Chapter9 Theory and Applications of Transmission Lines

Application: Signal Transmission

Models

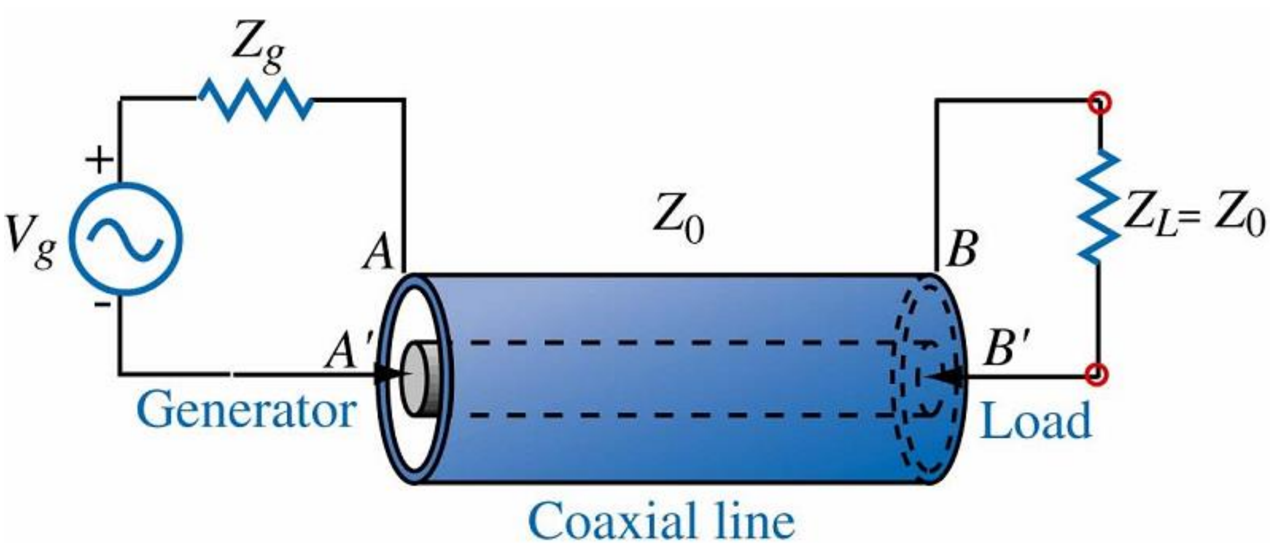
Electromagnetic theory

General method

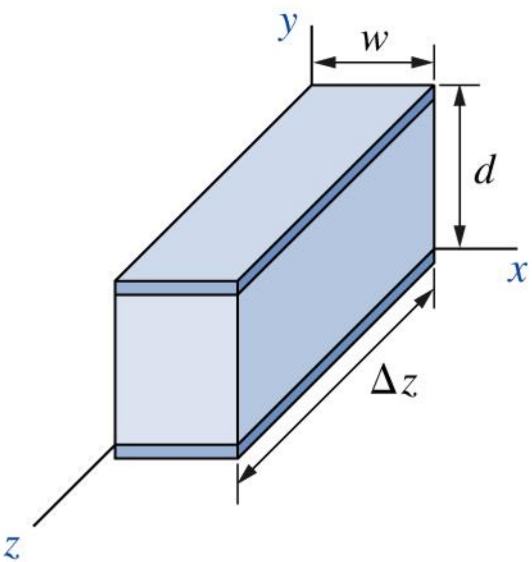
Very complicate

Lump-element model

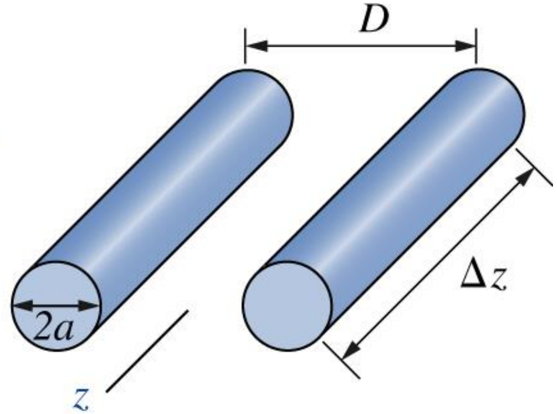
Applied to transmission line only



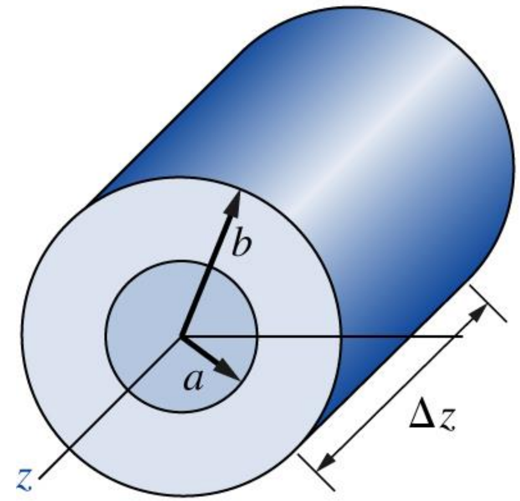
Common types of transmission lines



Parallel-plate
transmission line



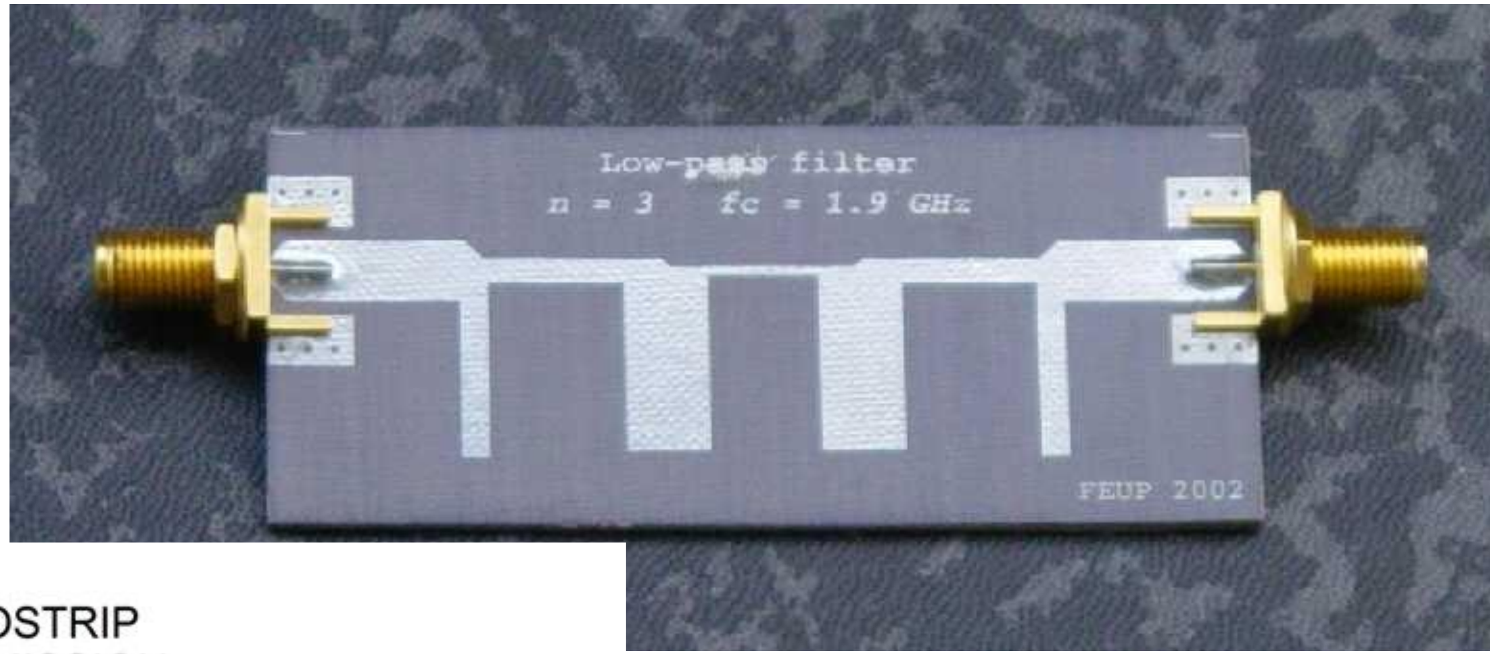
Two-wire (twin-lead)
transmission line

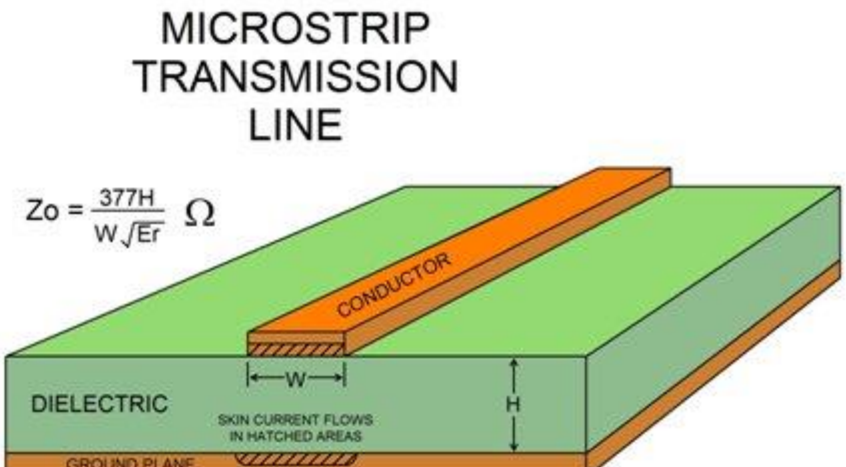


Coaxial
transmission line

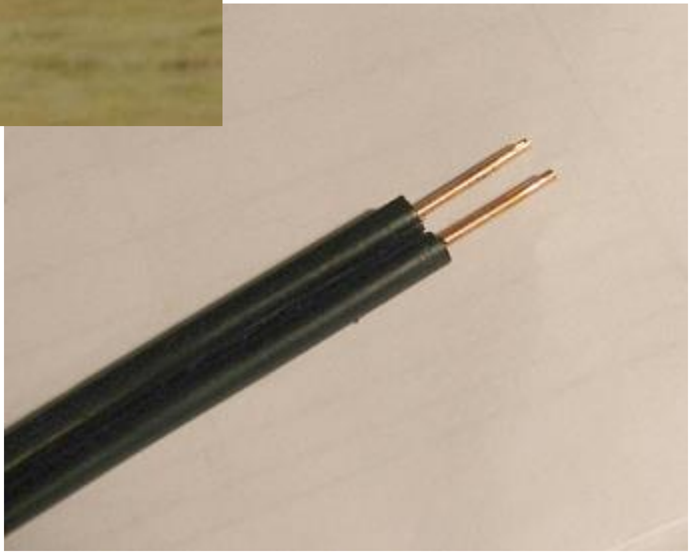
Each structure (including the twin lead) may have a dielectric between two conductors used to keep the separation between the metallic elements constant, so that the electrical properties would be constant.

