Knut Wicksell's triangle. Wicksell's fable was notion of spatial separation of traders and the lack of a single Walrasian market where trades could be supported. In the OLG parable, we may have Walrasian markets in each period but agents do not coexist temporally.

In class we showed that, in an extreme environment, in the absence of any means of saving (e.g. capital or storage), a competitive equilibrium outcome is Pareto inefficient. That is, compared to a hypothetical planner's benevolent allocation, the competitive equilibrium allocation was such that if possible, some agents could be made better off without making some others worse off. We next supposed that old agents were endowed with some $H > 0$ amount of intrinsically worthless currency. Note that no one in the model, including the issuing government, has promised to back this worthless paper by redeeming it for goods. Nevertheless, Samuelson showed that there exists a belief system in equilibrium that makes the currency have indirect value. This was an outcome where currency is backed by expectations that it has value despite no promise of convertibility.

We could also have studied what happens to the demand for money in equilibrium if there were a competing storage mechanism. We get three kinds of possible cases depending on the relative return to money (i.e. gross deflation versus the return on storage). If the relative rate of return to money is larger than one then agents hold only money as the means of intertemporal trade. Otherwise, if this is less than one, then they choose storage. The indeterminate case arises when the two assets have the same return. Then we have indeterminacy in the portfolio composition of the two assets held by agents. With the introduction of money supply growth, and therefore inflation, we next saw that, in general, the monetary equilibrium is no longer Pareto efficient. In the long run, inflation is not superneutral.

In this tutorial we do a few exercises extending on the framework that we have studied in class.

Exercise 0 (DIY Brain Flexercise). You should recall how to rewrite a general constrained optimization problem with inequality constraints and the associated Lagrange-Karush-Kuhn-Tucker theorems. Apply these to verify that you get the Euler (weak) inequalities, which we presented in class as the first-order conditions in the model with money and a competing asset k_t.

Exercise 1. Suppose time is indexed by $t \in \mathbb{N} = \{0, 1, 2...\}$ and all agents live for two periods. There is a continuum of young agents on $[0, 1]$. There is no population growth, so the size of the population of each type of agent differentiated by age is always 1.

Suppose the old agents pre-existing in time $t = 0$ were issued fiat currency $H > 0$ and the young agents in every period $t \geq 1$ young agents purchase M_t^d units of currency at a price $1/P_t$ of units of the time t consumption good. At each $t \geq 1$, each old agent exchanges her holdings of currency for the time t consumption good, c_t^{t-1}. Suppose the young and the old at time $t \geq 1$ are, respectively, endowed with y_t^t and y_t^{t-1}, where the superscripted time indexes refer to the agents period of birth.

1. Write down the sequence of budget constraints for an agent currently young in period $t \geq 1$.
2. **Definition**: A *monetary equilibrium* with valued fiat money is a feasible allocation $\{c_t^t, c_{t+1}^t\}_{t=1}^{\infty}$ supported by a nominal price sequence $\{P_t\}_{t=1}^{\infty}$ such that for all $t \geq 1$, $P_t < +\infty$, the allocation solves the agents' problem for each generation of agents.

Derive the necessary and sufficient conditions for characterizing a *monetary equilibrium*, given the assumption that agent's lifetime preferences are represented by the utility function

$$U(c_t^t) + \beta U(c_{t+1}^t),$$

where U is strictly increasing, strictly concave, and twice continuously differentiable. Provide an economic interpretation of these conditions.

Exercise 2. This follows from the last question. Now suppose $U(c) = \ln(c)$, and that the endowments are constant $y_t^t = e_y > 0$ and $y_{t+1}^t = e_o > 0$. Derive the explicit conditions that describe a monetary equilibrium. What is the sufficient condition for there to exist a monetary equilibrium. Then solve for a monetary equilibrium (i.e. find the trajectory of P_t that satisfies the condition for a monetary equilibrium).