Money, Co-existence of Assets, Inflation: An OLG Interpretation

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Outline

1. Previously
2. Competing Assets
3. Inflation
   - Neutrality
   - Non-Superneutrality
   - Pareto Efficiency?
   - Superneutrality

Roadmap

- Study competitive equilibrium allocation in simple endowment OLG economy.
- Competitive allocation is Pareto inefficient.
- Introduce fiat money.
- Derive equilibrium demand for money.
- Is money essential? Does it improve allocations/welfare in a Pareto sense? Yes.
- Under some condition, possible for monetary (competitive) equilibrium to replicate Pareto application.

Previous Endowment OLG Economy

- Agents endowed with 1 unit of good when young.
- No endowment when old.
- Perishable goods (equivalently, net return to storage $r = -1$), so no storage.
- No uncertainty, perfect foresight.

Competing Assets

- Agents endowed with 1 unit of good when young.
- No endowment when old.
- Storage exists: (equivalently, net return to storage $r \neq -1$).
- No uncertainty, perfect foresight.
- Two cases: $r < n$ and $r > n$.
- Stored goods: k_t.
- Now M and k are competing stores of value – vehicles for transferring resources intertemporally.

Agent's decision problem

Agent young at t solves

$$\max \ln(c_t^t) + \beta \ln(c_{t+1}^t)$$

such that

$$c_t^t \leq 1 - k_t - \frac{M_t}{P_t}$$

and

$$c_{t+1}^t \leq (1+r)k_t + \frac{P_t}{P_{t+1}} \frac{M_t}{P_t}.$$

The Karush-Kuhn-Tucker (KKT) FONCs for k_t and $\frac{M_t}{P_t}$ are, respectively:

$$-\frac{1}{c_t^t} + \beta(1+r)\frac{1}{c_{t+1}^t}\begin{cases} < 0 & \text{if } k_t = 0 \\ \\ = 0 & \text{if } k_t > 0 \end{cases},$$

and,

$$-\frac{1}{c_t^t} + \beta\frac{P_t}{P_{t+1}}\frac{1}{c_{t+1}^t}\begin{cases} < 0 & \text{if } \frac{M_t}{P_t} = 0 \\ \\ = 0 & \text{if } \frac{M_t}{P_t} > 0 \end{cases},$$

What the KKT conditions say:

- Preference set is strictly convex on the set of allocation (c_t^t, c_{t+1}^t)
- This is implied by Inada conditions for $U(c_t^t, c_{t+1}^t)$
- Our log utility example satisfies these conditions
- Then we have that any optimal choice (c_t^t, c_{t+1}^t) is a strictly non-zero bundle.
- Which implies saving (so, either $k_t > 0$ or $\frac{M_t}{P_t} > 0$).

- Comparing the FONCs gives that if

$$\frac{P_t}{P_{t+1}} < 1 + r$$

then ...

$$\frac{M_t}{P_t} = 0 \text{ and } k_t > 0.$$

- In words: If money earns a better (worse) rate of return than the storage technology, then real money balances will be held, and none of the storage technology will be.
- If the two rates are equal, then the agent is indifferent between the two.

Discussion

- Long-standing problem with monetary models.
- Competing assets: asset with dominating rate of return survives existence problem.
- If both assets have same rate of return, then indeterminacy in the composition of these assets held.
- How to have a determinate distribution of, and relative price, for these assets?
- More microeconomic foundations from information economics: e.g. asymmetric information re: asset quality; limited commitment to repaying. Beyond the scope of our study here.

Inflation

- So far, we have assumed a constant nominal money supply H.
- Now, allow money growth $H_{t+1} = (1 + \sigma)H_t$.
- We'll see that at steady state, we will have gross inflation $\frac{P_{t+1}}{P_t} = \sigma - n$.
- Suppose that new money is given to the old agents via lump sum transfer, T_t, at time t.

Now agent young at t solves

$$\max \ln(c_t^t) + \beta \ln(c_{t+1}^t)$$

such that

$$c_t^t \leq 1 - k_t - \frac{M_t}{P_t}$$

and

$$c_{t+1}^t \leq (1+r)k_t + \frac{P_t}{P_{t+1}}\frac{M_t}{P_t} + \frac{T_{t+1}}{P_{t+1}}.$$

The right-most term being the new real money balances.

