Field and Wave Electromagnetics

Theory and Applications of Transmission Lines

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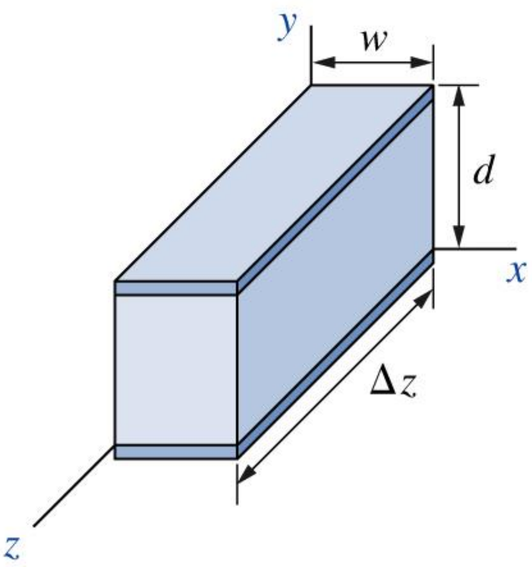
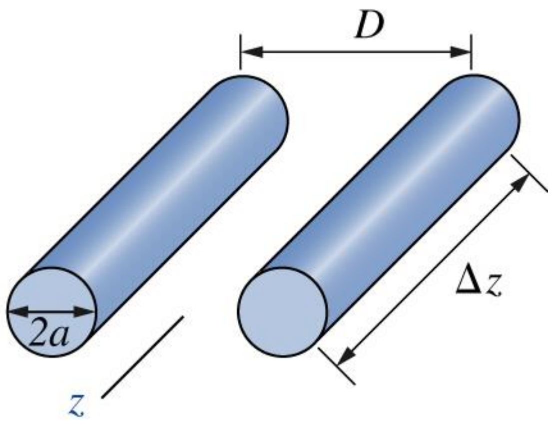
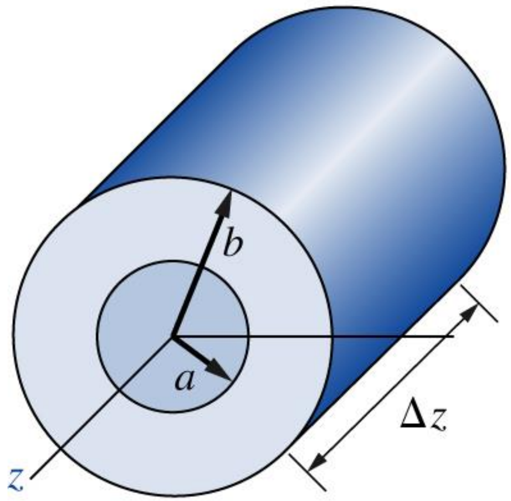
Common types of transmission lines



