# For a power analysis

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#### R Markdown

With my thesis advisor, we wanted to conduct a power analysis in GPower. The parameter (i.e., the  $\eta_p^2$ ) that we wanted to use comes from a previous paper in which an interaction between a between-subjects variable (Group) and a within-subjects variable (Contingency) was found, the size of the effect was found to be  $\eta_p^2$  = .181. Therefore, it seems that we have to use "ANOVA: Repeated measures, within-between interaction", hence, we can fill in the  $\eta_p^2$  = .181. However, a main issue appears when we have to select a value for correlation among repeated measures. In fact, we were lost regarding what one has to fill in for this value. Regarding this correlation among repeated measures, we know that the greater the correlation among repeated measures, the more the power. In fact, an effect size for a within-subjects design cannot be equal to an effect size for a betweensubjects design even if they were computed from the same data, because the former takes into account the correlation among repeated measures. As a consequence, if the correlation is strong, it inflates the effect size (e.g., a Cohen's  $d_D$  is a merge between the standardized mean difference [i.e., a Cohen's  $d_p$ , that is the standardized mean difference that one can find in a between-subject design] and the correlation between repeated measures; Dunlap et al., 1996; see below), offering more power (Lakens, 2013; Cousineau & Goulet-Pelletier, 2021). Now, to deal with such a problem and eliminate the influence of correlation, one can estimate an effect size for a given within-subjects variable in the same way as one would be done with a between-subject variable (regarding Cohen's d see Goulet-Pelletier &

Cousineau, 2018; Fitts, 2020; Cousineau & Goulet-Pelletier, 2021). Before continuing, our solution for the power analysis seems to take the more conservative mean to perform our analysis as suggested here:

https://stats.stackexchange.com/questions/44134/correlation-among-repeated-measures-i-need-an-explanation

one can simply fill in 0, but we will see that it can be problematic.

If I correctly understand the issue with GPower, it requires a  $\eta_p^2$  that does not take into account the correlation among repeated measures (thus, an effect size that was estimated to be comparable across the designs; between a between-subjects variable and a within-subjects variable) and the program will include the correlation among the repeated measures later in order to offer more power, that why it is demanded. Hence, we better understand the main issue with a  $\eta_p^2$  estimated in SPSS, since it directly returns an effect size that is influenced by the correlation among repeated measures (Lakens, 2013).

Let's take a reminder in Lakens (2013):

Although  $\eta_p^2$  is more useful when the goal is to compare effect sizes across studies, it is not perfect, because  $\eta_p^2$  differs when the same two means are compared in a within-subjects design or a between-subjects design. In a within-subjects ANOVA, the error sum of squares can be calculated around the mean of each measurement, but also around the mean of each individual when the measurements are averaged across individuals. This allows researchers to distinguish variability due to individual differences from variability due to the effect in a within-subjects design, whereas this differentiation is not possible in a between-subjects design. As a consequence,

whenever the two groups of observations are positively correlated,  $\eta_p^2$  will be larger in a within-subjects design than in a between-subjects design. This is also the reason a within-subjects ANOVA typically has a higher statistical power than a between-subjects ANOVA.

To illustrate the issue, I simulated a 2X2 ANOVA followed by two one-way ANOVAs, though the latter is not optimal.

```
## anova simulation
library(MOTE)
library(MBESS)
library(data.table)
report<-function(anova){</pre>
  a<-as.data.table(anova)</pre>
  b<-nrow(a)
  test <-
data.table(Factor=factor(b),DFn=integer(b),DFd=integer(b),F=integer(
b), MSE=integer(b), p=integer(b), ges=integer(b), eta=integer(b),
eta.Ll =integer(b), eta.Ul=integer(b))
  test$Factor<-a$ANOVA.Effect
  test$DFn<-a$ANOVA.DFn
  test$DFd<-a$ANOVA.DFd
  test$F<-round(a$ANOVA.F,2)
  test$p<-round(a$ANOVA.p,3)
  test$ges<-round(a$ANOVA.ges, 3)</pre>
  loweretasquared <- c()</pre>
  upperetasquared <- c()</pre>
  for(i in 1:b){
    test$MSE[i]=a$ANOVA.SSd[i]/a$ANOVA.DFd[i]
test$eta[i]=round((a$ANOVA.DFn[i]*a$ANOVA.F[i])/((a$ANOVA.DFn[i]*a$A
NOVA.F[i])+a$ANOVA.DFd[i]),3)
    Lims <- conf.limits.ncf(F.value = a$ANOVA.F[i], conf.level =</pre>
0.90, df.1 <- a$ANOVA.DFn[i], df.2 <- a$ANOVA.DFd[i])</pre>
    Lower.lim <- Lims$Lower.Limit/(Lims$Lower.Limit + df.1 + df.2 +</pre>
1)
    Upper.lim <- Lims$Upper.Limit/(Lims$Upper.Limit + df.1 + df.2 +</pre>
1)
    if (is.na(Lower.lim)) {
      Lower.lim <- 0
    if (is.na(Upper.lim)) {
      Upper.lim <- 1</pre>
```

```
loweretasquared <- c(loweretasquared,Lower.lim)</pre>
    upperetasquared <- c(upperetasquared, Upper.lim)
  }
  test$eta.L1 <- round(loweretasquared, 3)</pre>
  test$eta.Ul <- round(upperetasguared, 3)
  return(test)
}
```

