Money: An OLG Interpretation

Timothy Kam

Outline

Motivation

Recall:

- OLG as a kind of dynamic model with within-period heterogeneity.
- Competitive equilibrium of OLG not Pareto efficient in some cases.
- Inefficiency arises from self-interested myopia (Weil, 1989):
  1. time is infinite $t \in \mathbb{N} = \{0, 1, 2, \dots\}$, and
  2. Agents $i = 0, 1, 2, \dots$ are finitely lived.
  3. Dynamic inefficiency: in the long run, steady state k^* can be greater than k_{golden}.

What is Money?

- (Fiat) money: intrinsically worthless object, but accepted as
  1. medium of exchange
  2. unit of account
  3. store of value
- People do not value money in order to consume it.
- They *indirectly* value money because it helps them exchange for goods and services that can be consumed.
- In short, money should not be modelled as delivering direct/primitive utility, but it may yield indirect utility as an (equilibrium) outcome. (a.k.a. "the Wallace dictum").
- So money is not like any other good. (Although historically, commodity money can be consumed too!)

Why Money?

If economies behaved in a "Arrow-Debreu-Walrasian" way:

- Markets are "complete"
  - Meaning: exists a complete set of contingent claims traded against all possible trader-specific events
- Trader histories are not private information
- Contracts are enforceable
- No lack of commitment to honor contractual obligations

Then "money" is an inessential object in equilibrium.

Why Money?

But we do observe trades being conducted where fiat money is means of payment.

- Why?
- What makes people derive indirect utility or value from money?
- Even if they have no direct utility over money?
- What can be plausible explanations?

Modeling Money

Frictions? i.e. Non-existence of Walrasian markets?

- Missing markets
- Spatial separation/Decentralized trading processes – limits to barter exchange
- Information frictions – limits to contracts

A good model capturing monetary phenomena has trading costs as equilibrium outcomes, given frictions in

- trading enviroment, and/or
- information

Frictions should not be assumed in primitive utility descriptions! [c.f. models with money-in-utility; utility cost of shopping].

Roadmap

- Study competitive equilibrium allocation in simple endowment OLG economy.
- Introduce fiat money.
- Derive equilibrium demand for money.
- Is money essential? Does it improve allocations/welfare in a Pareto sense?

Simple Endowment OLG Economy

- Agents endowed with 1 unit of good when young.
- No endowment when old.
- Perishable goods (equivalently, net return to storage $r = -1$), so no storage.
- No uncertainty, perfect foresight.

Notation

- Consumption at time t of (superscript) agent with birth date t, c_t^t
- Consumption at time $t+1$ of (superscript) agent with birth date t, c_t^{t+1}
- Subjective discount factor, $\in (0,1)$
- Per-period felicity function, $u : \mathbb{R}_+ \to \mathbb{R}$
- Current population of newborns N_t. Assume $N_t = (1+n)^t N_0$, with $N_0 := 1$.

- Lifetime utility of time t newborn:

$$U(c_t^t, c_{t+1}^t) = u(c_t^t) + \beta u(c_{t+1}^t).$$

- Assume example of $u(c) = \ln(c)$, so then $c \in \mathbb{R}_{++}$.

Feasible allocations must satisfy

$$N_t c_t^t + N_{t-1} c_t^{t-1} \leq N_t \cdot 1$$

for all $t \in \mathbb{N}$.

Or, applying population transition law,

$$c_t^t + \frac{c_t^{t-1}}{1+n} \leq 1,$$

for all $t \in \mathbb{N}$.

Feasible set in (c_t^t, c_{t+1}^t)-space.

Benchmark Pareto allocation

A benevolent, omnipotent and omnipresent planner solves:

$$\max_{c_t^t, c_{t+1}^t} u(c_t^t) + \beta u(c_{t+1}^t)$$

such that

$$c_t^t + \frac{c_{t+1}^t}{1+n} \leq 1.$$

Pareto allocation in (c_t^t, c_{t+1}^t)-space.