Parallel-plate
transmission line

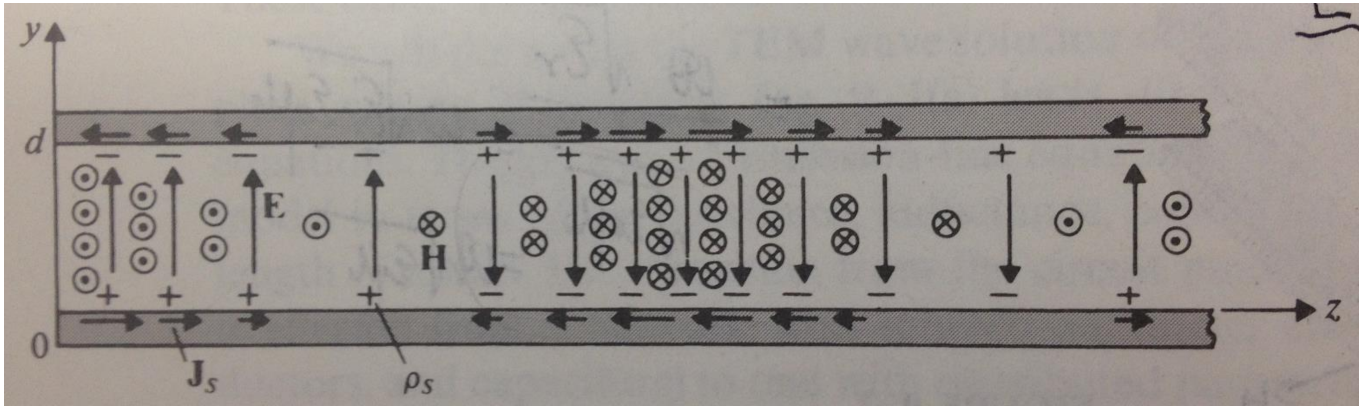


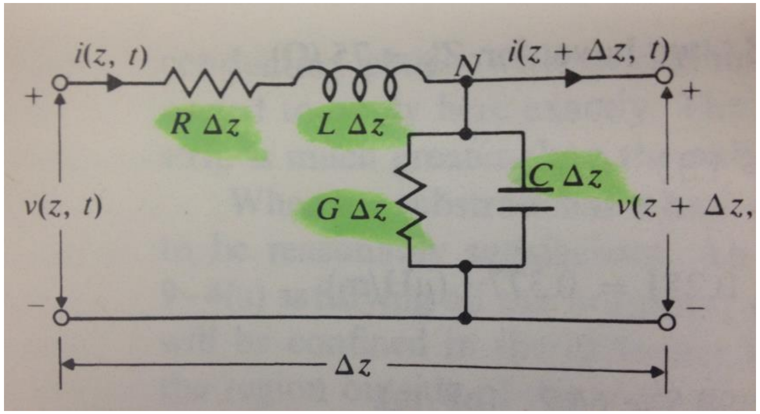
Two-wire (twin-lead)
transmission line



Coaxial
transmission line

Finite Transmission line terminated with load impedance

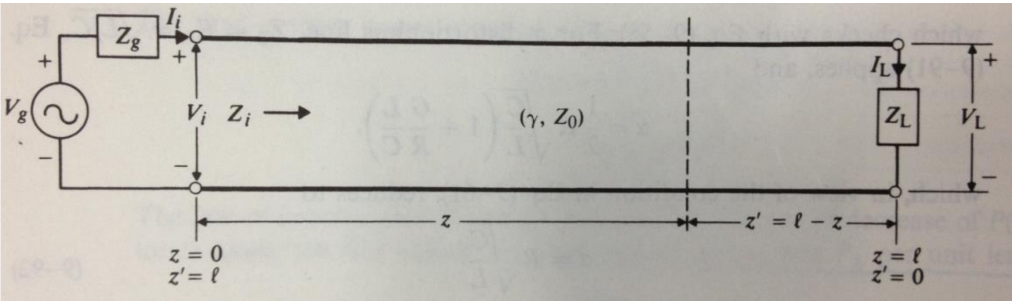


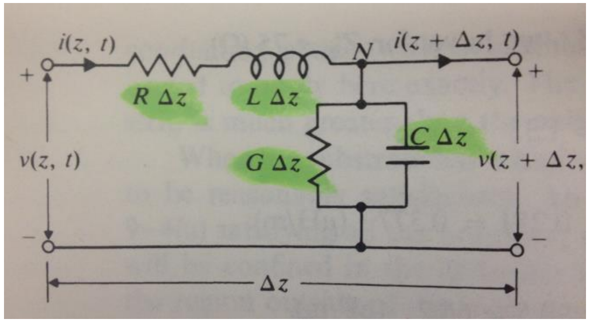


Distributed Parameters of Parallel-Plate Transmission Line (Width = w, Separation = d)

Parameter	Formula	Unit	
R	$\frac{2}{w}\sqrt{\frac{\pi f \mu_c}{\sigma_c}}$	Ω/m	conductor
L	$\mu \frac{d}{w}$	H/m	
G	$\sigma \frac{w}{d}$	S/m	dielectric
C	$\epsilon \frac{w}{d}$	F/m	

Finite Transmission line terminated with load impedance





Finite transmission line

Propagation constant $$\gamma = \alpha + j\beta = \sqrt{(R + j\omega L)(G + j\omega C)}$$