- Denote:
  - g as gross deflation rate,
  - $m = M/P$ as real money demand, and
  - $t = T/P$ as new real money balance.
- We can rewrite this as

$$\max_{k_t, m_t} \Big\{ \ln(1 - k_t - m_t) + \beta \ln[(1+r)k_t + (1+g_t)m_t + t_{t+1}] \Big\}.$$

As before, the Karush-Kuhn-Tucker (KKT) FONCs for k_t and $\frac{M_t}{P_t}$ are, respectively:

$$-\frac{1}{c_t^t} + \beta(1+r)\frac{1}{c_{t+1}^t} \begin{cases} < 0 & \text{if } k_t = 0 \\ \\ = 0 & \text{if } k_t > 0 \end{cases},$$

and,

$$-\frac{1}{c_t^t} + \beta\frac{P_t}{P_{t+1}}\frac{1}{c_{t+1}^t} \begin{cases} < 0 & \text{if } \frac{M_t}{P_t} = 0 \\ \\ = 0 & \text{if } \frac{M_t}{P_t} > 0 \end{cases},$$

Steady State

- In a steady state, $m_{t+1} = m_t$, for all t, and so gross inflation rate is

$$\frac{P_{t+1}}{P_t} = \frac{N_t}{N_{t+1}} \frac{H_{t+1}}{H_t}$$
$$= \frac{1}{1+n}(1+\sigma) \approx \sigma - n, \qquad \text{for } (\sigma, n) \text{ small.}$$

- Alternatively, in terms of the gross return on money (i.e., deflation), at steady state,

$$\frac{P_t}{P_{t+1}} = \frac{1+n}{1+\sigma}$$

so that $g \approx n - \sigma$.

Special Case

- Assume that $\frac{1+n}{1+\sigma} \geq 1+r$: Money weakly dominates storage in RoR.
- Consumers' FONCs imply that:
    - $k_t = 0$,
    - $\frac{c_{t+1}^t}{\beta c_t^t} > 1+r$,
    - $m_t > 0$, and

$$\begin{aligned}
\frac{1}{c_t^t} &= \frac{1}{1-m_t} \\
&= \frac{\beta(1+g_t)}{(1+g_t).m_t + (1+g_t)\frac{T_{t+1}}{P_t}} \\
&= \frac{\beta.(1+g_t)}{c_{t+1}^t} \\
&= \frac{\beta}{m_t + \frac{T_{t+1}}{P_t}}.
\end{aligned}$$

- Now, since

$$T_{t+1} = \frac{\sigma H_t}{N_t},$$

then

$$\frac{T_{t+1}}{P_t} = \sigma m_t.$$

- Getting back to the FONC above gives that

$$\frac{1}{1-m_t} = \frac{\beta}{m_t + \sigma m_t}$$

so that in steady state, real money balance is

$$m_t = \frac{\beta}{1+\sigma+\beta},$$

consumption for young agent is

$$c_t^t = \frac{1+\sigma}{1+\sigma+\beta},$$

and, for old agent is

$$c_{t+1}^t = \frac{(1+n)\beta}{1+\sigma+\beta}.$$

Money Neutrality

- Note that

$$\frac{P_{t+1}}{P_t} = \left(\frac{1+n}{1+\sigma}\right)^{-1}.$$

- For constant population growth rate, n,

$$\frac{P_{t+1}}{P_t} \propto 1+\sigma = \frac{H_{t+1}}{H_t}.$$

- Prices will adjust at the same rate as money supply growth.

Proposition

In a monetary equilibrium in this OLG model, for fixed σ, doubling money supply merely doubles the price level – i.e., Money is neutral.

Discussion

- What does the money neutrality imply for “real-world” monetary policy?
- In monetary policy really neutral in the “real world”?

Non-superneutrality

- Recall, we showed that if a monetary equilibrium exists (in log-utility model), then in steady state, we have consumption for young agent is

$$c_t^t = \frac{1+\sigma}{1+\sigma+\beta},$$

and, for old agent is

$$c_{t+1}^t = \frac{(1+n)\beta}{1+\sigma+\beta}.$$

Proposition

Money supply is not super-neutral — changing its growth rate σ has real effects.

(In)Efficiency of Monetary Equilibrium

- Monetary equilibrium is no longer Pareto Optimal if $\sigma > 0$.
- To see this, consider the steady state equilibrium with $k_t = 0$.
- The FONCs are

$$\frac{c_{t+1}^t}{\beta c_t^t} > 1 + r$$

and

$$\frac{c_{t+1}^t}{\beta c_t^t} = \frac{1+n}{1+\sigma}.$$

Exercise

What happens if money earns interest? Before, RoR on money is gross deflation. Now suppose the $t+1$ *budget constraint for agent* t *is*

$$c_{t+1}^{t} = (1+\sigma)\frac{M_t}{P_t}.\frac{P_t}{P_{t+1}} + (1+r).k_t$$

and everything else is as before. Show that money is now super-neutral.