Chapter 5
Laser

In previous Chapter (1)   Gain

the Gain coefficient of laser medium $\gamma(\nu)$:

$$\gamma(\nu) = N\sigma(\nu) = N\frac{\lambda^2}{8\pi t_{sp}} g(\nu)$$

So the photon flux

$$\phi(z) = \phi(0)\exp[\gamma(\nu)z]$$

Gain

$$G(\nu) = \exp[\gamma(\nu)d]$$

Lorentzian lineshape

$$\gamma(\nu) = \gamma(\nu_0)\frac{(\Delta\nu/2)^2}{(\nu-\nu_0)^2 + (\Delta\nu/2)^2}$$

$$\gamma(\nu_0) = N(\lambda^2/4\pi^2 t_{sp}\,\Delta\nu)$$

Phase change in amplification

$$\varphi(\nu) = \frac{\nu-\nu_0}{\Delta\nu}\gamma(\nu)$$

In previous Chapter (2) Rate equations

Lifetime of atoms in energy level τ

$$N(t) = N_0 e^{-\frac{t}{\tau}}$$

Steady-state population difference No radiation case:

$$N_0 = R_2\tau_2(1-\frac{\tau_1}{\tau_{21}}) + R_1\tau_1$$

Steady-state population difference with radiation case:

$$N = \frac{N_0}{1+\tau_s W_i}$$

$$\tau_s = \tau_2 + \tau_1(1-\frac{\tau_2}{\tau_{21}})$$

Saturation time constant

For 4 level system $\tau_s \approx t_{sp}$

For 3 level system $\tau_s = 2\tau_{21} \approx 2t_{sp}$

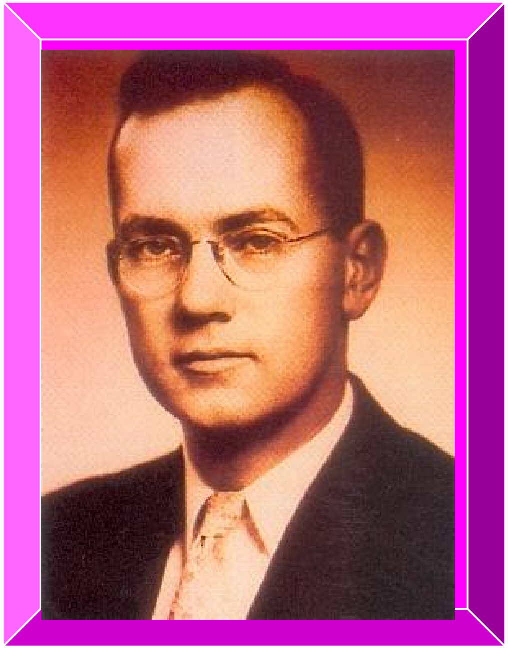
$$\gamma(\nu) = \frac{\gamma_0(\nu)}{1+\phi/\phi_s(\nu)}$$

$$\phi_s(\nu) = 1/\tau_s\sigma(\nu)$$

$\alpha(v) = \alpha_0(v)/[l + \phi/\phi_s(v)]$

$$\Delta\nu_s = \Delta\nu[1+\frac{\phi}{\phi_s(\nu_0)}]^{1/2}$$

LASERS



In 1958 Arthur Schawlow, together with Charles Townes, showed how to extend the principle of the maser to the optical region. He shared the 1981 Nobel Prize with Nicolaas Bloembergen. Maiman demonstrated the first successful operation of the ruby laser in 1960.

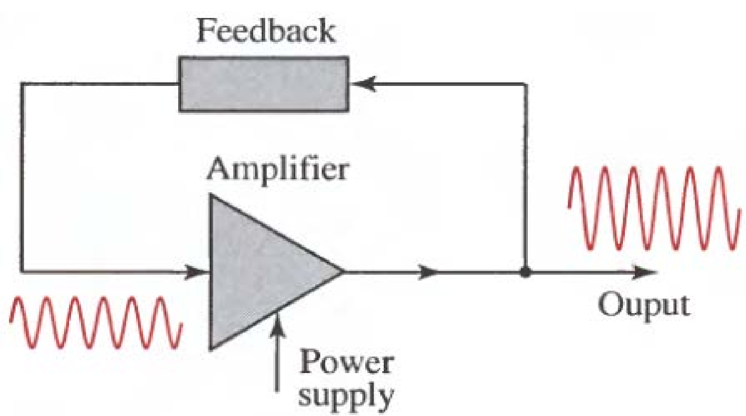
Outline

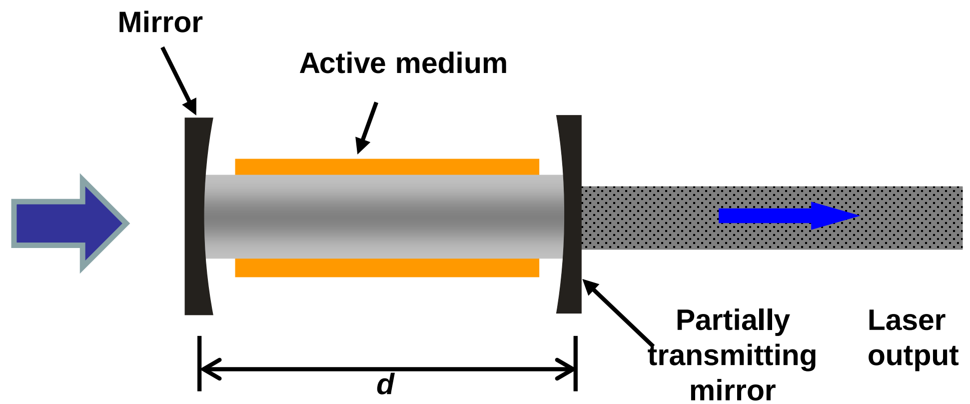
5.1 THEORY OF LASER OSCILLATION
- A. Optical Amplification and Feedback
- B. Conditions for Laser Oscillation

5.2 CHARACTERISTICS OF THE LASER OUTPUT
- A. Power
- B. Spectral Distribution
- C. Spatial Distribution and Polarization
- D. Mode Selection
- E. Characteristics of Common Lasers

5.3 PULSED LASERS
- A. Methods of Pulsing Lasers
- *B. Analysis of Transient Effects
- *C. Q-Switching
- D. Mode Locking

LASERS



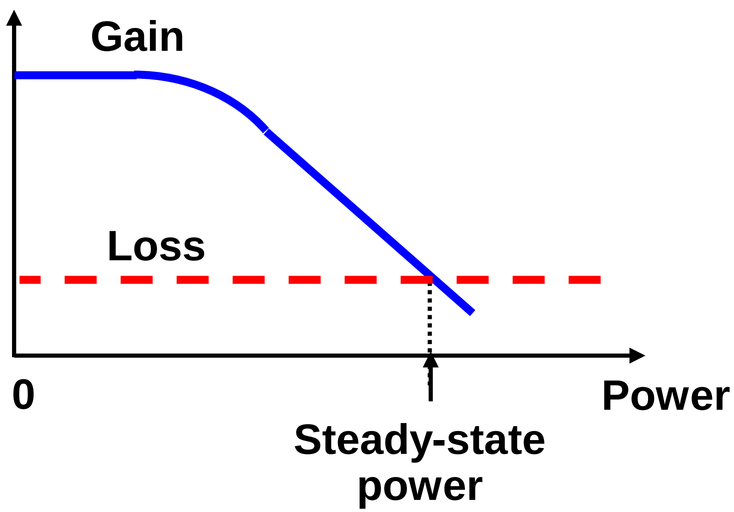


an oscillator is an amplifier with positive feedback

Two conditions for an oscillation:

1. Gain greater than loss: net gain
2. Phase shift in a round trip is a multiple of 2π

Stable condition 2: gain = loss



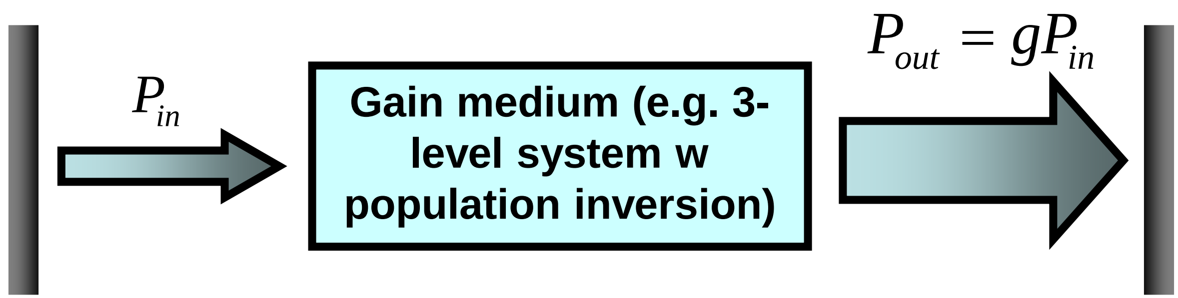
If the initial amplifier gain is greater than the loss, oscillation may initiate. The amplifier then satuates whereupon its gain decreases.

A steady-state condition is reached when the gain just equals the loss.

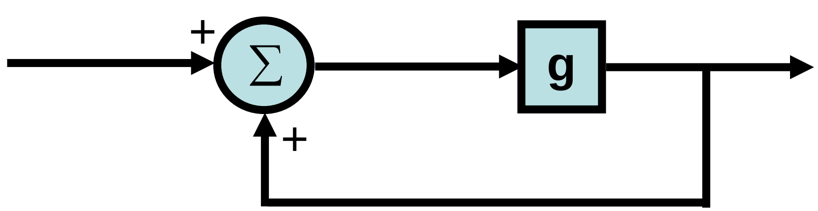
An oscillator comprises:

- **An amplifier with a gain-saturation mechanism**
- **A feedback system**
- **A frequency-selection mechanism**
- **An output coupling scheme**

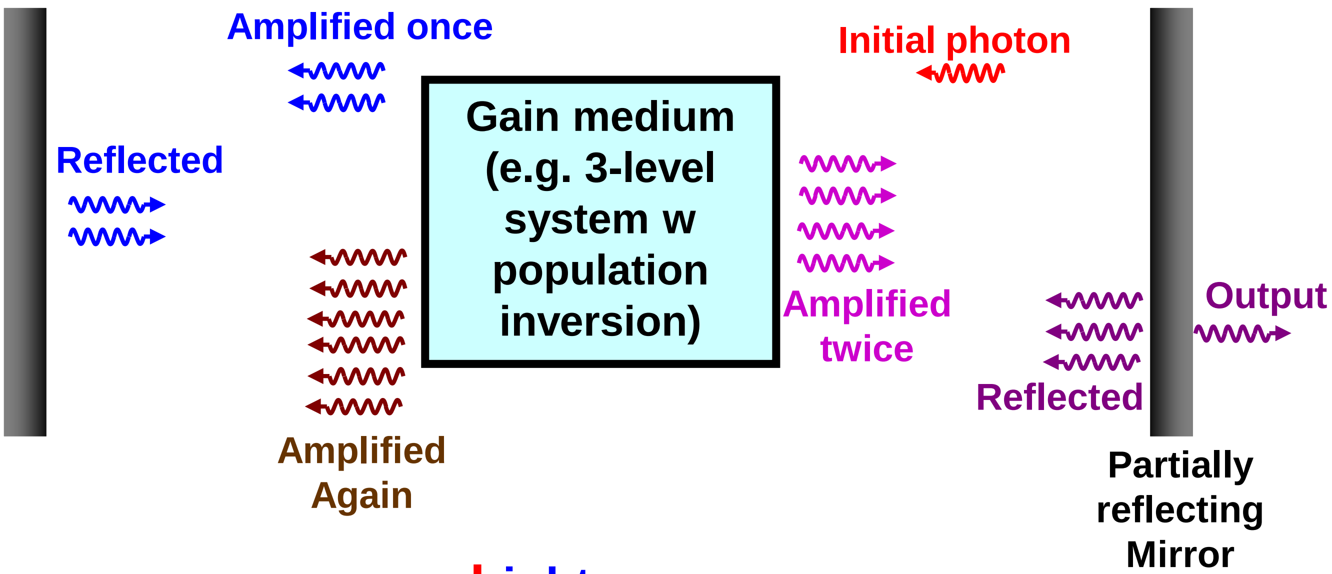
Light amplifier with positive feedback



When the gain exceeds the roundtrip losses, the system goes into oscillation



LASERS



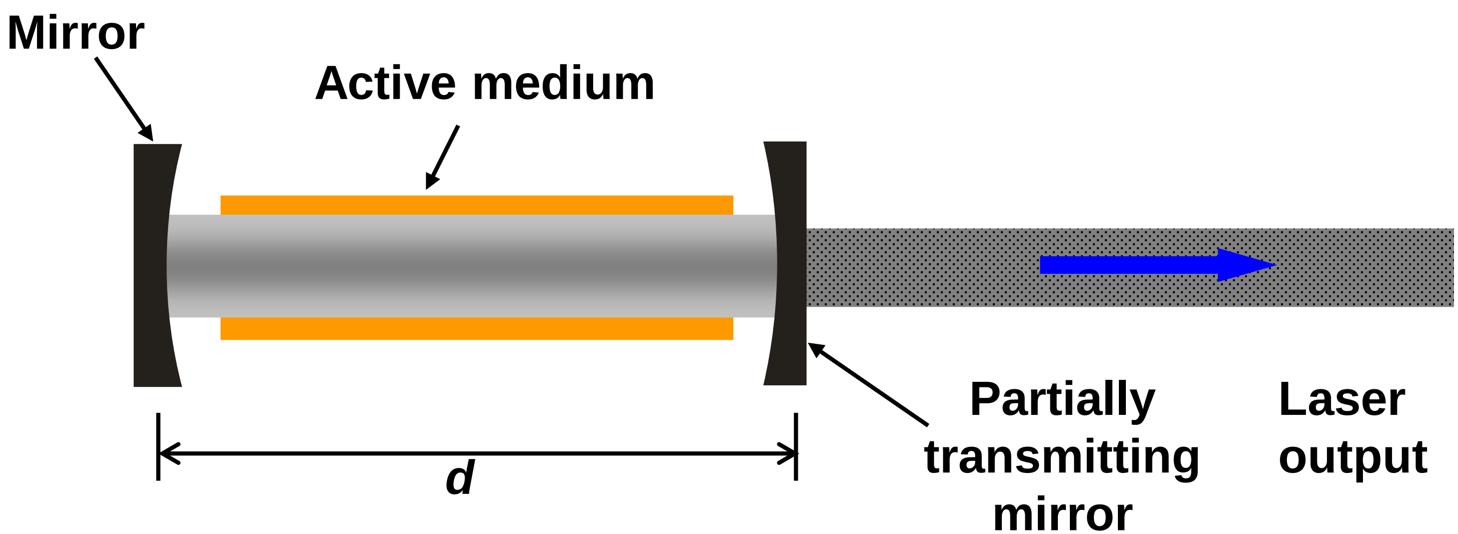
Light

Amplification through

Stimualted

Emission

Radiation



A laser consists of an optical amplifier (employing an active medium) placed within an optical resonator. The output is extracted through a partially transmitting mirror.

