Control Systems LAB Digital Assignment 5

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School of Electrical Engineering

Faculty: **Professor Dhanamjayalu C**

Course: **EEE-3001**

Course Name: **Control Systems Lab**

Lab Slot: **L45 + L46**

Study of First Order System

Exp No: 5

Date: 02-03-2022

AIM

1. To obtain the time and frequency response for a step input of a second order electrical system and correlate experimental results with theoretical.

APPARATUS REQUIRED

S.No.	Name of the apparatus / equipment	Specification / Range	Quantity
1	Decade Resistance Box	-----	1
2	Decade Inductance box	-----	1
3	Decade Capacitance box	-----	1
4	Function generator	-----	1
5	Cathode ray oscilloscope	-------	1

THEORY

1. TIME RESPONSE

An RLC circuit is considered as a simple second system. Fig.1 shows a second order system whose input is e_i and output is e_o.

$$e_i(t) = Ri(t) + L\frac{di}{dt} + \frac{1}{C}\int idt$$

$$e_o(t) = \frac{1}{C}\int idt$$

Taking laplace transform of the above equations, we get

$$E_i(s) = I(s)\left(R + Ls + \frac{1}{Cs}\right)$$

$$E_o(s) = I(s)\frac{1}{Cs}$$

$$\therefore TransferFunction = \frac{E_0(s)}{E_i(s)} = \left(\frac{1}{s^2LC + RCs + 1}\right)$$

$$=\left(\frac{1}{Lc}\right)\left(\frac{1}{s^2+\frac{R}{L}s+\frac{1}{LC}}\right) \qquad \text{------------} \quad (1)$$

A general form of second order system transfer function is given by

$$\frac{E_o(s)}{E_i(s)}=\frac{\omega_n^2}{s^2+2\zeta\omega_n s+\omega_n^2} \qquad \text{------------} \quad (2)$$

where ω_n is the undamped natural frequency.

ζ is the damping ratio of the system

Also $\zeta\omega_n = \sigma$ is called attenuation

Comparing equations (1) and (2), we get

$$\omega_n=\frac{1}{\sqrt{LC}} \quad \text{and} \quad 2\zeta\omega_n=\frac{R}{L}$$

$$\therefore \zeta=\frac{R}{2L}\times\sqrt{LC}=\frac{R}{2}\sqrt{\frac{C}{L}}$$

Dynamic behaviour of second order system is defined in terms of two parameters ζ and ω_n .If $0<\zeta<1$ the closed loop poles are complex conjugates and lie in the left half of s plane. Then, the system is under-damped and the transient response is oscillatory. If $\zeta=1$, the system is called critically damped. Over-damped system corresponds to $\zeta>1$. In the under-damped and over-damped case the transient response is not oscillatory. For $\zeta<1$ the step response of the system is given by

$$e_o(t)=1-\frac{e^{-\zeta\omega_n t}}{\sqrt{1-\zeta^2}}\sin(\omega_d t+\phi)$$

where ω_d = damped natural frequency $=\omega_n\sqrt{1-\zeta^2}$

$$\phi=\tan^{-1}\left(\frac{\sqrt{1-\zeta^2}}{\zeta}\right)$$

The frequency of oscillation is the damped natural frequency ω_d & this varies with damping ratio ζ.Fig. 2 shows the time response of the under-damped second order system along with the following time-domain specifications.

DELAY TIME: Delay time is the time required for the response to reach from 0% to 50% of the steady state value.

RISE TIME: Rise time is the time required for the response to reach from 10% to 90% of the steady state value for under-damped systems.

$$t_r = \frac{\pi - \phi}{\omega_d} \text{sec.}$$

PEAK TIME: Peak time is the time required to reach the response first peak of the overshoot.

$$t_p = \frac{\pi}{\omega_d} \text{sec.}$$

% PEAK OVERSHOOT: Maximum overshoot is the maximum value of the response measured from unity.

$$M_p = e^{-\zeta\pi/\sqrt{1-\zeta^2}} \times 100$$

If the final steady state is different from unity maximum overshoot is given by

$$\frac{c(t_p) - c(\infty)}{c(\infty)} \times 100$$

SETTLING TIME: Settling time is for the response to settle around the steady state value with the variation not exceeding a permissible tolerance level.