Microstrip line

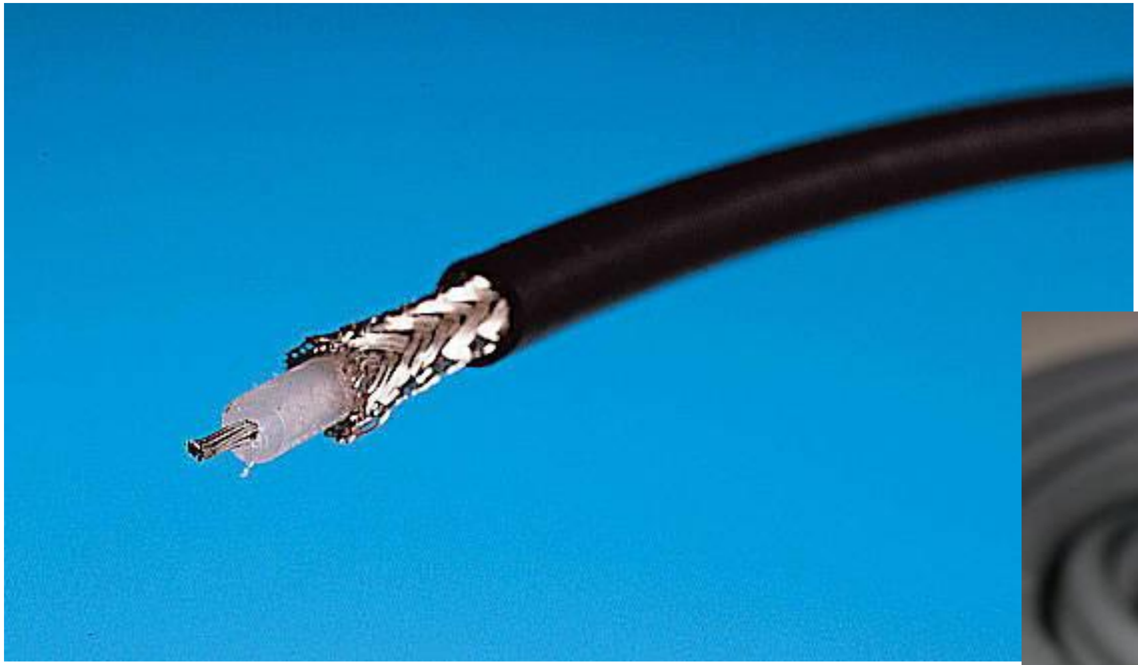




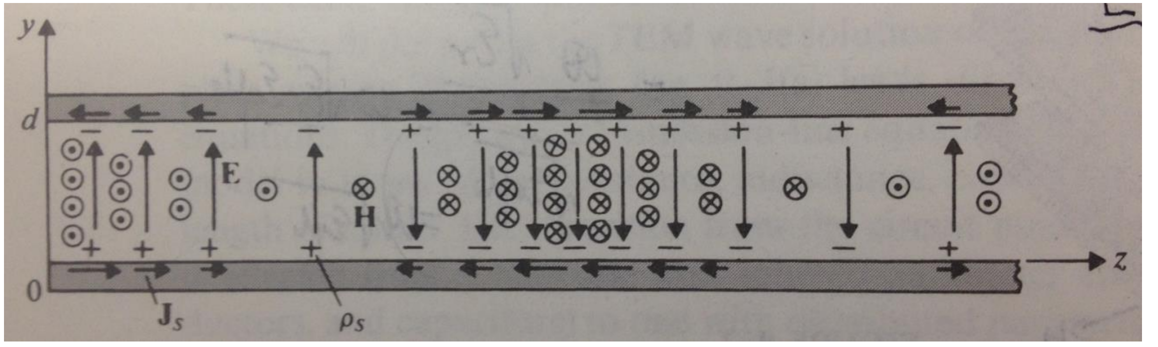
Twin lead



Coaxial cable



9-2 Parallel-Plate Transmission Line



$$\nabla^2\mathbf{E} - \frac{1}{c^2}\frac{\partial^2\mathbf{E}}{\partial t^2} = 0 \quad \text{since} \quad \mathbf{E} \propto e^{i\omega t} \;\rightarrow\; \nabla^2\mathbf{E} + \frac{\omega^2}{c^2}\mathbf{E} = 0\,(\nabla^2\mathbf{E} + k_0^2\mathbf{E} = 0)$$

Assume it's a plane wave propagate in the z with polarization in y direction.

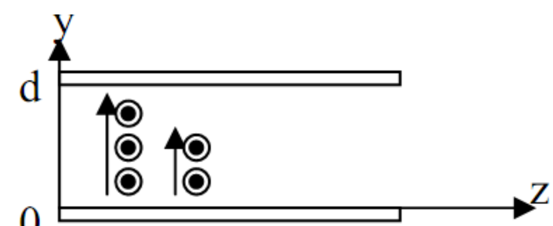
$$\frac{d^2}{dz^2}E_y + k_0^2 E_y = 0 \;\rightarrow\; \mathbf{E} = \hat{y}\widetilde{E}_0 e^{-ik_0 z + i\omega t}$$

$$\mathbf{H} = -\hat{x}\frac{1}{\sqrt{\frac{\mu}{\varepsilon}}}\widetilde{E}_0 e^{-ikz+i\omega t} = H_x\hat{x}$$

9-2 Parallel-Plate Transmission Line

Crossing the boundary from dielectric medium to the perfect conduction plates:

$$\nabla\times\vec{E}=-\frac{\partial\vec{B}}{\partial t}\ \ \&\ \ \nabla\times\vec{B}=\mu\varepsilon\frac{\partial\vec{E}}{\partial t}$$



$$\rightarrow\ \nabla\times\vec{E}=-i\omega\mu\vec{H}\ \ \&\ \ \nabla\times\vec{H}=i\omega\varepsilon\vec{E}$$

$$\mathbf{E}=\hat{y}\widetilde{E}_y(z,t),\ \ \mathbf{H}=\hat{x}\widetilde{H}_x(z,t)\ \ (\vec{E}\rightarrow V,\vec{H}\rightarrow\sigma\rightarrow I)$$

basic differential equations

$$\begin{vmatrix}\hat{i} & \hat{j} & \bar{k}\\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z}\\ 0 & E_y & 0\end{vmatrix}=-i\omega\mu H_x\ \rightarrow\ \frac{dE_y}{dz}=i\omega\mu H_x\ \ \&\ \ \frac{dH_x}{dz}=i\omega\varepsilon E_y$$

9-2 Parallel-Plate Transmission Line

$$\int_0^d \frac{dE_y}{dz} dy = i\omega\mu \int_0^d H_x dy \quad \rightarrow \quad \boxed{-\frac{dV(z)}{dz} = i\omega L I(z)}$$

$$L = \mu \frac{d}{w} \quad \text{is the inductance per unit length}$$

$$\int_0^w \frac{dH_x}{dz} dx = i\omega\varepsilon \int_0^w E_y dx \quad \rightarrow \quad \boxed{-\frac{dI(z)}{dz} = i\omega C V(z)}$$

$$C = \varepsilon \frac{w}{d} \quad \text{is the capacitance per unit length}$$

9-2 Parallel-Plate Transmission Line

$$\frac{d^2V(z)}{dz^2} = -\omega^2 LCV(z) \qquad V(z) = V_0 e^{-ikz}$$

$$\frac{d^2I(z)}{dz^2} = -\omega^2 LCI(z) \qquad I(z) = I_0 e^{-ikz}$$

impedance

$$Z_0 = \frac{V(z)}{I(z)} = \frac{V_0}{I_0} = \frac{\omega L I_0}{k I_0} = \sqrt{\frac{L}{C}} = \sqrt{\frac{\mu d / w}{\varepsilon w / d}} = \frac{d}{w}\sqrt{\frac{\mu}{\varepsilon}}$$

velocity of propagation along the line

$$v = \frac{\omega}{k} = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{(\mu d / w)(\varepsilon w / d)}} = \frac{1}{\sqrt{\mu\varepsilon}}$$

9-2 Parallel-Plate Transmission Line

Dielectric medium between two conductors having a perimittivity and a conductivity → Conductance C

The two conductors have a very large but finite conductivity → Resistance R

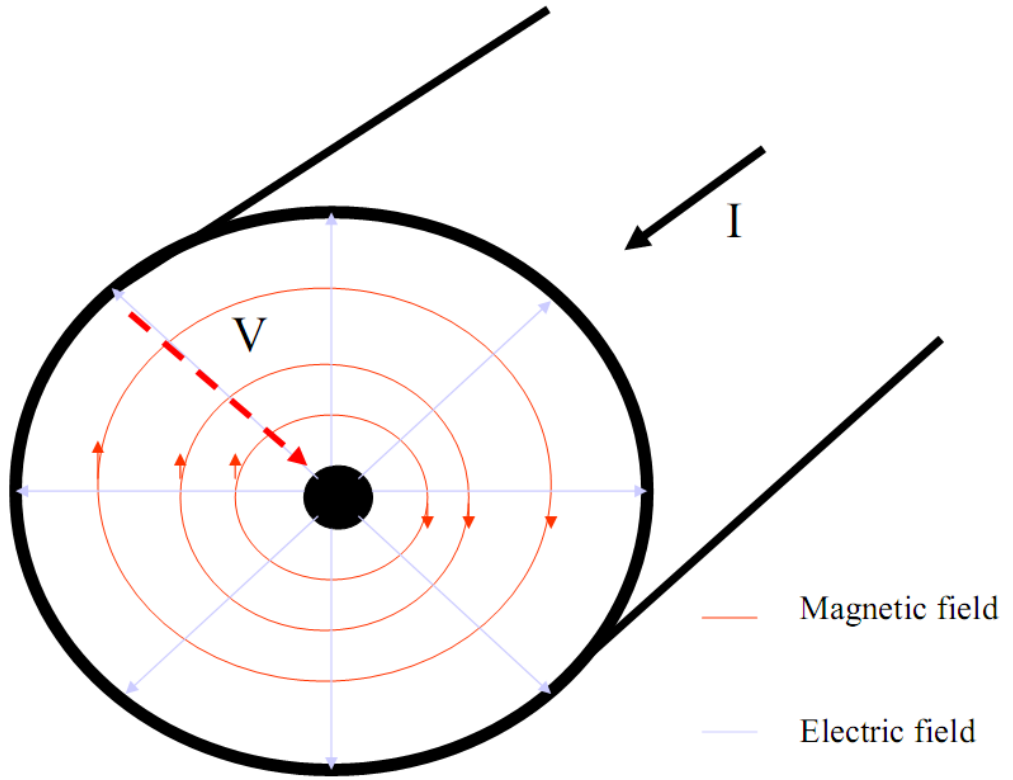
Distributed Parameters of Parallel-Plate Transmission Line (Width = *w*, Separation = *d*)

Parameter	Formula	Unit	
R	$\frac{2}{w}\sqrt{\frac{\pi f \mu_c}{\sigma_c}}$	Ω/m	conductor
L	$\mu \frac{d}{w}$	H/m	
G	$\sigma \frac{w}{d}$	S/m	dielectric
C	$\epsilon \frac{w}{d}$	F/m	

Coaxial transmission line
Example: E,H,V,I

$$V = -\int_b^a \mathbf{E} \cdot \mathbf{dr}$$

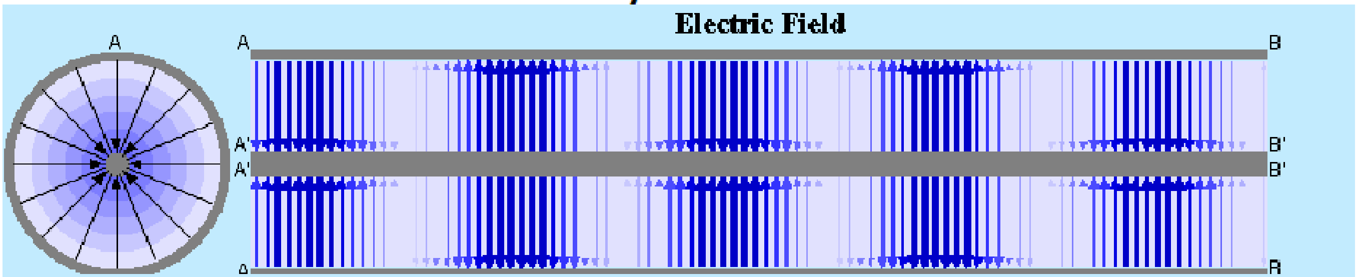
$$I = \oint_l \mathbf{H} \cdot \mathbf{dl}$$