#### Simulate three bivariate normal distributions

```
We will generate three measures from bivariate normal distributions:
data \sim N(\mu, \Sigma)
where, \Sigma = \begin{pmatrix} \sigma^2 & \rho\sigma^2 \\ \rho\sigma^2 & \sigma^2 \end{pmatrix}, where \sigma=1 and \rho=0.7, across the three simulated samples.
library(MASS)
## Generate the data
set.seed(123)
n <- 30
## set the different population means:
mu1 \leftarrow c(0.36, 0.36)
mu2 \leftarrow c(1.4, 1.4)
mu3 < - c(-0.8, 0.8)
## create covariance matrix with correlation = 0.7
\# sd = 1 \longrightarrow var = 1^2; rho = 0.7
cov_matrix \leftarrow array(c(1,0.7,0.7,1), dim = c(2,2))
cov_matrix
##
          [,1] [,2]
## [1,] 1.0 0.7
## [2,] 0.7 1.0
gp1 <- mvrnorm(n, mu = mu1, Sigma = cov matrix)</pre>
gp2 <- mvrnorm(n, mu = mu2, Sigma = cov matrix)</pre>
gp3 <- mvrnorm(n, mu = mu3, Sigma = cov matrix)</pre>
cor1 \leftarrow cor(x = gp1[,1], y = gp1[,2]); print(cor1)
## [1] 0.7771165
cor2 \leftarrow cor(x = gp2[,1], y = gp2[,2]); print(cor2)
## [1] 0.6780823
cor3 \leftarrow cor(x = gp3[,1], y = gp3[,2]); print(cor3)
```