$$t_s = \frac{4}{\zeta\omega_n} \quad \text{for 2\% tolerance}$$

$$t_s = \frac{3}{\zeta\omega_n} \quad \text{for 5\% tolerance}$$

Frequency Response

Frequency response of the system is given by

$$G(j\omega) = \frac{1}{LC(j\omega)^2 + RC(j\omega) + 1} = \frac{1}{1 - \left(\frac{\omega}{\omega_n}\right)^2 + 2\zeta\left(\frac{j\omega}{\omega_n}\right)} = \frac{1}{1 - u^2 + j2\zeta u}$$

where $u = \frac{\omega}{\omega_n}$

$\therefore$ Magnitude of G(jω) = $\frac{1}{\sqrt{(1-u^2)^2 + (2\zeta u)^2}}$

$\therefore$ Phase angle of G(jω) = $\phi = \tan^{-1}\left(\frac{2\zeta u}{1-u^2}\right)$

This sinusoidal transfer function can be represented by Bode diagram as shown in fig.3.

PROCEDURE

1. Connections are made as shown in figure 1.
2. Keep the appropriate value for resistance and inductance using Decade Resistance Box (DRB) and Decade inductance Box respectively.
3. A step input (square pulse of very low frequency i.e. large time period)) is given at input and output is observed across the capacitor using CRO.
4. Output shows a damped oscillation before it comes to steady state. Maximum overshoot or peak overshoot is noted.
5. A graph is plotted showing the variation of output voltage with time.
6. To get frequency response a sinusoidal signal is given as input and output (peak to peak value) is noted.
7. The input voltage is kept constant and output is noted for different frequency. Also the phase angle is noted, the output waveforms are noted using CRO.
8. The magnitude in decibel and phase angle in degree is plotted as a function of frequency ran/sec. In the semilog graph sheet.