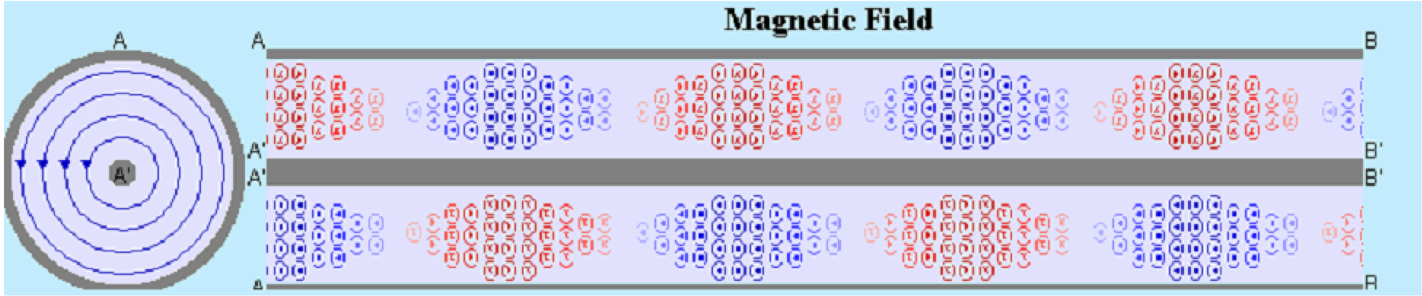
Cross-section of a coaxial cable showing the electric and magnetic fields

Coaxial transmission line

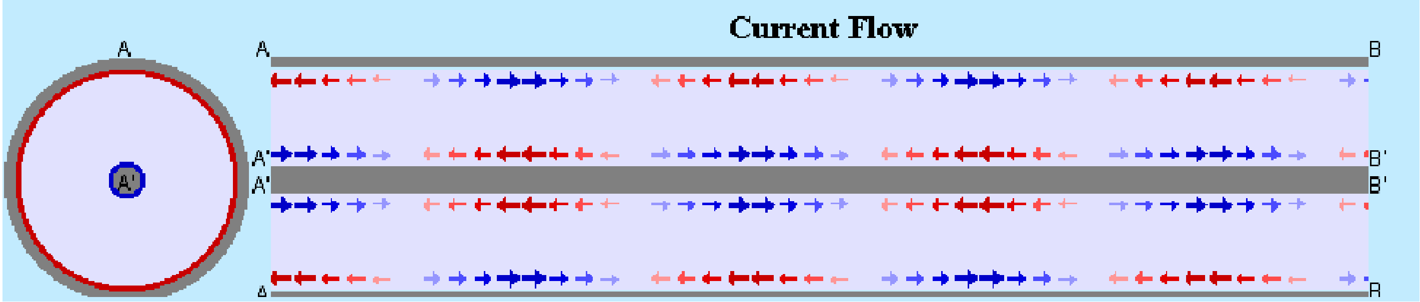
D2.1A Electric field intensity inside a coaxial cable



D2.3A Magnetic field inside a coaxial cable



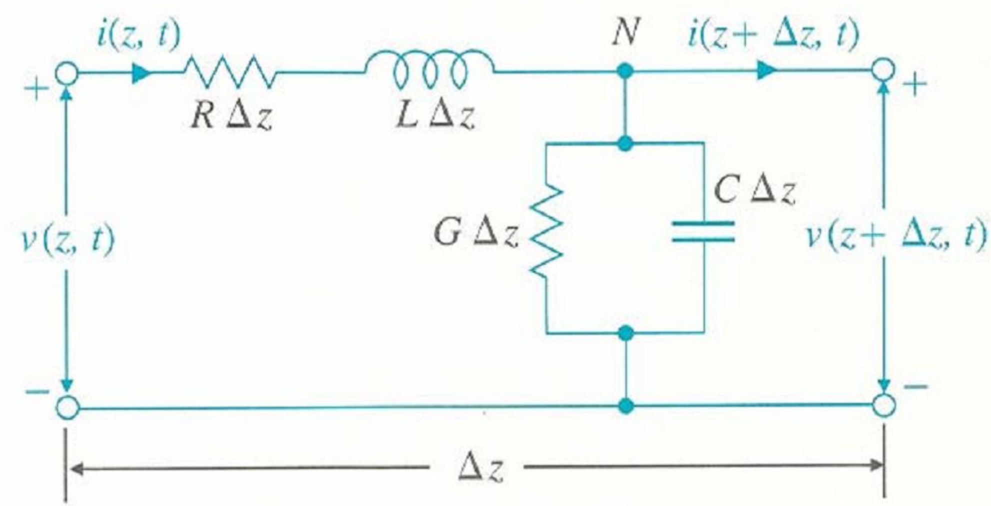
D2.4A Electric current density inside a coaxial cable



9-3 General Transmission-line equations Lumped-element model

Small section of a transmission line (length = Δz)

represented by an equivalent circuit



R: Resistance of both conductors per unit length (Ω/m)

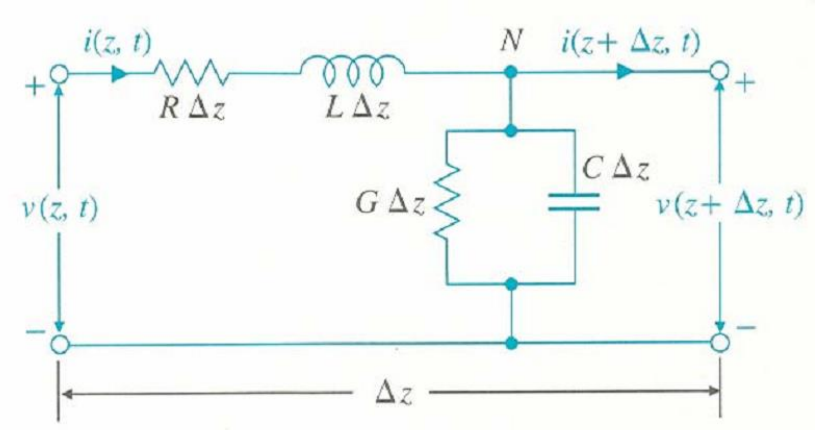
L: Inductance of both conductors per unit length (H/m)

G: Conductance of the insulation medium per unit length (S/m)

C: Capacitance of the two conductors per unit length (F/m)

9-3 General Transmission-line equations

1. Applying Kirchhoff's voltage law



$$v(z,t) = v_R + v_L + v(z+\Delta z, t)$$

$$= [R\Delta z]i(z,t) + [L\Delta z]\frac{\partial i(z,t)}{\partial t} + v(z+\Delta z, t)$$

$$\Rightarrow \frac{v(z+\Delta z,t) - v(z,t)}{\Delta z} = -Ri(z,t) - L\frac{\partial i(z,t)}{\partial t}$$

at node N

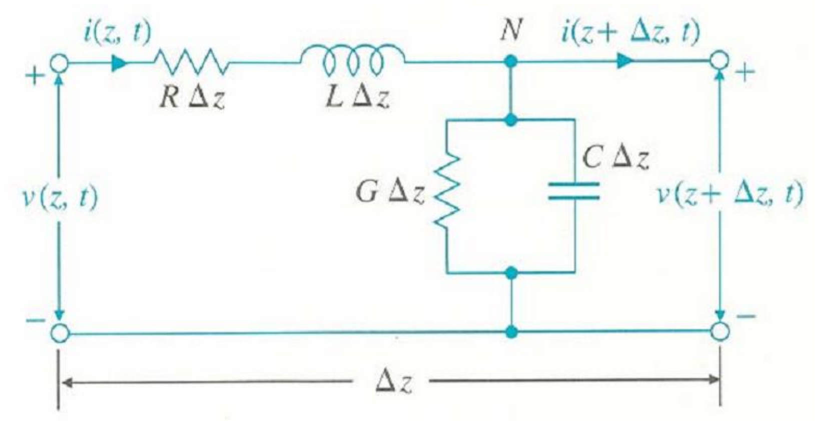
$$i(z,t) = i_G + v_C + i(z+\Delta z, t)$$

$$= [G\Delta z]v(z+\Delta z,t) + [C\Delta z]\frac{\partial v(z+\Delta z,t)}{\partial t} + i(z+\Delta z, t)$$

$$\Rightarrow \frac{i(z+\Delta z,t) - i(z,t)}{\Delta z} = -Gv(z+\Delta z,t) - C\frac{\partial v(z+\Delta z,t)}{\partial t}$$

9-3 General Transmission-line equations

2. Taking the limit as Δz tends to zero, we have



$$\frac{\partial v(z,t)}{\partial z} = -Ri(z,t) - L\frac{\partial i(z,t)}{\partial t}$$

$$\frac{\partial i(z,t)}{\partial z} = -Gv(z,t) - C\frac{\partial v(z,t)}{\partial t}$$

Time-domain form of the Transmission-line equations

Solving this equation with the appropriate initial conditions and boundary condition, we can determined the voltage and current

9-3 General Transmission-line equations

For sinusoidal steady-state conditions, phasors can be used

$$v(z,t) = \mathrm{Re}\left[V(z)e^{j\omega t}\right]$$

$$i(z,t) = \mathrm{Re}\left[I(z)e^{j\omega t}\right]$$

$$\frac{\partial v(z,t)}{\partial t} = \frac{\partial\, \mathrm{Re}\left[V(z)e^{j\omega t}\right]}{\partial t}$$

$$= \mathrm{Re}\left[V(z)\frac{\partial e^{j\omega t}}{\partial t}\right]$$

$$= \mathrm{Re}\left[\boxed{j\omega V(z)}e^{j\omega t}\right]$$

$$\frac{\partial v(z,t)}{\partial z} = \frac{\partial\, \mathrm{Re}\left[V(z)e^{j\omega t}\right]}{\partial z}$$

$$= \mathrm{Re}\left[\frac{dV(z)}{dz}e^{j\omega t}\right]$$

Similarly,

$$\frac{\partial i(z,t)}{\partial t} = \mathrm{Re}\left[j\omega I(z)e^{j\omega t}\right]$$

$$\frac{\partial i(z,t)}{\partial z} = \mathrm{Re}\left[\frac{dI(z)}{dz}e^{j\omega t}\right]$$

9-3 General Transmission-line equations

Therefore, the transmission-line equations becomes

$$-\frac{dV(z)}{dz} = (R + i\omega L)I(z)$$

$$-\frac{dI(z)}{dz} = (G + i\omega C)V(z)$$

- Transmission-line equations in phasor form
- The solution of the equations is the sinusoidal excited steady-state voltage and current phasor along the transmission line

9-3 General Transmission-line equations

$$-\frac{dV(z)}{dz} = (R + i\omega L)I(z) \qquad -\frac{dI(z)}{dz} = (G + i\omega C)V(z)$$

$$\downarrow$$

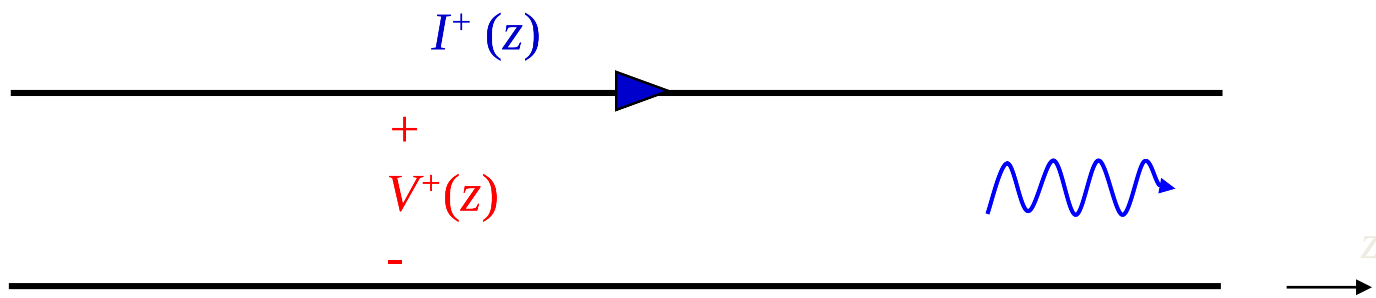
$$\frac{d^2V}{dz^2} = \frac{d}{dz}\left(-(R + i\omega L)I\right) = (R + i\omega L)\left(-\frac{dI}{dz}\right) = (R + i\omega L)(G + i\omega C)V(z)$$

$$\text{let } \; V(z) = e^{-kz} \;\; \& \;\; \frac{d^2V}{dz^2} = k^2 V(z)$$

$$\rightarrow \qquad \boxed{k = \alpha + i\beta = \sqrt{(R + i\omega L)(G + i\omega C)}}$$