Optical amplification and feedback

★ Gain medium

The laser amplifier is a distributed-gain device characterized by its gain coefficient

$$\gamma_0(\nu) = N_0\sigma(\nu) = N_0 \frac{\lambda^2}{8\pi t_{sp}} g(\nu)$$

Small signal Gain Coefficient

$$\gamma(\nu) = \frac{\gamma_0(\nu)}{1+\phi/\phi_s(\nu)}$$

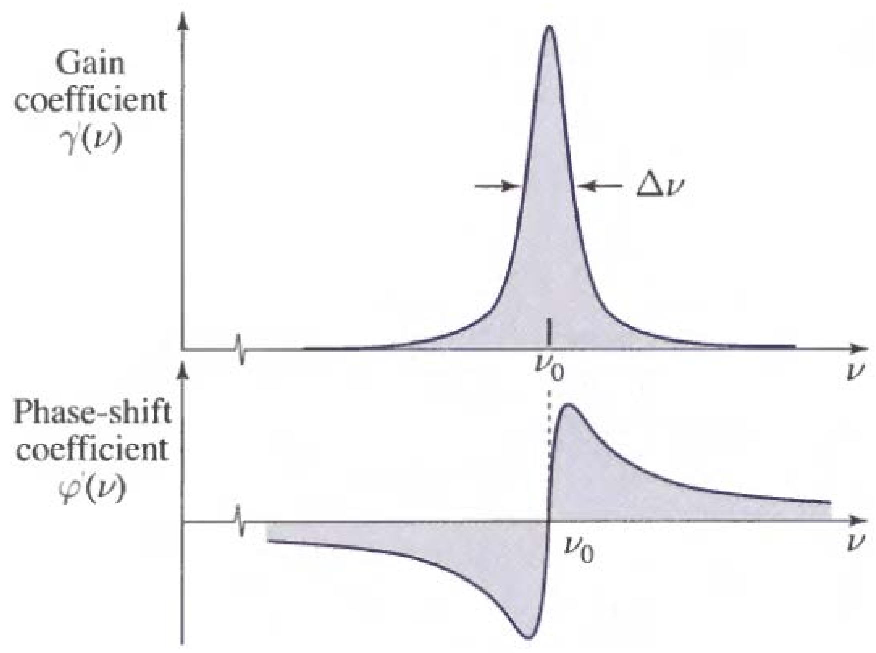
Saturated Gain Coefficient

where $\phi_s(\nu) = [\tau_s\sigma(\nu)]^{-1}$ = saturation photon-flux density

For 4 level system $\tau_s = t_{sp}$, for 3 level system $\tau_s = 2t_{sp}$

$$\varphi(\nu) = \frac{\nu - \nu_0}{\Delta \nu} \gamma(\nu)$$

Phase-shift Coefficient (Lorentzian Lineshape)



Spectral dependence of the gain and phase-shift coefficients for an optical amplifier with Lorentzian lineshape function

Optical Feedback-Optical Resonator

Feedback and Loss: The optical resonator

Optical feedback is achived by placing the active medium in an optical resonator. A Fabry-Perot resonator, comprising two mirrors separated by a distance d, contains the medium (refractive index n) in which the active atoms of the amplifier reside. Travel through the medium introduces a phase shift per unit length equal to the wavenumber

$$\boxed{k = \frac{2\pi\nu}{c}}$$

The resonator also contributes to losses in the system. Absorption and scattering of light in the medium introduces a distributed loss characterized by the attenuation coefficient α_s, (loss per unit length). In traveling a round trip through a resonator of length d, the photon-flux density is reduced by the factor $R_1R_2\exp(-2\alpha_s d)$, where R_1 and R_2 are the reflectances of the two mirrors. The overall loss in one round trip can therefore be described by a total effective distributed loss coefficient α_r, where

$$\boxed{\exp(-2\alpha_r d) = R_1 R_2 \exp(-2\alpha_s d)}$$

Loss coefficient

$$\alpha_r = \alpha_s + \alpha_{m1} + \alpha_{m2}$$

$$\alpha_{m1} = \frac{1}{2d} \ln \frac{1}{R_1}$$

$$\alpha_{m2} = \frac{1}{2d} \ln \frac{1}{R_2}$$

$$\alpha_m = \alpha_{m1} + \alpha_{m2} = \frac{1}{2d} \ln \frac{1}{R_1 R_2}$$

Photon lifetime

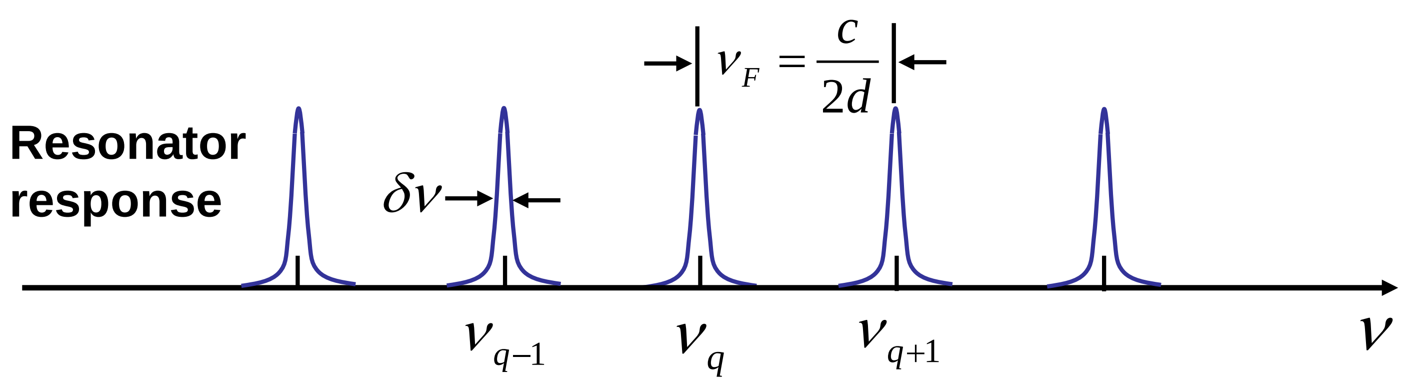
$$\tau_p = \frac{1}{\alpha_r c}$$

α_r represents the total loss of energy (or number of photons) per unit length, $\alpha_r c$ represents the loss of photons per second

$$\nu_q = q\nu_F, q = 1, 2, \ldots,$$

$$\delta\nu \approx \frac{\nu_F}{F}, \nu_F = c/2d$$

$$F \approx \frac{\pi}{\alpha_r d} = 2\pi\tau_p\nu_F$$



Resonator modes are separated by the frequency $\nu_F = c/2d$ and have linewidths $\delta\nu = \nu_F / F = 1/2\pi\tau_p$.

Conditions for laser oscillation

Condition 1: Gain condition, Laser threshold

$$\gamma_0(\nu) > \alpha_r$$

Threshold Gain Condition

because

$$\gamma_0(\nu) = N_0\sigma(\nu)$$

$$N_0 = \gamma_0(\nu)/\sigma(\nu) > \alpha_r/\sigma(\nu)$$

$$N_0 > N_t$$

where

$$N_t = \frac{\alpha_r}{\sigma(\nu)}$$

or

$$N_t = \frac{1}{c\tau_p\sigma(\nu)}$$

$$N_t = \frac{8\pi}{\lambda^2}\frac{t_{sp}}{\tau_p}\frac{1}{g(\nu)}$$

Threshold Population Difference

For a Lorentzian lineshape function, @ $v = v_0$, as $g(\nu_0) = 2/\pi\Delta\nu$

$$N_t = \frac{2\pi}{\lambda^2 c}\frac{2\pi\Delta\nu t_{sp}}{\tau_p}$$

If the transition is limited by lifetime broadening with a decay time t_{sp}

At the center frequency ν_0, with *$g(\nu_0)=2/(\pi\Delta\nu)$*, then assuming *$\Delta\nu=1/(2\pi t_{sp})$*

$$N_t = \frac{2\pi}{\lambda^2 c\tau_p} = \frac{2\pi\alpha_r}{\lambda^2}$$

As a numerical example, if $\lambda_0=1$ μm, $\tau_p=1$ ns, and the refractive index n=1, we obtain $N_t=2.1\times10^7$ cm^{-3}

Conditions for laser oscillation(2)

Condition 2: Phase condition, Laser Frequencies

$$2kd + 2\varphi(\nu)d = 2\pi q, q = 1, 2\cdots\cdots$$

Frequency Pulling

$$\nu + \frac{c}{2\pi}\frac{\nu - \nu_0}{\Delta\nu}\gamma(\nu) = \nu_q$$

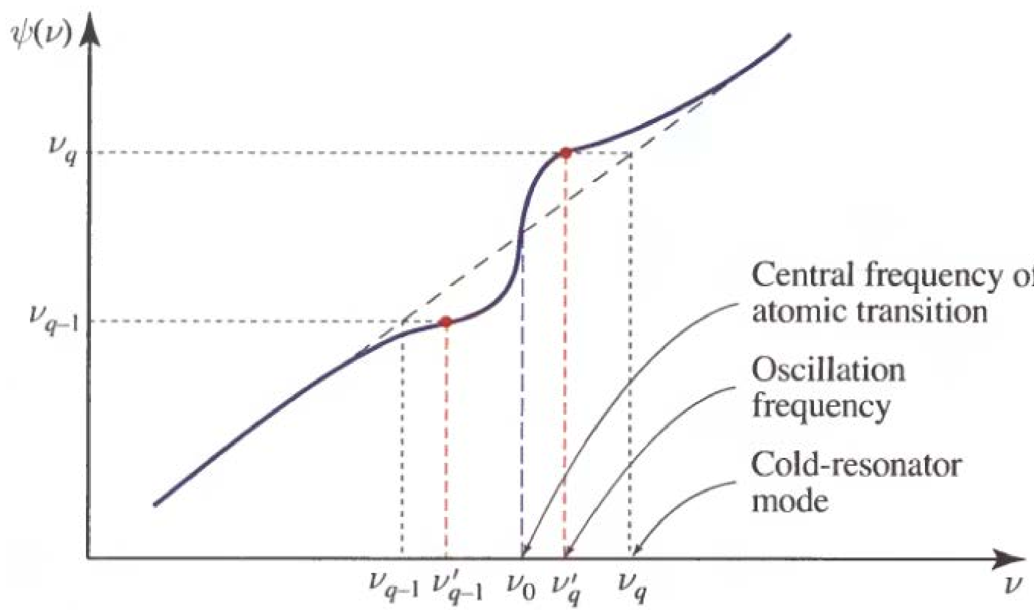
or

$$\nu = \nu_q - \frac{c}{2\pi}\frac{\nu - \nu_0}{\Delta\nu}\gamma(\nu)$$

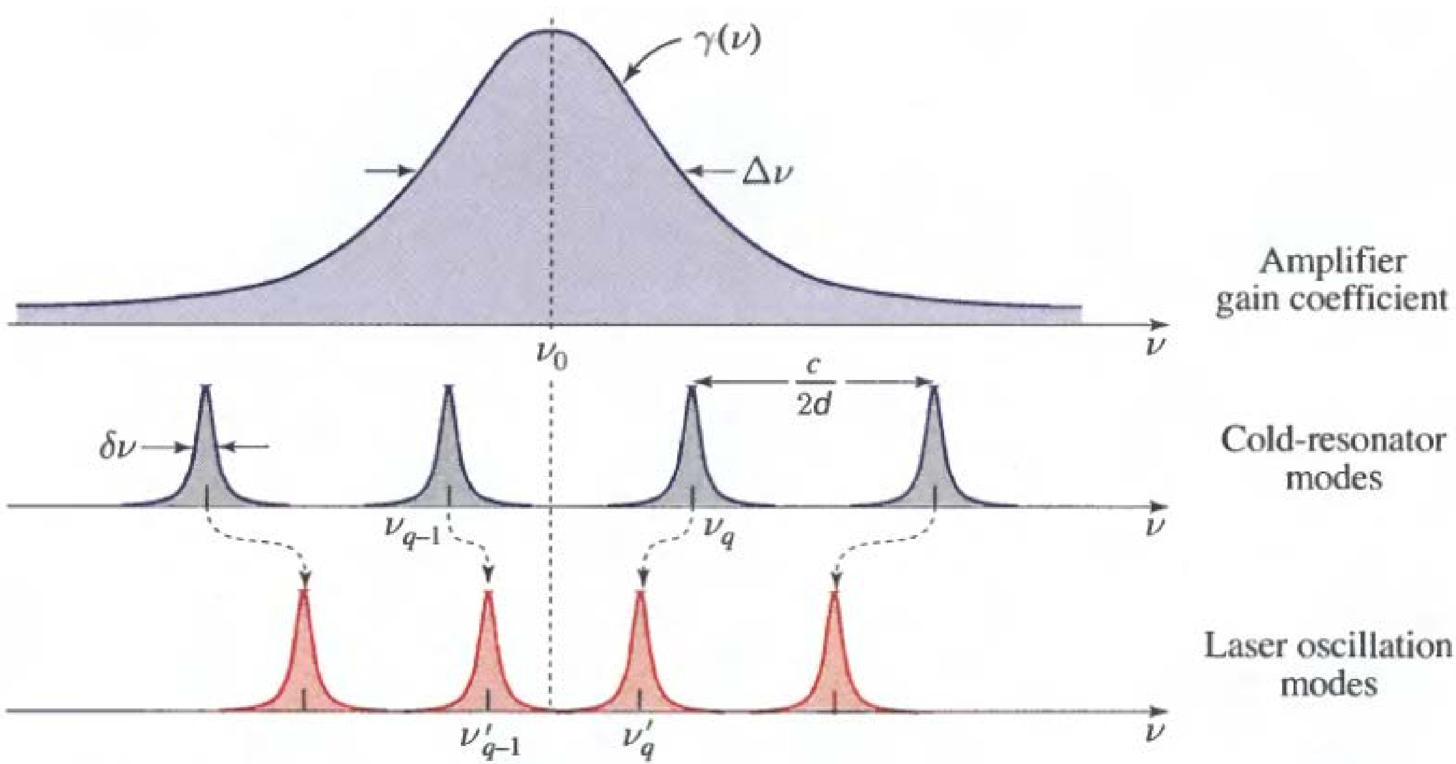
$$\nu = \nu_q{}' = \nu_q$$

$$\nu_q^{'} = \nu_q - \frac{c}{2\pi}\frac{\nu_q - \nu_0}{\Delta\nu}\gamma(\nu_q)$$

$$\nu_q' = \nu_q - (\nu_q - \nu_0)\frac{\delta\nu}{\Delta\nu}$$ **Laser Frequencies**



The $\psi(\nu)$, plotted as a function of ν. The frequency ν for which $\psi(\nu) = \nu_q$ is the solution Each "cold" resonator frequency ν_q corresponds to a "hot" resonator frequency ν_q', which is shifted in the direction of the atomic resonance central frequency ν_0.



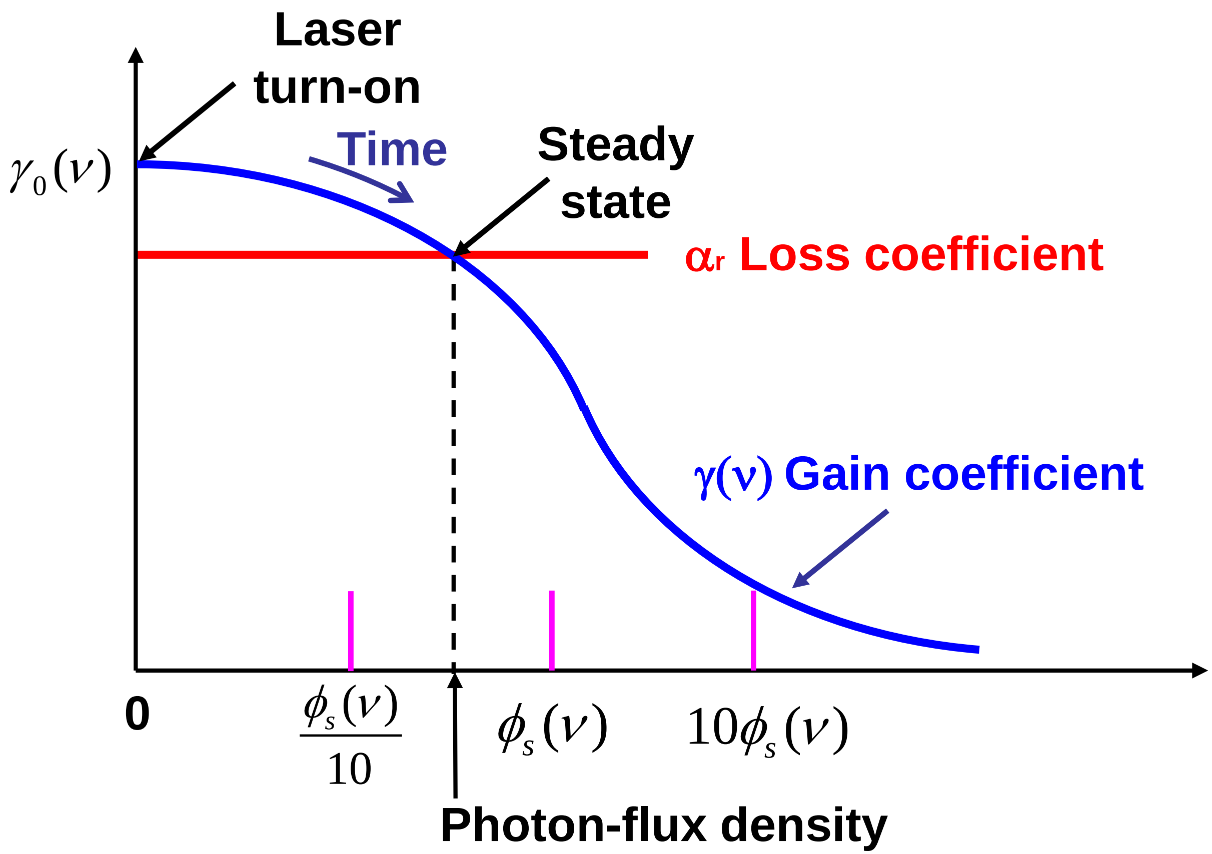
The laser oscillation frequencies fall near the cold-resonator modes; they are pulled slightly toward the atomic resonance central frequency ν_0.

Characteristics of the laser output

Internal Photon-Flux Density

Gain Clamping

$$\gamma_0(\nu)/[1+\phi/\phi_s(\nu)]=\alpha_r$$



Determination of the steady-state laser photon-flux density ϕ. At the time of laser turn on, $\phi=0$ so that $\gamma(\nu)=\gamma_0(\nu)$. As the oscillation builds up in time, the increase in ϕ causes $\gamma(\nu)$ to decrease through gain saturation. When γ reached α_r , the photon-flux density causes its growth and steady-state conditions are achieved. The smaller the loss, the greater the value of ϕ.

