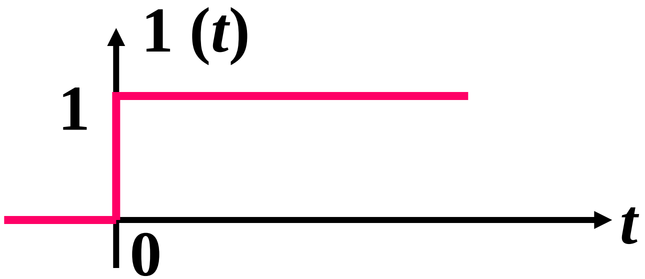
第7章　动态电路的暂态分析

(dynamic circuit) (transient analysis)

§ 7-5 一阶电路的阶跃响应和冲激响应

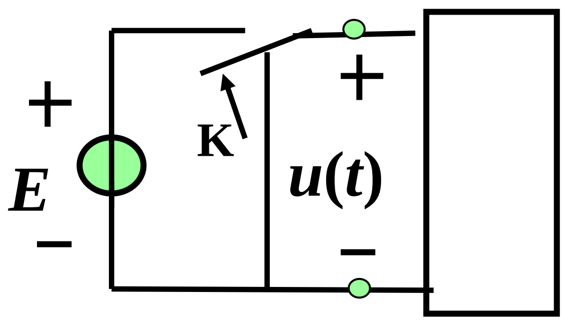
一　单位阶跃函数

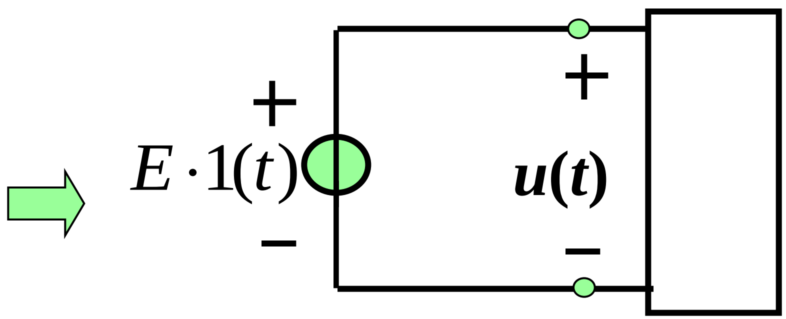
1. 定义　$1(t)=\begin{cases}0 & t<0\\ 1 & t>0\end{cases}$

