HW1_Answer

P.3-11

Spherical symmetry: $\vec{E} = \overrightarrow{a_R} E_R$ Apply Gauss's Law （球面作为高斯面）

1) **0⩽R⩽b** $E_R = \dfrac{\rho_0}{\varepsilon_0} R(\dfrac{1}{3} - \dfrac{R^2}{5b^2})$

2) **b⩽R＜Ri** $E_R = \dfrac{2\rho_0 b^3}{15R^2 \varepsilon_0}$

3) **Ri⩽R＜R0** $E_R = 0$

4) **R＞R0** $E_R = \dfrac{2\rho_0 b^3}{15R^2 \varepsilon_0}$

P.3-12

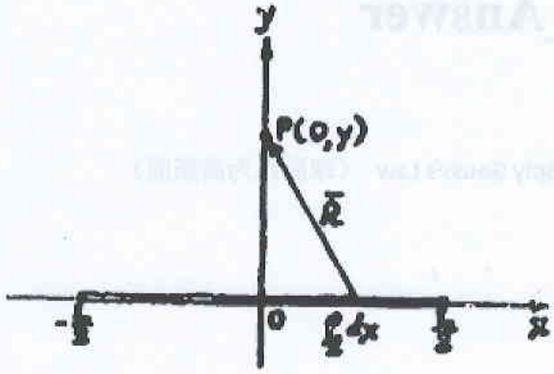
a) r<a $E=0$

a<r<b $E = \dfrac{a\rho_{sa}}{\varepsilon_0 r}$

b<r $E = \dfrac{a\rho_{sa} + b\rho_{sb}}{\varepsilon_0 r}$

b) $b = -a\dfrac{\rho_{sa}}{\rho_{sb}}$

P.3-16



a) $V = 2\int_0^{L/2} \dfrac{\rho_l dx}{4\pi\varepsilon_0\sqrt{x^2+y^2}}$ （换元得到 $\sec\theta d\theta$ 积分，进一步转换成关于 $d(\sin\theta)$ 的积分）

$$V = \frac{\rho_l}{2\pi\varepsilon_0}\ln\frac{\sqrt{L^2+4y^2}+L}{2y}$$

参考答案此处计算的是y轴上的电势与电场
实际应该计算x=0（即yz平面），所以答案只需将y改为R，R为yz平面上某点到0点的距离，即：

(a) $V = \dfrac{\rho_l}{2\pi\varepsilon_0}\ln\dfrac{\sqrt{L^2+4R^2}+L}{2R}$

b) $E = 2\int_0^{L/2}\dfrac{\rho_l y dx}{4\pi\varepsilon_0 R^3} = \dfrac{\rho_l}{2\pi\varepsilon_0 y}\dfrac{L}{\sqrt{L^2+4y^2}}$ $\qquad \overline{E} = E\overline{q}$

(b)

$$\vec{E} = \frac{\rho_l\cdot y\cdot L}{2\pi\varepsilon_0 R^2\sqrt{L^2+4R^2}}\vec{a}_y + \frac{\rho_l\cdot z\cdot L}{2\pi\varepsilon_0 R^2\sqrt{L^2+4R^2}}\vec{a}_z$$

c) $\overline{E} = -\overline{\nabla}V = \dfrac{dV}{dy}\overline{a_y}$ gives the same answer.

P.3-44

a) Region [0, x] $\qquad \overline{E} = -\overline{\nabla}V = -\overline{a_y}\dfrac{V_0}{d}$

$\overline{D} = \varepsilon\overline{E} = -\overline{a_y}\dfrac{\varepsilon_0\varepsilon_r V_0}{d}$ $\qquad$ Apply formula (3-121b) $\qquad \rho_s = \dfrac{\varepsilon_0\varepsilon_r V_0}{d}$

Region (x, l] $\qquad \overline{E} = -\overline{\nabla}V = -\overline{a_y}\dfrac{V_0}{d}$

$\overline{D} = \varepsilon\overline{E} = -\overline{a_y}\dfrac{\varepsilon_0 V_0}{d}$ $\qquad \rho_s = \dfrac{\varepsilon_0 V_0}{d}$

b) Apply formula (3-176c) $\qquad W_e = \dfrac{1}{2}\int_{V'}\varepsilon E^2 dv$

To simplify, $\varepsilon_0\varepsilon_r x = \varepsilon_0(L-x)$

We obtain $\quad x = \dfrac{L}{1+\varepsilon_r}$

P.4-1 Use subscripts d and a to denote dielectric and air regions respectively. $\nabla^2 V = 0$ in both regions.

$$V_d = c_1 y + c_2, \quad \bar{E}_d = -\bar{a}_y c_1, \quad \bar{D}_d = -\bar{a}_y \epsilon_0 \epsilon_r c_1.$$

$$V_a = c_3 y + c_4, \quad \bar{E}_a = -\bar{a}_y c_3, \quad \bar{D}_a = -\bar{a}_y \epsilon_0 c_3.$$

B.C.: At $y=0$, $V_d = 0$; at $y=d$, $V_a = V_0$;

at $y = 0.8d$: $V_d = V_a$, $\bar{D}_d = \bar{D}_a$.

Solving: $c_1 = \dfrac{V_0}{(0.8+0.2\epsilon_r)d}$, $c_2 = 0$, $c_3 = \dfrac{\epsilon_r V_0}{(0.8+0.2\epsilon_r)d}$, $c_4 = \dfrac{(1-\epsilon_r)V_0}{1+0.25\epsilon_r}$.

a) $V_d = \dfrac{5yV_0}{(4+\epsilon_r)d}$, $\quad \bar{E}_d = -\bar{a}_y \dfrac{5V_0}{(4+\epsilon_r)d}$.

b) $V_a = \dfrac{5\epsilon_r y - 4(\epsilon_r - 1)d}{(4+\epsilon_r)d} V_0$, $\quad \bar{E}_a = -\bar{a}_y \dfrac{5\epsilon_r V_0}{(4+\epsilon_r)d}$.

c) $(\rho_s)_{y=d} = -(D_a)_{y=d} = \dfrac{5\epsilon_0 \epsilon_r V_0}{(4+\epsilon_r)d}$.

$(\rho_s)_{y=0} = (D_d)_{y=0} = -\dfrac{5\epsilon_0 \epsilon_r V_0}{(4+\epsilon_r)d}$.

$\because \epsilon_r = 6 \Rightarrow$

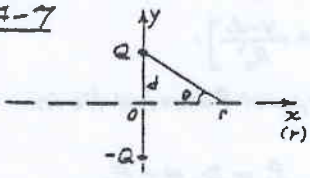
$\therefore$ a) $V_d = \dfrac{yV_0}{2d}$, $\vec{E}_d = -\vec{a}_y \cdot \dfrac{V_0}{2d}$

b) $V_a = \dfrac{3V_0}{d} y - 2V_0$, $\vec{E}_a = -\dfrac{3V_0}{d} \cdot \vec{a}$

c) $(\rho_s)_{y=d} = \dfrac{3V_0 \epsilon_0}{d}$

$(\rho_s)_{y=0} = -\dfrac{3V_0 \epsilon_0}{d}$.

P.4-7

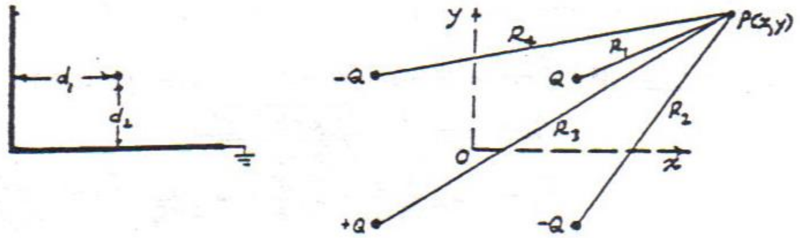


$$\bar{E}\Big|_{y=0} = -\bar{a}_y \frac{Q}{4\pi\epsilon R^2}(2\sin\theta) = -\bar{a}_y \frac{Qd}{2\pi\epsilon (d^2+r^2)^{3/2}}.$$

a) $\rho_s = \bar{a}_y \cdot \epsilon \bar{E}\Big|_{y=0} = -\dfrac{Qd}{2\pi (d^2+r^2)^{3/2}}$.

b) $\displaystyle\int_0^\infty \rho_s \, 2\pi r \, dr = -Q$.

P.4-8



Consider the conditions in the xy-plane $(z=0)$.

a) $V_P = \frac{Q}{4\pi\epsilon}\left(\frac{1}{R_1} - \frac{1}{R_2} + \frac{1}{R_3} - \frac{1}{R_4}\right)$, where

$$R_1 = [(x-d_1)^2 + (y-d_2)^2]^{1/2}, \qquad R_2 = [(x-d_1)^2 + (y+d_2)^2]^{1/2},$$
$$R_3 = [(x+d_1)^2 + (y+d_2)^2]^{1/2}, \qquad R_4 = [(x+d_1)^2 + (y-d_2)^2]^{1/2}.$$

$$\bar{E}_P = -\bar{\nabla} V_P = -\bar{a}_x \frac{\partial V_P}{\partial x} - \bar{a}_y \frac{\partial V_P}{\partial y}$$
$$= \bar{a}_x \frac{Q}{4\pi\epsilon}\left[-\frac{x-d_1}{R_1^3} + \frac{x-d_1}{R_2^3} - \frac{x+d_1}{R_3^3} + \frac{x+d_1}{R_4^3}\right]$$
$$+ \bar{a}_y \frac{Q}{4\pi\epsilon}\left[-\frac{y-d_2}{R_1^3} + \frac{y+d_2}{R_2^3} - \frac{y+d_2}{R_3^3} + \frac{y-d_2}{R_4^3}\right].$$

E_P will have a z-component if the point P does not lie in the xy-plane.

b) On the conducting half-planes, $\rho_s = D_n = \epsilon E_n$.
Along the x-axis, $y=0$: $R_1 = [(x-d_1)^2 + d_2^2]^{1/2} = R_2$,
and $R_3 = [(x+d_1)^2 + d_2^2]^{1/2} = R_4$.

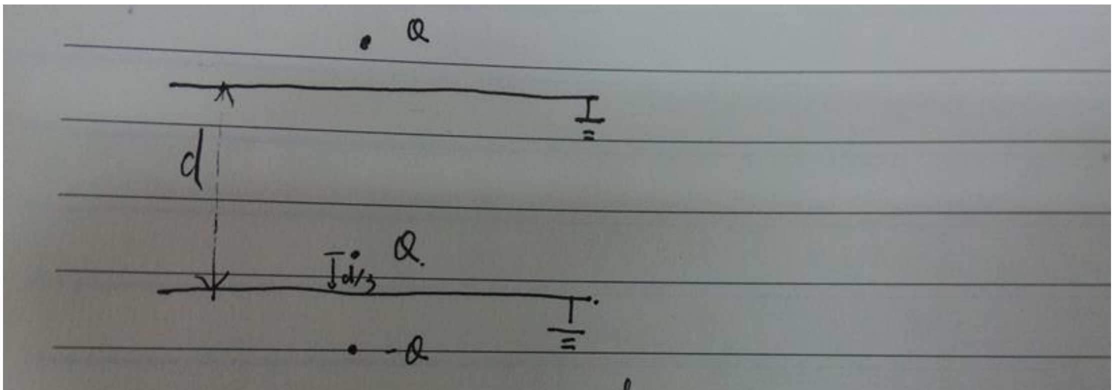
$$E_x = 0, \quad E_y = \frac{Q}{2\pi\epsilon}\left[\frac{d_2}{R_1^3} - \frac{d_2}{R_3^3}\right].$$

$$\therefore \rho_s(y=0) = \frac{Qd_2}{2\pi}\left\{\frac{1}{[(x-d_1)^2+d_2^2]^{3/2}} - \frac{1}{[(x+d_1)^2+d_2^2]^{3/2}}\right\}$$
$$= \begin{cases} 0, & \text{at } x=0. \\ \text{max.}, & \text{at } x=d_1. \end{cases}$$

Similarly for $\rho_s(x=0)$ on the vertical conducting conducting plane by changing x to y and $d_1 \leftrightarrow d_2$.

P4-9

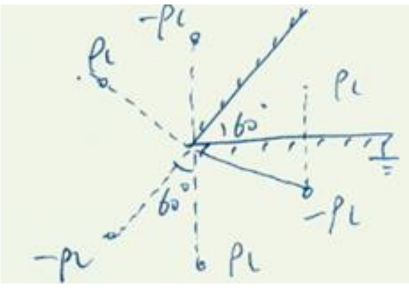
a)



$-Q$ at $y=(5+6n)\cdot\frac{d}{3}$ $\quad n=0,\pm1,\pm2\ldots$

Q at $y=(1+6n)\cdot\frac{d}{3}$ $\quad n=\pm1,\pm2\ldots$

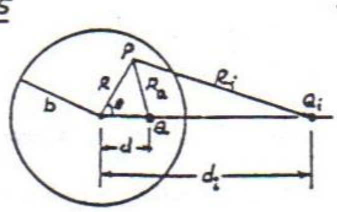
b)



ρ_l at $\theta=150°, -90°$

$-\rho_l$ at $\theta=90°, -30°, -150°$

P.4-15



$Q_i=-\frac{b}{d}Q, \qquad d_i=\frac{b^2}{d}.$

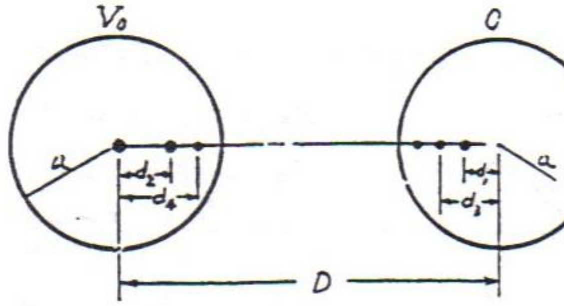
a) $V_P=\frac{Q}{4\pi\epsilon_0}\left(\frac{1}{R_Q}-\frac{b}{dR_i}\right);$

$R_Q=(R^2+d^2-2Rd\cos\theta)^{1/2},$

$R_i=(R^2+d_i^2-2Rd_i\cos\theta)^{1/2}.$

b) $\rho_s=-\epsilon_0\frac{\partial V}{\partial R}\Big|_{R=b}=-\frac{Q(b^2-d^2)}{4\pi b(b^2+d^2-2bd\cos\theta)^{3/2}}.$

P. 4-16



a)
Q_0 and system of image charges:

In left sphere	In right sphere
Q_0 at $d_0 = 0$.	$-Q_1 = -\frac{a}{D}Q_0$ at d_1.
$Q_2 = \frac{a^2}{D(D-d_1)}Q_0$ at d_2.	$-Q_3 = -\frac{a}{D-d_2}Q_2 = -\frac{a^3}{D(D-d_1)(D-d_2)}Q_0$ at d_3.
$Q_4 = \frac{a^4}{D(D-d_1)(D-d_2)(D-d_3)}Q_0$ at d_4.	$\vdots$
$\vdots$	
$Q_{Ln} = Q_0 \prod_{m=1}^{n} \frac{a}{D-d_{m-1}}$ at d_n. $(n = 2, 4, 6, \cdots)$	$-Q_{Rn} = -Q_0 \prod_{m=1}^{n} \frac{a}{D-d_{m-1}}$ at d_n. $(n = 1, 3, 5, \cdots)$

$$d_m = \frac{a^2}{D-d_{m-1}}, \quad m = 1, 2, 3, \cdots ; \; d_0 = 0.$$

b) $$C = \frac{Q_0 + \sum_n Q_{Ln}}{V_0} = 4\pi\epsilon_0 a\left[1 + \sum_{n=2,4,\cdots}\left(\prod_{m=1}^{n} \frac{a}{D-d_{m-1}}\right)\right].$$