α : attenuation constant $\qquad$ β : phase constant

9-3 General Transmission-line equations



A wave is traveling in the positive *z* direction.

$$V(z) = V_0^+ e^{-kz} + V_0^- e^{+kz} \qquad v(z,t) = V_0^+ e^{-kz+i\omega t} + V_0^- e^{+kz+i\omega t}$$

$$I(z) = I_0^+ e^{-kz} + I_0^- e^{+kz} \qquad i(z,t) = I_0^+ e^{-kz+i\omega t} + I_0^- e^{+kz+i\omega t}$$

$$\rightarrow \qquad \frac{V_0^+}{I_0^-} = -\frac{V_0^-}{I_0^-} = \frac{R+i\omega L}{k}$$

$$\boxed{\text{characteristic impetance: } Z_0 = \frac{R+i\omega L}{k} = \sqrt{\frac{R+i\omega L}{G+i\omega C}} \left(\propto \frac{V/l_z}{I/l_z} \right)}$$

9-3 General Transmission-line equations

Three limiting cases

1. Lossless Line ($R = 0$, $G = 0$. There is no real part in k.)

(a) Propagation constant: $k = i\omega\sqrt{LC}$, $\alpha = 0$, $\beta = \omega\sqrt{LC}$

(b) Phase velocity: $v_{phase} = \dfrac{\omega}{\beta} = \dfrac{1}{\sqrt{LC}}$

(c) Characteristic impedance: $Z_0 = R_0 + iX_0 = \sqrt{\dfrac{R + i\omega L}{G + i\omega C}} = \sqrt{\dfrac{L}{C}}$, $R_0 = \sqrt{\dfrac{L}{C}}$,

$X_0 = 0$

9-3 General Transmission-line equations

Three limiting cases

2. Low-Loss Line ($R << \omega L$, $G << \omega C$)

(a) Propagation constant:

$$k = i\omega\sqrt{LC}\sqrt{\left(1-i\frac{R}{\omega L}\right)\left(1-i\frac{G}{\omega C}\right)} \cong i\omega\sqrt{LC}\left(1-i\frac{R}{2\omega L}-i\frac{G}{2\omega C}\right)$$

$$= \frac{R}{2}\sqrt{\frac{C}{L}} + \frac{G}{2}\sqrt{\frac{L}{C}} + i\omega\sqrt{LC}$$

$$\alpha \cong \frac{R}{2}\sqrt{\frac{C}{L}} + \frac{G}{2}\sqrt{\frac{L}{C}}, \quad \beta \cong \omega\sqrt{LC}$$

(b) Phase velocity: $v_{phase} = \dfrac{\omega}{\beta} \cong \dfrac{1}{\sqrt{LC}}$

(c) Characteristic impedance: $Z_0 = R_0 + iX_0 = \sqrt{\dfrac{R+i\omega L}{G+i\omega C}}$ →

$$Z_0 = \sqrt{\frac{L}{C}}\left(1-i\frac{R}{\omega L}\right)^{1/2}\left(1-i\frac{G}{\omega C}\right)^{-1/2} \cong \sqrt{\frac{L}{C}}\left(1-i\frac{R}{2\omega L}+i\frac{G}{2\omega C}\right), \quad R_0 = \sqrt{\frac{L}{C}},$$

$$X_0 = \frac{G}{2\omega C}\sqrt{\frac{L}{C}} - \frac{R}{2\omega}\frac{1}{\sqrt{LC}}$$

9-3 General Transmission-line equations

Three limiting cases

$$\boxed{\text{3. Distortionless Line } (R/L = G/C)}$$

(a) Propagation constant: $k = i\omega\sqrt{LC}\left(1 - i\dfrac{R}{\omega L}\right) = \sqrt{\dfrac{C}{L}}R + i\omega\sqrt{LC}$

$$\alpha \cong R\sqrt{\frac{C}{L}}, \quad \beta \cong \omega\sqrt{LC}$$

(b) Phase velocity: $v_{phase} = \dfrac{\omega}{\beta} \cong \dfrac{1}{\sqrt{LC}}$

(c) Characteristic impedance: $Z_0 = R_0 + iX_0 = \sqrt{\dfrac{R + i\omega L}{G + i\omega C}} = \sqrt{\dfrac{R}{G}} = \sqrt{\dfrac{L}{C}} \rightarrow,$

$$R_0 = \sqrt{\frac{L}{C}}, \quad X_0 = 0$$

9-3 General Transmission-line equations

Examples

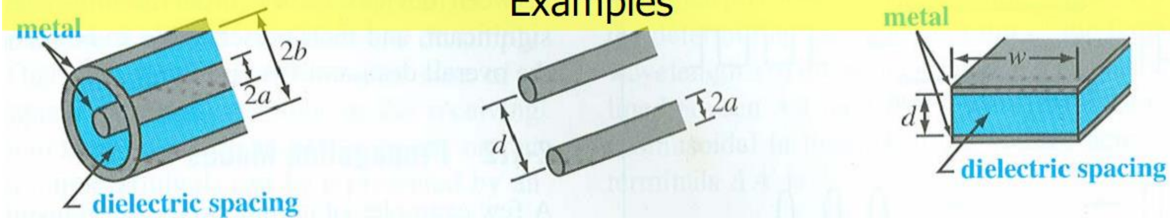
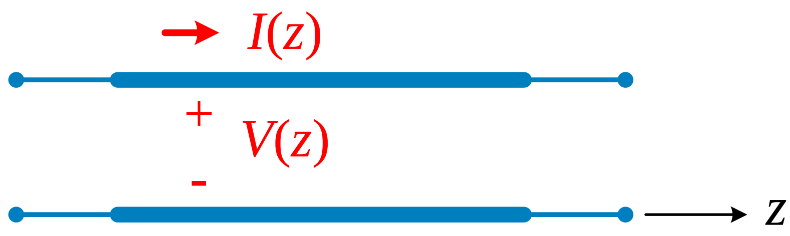


Table 2-1: Transmission-line parameters R', L', G', and C' for three types of lines.

Parameter	Coaxial	Two Wire	Parallel Plate	Unit
R'	$\dfrac{R_s}{2\pi}\left(\dfrac{1}{a}+\dfrac{1}{b}\right)$	$\dfrac{R_s}{\pi a}$	$\dfrac{2R_s}{w}$	Ω/m
L'	$\dfrac{\mu}{2\pi}\ln(b/a)$	$\dfrac{\mu}{\pi}\ln\left[(d/2a)+\sqrt{(d/2a)^2-1}\right]$	$\dfrac{\mu d}{w}$	H/m
G'	$\dfrac{2\pi\sigma}{\ln(b/a)}$	$\dfrac{\pi\sigma}{\ln\left[(d/2a)+\sqrt{(d/2a)^2-1}\right]}$	$\dfrac{\sigma w}{d}$	S/m
C'	$\dfrac{2\pi\varepsilon}{\ln(b/a)}$	$\dfrac{\pi\varepsilon}{\ln\left[(d/2a)+\sqrt{(d/2a)^2-1}\right]}$	$\dfrac{\varepsilon w}{d}$	F/m

Notes: (1) Refer to Fig. 2-4 for definitions of dimensions. (2) μ, ε, and σ pertain to the insulating material between the conductors. (3) $R_s=\sqrt{\pi f\mu_c/\sigma_c}$. (4) μ_c and σ_c pertain to the conductors. (5) If $(d/2a)^2 \gg 1$, then $\ln\left[(d/2a)+\sqrt{(d/2a)^2-1}\right] \simeq \ln(d/a)$.

Summary of Basic TL formulas



$$V(z) = V_0^+ e^{-\gamma z} + V_0^- e^{+\gamma z}$$

$$I(z) = \frac{V_0^+}{Z_0} e^{-\gamma z} - \frac{V_0^-}{Z_0} e^{+\gamma z}$$

$$\gamma = \alpha + j\beta = \left[(R + j\omega L)(G + j\omega C)\right]^{1/2}$$

$$Z_0 = \left(\frac{R + j\omega L}{G + j\omega C}\right)^{1/2}$$

guided wavelength $\equiv \lambda_g$

$$\lambda_g = \frac{2\pi}{\beta} \text{ [m]}$$

phase velocity $\equiv v_p$

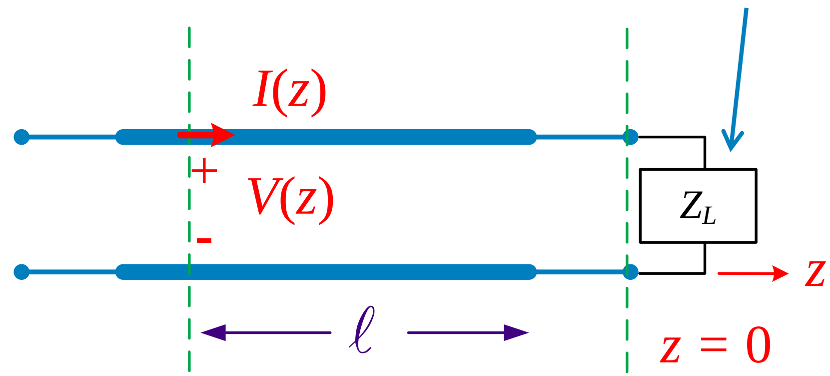
$$v_p = \frac{\omega}{\beta} \text{ [m/s]}$$

9-4 Terminated Transmission Line

$$V(z) = V_0^+ e^{-\gamma z} + V_0^- e^{+\gamma z}$$

Ampl. of voltage wave propagating in positive z direction at $z = 0$.

Ampl. of voltage wave propagating in negative z direction at $z = 0$.



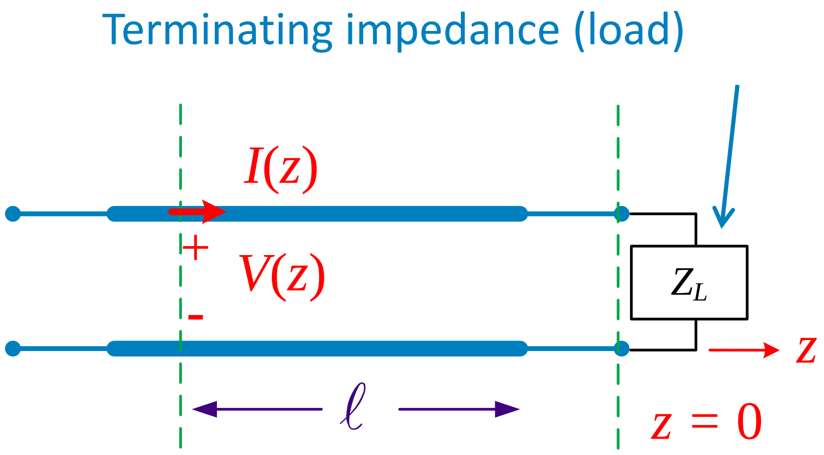
Where do we assign $z = 0$?

The usual choice is at the load.