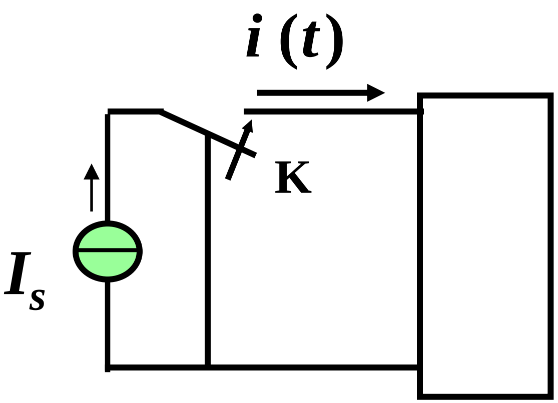


用 $1(t)$ 来描述开关的动作

$t=0$合闸　$u(t)=E\cdot 1(t)$

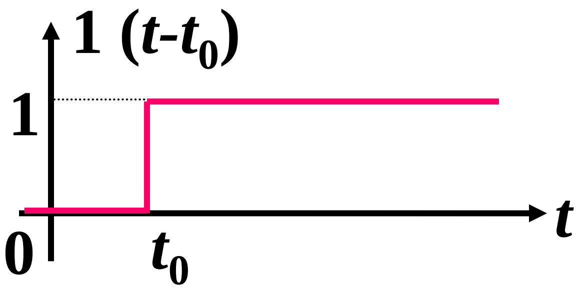






$t=0$合闸　$i(t)=I_s\cdot 1(t)$

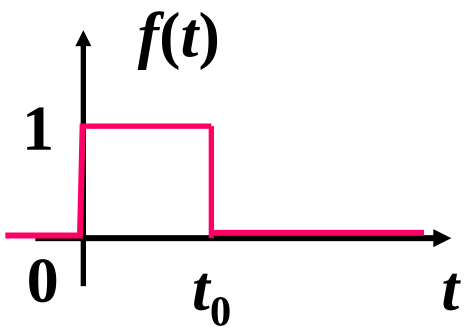
2. 单位阶跃函数的延迟

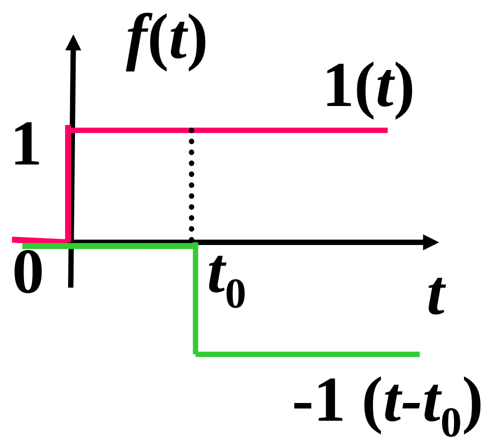


$$1(t-t_0)=\begin{cases}0 & (t<t_0)\\ 1 & (t>t_0)\end{cases}$$

3. 由单位阶跃函数可组成复杂的信号

例 1





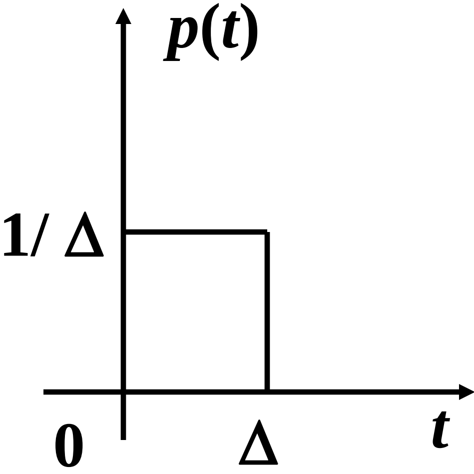
$$f(t)=1(t)-1(t-t_0)$$

二 单位冲激函数

1. 单位脉冲函数 $p(t)$

$$p(t)=\frac{1}{\Delta}[1(t)-1(t-\Delta)]$$

$$\int_{-\infty}^{\infty}p(t)\mathrm{d}t=1$$

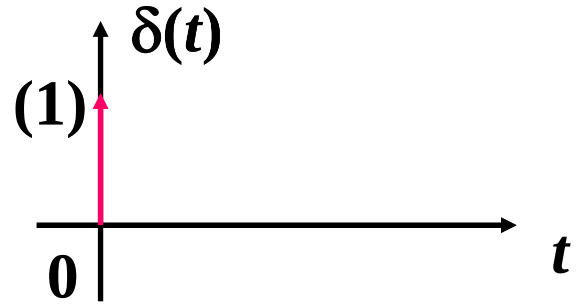


2. 单位冲激函数 $\delta(t)$

$$\Delta\to 0 \qquad \frac{1}{\Delta}\to\infty$$

$$\lim_{\Delta\to 0}p(t)=\delta(t)$$

定义

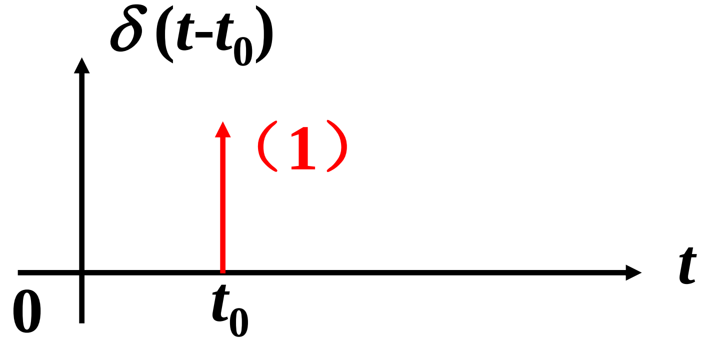


$$\delta(t)=\begin{cases}0 & (t<0)\\ 0 & (t>0)\end{cases}$$

$$\int_{-\infty}^{\infty}\delta(t)\mathrm{d}t=1$$

3. 单位冲激函数的延迟 $\delta(t-t_0)$

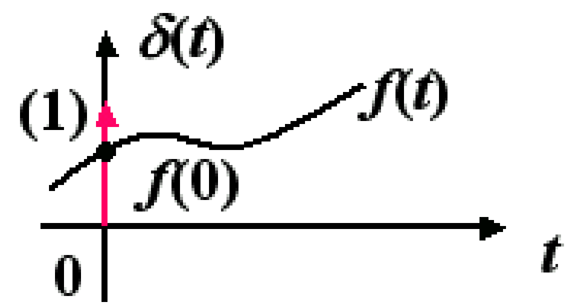
$$\begin{cases} \delta(t-t_0)=0 \quad (t \neq t_0) \\ \int_{-\infty}^{\infty} \delta(t-t_0)\mathrm{d}t = 1 \end{cases}$$



4. δ 函数的筛分性

$$\int_{-\infty}^{\infty} \underbrace{f(t)\delta(t)}_{f(0)\delta(t)}\mathrm{d}t = f(0)\int_{-\infty}^{\infty}\delta(t)\mathrm{d}t = f(0)$$