P. 4-17 Required boundary conditions at $x=0$: $V_1 = V_2$, and $\epsilon_1\frac{\partial V_1}{\partial x} = \epsilon_2\frac{\partial V_2}{\partial x}$.
From Fig. 4-23 and the hypotheses in parts a) and b):

$$V_1 = \frac{Q}{4\pi\epsilon_1\sqrt{(x-d)^2+y^2+z^2}} - \frac{Q_1}{4\pi\epsilon_1\sqrt{(x+d)^2+y^2+z^2}},$$

$$V_2 = \frac{Q+Q_2}{4\pi\epsilon_2\sqrt{(d-x)^2+y^2+z^2}}.$$

In order to satisfy the b.c.'s at $x=0$, we require

$$\frac{Q-Q_1}{\epsilon_1} = \frac{Q+Q_2}{\epsilon_2} \text{ and } Q+Q_1 = Q+Q_2 \longrightarrow Q_1 = Q_2 = \frac{\epsilon_2-\epsilon_1}{\epsilon_2+\epsilon_1}Q.$$

P. 4-21 $$V(x,y) = \sum_n \sin\frac{n\pi}{a}x\left[A_n \sinh\frac{n\pi}{a}y + B_n\cosh\frac{n\pi}{a}y\right].$$

At $y=0$, $V(x,0) = V_2 = \sum_n B_n \sin\frac{n\pi}{a}x \longrightarrow B_n = \begin{cases}\frac{4V_2}{n\pi}, & n = \text{odd}.\\ 0, & n = \text{even}.\end{cases}$

At $y=b$, $V(x,b) = V_1 = \sum_n \sin\frac{n\pi}{a}x\left[A_n \sinh\frac{n\pi}{a}b + B_n\cosh\frac{n\pi}{a}b\right]$

$$\longrightarrow A_n \sinh\frac{n\pi}{a}b + B_n\cosh\frac{n\pi}{a}b = \begin{cases}\frac{4V_1}{n\pi}, & n = \text{odd}.\\ 0, & n = \text{even}.\end{cases}$$

$$\therefore A_n = \begin{cases}\frac{4}{n\pi \sinh(n\pi b/a)}\left(V_1 - V_2\cosh\frac{n\pi}{a}b\right), & n = \text{odd}.\\ 0, & n = \text{even}.\end{cases}$$

P.4-23 Solution: $V(\phi) = A_0\phi + B_0$.

a) B.C. ①: $V(0) = 0 \longrightarrow B_0 = 0$.

B.C. ②: $V(\alpha) = V_0 = A_0\alpha \longrightarrow A_0 = \frac{V_0}{\alpha}$.

$$\left.\begin{matrix} \\ \end{matrix}\right\} \therefore V(\phi) = \frac{V_0}{\alpha}\phi, \quad 0 \le \phi \le \alpha.$$

b) B.C. ①: $V(\alpha) = V_0 = A_1\alpha + B_1$

B.C. ②: $V(2\pi) = 0 = 2\pi A_1 + B$

$$\left.\begin{matrix} \\ \end{matrix}\right\} \longrightarrow A_1 = -\frac{V_0}{2\pi-\alpha}, \quad B_1 = \frac{2\pi V_0}{2\pi-\alpha}.$$

$$\therefore V(\phi) = \frac{V_0}{2\pi-\alpha}(2\pi-\phi), \quad \alpha \le \phi \le 2\pi.$$

P.5-6 $\rho_0 = \frac{Q_0}{(4\pi/3)b^3} = \frac{10^{-3}}{(4\pi/3)(0.1)^3} = 0.239\ (C/m^3)$, $\rho = \rho_0 e^{-(\sigma/\epsilon)t}$.

a) $R < b$: $\bar{E}_i = \bar{a}_R \frac{(4\pi/3)R^3\rho}{4\pi\epsilon R^2} = \bar{a}_R \frac{\rho_0 R}{3\epsilon} e^{-(\sigma/\epsilon)t} = \bar{a}_R\, 7.5\times10^{9} R\, e^{-9.42\times10^{11}t}$ (V/m)

$R > b$: $\bar{E}_o = \bar{a}_R \frac{Q_0}{4\pi\epsilon_0 R^2} = \bar{a}_R \frac{9}{R^2}\times10^{6}$ (V/m).

b) $R < b$: $\bar{J}_i = \sigma\bar{E}_i = \bar{a}_R\, 7.5\times10^{10} R\, e^{-9.42\times10^{11}t}$

$R > b$: $\bar{J}_o = 0$.

P.5-9 a) Eq. (3-118): $E_{1t} = E_{2t} \longrightarrow E_2\sin\alpha_2 = E_1\sin\alpha_1$.

Eq. (5-58): $J_{1n} = J_{2n} \longrightarrow \sigma_1 E_{1n} = \sigma_2 E_{2n} \longrightarrow \sigma_2 E_2\cos\alpha_2 = \sigma_1 E_1\cos\alpha_1$.

$$\therefore\ E_2 = E_1\sqrt{\sin^2\alpha_1 + \left(\frac{\sigma_1}{\sigma_2}\cos\alpha_1\right)^2}. \qquad ①$$

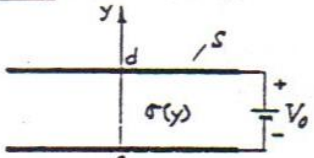
$$\tan\alpha_2 = \frac{\sigma_2}{\sigma_1}\tan\alpha_1 \longrightarrow \alpha_2 = \tan^{-1}\left(\frac{\sigma_2}{\sigma_1}\tan\alpha_1\right). \qquad ②$$

b) Eq. (3-121b): $D_{2n} - D_{1n} = \rho_s \longrightarrow \epsilon_2 E_{2n} - \epsilon_1 E_{1n} = \rho_s$.

$$\rho_s = \left(\frac{\sigma_1}{\sigma_2}\epsilon_2 - \epsilon_1\right)E_{1n} = \left(\frac{\sigma_1}{\sigma_2}\epsilon_2 - \epsilon_1\right)E_1\cos\alpha_1.$$

c) If both media are perfect dielectrics, $\sigma_1 = \sigma_2 = 0$, Eqs. ① and ② revert to Eqs. (3-130) and (3-129) respectively and $\rho_s = 0$.

P.5-10



$$\sigma(y) = \sigma_1 + (\sigma_2 - \sigma_1)\frac{y}{d}$$

a) Neglecting fringing effect and assuming a current density $\bar{J} = -\bar{a}_y J_0 \longrightarrow \bar{E} = \frac{\bar{J}}{\sigma} = -\bar{a}_y \frac{J_0}{\sigma(y)}$.

$$V_0 = -\int_0^d \bar{E}\cdot\bar{a}_y\, dy = \int_0^d \frac{J_0\, dy}{\sigma_1 + (\sigma_2-\sigma_1)\frac{y}{d}} = \frac{J_0 d}{\sigma_2 - \sigma_1}\ln\frac{\sigma_2}{\sigma_1}.$$

$$R = \frac{V_0}{I} = \frac{V_0}{J_0 S} = \frac{d}{(\sigma_2-\sigma_1)S}\ln\frac{\sigma_2}{\sigma_1}.$$

b) $(\rho_s)_u = \epsilon_0 E_y(d) = \frac{\epsilon_0 J_0}{\sigma_2} = \frac{\epsilon_0(\sigma_2-\sigma_1)V_0}{\sigma_2 d\ln(\sigma_2/\sigma_1)}$ on upper plate,

$(\rho_s)_l = -\epsilon_0 E_y(0) = -\frac{\epsilon_0 J_0}{\sigma_1} = -\frac{\epsilon_0(\sigma_2-\sigma_1)V_0}{\sigma_1 d\ln(\sigma_2/\sigma_1)}$ on lower plate.

c) $\rho = \bar{\nabla}\cdot\bar{D} = \frac{d}{dy}(\epsilon_0 E) = -\epsilon_0 J_0 \frac{d}{dy}\left[\frac{1}{\sigma_1 + (\sigma_2-\sigma_1)y/d}\right] = \epsilon_0 J_0 \frac{(\sigma_2-\sigma_1)/d}{[\sigma_1 + (\sigma_2-\sigma_1)y/d]^2}$.

P.5-12 Refer to Fig. 5-6. In the transient state, the equation of continuity must be satisfied at the interface.

$$-\frac{\partial\rho_{si}}{\partial t} = J_2 - J_1 = \sigma_2 E_2 - \sigma_1 E_1 \qquad ①$$

Now

$$E_1 d_1 + E_2 d_2 = V \qquad ②$$

$$\epsilon_2 E_2 - \epsilon_1 E_1 = \rho_{si} \qquad ③$$

Solving ② and ③ for E_1 and E_2 in terms of $\mathcal{V}$ and ρ_{si}:

$$E_1 = \frac{\epsilon_2\mathcal{V} - d_2\rho_{si}}{\epsilon_2 d_1 + \epsilon_1 d_2} \quad ④; \qquad E_2 = \frac{\epsilon_1\mathcal{V} + d_1\rho_{si}}{\epsilon_2 d_1 + \epsilon_1 d_2} \quad ⑤.$$

a) Substituting ④ and ⑤ in ①:

$$-\frac{\partial\rho_{si}}{\partial t} = \frac{\sigma_2 d_1 + \sigma_1 d_2}{\epsilon_2 d_1 + \epsilon_1 d_2}\rho_{si} + \frac{\epsilon_1\sigma_2 - \epsilon_2\sigma_1}{\epsilon_2 d_1 + \epsilon_1 d_2}\mathcal{V}. \qquad ⑥$$

Solution of ⑥:

$$\rho_{si} = \left(\frac{\epsilon_2\sigma_1 - \epsilon_1\sigma_2}{\sigma_2 d_1 + \sigma_1 d_2}\right)\mathcal{V}\left[1 - e^{-t/\tau}\right]. \qquad ⑦$$

where $\tau = \text{Relaxation time} = \frac{\epsilon_2 d_1 + \epsilon_1 d_2}{\sigma_2 d_1 + \sigma_1 d_2}$. ⑧

b) Using ④ and ⑤:

$$E_1 = \frac{\sigma_2\mathcal{V}}{\sigma_2 d_1 + \sigma_1 d_2}\left(1 - e^{-t/\tau}\right) + \frac{\epsilon_2\mathcal{V}}{\epsilon_2 d_1 + \epsilon_1 d_2}e^{-t/\tau};$$

$$E_2 = \frac{\sigma_1\mathcal{V}}{\sigma_2 d_1 + \sigma_1 d_2}\left(1 - e^{-t/\tau}\right) + \frac{\epsilon_1\mathcal{V}}{\epsilon_2 d_1 + \epsilon_1 d_2}e^{-t/\tau}.$$

<u>P.5-14</u> $\quad \nabla^2 V = 0 \longrightarrow \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial V}{\partial r}\right) = 0.$

Solution: $V(r) = c_1 \ln r + c_2$.

Boundary conditions: $V(a) = V_0$; $\quad V(b) = 0$.

$$\longrightarrow V(r) = V_0\frac{\ln(b/r)}{\ln(b/a)}.$$

$$\bar{E}(r) = -\bar{a}_r\frac{\partial V}{\partial r} = \bar{a}_r\frac{V_0}{r\ln(b/a)}.$$

$$\bar{J}(r) = \sigma\bar{E}(r).$$

$$I = \int_S \bar{J}\cdot d\bar{s} = \int_0^{\pi/2}\bar{J}\cdot(\bar{a}_r h r\,d\phi) = \frac{\pi\sigma h V_0}{2\ln(b/a)}.$$

$$R = \frac{V_0}{I} = \frac{2\ln(b/a)}{\pi\sigma h}.$$

<u>P.5-17</u> Assume I. $\quad \bar{J}(R) = \bar{a}_R\frac{I}{S(R)}$.

$$S(R) = \int_0^{2\pi}\int_0^{\theta_0} R^2\sin\theta\,d\theta\,d\phi = 2\pi R^2(1-\cos\theta_0).$$

$$\bar{E}(R) = \frac{1}{\sigma}\bar{J}(R) = \bar{a}_R\frac{I}{2\pi\sigma R^2(1-\cos\theta_0)}.$$

$$V_0 = -\int_{R_2}^{R_1} E(R)\,dR = \frac{I(R_2-R_1)}{2\pi\sigma R_1R_2(1-\cos\theta_0)}.$$

$$R = \frac{V_0}{I} = \frac{1}{2\pi(1-\cos\theta_0)}\left(\frac{1}{R_1} - \frac{1}{R_2}\right).$$

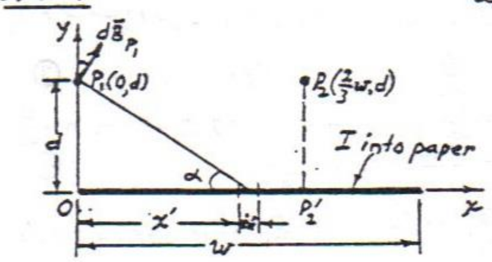
P.6-3 Application of Ampère's circuital law.

$$0 \le r \le a, \quad \bar{B} = \bar{a}_\phi \frac{\mu_0 r I}{2\pi a^2}.$$

$$a \le r \le b, \quad \bar{B} = \bar{a}_\phi \frac{\mu I}{2\pi r}.$$

$$b \le r \le c, \quad \bar{B} = \bar{a}_\phi \left(\frac{c^2 - r^2}{c^2 - b^2}\right) \frac{\mu I}{2\pi r}.$$

P.6-4



a) Using Eq. (6-35c):

$$d\bar{B}_{P_1} = \bar{a}_x\, dB_x + \bar{a}_y\, dB_y$$
$$= \bar{a}_x (dB_1)\sin\alpha + \bar{a}_y (dB_1)\cos\alpha,$$
$$dB_{P_1} = \frac{\mu_0 (I/w)\, dx'}{2\pi (x'^2 + d^2)^{1/2}},$$
$$\sin\alpha = \frac{d}{(x'^2+d^2)^{1/2}}, \quad \cos\alpha = \frac{x'}{(x'^2+d^2)^{1/2}}.$$
$$\therefore \bar{B}_{P_1} = \bar{a}_x B_x + \bar{a}_y B_y,$$

where $B_x = \dfrac{\mu_0 I d}{2\pi w}\displaystyle\int_0^w \frac{dx'}{x'^2+d^2} = \frac{\mu_0 I}{2\pi w}\tan^{-1}\left(\frac{w}{d}\right),$

$$B_y = \frac{\mu_0 I}{2\pi w}\int_0^w \frac{x'\,dx'}{x'^2+d^2} = \frac{\mu_0 I}{4\pi w}\ln\left(1+\frac{w^2}{d^2}\right).$$

b) To find B at $P_2(\frac{2}{3}w, d)$, we add vectorially the contributions of the current strips to the right and to the left of point P_2' using the result in part (a).

$$\bar{B}_{P_2} = \bar{B}_{2R} + \bar{B}_{2L}.$$
$$\bar{B}_{2R} = \frac{\mu_0 I}{2\pi w}\left[\bar{a}_x \tan^{-1}\left(\frac{w}{3d}\right) + \bar{a}_y \frac{1}{2}\ln\left(1+\frac{w^2}{9d^2}\right)\right].$$
$$\bar{B}_{2L} = \frac{\mu_0 I}{2\pi w}\left[\bar{a}_x \tan^{-1}\left(\frac{2w}{3d}\right) - \bar{a}_y \frac{1}{2}\ln\left(1+\frac{4w^2}{9d^2}\right)\right].$$
$$\therefore \bar{B}_{P_2} = \frac{\mu_0 I}{2\pi w}\left[\bar{a}_x\left(\tan^{-1}\frac{w}{3d} + \tan^{-1}\frac{2w}{3d}\right) - \bar{a}_y \ln\sqrt{\frac{1+(2w/3d)^2}{1+(w/3d)^2}}\right].$$

P.6-6 The problem can be decomposed into two subproblems (assuming b = radius of solenoid):

1. A cylindrical tube carrying a uniformly distributed longitudinal surface current $2\pi b n I \sin\alpha$.

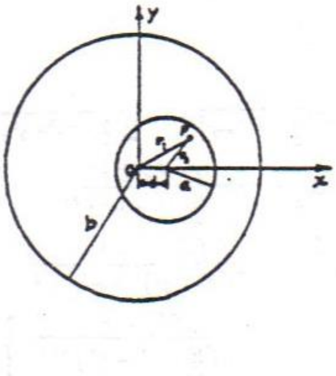
$$\longrightarrow \bar{B}_1 = \begin{cases} 0, & 0<r<b, \\ \bar{a}_\phi \dfrac{bnI}{r}\sin\alpha, & r>b. \end{cases}$$

2. A solenoid with n turns per unit length carrying a current $I\cos\alpha$.

$$\longrightarrow \bar{B}_2 = \begin{cases} \bar{a}_z \mu_0 n I \cos\alpha, & 0<r<b, \\ 0, & r>b. \end{cases}$$

Total $\bar{B} = \bar{B}_1 + \bar{B}_2$.