FOCs characterizing Pareto allocation. For all $t \in \mathbb{N}$,

$$\frac{u'(c^t_{t+1})}{u'(c^t_t)} = \frac{1}{\beta(1+n)},$$

and,

$$c^t_t + \frac{c^t_{t+1}}{1+n} \leq 1.$$

Proposition

A benevolent planner optimally allocates positive consumption between young and old age for every agent born in time $t \in \mathbb{N}$.

Proof.

Easy exercise. $\square$

Example

If $u(c) = \ln(c)$, then the Pareto allocation $\{c_t^t, c_{t+1}^t\}_{t=0}^{\infty}$ is given by:

$$c_t^t = \frac{1}{1+\beta} > 0, \qquad c_{t+1}^t = \frac{\beta(1+n)}{1+\beta} > 0, \qquad \forall t \in \mathbb{N}$$

Competitive allocation

Each time t young solves a decentralized problem of

$$\max_{c_t^t, c_{t+1}^t} u(c_t^t) + \beta u(c_{t+1}^t)$$

such that

$$c_t^t \leq 1, \qquad c_t^{t+1} \geq 0.$$

Proposition

In a decentralized equilibrium, every time $t \in \mathbb{N}$ young agent chooses:

- $c_t^t = 1$, *and*
- $c_t^{t+1} = 0$.

Comments:

1. Pareto optimum is a feasible social allocation, but not attainable through two-sided (bilateral) trade. Why?
2. To get to this point, the young at time t need to trade goods this period for goods next period.
3. But the only trading partner (old at time t) have no claims over the next period (they'll be dead!).
4. There is no one to deliver c_{t+1}^t.
5. Hence competitive equilibrium is where $c_t^t = 1$, $c_{t+1}^t = 0$, which is not Pareto optimal (PO).
6. Note in this setting, absence of a storage technology like capital. We shut this down to make clear the point of money.

Monetary Economy

- Previously, competitive equilibrium (CE) allocation: $c_t^t = 1$, $c_{t+1}^t = 0$.
- CE is not PO.
- Consider $u(c) = \ln(c)$ example. If at every time t young transfers c_t^{t-1} to old at t equal to $\frac{\beta(1+n)}{1+\beta}$, and if this is sustainable in a CE, everybody is strictly better off.
- Missing market makes this scheme unsustainable as a CE.

Monetary Economy

- Suppose now there exists fiat money mandated by a reputable government.
- The "government" prints and gives old at $t = 0$ H units of money.
- Let P_t be the price of time t good in terms of money. (So purchasing power of money is $1/P_t$.)
- Suppose agents (old at time t and all generations thereafter) believe that $P_t < \infty$ so that $\frac{1}{P_t} > 0$.

The individual born at time t now solves

$$\max_{c_t^t, c_{t+1}^t} \ln(c_t^t) + \beta \ln(c_{t+1}^t)$$

such that

$$c_t^t + \frac{M_t^d}{P_t} \leq 1$$

and

$$c_{t+1}^t \leq \frac{M_t^d}{P_t} \frac{P_t}{P_{t+1}}.$$

The FONC is

$$\frac{M_t^d}{P_t} = L\left(\frac{P_t}{P_{t+1}}\right)$$

for some function L, along with,

$$c_t^t + \frac{M_t^d}{P_t} = 1$$

and

$$c_{t+1}^t = \frac{M_t^d}{P_t}\frac{P_t}{P_{t+1}}.$$

Example

Suppose $u(c) = \ln(c)$. We can solve for L explicitly so we have:

$$\frac{M_t^d}{P_t} = \frac{\beta}{1+\beta},$$

so that

$$c_t^t = \frac{1}{1+\beta}$$

and

$$c_{t+1}^t = \frac{P_t}{P_{t+1}} \frac{\beta}{1+\beta}.$$

Remarks:

- Note that only the young demand money.
- Rate of return on money given by $\frac{P_t}{P_{t+1}}$ (gross deflation).
- In general, effect of gross deflation P_t/P_{t+1} on liquidity demand $L(P_t/P_{t+1})$ is ambiguous.
- In the log-utility example, intertemporal substitution and income effects of a change in relative price cancel out.
- Therefore optimal demand for real money balance is constant – does not depend on gross deflation P_t/P_{t+1}.