同理有： $\int_{-\infty}^{\infty} f(t)\delta(t-t_0)\mathrm{d}t = f(t_0)$



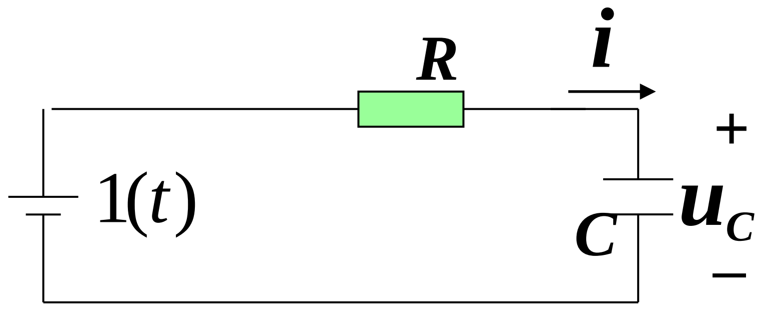
* $f(t)$在 t_0 处连续

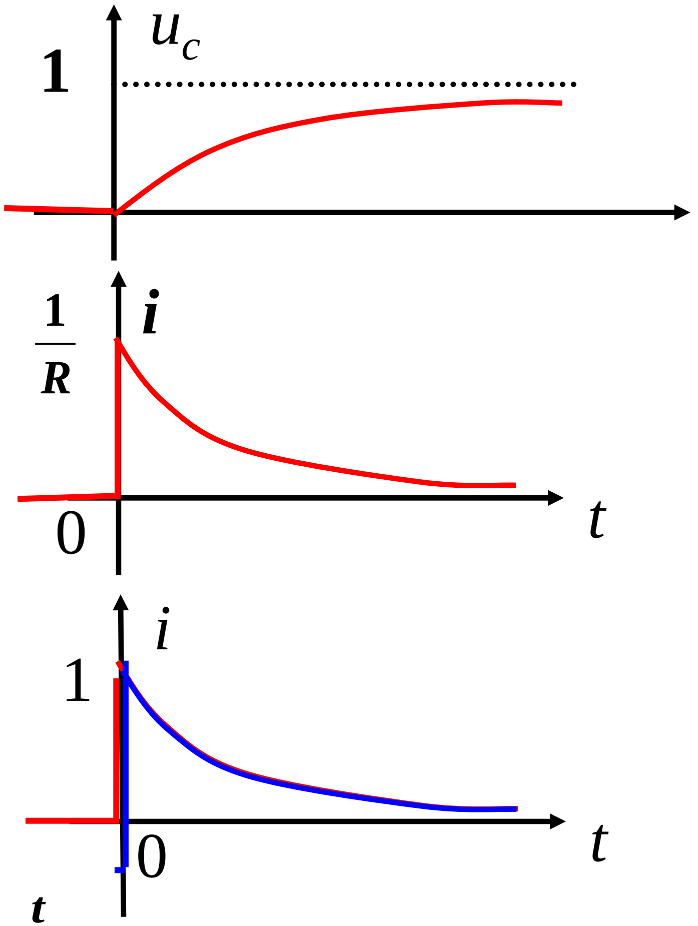
5. $\delta(t)$与1(t)的关系

$$\delta(t) = \frac{d1(t)}{dt}$$

三　阶跃和冲激响应

1、阶跃响应s(t)： 单位阶跃函数激励下电路中产生的零状态响应



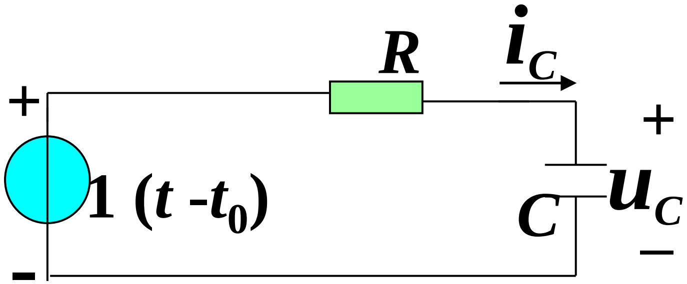


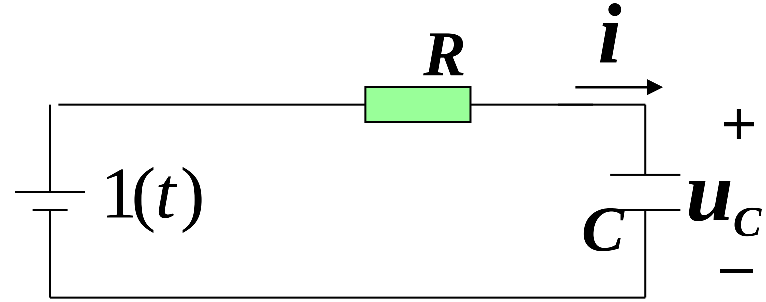
$u_C(0^-)=0$

$$u_C(t)=(1-e^{-\frac{t}{RC}})\ 1(t)$$

$$i(t)=\frac{1(t)-u_C}{R}=\frac{1}{R}e^{-\frac{t}{RC}}\ 1(t)$$

注意　$i=e^{-\frac{t}{RC}}\ 1(t)$ 和 $i=e^{-\frac{t}{RC}}$　$t\geq 0$ 的区别



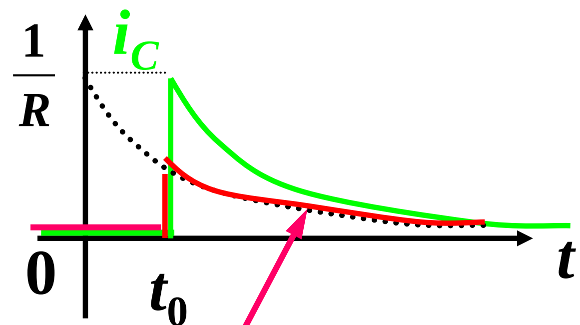


$u_C(0^-)=0$

激励在 $t = t_0$ 时加入，
则响应从$t=t_0$开始。

$$i(t)=\frac{1}{R}e^{-\frac{t}{RC}}1(t)$$

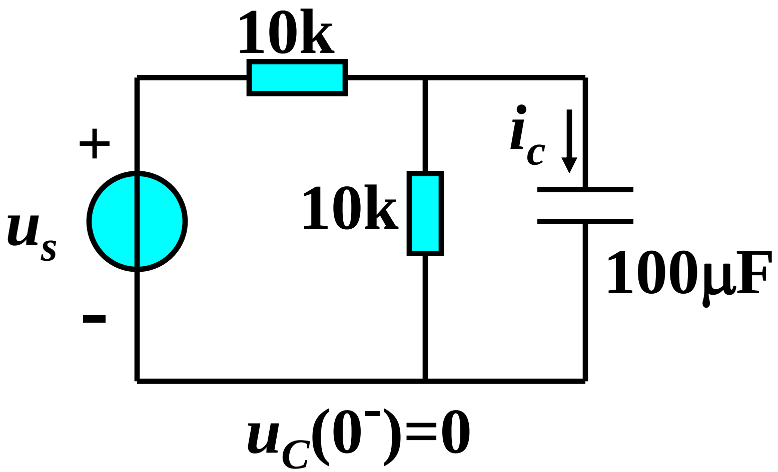
$$i_C=\frac{1}{R}e^{-\frac{t-t_0}{RC}}1(t-t_0)$$