P.6-15 $\bar{J} = \bar{a}_z J.$ $\oint \bar{B}\cdot d\bar{\ell} = \mu_0 I.$



If there is no hole,

$$2\pi r_1 B_{\phi 1} = \mu_0 \pi r_1^2 J$$

$$\longrightarrow B_{\phi 1} = \frac{\mu_0 r_1}{2} J \longrightarrow \begin{cases} B_{x1} = -\frac{\mu_0 J}{2} y_1, \\ B_{y1} = +\frac{\mu_0 J}{2} x_1. \end{cases}$$

For $-\bar{J}$ in the hole portion:

$$B_{\phi 2} = -\frac{\mu_0 r_2}{2} J \longrightarrow \begin{cases} B_{x2} = +\frac{\mu_0 J}{2} y_2. \\ B_{y2} = -\frac{\mu_0 J}{2} x_2. \end{cases}$$

Superposing $B_{\phi 1}$ and $B_{\phi 2}$ and noting that $y_1 = y_2$ and $x_1 = x_2 + d$,

we have $B_x = B_{x1} + B_{x2} = 0$, and $B_y = B_{y1} + B_{y2} = \frac{\mu_0 J}{2} d.$

P.6-18 Eq. (6-34) for one wire: $\bar{A} = \bar{a}_z \frac{\mu_0 I}{4\pi} \ln \frac{\sqrt{L^2+r^2}+L}{\sqrt{L^2+r^2}-L}.$

For two wires carrying equal and opposite currents:

a) $$\bar{A} = \bar{a}_z \frac{\mu_0 I}{4\pi} \ln\left[\frac{\sqrt{L^2+r_2^2}+L}{\sqrt{L^2+r_2^2}-L}\,\frac{\sqrt{L^2+r_1^2}-L}{\sqrt{L^2+r_1^2}+L}\right] = \bar{a}_z \frac{\mu_0 I}{2\pi} \ln\left[\frac{r_1}{r_2}\,\frac{\sqrt{L^2+r_2^2}-L}{\sqrt{L^2+r_1^2}+L}\right].$$

b) For a very long two-wire transmission line, $L \to \infty$:

$$\bar{A} = \bar{a}_z \frac{\mu_0 I}{2\pi} \ln\left(\frac{r_1}{r_2}\right) = \bar{a}_z \frac{\mu_0 I}{4\pi} \ln \frac{(\frac{d}{2}+y)^2 + x^2}{(\frac{d}{2}-y)^2 + x^2}.$$

c) $$\bar{B} = \bar{\nabla}\times\bar{A} = \bar{a}_x \frac{\partial A_z}{\partial y} - \bar{a}_y \frac{\partial A_z}{\partial x}$$

$$= \bar{a}_x \frac{\mu_0 I}{2\pi}\left[\frac{\frac{d}{2}+y}{(\frac{d}{2}+y)^2+x^2} - \frac{\frac{d}{2}-y}{(\frac{d}{2}-y)^2+x^2}\right] - \bar{a}_y \frac{\mu_0 I}{2\pi}\left[\frac{x}{(\frac{d}{2}+y)^2+x^2} - \frac{x}{(\frac{d}{2}-y)^2+x^2}\right]$$

$$= \frac{\mu_0 I}{2\pi}\left[\bar{a}_{\phi 1}\frac{1}{r_1} - \bar{a}_{\phi 2}\frac{1}{r_2}\right].$$

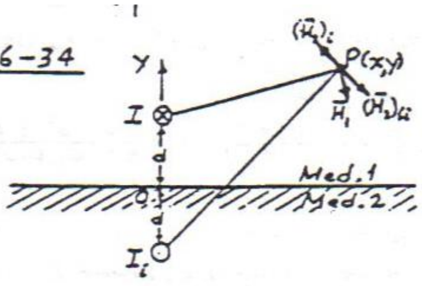
d) To find the equation for magnetic flux lines:

$$\frac{dx}{B_x} = \frac{dy}{B_y} \longrightarrow \frac{\partial A}{\partial x} dx + \frac{\partial A}{\partial y} dy = 0$$

$$\longrightarrow dA = 0 \longrightarrow A = \text{constant}.$$

Thus, $$\frac{r_1^2}{r_2^2} = \frac{(\frac{d}{2}+y)^2 + x^2}{(\frac{d}{2}-y)^2 + x^2} = K.$$

P.6-34



a) (i) If $\sigma_2 \to \infty$, $\bar{B}_2 = \bar{H}_2 = 0$.

B_n continuous $\longrightarrow B_{1n} = H_{1n} = 0$;

$\bar{a}_y \times \bar{H}_1 = \bar{J}_s \longrightarrow \bar{J}_s = -\bar{a}_z H_{1x}$.

Image $I_i\ (= -I)$ flowing out of the paper.

(ii) If $\mu_2 \to \infty$, $\bar{H}_2 = 0$, but $\bar{B}_2$ is finite.

No surface current. $\longrightarrow H_{1t} = H_{2t} = 0$;

B_n continuous $\longrightarrow B_{1n} = B_{2n}$.

Image $I_i\ (= I)$ flowing into the paper.

b) (i) $\bar{H}_P = \bar{H}_1 + (\bar{H}_2)_i$, where $\bar{H}_1 = \frac{I}{2\pi}\left[\bar{a}_x \frac{y-d}{x^2+(y-d)^2} - \bar{a}_y \frac{x}{x^2+(y-d)^2}\right]$,

$$(\bar{H}_2)_i = \frac{I}{2\pi}\left[-\bar{a}_x \frac{y+d}{x^2+(y+d)^2} + \bar{a}_y \frac{x}{x^2+(y+d)^2}\right].$$

(ii) $\bar{H}_P' = \bar{H}_1 + (\bar{H}_2)_{ii} = \bar{H}_1 - (\bar{H}_2)_i$.

c) (i) $\bar{J}_s = -\bar{a}_z (H_P)_x \Big|_{y=0} = \bar{a}_z \left(\frac{Id}{x^2+d^2}\right)$.

(ii) $\bar{J}_s = 0$.

P.7-6 a) Flux enclosed in the ring in Fig. 7-12(a): $\Phi = \pi r^2 B(t)$. ①

The induced emf in the ring referring to the assigned direction for current: $\mathcal{V} = iR_r = \frac{d\Phi}{dt} = \pi r^2 \frac{dB(t)}{dt}$. ②

Resistance of differential circular ring: $R_r = \frac{2\pi r}{\sigma h\, dr}$. ③

Combining ② and ③: $i = \frac{\pi r^2}{R_r}\frac{dB(t)}{dt} = \frac{\sigma h}{2} r\, dr \left(\frac{dB}{dt}\right)^2$. ④

$$dp = i^2 R_r = \frac{\pi\sigma h}{2} r^3 dr \left(\frac{dB}{dt}\right)^2. \quad ⑤$$

$$p = \int dp = \frac{\pi\sigma h}{8} a^4 \left(\frac{dB}{dt}\right)^2 = \frac{\pi\sigma h}{8} a^4 \omega^2 B_0^2 \cos^2\omega t. \quad ⑥$$

$$P_{av.} = \frac{\pi\sigma h}{16} a^4 \omega^2 B_0^2. \quad ⑦$$

b) For N insulated filamentary parts, each with an area

$$S = \frac{0.95\pi a^2}{N} = \pi b^2 \longrightarrow b = \sqrt{\frac{S}{\pi}} = a\sqrt{\frac{0.95}{N}}.$$

Power loss in N filaments in Fig. 7-12(b) from ⑥ $p' = N\left(\frac{\pi\sigma h}{8}\right)\left(a\sqrt{\frac{0.95}{N}}\right)^4 \omega^2 B_0^2 \cos^2\omega t = \frac{0.95^2}{N} p$.

$$P'_{av} = \frac{0.95^2}{N} P_{av}.$$

P.7-7 $\Phi(t) = \bar{B}(t)\cdot\bar{S}(t) = -(5\cos\omega t)\times 0.2(0.7-x)$

$$= -0.35\cos\omega t\,(1+\cos\omega t) \quad (mT),$$

$$i = -\frac{1}{R}\frac{d\Phi}{dt} = -\frac{1}{R}\, 0.35\,\omega(\sin\omega t + \sin 2\omega t)$$

$$= -1.75\,\omega\sin\omega t\,(1+2\cos\omega t) \quad (mA).$$

P.7-10 a) $\mu_r = 1+\chi_m$, $\chi_m = 5000-1 = 4999$,

$$\bar{H} = \frac{\bar{M}}{\chi_m} = \bar{a}_z\frac{M_0}{4999}; \quad \bar{B} = \mu_0\mu_r\bar{H} = \bar{a}_z\frac{5000}{4999}\mu_0 M_0.$$

b) $V_0 = \oint \bar{u}\times\bar{B}\cdot d\bar{\ell} = \int_b^a (\bar{a}_\phi\,\omega r)\times(\bar{a}_z B)\cdot\bar{a}_r\, dr$

$$= -\frac{\omega B}{2}(b^2-a^2) = -\frac{2500}{4999}\mu_0 M_0\,\omega(b^2-a^2).$$

c) $\bar{E}_r = \bar{E}_r' - \bar{u}\times\bar{B} = \frac{\bar{J}_r}{\sigma} - (\bar{a}_\phi u)\times(\bar{a}_z B) = \bar{a}_r\left(\frac{i}{2\pi r h\sigma} - \omega r B\right)$

at r

Induced voltage $V = \int_a^b E_r\, dr = \frac{i}{2\pi h\sigma}\ln\frac{b}{a} - \frac{\omega B}{2}(b^2-a^2)$.

$$= iR + V_0.$$

Short circuit: $V_0 = 0$, $i_{sc} = \frac{\omega B}{2R}(b^2-a^2)$, where $R = \frac{\ln(b/a)}{2\pi h\sigma}$.

P.7-18 a) Eq. (3-89): $\bar{\nabla}\cdot\bar{P} = -\rho_p \longrightarrow \bar{a}_{n2}\cdot(\bar{P}_1-\bar{P}_2) = -\rho_{ps}$.

b) $\epsilon_0\bar{E} = \bar{D}-\bar{P}$ $\begin{cases} \bar{\nabla}\cdot\bar{D} = \rho_f \longrightarrow \bar{a}_{n2}\cdot(\bar{D}_1-\bar{D}_2) = \rho_{fs} & ① \\ -\bar{\nabla}\cdot\bar{P} = \rho_p \longrightarrow \bar{a}_{n2}\cdot(\bar{P}_1-\bar{P}_2) = -\rho_{ps} & ② \end{cases}$

$\xrightarrow[①\,\&\,②]{\text{Combining}}$ $\bar{a}_{n2}\cdot(\bar{E}_1-\bar{E}_2) = \frac{1}{\epsilon_0}(\rho_{fs}+\rho_{ps})$. Subscript f signifies free charge.

P.7-19 Medium 1: Free space.

Medium 2: $\mu_2 \rightarrow \infty$. H_2 must be zero so that B_2 is not infinite.

Boundary Conditions: $\bar{a}_{n2} \times \bar{H}_1 = \bar{J}_s$, $B_{1n} = B_{2n}$.

$E_{1t} = E_{2t}$, $\bar{a}_{n2} \cdot (\bar{D}_1 - \bar{D}_2) = \rho_s$.

P.7-23 $\bar{E}_1(z,t) = \bar{a}_x 0.03 \sin 10^8\pi(t - \frac{z}{c}) = \bar{a}_x \mathcal{R}_e\left[0.03 e^{-j\pi/2} e^{j10^8\pi(t-z/c)}\right]$,

$\bar{E}_2(z,t) = \bar{a}_x 0.04 \cos\left[10^8\pi(t - \frac{z}{c}) - \frac{\pi}{3}\right] = \bar{a}_x \mathcal{R}_e\left[0.04 e^{-j\pi/3} e^{j10^8\pi(t-z/c)}\right]$.

Phasors: $\bar{E} = \bar{E}_1 + \bar{E}_2 = \bar{a}_x\left[0.03 e^{-j\pi/2} + 0.04 e^{-j\pi/3}\right]$

$= \bar{a}_x\left[-j0.03 + (0.02 - j0.02\sqrt{3})\right] = \bar{a}_x(0.068 e^{-j1.27}) = \bar{a}_x E_0 e^{j\theta}$.

$\therefore E_0 = 0.065$, $\theta = -1.27$ (rad), or $-72.8°$.