Steady Photon Density

$$\phi = \phi_s(\nu)[\frac{\gamma_0(\nu)}{\alpha_r} - 1], \qquad \gamma_0(\nu) > \alpha_r$$

$$\phi = 0, \qquad \gamma_0(\nu) \le \alpha_r$$

$$\phi_s(\nu) = [\tau_s \sigma(\nu)]^{-1}$$

$\tau_s = t_{sp}$ **For four levels system**

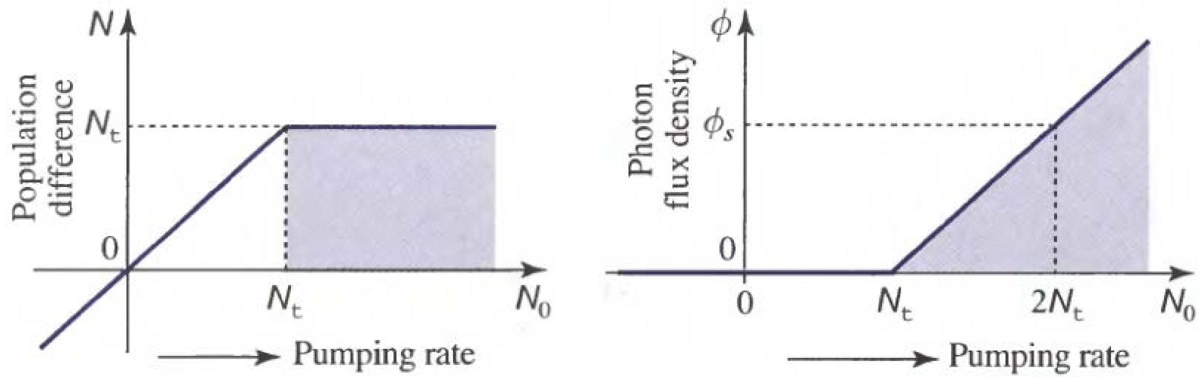
$\tau_s = 2t_{sp}$ **For three levels system**

Since $\gamma_0(\nu) = N_0 \sigma(\nu)$ **and** $\alpha_r = N_t \sigma(\nu)$

$$\phi = \phi_s(\nu)(\frac{N_0}{N_t} - 1), \; N_0 > N_t$$

$$\phi = 0, \; N_0 \le N_t$$

Steady-State Laser Internal Photon-Flux Density



Steady-state values of the population difference N, and the laser internal photon-flux density ϕ, as functions of N_0 (the population difference in the absence of radiation; N_0, increases with the pumping rate R). Laser oscillation occurs when N_0 exceeds N_t; the steady-state value of N_t then saturates, clamping at the value N_t, [just as $r_0(v)$ is clamped at α_r]. Above threshold, ϕ is proportional to N_t-N_0.

Output photon-flux density

$$\phi_0 = \frac{T\phi}{2}$$

Optical Intensity of Laser Output

$$I_0 = \frac{h\nu T\phi}{2}$$

Optimization of the output photon-flux density

From $$\alpha_{m1} = \frac{1}{2d}\ln\frac{1}{R_1} = -\frac{1}{2d}\ln(1-T)$$

We obtain $$\alpha_r = \alpha_s + \alpha_{m2} - \frac{1}{2d}\ln(1-T)$$

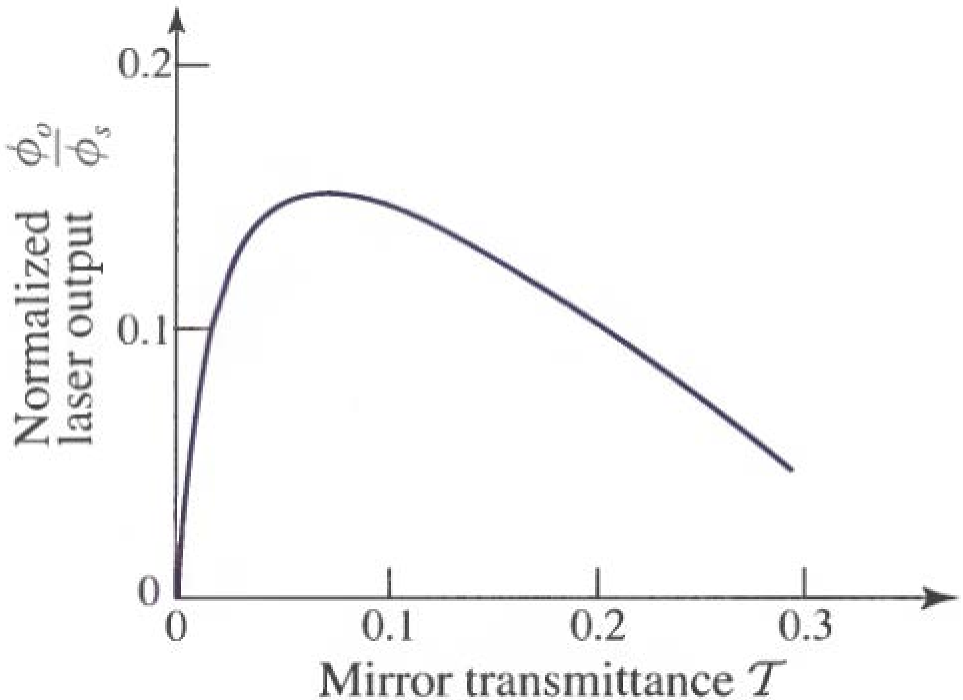
$$\because \quad \phi = \phi_s\left[\frac{\gamma_0}{\alpha_r} - 1\right]$$

$$\phi_0 = \frac{T\phi}{2} \Rightarrow \phi_0 = \frac{1}{2}\phi_s T\left[\frac{g_0}{L-\ln(1-T)} - 1\right],\ g_0 = 2\gamma_0(\nu)d,\ L = 2(\alpha_s + \alpha_{m2})d$$

When, $T << 1$

use the approximation $\ln(1-T) \approx -T$

Then $T_{op} \approx (g_0 L)^{1/2} - L$



Dependence of the transmitted steady-state photon-flux density ϕ_o on the mirror transmittance $\mathscr{T}$. For the purposes of this illustration, the gain factor $g_0 = 2\gamma_0 d$ has been chosen to be 0.5 and the loss factor $L = 2(\alpha_s + \alpha_{m2})d$ is 0.02 (2%). The optimal transmittance $\mathscr{T}_{\text{op}}$ turns out to be 0.08.

Internal Photon-Number Density

The steady-state number of photons per unit volume inside the resonator $n = \dfrac{\phi}{c}$

The steady-state internal photon-number density $n = n_s(\dfrac{N_0}{N_t} - 1), \quad N_0 > N_t$

Where $n_s = \phi_s(v)/c$ is the photon-number density saturation value

Because: $\alpha_r = 1/c\tau_p \quad \phi_s(v) = [\tau_s \sigma(v)]^{-1} \quad \gamma(v) = N\sigma(v) = N_t \sigma(v)$

We have $$n = (N_0 - N_t)\frac{\tau_p}{\tau_s}, \qquad N_0 > N_t$$

For 4 level system, there are $\tau_s = t_{sp}$ **and** $N_0 \approx R t_{sp}$ **so that**

$$\frac{n}{\tau_p} = R = R_{t,} \qquad R > R_t$$ ***R*** **is pumping rate ($s^{-1}cm^{-3}$)**

Where $R_t = N_t / t_{sp}$ **Is the thresh value of pumping rate.**

Output Photon Flux and Efficiency

Ideal state

$$\phi_0 = (R - R_t)V, R > R_t$$

With loss in cavity and outlet mirror

$$\phi_0 = \eta_e (R - R_t)V$$

where

$$\eta_e = \frac{\alpha_{m1}}{\alpha_r} = \frac{c}{2d}\tau_p \ln\frac{1}{R_1}$$

有效输出损耗与腔内总损耗之比

$$\eta_e \approx \frac{\tau_p}{T_F} T \qquad T_F = 2d/c$$

η_e 光子寿命与腔内来回一周时间之比乘以输出镜的透射率

$$P_o = hv\phi_o = \eta_e hv(R - R_t)V$$

Spectral Distribution

Determined both by the atomic lineshape and by the resonant modes

$$M \approx \frac{B}{\nu_F}$$

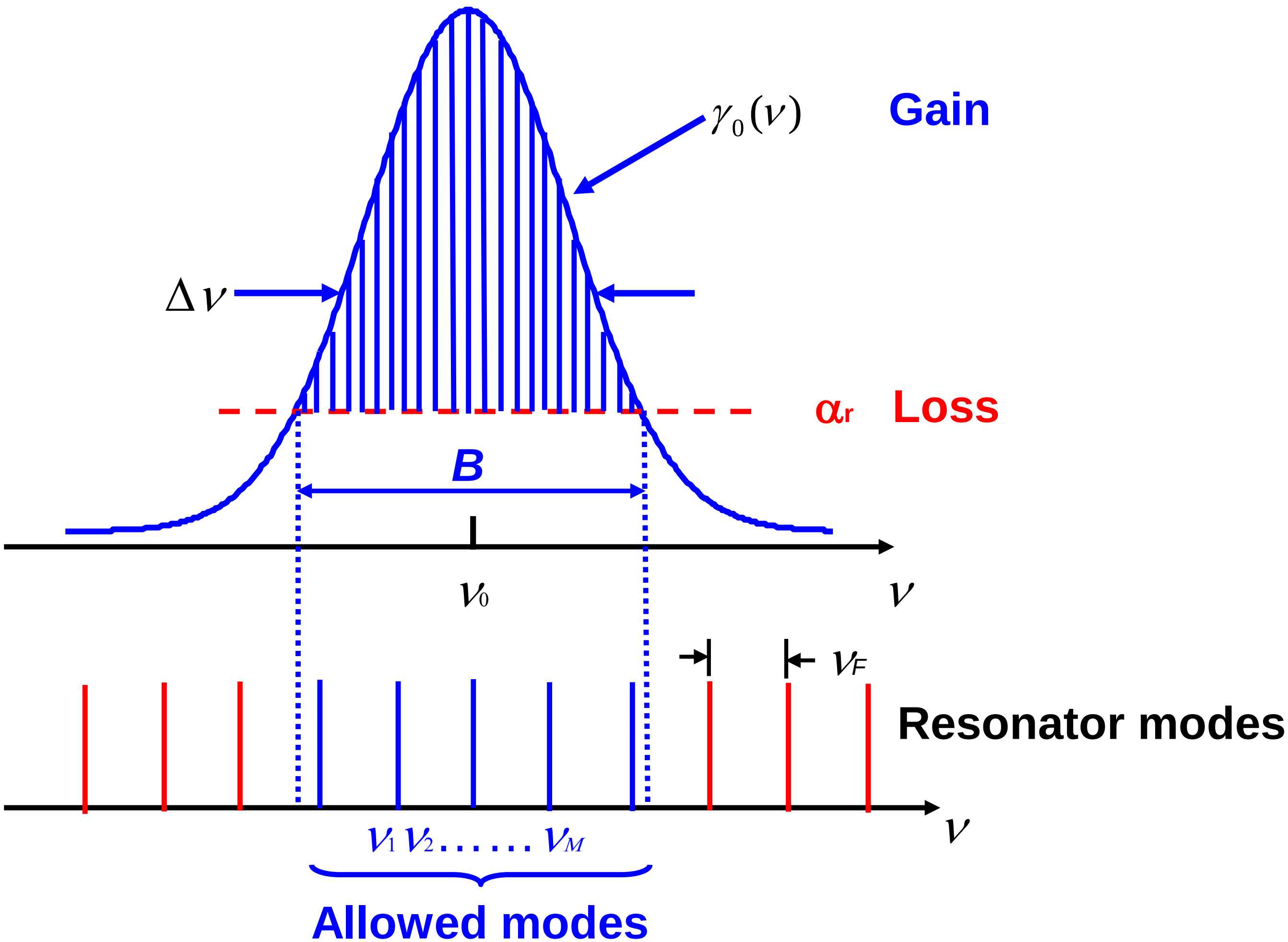
Number of Possible Laser Modes

Where B is pectral band of width, V_F mode interval

Linewidth $\approx \delta\nu$?

Schawlow-Tones limit

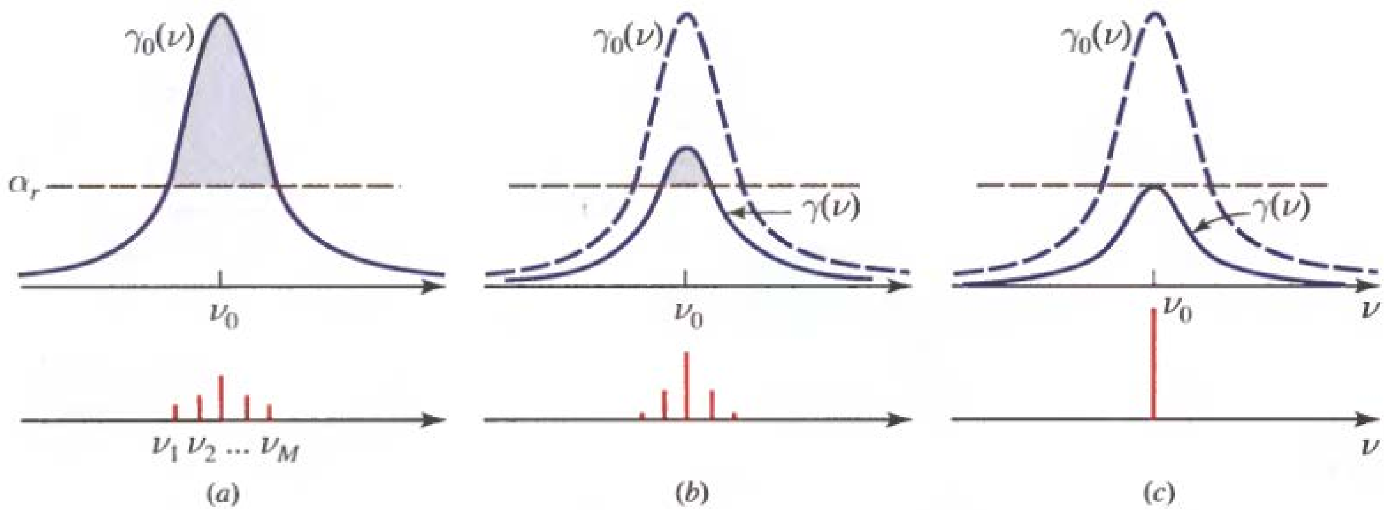
随输出功率的加大，激光带宽不断减小



(a) Laser oscillation can occur only at frequencies for which the gain coefficient is greater than the loss coefficient (stippled region). (b) Oscillation can occur only within $\delta\nu$ of the resonator modal frequencies (which are represented as lines for simplicity of illustration).

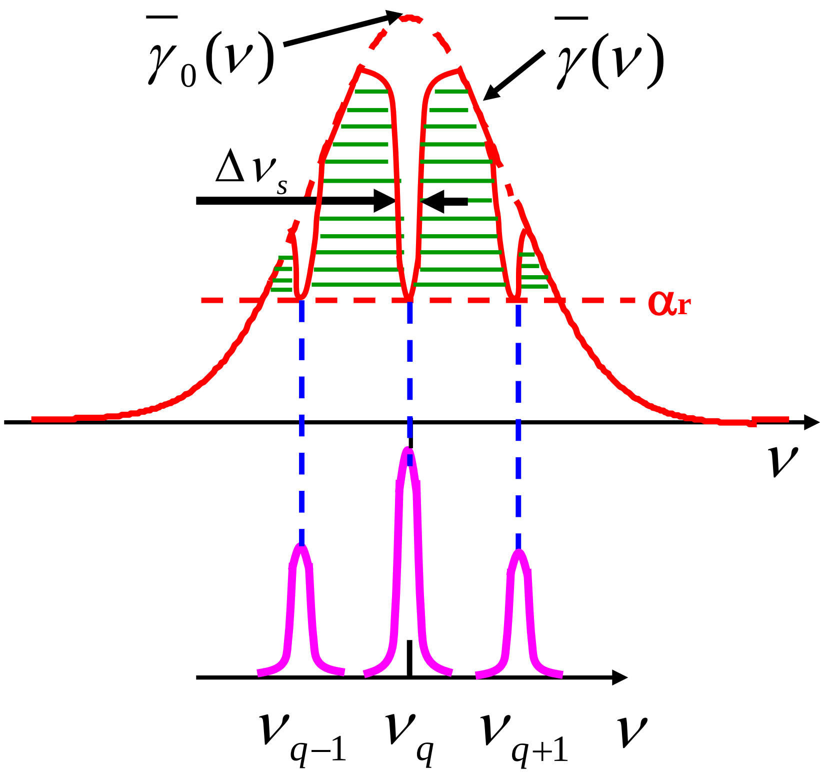
Homogeneously Broadened Medium

$$\gamma(\nu) = \frac{\gamma_0(\nu)}{1+\sum_{i=1}^{M}\phi_j / \phi_s(\nu_j)}$$

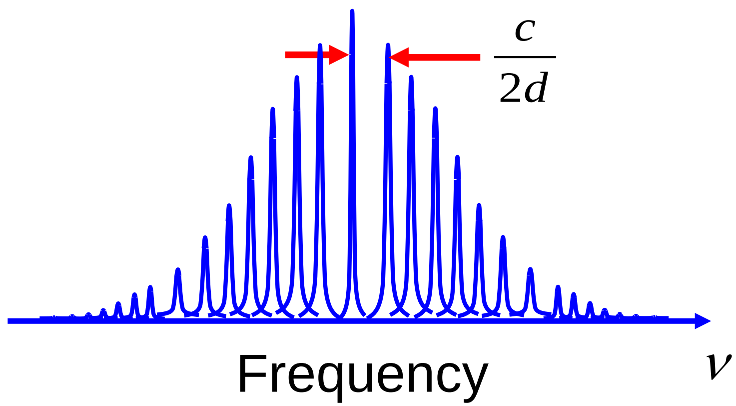


Growth of oscillation in an ideal homogeneously broadened medium. (*a*) Immediately following laser turn-on, all modal frequencies $\nu_1, \nu_2, \ldots, \nu_M$, for which the gain coefficient exceeds the loss coefficient, begin to grow, with the central modes growing at the highest rate. (*b*) After a short time the gain saturates so that the central modes continue to grow while the peripheral modes, for which the loss has become greater than the gain, are attenuated and eventually vanish. (*c*) In the absence of spatial hole burning, only a single mode survives.