Note: The length ℓ measures distance from the load: $\ell = -z$

9-4 Terminated Transmission Line

$$V(z) = V_0^+ e^{-\gamma z} + V_0^- e^{+\gamma z}$$



What if we know

V^+ and V^- @ $z = -\ell$

Can we use $z = -\ell$ as a reference plane?

$$V_0^+ = V^+(0) = V^+(-\ell) e^{-\gamma\ell}$$

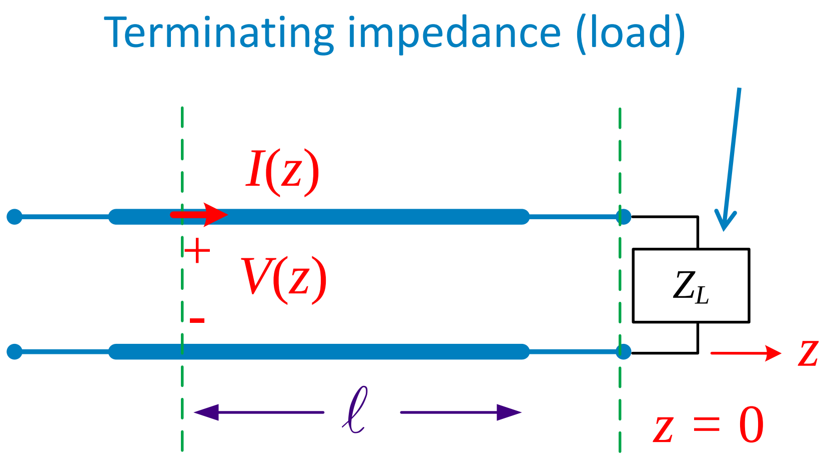
$$V^-(-\ell) = V^-(0) e^{-\gamma\ell}$$

$$\Rightarrow V_0^- = V^-(0) = V^-(-\ell) e^{\gamma\ell}$$

Hence

$$V(z) = V^+(-\ell) e^{-\gamma(z+\ell)} + V^-(-\ell) e^{\gamma(z+\ell)}$$

9-4 Terminated Transmission Line



Compare:

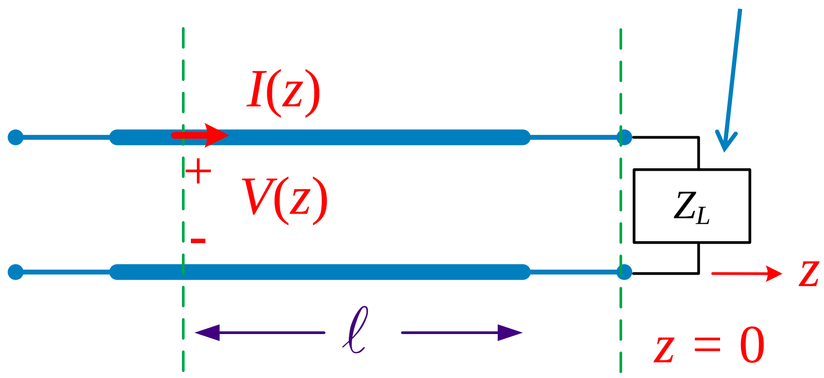
$$V(z) = V^{+}(0)e^{-\gamma z} + V^{-}(0)e^{+\gamma z}$$

$$V(z) = V^{+}(-\ell)e^{-\gamma(z-(-\ell))} + V^{-}(-\ell)e^{\gamma(z-(-\ell))}$$

Note: This is simply a change of reference plane, from $z = 0$ to $z = -\ell$.

9-4 Terminated Transmission Line

$$V(z) = V_0^+ e^{-\gamma z} + V_0^- e^{+\gamma z}$$



What is $V(-\ell)$?

$$V(-\ell) = V_0^+ e^{\gamma \ell} + V_0^- e^{-\gamma \ell}$$

propagating forwards

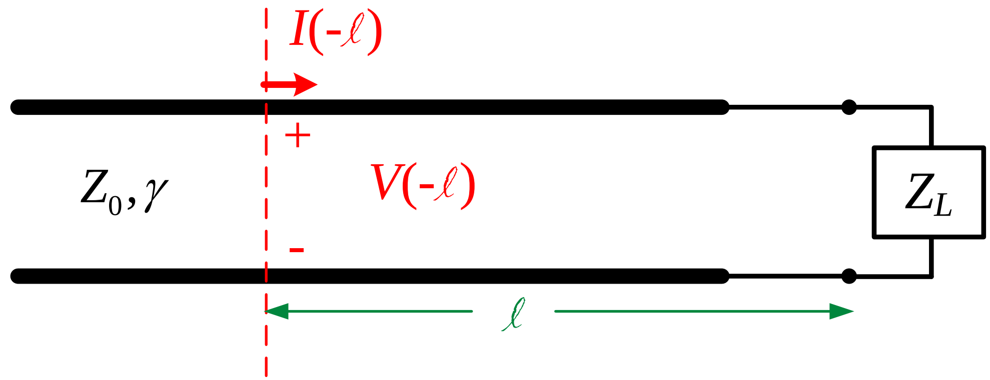
propagating backwards

The current at $z = -\ell$ is then

$$I(-\ell) = \frac{V_0^+}{Z_0} e^{\gamma \ell} - \frac{V_0^-}{Z_0} e^{-\gamma \ell}$$

$\ell \equiv$ distance away from load

9-4 Terminated Transmission Line



Total volt. at distance ℓ from the load

$$V(-\ell) = V_0^+ e^{\gamma\ell} + V_0^- e^{-\gamma\ell} = V_0^+ e^{\gamma\ell}\left(1+\frac{V_0^-}{V_0^+}e^{-2\gamma\ell}\right)$$

Ampl. of volt. wave prop. towards load, at the load position (z = 0).

Ampl. of volt. wave prop. away from load, at the load position (z = 0).

$\Gamma_L \equiv$ Load reflection coefficient

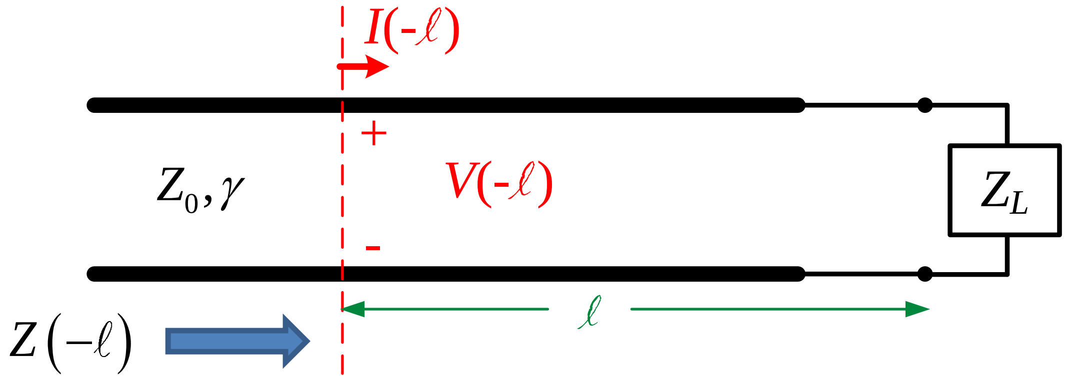
$\Gamma_\ell \equiv$ Reflection coefficient at z = - ℓ

$$= V_0^+ e^{\gamma\ell}\left(1+\Gamma_L e^{-2\gamma\ell}\right)$$

Similarly,

$$I(-\ell) = \frac{V_0^+}{Z_0} e^{\gamma\ell}\left(1-\Gamma_L e^{-2\gamma\ell}\right)$$

9-4 Terminated Transmission Line



$$V(-\ell) = V_0^+ e^{\gamma\ell}\left(1 + \Gamma_L e^{-2\gamma\ell}\right)$$

$$I(-\ell) = \frac{V_0^+}{Z_0} e^{\gamma\ell}\left(1 - \Gamma_L e^{-2\gamma\ell}\right)$$

$$Z(-\ell) = \frac{V(-\ell)}{I(-\ell)} = Z_0\left(\frac{1 + \Gamma_L e^{-2\gamma\ell}}{1 - \Gamma_L e^{-2\gamma\ell}}\right)$$

Input impedance seen “looking” towards load at $z = -\ell$.

9-4 Terminated Transmission Line

At the load ($\ell = 0$):

Voltage reflection coefficient of the load impedance

$$Z(0) = Z_0\left(\frac{1+\Gamma_L}{1-\Gamma_L}\right) \equiv Z_L \quad \Rightarrow \Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0} = |\Gamma| e^{j\theta_\Gamma}$$

Recall $$Z(-\ell) = Z_0\left(\frac{1+\Gamma_L e^{-2\gamma\ell}}{1-\Gamma_L e^{-2\gamma\ell}}\right)$$

Thus,

$$Z(-\ell) = Z_0\left(\frac{1+\left(\frac{Z_L - Z_0}{Z_L + Z_0}\right)e^{-2\gamma\ell}}{1-\left(\frac{Z_L - Z_0}{Z_L + Z_0}\right)e^{-2\gamma\ell}}\right)$$

9-4 Terminated Transmission Line

Simplifying, we have

$$Z(-\ell) = Z_0 \left(\frac{1 + \left(\dfrac{Z_L - Z_0}{Z_L + Z_0} \right) e^{-2\gamma\ell}}{1 - \left(\dfrac{Z_L - Z_0}{Z_L + Z_0} \right) e^{-2\gamma\ell}} \right) = Z_0 \left(\frac{(Z_L + Z_0) + (Z_L - Z_0) e^{-2\gamma\ell}}{(Z_L + Z_0) - (Z_L - Z_0) e^{-2\gamma\ell}} \right)$$

$$= Z_0 \left(\frac{(Z_L + Z_0) e^{+\gamma\ell} + (Z_L - Z_0) e^{-\gamma\ell}}{(Z_L + Z_0) e^{+\gamma\ell} - (Z_L - Z_0) e^{-\gamma\ell}} \right)$$

$$= Z_0 \left(\frac{Z_L \cosh(\gamma\ell) + Z_0 \sinh(\gamma\ell)}{Z_0 \cosh(\gamma\ell) + Z_L \sinh(\gamma\ell)} \right)$$

Hence, we have

$$\boxed{Z(-\ell) = Z_0 \left(\frac{Z_L + Z_0 \tanh(\gamma\ell)}{Z_0 + Z_L \tanh(\gamma\ell)} \right)}$$

9-4 Terminated Transmission Line

$$\gamma = \cancel{\alpha} + j\beta = j\beta$$

$$V(-\ell) = V_0^+ e^{j\beta\ell}\left(1 + \Gamma_L e^{-2j\beta\ell}\right)$$

$$I(-\ell) = \frac{V_0^+}{Z_0} e^{j\beta\ell}\left(1 - \Gamma_L e^{-2j\beta\ell}\right)$$

$$Z(-\ell) = Z_0\left(\frac{1 + \Gamma_L e^{-2j\beta\ell}}{1 - \Gamma_L e^{-2j\beta\ell}}\right)$$

$$\boxed{Z(-\ell) = Z_0\left(\frac{Z_L + jZ_0\tan(\beta\ell)}{Z_0 + jZ_L\tan(\beta\ell)}\right)}$$

Impedance is periodic with period $\lambda_g/2$

tan repeats when

$$\beta\ell = \pi$$

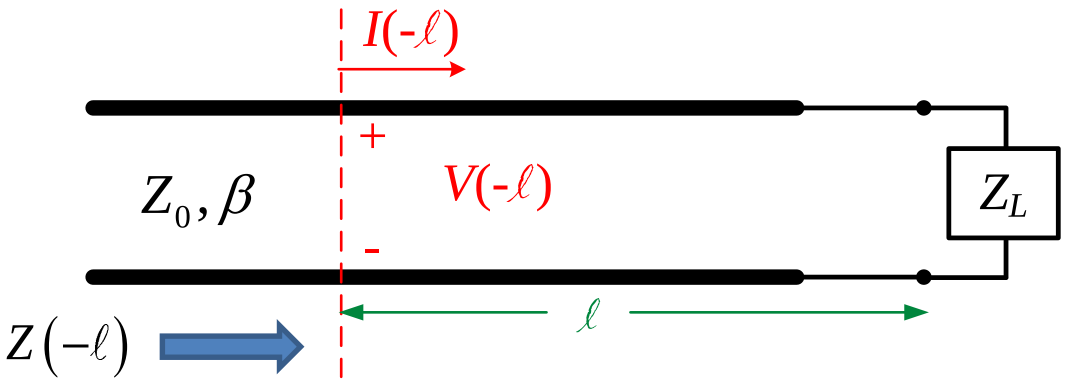
$$\frac{2\pi}{\lambda_g}\ell = \pi$$

$$\Rightarrow \ell = \lambda_g / 2$$

Note: $\tanh(\gamma\ell) = \tanh(j\beta\ell) = j\tan(\beta\ell)$

9-4 Terminated Transmission Line

For the remainder of our transmission line discussion we will assume that the transmission line is lossless.