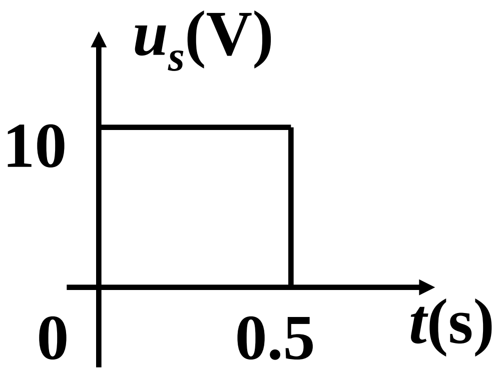


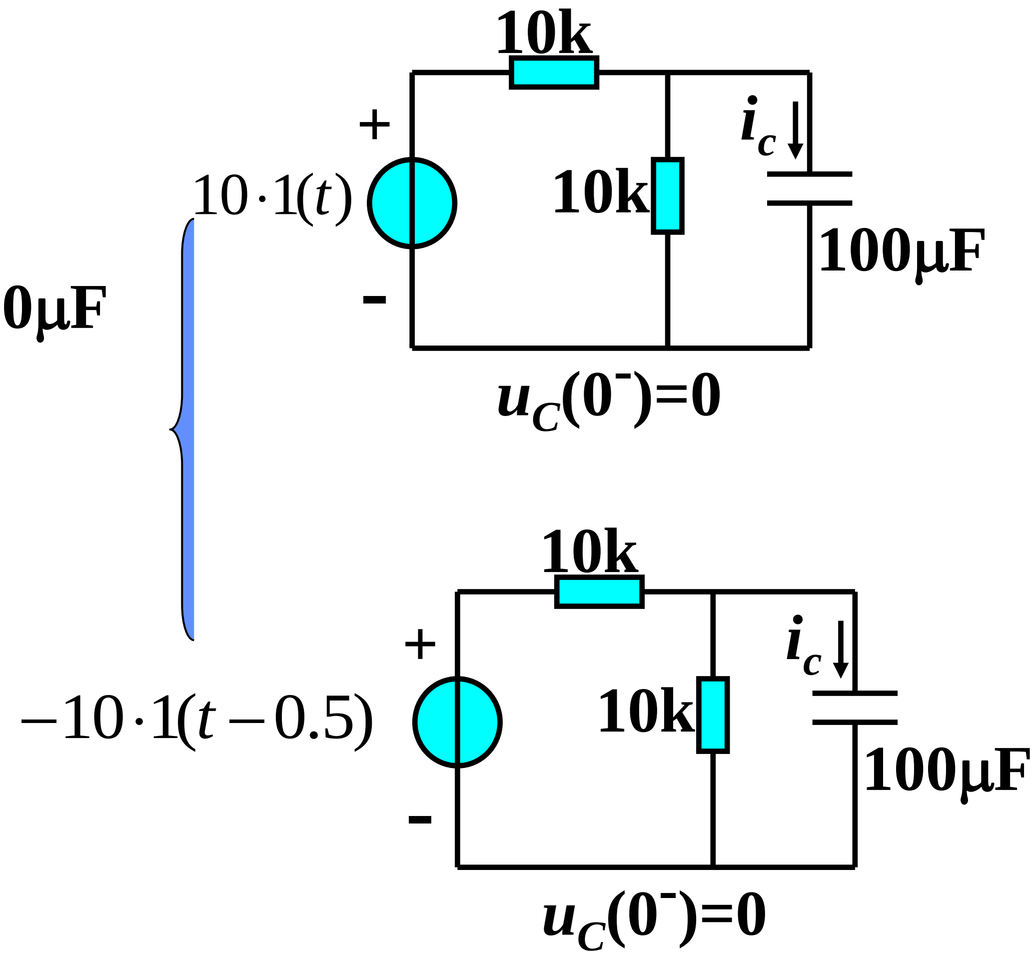
注意

不要写为 $\frac{1}{R}e^{-\frac{t}{RC}}1(t-t_0)$

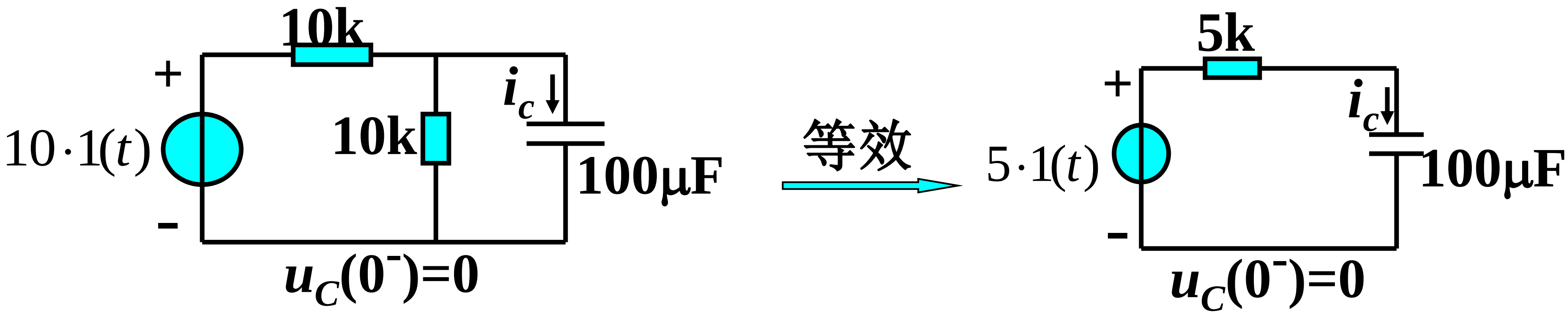
例 求图示电路中电流 $i_C(t)$





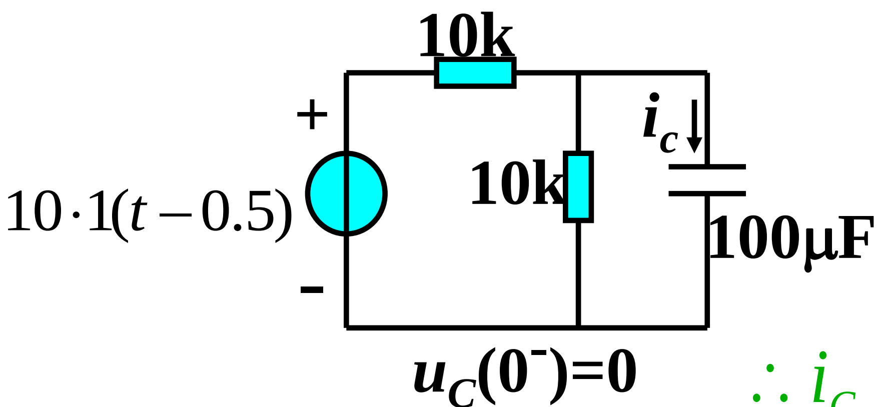


$$u_S = 10\cdot 1(t) - 10\cdot 1(t-0.5)$$



$$\tau = RC = 100\times10^{-6}\times5\times10^{-3} = 0.5\text{s}$$

$$i_C = e^{-2t}1(t) \quad \text{mA}$$



$$i_C = e^{-2(t-0.5)}1(t-0.5)\ \text{mA}$$

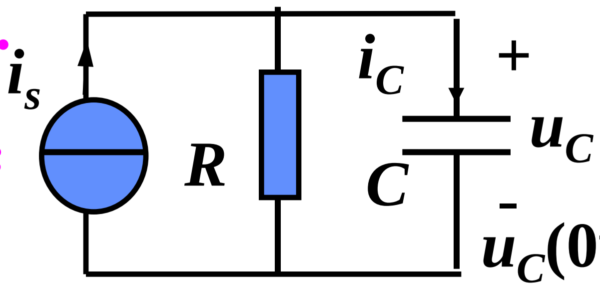
$$\therefore i_C = e^{-2t}1(t) - e^{-2(t-0.5)}1(t-0.5) \quad \text{mA}$$

2、冲激响应：单位冲激函数激励下电路中产生的零状态响应h(t)

1）分二个时间段来考虑冲激响应

例1.

解法1:



$u_C(0^-)=0$

$t \begin{cases} 0^- \longrightarrow 0^+ & \text{电容充电} \\ 0^+ \longrightarrow \infty & \text{零输入响应} \end{cases}$

(1). t 在 $0^- \longrightarrow 0^+$间

$$C\frac{du_c}{dt}+\frac{u_c}{R}=\delta(t)$$

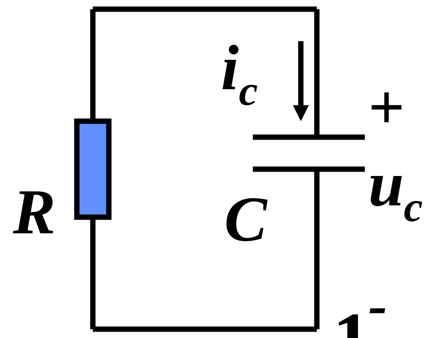
u_c 不可能是冲激函数，否则KCL不成立

$$\int_{0^-}^{0^+} C\frac{du_c}{dt}dt+\underbrace{\int_{0^-}^{0^+}\frac{u_c}{R}dt}_{=0}=\underbrace{\int_{0^-}^{0^+}\delta(t)dt}_{=1}$$

$$C[u_c(0^+)-u_c(0^-)]=1 \qquad u_c(0^+)=\frac{1}{C} \neq u_C(0^-)$$

电容中的冲激电流使电容电压发生跳变

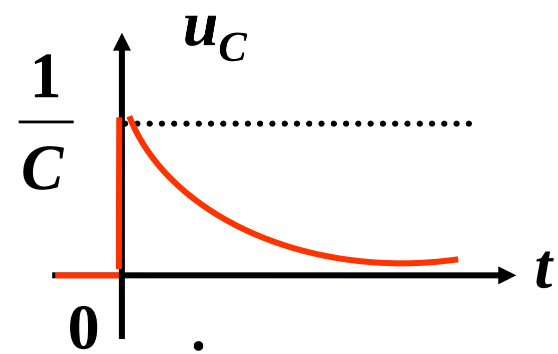
(2). $t > 0^+$ 零输入响应 （RC放电）



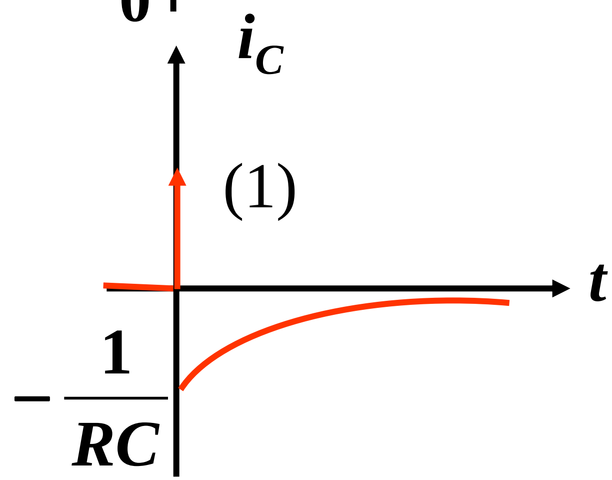
$$u_c(0^+) = \frac{1}{C}$$

$$u_c = \frac{1}{C} e^{-\frac{t}{RC}} \quad t \geq 0^+$$

$$i_c = -\frac{u_c}{R} = -\frac{1}{RC} e^{-\frac{t}{RC}} \quad t \geq 0^+$$



$$\begin{cases} u_c = \dfrac{1}{C} e^{-\frac{t}{RC}} 1(t) \\ i_c = \delta(t) - \dfrac{1}{RC} e^{-\frac{t}{RC}} 1(t) \end{cases}$$



2）由单位阶跃响应求单位冲激响应

$e(t) \longrightarrow$ 零状态 $\longrightarrow R(t)$

零状态相应与激励线性相关

单位阶跃 $1(t)$ $\Longrightarrow$ 单位阶跃响应 $s(t)$

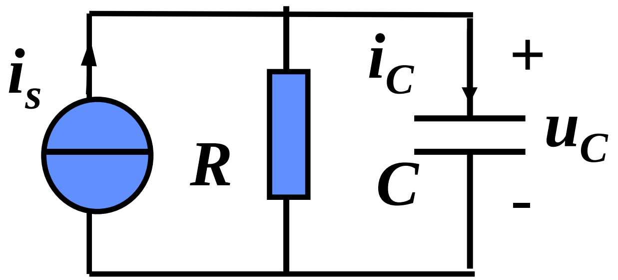
$$\delta(t) = \frac{d1(t)}{dt}$$

$$h(t) = \frac{d}{dt}s(t)$$

?