#### Create the data frame

```
factor1 <- c(gp1[,1], gp2[,1], gp3[,1])
  factor2 <- c(gp1[,2], gp2[,2], gp3[,2])
id <- 1:90
group \leftarrow c(rep(1, 30), rep(2, 30), rep(3, 30))
data <- cbind(id, group, factor1, factor2)</pre>
df <- as.data.frame(data); print(df)</pre>
##
      id group
                   factor1
                                 factor2
## 1
       1
             1 -0.32190190
                             0.008435869
## 2
       2
             1
                0.26206753
                             0.033506146
## 3
       3
             1
                1.45037738
                             2.143738738
## 4
       4
             1
                0.08490589
                             0.765105162
## 5
       5
             1 0.16100042 0.797394387
## 6
       6
             1
                1.67450257
                             2.207921013
## 7
       7
             1
                0.57041236 0.999475130
## 8
       8
             1 -0.78235053 -0.830307132
## 9
       9
             1 -0.15474821 -0.391745870
## 10 10
                0.09647575 -0.198235820
             1
## 11 11
             1
                1.75760651
                            1.219488799
## 12 12
             1 0.77225797
                             0.611205942
## 13 13
                1.21957892 0.239407121
             1
## 14 14
             1 -0.37798861
                             1.302077455
## 15 15
             1 -0.62030188
                             0.315381465
## 16 16
                2.44243059
                             1.572474427
             1
               0.97503209
## 17 17
                             0.662958836
             1
## 18 18
             1 -1.27239659 -1.633866272
## 19 19
               0.70453900
                            1.308697383
             1
## 20 20
             1 -0.04360344 -0.108180841
## 21 21
             1 -0.72259465 -0.526374974
## 22 22
                0.17009317
                             0.147980948
## 23 23
             1 -0.56932571 -0.602533019
## 24 24
             1 -0.84206189
                             0.218052876
## 25 25
             1 -0.12881700 -0.303698459
## 26 26
             1 -1.78238094 -0.607727857
                1.73223087
## 27 27
                             0.532572108
             1
                0.27498310
## 28 28
             1
                             0.727822959
## 29 29
             1 -0.73727895 -0.641341867
## 30 30
                1.43232643
                             1.599594051
             1
## 31 31
             2
                1.36522792
                             2.134792698
## 32 32
             2
                0.72448743
                             1.149273888
## 33 33
             2
                1.00033757
                             1.185258375
## 34 34
             2
                0.70410687
                             0.217732920
## 35 35
             2 -0.11512111
                             0.938835741
## 36 36
             2
                1.91231912
                             1.447360042
             2 0.96607857 2.660379423
## 37 37
```

```
## 38 38
              2
                 0.85528994
                              2.042445026
## 39 39
              2
                 2.34157495
                              2.159002236
              2
## 40 40
                 3.68761580
                              2.892553585
## 41 41
              2
                 1.22243091
                              0.672152354
## 42 42
              2
                -0.82843914
                             -0.629457878
## 43 43
              2
                 2.42278846
                              2.231701750
              2
## 44 44
                 0.88075187
                              0.611546534
## 45 45
              2
                 1.13424768
                              0.397127111
              2
## 46 46
                 2.36296925
                              2.328090921
## 47 47
              2
                 1.4414445
                              0.833460066
              2
## 48 48
                 0.92054501
                             -0.371437256
## 49 49
              2
                 1.71441465
                              1.419892451
## 50 50
              2
                 0.91602263
                              1.627874347
## 51 51
              2
                 1.62814524
                              1.182483396
              2
## 52 52
                 1.51974741
                              1.990674548
## 53 53
              2
                 1.68487161
                              0.431665057
## 54 54
              2
                 2.01560488
                              1.972566767
## 55 55
              2
                 0.99555589
                              1.397886979
## 56 56
              2
                 1.58925166
                              1.822524052
## 57 57
              2
                 2.37030739
                              2.452163818
## 58 58
              2
                 2.04936188
                              1.553073140
## 59 59
              2
                 1.42859500
                              0.770416848
## 60 60
              2
                 2.85579167
                              2.062504916
## 61 61
              3
               -0.99662514
                              1,213554747
## 62 62
              3
               -1.97137721
                              0.224320346
## 63 63
               -1.38093312
              3
                              0.476389889
## 64 64
              3
               -0.64556275
                              0.173352084
## 65 65
                 0.94622057
                              2.453692977
              3
## 66 66
              3
               -1.29247146
                              0.090335243
## 67 67
              3 -0.80102921
                              1.235060605
## 68 68
              3
               -0.58387874
                              0.727631442
## 69 69
                              0.291592166
              3 -2.06516817
                              0.589182651
## 70 70
              3
               -0.72066826
## 71 71
              3
                              2.539523483
                 0.12409669
## 72 72
              3
                 0.02261068
                              0.809921662
## 73 73
              3 -0.27392910
                              0.349958848
## 74 74
              3
               -2.44477220
                              1.665726536
## 75 75
              3 -2.53155215 -1.254448654
## 76 76
              3
                 0.12753832
                              1.958544423
## 77 77
              3
               -2.39318598 -0.300101232
## 78 78
              3
                 0.06956533
                              1.294830467
## 79 79
              3
                 0.75992671
                              2.760286332
## 80 80
              3 -2.27410307 -0.388304372
## 81 81
              3 -0.06957030
                              1.363596676
## 82 82
              3 -1.06702202
                              0.583553742
## 83 83
              3 -2.23625111 -0.662639487
## 84 84
              3 -3.02080045
                              0.227891299
## 85 85
              3 -1.98942516 -0.963661631
## 86 86
              3 -0.86499410 -0.113949157
```

```
## 87 87
             3 -2.16230744 -0.533036676
## 88 88
             3 -0.28602075 1.554476604
## 89 89
             3 0.96713996 2.905269591
## 90 90
             3 -1.80905934 -0.564107599
library(reshape2)
##
## Attachement du package : 'reshape2'
## Les objets suivants sont masqués depuis 'package:data.table':
##
##
       dcast, melt
data_long <- melt(df, id.vars = c("id", "group"), measure.vars =</pre>
c("factor1", "factor2"), variable.name = "factor", value.name =
"DV")
```