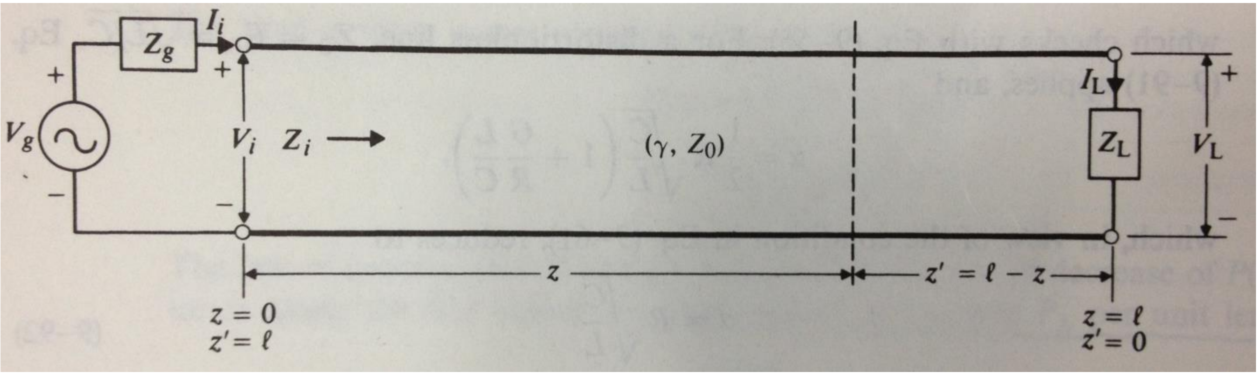
Characteristic impedance $$Z_0 = R_0 + jX_0 = \sqrt{\frac{R + j\omega L}{G + j\omega C}}$$

$\boldsymbol{\gamma}$ **and $\boldsymbol{Z_0}$ are characteristic properties of a transmission line regardless of the length of the line**

Load impedance $\boldsymbol{Z_L}$

Input impedance $\boldsymbol{Z_i}$ $$Z_i = (Z)_{\substack{z=0 \\ z'=l}} = Z_0 \frac{Z_L + Z_0 \tanh \gamma l}{Z_0 + Z_L \tanh \gamma l}$$

Finite Transmission line terminated with load impedance



Lossless transmission line ***R*=0 *G*=0**

Propagation constant $$\gamma = j\beta$$

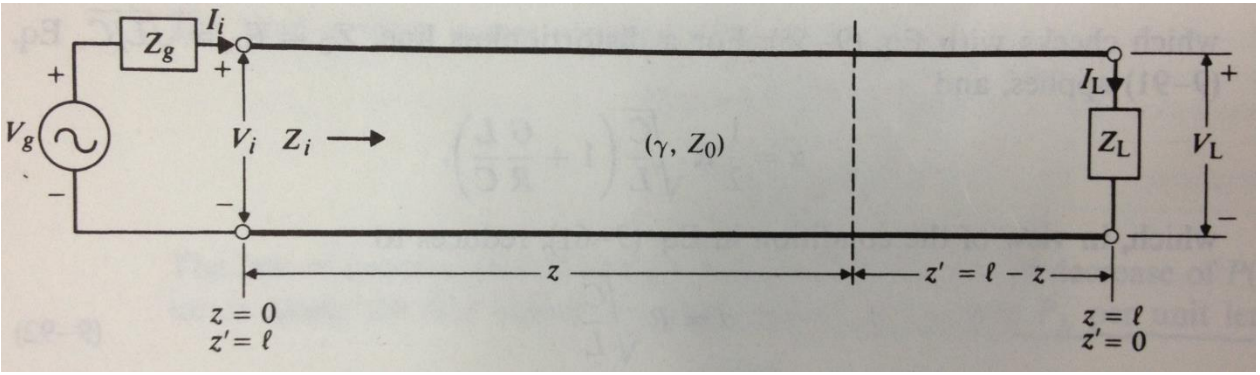
Characteristic impedance $$Z_0 = R_0$$

$$\tanh \gamma l = \tanh j\beta l = j \tan \beta l$$

Input impedance $\boldsymbol{Z_i}$

$$Z_i = (Z)_{\substack{z=0 \\ z'=l}} = R_0 \frac{Z_L + jR_0 \tan \beta l}{R_0 + jZ_L \tan \beta l}$$