Inhomogeneously Broadened Medium



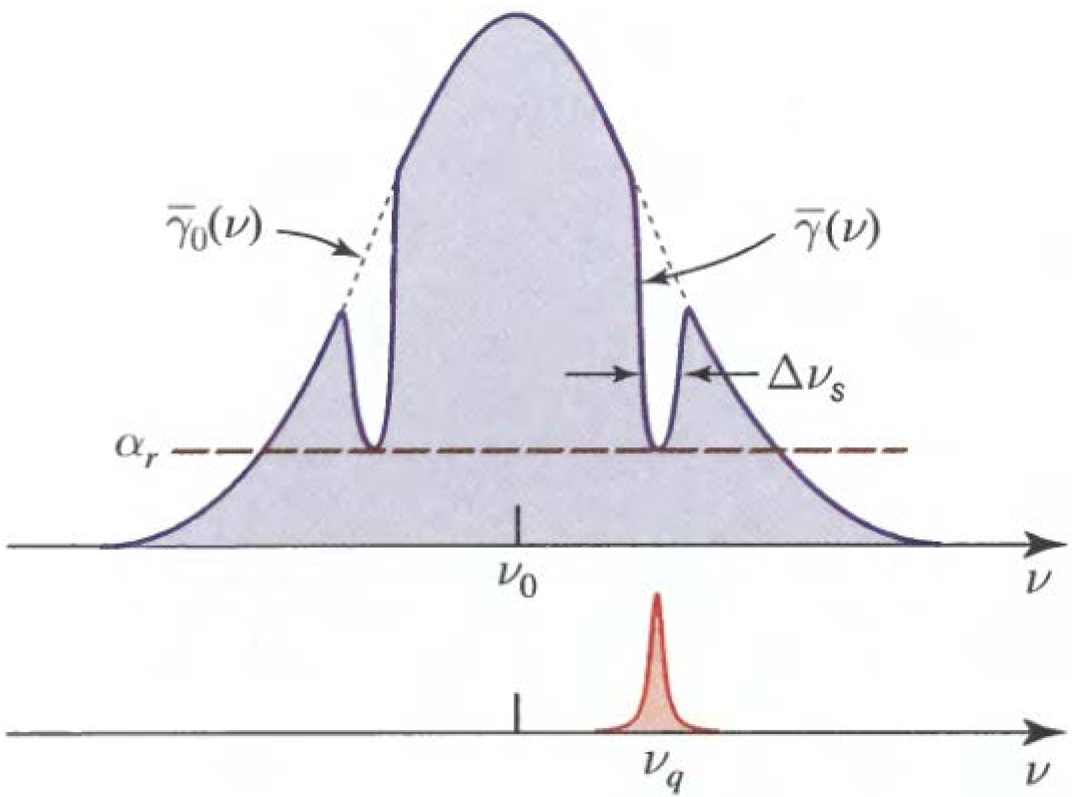
(a)



(b)

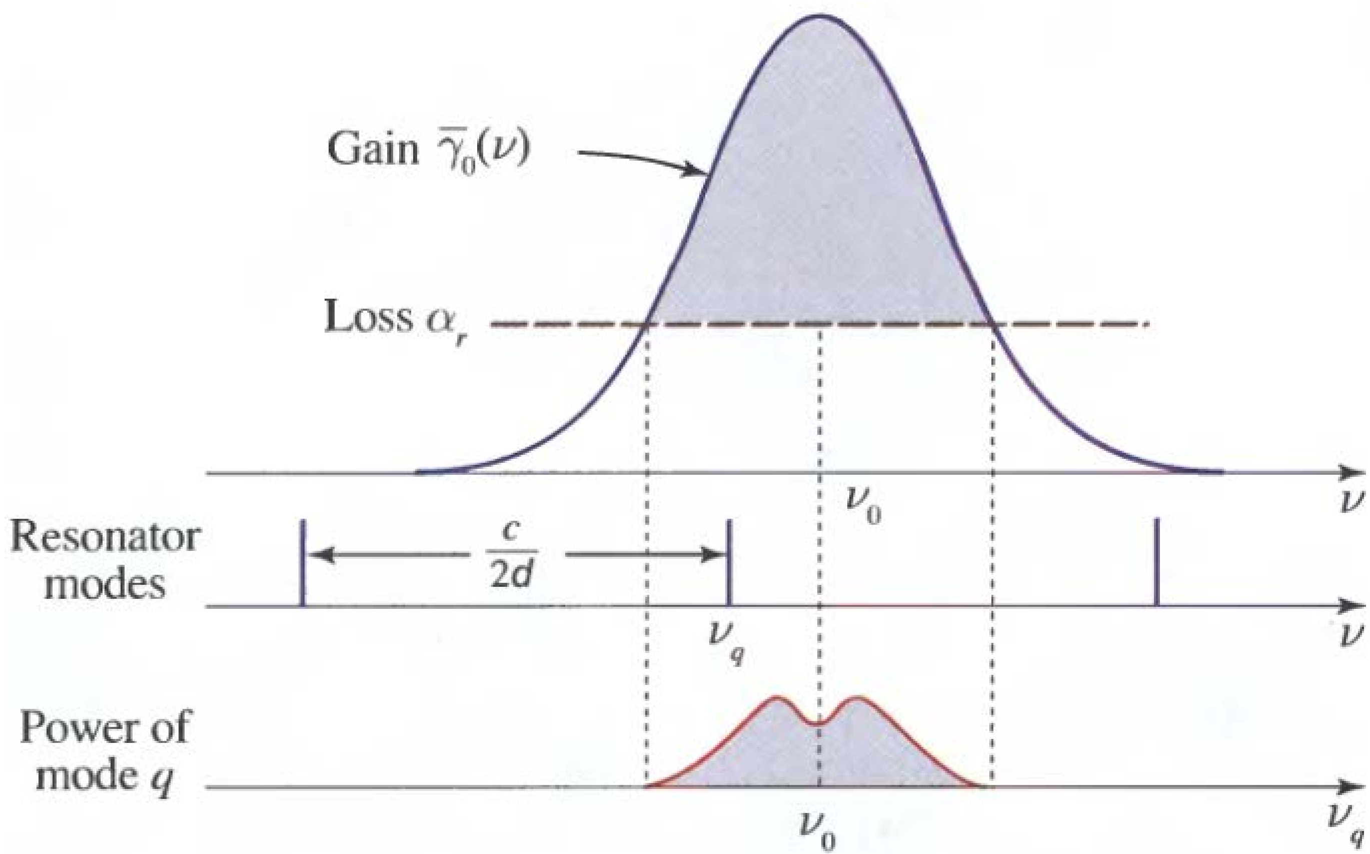
(a) Laser oscillation occurs in an inhomogeneously broadened medium by each mode independently burning a hole in the overall spectral gain profile. The gain provided by the medium to one mode does not influence the gain it provides to other modes. The central modes garner contributions from more atoms, and therefore carry more photons than do the perpheral modes. (b) Spectrum of a typical inhomogeneously broadened multimode gas laser.

Hole burning in a Doppler-broadened medium



Hole burning in a Doppler-broadened medium. A probe wave at frequency ν_q saturates those atomic populations with velocities $v = \pm c(\nu_q/\nu_0 - 1)$ on both sides of the central frequency, burning two holes in the gain profile.

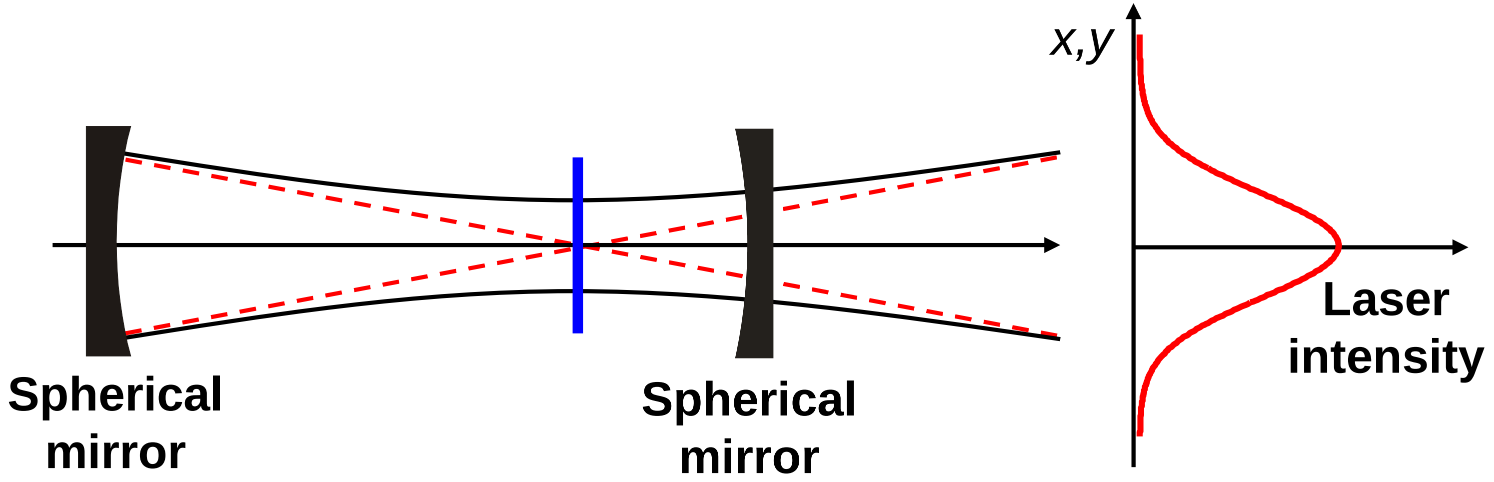
Hole burning in a Doppler-broadened medium



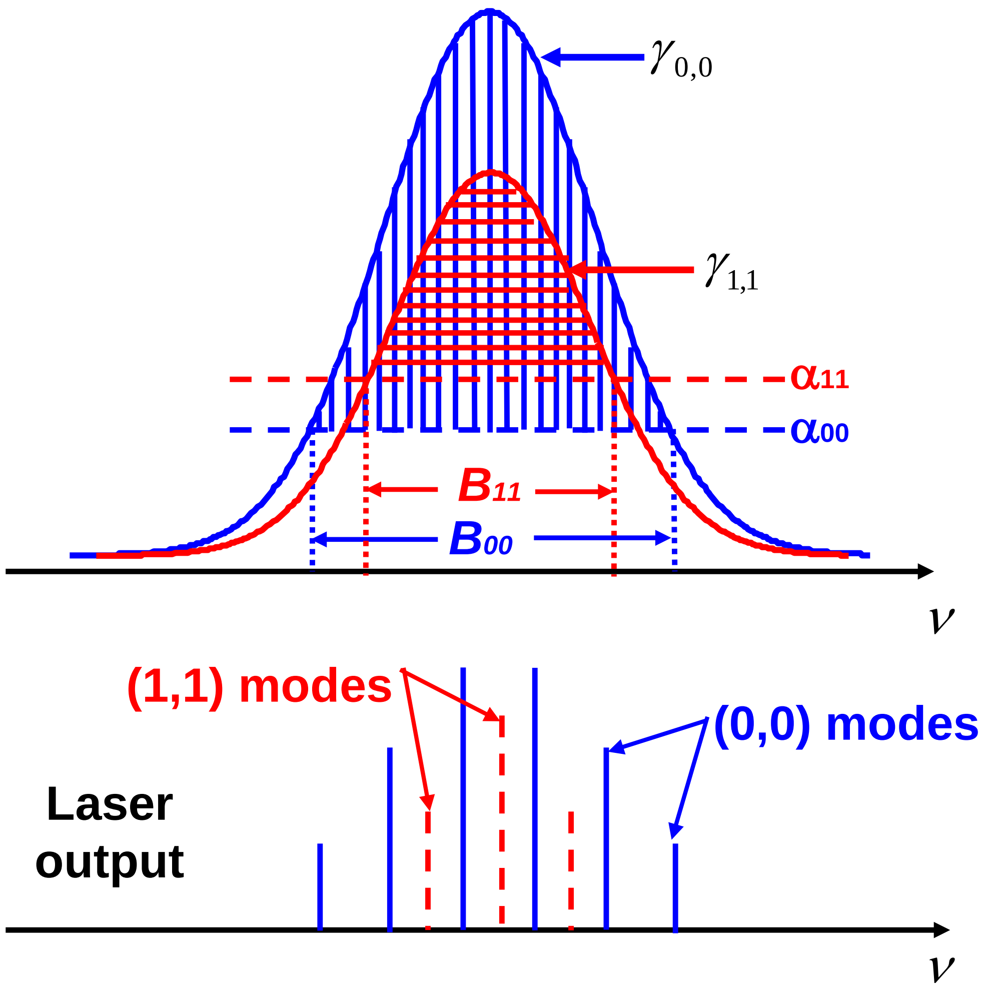
Power in a single laser mode of frequency ν_q in a Doppler-broadened medium whose gain coefficient is centered about ν_0. Rather than providing maximum power at $\nu_q = \nu_0$, it exhibits the Lamb dip.

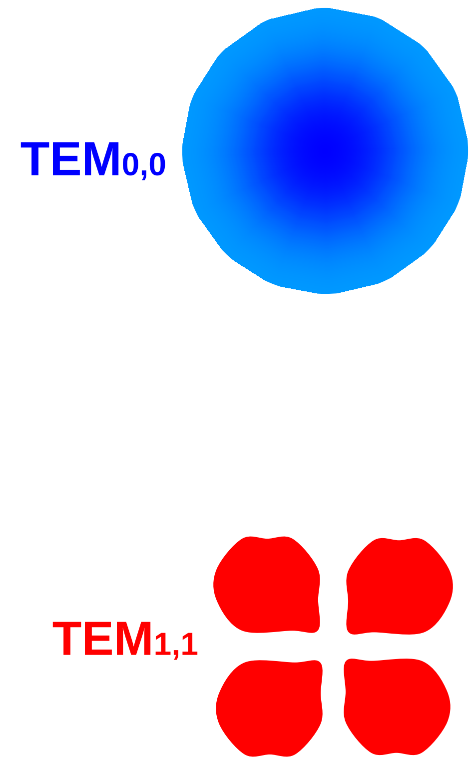
Spatial distribution and polarization

Spatial distribution



The laser output for the (0,0) transverse mode of a spherical-mirror resonator takes the form of a Gaussian beam.





The gains and losses for two transverse modes, say (0,0) and (1,1), usually differ because of their different spatial distributions. A mode can contribute to the output if it lies in the spectral band (of width B) within the gain coefficient exceeds the loss coefficient. The allowed longitudinal modes associated with each transverse mode are shown.

Two Issues: Polarization, Unstable Resonators

Polarizations

- Each *(l, m, q)* mode has two degrees of freedom, corresponding to two independent orthogonal polarizations.
- These two polarizations are regarded as two independent modes.

Unstable Resonators

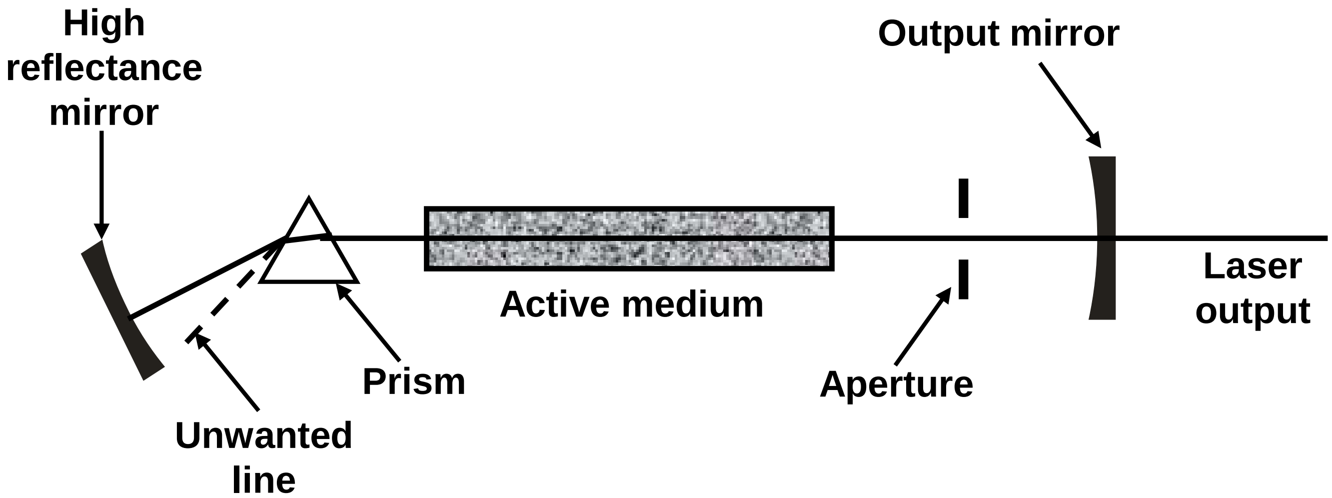
Unstable resonators offers a number of advantages in the operation of high-power lasers.