单位冲激 $\delta(t)$ $\Longrightarrow$ 单位冲激响应 $h(t)$

例1解法2



已知： $u_c(0^-)=0$

求： $i_s(t)$为单位冲激时电路响应 $u_C(t)$和 $i_C(t)$

先求单位阶跃响应　令　$i_s(t)=1(t)$

$u_{Cs}(0^+)=0$　　$u_{Cs}(\infty)=R$　　$\tau=RC$　　$i_{Cs}(0^+)=1$　　$i_{Cs}(\infty)=0$

$$u_{Cs}(t)=R(1-e^{-\frac{t}{RC}})1(t)$$

$$i_{Cs}=e^{-\frac{t}{RC}}1(t)$$

再求单位冲激响应　令　$i_s(t)=\delta(t)$

$$u_{Ch}=\frac{d}{dt}R(1-e^{-\frac{t}{RC}})1(t)=\underbrace{R(1-e^{-\frac{t}{RC}})\delta(t)}_{0}+\frac{1}{C}e^{-\frac{t}{RC}}1(t)$$

$$=\frac{1}{C}e^{-\frac{t}{RC}}1(t)$$

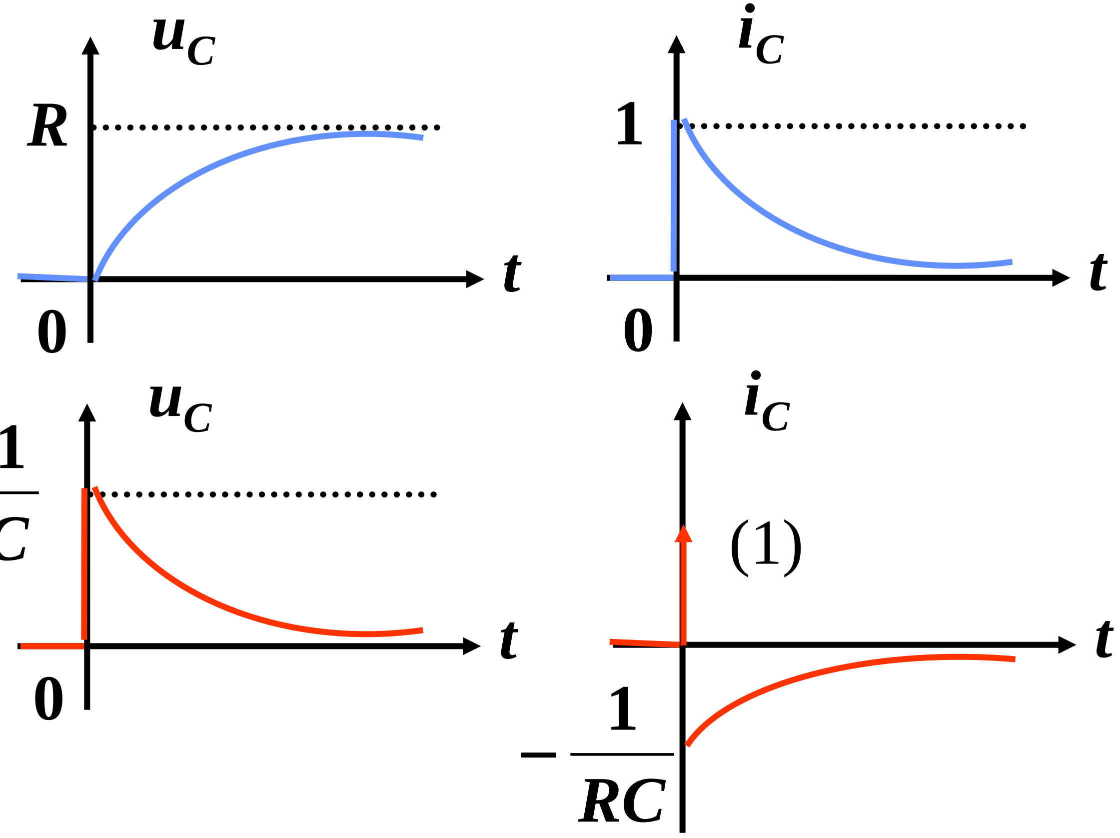
$f(t)\delta(t)$
$=f(0)\delta(t)$

$$i_c = \frac{\mathrm{d}}{\mathrm{d}t}[e^{-\frac{t}{RC}}1(t)] \quad = e^{-\frac{t}{RC}}\delta(t) - \frac{1}{RC}e^{-\frac{t}{RC}}1(t)$$

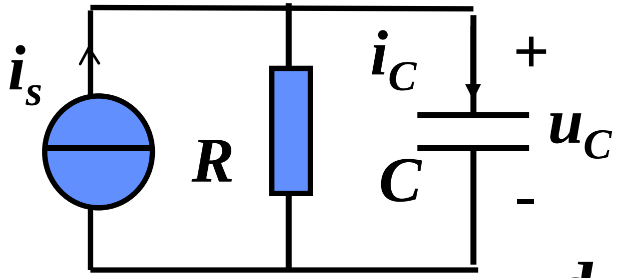
$$= \delta(t) - \frac{1}{RC}e^{-\frac{t}{RC}}1(t)$$

阶跃响应

冲激响应



例1解法3



已知： $u_c(0^-)=0$

求： $i_s(t)$为单位冲激时电路响应 $u_C(t)$和 $i_C(t)$

$$C\frac{du_c}{dt}+\frac{u_c}{R}=\delta(t) \qquad u_c=\alpha\delta(t)+Ae^{-\frac{t}{RC}}\varepsilon(t)$$

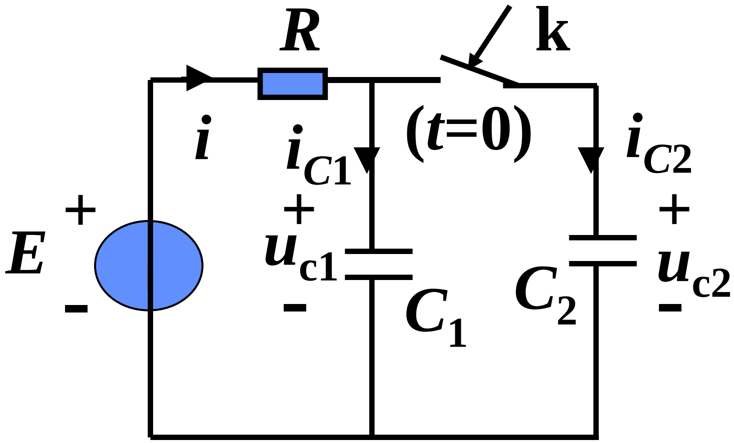
$$C\alpha\frac{d\delta(t)}{dt}+CAe^{-\frac{t}{RC}}\frac{d\varepsilon(t)}{dt}-\frac{1}{RC}CAe^{-\frac{t}{RC}}\varepsilon(t)$$

$$+\frac{1}{R}\left(\alpha\delta(t)+Ae^{-\frac{t}{RC}}\varepsilon(t)\right)=\delta(t)$$

$$\alpha=0$$

$$CA+\frac{1}{R}\alpha=1$$

$$\Longrightarrow \quad u_C=\frac{1}{C}e^{-\frac{t}{RC}}\varepsilon(t)$$



例：奇异电路的暂态响应

已知：E=1V，R=1Ω，C_1=0.25F，C_2=0.5F，t = 0时合k。

求：u_{C1}，u_{C2}。

解 合k前 $u_{C1}(0^-) = E = 1\text{V}$ $u_{C2}(0^-) = 0$

合k后 $u_{C1}(0^+) = u_{C2}(0^+) = u_C(0^+)$

电容电压初值一定会发生跳变。

$$\begin{cases} C_1 u_{C1}(0^+) + C_2 u_{C2}(0^+) = C_1 u_{C1}(0^-) + C_2 u_{C2}(0^-) \\ u_{C1}(0+) = u_{C2}(0+) \end{cases}$$