$$V(-\ell) = V_0^+ e^{j\beta\ell}\left(1+\Gamma_L e^{-2j\beta\ell}\right)$$

$$I(-\ell) = \frac{V_0^+}{Z_0} e^{j\beta\ell}\left(1-\Gamma_L e^{-2j\beta\ell}\right)$$

$$\Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0}$$

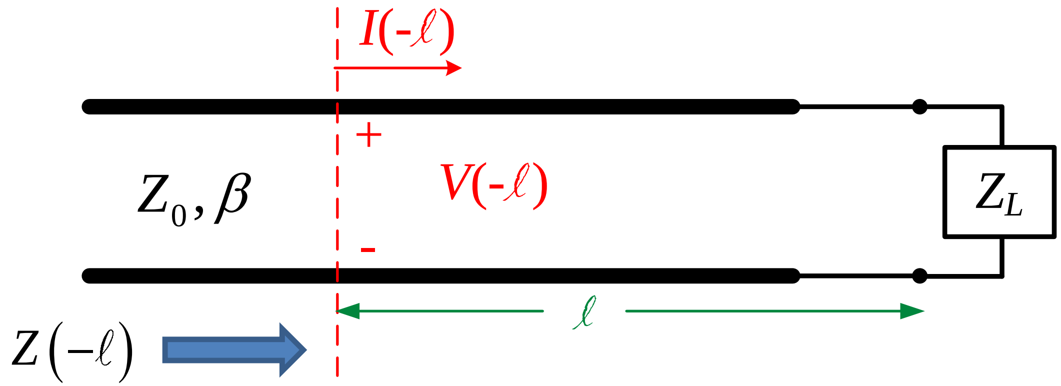
$$\lambda_g = \frac{2\pi}{\beta}$$

$$Z(-\ell) = \frac{V(-\ell)}{I(-\ell)} = Z_0\left(\frac{1+\Gamma_L e^{-2j\beta\ell}}{1-\Gamma_L e^{-2j\beta\ell}}\right)$$

$$= Z_0\left(\frac{Z_L + jZ_0\tan(\beta\ell)}{Z_0 + jZ_L\tan(\beta\ell)}\right)$$

$$v_p = \frac{\omega}{\beta}$$

9-4 Terminated Transmission Line Matched Load



Ⓐ Matched load: (Z_L=Z_0)

$$\Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0} = 0$$

No reflection from the load

$$\Rightarrow V(-\ell) = V_0^+ e^{+j\beta\ell}$$

$$I(-\ell) = \frac{V_0^+}{Z_0} e^{+j\beta\ell}$$

$$\Rightarrow Z(-\ell) = Z_0$$

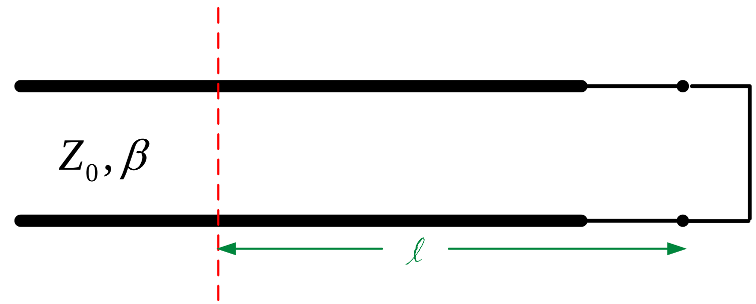
For any ℓ

9-4 Terminated Transmission Line
Short-Circuit Load

(B) Short circuit load: ($Z_L = 0$)

$$\Gamma_L = \frac{0 - Z_0}{0 + Z_0} = -1$$

$$\Rightarrow Z(-\ell) = jZ_0 \tan(\beta\ell)$$

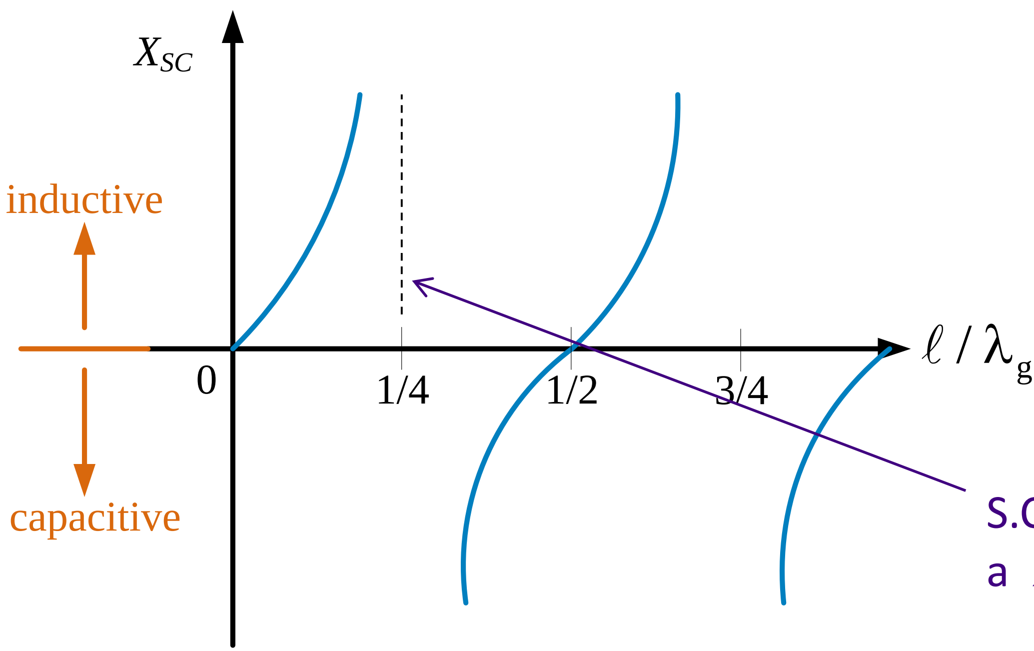


Note: $\beta\ell = 2\pi \dfrac{\ell}{\lambda_g}$

Always imaginary!

$$\Rightarrow Z(-\ell) = jX_{sc}$$

$$X_{sc} = Z_0 \tan(\beta\ell)$$

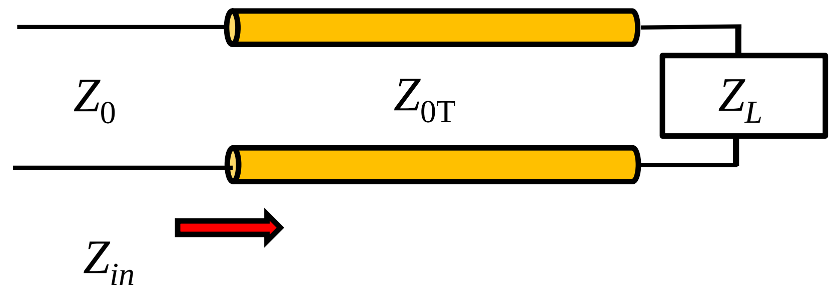


S.C. can become an O.C. with a $\lambda_g/4$ trans. line

9-4 Terminated Transmission Line

(c) Quarter-Wave Transformer

$$Z_{in} = Z_{0T}\left(\frac{Z_L + jZ_{0T}\tan\beta\ell}{Z_{0T} + jZ_L\tan\beta\ell}\right)$$



$$\beta\ell = \beta\frac{\lambda_g}{4} = \frac{2\pi}{\lambda_g}\frac{\lambda_g}{4} = \frac{\pi}{2}$$

$$\Gamma_{in} = 0 \quad \Rightarrow \quad Z_{in} = Z_0$$

$$\Rightarrow \quad Z_0 = \frac{Z_{0T}^2}{Z_L}$$

$$\Rightarrow Z_{in} = Z_{0T}\left(\frac{jZ_{0T}}{jZ_L}\right)$$

This requires Z_L to be real.

so

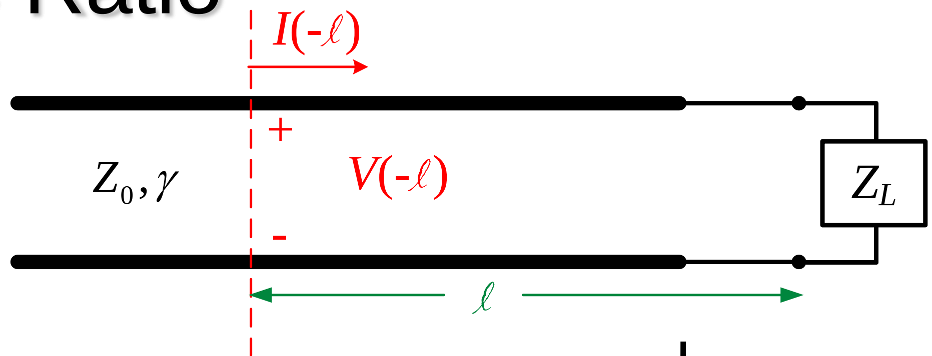
$$\boxed{Z_{in} = \frac{Z_{0T}^2}{Z_L}}$$

Hence

$$\boxed{Z_{0T} = \left[Z_0 Z_L\right]^{1/2}}$$

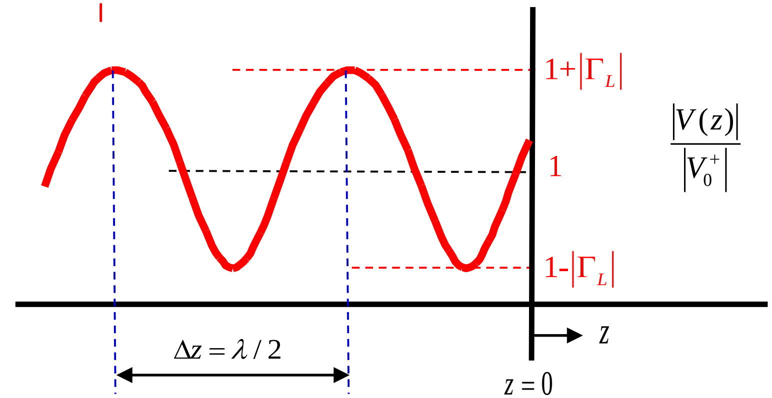
9-4 Terminated Transmission Line

Voltage Standing Wave Ratio



$$V(-\ell) = V_0^+ e^{j\beta\ell}\left(1+\Gamma_L e^{-2j\beta\ell}\right)$$
$$= V_0^+ e^{j\beta\ell}\left(1+\left|\Gamma_L\right| e^{j\phi_L} e^{-2j\beta\ell}\right)$$

$$\left|V(-\ell)\right| = \left|V_0^+\right|\left|1+\left|\Gamma_L\right| e^{j\phi_L} e^{-j2\beta\ell}\right|$$



$$V_{\max} = \left|V_0^+\right|\left(1+\left|\Gamma_L\right|\right)$$
$$V_{\min} = \left|V_0^+\right|\left(1-\left|\Gamma_L\right|\right)$$

$$\text{Voltage Standing Wave Ratio (SWR)} = \frac{V_{\max}}{V_{\min}}$$

$$S = \frac{1+\left|\Gamma_L\right|}{1-\left|\Gamma_L\right|}$$

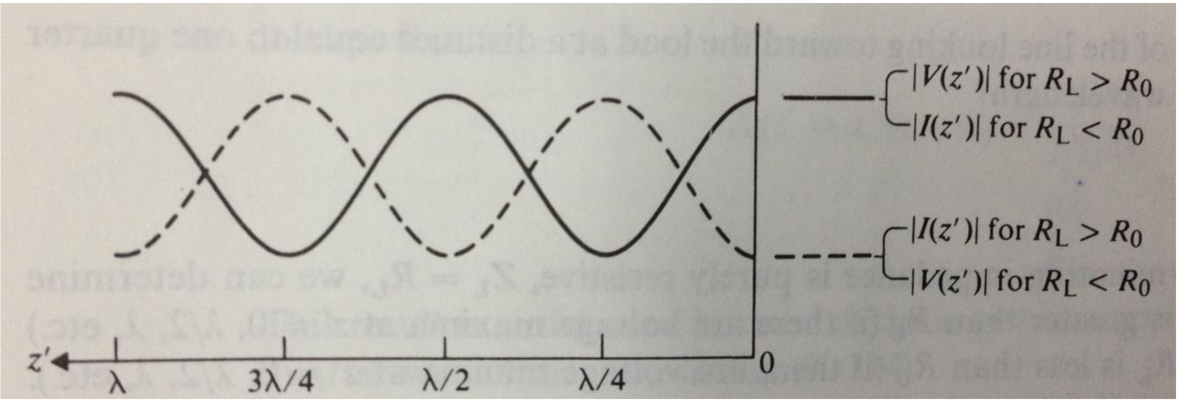
$\Gamma = 0,\quad S = 1\quad$ for $Z_L = Z_0$ matched load