Finite Transmission line terminated with load impedance



Voltage reflection coefficient of the load impedance

Ratio of Complex amplitudes of the reflected and incident voltage at the load

$$\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0} = |\Gamma| e^{j\theta_\Gamma}$$

Standing-wave ratio

$$S = \frac{|V_{\max}|}{|V_{\min}|} = \frac{1+|\Gamma|}{1-|\Gamma|}$$

$$|\Gamma| = \frac{S-1}{S+1}$$

$\Gamma = 0,\quad S = 1\quad$ for $Z_L = Z_0$ matched load

$\Gamma = -1,\quad S \to \infty\quad$ for $Z_L = 0$ short circuit

$\Gamma = +1,\quad S \to \infty\quad$ for $Z_L \to \infty$ open circuit

FIELD AND WAVE ELECTROMAGNETICS

Waveguides and Cavity Resonators

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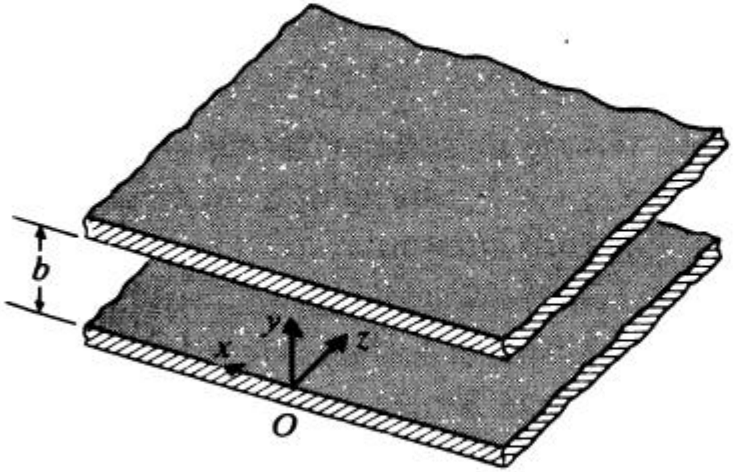
Parallel-plate waveguide

***x*-direction: infinite; fields do not vary**
***z*-direction: propagation direction**

***TM: H*$_z$=0**



Two perfectly conducting plates

$$E_z^{\,0}(y) = A_n \sin(\frac{n\pi y}{b})$$

$$E_y^{\,0}(y) = -\frac{\gamma}{h} A_n \cos(\frac{n\pi y}{b})$$

$$H_x^{\,0}(y) = \frac{j\omega\varepsilon}{h} A_n \cos(\frac{n\pi y}{b})$$

$$\gamma = j\beta = \sqrt{(\frac{n\pi}{b})^2 - \omega^2\mu\varepsilon}$$

$$h = \frac{n\pi}{b}$$

$$E_z^{\,0}(y=0) = 0$$

Cutoff frequency

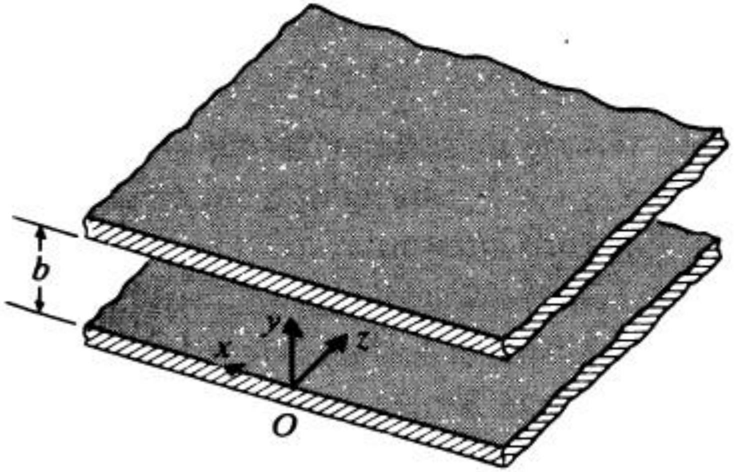
$$\gamma = 0 \qquad f_c = \frac{n}{2b\sqrt{\mu\varepsilon}}$$

$$\gamma = h\sqrt{1-(\frac{f}{f_c})^2}$$

The dominant mode is the TEM mode (n=0, Ez=0)