$u_C(\infty)$ =E= 1V

可解得 $u_C(0^+) = u_{C1}(0^+) = u_{C2}(0^+) = \dfrac{1}{3}\text{V}$

$\tau = R(C_1+C_2) = \dfrac{3}{4}\text{s}$

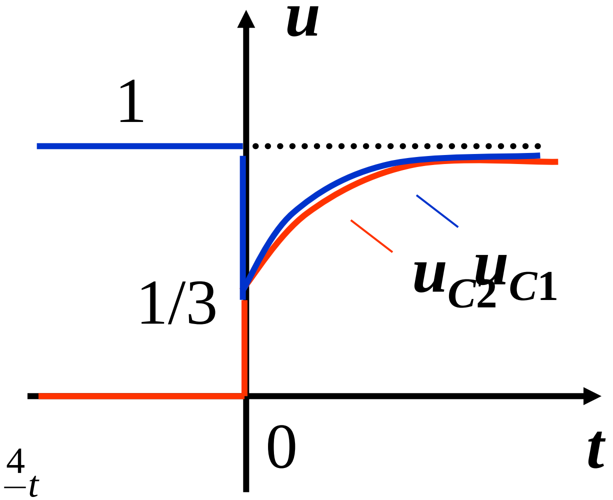
$$u_C(t) = 1 + (\frac{1}{3} - 1)e^{-\frac{4}{3}t} = 1 - \frac{2}{3}e^{-\frac{4}{3}t} \quad t \geq 0^+$$

$$u_C(t) = 1 + (\frac{1}{3} - 1)e^{-\frac{4}{3}t} = 1 - \frac{2}{3}e^{-\frac{4}{3}t} \quad t \geq 0^+$$

$$t \leq 0^- \quad u_{C1}(0^-) = 1 \quad u_{C2}(0^-) = 0$$

$$u_{C1}(t) = 1(-t) + (1 - \frac{2}{3}e^{-\frac{4}{3}t})\,1(t)$$

$$u_{C2}(t) = (1 - \frac{2}{3}e^{-\frac{4}{3}t})\,1(t)$$



$$i_{C1} = C_1 \frac{\mathrm{d}u_1}{\mathrm{d}t}$$

$$= 0.25[-\delta(-t) + \frac{8}{9}e^{-\frac{4}{3}t}1(t) + (1 - \frac{2}{3}e^{-\frac{4}{3}t})\delta(t)]$$

$$= -\frac{1}{6}\delta(t) + \frac{2}{9}e^{-\frac{4}{3}t}1(t)$$

$$u_{C1}(t)=1(-t)+(1-\frac{2}{3}e^{-\frac{4}{3}t})1(t)$$

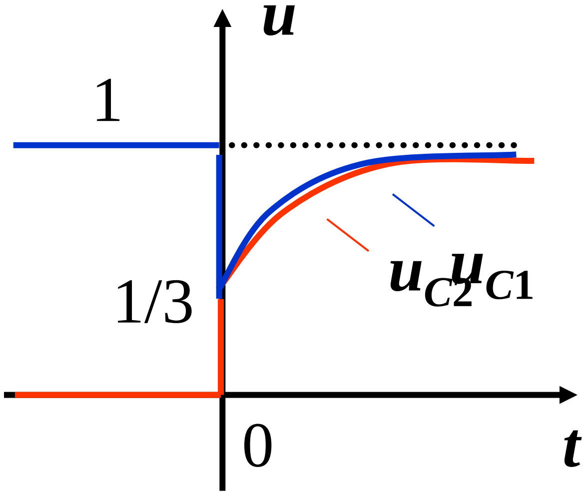
$$i_{C1}(t)=-\frac{1}{6}\delta(t)+\frac{2}{9}e^{-\frac{4}{3}t}1(t)$$

$$u_{C2}(t)=(1-\frac{2}{3}e^{-\frac{4}{3}t})1(t)$$

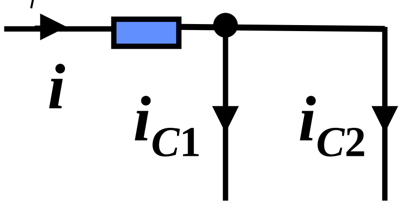
$$i_{C2}(t)=C_2\frac{\mathrm{d}u_2}{\mathrm{d}t}$$

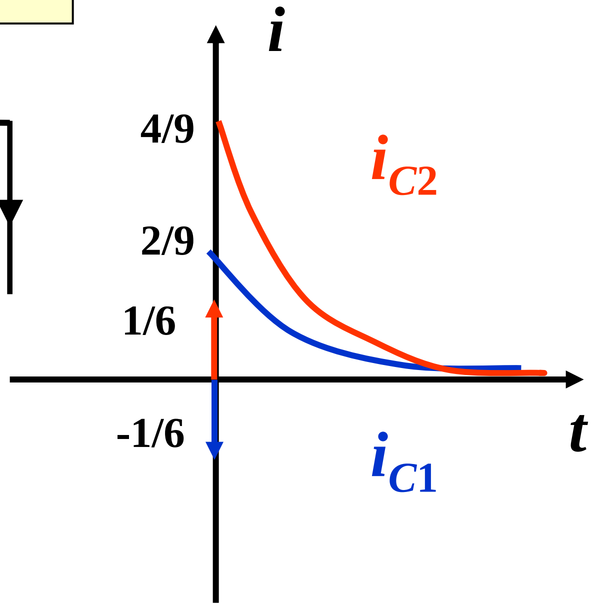
$$=0.5\,[\frac{8}{9}e^{-\frac{4}{3}t}1(t)+(1-\frac{2}{3}e^{-\frac{4}{3}t})\delta(t)]$$

$$=\frac{1}{6}\delta(t)+\frac{4}{9}e^{-\frac{4}{3}t}1(t)$$



***i* 无冲激**





根据物理概念求电容电流

$0^- \rightarrow 0^+$

$$\Delta u_{C1} = \frac{1}{3} - 1 = -\frac{2}{3}$$

转移的电荷 $\Delta q_1 = 0.25 \times (-2/3) = -1/6$

$$\Delta u_{C2} = 1/3 - 0 = 1/3$$

$-1/6\ \delta(t)$ 冲激电流

转移的电荷 $\Delta q_2 = 0.5 \times 1/3 = 1/6$

$1/6\ \delta(t)$ 冲激电流

$t > 0^+$

$$i_{C1} = 0.25 \frac{\mathrm{d}(1 - \frac{2}{3} e^{-\frac{4}{3}t})}{\mathrm{d}t} = \frac{2}{9} e^{-\frac{4}{3}t}$$

$$i_{C2} = 0.5 \frac{\mathrm{d}(1 - \frac{2}{3} e^{-\frac{4}{3}t})}{\mathrm{d}t} = \frac{4}{9} e^{-\frac{4}{3}t}$$

$$\begin{cases} i_{C1} = -\frac{1}{6}\delta(t) + \frac{2}{9} e^{-\frac{4}{3}t} 1(t) \\ i_{C2} = \frac{1}{6}\delta(t) + \frac{4}{9} e^{-\frac{4}{3}t} 1(t) \end{cases}$$

作业

- 初值：7-2，4，5，6*，7*
- 一阶电路：7-8，11，14*，17，16（冲激）
- ~~不要：7.16，19，21，24，25，32，33，34~~
- 二阶：7.20，22，24

