OLG: Economic Policy

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Outline

1. Signpost
2. Lump-sum transfers
   - RCE with lump-sum transfers
   - Competitive Equilibrium

Recap

Previous lectures:

- Diamond-SamuelsonOLG model
- Recursive competitive equilibrium (RCE) characterization (with tractable example)
- Welfare properties of RCE in the OLG model (in a steady state) – general and tractable example
- (Steady state) Competitive equilibrium of OLG model may/may not be Pareto optimal

Overview: this lecture series

- OLG: natural model to consider many policy issues involving intergenerational redistribution of resources
- Here focus on per-period balanced-budget policies
  1. Equilibrium characterization with lump-sum transfers; Existence of equilibrium
  2. Decentralization of Pareto allocation if lump sum taxes available: Second Welfare Theorem
  3. Lump-sum transfers and pensions; effect on capital accumulation:
     - Unfunded pensions: PAYG social security
     - Fully funded social security

RCE with lump-sum transfers I

Our goal for this lecture:

- Extend previous OLG model: now assume $\exists$ a transfer system in place:
  - Lump sum taxes on young: a_t
  - Lump sum transfers to old: z_t
- Use this extended vehicle to study various transfer (fiscal) policies.
- Consider for now, *per-period* balanced-budget policies.

RCE with lump-sum transfers II

Current Young-age budget constraint. Time-t young's budget constraint is now:

$$w_t - a_t = c_t^y + s_t.$$

Income from supplying labor (inelastically 1 unit) net of lump-sum tax finances young-age consumption and saving.

RCE with lump-sum transfers III

Expected old-age budget constraint. When old in time $t+1$ the budget constraint faced by this agent is

$$c^o_{t+1} = R^e_{t+1} s_t + z^e_{t+1}.$$

(Refined) notation used:

- R^e_{t+1} is expected return to savings.
- z^e_{t+1} is expected lump-sum transfer when old.

RCE with lump-sum transfers IV

Remarks:

- Off-equilibrium agent's beliefs about future need not be equal to actual/equilibrium outcome
- In a perfect-foresight equilibrium, these quantities are required to be consistent (i.e.exactly equal)
- Old-school terminology: "Intertemporal equilibrium" or "Intertemporal equilibrium with perfect foresight"
- Precursor to idea of rational expectations equilibrium in stochastic environments (more on this later!)

RCE with lump-sum transfers V

Government budget constraint. The budget of the transfer system is assumed to be balanced:

$$N_t a_t = N_{t-1} z_t,$$

for every $t \in \mathbb{N}$.

Equivalently written as

$$z_t = (1+n)a_t.$$

Feasibility of the system also requires $a_t < w_t$.

... (What does this mean?)

RCE with lump-sum transfers VI

Optimal agent savings behavior. Optimal savings rule $\tilde{s}$ such that

$$\tilde{s}(w_t, a_t, z^e_{t+1}, R^e_{t+1}) = \arg\max_s \left\{ U(w_t - a_t - s) + \beta U(R^e_{t+1} s + z^e_{t+1}) \right\}.$$

An interior solution*, $s_t = \tilde{s}(w_t, a_t, z^e_{t+1}, R^e_{t+1})$, satisfies the FONC:

$$U'(w_t - a_t - s_t) = \beta R^e_{t+1} U'(R^e_{t+1} s_t + z^e_{t+1}).$$

What does this condition say, economically?

RCE with lump-sum transfers VII

Exercise (*)

What assumptions on $U : \mathbb{R}_+ \to \mathbb{R}$ *will ensure that the solution to the agent's lifetime utility maximization problem is interior?*

RCE with lump-sum transfers VIII

Let: $\omega_1 := w - a$ (period-1 income), and, $\omega_2 := z'$ (period-2 non-capital income).

On the optimal savings function:

- For all $R' > 0$ and positive P.V. lifetime income, $\omega_1 + \omega_2/R > 0$, optimal savings function $\tilde{s}$ is well-defined and has property:

  $$-\omega_2/R < \tilde{s}(\omega_1, \omega_2, R) < \omega_1$$

  Recall: Exercise (*).
- Inequalities above ensure that $c_t^y > 0$ and $c_{t+1}^o > 0$.

RCE with lump-sum transfers IX

The agent's lifetime payoff at (ω_1, ω_2, R) at the optimal $\tilde{s}$ is

$$U(\omega_1 - \tilde{s}) + \beta U(\omega_2 + R\tilde{s}).$$

Equivalently,

$$U(\omega_1 + \omega_2/R - (\tilde{s} + \omega_2/R)) + \beta U(R(\tilde{s} + \omega_2/R)).$$

This implies that we can write the optimal savings function, equivalently, as

$$s(\omega_1 + \omega_2/R, R) = \tilde{s}(\omega_1, \omega_2, R) + \frac{\omega_2}{R}.$$

First term on LHS looks similar to the regular savings function we saw before, in the economy without a transfer system ...

RCE with lump-sum transfers X

Then from what we did before, we can deduce properties of this "new" savings function, $\tilde{s}$ ...

Marginal best response of savings to ...