#### Run the ANOVAs

```
options(contrasts = c("contr.sum", "contr.poly"))
library(ez)
data long$id <- as.factor(data long$id)</pre>
data_long$group <- as.factor(data_long$group)</pre>
data long$factor <- as.factor(data long$factor)</pre>
data long$DV <- as.numeric(data long$DV)</pre>
Model <- ezANOVA(data long,</pre>
                  dv = . (DV), wid = .(id),
                  within =. (factor),
                  between =. (group),
                  type = 3,
                  detailed = T,
                  return aov = T)
report(Model)
## Warning in as.data.table.list(anova): Item 2 has 3 rows but
longest item has 4;
## recycled with remainder.
##
            Factor DFn DFd
                                F
                                         MSE p
                                                 ges
                                                        eta eta.Ll
eta.Ul
## 1:
       (Intercept)
                         87 28.63 1.7215142 0 0.224 0.248 0.125
                      1
0.362
## 2:
                      2 87 23.19 1.7215142 0 0.319 0.348
             group
                                                             0.207
0.450
## 3:
                      1 87 66.16 0.2406462 0 0.085 0.432 0.300
            factor
0.531
## 4: group:factor
                      2 87 59.78 0.2406462 0 0.144 0.579 0.458
0.653
```

Regarding the output, the returned  $\eta_p^2$  are greater than the  $\eta_G^2$  for our two variables and the interaction as well. The  $\eta_p^2$  estimates are inflated intrinsically, not only due to correlation among repeated measures since the *group* factor is a between subjects-variable. Lakens (2013) stated that "For designs where all factors are manipulated between participants,  $\eta_p^2$  and  $\eta_G^2$  are identical". Therefore, one can run a one-way ANOVA on group, confirming that both effect sizes are identical.

```
Model2 <- ezANOVA(data_long,</pre>
                 dv = . (DV), wid = .(id),
                 between =. (group),
                 type = 3,
                  detailed = T,
                 return aov = T); report(Model2)
## Warning: Collapsing data to cell means. *IF* the requested
effects are a subset
## of the full design, you must use the "within full" argument, else
results may be
## inaccurate.
## Coefficient covariances computed by hccm()
## Warning in as.data.table.list(anova): Item 1 has 2 rows but
longest item has 13;
## recycled with remainder.
           Factor DFn DFd F
##
                                      MSE p
                                              ges
                                                    eta eta.Ll
eta.Ul
## 1: (Intercept) 1 87 28.63 0.8607571 0 0.248 0.248 0.125
0.362
                    2 87 23.19 0.8607571 0 0.348 0.348 0.207
## 2:
             group
0.450
```

One can perform the same thing on *factor* (though it is not optimal given the fact that one has to carry out a t-test), hence, a part of the difference between the  $\eta_p^2$  and the  $\eta_g^2$  will reflect the impact of correlation among repeated measures on the  $\eta_p^2$  magnitude. Before, running the last ANOVA, we can just compute the correlation between factor 1 and factor 2 to have an intuitive idea of this impact.

```
corG <- cor(x = factor1, y = factor2); print(corG)</pre>
## [1] 0.6746506
Model3 <- ezANOVA(data long,</pre>
                  dv = . (DV), wid = .(id),
                  within =. (factor),
                  type = 3,
                  detailed = T,
                  return aov = T); report(Model3)
## Warning in as.data.table.list(anova): Item 1 has 2 rows but
longest item has 3;
## recycled with remainder.
##
           Factor DFn DFd
                               F
                                       MSE p
                                                      eta eta.Ll
                                               ges
eta.Ul
                    1 89 19.10 2.5800997 0 0.150 0.177
## 1: (Intercept)
0.290
## 2:
           factor
                    1 89 28.51 0.5585151 0 0.054 0.243
                                                           0.122
0.356
```