- a greater portion of the gain medium contributing to the laser output power， so a larger modal volume;
- higher output powers attained from operation on the lowest-order transverse mode, rather than on higher-order transverse modes as in the case of stable resonators;
- high output power with minimal optical damage to the resonator mirrors, the use of purely reflective optics that permits the laser light to spill out around the mirror edges

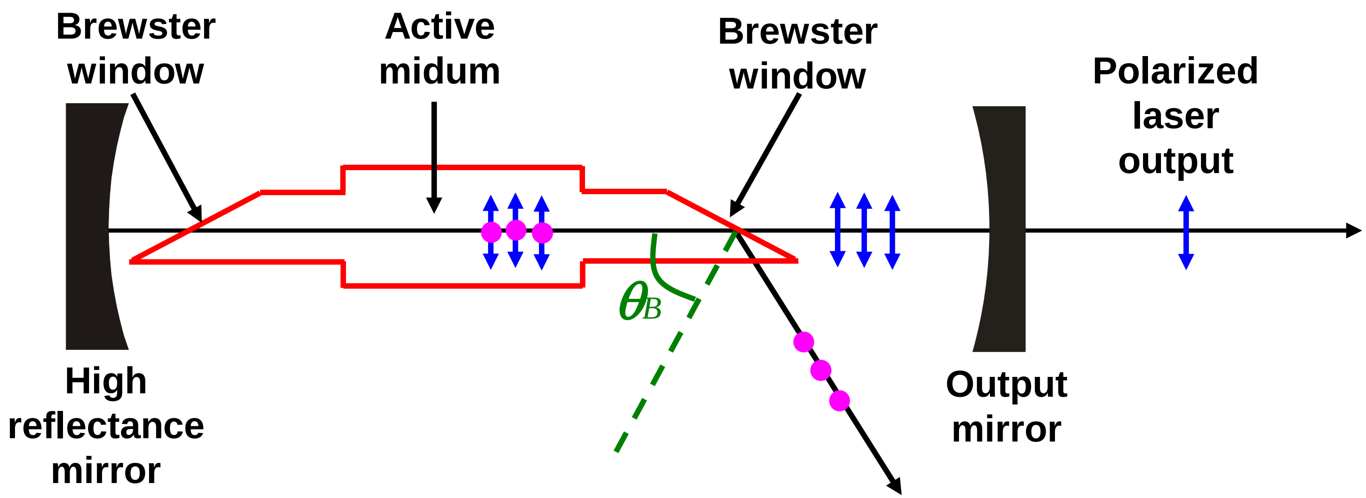
Mode Selection

Selection of

1. **Laser Line**
2. **Transverse Mode**
3. **Polarization**
4. **Longitudinal Mode**

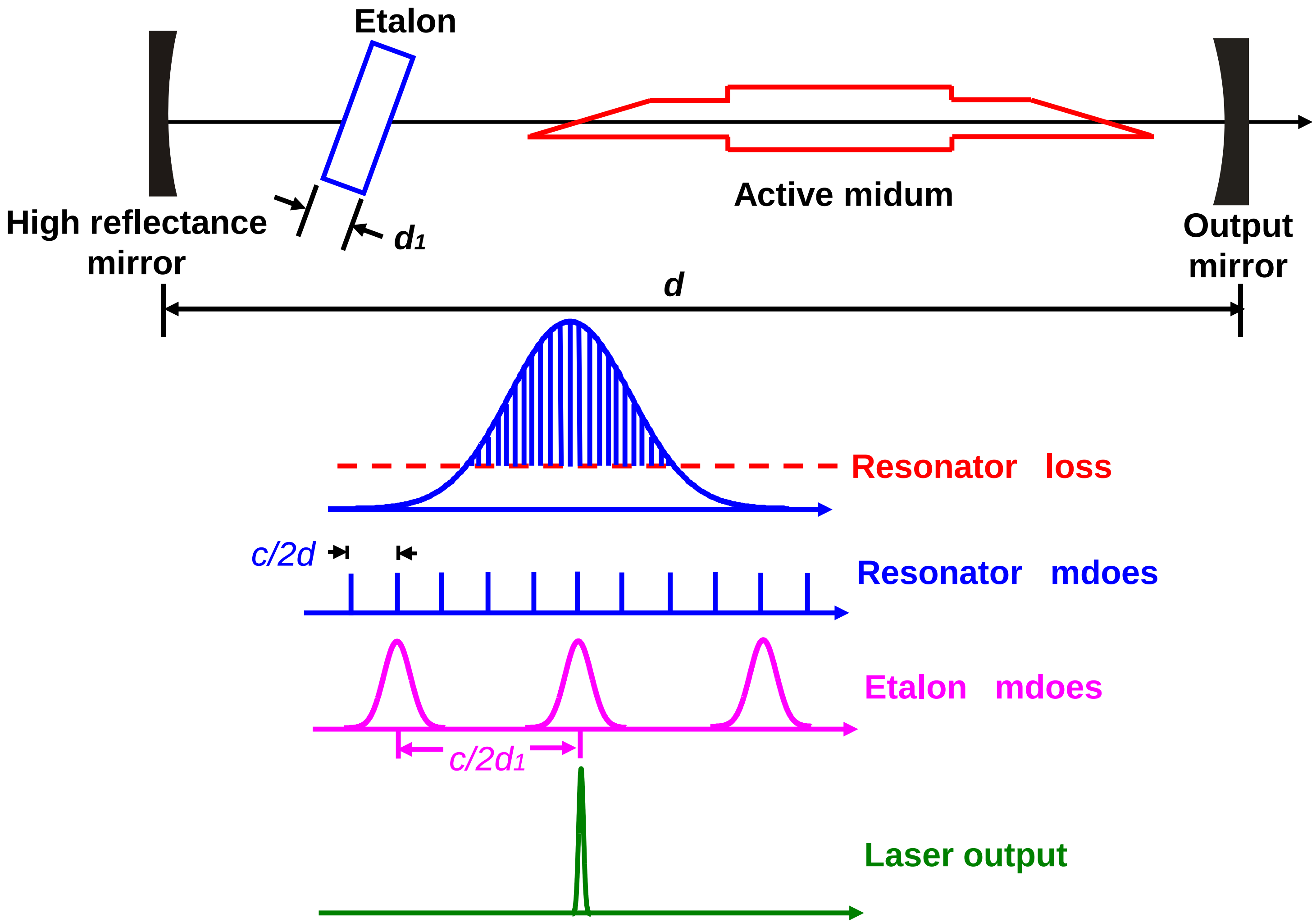


A particular atomic line may be selected by the use of a prism placed inside the resonator. A transverse mode may be selected by means of a spatial aperture of carefully chosen shaped and size.



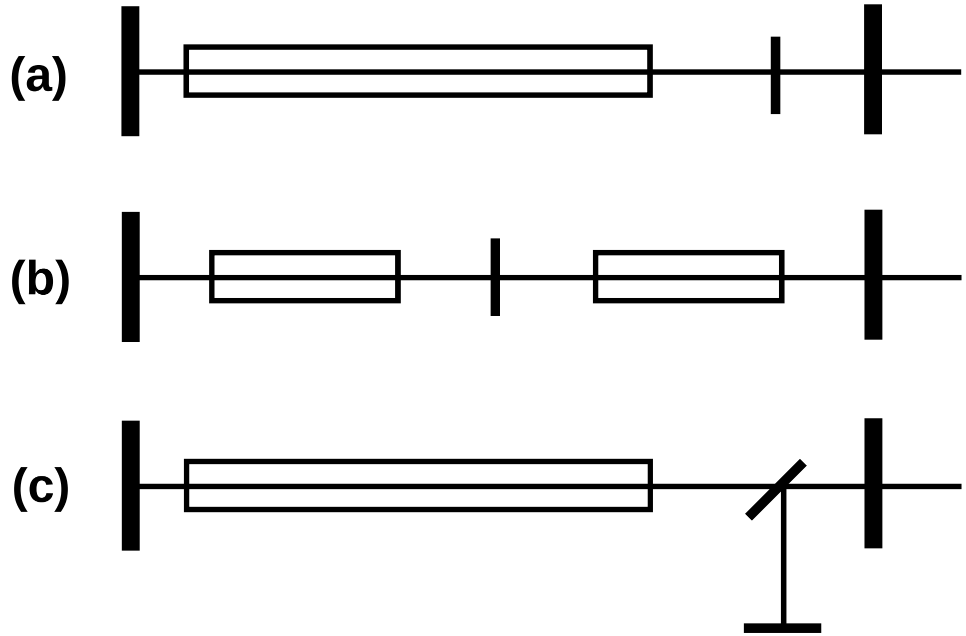
The use of Brewster windows in a gas laser provides a linearly polarized laser beam. Light polarized in the plane of incidence (the TM wave) is transmitted without reflection loss through a window placed at the Brewster angle. The orthogonally polarized (TE) mode suffers reflection loss and therefore does not oscillate.

Selection of Longitudinal Mode



Longitudianl mode selection by the use of an intracavity etalon. Oscillation occurs at frequencies where a mode of the resonator coincides with an etalon mode; both must, of course, lie within the spectral window where the gain of the medium exceeds the loss.

Multiple Mirror Resonators



Longitudinal mode selection by use of (a) two coupled resonators (one passive and one active); (b) two coupled active resonators; (c) a coupled resonator-interferometer.

Characteristics of Common Lasers

Solid State Lasers: Ruby, Nd^{3+}:YAG, Nd^{3+}:Silica, Er^{3+}:Fiber, Yb^{3+}:Fiber

Gas Lasers: He-Ne, Ar^{+}; CO_2, CO, KF;

Liquid Lasers: Dye

Plasma X-Ray Lasers

Free Electron Lasers

Laser Medium	Transition Wavelength λ_o	Single Mode (S) or Multimode (M)	CW or Pulsed[b]	Approximate Overall Efficiency $\eta_c(\%)^c$	Output Power or Energy[d]	Energy-Level Diagram
Ag^{19+} (p)	13.9 nm	M	Pulsed	0.0002	25 μJ	
C^{5+} (p)	18.2 nm	M	Pulsed	0.0005	2 mJ	Fig. 13.1-1
ArF Excimer (g)	193 nm	M	Pulsed	1.	200 mJ	
KrF Excimer (g)	248 nm	M	Pulsed	1.	500 mJ	
Er^{3+}:Silica fiber (s)	1550 nm	S/M	CW	10.	100 W	Fig. 14.3-6
Tm^{3+}:Fluoride fiber (s)	1.8–2.1 μm	S/M	CW	5.	150 W	
He–Ne (g)	3.39 μm	S/M	CW	0.05	20 mW	Fig. 13.1-2
CO_2 (g)	10.6 μm	S/M	CW	10.	500 W	Fig. 13.1-4
H_2O (g)	28 μm	S/M	CW	0.02	100 mW	
FEL at UCSB	60 μm–2.5 mm	M	Pulsed	0.5	5 mJ	
H_2O (g)	118.7 μm	S/M	CW	0.01	50 mW	
CH_3OH (g)	118.9 μm	S/M	CW	0.02	100 mW	
HCN (g)	336.8 μm	S/M	CW	0.01	20 mW	

[a] Gas (g), solid (s), liquid (l), plasma (p).

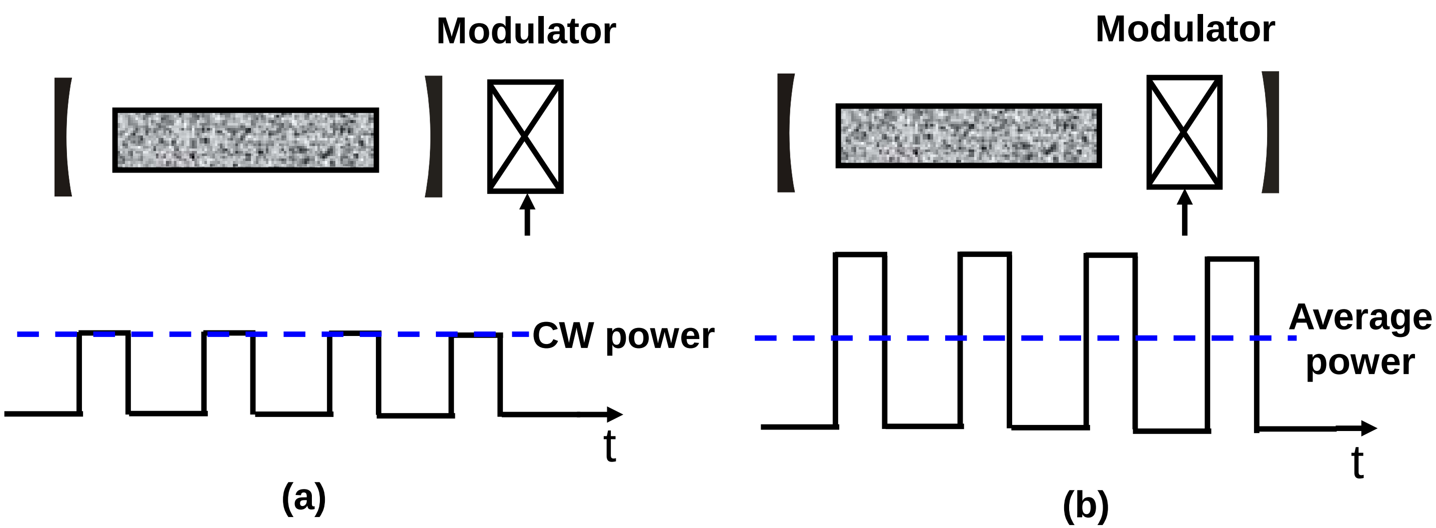
[b] Lasers designated "CW" can, of course, be operated in a pulsed mode; lasers designated "pulsed" are usually operated in that mode.

[c] The power-conversion efficiency η_c (also called the overall efficiency and wall-plug efficiency) is the ratio of output light power to input electrical power (for pulsed lasers, the ratio of output light energy to input electrical energy). Values reported have substantial uncertainty since in some cases they include the electrical power consumed for overhead functions such as cooling and monitoring. Laser diodes exhibit the highest efficiencies, readily exceeding 50%, as discussed in Sec. 17.4C.

[d] The output power (for CW systems) and output energy per pulse (for pulsed systems) vary over a substantial range, in part because of the wide range of pulse durations; representative values are provided.

Pulsed Lasers

Method of pulsing lasers → **External Modulator or Internal Modulator?**

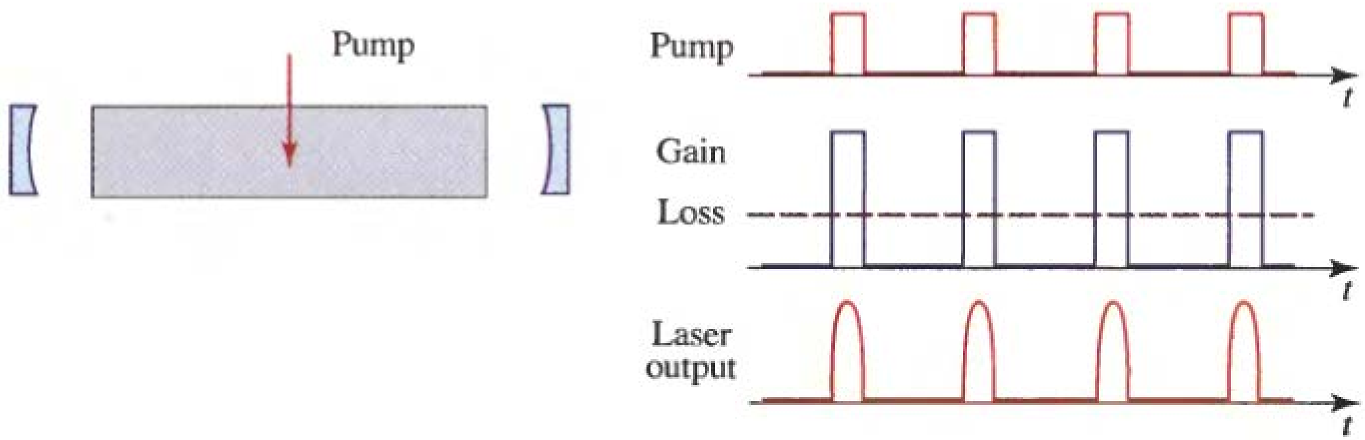


Comparison of pulsed laser outputs achievable with (a) an external modulator, and (b) an internal modulator