P.7-29 $\bar{H} = j\omega\epsilon_0 \bar{\nabla} \times \bar{\pi}_e$. ①

$\bar{\nabla} \times \bar{E} = -j\omega\mu_0\bar{H} = \omega^2\mu_0\epsilon_0 \bar{\nabla} \times \bar{\pi}_e$,

$\longrightarrow \bar{\nabla} \times (\bar{E} - k_0^2\bar{\pi}_e) = 0$. Let $\bar{E} - k_0^2\bar{\pi}_e = \bar{\nabla} V_e$. ②

$\bar{\nabla} \times \bar{H} = j\omega\bar{D} = j\omega(\epsilon_0\bar{E} + \bar{P}) = j\omega\epsilon_0(\bar{E} + \frac{\bar{P}}{\epsilon_0})$. ③

Substituting ① and ② in ③:

$j\omega\epsilon_0 \bar{\nabla} \times \bar{\nabla} \times \bar{\pi}_e = j\omega\epsilon_0(k_0^2\bar{\pi}_e + \bar{\nabla} V_e + \frac{\bar{P}}{\epsilon_0})$

$= j\omega\epsilon_0(\bar{\nabla}\bar{\nabla}\cdot\bar{\pi}_e - \bar{\nabla}^2\bar{\pi}_e)$. ④

Choose $\bar{\nabla}\cdot\bar{\pi}_e = V_e$. Eq. ④ becomes

b) $$\bar{\nabla}^2\bar{\pi}_e + k_0^2\bar{\pi}_e = -\frac{\bar{P}}{\epsilon_0} \qquad (7\text{-}119)$$

a) Eq. ② becomes

$\bar{E} = k_0^2\bar{\pi}_e + \bar{\nabla}\bar{\nabla}\cdot\bar{\pi}_e$

$= k_0^2\bar{\pi}_e + (\bar{\nabla}^2\bar{\pi}_e + \bar{\nabla} \times \bar{\nabla} \times \bar{\pi}_e)$. ⑤

Combination of Eqs. (7-119) and ⑤ gives

$$\bar{E} = \bar{\nabla} \times \bar{\nabla} \times \bar{\pi}_e - \frac{\bar{P}}{\epsilon_0}.$$

P.7-30

a) $$\left|\frac{\text{Displacement current}}{\text{Conduction current}}\right| = \frac{\omega\epsilon}{\sigma} = \frac{(2\pi\times 100\times 10^{9})\times\frac{1}{36\pi}\times 10^{-9}}{5.70\times 10^{7}}$$

$$= 9.75\times 10^{-8}.$$

b) In a source-free conductor:

$$\bar{\nabla}\times\bar{H} = \sigma\bar{E}, \qquad (1)$$

$$\bar{\nabla}\times\bar{E} = -j\omega\mu\bar{H}. \qquad (2)$$

$\bar{\nabla}\times$ (1): $$\bar{\nabla}\times\bar{\nabla}\times\bar{H} = \bar{\nabla}(\bar{\nabla}\cdot\bar{H})-\bar{\nabla}^2\bar{H} = \sigma\bar{\nabla}\times\bar{E}. \qquad (3)$$

But $\bar{\nabla}\cdot\bar{H}=0$, Eq. (3) becomes

$$\bar{\nabla}^2\bar{H} + \sigma\bar{\nabla}\times\bar{E} = 0. \qquad (4)$$

Combining (1) and (4):

$$\bar{\nabla}^2\bar{H} - j\omega\mu\sigma\bar{H} = 0.$$

P. 8-6 Phasor: $\bar{E} = \bar{a}_x 2 e^{-jz/\sqrt{3}} + \bar{a}_y j e^{-jz/\sqrt{3}}$ (V/m).

a) $\omega = 10^8$ (rad/s) $\longrightarrow$ $f = 10^8/2\pi = 1.59\times 10^7$ (Hz),

$\beta = 1/\sqrt{3}$ (rad/m) $\longrightarrow$ $\lambda = 2\pi/\beta = 2\sqrt{3}\,\pi$ (m).

b) $u = \frac{c}{\sqrt{\epsilon_r}} = \frac{\omega}{\beta} \longrightarrow \epsilon_r = \left(\frac{\beta c}{\omega}\right)^2 = 3.$

c) Left-hand elliptically polarized.

d) $\eta = \sqrt{\frac{\mu}{\epsilon}} = \frac{120\pi}{\sqrt{\epsilon_r}} = \frac{120\pi}{\sqrt{3}}$ (Ω),

$\bar{H} = \frac{1}{\eta}\bar{a}_z \times \bar{E} = \frac{\sqrt{3}}{120\pi}\left(\bar{a}_y 2 e^{-jz/\sqrt{3}} - \bar{a}_x j e^{-jz/\sqrt{3}}\right),$

$\bar{H}(z,t) = \frac{\sqrt{3}}{120\pi}\left[\bar{a}_x \sin(10^8 t - z/\sqrt{3}) + \bar{a}_y \cos(10^8 t - z/\sqrt{3})\right]$ (A/m).

P. 8-9 For conducting media: $k_c = \beta - j\alpha$.

$$k_c^2 = \beta^2 - \alpha^2 - 2j\alpha\beta$$
$$= \omega^2\mu\epsilon_c = \omega^2\mu\epsilon\left(1 - j\frac{\sigma}{\omega\epsilon}\right).$$
$$\therefore\quad \beta^2 - \alpha^2 = \mathcal{R}e(k_c^2) = \omega^2\mu\epsilon, \qquad ①$$
$$\beta^2 + \alpha^2 = |k_c^2| = \omega^2\mu\epsilon\sqrt{1+\left(\frac{\sigma}{\omega\epsilon}\right)^2}. \qquad ②$$

From ① and ② we obtain

$$\alpha = \omega\sqrt{\frac{\mu\epsilon}{2}}\left[\sqrt{1+\left(\frac{\sigma}{\omega\epsilon}\right)^2} - 1\right]^{1/2}, \quad \beta = \omega\sqrt{\frac{\mu\epsilon}{2}}\left[\sqrt{1+\left(\frac{\sigma}{\omega\epsilon}\right)^2} + 1\right]^{1/2}.$$

P.8-10 All three metals are good conductors, $\left(\frac{\sigma}{\omega\epsilon}\right)^2 \gg 1$.

$\alpha = \sqrt{\pi f \mu \sigma}$, $\quad \delta = \frac{1}{\alpha}$, $\quad \eta_c = (1+j)\frac{\alpha}{\sigma}$.

a) $f = 60$ (Hz)

	η_c (Ω)	α (Np/m)	α (dB/m)	δ (m)
Copper	$2.02(1+j)\times10^{-6}$	0.117×10^3	1.02×10^3	8.53×10^{-3}
Silver	$2.08(1+j)\times10^{-6}$	0.121×10^3	1.05×10^3	8.29×10^{-3}
Brass	$3.86(1+j)\times10^{-6}$	0.061×10^3	0.53×10^3	16.3×10^{-3}

b) $f = 1$ (MHz)

	η_c (Ω)	α (Np/m)	α (dB/m)	δ (m)
Copper	$2.61(1+j)\times10^{-4}$	1.51×10^4	1.31×10^5	6.61×10^{-5}
Silver	$2.57(1+j)\times10^{-4}$	1.58×10^4	1.35×10^5	6.32×10^{-5}
Brass	$4.98(1+j)\times10^{-4}$	0.79×10^4	0.69×10^5	12.6×10^{-5}

c) $f = 1$ (GHz)

	η_c (Ω)	α (Np/m)	α (dB/m)	δ (m)
Copper	$8.25(1+j)\times10^{-3}$	4.79×10^5	4.16×10^6	2.09×10^{-6}
Silver	$8.01(1+j)\times10^{-3}$	4.93×10^5	4.28×10^6	2.03×10^{-6}
Brass	$15.8(1+j)\times10^{-3}$	2.51×10^5	2.18×10^6	3.99×10^{-6}

P.8-11 $f = 3\times10^9$ (Hz), $\quad \epsilon_r = 2.5$, $\quad \tan\delta_c = \frac{\epsilon''}{\epsilon'} = 10^{-2}$.

a) Eq. (8-48): $\alpha = \frac{\omega\epsilon''}{2}\sqrt{\frac{\mu}{\epsilon'}} = \frac{\omega}{2}\left(\frac{\epsilon''}{\epsilon'}\right)\frac{\sqrt{\epsilon_r}}{c} = 0.497$ (Np/m).

$$e^{-\alpha x} = \frac{1}{2} \longrightarrow x = \frac{1}{\alpha}\ln 2 = 1.395 \text{ (m)}.$$

b) Eq. (8-50): $\eta_c = \frac{1}{\sqrt{\epsilon_r}}\sqrt{\frac{\mu_0}{\epsilon_0}}\left(1 + j\frac{\epsilon''}{2\epsilon'}\right) = 238(1+j0.005) = 238\underline{/0.29^\circ}$ (Ω),

Eq. (8-49): $\beta = \omega\sqrt{\mu\epsilon}\left[1 + \frac{1}{8}\left(\frac{\epsilon''}{\epsilon'}\right)^2\right] = 31.6\pi$ (rad/m).

$\lambda = \frac{2\pi}{\beta} = 0.063$ (m),

$u_p = \frac{\omega}{\beta} = 1.8973\times10^8$ (m/s).

$u_g = \frac{1}{\frac{d\beta}{d\omega}} = \frac{c}{\sqrt{\epsilon_r}}\left[1 + \frac{1}{8}\left(\frac{\epsilon''}{\epsilon'}\right)^2\right] = 1.8975\times10^8$ (m/s).

c) At $x = 0$, $\bar{E} = \bar{a}_y e^{j\pi/3}$.

$\bar{H} = \frac{1}{\eta_c}\bar{a}_x \times \bar{E} = \bar{a}_z\, 0.210\, e^{j(\frac{\pi}{3} - 0.0016\pi)}$.

$\bar{H}(x,t) = \bar{a}_z\, 0.210\, e^{-0.497x}\sin(6\pi10^9 t - 31.6\pi x + 0.332\pi)$ (A/m).

P.8-12 $\sigma/\omega\epsilon = 4/10^{10}\pi \times 80 \times (\frac{1}{36}\times 10^{-9}) = 0.18$.

a) $\alpha = \omega\sqrt{\frac{\mu\epsilon}{2}}\left[\sqrt{1+(\frac{\sigma}{\omega\epsilon})^2}-1\right]^{1/2} = 84\ (Np/m)$,

$\beta = \omega\sqrt{\frac{\mu\epsilon}{2}}\left[\sqrt{1+(\frac{\sigma}{\omega\epsilon})^2}+1\right]^{1/2} = 300\pi\ (rad/m)$,

$\eta_c = \sqrt{\frac{\mu}{\epsilon_c}} = \frac{120\pi}{\sqrt{\epsilon_r}\left[1+(\sigma/\omega\epsilon)^2\right]^{1/4}} e^{j\frac{1}{2}\tan^{-1}(\sigma/\omega\epsilon)} = 41.8\, e^{j0.0283\pi}\ (\Omega)$,

$u_p = \omega/\beta = 33.3\times 10^6\ (m/s)$, $\lambda = 2\pi/\beta = 0.67\ (cm)$, $\delta = \frac{1}{\alpha} = 1.19\ (cm)$.

b) $e^{-\alpha y} = \frac{1}{10}$, $y = \frac{1}{\alpha}\ln 10 = 2.74\ (cm)$.

c) $\bar{H}(y,t) = \bar{a}_x 0.1\, e^{-84\times 0.5}\sin(10^{10}\pi t - 300\pi\times 0.5 - \pi/3)$

$= \bar{a}_x 5.75\times 10^{-20}\cdot \sin(10^{10}\pi t - \pi/3)\quad (A/m)$,

$\bar{E}(y,t) = \mathcal{I}m\left[\eta_c \bar{H}(y)\times\bar{a}_y\right]e^{j\omega t} = \bar{a}_z 2.41\times 10^{-18}\sin(10^{10}\pi t - \frac{\pi}{3} + 0.0283\pi)$ (V/m).

P.8-16 $\mathcal{P}_{av} = |E|^2/2\eta_0 = 10^{-2}\ (W/cm^2)$.

a) $|E| = \sqrt{0.02\eta_0} = 2.75\ (V/cm) = 275\ (V/m)$,

$|H| = |E|/\eta_0 = 7.28\times 10^{-3}\ (A/cm) = 0.728\ (A/m)$.

b) $\mathcal{P}_{av} = |E|^2/2\eta_0 = 1300\ (W/m^2)$.

$|E| = 990\ (V/m)$, $|H| = 2.63\ (A/m)$.

P.8-17 Assume circularly polarized plane wave:

$\bar{E}(z,t) = \bar{a}_x E_0\cos(\omega t - kz + \phi) + \bar{a}_y E_0 \sin(\omega t - kz + \phi)$,

$\bar{H}(z,t) = \bar{a}_y \frac{E_0}{\eta}\cos(\omega t - kz + \phi) - \bar{a}_x\frac{E_0}{\eta}\sin(\omega t - kz + \phi)$.

Poynting vector, $\bar{\mathcal{P}} = \bar{E}\times\bar{H} = \bar{a}_z\frac{E_0^2}{\eta}\left[\cos^2(\omega t - kz+\phi) + \sin^2(\omega t - kz + \phi)\right]$

$= \bar{a}_z\frac{E_0^2}{\eta}$, a constant independent of t and z.

P.8-18 $\bar{E} = \bar{a}_\theta E_\theta + \bar{a}_\phi E_\phi$,

$\bar{H} = \frac{1}{\eta}\bar{a}_R\times\bar{E} = \frac{1}{\eta}(\bar{a}_\phi E_\theta - \bar{a}_\theta E_\phi)$.

$\bar{\mathcal{P}}_{av} = \frac{1}{2}\mathcal{R}e(\bar{E}\times\bar{H}^*) = \bar{a}_z\frac{1}{2\eta}(|E_\theta|^2 + |E_\phi|^2)$.

P.8-21 Given $\bar{E}_i = E_0(\bar{a}_x - j\,\bar{a}_y)\,e^{-j\beta z}$.

a) Assume reflected $\bar{E}_r(z) = (\bar{a}_x E_{rx} + \bar{a}_y E_{ry})\,e^{j\beta z}$.

Boundary condition at $z=0$: $\bar{E}_i(0) + \bar{E}_r(0) = 0$.

$\longrightarrow \bar{E}_r(z) = E_0(-\bar{a}_x + j\bar{a}_y)\,e^{j\beta z}$, a left-hand circularly polarized wave in $-z$ direction.

b) $\bar{a}_{n2} \times (\bar{H}_1 - \bar{H}_2) = \bar{J}_s \longrightarrow -\bar{a}_z \times [\bar{H}_i(0) + \bar{H}_r(0)] = \bar{J}_s$. ($\bar{H}_2 = 0$ in perfect conductor)

$\bar{H}_i(0) = \frac{1}{\eta_0}\bar{a}_z \times \bar{E}_i(0) = \frac{E_0}{\eta_0}(j\bar{a}_x + \bar{a}_y)$, $\bar{H}_r(0) = \frac{1}{\eta_0}(-\bar{a}_z)\times\bar{E}_r(0) = \frac{E_0}{\eta_0}(j\bar{a}_x + \bar{a}_y)$.

$\bar{H}_1(0) = \bar{H}_i(0) + \bar{H}_r(0) = \frac{2E_0}{\eta_0}(j\bar{a}_x + \bar{a}_y)$,

$\bar{J}_s = -\bar{a}_z \times \bar{H}_1(0) = \frac{2E_0}{\eta_0}(\bar{a}_x - j\bar{a}_y)$.

c) $\bar{E}_1(z,t) = \mathcal{R}e\,[\bar{E}_i(z) + \bar{E}_r(z)]\,e^{j\omega t}$

$= \mathcal{R}e\; E_0\left[(\bar{a}_x - j\bar{a}_y)\,e^{-j\beta z} + (-\bar{a}_x + j\bar{a}_y)\,e^{j\beta z}\right]e^{j\omega t}$

$= \mathcal{R}e\; E_0\left[-2j(\bar{a}_x - j\bar{a}_y)\sin\beta z\right]e^{j\omega t}$

$= 2E_0 \sin\beta z\,(\bar{a}_x \sin\omega t - \bar{a}_y \cos\omega t)$.

P.8-22 Given $\bar{E}_i(x,z) = \bar{a}_y 10\, e^{-j(6x+8z)}$ (V/m).

a) $k_x = 6,\ k_z = 8 \longrightarrow k = \beta = \sqrt{k_x^2 + k_z^2} = 10$ (rad/m).

$\lambda = 2\pi/k = 2\pi/10 = 0.628$ (m); $f = c/\lambda = 4.78\times10^8$ (Hz); $\omega = kc = 3\times10^9$ (rad/s)

b) $\bar{E}_i(x,z;t) = \bar{a}_y 10\cos(3\times10^9 t - 6x - 8z)$ (V/m).