Remarks:

- If agents are willing to hold money, they can improve on their consumption allocations over time $(t, t+1)$ compared to the competitive equilibrium where money does not exist.
- Individuals are willing to hold money only if the purchasing power of money is positive, $1/P_t > 0$.
- Important: Notice that there is an equilibrium/optimal demand function L for money, even though direct utility representation does not involve money.
- Example of money as unit of account, store of value and medium of exchange.

Money Market Equilibrium

- But what determines value of money?
- We have derived demand L for real money balances.
- Need to specify supply of money over time. Know initial stock of money supplied is fixed at H. Suppose this does not change.
- Suppose government policy is described by the variable g_t, the deflation rate in price.

$$\frac{P_t}{P_{t+1}} = 1 + g_t.$$

- Money market clearing requires that

$$N_t P_t \frac{M_t^d}{P_t} = H$$

where H is the constant money supply.

- Equivalently,

$$N_t P_t L\left(\frac{P_t}{P_{t+1}}\right) = H.$$

- This implies

$$(1+n)(1+g_t)^{-1} = \frac{L(1+g_t)}{L(1+g_{t+1})}.$$

Definition

Given government policy $\{g_t\}_{t=0}^{\infty}$, $N_{t+1} = (1+n)N_t$, (N_0, P_0). A competitive monetary equilibrium in this endowment economy is a sequence of allocations $\{\frac{M_t}{P_t}, c_t^t, c_{t+1}^t\}_{t=0}^{\infty}$ and relative prices $\{P_{t+1}/P_t\}_{t=0}^{\infty}$, such that

1. Agent's optimize to derive:
   - Demand for real balances: $\frac{M_t^d}{P_t} = L\left(\frac{P_t}{P_{t+1}}\right)$,
   - Demand for consumption goods: $c_t^t + \frac{M_t^d}{P_t} = 1$ (or $c_{t+1}^t = \frac{M_t^d}{P_t}\frac{P_t}{P_{t+1}}$);
2. Money market clears: $N_t L\left(\frac{P_t}{P_{t+1}}\right) = \frac{H}{P_t}$; and
3. Policy rule: $\frac{P_t}{P_{t+1}} = 1 + g_t$.

Monetary Equilibrium allocation in (c_t^t, c_{t+1}^t)-space

Steady state

- Consider $u(c) = \ln(c)$ example.
- Consider steady state, $g_{t+1} = g_t = g$.
- Assume $g = n$; prices must fall at a rate equal to the population growth rate.
- Consumption is then given by,
  - $c_t^t = \frac{1}{1+\beta}$ and
  - $c_{t+1}^t = \frac{\beta(1+n)}{1+\beta}$

  which is in fact the Pareto optimal solution.

Steady-state Monetary Equilibrium allocation in (c_t^t, c_{t+1}^t)-space

Proposition

Money is essential: the introduction of money moves the competitive equilibrium to a Pareto optimum. In particular if government policy is such that $g_t = n$ *for all* t*, the monetary equilibrium allocation is also a Pareto efficient allocation.*

Exercise

This result requires an infinite time horizon, a monetary equilibrium cannot exist. Prove this statement.

Conclusion

- Illustrate essentiality of money when exists missing markets in competitive equilibrium.
- Absent money, competitive equilibrium allocation is not Pareto efficient.
- Agents have lower welfare than what a planner could hypothetically provide.

Conclusion

- With fiat money, which is intrinsically worthless, monetary equilibrium can exist (if $1/P_t > 0$), and money acts as (imperfect) substitute for missing market between current young and current old.
- Depending on government policy, and therefore inflation rate, monetary equilibrium can deliver allocation that is Pareto optimal.
- Illustration done by shutting down alternative means of storage/saving. What happens when we also have another asset in the economy, e.g. capital?