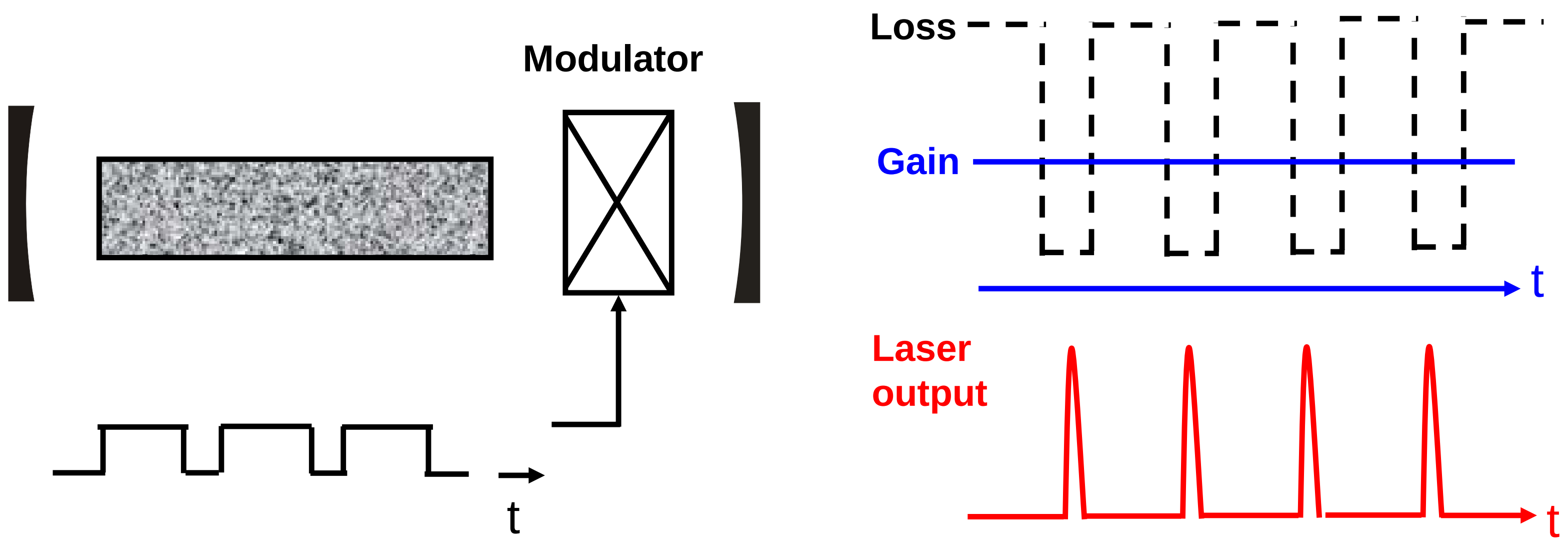
1. Gain switching **2. Q-Switching**

3. Cavity Dumping **4.Mode Locking**

Gain Switching

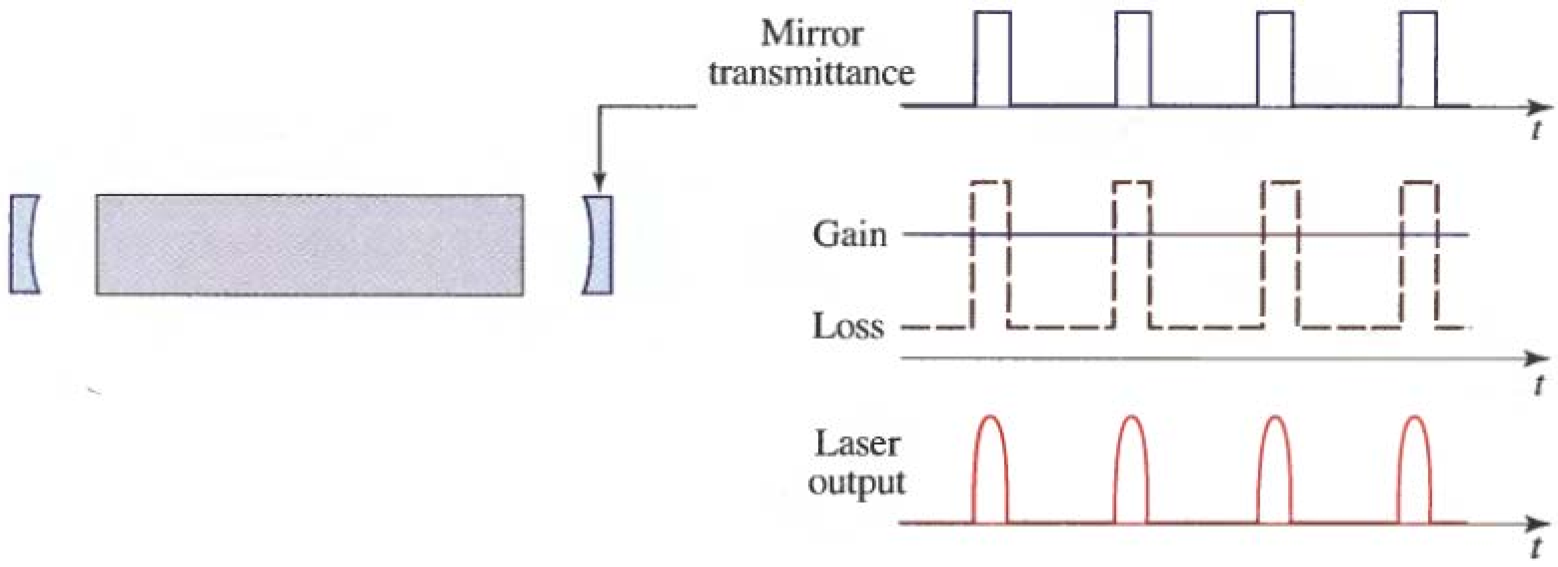


Q- Switching



Q-switching.

Cavity Dumping



Cavity dumping. One of the mirrors is removed altogether to dump the stored photons as useful light.

Rate equation for the photon-number density

$$\frac{dn}{dt} = -\frac{n}{\tau_p} + NW_i$$

τ_p **photon lifetime**

$$W_i = \phi\sigma(\nu) = cn\sigma(\nu)$$

From $\sigma(\nu) = 1/c\tau_p N_t$ **Probability density for induced absorption/emission**

We have

$$\frac{dn}{dt} = -\frac{n}{\tau_p} + \frac{N}{N_t}\frac{n}{\tau_p}$$

Photon-Number Rate Equation

Rate equation for the photon-number density

For a three level system

$$\frac{dN_2}{dt} = R - \frac{N_2}{t_{sp}} - W_i(N_2 - N_1)$$

Note $N_1 = (N_a - N)/2, N_2 = (N_a + N)/2, N = N_2 - N_1$

For 3 level system the small signal population difference

$$N_0 = 2Rt_{sp} - N_a$$

Then

$$\frac{dN}{dt} = \frac{N_0}{t_{sp}} - \frac{N}{t_{sp}} - 2W_i N$$

$\because\ W_i = \phi\sigma(v) = cn\sigma(v)$ **and** $N_t = \alpha_r / \sigma(v) = 1 / c\tau_p\sigma(v)$ **then** $W_i = n / N_t\tau_p$

We have

$$\frac{dN}{dt} = \frac{N_0}{t_{sp}} - \frac{N}{t_{sp}} - 2\frac{N}{N_t}\frac{n}{\tau_p}$$

Population-difference rate equation (Three-level system)

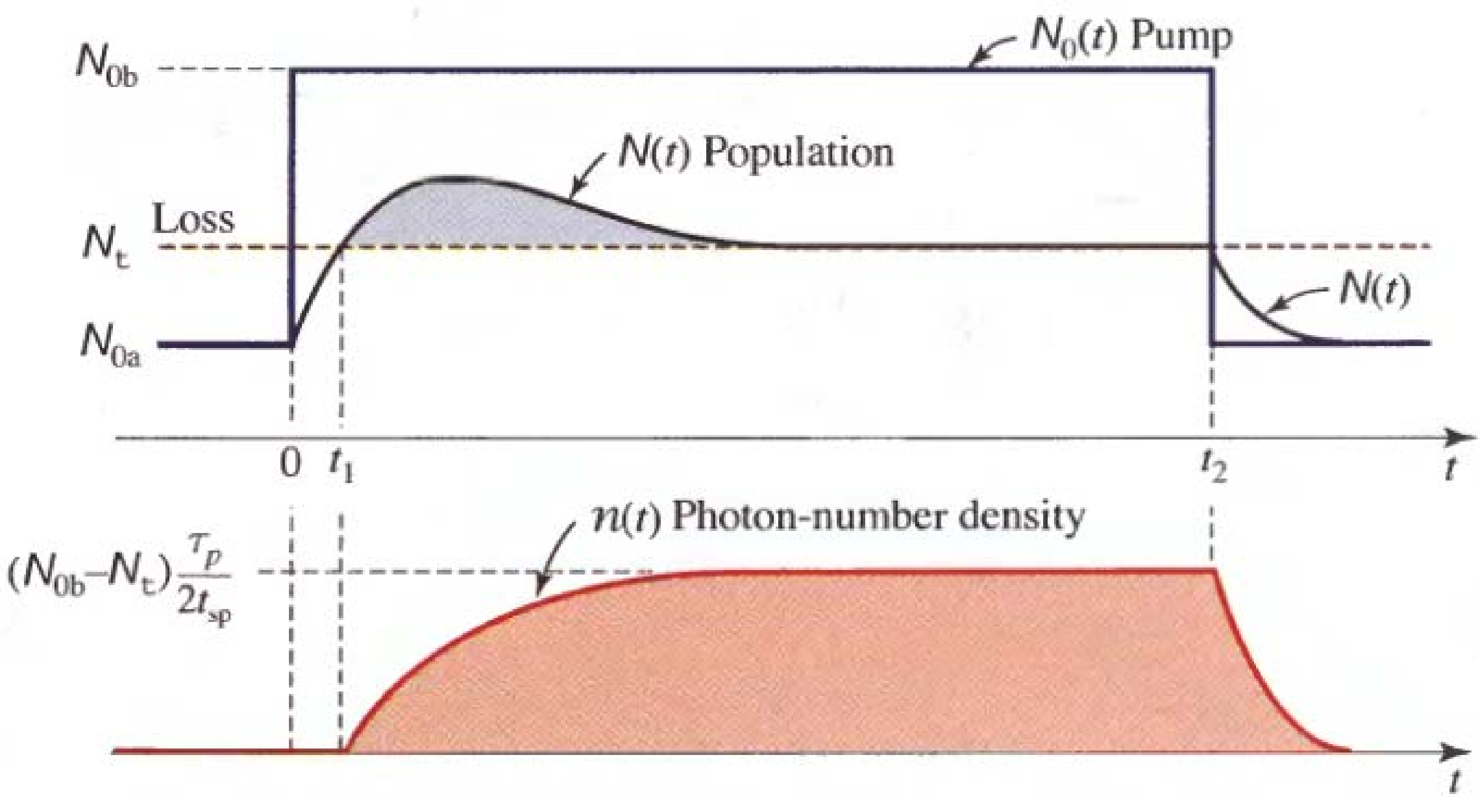
- For 3 level system， we have these tow equations

$$
\begin{cases}
\dfrac{dn}{dt} = -\dfrac{n}{\tau_p} + \dfrac{N}{N_t}\dfrac{n}{\tau_p} \\[2ex]
\dfrac{dN}{dt} = \dfrac{N_0}{t_{sp}} - \dfrac{N}{t_{sp}} - 2\dfrac{N}{N_t}\dfrac{n}{\tau_p}
\end{cases}
$$

Photon-Number Rate Equation

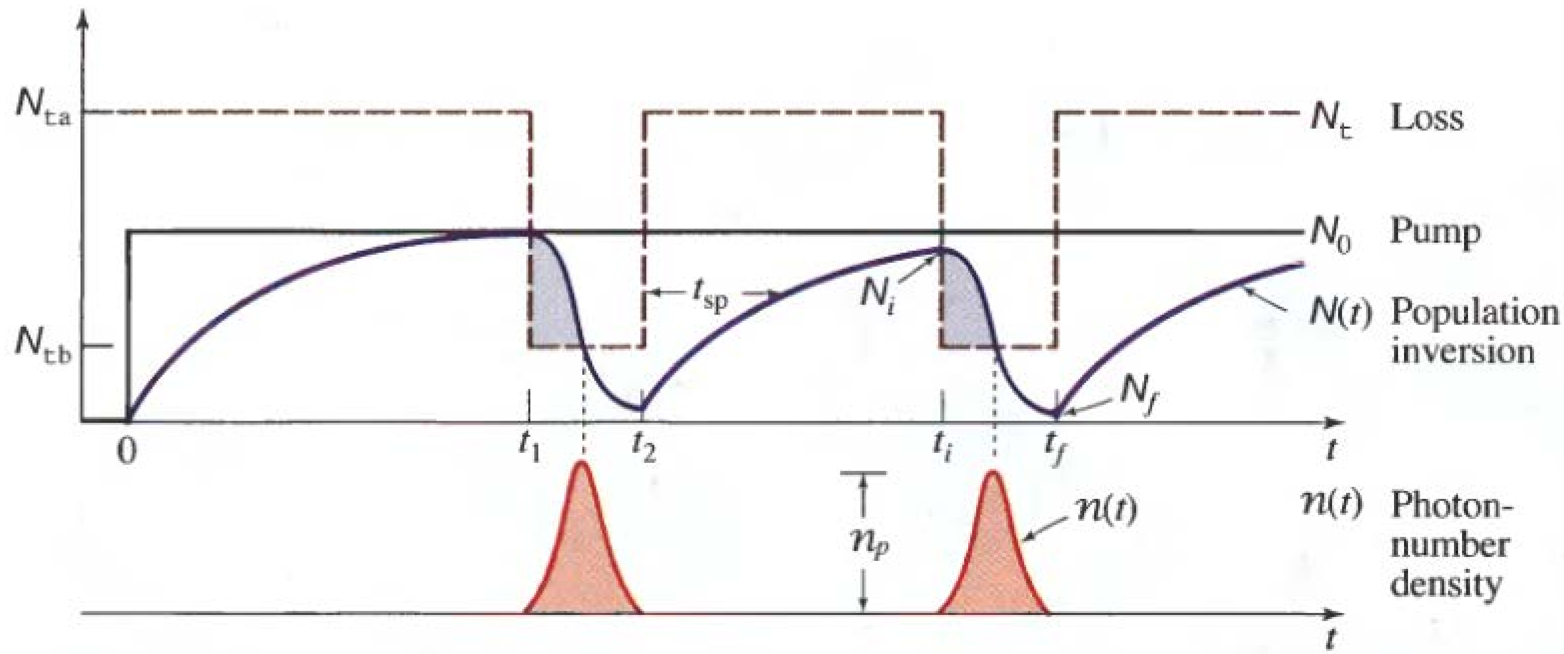
Population-difference rate equation

Situation for the gain switching



Variation of the population difference $N(t)$ and the photon-number density $n(t)$ with time, as a square pump pulse results in N_0 suddenly increasing from a low value N_{0a} to a high value N_{0b}, and then decreasing back to a low value N_{0a}.

Situation for the Q-switching



Operation of a Q-switched laser. Variation of the population threshold N_t (which is proportional to the resonator loss), the pump parameter N_0, the population difference $N(t)$, and the photon number $n(t)$.

Determination of the peak power, energy, width and shape of the optical pulse

For 3 level system, in case of the pulse is very short less than the t_{sp}, in the short time, we can neglect the R and p_{sp}, and have:

$$\begin{cases} \dfrac{dn}{dt} = \left(\dfrac{N}{N_t} - 1\right)\dfrac{n}{\tau_p} \\ \dfrac{dN}{dt} = -2\dfrac{N}{N_t}\dfrac{n}{\tau_p} \end{cases}$$

Dividing

$$\frac{dn}{dN} \approx \frac{1}{2}\left(\frac{N_t}{N} - 1\right)$$

$$n \approx \frac{1}{2} N_t \ln(N) - \frac{1}{2} N + cons\tan t$$

At initial time, we name the N as N_i,(the initial population inversion) and n=0, so we get the photon density

$$n \approx \frac{1}{2} N_t \ln \frac{N}{N_i} - \frac{1}{2}(N - N_i)$$

Power

$$P_0 = h\nu A\phi_0 = \frac{1}{2} h\nu cTAn = h\nu T \frac{c}{2d} Vn$$

V the volume of mode

Peak pulse power

$$\frac{dn}{dt} = 0 \quad n_{peak} \to max$$

$$n_p = \frac{1}{2} N_i \left(1 + \frac{N_t}{N_i} \ln \frac{N_t}{N_i} - \frac{N_t}{N_i}\right)$$

so

$$P_p = h\nu T \frac{c}{2d} Vn_p$$

For high power pulse laser , needs N_i>>N_t

$$n_p = \frac{1}{2} N_i \left(1 + \frac{N_t}{N_i} \ln \frac{N_t}{N_i} - \frac{N_t}{N_i}\right) \Rightarrow n_p \approx \frac{1}{2} N_i$$

So

$$P_p \approx \frac{1}{2} h\nu T \frac{c}{2d} VN_i$$

The larger the initial population inversion, the higher the Q-switched pulse peak power.

c. Pulse energy:

$$E = h\nu T \frac{c}{2d} V \int_{t_i}^{t_f} n(t)dt = h\nu T \frac{c}{2d} V \int_{N_i}^{N_f} n(t) \frac{dt}{dN} dN$$

$$E = \frac{1}{2} h\nu T \frac{c}{2d} V N_t \tau_p \int_{N_f}^{N_i} \frac{dN}{N}$$

$$E = \frac{1}{2} h\nu T \frac{c}{2d} V N_t \tau_p \ln \frac{N_i}{N_f}$$

The final population difference *N_f*

from $n \approx \frac{1}{2} N_t \ln \frac{N}{N_i} - \frac{1}{2}(N - N_i)$ **When n =0, *$N=N_f$*, then** $\ln \frac{N_i}{N_f} = \frac{N_i - N_f}{N_t}$

$$E = \frac{1}{2} h\nu T \frac{c}{2d} V N_t \tau_p (N_i - N_f)$$

d. Pulse width:

A rough estimation of the pulse width is *the ratio of the pulse energy to the peak pulse power.*

$$\tau_{pulse} = \tau_p \frac{N_i / N_t - N_f / N_t}{N_i / N_t - \ln(N_i / N_t) - 1}$$

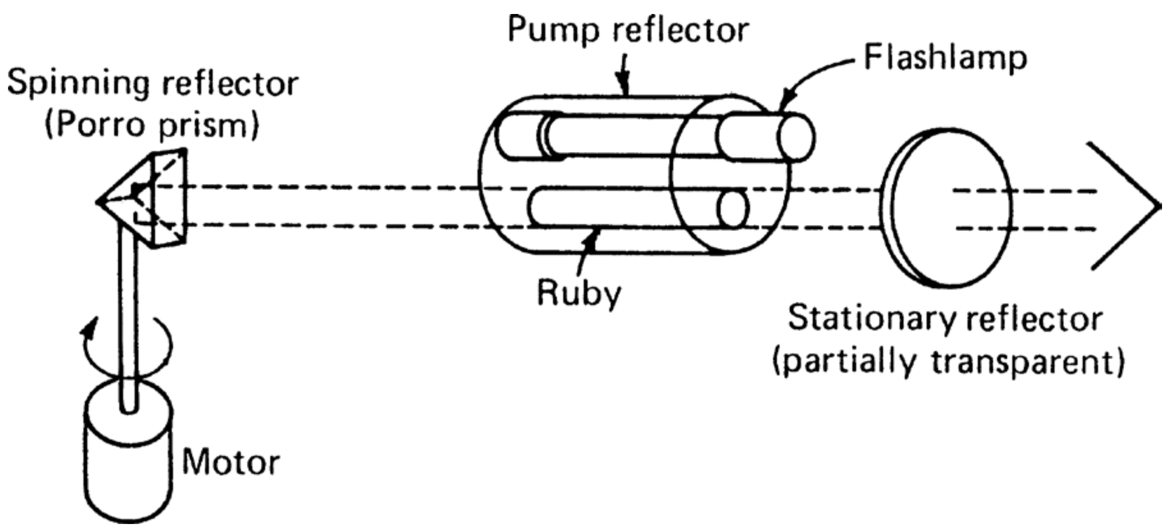
When $N_i >> N_{th}$ and $N_i >> N_f$

$$\tau_{pulse} \approx \tau_p$$

The shorter the photon life time, the shorter the Q-switched pulses.