$\bar{H}_i(x,z) = \frac{1}{\eta_0}\bar{a}_{ni}\times\bar{E}_i$ $\quad\left(\bar{a}_{ni} = \frac{\bar{k}}{k} = \bar{a}_x 0.6 + \bar{a}_z 0.8\right)$

$= \frac{1}{120\pi}(\bar{a}_x 0.6 + \bar{a}_z 0.8)\times\bar{a}_y 10\,e^{-j(6x+8z)} = \left(-\bar{a}_x\frac{1}{15\pi} + \bar{a}_z\frac{1}{20\pi}\right)e^{-j(6x+8z)}$

$\bar{H}_i(x,z;t) = \left(-\bar{a}_x\frac{1}{15\pi} + \bar{a}_z\frac{1}{20\pi}\right)\cos(3\times10^9 t - 6x - 8z)$ (A/m).

c) $\cos\theta_i = \bar{a}_{ni}\cdot\bar{a}_z = \left(\frac{\bar{k}}{k}\right)\cdot\bar{a}_z = 0.8 \longrightarrow \theta_i = \cos^{-1}0.8 = 36.9°$.

d) $\bar{E}_i(x,0) + \bar{E}_r(x,0) = 0 \longrightarrow \bar{E}_r(x,z) = -\bar{a}_y 10\,e^{-j(6x-8z)}$

$\bar{H}_r(x,z) = \frac{1}{\eta_0}\bar{a}_{nr}\times\bar{E}_r(x,z)$ $\quad(\bar{a}_{nr} = \bar{a}_x 0.6 - \bar{a}_z 0.8)$

$= -\left(\bar{a}_x\frac{1}{15\pi} + \bar{a}_z\frac{1}{20\pi}\right)e^{-j(6x-8z)}$

e) $\bar{E}_1(x,z) = \bar{E}_i(x,z) + \bar{E}_r(x,z) = \bar{a}_y 10\,(e^{-j8z} - e^{j8z})\,e^{-j6x}$

$= -\bar{a}_y j20\,e^{-j6x}\sin 8z$ (V/m).

$\bar{H}_1(x,z) = \bar{H}_i(x,z) + \bar{H}_r(x,z) = -\left(\bar{a}_x\frac{2}{15\pi}\cos 8z + \bar{a}_z\frac{j}{10\pi}\sin 8z\right)e^{-j6x}$ (A/m).

P.8-23 Given $\bar{E}_i(y,z) = 5(\bar{a}_y + \bar{a}_z\sqrt{3})e^{j6(\sqrt{3}y-z)}$ (V/m).

a) $k_y = -6\sqrt{3}$, $k_z = 6 \longrightarrow k = \sqrt{k_y^2+k_z^2} = 12$ (rad/m).

$\lambda = 2\pi/k = \pi/6 = 0.524$ (m); $f = c/\lambda = 5.73\times10^8$ (Hz); $\omega = kc = 3.60\times10^9$ (rad/s).

b) $\bar{E}_i(y,z;t) = 5(\bar{a}_y + \bar{a}_z\sqrt{3})\cos(3.60\times10^9 t + 6\sqrt{3}y - 6z)$ (V/m).

$\bar{H}_i(y,z) = \frac{1}{\eta_0}\bar{a}_{ni}\times\bar{E}_i = \frac{1}{120\pi}\left(-\bar{a}_y\frac{\sqrt{3}}{2} + \bar{a}_z\frac{1}{2}\right)\times 5(\bar{a}_y + \bar{a}_z\sqrt{3})e^{j6(\sqrt{3}y-z)}$

$= \bar{a}_x\left(-\frac{1}{12\pi}\right)e^{j6(\sqrt{3}y-z)}$

$\bar{H}_i(y,z;t) = \bar{a}_x\left(-\frac{1}{12\pi}\right)\cos(3.60\times10^9 t + 6\sqrt{3}y - 6z)$ (A/m).

c) $\cos\theta_i = \bar{a}_{ni}\cdot\bar{a}_z = \frac{1}{2} \longrightarrow \theta_i = \cos^{-1}\left(\frac{1}{2}\right) = 60°$.

d) Conditions $\bar{a}_{nr}\cdot\bar{E}_r(y,z) = 0$ and $E_{iy}(y,0) + E_{ry}(y,0) = 0$ lead to:

$\bar{E}_r(y,z) = 5(-\bar{a}_y + \bar{a}_z\sqrt{3})e^{j6(\sqrt{3}y+z)}$ (V/m),

$\bar{H}_r(y,z) = \frac{1}{\eta_0}\bar{a}_{nr}\times\bar{E}_r(y,z) = \frac{1}{120\pi}\left(-\bar{a}_y\frac{\sqrt{3}}{2} - \bar{a}_z\frac{1}{2}\right)\times 5(-\bar{a}_y + \bar{a}_z\sqrt{3})e^{j6(\sqrt{3}y+z)}$

$= \bar{a}_x\left(-\frac{1}{12\pi}\right)e^{j6(\sqrt{3}y+z)}$ (A/m).

e) $\bar{E}_1(y,z) = \bar{E}_i(y,z) + \bar{E}_r(y,z) = (-\bar{a}_y j10\sin 6z + \bar{a}_z 10\sqrt{3}\cos 6z)e^{j6\sqrt{3}y}$ (V/m),

$\bar{H}_1(y,z) = \bar{H}_i(y,z) + \bar{H}_r(y,z) = \bar{a}_x\left(-\frac{1}{6\pi}\right)\cos 6z\cdot e^{j6\sqrt{3}y}$ (A/m).

P.8-24 a) From Eqs. (8-113) and (8-114):

$\bar{E}_1(x,z;t) = \bar{a}_y 2E_{i0}\sin(\beta_1 z\cos\theta_i)\sin(\omega t - \beta_1 x\sin\theta_i)$,

$$\bar{H}_1(x,z;t) = \frac{2E_{i0}}{\eta_1}\Big[-\bar{a}_x\cos\theta_i\cos(\beta_1 z\cos\theta_i)\cos(\omega t - \beta_1 x\sin\theta_i) + \bar{a}_z\sin\theta_i\sin(\beta_1 z\cos\theta_i)\sin(\omega t - \beta_1 x\sin\theta_i)\Big].$$

b) $\bar{\mathcal{P}}_{av} = \frac{1}{2}\mathcal{R}e(\bar{E}\times\bar{H}^*) = \bar{a}_x\frac{2E_{i0}^2}{\eta_1}\sin\theta_i\sin^2(\beta_1 z\cos\theta_i)$.

P.8-25 a) From Eqs. (8-128) and (8-129):

$$\bar{E}_1(x,z;t) = -2E_{i0}\Big[\bar{a}_x\cos\theta_i\sin(\beta_1 z\cos\theta_i)\cos(\omega t - \beta_1 x\sin\theta_i) + \bar{a}_z\sin\theta_i\cos(\beta_1 z\cos\theta_i)\sin(\omega t - \beta_1 x\sin\theta_i)\Big],$$

$\bar{H}_1(x,z;t) = \bar{a}_y\frac{2E_{i0}}{\eta_1}\cos(\beta_1 z\cos\theta_i)\sin(\omega t - \beta_1 x\sin\theta_i)$.

b) $\bar{\mathcal{P}}_{av} = \frac{1}{2}\mathcal{R}e(\bar{E}\times\bar{H}^*) = \bar{a}_x\frac{2E_{i0}^2}{\eta_1}\sin\theta_i\cos^2(\beta_1 z\cos\theta_i)$.

<u>P.8-27</u> a) In the lossy medium (medium 2):

$$\bar{E}_t = \bar{a}_x E_{t0} e^{-\alpha_2 z} e^{-j\beta_2 z},$$

where, from Problem P.8-9, $\alpha_2 = \omega\sqrt{\frac{\mu_2\epsilon_2}{2}}\left[\sqrt{1+\left(\frac{\sigma_2}{\omega\epsilon_2}\right)^2}-1\right]^{\frac{1}{2}}$, $\beta_2 = \omega\sqrt{\frac{\mu_2\epsilon_2}{2}}\left[\sqrt{1+\left(\frac{\sigma_2}{\omega\epsilon_2}\right)^2}+1\right]^{\frac{1}{2}}$.

Given: $\beta_1 = 6\ (\text{rad/m}) \longrightarrow \omega = \beta_1 c = 1.8\times 10^9\ (\text{rad/s})$.

$\tan\delta_c = \frac{\sigma_2}{\omega\epsilon_2} = 0.5 \longrightarrow \alpha_2 = 2.30\ (\text{Np/m}),\ \beta_2 = 9.76\ (\text{rad/m})$

$\eta_2 = \sqrt{\frac{\mu_2}{\epsilon_2}} = \frac{120\pi}{\sqrt{\epsilon_{r2}}(1+\tan^2\delta_c)^{1/4}} e^{j\frac{1}{2}\tan^{-1}(\sigma_2/\omega\epsilon_2)} = 225\, e^{j13.3^\circ}$.

$\bar{E}_t = \bar{a}_x E_{t0} e^{-2.30z} e^{-j9.76z}$, $\quad \bar{H}_t = \bar{a}_z \times \frac{\bar{E}_t}{\eta_2} = \bar{a}_y \frac{E_{t0}}{225} e^{-j13.3^\circ} e^{-2.30z} e^{-j9.7}$

Let $\bar{E}_r = \bar{a}_x E_{r0} e^{j6z} \longrightarrow \bar{H}_r = -\bar{a}_y \frac{E_{r0}}{120\pi} e^{j6z}$. $\quad \bar{H}_i = \bar{a}_y \frac{10}{120\pi} e^{-j6z}$.

Boundary conditions for $\bar{E}$ and $\bar{H}$ at $z=0$: $\begin{cases} 10 + E_{r0} = E_{t0}, \\ 10 - E_{r0} = E_{t0}\sqrt{\epsilon_{r2}}\,(1+\tan^2\delta_c)^{1/4} e^{-j13.3^\circ}. \end{cases}$

$\longrightarrow E_{r0} = 2.77\, e^{j157^\circ};\quad E_{t0} = 7.53\, e^{-j172^\circ}$

$\therefore$
$\bar{E}_r(z,t) = \bar{a}_x 2.77 \cos(1.8\times 10^9 t + 6z + 157^\circ)$ (V/m),
$\bar{H}_r(z,t) = -\bar{a}_y 0.073 \cos(1.8\times 10^9 t + 6z + 157^\circ)$ (A/m),
$\bar{E}_t(z,t) = \bar{a}_x 7.53\, e^{-2.30z} \cos(1.8\times 10^9 t - 9.76z - 172^\circ)$ (V/m),
$\bar{H}_t(z,t) = \bar{a}_y 0.033\, e^{-2.30z} \cos(1.8\times 10^9 t - 9.76z + 174.7^\circ)$ (A/m).

b) $(\bar{\mathcal{P}}_{av})_1 = \bar{a}_z\left(\frac{10^2}{2\times 120\pi} - \frac{2.77^2}{2\times 120\pi}\right) = \bar{a}_z 0.122$ (W/m²),

$(\bar{\mathcal{P}}_{av})_2 = \bar{a}_z \frac{7.53^2}{2\times 225}(\cos 13.3^\circ) e^{-4.60z} = \bar{a}_z 0.122\, e^{-4.60z}$ (W/m²).

<u>P.8-28</u> a) $\Gamma = \frac{E_r}{E_i} = \frac{\eta_c - \eta_0}{\eta_c + \eta_0}$; $\quad \eta_0 = \sqrt{\frac{\mu_0}{\epsilon_0}} = 120\pi,\ \eta_c = \sqrt{j\omega\mu/\sigma}$.

$|\eta_c| \ll \eta_0$.

b) $|\Gamma|^2 = \left|\frac{\eta_c - \eta_0}{\eta_c + \eta_0}\right|^2 = \left|\frac{1-\eta_c/\eta_0}{1+\eta_c/\eta_0}\right|^2 \cong \left|1 - 2\eta_c/\eta_0\right|^2$

$= (1-2\eta_c/\eta_0)(1-2\eta_c^*/\eta_0) \cong 1 - 4\mathcal{R}_e(\eta_c)/\eta_0$.

Fraction of power absorbed, $F = 1 - |\Gamma|^2 = \frac{4}{\eta_0}\mathcal{R}_e\sqrt{\frac{j\omega\mu}{\sigma}}$

$= \frac{4}{\eta_0}\sqrt{\frac{\omega\mu}{2\sigma}}$.

c) $\omega = 2\pi\times 10^6$ (Hz). For iron: $\mu = 4{,}000\times(4\pi 10^{-7})$ (H/m), $\sigma = 10^7$ (S/m).

$F = 4.21\times 10^{-4}$, or 0.0421%.

P.8-32

$$\bar{E}_1 = \bar{a}_x \left(E_{io} e^{-j\beta_1 z} + E_{ro} e^{j\beta_1 z} \right),$$

$$\bar{H}_1 = \bar{a}_y \frac{1}{\eta_1} \left(E_{io} e^{-j\beta_1 z} - E_{ro} e^{j\beta_1 z} \right).$$

$$\bar{E}_2 = \bar{a}_x \left(E_2^+ e^{-j\beta_2 z} + E_2^- e^{j\beta_2 z} \right).$$

$$\bar{H}_2 = \bar{a}_y \frac{1}{\eta_2} \left(E_2^+ e^{-j\beta_2 z} - E_2^- e^{j\beta_2 z} \right).$$

At $z = d$, $\bar{E}_2 = 0 \longrightarrow E_2^- = -E_2^+ e^{-j2\beta_2 d}$.

$$\bar{E}_2 = \bar{a}_x E_2^+ \left[e^{-j\beta_2 z} - e^{j\beta_2 (z-2d)} \right],$$

$$\bar{H}_2 = \bar{a}_y \frac{E_2^+}{\eta_2} \left[e^{-j\beta_2 z} + e^{j\beta_2 (z-2d)} \right].$$

Boundary conditions at $z=0$: $E_1(0) = E_2(0) \longrightarrow E_{io} + E_{ro} = E_2^+ \left(1 - e^{-j2\beta_2 d}\right)$,
$H_1(0) = H_2(0) \longrightarrow E_{io} - E_{ro} = E_2^+ \frac{\eta_0}{\eta_2} \left(1 + e^{-j2\beta_2 d}\right)$.

$$E_2^+ = \frac{2\eta_2 E_{io}}{(\eta_0 + \eta_2) + (\eta_0 - \eta_2) e^{-j2\beta_2 d}}.$$

$$E_{ro} = -\left(\frac{\eta_0 - j\eta_2 \tan \beta_2 d}{\eta_0 + j\eta_2 \tan \beta_2 d} \right) E_{io}.$$

a) $\bar{E}_r(z,t) = \bar{a}_x E_{io} \cos\left[\omega\left(t - \frac{z}{u_p}\right) + \theta\right]$, $\theta = \pi - 2\tan^{-1}\left(\frac{\eta_2}{\eta_0} \tan \beta_2 d\right)$.

b) $\bar{E}_1(z,t) = \bar{a}_x E_{io} \left\{ \cos\omega\left(t - \frac{z}{u_p}\right) + \cos\left[\omega\left(t - \frac{z}{u_p}\right) + \theta\right] \right\}$.

c) $$\bar{E}_2(z,t) = \bar{a}_x \frac{2\eta_2 E_{io}}{\sqrt{2[(\eta_0^2 + \eta_2^2) + (\eta_0^2 - \eta_1^2)\cos 2\beta_2 d]}} \left\{ \cos\left[\omega\left(t - \frac{z}{u_{p2}}\right) + \psi\right] - \cos\left[\omega\left(t + \frac{z}{u_{p2}}\right) - \frac{2\omega d}{u_{p2}} + \psi\right] \right\},$$

$$\psi = \tan^{-1}\left[\frac{(\eta_0 - \eta_2)\sin 2\beta_2 d}{(\eta_0 + \eta_2) + (\eta_0 - \eta_2)\cos 2\beta_2 d} \right].$$

d) $(\bar{\mathcal{P}}_{av})_1 = \frac{1}{2} \mathcal{R}e\left(\bar{E}_1 \times \bar{H}_1^*\right) = 0$.

e) $(\bar{\mathcal{P}}_{av})_2 = 0$.

f) Let $E_r = -E_{io} \longrightarrow \tan\beta_2 d = 0 \longrightarrow d = n\lambda_2/2$, $n = 0, 1, 2, \cdots$

P.8-34 Given $f = f_p/2$ and $\theta_i = 60°$.

$$\longrightarrow \eta_p = \eta_0/\sqrt{1-(f_p/f)^2} = -j\eta_0/\sqrt{3}, \quad \eta_p/\eta_0 = -j/\sqrt{3}.$$

From Eq.(8-185): $\sin\theta_t = \frac{\eta_p}{\eta_0}\sin\theta_i = -j/2$, $\cos\theta_t = \sqrt{5}/2$, $\cos\theta_i = 1/2$.

a) From Eq. (8-206): $\Gamma_\perp = \frac{(\eta_p/\eta_0)\cos\theta_i - \cos\theta_t}{(\eta_p/\eta_0)\cos\theta_i + \cos\theta_t} = e^{j209°}$.