讨论：7-9，8，34

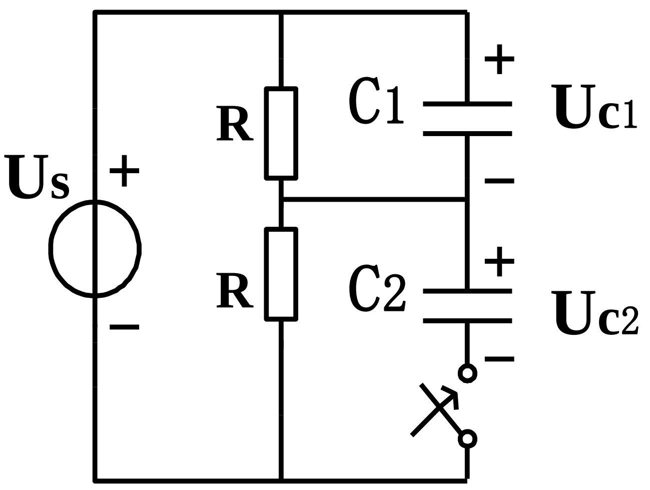
竞答：7-37，38，41，43

例7： 如图电路，$R=10\Omega$，$C_1=0.01F$

$C_2=0.02F$，$U_S=10V$，$U_{C2}(0\_)=0$

开关打开已久。求开关闭合后

$U_{C1}(t)$和$U_{C2}(t)$ 。



解：用三要素法求解

1）求初值 $t=0^+$ 电容电压跳变

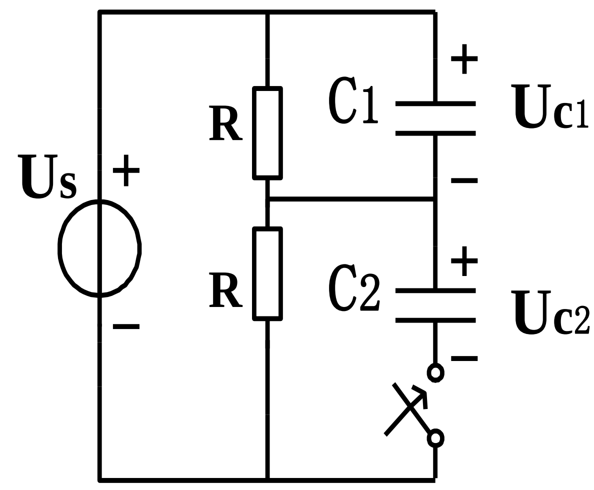
$$U_{C1}(0^-)=\frac{U_S}{2}=5V,\quad U_{C2}(0^-)=0$$

$$U_S=U_{C1}(0^+)+U_{C2}(0^+)$$

$$-C_1\times U_{C1}(0^-)+C_2\times U_{C2}(0^-)=-C_1\times U_{C1}(0^+)+C_2\times U_{C2}(0^+)$$

$$U_{C2}(0^+) = \frac{-C_1 \times U_{C1}(0^-) + C_2 \times U_{C2}(0^-) + C_1 \times U_S}{C_1 + C_2}$$

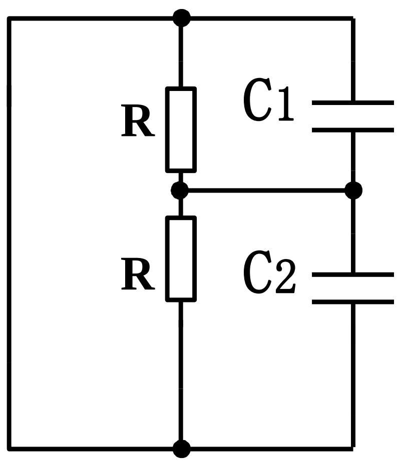
$$= \frac{5}{3}V$$



$$U_{C1}(0^+) = U_S - U_{C2}(0^+) = \frac{25}{3}V$$

2）电容电压稳态值

$$U_{C1}(\infty) = U_{C2}(\infty) = \frac{U_S}{2} = 5V$$



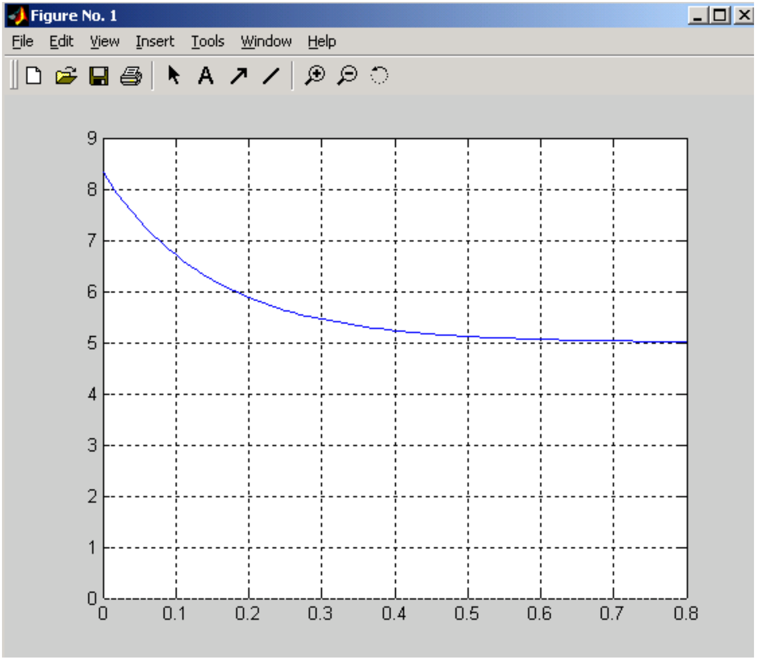
3）电路时间常数

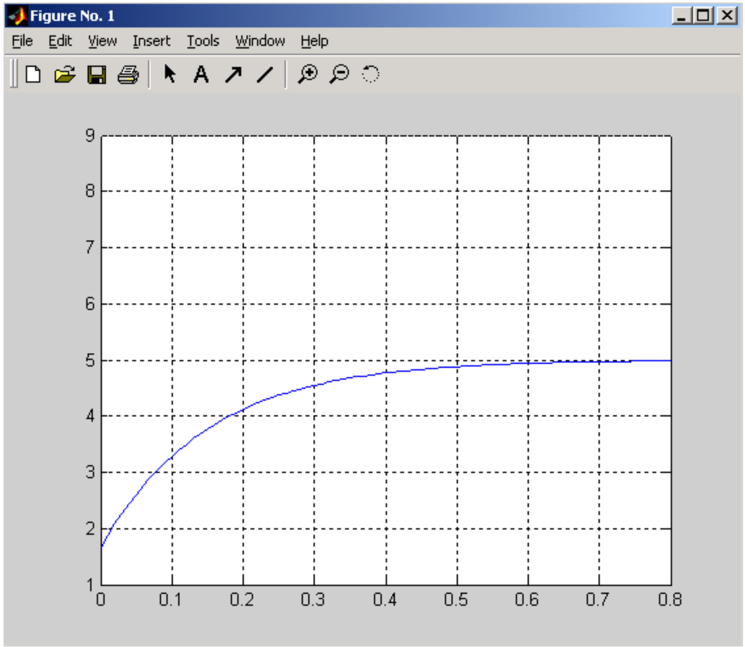
$$\tau = R^{'}C^{'} = \frac{R}{2} \times (C_1 + C_2) = \frac{3}{20}$$

$$U_{C1}(t) = U_{C1}(\infty) + [U_{C1}(0^+) - U_{C1}(\infty)]e^{-\frac{t}{\tau}}$$