- first-period wealth: $\tilde{s}_{\omega_1} = s_w > 0$
- second-period wealth: $\tilde{s}_{\omega_2} = R^{-1}(s_w - 1) < 0$, since $s_w \in (0,1)$
- return on saving: $\tilde{s}_R = s_R + \omega_2(1 - s_w)/R^2$. (Ambiguous sign.)

What do these partial derivatives mean, economically?

RCE with lump-sum transfers XI

... in words:

- $\tilde{s}_{\omega_1} > 0$, since period-two consumption is also normal good.
- $\tilde{s}_{\omega_2} < 0$: expected increase in terminal period wealth means need to save less.
- $\tilde{s}_R$ (sign?): ... we know $s_R \lesseqqgtr 0$
  - If $\omega_2 > 0$ then $\tilde{s}_R > s_R$
  - If $\omega_2 < 0$ then $\tilde{s}_R < s_R$

  i.e. we at least know that savings reacts more positive (negatively) than in the previous model without transfer system.

RCE with lump-sum transfers XII

Firm's problem. ... Same as before.

Market clearing. ... Same as before.

RCE with lump-sum transfers I

Now we are ready to define equilibrium.

Definition (RCE)

Given k_0 and a sequence of lump-sum transfers $\{a_t\}_{t\in\mathbb{N}}$, a RCE (with perfect foresight) and lump-sum transfers is a sequence of allocations $\{k_{t+1}\}_{t\in\mathbb{N}}$ and relative prices $\{R_{t+1}, w_t\}_{t\in\mathbb{N}}$ such that for all $t \in \mathbb{N}$,

1. $w_t = f(k_t) - f'(k_t)k_t \equiv w(k_t)$;
2. $(1+n)k_{t+1} = \tilde{s}\left(w_t - a_t, z^e_{t+1}, R^e_{t+1}\right) > 0$;
3. $R^e_{t+1} = R_{t+1} = f'(k_{t+1}) + 1 - \delta$; and
4. $z^e_{t+1} = (1+n)a_{t+1}$.

RCE with lump-sum transfers II

- Conditions 1 and 3: firm maximizes profit
- Condition 2: Capital market clears
- Condition 4: Transfer system's (or "government") budget constraint satisfied

RCE with lump-sum transfers III

RCE map. The RCE conditions yield a first-order difference equation in k which characterizes *an* equilibrium trajectory:

$$(1+n)k_{t+1} = \tilde{s}\left(w(k_t) - a_t, (1+n)a_{t+1}, f'(k_{t+1}) + 1 - \delta\right),$$

given a feasible policy sequence $\{a_t\}_{t\in\mathbb{N}}$.

Existence of RCE: Does the equation above have a solution? Depends on z. Generally, z cannot be "too big".

Gnarly to characterize ... so ... we study a specific example.

RCE and lump-sum transfers: Existence I

... Gnarly to characterize in general. We study a concrete example.

Example

- Preference representation: $U(c) = \ln(c)$ so that lifetime utility is

$$\ln(c_t^y) + \beta \ln(c_{t+1}^o),$$

  and, $\beta \in (0, 1)$.
- Production technology: $f(k) = k^\alpha$, $\alpha \in (0, 1)$.
- $\delta = 1$.

RCE and lump-sum transfers: Existence II

Example (cont'd)

The *equilibrium* savings function $\tilde{s}(\omega_1, \omega_2, R)$ in this example is ...

$$\begin{aligned} s_t &= \tilde{s}(w_t - a_t, z^e_{t+1}, R^e_{t+1}) \\ &= \frac{\beta}{1+\beta}(w_t - a_t) - \frac{1}{1+\beta}\frac{z^e_{t+1}}{R^e_{t+1}}, \end{aligned}$$

where in a perfect-foresight RCE,

- $z^e_{t+1} = (1+n)a_{t+1}$, and
- $R^e_{t+1} = R_{t+1} = \alpha k^{\alpha-1}_{t+1}$.

RCE and lump-sum transfers: Existence III

Example (cont'd)

Given k_0 and $\{a_t\}_{t\geq 0}$, the RCE is a sequence of allocations $\{k_{t+1}\}_{t\geq 0}$ satisfying the equilibrium map:

$$\begin{aligned}(1+n)k_{t+1} = \frac{\beta}{1+\beta}\left[(1-\alpha)k_t^{\alpha} - \frac{z_t}{1+n}\right] \\ - \frac{z_{t+1}}{(1+\beta)\alpha k_{t+1}^{\alpha-1}} \\ \equiv g(k_t; a_t) > 0.\end{aligned}$$

where $z_t = (1+n)a_t$.

RCE and lump-sum transfers: Existence IV

Remarks:

- If along RCE path, ...

  ... net wage is positive, $w_t - a_t = (1-\alpha)k_t^{\alpha} - \frac{z_{t+1}}{1+n} > 0$, for all a_t (or all z_{t+1}), ...

  ... then the RCE map $\equiv g(k_t; a_t) > 0 \Rightarrow k_{t+1} > 0$, for all $t \geq 0$.

- i.e. there is a solution $\{k_{t+1}\}_{t\geq 0}$ to this difference equation.

Next:

- Decentralization of Pareto optimal allocation: 2nd Welfare Theorem
- Direction of optimal tranfers in the long run
- Pension systems
  - Fully funded
  - Unfunded/PAYG