From Eq. (8-207): $\tau_\perp = \frac{2(\eta_p/\eta_0)\cos\theta_i}{(\eta_p/\eta_0)\cos\theta_i + \cos\theta_t} = 0.5\, e^{-j75.5°}$.

b) From Eq. (8-221): $\Gamma_\parallel = \frac{(\eta_p/\eta_0)\cos\theta_t - \cos\theta_i}{(\eta_p/\eta_0)\cos\theta_t + \cos\theta_i} = e^{j76°}$.

From Eq. (8-222): $\tau_\parallel = \frac{2(\eta_p/\eta_0)\cos\theta_i}{(\eta_p/\eta_0)\cos\theta_t + \cos\theta_i} = 0.177\, e^{-j38°}$.

$|\Gamma_\perp| = |\Gamma_\parallel| = 1$, but the phase shift of the reflected wave depends on the polarization of the incident wave. There are standing waves in the air and exponentially decaying transmitted waves in the ionosphere.

P.8-35 $k_{2x}^2 + k_{2z}^2 = k_2^2 = \omega^2\mu_0\epsilon_2 - j\omega\mu_0\sigma_2$. ①

Continuity conditions at $z=0$ for all x and y require:

$$k_{2x} = k_{1x} = \omega\sqrt{\mu_0\epsilon_0}\sin\theta_i = \beta_x = 2.09\times10^{-4}. \quad ②$$

$$k_{2z} = \beta_{2z} - j\alpha_{2z}. \quad ③$$

Combining ①, ② and ③, we can solve for α_{2z} and β_{2z} in terms of $\omega, \mu_0, \epsilon_2, \sigma_2$, and β_x. But, since

$$\beta_x^2 << \omega^2\mu_0\epsilon_2,$$

we have $\alpha_2 = \alpha_{2z} \cong \beta_{2z} \cong \frac{1}{\delta} = \sqrt{\pi f\mu_0\sigma_2} = 0.3974\ (m^{-1})$.

a) $\theta_t = \tan^{-1}\frac{\beta_x}{\beta_{2z}} \cong \tan^{-1}\frac{2.09}{0.3974}\times10^{-4} \cong 5.26\times10^{-4}$ (rad) $= 0.03°$.

b) $\Gamma_\parallel = \frac{2\eta_2\cos\theta_i}{\eta_2\cos\theta_t + \eta_0\cos\theta_i}$ $\quad \eta_2 = \frac{\alpha_2}{\sigma_2}(1+j) = 0.0993(1+j)$.

$= \frac{2\times0.0993(1+j)}{0.0993(1+j) + 377\cos 38°}$ $\quad \cos\theta_t = \cos 0.03° \cong 1$.

$\cong 0.0151(1+j) = 0.0214\, e^{j\pi/4}$.

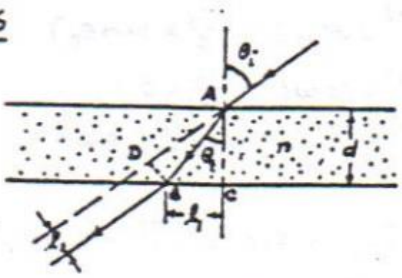
c) $(\mathcal{P}_{av})_i = \frac{E_{i0}^2}{2\eta_0}$.

$E_{t0} \cong 2E_{i0}\frac{\eta_2}{\eta_0}$, $\quad H_{t0} \cong \frac{2E_{i0}}{\eta_0} \longrightarrow (\mathcal{P}_{av})_t = 2\frac{E_{i0}^2\alpha_2}{\eta_0^2\sigma_2}e^{-2\alpha_2 z}$.

$\therefore \frac{(\mathcal{P}_{av})_t}{(\mathcal{P}_{av})_i} = \frac{4\alpha_2}{\eta_0\sigma_2}e^{-2\alpha_2 z} = 1.054\times10^{-3}e^{-0.795z}$

d) $20\log_{10}e^{-\alpha_2 z} = -30. \longrightarrow z = \frac{1.5}{\alpha_2\log_{10}e} = 8.69$ (m).

P. 8-36



a) Snell's law:

$$\frac{\sin\theta_t}{\sin\theta_i} = \frac{1}{n},$$

$$\theta_t = \sin^{-1}\left(\frac{1}{n}\sin\theta_i\right).$$

b) $\cos\theta_t = \sqrt{1-\left(\frac{1}{n}\sin\theta_i\right)^2}.$

$$\ell_1 = \overline{BC} = \overline{AC}\tan\theta_t = d\frac{\sin\theta_t}{\cos\theta_t} = \frac{d\sin\theta_i}{\sqrt{n^2-\sin^2\theta_i}}.$$

c) $$\ell_2 = \overline{BD} = \overline{AC}\sin(\theta_i-\theta_t) = \frac{d}{\cos\theta_t}(\sin\theta_i\cos\theta_t - \cos\theta_i\sin\theta_t)$$
$$= d\sin\theta_i\left[1-\frac{\cos\theta_i}{\sqrt{n^2-\sin^2\theta_i}}\right].$$

P. 8-37

a) $\sin\theta_c = \sqrt{\frac{\epsilon_2}{\epsilon_1}} \longrightarrow \sin\theta_t = \sqrt{\frac{\epsilon_1}{\epsilon_2}}\sin\theta_i > 1$ for $\theta_i > \theta_c$,

$$\cos\theta_t = -j\sqrt{\left(\frac{\epsilon_1}{\epsilon_2}\right)\sin^2\theta_i - 1}.$$

From Eqs. (8-200) and (8-201):

$$\bar{E}_t(x,z) = \bar{a}_y E_{t0} e^{-\alpha_2 z} e^{-j\beta_{2x}x},$$

$$\bar{H}_t(x,z) = \frac{E_{t0}}{\eta_2}\left(\bar{a}_x j\alpha_2 + \bar{a}_z\sqrt{\frac{\epsilon_1}{\epsilon_2}}\sin\theta_i\right)e^{-\alpha_2 z}e^{-j\beta_{2x}x},$$

where $\beta_{2x} = \beta_2\sin\theta_t = \beta_2\sqrt{\frac{\epsilon_1}{\epsilon_2}}\sin\theta_i,$

$$\alpha_2 = \beta_2\sqrt{\left(\frac{\epsilon_1}{\epsilon_2}\right)\sin^2\theta_i - 1},$$

$$E_{t0} = \frac{2\eta_2\cos\theta_i E_{i0}}{\eta_2\cos\theta_i - j\eta_1\sqrt{\left(\frac{\epsilon_1}{\epsilon_2}\right)\sin^2\theta_i - 1}} \quad \text{from Eq. (8-207).}$$

b) $(\mathcal{P}_{av})_{2z} = \frac{1}{2}\mathcal{R}e\,(E_{ty}H_{tx}^*) = 0.$

P. 8-38 Given $\theta_i = \theta_c \longrightarrow \theta_t = \pi/2$, $\cos\theta_t = 0$.

a) From Eq. (8-207): $(E_{t0}/E_{i0})_\perp = 2.$

b) From Eq. (8-221): $(E_{t0}/E_{i0})_\parallel = 2\eta_2/\eta_1.$

c) $\bar{E}_i(x,z;t) = \bar{a}_y E_{i0}\cos\omega\left[t - \frac{n_1}{c}(x\sin\theta_i + z\cos\theta_i)\right],$

$$\bar{E}_t(x,z;t) = \bar{a}_y 2E_{i0}e^{-\alpha z}\cos\omega\left(t - \frac{n_2}{c}x\sin\theta_t\right)$$
$$= \bar{a}_y 2E_{i0}e^{-\alpha z}\cos\omega\left(t - \frac{n_1}{c}x\sin\theta_i\right),$$

where $\alpha = \frac{n_2\omega}{c}\sqrt{\left(\frac{n_1}{n_2}\sin\theta_i\right)^2 - 1} = 0$ when $\theta = \theta_c$.

P.8-39. a) $\theta_c = \sin^{-1}\sqrt{\epsilon_{r2}/\epsilon_{r1}} = \sin^{-1}\sqrt{1/81} = 6.38°.$

b) $\theta_i = 20° > \theta_c. \qquad \sin\theta_t = \sqrt{\frac{\epsilon_1}{\epsilon_2}}\sin\theta_i = 3.08., \quad \cos\theta_t = -j2.91.$

$$\Gamma_\perp = \frac{\sqrt{\epsilon_{r1}}\cos\theta_i - \cos\theta_t}{\sqrt{\epsilon_{r1}}\cos\theta_i + \cos\theta_t} = e^{j38°} = e^{j0.66}$$

c) $$\tau_\perp = \frac{2\sqrt{\epsilon_{r1}}\cos\theta_i}{\sqrt{\epsilon_{r1}}\cos\theta_i + \cos\theta_t} = 1.89\,e^{j19°} = 1.89\,e^{j0.33}$$

d) The transmitted wave in air varies as $e^{-\alpha_2 z}e^{-j\beta_{2x}x}$.
where $\alpha_2 = \beta_2\sqrt{\left(\frac{\epsilon_1}{\epsilon_2}\right)\sin^2\theta_i - 1} = \frac{2\pi}{\lambda_0}(2.91).$

Attenuation in air for each wavelength
$= 20\log_{10} e^{-\alpha_2\lambda_0} = 159$ (dB).

P.8-40 When the incident light first strikes the hypotenuse surface, $\theta_i = \theta_t = 0$, $\tau_1 = \frac{2\eta_2}{\eta_2 + \eta_0}$.

$$\frac{(\mathcal{P}_{av})_{t1}}{(\mathcal{P}_{av})_i} = \frac{\eta_0}{\eta_2}\tau_1^2 = \frac{4\eta_0\eta_2}{(\eta_2 + \eta_0)^2}.$$

Total reflections occur inside the prism at both slanting surfaces because

$$\theta_i = 45° > \theta_c = \sin^{-1}\left(\tfrac{1}{2}\right) = 30°.$$

On exit from the prism, $\tau_2 = \frac{2\eta_0}{\eta_2 + \eta_0}$.

$$\frac{(\mathcal{P}_{av})_o}{(\mathcal{P}_{av})_{t1}} = \frac{\eta_2}{\eta_0}\tau_2^2 = \frac{4\eta_0\eta_2}{(\eta_2 + \eta_0)^2}.$$

$$\therefore \quad \frac{(\mathcal{P}_{av})_o}{(\mathcal{P}_{av})_i} = \left[\frac{4\eta_0\eta_2}{(\eta_2 + \eta_0)^2}\right]^2 = \left[\frac{4\sqrt{\epsilon_r}}{(1+\sqrt{\epsilon_r})^2}\right]^2 = 0.79.$$

P.8-41 a) $n_0\sin\theta_a = n_1\sin(90° - \theta_c) = n_1\cos\theta_c$

$$= n_1\sqrt{1 - \sin^2\theta_c} = n_1\sqrt{1 - (n_2/n_1)^2} = \sqrt{n_1^2 - n_2^2}.$$

$$\sin\theta_a = \frac{1}{n_0}\sqrt{n_1^2 - n_2^2} = \sqrt{n_1^2 - n_2^2}. \qquad (n_0 = 1)$$

b) $N.A. = \sin\theta_a = \sqrt{2^2 - 1.74^2} = 0.9861,$

$$\theta_a = \sin^{-1}0.9861 = 80.4°.$$

P.8-42 Eq.(8-185): $\dfrac{\eta_2}{\eta_1} = \dfrac{\sin\theta_t}{\sin\theta_i}$.

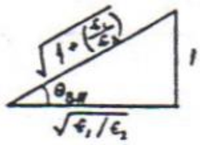
a) Eq.(8-206): $$\Gamma_\perp = \frac{(\eta_2/\eta_1)\cos\theta_i - \cos\theta_t}{(\eta_2/\eta_1)\cos\theta_i + \cos\theta_t} = \frac{\sin\theta_t\cos\theta_i - \cos\theta_t\sin\theta_i}{\sin\theta_t\cos\theta_i + \cos\theta_t\sin\theta_i} = \frac{\sin(\theta_t - \theta_i)}{\sin(\theta_t + \theta_i)}.$$

Eq.(8-207): $$\tau_\perp = \frac{2(\eta_2/\eta_1)\cos\theta_i}{(\eta_2/\eta_1)\cos\theta_i + \cos\theta_t} = \frac{2\sin\theta_t\cos\theta_i}{\sin(\theta_t + \theta_i)}.$$

b) Eq. (8-221): $$\Gamma_\parallel = \frac{(\eta_2/\eta_1)\cos\theta_t - \cos\theta_i}{(\eta_2/\eta_1)\cos\theta_t + \cos\theta_i} = \frac{\sin\theta_t\cos\theta_t - \sin\theta_i\cos\theta_i}{\sin\theta_t\cos\theta_t + \sin\theta_i\cos\theta_i} = \frac{\sin 2\theta_t - \sin 2\theta_i}{\sin 2\theta_t + \sin 2\theta_i}.$$

Eq. (8-222): $$\tau_\parallel = \frac{2(\eta_2/\eta_1)\cos\theta_i}{(\eta_2/\eta_1)\cos\theta_t + \cos\theta_i} = \frac{4\sin\theta_t\cos\theta_i}{\sin 2\theta_t + \sin 2\theta_i}.$$

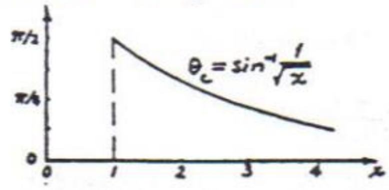
P.8-44 a) $\sin\theta_c = \sqrt{\dfrac{\epsilon_2}{\epsilon_1}}$; $\sin\theta_{B\parallel} = \dfrac{1}{\sqrt{1 + \left(\frac{\epsilon_1}{\epsilon_2}\right)}}$

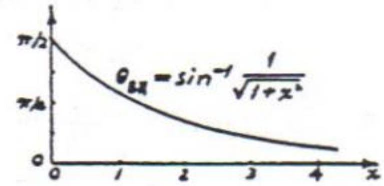


$\longrightarrow \tan\theta_{B\parallel} = \sqrt{\dfrac{\epsilon_2}{\epsilon_1}}$.

$\therefore\ \sin\theta_c = \tan\theta_{B\parallel}$. $(\theta_c > \theta_{B\parallel})$

b) Let $\epsilon_1/\epsilon_2 = x$.