Given the fact, again, that "For designs where all factors are manipulated between participants,  $\eta_p^2$  and  $\eta_G^2$  are identical", a  $\eta_p^2$  computed with factor as a between-subjects variable should equal the  $\eta_G^2$ , hence, the correlation between measures inflated well the  $\eta_p^2$  depicted above. To be sure about that, we have just to estimate the t-statistic when factor is treated as a between-subjects variable and as a within-subjects variable and convert them into  $\eta_p^2$ , using  $\eta_p^2 = \frac{t^2}{t^2 + dferror}$ 

The returned values correspond well to both the  $\eta_p^2$  and the  $\eta_G^2$  estimated from the last ANOVA, thus, the returned  $\eta_p^2$  is inflated *in part* due to the correlation (since  $t_{\text{within}} = d_D^* \sqrt{n} = \lambda$ , see below) *as in SPSS*.

## Effect sizes and correlation among repeated measures

Above, I described that the returned  $\eta_p^2$  is inflated due to correlation among repeated measures (regardless of whether it was computed from a variance table or from the t statistics; as a precision:  $F = t^2$ ). In fact, consider a within-subjects variable, as highlighted earlier  $\eta_p^2 = \frac{t_{within}^2}{t_{within}^2 + dferror}$ , thus, given that  $t_{within} = d_D * \sqrt{n}$ , which is the noncentrality parameter  $\lambda$  (Fitts, 2021, 2022), where  $d_D = \frac{D}{S_D}$ , where  $D = m_x - m_y$ , and sd = thestandard deviation of the difference scores, such that  $\frac{D}{S_D} * \sqrt{n} \sim t_{n-1} \ (\frac{\Delta}{\sigma_D} * \sqrt{n} = \lambda)$ , where  $\frac{\Delta}{\sigma_D} = \delta_D$ , where  $\delta_D = \frac{\mu_X - \mu_Y}{\sqrt{\sigma_X^2 + \sigma_V^2 - 2*\rho*\sigma_X*\sigma_Y}}$  (Cousineau & Goulet-Pelletier, 2021), hence, the standardizer for the sample is equivalent, so that  $S_D = \sqrt{S_1^2 + S_2^2 - 2 * r * S_1 * S_2}$ , where ris the Pearson correlation coefficient, we better understand the relationship between the correlation coefficient and the Cohen's d<sub>D</sub> (see Cousineau & Goulet-Pelletier, 2021) and so with the returned  $\eta_p^2$ . In that, "As the correlation between measures increases, the standard deviation of the difference scores decreases" (Lakens, 2013; see also Dunlap et al., 1996), thus a strong correlation will inflate the standardized mean difference, while a weak correlation will deflate it, thus, a correlation = 0.5 will return a Cohen's  $d_D$  that equals Cohen's  $d_p$ . However, this is not because that the correlation among repeated measures equals 0.5, making Cohen's  $d_D$  indistinguishable from Cohen's  $d_p$  that this will make  $\eta_p^2$ indistinguishable from  $\eta_G^2$  or a  $\eta_p^2$  estimated from a between-subjects variable, at

minimum it seems true for the latter effect size when "For designs where all factors are manipulated between participants,  $\eta_p^2$  and  $\eta_G^2$  are identical" (In other scenarios I don't know). Let's perform two analyses, in which, we will set different parameters:

```
\Delta = 15, \sigma = 15, \rho = 0.5 and \rho = 0
```

```
## Compute the different Cohen's d
# Cohen's d p
dp <- Delta/sdp; dp</pre>
## [1] 1
# Cohen's d av = Cohen's d p
dav <- Delta/sdav; dav
## [1] 1
# Cohen's d D with Rho = 0.5
dDHalf <- Delta/sdD[2]; dDHalf</pre>
## [1] 1
# Cohen's d D with Rho = 0
dDZero <- Delta/sdD[1]; dDZero</pre>
## [1] 0.7071068
library(psych)
ms \leftarrow c(n, n)
hm <- harmonic.mean(ms)</pre>
## compute the t-statistics
# between-subjects design
lambda_dp <- dp* sqrt(hm/2)</pre>
# within-subjects design, when Rho = 0.5
lambda dDH <- dDHalf*sqrt(n)</pre>
# within-subjects design, when Rho = 0
lambda_dDZ <- dDZero*sqrt(n)</pre>
## compute the degrees of freedom
dfp \leftarrow sum(n,n)-2
dfD <- n-1
```

```
## return the partial etas squared
eta <- function(t, df) {
   partial <- t^2/sum(t^2, df)
   return(partial)
}

# eta for a between subjects-variable that equals the generalized
one
etaB <- eta(lambda_dp, dfp); etaB

## [1] 0.203252

# eta for a within subjects-variable when Rho = 0.5
etawH <- eta(lambda_dDH, dfD); etawH

## [1] 0.5050505

# eta for a within subjects-variable when Rho = 0
etawZ <- eta(lambda_dDZ, dfD); etawZ

## [1] 0.3378378</pre>
```