$\Gamma = -1,\quad S \to \infty\quad$ for $Z_L = 0$ short circuit

$\Gamma = +1,\quad S \to \infty\quad$ for $Z_L \to \infty$ open circuit

9-4 Terminated Transmission Line

For resistance-terminated lossless lines



$$\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0} = |\Gamma| e^{j\theta_\Gamma}$$

$$Z_L = R_L \qquad Z_0 = R_0$$

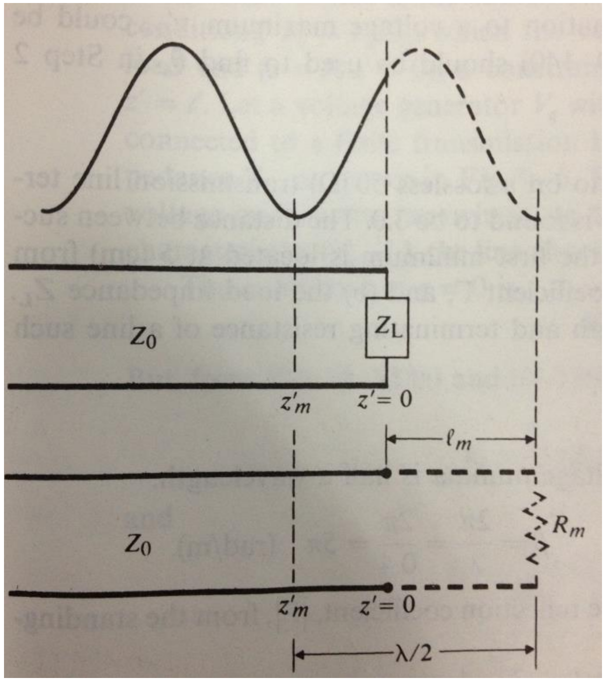
$$S = \frac{|V_{\max}|}{|V_{\min}|} = \frac{1+|\Gamma|}{1-|\Gamma|}$$

$$\boxed{R_L > R_0} \quad \theta_\Gamma = 0 \quad \boxed{S = \frac{R_L}{R_0}}$$

$$\boxed{R_L < R_0} \quad \theta_\Gamma = -\pi \quad \boxed{\frac{1}{S} = \frac{R_L}{R_0}}$$

9-4 Terminated Transmission Line

Line with arbitrary termination



$$Z_i = R_i + jX_i = R_0 \frac{R_m + jR_0 \tan \beta l_m}{R_0 + jR_m \tan \beta l_m}$$

Z_L can be determined by measuring S and the distance z'_m

$$l_m + z'_m = \lambda/2$$

Procedure

$$|\Gamma| = \frac{S-1}{S+1}$$

$$\theta_\Gamma = 2\beta z'_m - \pi$$

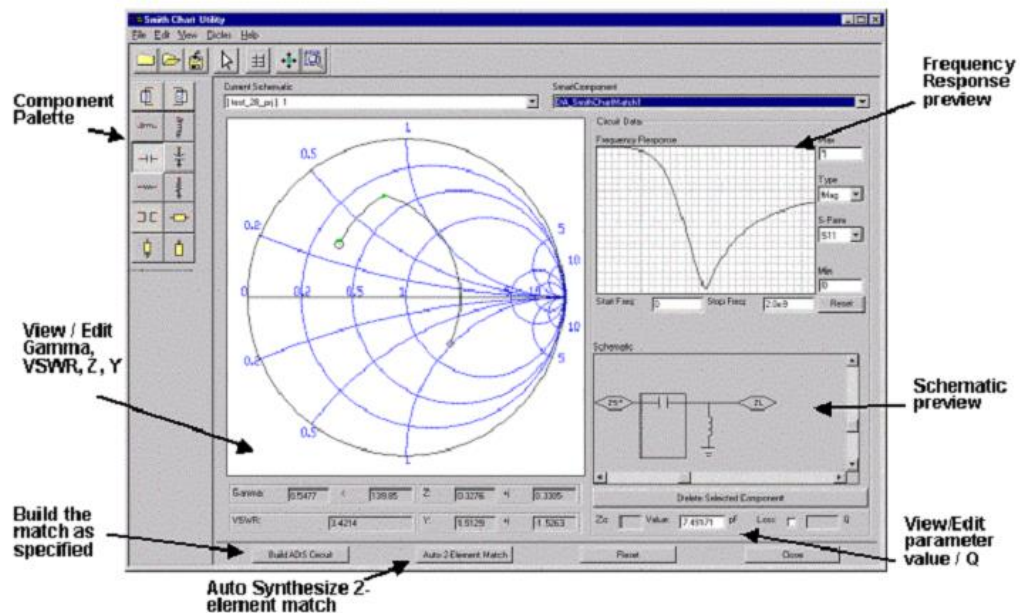
$$Z_L = R_L + jX_L = R_0 \frac{1+|\Gamma| e^{j\theta_\Gamma}}{1-|\Gamma| e^{j\theta_\Gamma}}$$

9-6 Smith Chart: Introduction

Introduction

A graphical tool used to solve transmission line problems.

Today, a presentation medium in computer-aided design (CAD) software and measuring equipment for displaying the performance of microwave circuits.



9-6 Smith-Chart

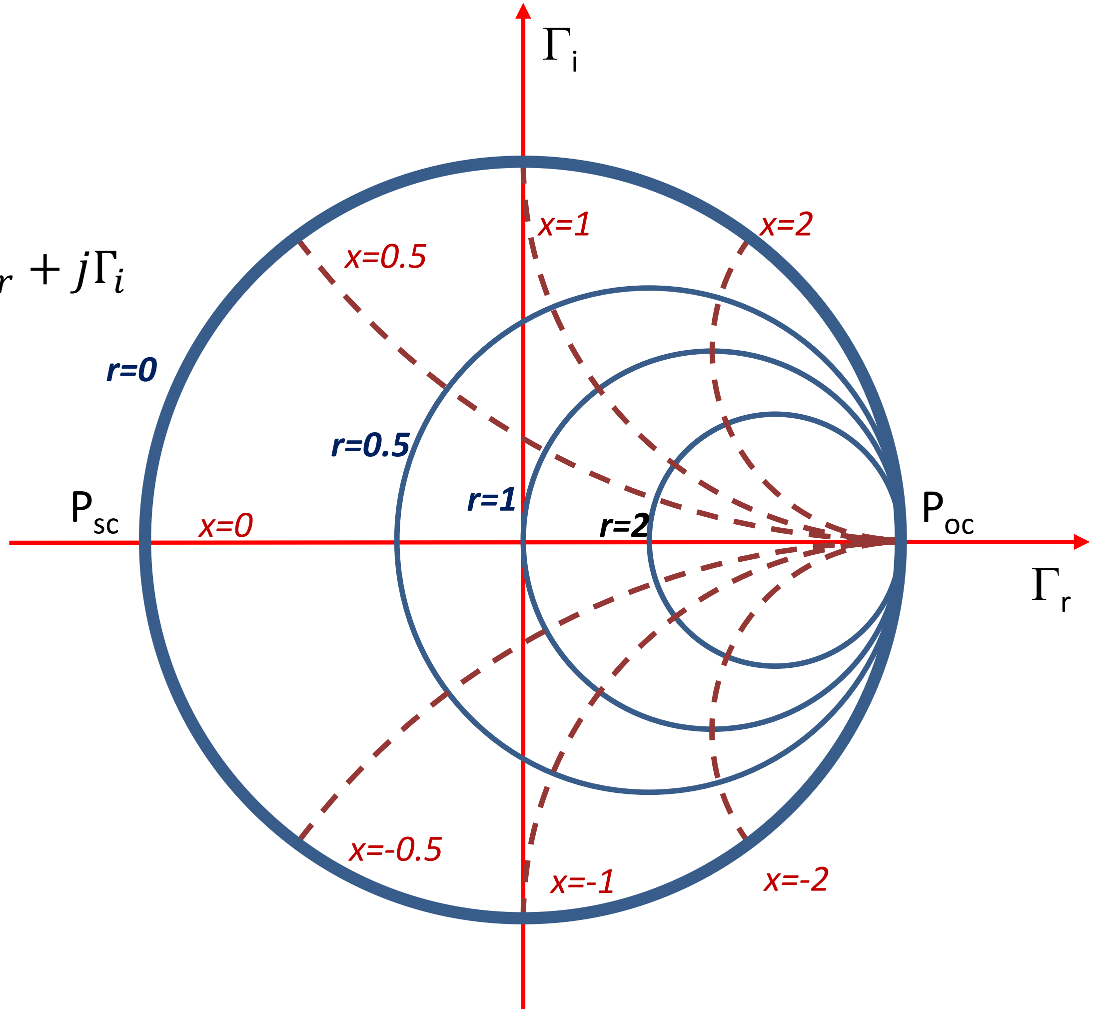
Normalized load impedance $z_L = \frac{Z_L}{R_0} = r + jx$

Voltage reflection coefficient $\Gamma = \frac{z_L - 1}{z_L + 1} = \Gamma_r + j\Gamma_i$

$$z_L = \frac{1+\Gamma}{1-\Gamma}$$

$$\left(\Gamma_r - \frac{r}{r+1}\right)^2 + {\Gamma_i}^2 = \left(\frac{1}{r+1}\right)^2$$

$$(\Gamma_r - 1)^2 + \left(\Gamma_i - \frac{1}{x}\right)^2 = \left(\frac{1}{x}\right)^2$$