P.8-45 a) For perpendicular polarization:

$$\Gamma_\perp = \frac{\sqrt{\epsilon_{r1}}\cos\theta_i - \sqrt{\epsilon_{r2}}\cos\theta_t}{\sqrt{\epsilon_{r1}}\cos\theta_i + \sqrt{\epsilon_{r2}}\cos\theta_t},$$

$$\sin\theta_t = \sqrt{\frac{\epsilon_{r1}}{\epsilon_{r2}}}\sin\theta_i, \quad \cos\theta_t = \sqrt{1 - \left(\frac{\epsilon_{r1}}{\epsilon_{r2}}\right)\sin^2\theta_i}.$$

$$\Gamma_\perp = \frac{\sqrt{\frac{\epsilon_{r1}}{\epsilon_{r2}}}\cos\theta_i - \sqrt{1 - \frac{\epsilon_{r1}}{\epsilon_{r2}}\sin^2\theta_i}}{\sqrt{\frac{\epsilon_{r1}}{\epsilon_{r2}}}\cos\theta_i + \sqrt{1 - \frac{\epsilon_{r1}}{\epsilon_{r2}}\sin^2\theta_i}},$$

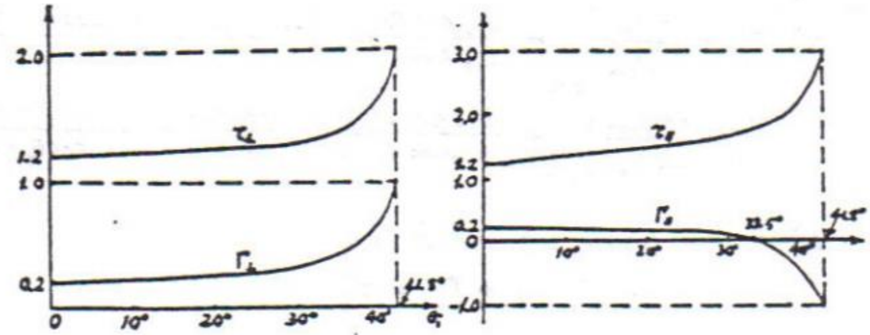
$$\tau_\perp = \frac{2\eta_2\cos\theta_i}{\eta_2\cos\theta_i + \eta_1\cos\theta_t} = \frac{2\sqrt{\frac{\epsilon_{r1}}{\epsilon_{r2}}}\cos\theta_i}{\sqrt{\frac{\epsilon_{r1}}{\epsilon_{r2}}}\cos\theta_i + \sqrt{1 - \frac{\epsilon_{r1}}{\epsilon_{r2}}\sin^2\theta_i}}.$$

For parallel polarization:

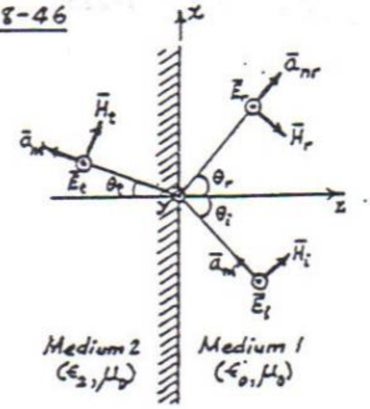
$$\Gamma_{\parallel} = \frac{\sqrt{\frac{\epsilon_{r1}}{\epsilon_{r2}}}\sqrt{1-\left(\frac{\epsilon_{r1}}{\epsilon_{r2}}\right)\sin^2\theta_i} - \cos\theta_i}{\sqrt{\frac{\epsilon_{r1}}{\epsilon_{r2}}}\sqrt{1-\left(\frac{\epsilon_{r1}}{\epsilon_{r2}}\right)\sin^2\theta_i} + \cos\theta_i}.$$

$$\tau_{\parallel} = \frac{2\sqrt{\frac{\epsilon_{r1}}{\epsilon_{r2}}}\cos\theta_i}{\sqrt{\frac{\epsilon_{r1}}{\epsilon_{r2}}}\sqrt{1-\left(\frac{\epsilon_{r1}}{\epsilon_{r2}}\right)\sin^2\theta_i} + \cos\theta_i}.$$

b) $\epsilon_{r1}/\epsilon_{r2} = 2.25$, $\sqrt{\epsilon_{r1}/\epsilon_{r2}} = 1.5 \longrightarrow \theta_c = \sin^{-1}\sqrt{\frac{\epsilon_2}{\epsilon_1}} = 41.8°$.



P. 8-46



Given: $\bar{E}_i(x,z) = \bar{a}_y E_{i0} e^{-jk_0(x\sin\theta_i - z\cos\theta_i)}$,

$\bar{a}_{ni} = \bar{a}_x \sin\theta_i - \bar{a}_z \cos\theta_i$.

$$\bar{H}_i(x,z) = \frac{1}{\eta_0}\bar{a}_{ni} \times \bar{E}_i(x,z)$$
$$= \frac{1}{\eta_0}(\bar{a}_x\cos\theta_i + \bar{a}_z\sin\theta_i)e^{-jk_0(x\sin\theta_i - z\cos\theta_i)}.$$

$$\epsilon_2 = \epsilon' - j\epsilon'',\quad k_2 = \omega\sqrt{\mu_0\epsilon_2} = \omega\sqrt{\mu_0\epsilon_0\left(\frac{\epsilon'}{\epsilon_0} - j\frac{\epsilon''}{\epsilon_0}\right)}$$
$$= k_0\sqrt{\epsilon_r' - j\epsilon_r''}.$$

a) From Eq. (8-207):

$$\tau_{\perp} = \frac{2(\eta_2/\eta_0)\cos\theta_i}{(\eta_2/\eta_0)\cos\theta_i + \cos\theta_t},$$

where $(\eta_2/\eta_0) = \sqrt{\epsilon_0/\epsilon_2} = 1/\sqrt{\epsilon_r' - j\epsilon_r''}$.

$$\bar{E}_t(x,z) = \bar{a}_y \tau_{\perp} E_{i0}\, e^{-jk_2(x\sin\theta_t - z\cos\theta_t)},$$

$$\bar{H}_t(x,z) = \frac{1}{\eta_2}\bar{a}_{nt} \times \bar{E}_t(x,z) = \frac{1}{\eta_2}(\bar{a}_x\cos\theta_t + \bar{a}_z\sin\theta_t)\tau_{\perp}\bar{E}_t(x,z).$$

b) From Eq. (8-185): $\sin\theta_t = \frac{\sin\theta_i}{\sqrt{\epsilon_r' - j\epsilon_r''}}$ (complex).

$\cos\theta_t = \sqrt{1-\sin^2\theta_t}$ (complex).

The x- and z-components of $\bar{H}_t(x,z)$ in part a) have different amplitudes and are out of phase, indicating that it is elliptically polarized.

P.9-2 a) $\bar{\nabla}\times(\bar{a}_x E_x+\bar{a}_y E_y) = -j\omega\mu(\bar{a}_x H_x+\bar{a}_y H_y)$.

$$\longrightarrow \begin{cases} \beta E_y = -\omega\mu H_x, & ① \\ \beta E_x = \omega\mu H_y. & ② \\ \dfrac{\partial E_y}{\partial x} = \dfrac{\partial E_x}{\partial y}. & ③ \end{cases}$$

$$\nabla\times(a_x H_x + a_y H_y) = j\omega\epsilon\,(a_x E_x + a_y E_y).$$

$$\longrightarrow \begin{cases} \beta H_y = \omega\epsilon E_x, & ④ \\ \beta H_x = -\omega\epsilon E_y, & ⑤ \\ \dfrac{\partial H_y}{\partial x} = \dfrac{\partial H_x}{\partial y}. & ⑥ \end{cases}$$

From ① and ⑤: $\beta = \omega\sqrt{\mu\epsilon}$. ⑦

From ② or ④: $\dfrac{E_x}{H_y} = \sqrt{\dfrac{\mu}{\epsilon}} = \eta$. ⑧

From ① or ⑤: $\dfrac{E_y}{H_x} = -\sqrt{\dfrac{\mu}{\epsilon}} = -\eta$. ⑨

b) From ③: $\dfrac{\partial^2 E_y}{\partial y\,\partial x} = \dfrac{\partial^2 E_x}{\partial y^2}$. ⑩

From ⑥, ⑧, and ⑨: $\dfrac{\partial E_x}{\partial x} = -\dfrac{\partial E_y}{\partial y} \longrightarrow \dfrac{\partial^2 E_x}{\partial x^2} = -\dfrac{\partial^2 E_y}{\partial x\,\partial y}$. ⑪

Combining ⑩ and ⑪, we have $\dfrac{\partial^2 E_x}{\partial x^2} + \dfrac{\partial^2 E_x}{\partial y^2} = 0$.

Similarly, $\dfrac{\partial^2 H_x}{\partial x^2} + \dfrac{\partial^2 H_x}{\partial y^2} = 0$.

P.9-4 Given: $\sigma_c = 1.6\times10^7$ (S/m), $w = 0.02$ (m), $d = 2.5\times10^{-3}$ (m).
Lossy dielectric slab: $\mu = \mu_0$, $\epsilon_r = 3$, $\sigma = 10^{-3}$ (S/m).
$f = 5\times10^8$ (Hz).

a) $R = \dfrac{2}{w}\sqrt{\dfrac{\pi f\mu_c}{\sigma_c}} = 1.11$ (Ω/m)

$L = \mu\dfrac{d}{w} = 0.157$ (μH/m)

$G = \sigma\dfrac{w}{d} = 0.008$ (S/m)

$C = \epsilon\dfrac{w}{d} = 0.212$ (nF/m).

b) $\dfrac{|E_z|}{|E_y|} = \sqrt{\dfrac{\omega\epsilon}{\sigma_c}} = 4.167\times10^{-5}$.

c) $\omega L = 493.5 \gg R$, $\omega C = 0.667 \gg G$

$\gamma \cong j\omega\sqrt{LC}\left[1+\dfrac{1}{2j}\left(\dfrac{R}{\omega L}+\dfrac{G}{\omega C}\right)\right] = 0.129 + j18.14$ (m^{-1})

$Z_0 \cong \sqrt{\dfrac{L}{C}}\left[1+\dfrac{1}{2j}\left(\dfrac{R}{\omega L}-\dfrac{G}{\omega C}\right)\right] = 27.21 + j0.13$ (Ω).

P.9-17 a) From Eq. (9-117): $Z_{is} = Z_0 \tanh \gamma \ell = Z_0 \frac{1-e^{-2\gamma\ell}}{1+e^{-2\gamma\ell}}$.

For $\ell = \lambda/4$, $\beta\ell = \pi/2$, $\alpha\lambda/2 \ll 1$.

$$Z_{is} = Z_0 \frac{1-e^{-2\alpha(\lambda/4)}e^{-j\pi}}{1+e^{-2\alpha(\lambda/4)}e^{-j\pi}} \cong Z_0 \frac{1+(1-\alpha\lambda/2)}{1-(1-\alpha\lambda/2)}$$

$$\cong 4Z_0/\alpha\lambda.$$

b) From Eq (9-116): $Z_{io} = Z_0 \coth \gamma\ell = Z_0 \frac{1+e^{-2\gamma\ell}}{1-e^{-2\gamma\ell}}$.

For $\ell = \lambda/4$, $Z_{io} = Z_0 \frac{1+e^{-(\alpha\lambda/2)}e^{-j\pi}}{1-e^{-(\alpha\lambda/2)}e^{-j\pi}} \cong Z_0 \frac{1-(1-\alpha\lambda/2)}{1+(1-\alpha\lambda/2)}$

$$\cong Z_0 \alpha\lambda/4.$$

P.9-18 $\beta\ell = \frac{2\pi f}{c}\ell = \frac{8\pi}{3} = 480°$.

$\tan \beta\ell = \tan 480° = -1.732$,

$$Z_i = Z_0 \frac{Z_L + jZ_0 \tan\beta\ell}{Z_0 + jZ_L \tan\beta\ell} = 50\,\frac{(40+j30)+j50(-1.732)}{50+j(40+j30)(-1.732)}$$

$$= 26.3 - j9.87 \quad (\Omega).$$

P.9-23 a) $|\Gamma| = \frac{S-1}{S+1} = \frac{\left|\frac{Z_L}{Z_0}-1\right|}{\left|\frac{Z_L}{Z_0}+1\right|} = \frac{\sqrt{(r_L-1)^2+x_L^2}}{\sqrt{(r_L+1)^2+x_L^2}}$,

where $r_L = R_L/Z_0$ and $x_L = X_L/Z_0$.

$$\longrightarrow x_L = \pm\left[\frac{\left(\frac{S-1}{S+1}\right)^2(r_L+1)^2-(r_L-1)^2}{1-\left(\frac{S-1}{S+1}\right)^2}\right]^{1/2}.$$

When $S=3$, $x_L = \pm\sqrt{(10r_L - 3r_L^2 - 3)/3}$.

b) $S = 3$ and $r_L = 150/75 = 2 \longrightarrow x_L = \pm\sqrt{5/3}$.

$X_L = x_L Z_0 = \pm 96.8\ (\Omega)$.

c) From Eq. (9-147): $r_L + jx_L = \frac{r_m + jt}{1+jr_m t}$,

where $r_m = R_m/Z_0$, and $t = \tan\beta\ell_m$.

$$\longrightarrow r_m = \frac{(1+r_L^2+x_L^2)\pm\sqrt{(1+r_L^2+x_L^2)^2-4r_L^2}}{2r_L}$$

$= 3$ or $\frac{1}{3}$, for $r_L = 2$ and $x_L^2 = 5/3$.

Also, $x_L = \frac{(1-r_m^2)t}{1+r_m^2t^2} \longrightarrow t = \frac{1}{2x_L r_m^2}\left[(1-r_m^2)\pm\sqrt{(1-r_m^2)^2-4x_L^2r_m^2}\right]$.

$r_m = 3$ yields negative t (discard).

For $r_m = \frac{1}{3}$, $t = \begin{cases} 3\sqrt{3/5} \longrightarrow \ell_m = 0.1865\lambda \\ \text{or } \sqrt{15} \longrightarrow \ell_m = 0.2098\lambda \end{cases}$

Use $\ell_m = 0.2098\lambda$ to obtain V_{min} nearest to the load at $(0.5-0.2098)\lambda = 0.2902\lambda$.

P.9-24 a) $|\Gamma|^2 = \left|\frac{(R_L-Z_0)+jX_L}{(R_L+Z_0)+jX_L}\right| = \frac{(R_L-Z_0)^2+X_L^2}{(R_L+Z_0)^2+X_L^2}$

$$\frac{\partial|\Gamma|^2}{\partial Z_0}=0 \longrightarrow Z_0=\sqrt{R_L^2+X_L^2}.$$

If $Z_L = 40+j30\ (\Omega)$, $Z_0 = 50\ (\Omega)$.

b) $\text{Min.}\ |\Gamma| = \sqrt{\frac{Z_0-R_L}{Z_0+R_L}} = \sqrt{\frac{50-40}{50+40}} = \frac{1}{3}.$

$\text{Min.}\ S = \frac{1+\frac{1}{3}}{1-\frac{1}{3}} = 2.$

c) From Eq. (9-147): $r_i + jx_i = \frac{r_m+jt}{1+jr_m t} = 0.8+j0.6.$

$\longrightarrow t = \frac{1}{2x_i r_m^2}\left[(1-r_m^2) \pm \sqrt{(1-r_m^2)^2-4x_i^2 r_m^2}\right]$ (See problem P.9-23.)

At voltage minimum, $r_m = \frac{1}{S} = \frac{1}{2}.$

$t = 1$ (Use negative sign.)