We better see that the  $\eta_p^2$  is always inflated compared to the  $\eta_G^2$ , even when  $\rho$  = 0 and  $\rho$  = 0.5. The following function use the formula provided by Lakens (2013) to convert  $\eta_n^2 SPSS$  to  $\eta_n^2 GPower$  for a power analysis.

```
Spss_to_GP <-function(eta, N, k, m, Rho) {
  fsq <- eta/(1-eta)
  Gpower <- fsq*((N-k)/N)*((m-1)/m)*(1-Rho)
  etaGP <- GPower/sum(1, 1, GPower)
  return(test)
}</pre>
```

At this moment using this formula for the two latter  $\eta_p^2$  (i.e., from a within-subject variable when  $\rho=0.5$  and  $\rho=0$ ) returns well the former (i.e., the effect size for a within-subjects variable that is estimated in the same way as for a between-subjects variable). Thus,  $\eta_p^2 GPower \approx \eta_G^2$ . The main issue is that the  $\eta_p^2 GPower$  is not only free from the correlation among repeated measures (remember how to compute the error sum of squares for within-subjects designs). As a consequence, it seems to respond to another issue regarding the  $\eta_p^2$  estimated with SPSS: why use specific conversion from a  $\eta_p^2$  estimated in

SPSS to  ${\eta_p}^2$  for GPower (see Lakens, 2013) given the fact that one can use the former and fill

in 0 for the value in "correlation among repeated measures"? It seems a reasonable idea,

but it appears to be a trap. In fact, I have just demonstrated that a  $\eta_n^2$  for a within-subjects-

variable even with no correlation is inflated compared to a  ${\eta_p}^2$  that is estimated to be

comparable to a  $\eta_p^2$  estimated for a between-subjects variable ( $\approx \eta_G^2$ ), and GPower

requires the latter. So, even with a null correlation, using a  $\eta_p^2$  for a within-subjects variable

should not be used when selecting options: as in GPower and fill in  $\rho = 0$  is not well-suited

(see below).

Resolve the main issue

Now, let 's take the illustration results in Lakens (2013), in which he compared two

sampled data, consider the following parameters:  $r_{xy} = 0.726$ ;  $\eta_n^2 = 0.71$  (within-subjects

variable);  $\eta_G^2 = \eta_p^2 = 0.26$  (between-subjects variable). Following Lakens (2013) and the

lines cited above, if in Gpower we choose "Options: as in GPower 3.0", we have to use  $\eta_p^2 =$ 

0.26 (which is identical to the  $\eta_G^2$ ), and r = 0.726 for "correlation among repeated

measures", theoretically, it should return similar results to what we would find if we choose

"Options: as in SPSS", and that we use  $\eta_n^2 = 0.71$ . To test this prediction, I choose two

scenarios for ANOVA: repeated measures, within factors:

1) 1 group, and 3 repeated measurements

2) 2 group, and 2 repeated measurements

Options: as in GPower 3.0.

F tests - ANOVA: Repeated measures, within factors

Analysis:

A priori: Compute required sample size

Input:

Effect size f

= 0.5927490

 $\alpha$  err prob = 0.05 Power (1- $\beta$  err prob) = 0.95 Number of groups = 1 Number of measurements = 3 Corr among rep measures = 0.726 Nonsphericity correction  $\epsilon$  = 1

**Output:** Noncentrality parameter  $\lambda = 23.0814773$ 

 Critical F
 = 4.1028210 

 Numerator df
 = 2.0000000 

 Denominator df
 = 10.0000000 

Total sample size = 6

Actual power = 0.9633994

Options: as in SPSS.