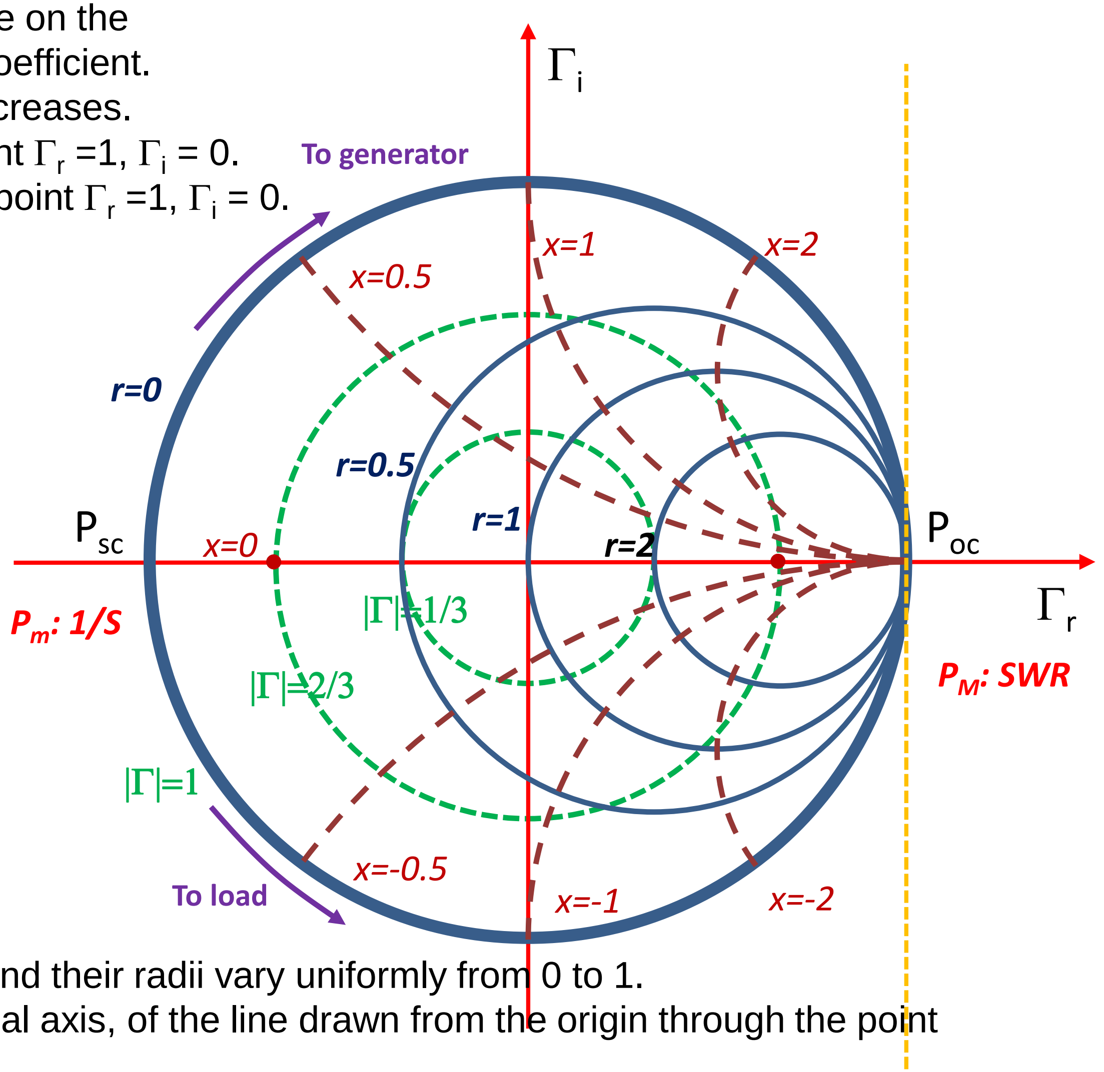


9-6 Smith-Chart

For the constant *r* circles:
1. The centers of all the constant *r* circles are on the horizontal axis – real part of the reflection coefficient.
2. The radius of circles decreases when *r* increases.
3. All constant *r* circles pass through the point $\Gamma_r = 1$, $\Gamma_i = 0$.
4. The normalized resistance $r = \infty$ is at the point $\Gamma_r = 1$, $\Gamma_i = 0$.

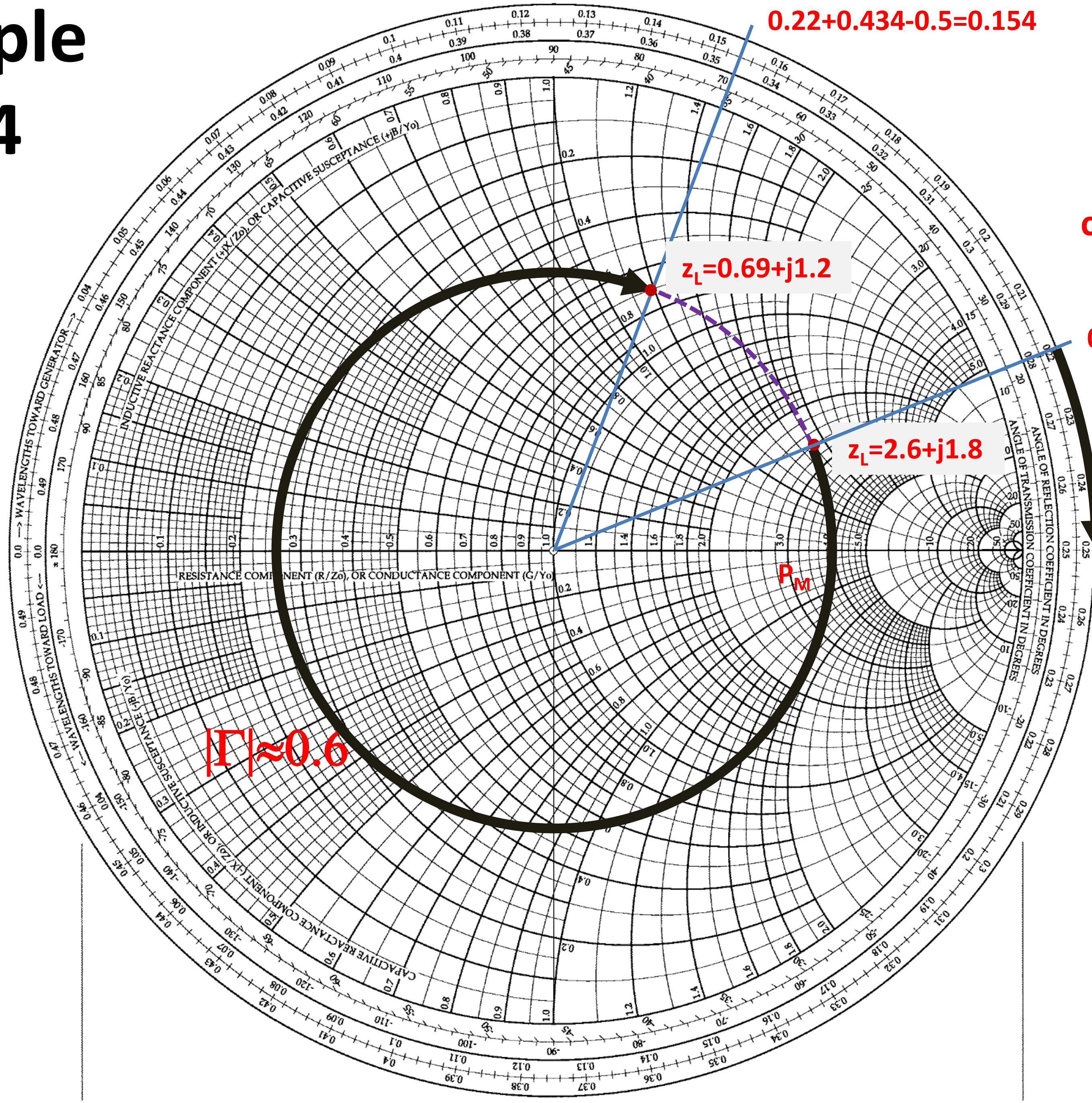
For the constant *x* (partial) circles:
1. The centers of all the constant *x* circles are on the $\Gamma_r = 1$ line. The circles with $x > 0$ (inductive reactance) are above the Γ_r axis; the circles with $x < 0$ (capacitive) are below the Γ_r axis.
2. The radius of circles decreases when absolute value of *x* increases.
3. The normalized reactances $x = \pm\infty$ are at the point $\Gamma_r = 1$, $\Gamma_i = 0$

The constant *r* circles are orthogonal to the constant *x* circles at every intersection.

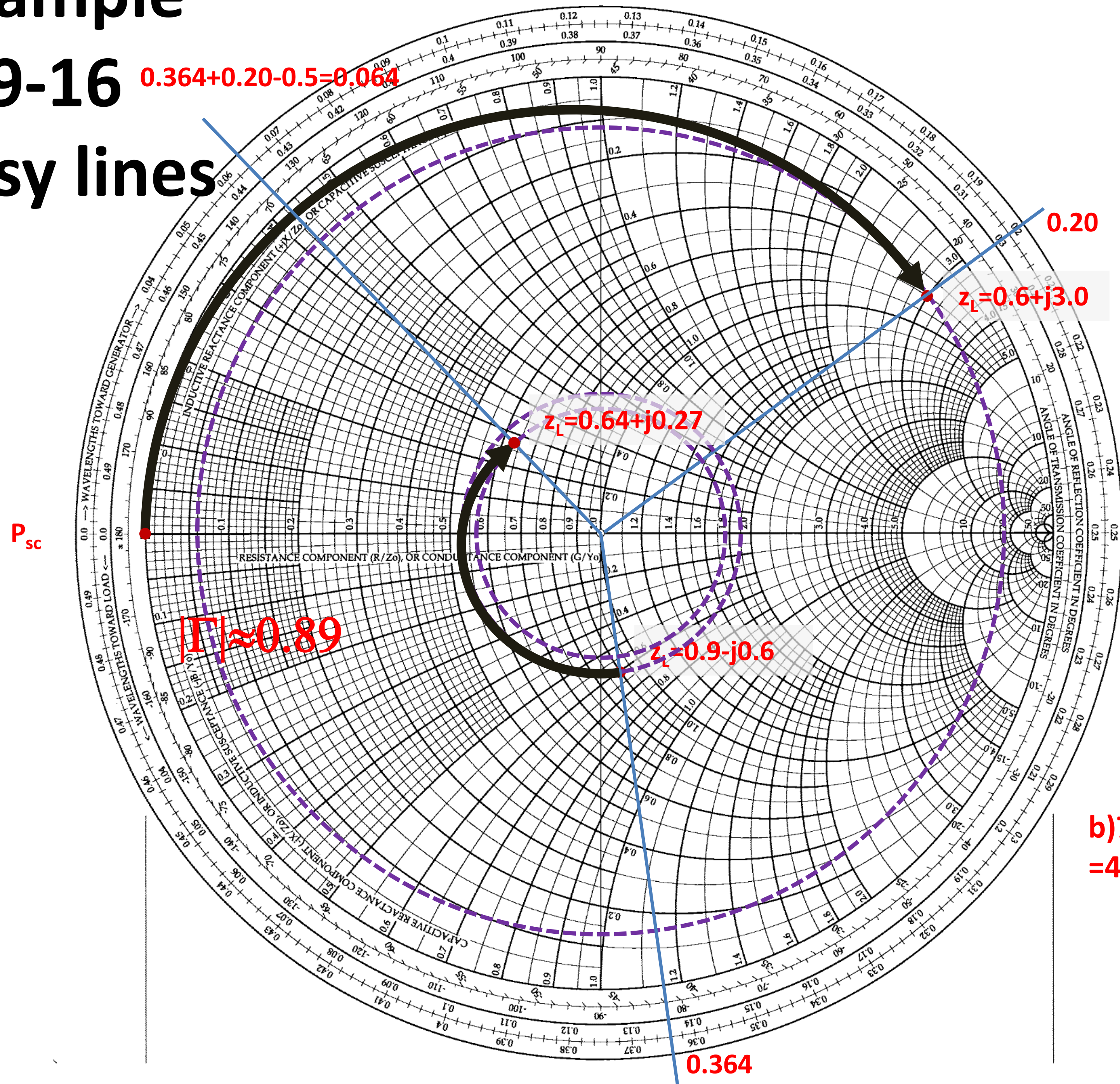


1. All $|\Gamma|$-circles are centered at the origin, and their radii vary uniformly from 0 to 1.
2. The angle, measured from the positive real axis, of the line drawn from the origin through the point representing z_L equals θ_Γ.
3. The value of the r-circle passing through the intersection of the $|\Gamma|$-circle and the positive real axis equals the standing-wave ratio S

Example 9-14


0.22+0.434-0.5=0.154
c) Z_i=69+j120
z_L=0.69+j1.2
0.22
a) Γ≈0.6∠21°
z_L=2.6+j1.8
d) 0.03λ
P_M
b) SWR=4
|Γ|≈0.6

Example 9-16 Lossy lines



a) $\alpha=0.029$

$\beta=0.2\pi$

b)75(0.64+j0.27)

=48.0+j20.3