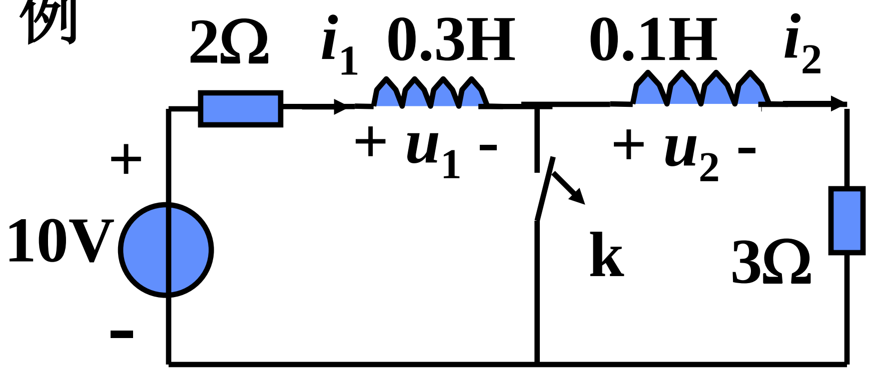
$$U_{C1}(t) = 5 + [\frac{25}{3} - 5]e^{-\frac{20}{3}t}$$

$$U_{C2}(t) = 5 + [\frac{5}{3} - 5]e^{-\frac{20}{3}t}$$





例



已知 如图 要打开开关K

求： i_1， i_2和 u_1 ，u_2。

解

$$i_1(0^-)=5\text{A} \qquad i_2(0^-)=0$$

而 $i_1(0^+)=i_2(0^+)=i(0^+)$

电感电流发生跃变

$$0.3i_1(0^+)+0.1i_2(0^+)=0.3i_1(0^-)+0.1i_2(0^-)$$

$$i(\infty)=2\text{A}$$

$$i(0^+)=\frac{0.3\times5}{0.3+0.1}=3.75\text{A}$$

$$\tau=(0.3+0.1)/(2+3)$$

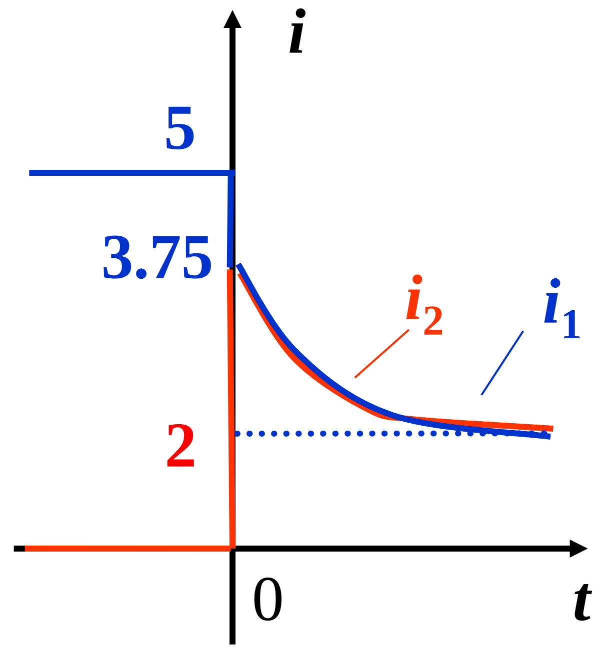
$$i(t)=2+(3.75-2)e^{-12.5t}=2+1.75e^{-12.5t} \quad t\geq0^+$$

$$i(t) = 2 + (3.75 - 2)e^{-12.5t} = 2 + 1.75e^{-12.5t} \quad t \geq 0^+$$

$$t \leq 0^- \quad i_1(0^-) = 5 \qquad\qquad i_2(0^-) = 0$$

$$i_1(t) = 5\,1(-t) + (2 + 1.75e^{-12.5t})1(t)$$

$$i_2(t) = (2 + 1.75e^{-12.5t})1(t)$$



$$u_1(t) = L_1 \frac{\mathrm{d}i_1}{\mathrm{d}t}$$

$$= 0.3[-5\delta(-t) - 21.875e^{-12.5t}1(t) + (2 + 1.75e^{-12.5t})\delta(t)]$$

$$= 0.3[-5\delta(-t) - 21.875e^{-12.5t}1(t) + 3.75\delta(t)]$$

$$= -0.375\delta(t) - 6.5625e^{-12.5t}1(t)$$

$$i_1(t) = 5\,1(-t) + (2 + 1.75e^{-12.5t})\,1(t)$$

$$u_1 = -0.375\delta(t) - 6.5625e^{-12.5t}\,1(t)$$

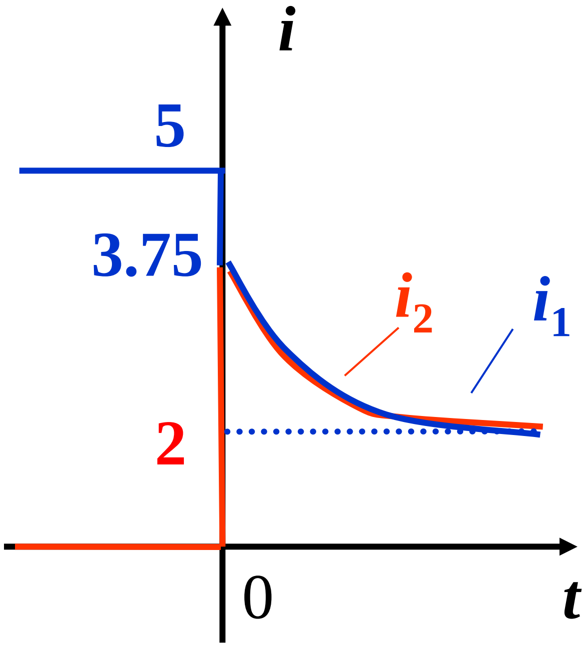
$$i_2(t) = (2 + 1.75e^{-12.5t})\,1(t)$$

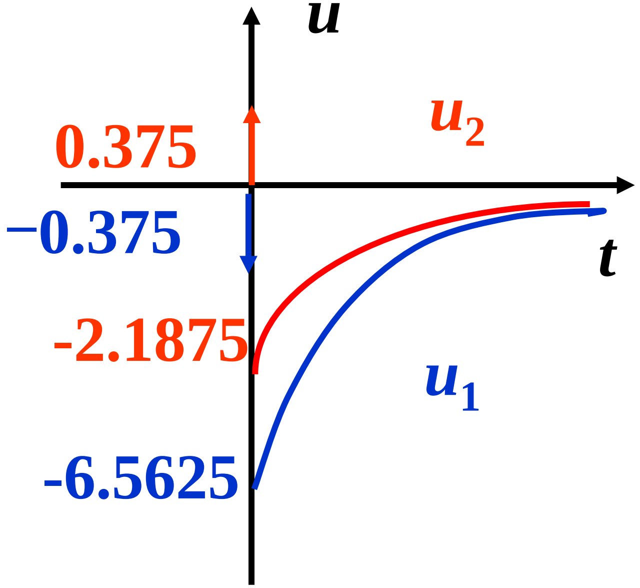
$$u_2 = L_2 \frac{\mathrm{d}i_2}{\mathrm{d}t}$$

$$= 0.1\,[-21.875e^{-12.5t}\,1(t) + 3.75\delta(t)]$$

$$= 0.375\delta(t) - 2.1875e^{-12.5t}\,1(t)$$

$u_1 + u_2$ 没有冲激





根据物理概念求电压

$0^- \rightarrow 0^+$

$$\Delta i_1 = 3.75 - 5 = -1.25$$

转移的磁链 $\Delta\psi_1 = 0.3 \times (-1.25) = -0.375$

$-0.375\,\delta(t)$ 冲激电压

$$\Delta i_2 = 3.75 - 0 = 3.75$$

转移的磁链 $\Delta\psi_2 = 0.1 \times 3.75 = 0.375$

$0.375\,\delta(t)$ 冲激电压

$t > 0^+$

$$u_1 = 0.3\frac{\mathrm{d}(2 + 1.75e^{-12.5t})}{\mathrm{d}t} = -6.5625e^{-12.5t}$$

$$u_2 = 0.1\frac{\mathrm{d}(2 + 1.75e^{-12.5t})}{\mathrm{d}t} = -2.1875e^{-12.5t}$$

$$\begin{cases} u_1 = -0.375\delta(t) - 6.5625e^{-12.5t}1(t) \\ u_2 = 0.375\delta(t) - 2.1875e^{-12.5t}1(t) \end{cases}$$