**F tests** – ANOVA: Repeated measures, within factors **Analysis:** A priori: Compute required sample size

Input: Effect size f(U) = 1.5646967

 $\begin{array}{lll} \alpha \ err \ prob & = & 0.05 \\ Power \ (1-\beta \ err \ prob) & = & 0.95 \\ Number \ of \ groups & = & 1 \\ Number \ of \ measurements & = & 3 \\ Nonsphericity \ correction \ \epsilon & = & 1 \end{array}$ 

Output: Noncentrality parameter  $\lambda = 24.4827576$ 

Critical F = 4.1028210 Numerator df = 2.0000000 Denominator df = 10.0000000

Total sample size = 6

Actual power = 0.9718118

Options: as in GPower 3.0.

**F tests** - ANOVA: Repeated measures, within factors

**Analysis:** A priori: Compute required sample size

Input: Effect size f = 0.5927490

 $\alpha$  err prob = 0.05

Power (1- $\beta$  err prob) = 0.95

Number of groups = 2

Number of measurements = 2

Corr among rep measures = 0.726

Nonsphericity correction  $\epsilon = 1$ 

Noncentrality parameter  $\lambda$  = 20.5168687 Critical F = 5.9873776 Numerator df = 1.0000000 Denominator df = 6.0000000

Total sample size = 8

Actual power = 0.9620289

Options: as in SPSS.

Output:

F tests - ANOVA: Repeated measures, within factors

Analysis: A priori: Compute required sample size

Input: Effect size f(U) = 1.5646967

 $\alpha$  err prob = 0.05 Power (1- $\beta$  err prob) = 0.95 Number of groups = 2 Number of measurements = 2 Nonsphericity correction  $\epsilon$  = 1

**Output:** Noncentrality parameter  $\lambda = 19.5862061$ 

 Critical F
 = 5.3176551

 Numerator df
 = 1.0000000

 Denominator df
 = 8.0000000

Total sample size = 10

Actual power = 0.9707300

Results seem comparable, the little gap between results could be due to a history of approximate values. In the following tests, we will take back the results from our simulated data and perform two power analyses, in which we will use the effect sizes estimated for the interaction between factor and group. I also imagine that for this interaction, the returned  $\eta_G^2$  is equivalent to the  $\eta_p^2$  that one would find for an interaction between two between-subjects variables (I think). As a consequence, if we fill in the  $\eta_G^2$  and the correlation among repeated measures, the total sample size needed may be equal the total sample size estimated with options: as in SPSS when we use the  $\eta_p^2$ .

Options: as in GPower 3.0.

F tests - ANOVA: Repeated measures, within-between interaction

Analysis: A priori: Compute required sample size

Input: Effect size f = 0.4101516

 $\alpha$  err prob = 0.05 Power (1- $\beta$  err prob) = 0.95 Number of groups = 3 Number of measurements = 2

Corr among rep measures = 0.6746506

Nonsphericity correction  $\epsilon = 1$ 

**Output:** Noncentrality parameter  $\lambda = 21.7164134$ 

Total sample size = 21

Actual power = 0.9759238

Options: as in SPSS.

F tests - ANOVA: Repeated measures, within-between interaction

Analysis: A priori: Compute required sample size

Input: Effect size f(U) = 1.1727305

 $\alpha$  err prob = 0.05 Power (1- $\beta$  err prob) = 0.95 Number of groups = 3 Number of measurements = 2 Nonsphericity correction  $\epsilon$  = 1

**Output:** Noncentrality parameter  $\lambda = 20.6294524$ 

Critical F = 3.6823203 Numerator df = 2.0000000 Denominator df = 15.0000000

Total sample size = 18

Actual power = 0.9638169

As we see results are fairly comparable. Apparently, one can choose the option as in GPower because it returns a greater a priori sample size which is desirable to better increase the power. The little gap between both results could be due to the fact that  $\eta_G^2$  is smaller than the  $\eta_p^2$  (though theoretically it should not be the case, cf. the citation from Lakens, but here I'm not sure because it is applied for an interaction), Hence, one can use the formula provided in Lakens (2013) to convert the  $\eta_p^2 SPSS$  into  $\eta_p^2 GPower$ , use the latter in GPower, and theoretically it should return similar results.