Techniques for Q-switching

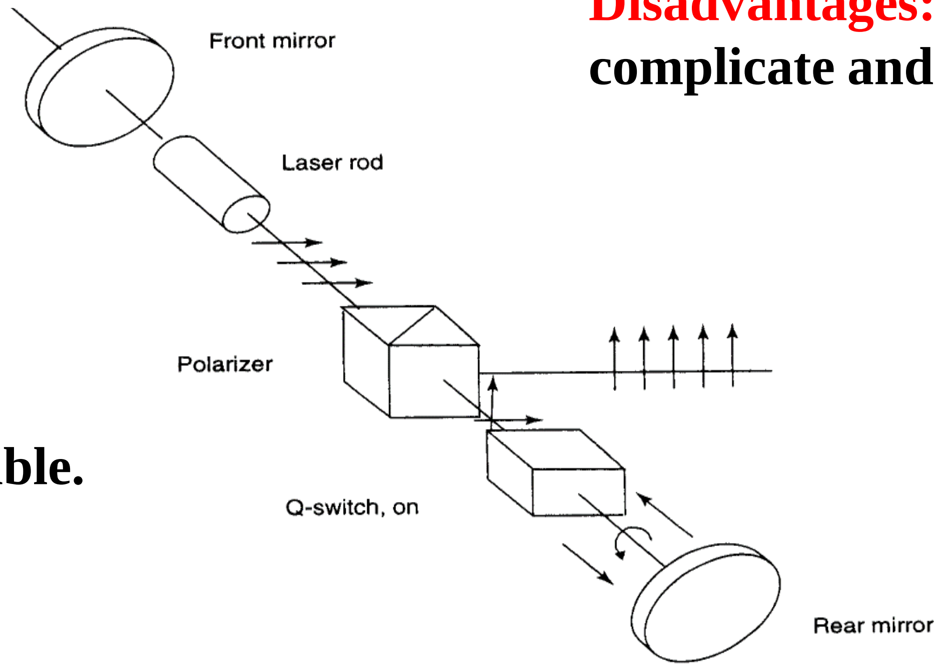
1. Mechanical rotating mirror method:



Q-switching principle: rotating the cavity mirror results in the cavity losses high and low, so the Q-switching is obtained.

Advantages: simple, inexpensive.
Disadvantages: very slow, mechanical vibrations.

2. Electro-optic Q-switching

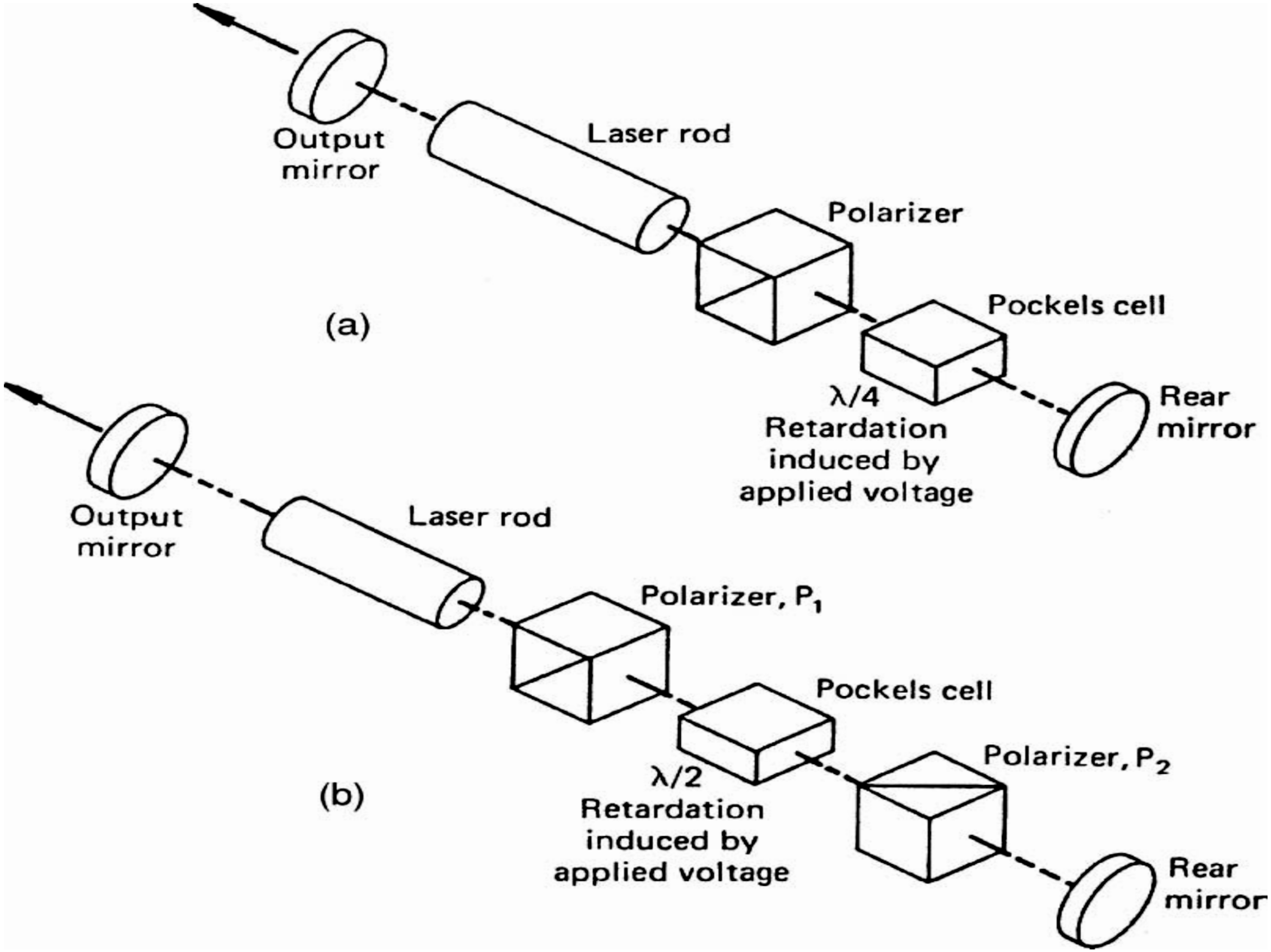


Disadvantages:
complicate and expensive

Advantages:
very fast and stable.

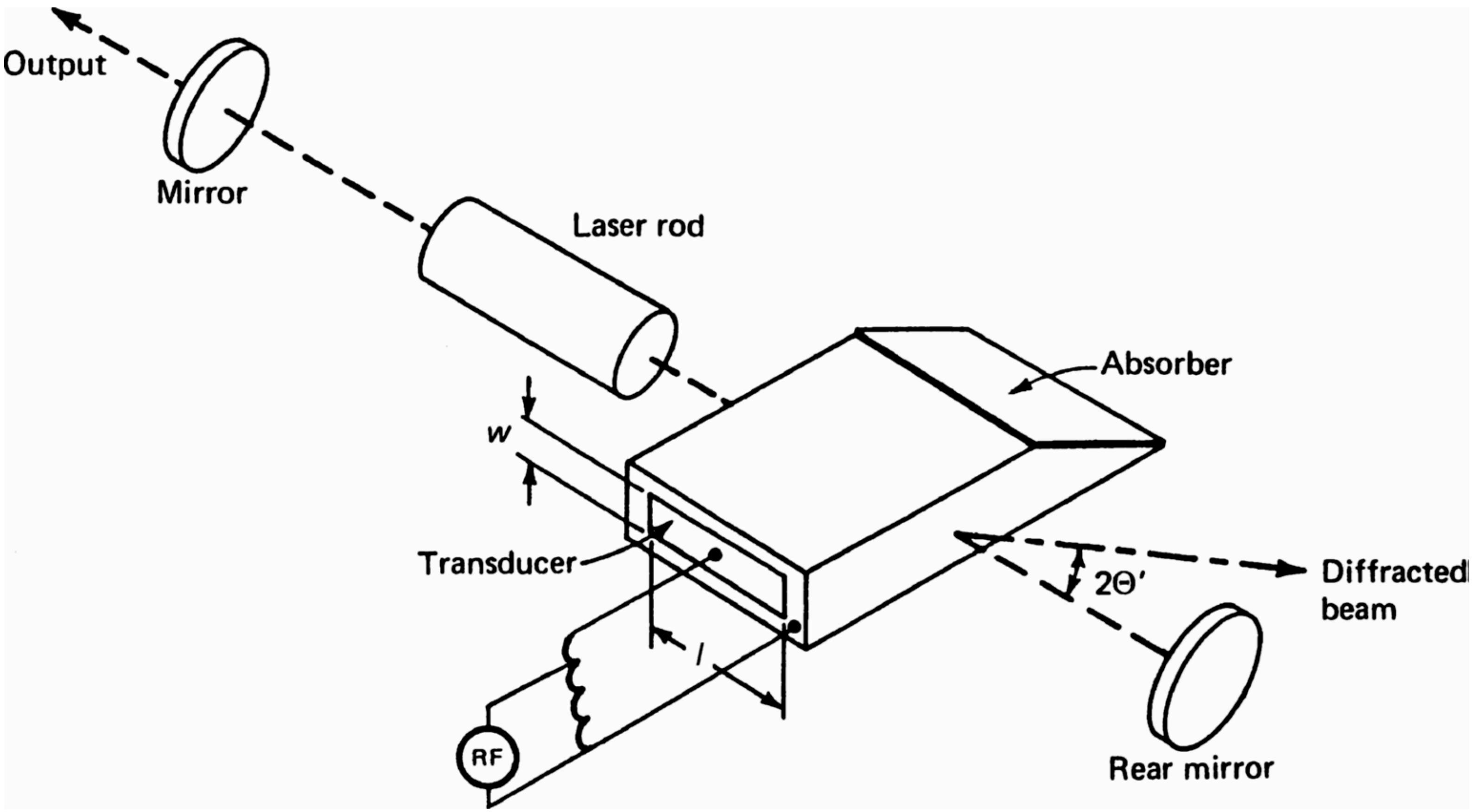
Pockels effect: applying electrical field in a uniaxial crystal results in additional birefringence, which changes the polarization of light when passing through it.

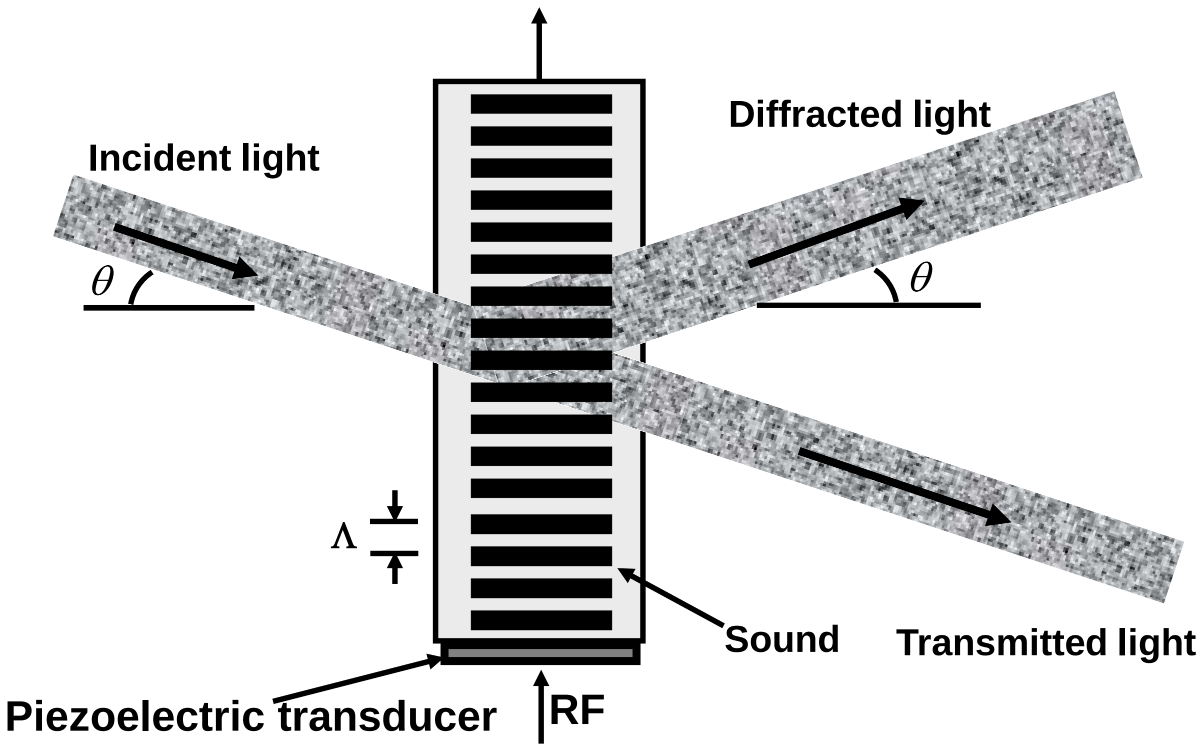
Q-switching principle: placing an electro-optic crystal between crossed polarizers comprises a Pockels switch. Turning on and off the electrical field results in high and low cavity losses.



Electro-optic Q-switch operated at (a) quarter-wave and (b) half-wave retardation voltage

3. Acousto-optic Q-switching





Bragg scattering: due to existence of the acoustic wave, light changes its propagation direction.

Q-switching principle: through switching on and off of the acoustic wave the cavity losses is modulated.

Advantages: works even for long wavelength lasers.
Disadvantages: low modulation depth and slow.

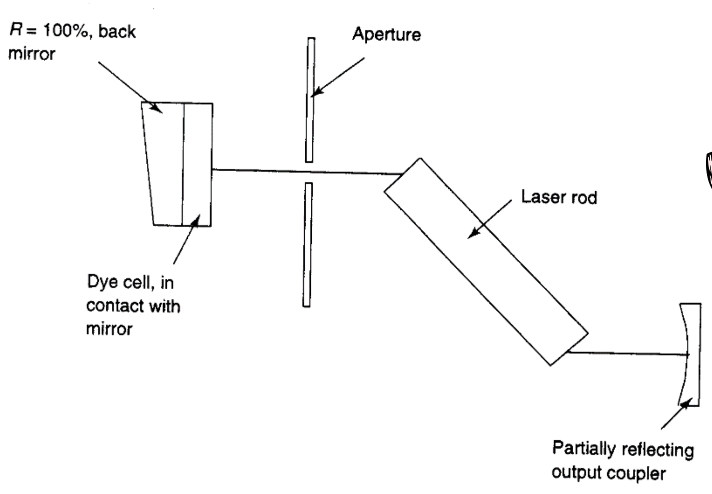
4. Saturable absorber Q-switching

What's a saturable absorber?