CIRCUIT DIAGRAM

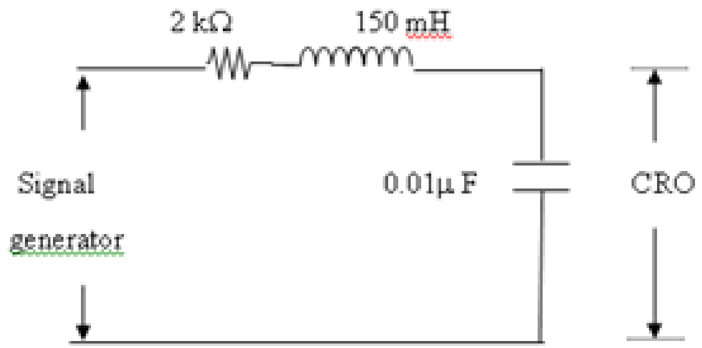
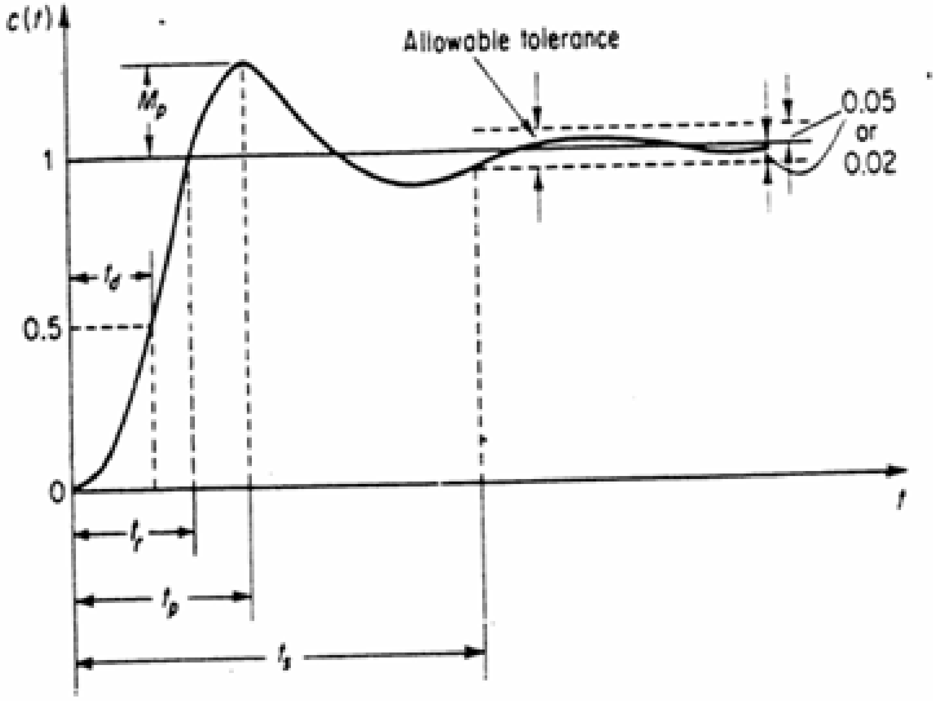
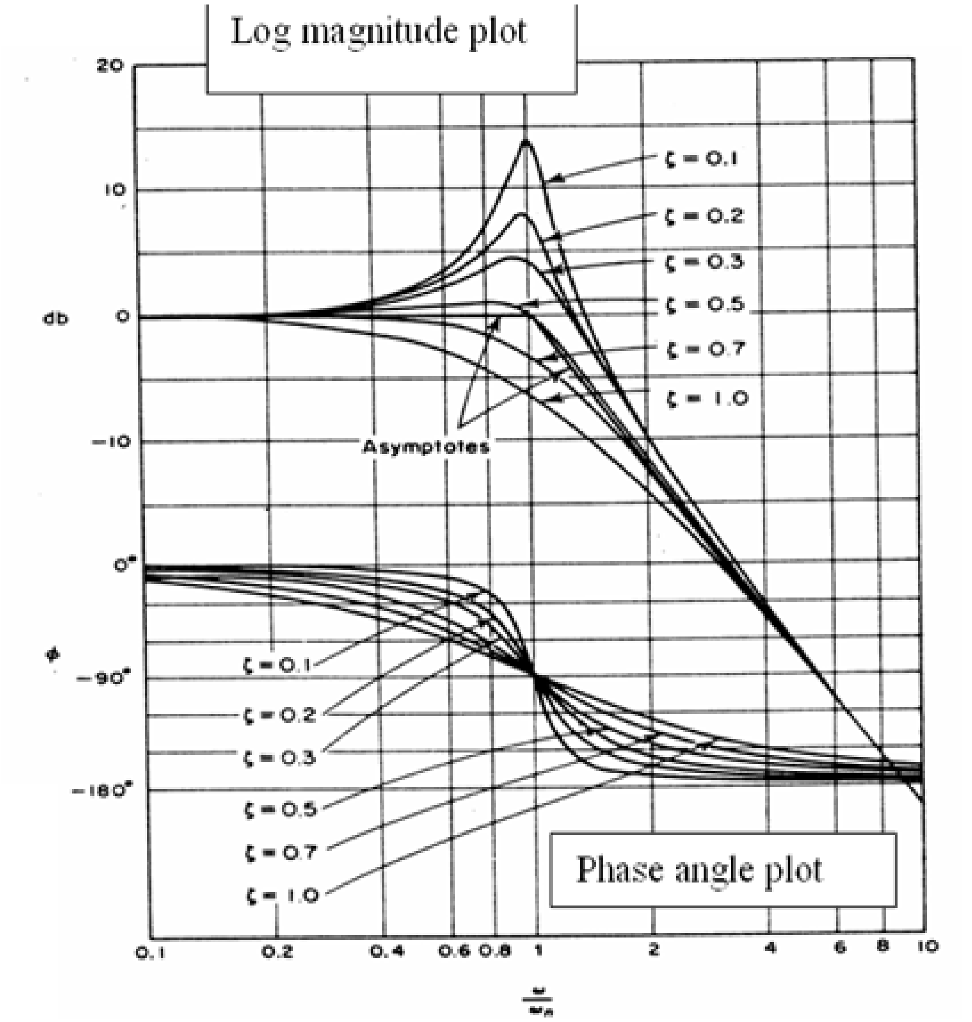


Fig. 1. Circuit Diagram for Second order System

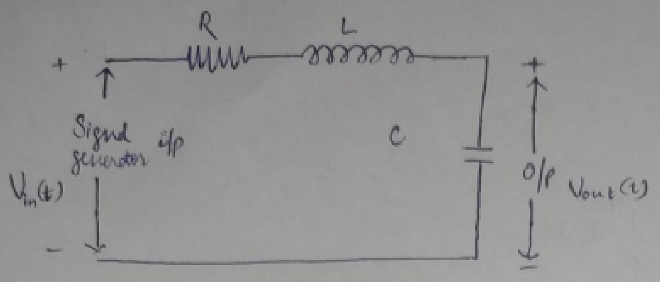
TIME RESPONSE



FREQUENCY RESPONSE



MODEL CALCULATION



Circuit diagram for second order system

wkt

$$\text{Transfer function} = \left.\frac{\text{Laplace Transform of output}}{\text{Laplace Transform of input}}\right|_{x(0)=0}$$