$\tan\beta\ell_m = \tan(2\pi\ell_m/\lambda) = 1 \longrightarrow \ell_m = \frac{\lambda}{8}.$

$\therefore$ Voltage minimum nearest to the load is $(\frac{\lambda}{2}-\frac{\lambda}{8})$ or $3\lambda/8$ from the load.

P.10-4 Field expressions for TM_n modes, from Eqs. (10-63, 64 & 65):

$$E_z^0(y) = A_n \sin(n\pi y/b),$$
$$H_x^0(y) = \frac{j\omega\epsilon}{h} A_n \cos(n\pi y/b),$$
$$E_y^0(y) = -\frac{\gamma}{h} A_n \cos(n\pi y/b).$$

Surface charge densities:

$$\rho_{sl} = \bar{a}_n \cdot \bar{D}\Big|_{y=0} = \epsilon E_y^0(0) = -\frac{\gamma\epsilon}{h} A_n,$$
$$\rho_{su} = \bar{a}_n \cdot \bar{D}\Big|_{y=b} = -\epsilon E^0(b) = (-1)^n \frac{\gamma\epsilon}{h} A_n.$$

Surface current densities:

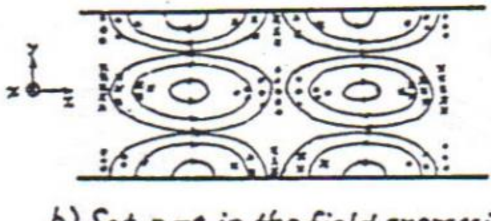
$$\bar{J}_{sl} = \bar{a}_n \times \bar{H}\Big|_{y=0} = \bar{a}_y \times \bar{H}(0) = -\bar{a}_z \frac{j\omega\epsilon}{h} A_n,$$
$$\bar{J}_{su} = \bar{a}_n \times \bar{H}\Big|_{y=b} = -\bar{a}_y \times \bar{H}(b) = \bar{a}_z (-1)^n \frac{j\omega\epsilon}{h} A_n = \begin{cases} \bar{J}_{sl} \text{ for } n \text{ odd.} \\ -\bar{J}_{sl} \text{ for } n \text{ even.} \end{cases}$$

P.10-5 Field expressions for TE_n modes, from Eqs. (10-83, 84 & 85):

$$H_z^0(y) = B_n \cos(n\pi y/b),$$
$$H_y^0(y) = \frac{\gamma}{h} B_n \sin(n\pi y/b),$$
$$E_x^0(y) = \frac{j\omega\mu}{h} B_n \sin(n\pi y/b).$$

$$\bar{J}_{sl} = \bar{a}_y \times \bar{H}(0) = \bar{a}_x B_n,$$
$$\bar{J}_{su} = -\bar{a}_y \times \bar{H}(b) = \bar{a}_x (-1)^{n+1} B_n = \begin{cases} \bar{J}_{sl} \text{ for } n \text{ odd,} \\ -\bar{J}_{sl} \text{ for } n \text{ even.} \end{cases}$$

P.10-6 a) Set $n=2$ in the field expressions in problem P.10-4.

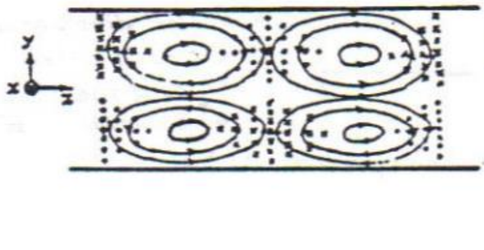


—— Electric field lines

x • Magnectic field lines

Equation for electric field lines: $\cos\beta z = \frac{\cos(2\pi y_0/b)}{\cos(2\pi y/b)}$.

b) Set $n=2$ in the field expressions in problem P.10-5.



x • Electric field lines

—— Magnetic field lines

Equation for magnetic field lines: $\cos\beta z = \frac{\sin(2\pi y_0/b)}{\sin(2\pi y/b)}$.

P.10-11 Parallel-plate waveguide: $b = 0.03$ (m), $f = 10^{10}$ (Hz).

a) TEM mode

From Eqs. (9-1a) and (9-1b):

$$\begin{cases} E_y^0 = E_0 . \\ H_x^0 = -\dfrac{E_0}{\eta_0} . \end{cases}$$

$$P_{av} = \frac{w}{2}\int_0^b -E_y^0 H_x^0 \, dy = \frac{wb}{2\eta_0} E_0^2 .$$

Dielectric strength of air: Max. $E_0 = 3\times 10^6$ (V/m).

$$Max.\left(\frac{P_{av}}{w}\right) = \frac{b}{2\eta_0}(3\times10^6)^2 = 358\times10^8 \text{ (W/m)} = 358 \text{ (MW/m)}.$$

b) TM₁ mode

From Eqs. (10-64) and (10-65):

$$\begin{cases} E_y^0(y) = E_0 \cos\left(\dfrac{\pi y}{b}\right) . \\ H_x^0(y) = -\dfrac{E_0}{\eta_0\sqrt{1-(f_c/f)^2}} \cos\left(\dfrac{\pi y}{b}\right) , \end{cases}$$

$$f_c = \frac{1}{2b\sqrt{\mu_0\epsilon_0}} = 5\times10^9 \text{ (Hz)}.$$

$$P_{av} = \frac{w}{2}\int_0^b -E_y^0(y) H_x^0(y)\, dy = \frac{wb\,E_0^2}{4\eta_0\sqrt{1-(f_c/f)^2}} .$$

$$Max.\left(\frac{P_{av}}{w}\right) = \frac{b\,(3\times10^6)^2}{4\eta_0\sqrt{1-(f_c/f)^2}} = 2.07\times10^8 \text{ (W/m)} = 207 \text{ (MW/m)}$$

c) TE₁ mode

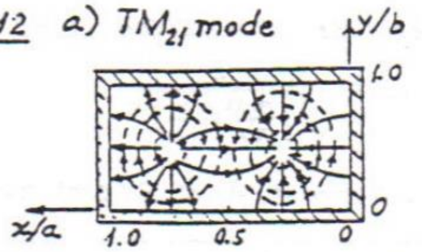
From Eqs. (10-84) and (10-85):

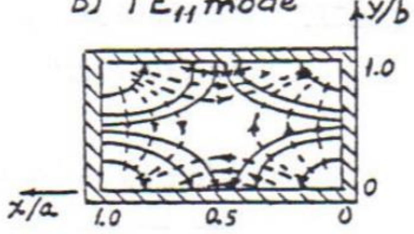
$$\begin{cases} E_x^0(y) = E_0 \sin\left(\dfrac{\pi y}{b}\right) , \\ H_y^0(y) = \dfrac{E_0}{\eta_0}\sqrt{1-\left(\dfrac{f_c}{f}\right)^2} \sin\left(\dfrac{\pi y}{b}\right) . \end{cases}$$

$$P_{av} = \frac{w}{2}\int_0^b E_x^0(y) H_y^0(y)\, dy = \frac{wbE_0^2}{4\eta_0}\sqrt{1-(f_c/f)^2} .$$

$$Max.\left(\frac{P_{av}}{w}\right) = \frac{b\,(3\times10^6)^2}{4\eta_0}\sqrt{1-(f_c/f)^2} = 1.55\times10^8 \text{ (W/m)} = 155 \text{ (MW/m)}.$$

P.10-12 a) TM_{21} mode b) TE_{11} mode





—— Electric field lines

- - - - Magnetic field lines

P.10-13 Equations (10-134) through (10-137) for TM_{11} mode:

$$E_x^0(x,y) = \frac{-j\beta_{11}}{h^2}\left(\frac{\pi}{a}\right)E_0\cos\left(\frac{\pi x}{a}\right)\sin\left(\frac{\pi y}{b}\right).$$

$$E_y^0(x,y) = \frac{-j\beta_{11}}{h^2}\left(\frac{\pi}{b}\right)E_0\sin\left(\frac{\pi x}{a}\right)\cos\left(\frac{\pi y}{b}\right).$$

$$E_z^0(x,y) = E_0\sin\left(\frac{\pi x}{a}\right)\sin\left(\frac{\pi y}{b}\right),$$

$$H_x^0(x,y) = \frac{j\omega\epsilon}{h^2}\left(\frac{\pi}{b}\right)E_0\sin\left(\frac{\pi x}{a}\right)\cos\left(\frac{\pi y}{b}\right),$$

$$H_y^0(x,y) = \frac{-j\omega\epsilon}{h^2}\left(\frac{\pi}{a}\right)E_0\cos\left(\frac{\pi x}{a}\right)\sin\left(\frac{\pi y}{b}\right).$$

a) Surface current densities:

$$\bar{J}_s(y=0) = \bar{a}_n\times\bar{H}\Big|_{y=0} = \bar{a}_y\times[\bar{a}_x H_x^0(x,0)+\bar{a}_y H_y^0(x,0)]$$

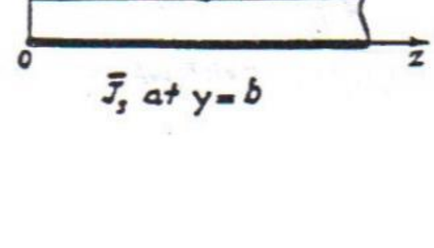
$$= -\bar{a}_z H_x^0(x,0) = -\bar{a}_z\frac{j\omega\epsilon}{h^2}\left(\frac{\pi}{b}\right)E_0\sin\left(\frac{\pi x}{a}\right)e^{-j\beta_{11}z}$$

$$= \bar{J}_s(y=b).$$

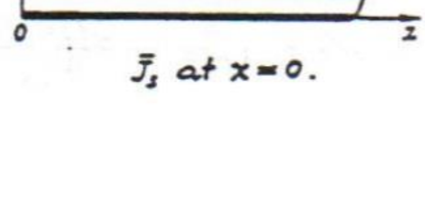
$$\bar{J}_s(x=0) = \bar{a}_n\times\bar{H}\Big|_{x=0} = \bar{a}_x\times[\bar{a}_x H_x^0(0,y)+\bar{a}_y H_y^0(0,y)]$$

$$= \bar{a}_z H_y^0(0,y) = -\bar{a}_z\frac{j\omega\epsilon}{h^2}\left(\frac{\pi}{a}\right)E_0\sin\left(\frac{\pi y}{b}\right)e^{-j\beta_{11}z}$$

$$= \bar{J}_s(x=a).$$



$\bar{J}_s$ at $y=b$



$\bar{J}_s$ at $x=0$.

P.10-14 Rectangular waveguide: $a = 7.21$ (cm), $b = 3.40$ (cm).

Eq.(10-140): $(\lambda_c)_{mn} = \dfrac{2}{\sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2}}$.

Modes with the shortest $\lambda_c < 5$ (cm) are:

Mode	TE_{10}	TE_{20}	TE_{01}	TE_{11}/TM_{11}
λ (cm)	14.4	7.20	6.80	6.15

a) For $\lambda = 10$ (cm), the only propagating mode is TE_{10}.

b) For $\lambda = 5$ (cm), the propagating modes are: TE_{10}, TE_{20}, TE_{01}, TE_{11}, and TM_{11}.

P.10-16 $(f_c)_{mn} = \dfrac{1}{2\sqrt{\mu\epsilon}}\sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2} = \dfrac{1}{2a\sqrt{\mu\epsilon}} F(m,n)$.

a) $a = 2b$, $F(m,n) = \sqrt{m^2 + 4n^2}$

Modes	$F(m,n)$
TE_{10}	1
TE_{01}, TE_{20}	2
TE_{11}, TM_{11}	$\sqrt{5}$
TE_{02}	4
TM_{12}	$\sqrt{17}$
TM_{22}	$\sqrt{20}$

b) $a = b$, $F(m,n) = \sqrt{m^2 + n^2}$.

Modes	$F(m,n)$
TE_{10}, TE_{01}	1
TE_{11}, TM_{11}	$\sqrt{2}$
TE_{02}, TE_{20}	2
TM_{12}	$\sqrt{5}$
TM_{22}	$2\sqrt{2}$

P.10-17 $f = 3 \times 10^9$ (Hz), $\lambda = c/f = 0.1$ (m).

Let $a = kb$, $1 < k < 2$. $(f_c)_{mn} = \dfrac{3\times 10^8}{2a}\sqrt{m^2 + k^2 n^2}$.

a) $(f_c)_{10} = \dfrac{1.5\times 10^8}{a}$ for the dominant TE_{10} mode.

For $f > 1.2\,(f_c)_{10}$: $a > 0.06$ (m).

The next higher-order mode is TE_{01} with $(f_c)_{01} = \dfrac{1.5\times 10^8}{b}$.

For $f < 0.8\,(f_c)_{01}$: $b < 0.04$ (m).

We choose $a = 6.5$ (cm) and $b = 3.5$ (cm).

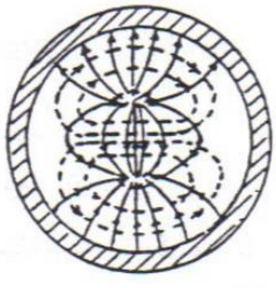
b) $u_p = \dfrac{c}{\sqrt{1-(\lambda/2a)^2}} = 4.70 \times 10^8$ (m/s),

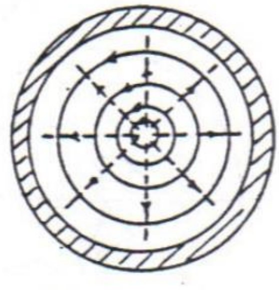
$\lambda_g = \dfrac{\lambda}{\sqrt{1-(\lambda/2a)^2}} = 0.157$ (m) $= 15.7$ (cm),

$\beta = \dfrac{2\pi}{\lambda_g} = 40.1$ (rad/m),

$(Z_{TE})_{10} = \dfrac{\eta_0}{\sqrt{1-(\lambda/2a)^2}} = 590$ (Ω).

P.10-27 a) TM_{11} b) TE_{01}





——— Electric field lines.

- - - - - Magnetic field lines.

c) Eq. (10-35): $f_c = \frac{h}{2\pi\sqrt{\mu\epsilon}} = \frac{1.5\times10^8}{\pi}h$

For TM_{11} mode, $(h)_{TM_{11}} = \frac{3.832}{a} \longrightarrow (f_c)_{TM_{11}} = \frac{1.83\times10^8}{a}$ (H

For TE_{01} mode, $(h)_{TE_{01}} = \frac{3.832}{a} \longrightarrow (f_c)_{TE_{01}} = (f_c)_{TM_{11}}$.

(Degenerate mode

P.10-38 From Eq. (10-301): $f_{mnp} = \frac{u}{2}\sqrt{\left(\frac{m}{a}\right)^2+\left(\frac{n}{b}\right)^2+\left(\frac{p}{d}\right)^2}$.

$f_{mnp} = 1.5\times10^{10}F(m,n,p)$, $F(m,n,p) = \sqrt{\left(\frac{m}{8}\right)^2+\left(\frac{n}{6}\right)^2+\left(\frac{p}{5}\right)^2}$.

Lowest-order modes and resonant frequencies:

Modes	$F(m,n,p)$	$(f_c)_{mnp}$ in (Hz)
TM_{110}	0.2083	3.125×10^9
TE_{101}	0.2358	3.538×10^9
TE_{011}	0.2603	3.905×10^9
TE_{111}, TM_{111}	0.2888	4.332×10^9
TM_{210}	0.3005	4.507×10^9
TE_{201}	0.3202	4.802×10^9
TM_{120}	0.3560	5.340×10^9
TE_{211}, TM_{211}	0.3609	5.414×10^9
TE_{021}	0.3887	5.831×10^9
TE_{121}, TM_{121}	0.4083	6.125×10^9