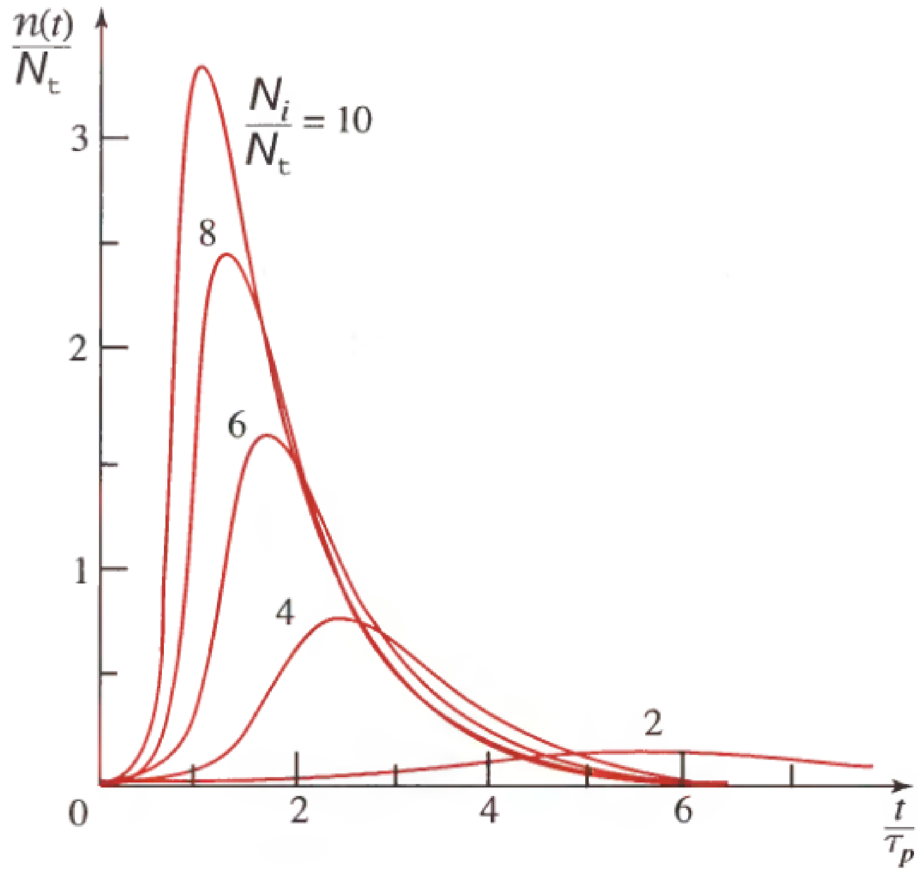
$$\alpha = \frac{\alpha_0}{1 + \frac{I}{I_s}}$$

Absorption coefficient of the material is reversely proportional to the light intensity. I_s: saturation intensity.

Saturable absorber Q-switching:



Insertion a saturable absorber in the laser cavity, the Q-switching will be automatically obtained.



Typical Q-switched pulse shapes obtained from numerical integration of the approximate rate equations. The photon-number density $n(t)$ is normalized to the threshold population difference $N_t = N_{th}$ and the time t is normalized to the photon lifetime τ_p. The pulse narrows and achieves a higher peak value as the ratio N_i/N_t increases. In the limit $N_i/N_t \gg 1$, the peak value of $n(t)$ approaches $\frac{1}{2}N_i$.

General characteristics of laser Q-switching

- **Pulsed laser output:**
  - Pulse duration – related to the photon lifetime.
  - Pulse energy - related to the upper level lifetime.
- **Laser operation mode:**
  - Single or multi-longitudinal modes.
- **Active verses passive Q-switching methods:**
  - Passive: simple, economic, pulse jitter and intensity fluctuations.
  - Active: stable pulse energy and repetition, expensive.
- **Comparison with chopped laser beams:**
  - Energy concentration in time axis.
- **Function of gain medium**
  - Energy storage

Laser mode-locking

Aims:

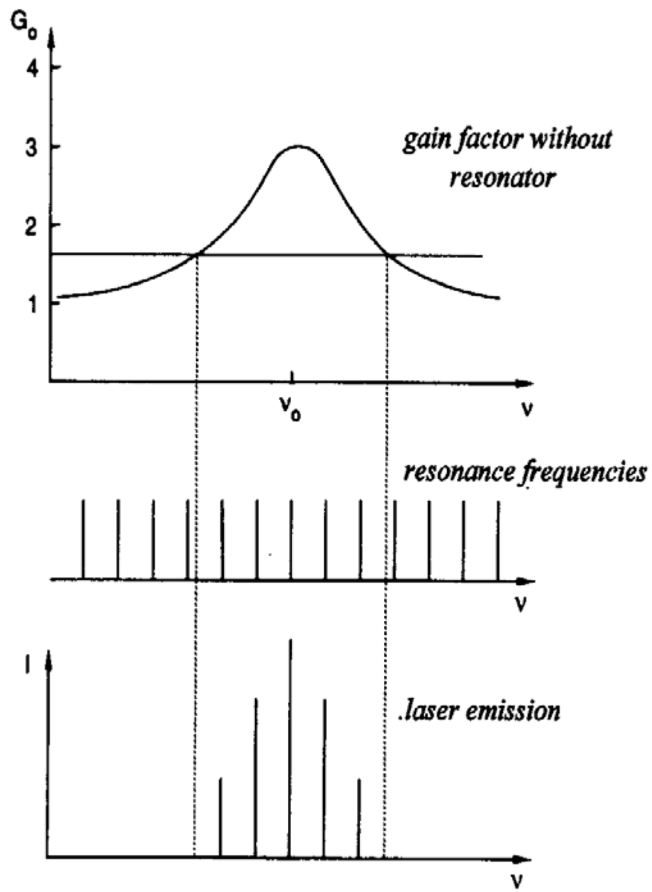
1. Familiarize with the principle of laser mode-locking.
2. Familiarize with different techniques of achieving laser Mode-locking.

Outlines:

1. Principle of laser mode-locking.
2. Methods of laser mode-locking.
3. Active mode-locking.
4. Passive mode-locking.
5. Transform-limited pulses.

Principle of laser mode-locking

1. **Lasing in inhomogeneously broadened lasers:**



i) Laser gain and spectral hole-burning.

ii) Cavity longitudinal mode frequencies.

iii) Multi-longitudinal mode operation.

2. **Laser multimode operation**:

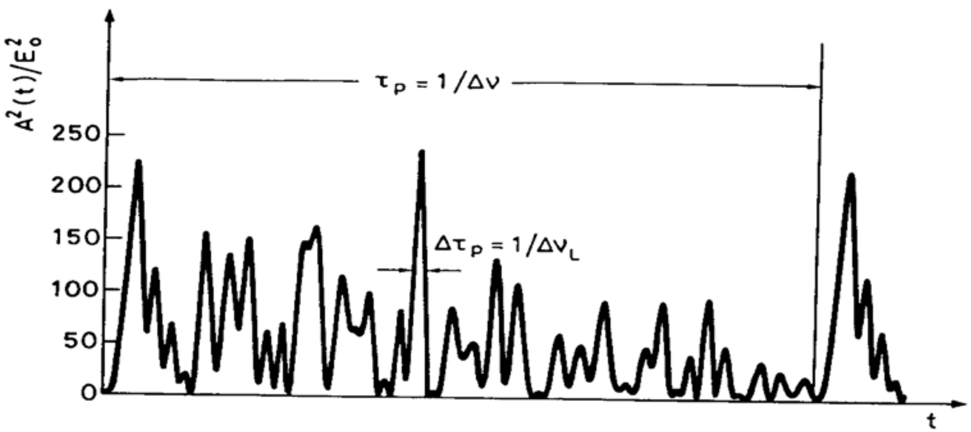
Single mode lasers: $E(t) = E_0 \cos[\omega_0 t + \varphi(t)]$

Multimode lasers: $E(t) = \sum_{i=1}^{M} E_i \cos\left[\omega_i t + \phi_i(t)\right]$

Mode-frequency separations: $\sim \dfrac{\pi c}{nd}$

Phase relation between modes: **Random and independent!**

Total laser intensity fluctuates with time !



The mean intensity of a multimode laser remains constant, however, its instant intensity varies with time.

3. Effect of mode-locking:

(i) Supposing that the phases of all modes are locked together:

$$\varphi_i(t) = \varphi_0 = 0$$

(ii) Supposing that all modes have the same amplitude:

$$E_i = E_0$$

purely for the convenience of the mathematical analysis

(iii) Under the above two conditions, the total electric field of the multimode laser is:

$$E(t) = \mathrm{Re}\left[\sum_{i=1}^{N} E_i e^{j\omega_i t}\right] \quad \text{where}$$

$$\omega_i = \omega_0 + \left[i - \frac{M+1}{2}\right]\Delta\omega_c \quad \Delta\omega_c = \frac{\pi c}{d}$$

ω_0 is the frequency of the central mode, M is the number of modes in the laser, $\Delta\omega_c$ is the mode frequency separation. ω_i is the frequency of the i-th mode.

Calculating the summation yields:

$$E(t) = E_0 \frac{\sin\left(M \dfrac{\Delta\omega_c t}{2}\right)}{\sin\left(\dfrac{\Delta\omega_c t}{2}\right)} \cos\omega_0 t$$

Note this is the optical field of the total laser Emission !

The optical filed can be thought to consist of a carrier wave of frequency ω_0 that amplitude modulated by the function

$$A_M(x) = \frac{\sin(Mx)}{\sin(x)}$$

The field of one modes: $$U(z,t)=\sum_q A_q \exp[j2\pi v_q(t-z/c)]$$

where $$v_q = v_0 + q v_F, \qquad v_F = c/2d$$

The sum of all the modes: $$U(z,t)=A(t)=\sum_q A_q \exp[\frac{jq2\pi t}{T_F}]$$

where $$T_F=\frac{1}{v_F}=\frac{2d}{c}$$

If all the mode have same phase

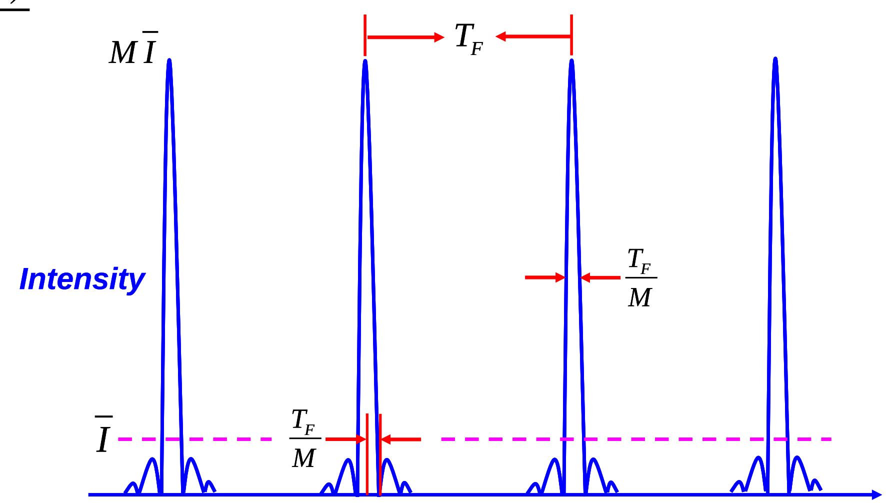
Then we have

$$A(t)=A\frac{\sin(M\pi t/T_F)}{\sin(\pi t/T_F)}$$

Where M is mode number

intensity

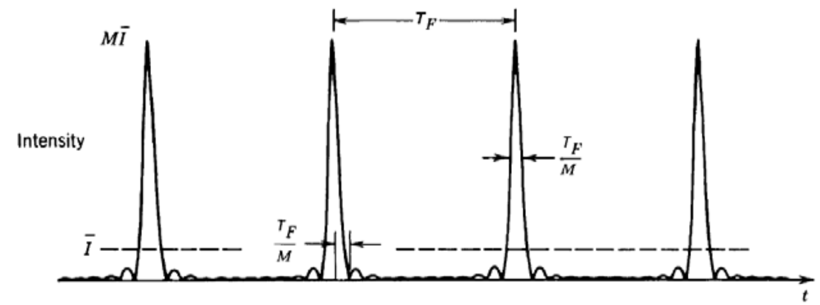
$$I(t,z)=|A|^2\frac{\sin^2[M\pi(t-z/c)/T_F]}{\sin^2[\pi(t-z/c)/T_F]}$$



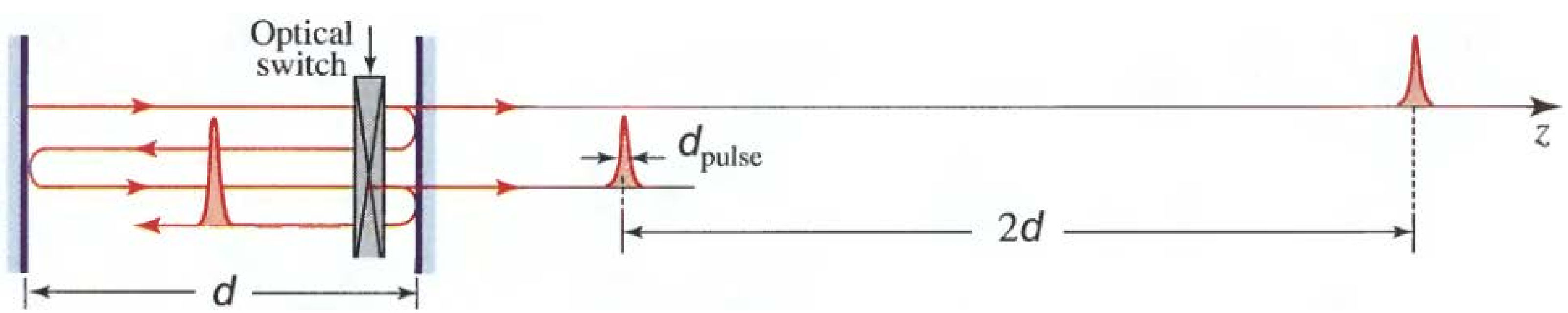
4. **Characteristics of the mode-locked lasers**:

The intensity of the laser field is:

$$I(t,z)=|A|^2\frac{\sin^2[M\pi(t-z/c)/T_F]}{\sin^2[\pi(t-z/c)/T_F]}$$



The output of a mode-locked laser consists of a series of pulses. The time separation between two pulses is determined by τ_{RT} and the pulse width of each pulse is Δt_p.



5. **Properties of mode-locked pulses:**

i) The pulse separation τ_{RT}:

$$\mathrm{Sin}^2\left(\frac{\Delta\omega_c t}{2}\right) = 0 \Longrightarrow \Delta\omega_c t = 2\pi \qquad \Delta\omega_c = 2\pi\nu_F$$

$$\tau_{RT} = \frac{2\pi}{\Delta\omega_c} = \frac{2d}{c} = T_F \Longrightarrow$$ ***The round-trip time of the cavity!***

ii) The peak power:

Average power

$$\Longrightarrow \quad I_{pulse} = M\bar{I} = M^2|A|^2 \qquad \bar{I} = M\;|A|^2$$

M times of the average power. M: number of modes.

The more the modes the higher the peak power of the Mode-locked pulses.

iii)The individual pulse width:

$$\sin\left(M\frac{\Delta\omega_c t}{2}\right)=0 \Rightarrow \Delta t_p=\frac{2\pi}{M\Delta\omega_c}$$

$$M\approx\frac{\Delta\omega_a}{\Delta\omega_c} \Rightarrow \Delta t_p\approx\frac{2\pi}{\Delta\omega_a}=\frac{1}{\Delta\nu_a}$$

$\Delta\nu_a$: bandwidth of the gain profile.

Narrower as M increases.

$$\Delta t_p\approx\frac{\tau_{RT}}{M}=\frac{T_F}{M}$$

The mode locked pulse width is reversely proportional to the gain band width, so the broader the gain profile, the shorter are the mode locked pulses.

summy

Temporal period	$T_F = \dfrac{2d}{c}$
Pulse width	$\tau_{\text{pulse}} = \dfrac{T_F}{M} = \dfrac{1}{M\nu_F}$
Spatial period	$2d$
Pulse length	$d_{\text{pulse}} = c\tau_{\text{pulse}} = \dfrac{2d}{M}$
Mean intensity	$\bar{I} = M\lvert A\rvert^2$
Peak intensity	$I_p = M^2\lvert A\rvert^2 = M\bar{I}$

Techniques of laser mode-locking

Active mode-locking:

Actively modulating the gain or loss of a laser cavity in a periodic way, usually at the cavity repetition frequency c/2nL to achieve mode-locking.

Amplitude modulation:

A modulator with a transmission function of

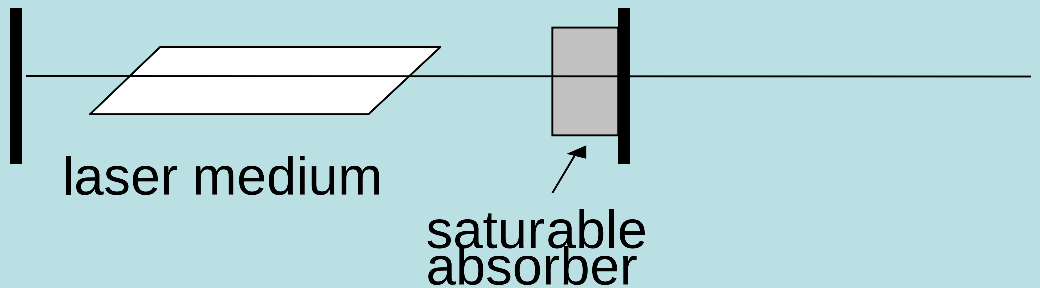
$$T = \left[1 - \delta\left(1 + \cos\left(\frac{2\pi t}{\tau_{RT}}\right)\right)\right]$$

is inserted in the laser cavity to modulate the light. Where δ is the modulation strength and $\delta < 0.5$. Under the influence of the modulation phases of the lasing modes become synchronized and as a consequence become mode-locked.

Passive mode-locking:

Inserting an appropriately selected saturable absorber inside the laser cavity. Through the mutual interaction between light, saturable absorber and gain medium to automatically achieve mode locking.

A typical passive mode locking laser configuration:



Mechanism of the mode-locking:

i) Interaction between saturable absorber and laser gain:

ii)Balance between the pulse shortening and pulse broadening: Final pulse width.

Home work

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10, 11, 14, 16, 18, 22