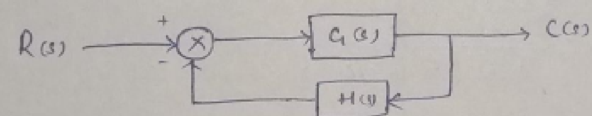
from the circuit (By Voltage division rule) —

$$V_{out}(s) = \frac{V_{in}(s) \times 1/sC}{R + sL + \frac{1}{sC}}$$

$$\frac{V_{out}(s)}{V_{in}(s)} = \frac{\cancel{1/sC}}{\cancel{1/sC}\,(1 + RCs + LCs^2)}$$

$$\frac{V_{out}(s)}{V_{in}(s)} = \frac{1}{1 + sRC + s^2 LC}$$

⇒



Standard form

$$\frac{C(s)}{R(s)} = \frac{\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2}$$

when

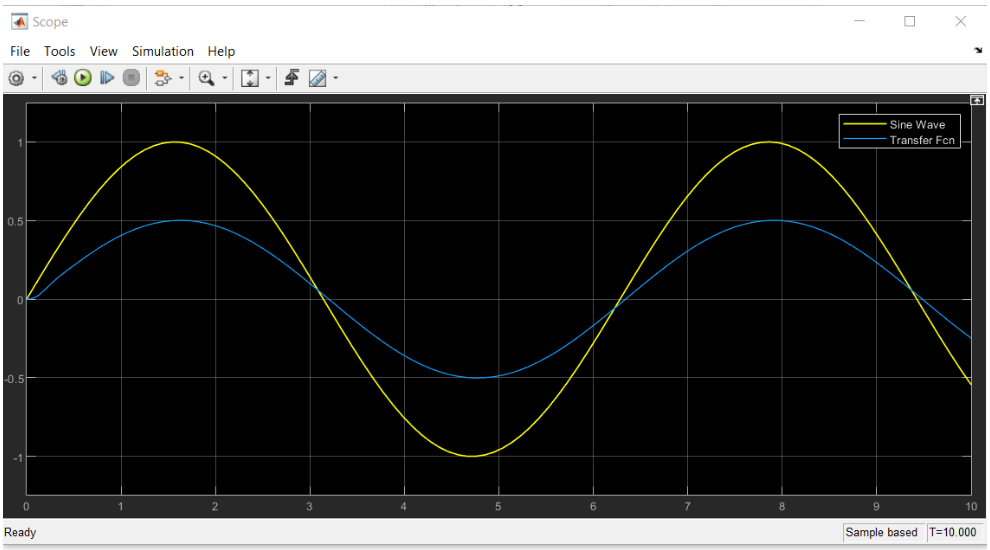
$\xi = 0$, undamped

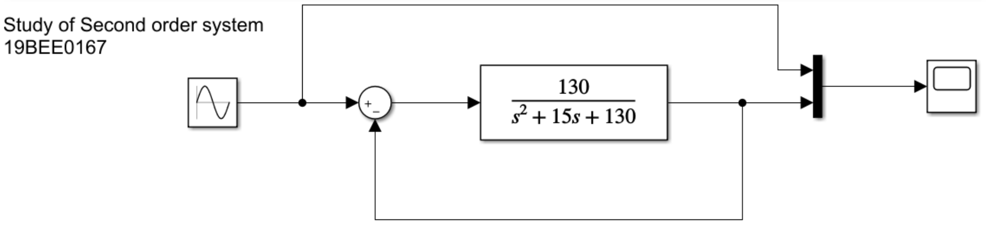
$0 < \xi < 1$, underdamped

$\xi = 1$, critically damped

$1 < \xi < \infty$, overdamped

RESULT





TYPICAL PROBING QUESTIONS

1. Give a general description of the relationship between the time constants and the settling time of a second order linear circuit

2. State the duals of the following terms as they apply to electric circuits : Resistance, Inductance , Voltage through.

3. In a series RLC circuit define the critical resistance and describe its importance.

4. Is it possible for the voltage that appears across the L or C elements in a series RLC circuit to exceed the voltage applied to the circuit.

5. What physical meaning can be given to the complex frequency as it arises in the characteristic equation of an underdamped second order system

6. What is meant by the characteristic equation of a second order system ?

7. Define order of a system ?

8. The real part of a pole generates what part of a response ?

9. What is the difference between the natural frequency and damped frequency of oscillations ?