**F tests –** ANOVA: Repeated measures, within-between interaction

Analysis: A priori: Compute required sample size

Input: Effect size f = 0.4650467  $\alpha \text{ err prob}$  = 0.05Power (1- $\beta$  err prob) = 0.95

Number of groups = 3
Number of measurements = 2

 $Corr \ among \ rep \ measures \qquad \qquad = \ \ 0.6746506$ 

Nonsphericity correction  $\epsilon = 1$ 

**Output:** Noncentrality parameter  $\lambda = 23.9301612$ 

 Critical F
 = 3.6823203

 Numerator df
 = 2.0000000

 Denominator df
 = 15.0000000

Total sample size = 18

Actual power = 0.9821984

Bingo, it returns nearly similar outputs. In my opinion the three options are fairly valid.

## The current experiment

For my experiment, we want to use the effect size estimated for the interaction between "Group" and "Contingency", that is, an interaction between a between-subject variable and a within-subject variable. However, I don't know whether the correlation between high and low contingency trials are taken into account the effect size of interest. It appears that this is the case since using the same descriptive statistics in a simulation and play with different correlation coefficient returns different  $\eta_p^2$  for an interaction in a same design, see the following link:

Chapter 5 Mixed ANOVA | Power Analysis with Superpower (aaroncaldwell.us)

So, from the paper of interest we have the following estimates:

 $r=0.8513636;~\eta_p{}^2SPSS=0.181;\eta_p{}^2GPower=0.01574769;\eta_G{}^2=0.01386743$  and we can perform the power analysis using three different means:

Options: as in GPower 3.0, with  $\eta_p^2 GPower$ 

F tests - ANOVA: Repeated measures, within-between interaction

Analysis: A priori: Compute required sample size

Input: Effect size f = 0.1264897 $\alpha \text{ err prob}$  = 0.05

Power (1- $\beta$  err prob) = 0.95 Number of groups = 3 Number of measurements = 2

Corr among rep measures = 0.8513636

Nonsphericity correction  $\epsilon = 1$ 

Output: Noncentrality parameter  $\lambda = 16.1464260$ 

 Critical F
 = 3.1239074

 Numerator df
 = 2.0000000

 Denominator df
 = 72.0000000

Total sample size = 75

Actual power = 0.9504779

Options: as in GPower 3.0, with  $\eta_G^2$ 

F tests - ANOVA: Repeated measures, within-between interaction

Analysis: A priori: Compute required sample size

Input: Effect size f = 0.1185852

 $\alpha$  err prob = 0.05 Power (1- $\beta$  err prob) = 0.95 Number of groups = 3 Number of measurements = 2

Corr among rep measures = 0.8513636

Nonsphericity correction  $\epsilon = 1$ 

Output: Noncentrality parameter  $\lambda = 16.4620930$ 

Critical F = 3.1051566 Numerator df = 2.0000000 Denominator df = 84.0000000

Total sample size = 87

Actual power = 0.9553120

Options: as in SPSS.

F tests - ANOVA: Repeated measures, within-between interaction

Analysis: A priori: Compute required sample size

Input: Effect size f(U) = 0.4701077

 $\begin{array}{lll} \alpha \ err \ prob & = & 0.05 \\ Power \ (1-\beta \ err \ prob) & = & 0.95 \\ Number \ of \ groups & = & 3 \\ Number \ of \ measurements & = & 2 \\ Nonsphericity \ correction \ \epsilon & = & 1 \\ \end{array}$ 

**Output:** Noncentrality parameter  $\lambda = 16.5750937$ 

Critical F = 3.1186421 Numerator df = 2.0000000 Denominator df = 75.0000000

Total sample size = 78

Actual power = 0.9557629

Thus, according to the power analysis at a total of 80 participants seems desirable for my following experiment.

A last power analysis can also be conducted to better illustrate that one has not to use: Set

the following parameters:  ${\eta_{p}}^{2}=0.181$  and p= 0.

F tests - ANOVA: Repeated measures, within-between interaction

Analysis: A priori: Compute required sample size

Number of groups = 0.

Number of measurements = 2Corr among rep measures = 0Nonsphericity correction  $\in = 1$ 

Output: Noncentrality parameter  $\lambda = 17.2380975$ 

 Critical F
 = 3.2594463 

 Numerator df
 = 2.0000000 

 Denominator df
 = 36.0000000 

Total sample size = 39

Actual power = 0.9547314

### References

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