Parallel-plate waveguide

x-direction: infinite; fields do not vary
z-direction: propagation direction



***TE: E*$_z$=0**

$$H_z^0(y) = B_n \cos(\frac{n\pi y}{b})$$

$$H_y^0(y) = \frac{\gamma}{h} B_n \sin(\frac{n\pi y}{b})$$

$$E_x^0(y) = \frac{j\omega\mu}{h} B_n \sin(\frac{n\pi y}{b})$$

$$\gamma = j\beta = \sqrt{(\frac{n\pi}{b})^2 - \omega^2 \mu\varepsilon}$$

$$h = \frac{n\pi}{b}$$

Two perfectly conducting plates

$$E_x^0(y=0) = 0$$

$$f_c = \frac{n}{2b\sqrt{\mu\varepsilon}}$$

$$\gamma = h\sqrt{1-(\frac{f}{f_c})^2}$$

10-3 a) Write the instantaneous field expressions for TM_1 mode in a parallel-plate waveguide.
(b) Sketch the electric and magnetic field lines in the *yz*-plane.

Multiplying the phasor expressions with $e^{j(\omega t-\beta z)}$ and taking the real part

***z*-direction: propagation direction**

H only Hx component

$$E_z(y,z;t) = A_1 \sin(\frac{\pi y}{b})\cos(\omega t-\beta z),$$

$$E_y(y,z;t) = \frac{\beta b}{\pi} A_1 \cos(\frac{\pi y}{b})\sin(\omega t-\beta z),$$

$$H_x(y,z;t) = -\frac{\omega\varepsilon b}{\pi} A_1 \cos(\frac{\pi y}{b})\sin(\omega t-\beta z),$$

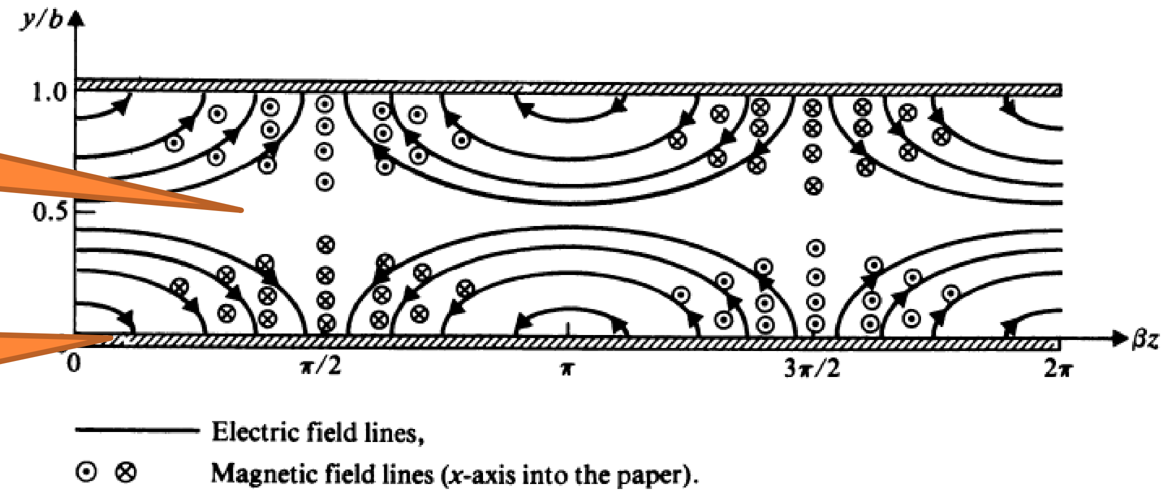
$$\beta = \sqrt{\omega^2\mu\varepsilon-(\frac{\pi}{b})^2}.$$

$$H_x(y,z;0) = \frac{\omega\varepsilon b}{\pi} A_1 \cos(\frac{\pi y}{b})\sin\beta z.$$

$$t=0$$

y=0.5b Ey=0

Perpendicular with respect to the plate



10-5 (a) Write the instantaneous field expressions for TE_1 mode in a parallel-plate waveguide.
(b) Sketch the electric and magnetic field lines in the *yz*-plane.

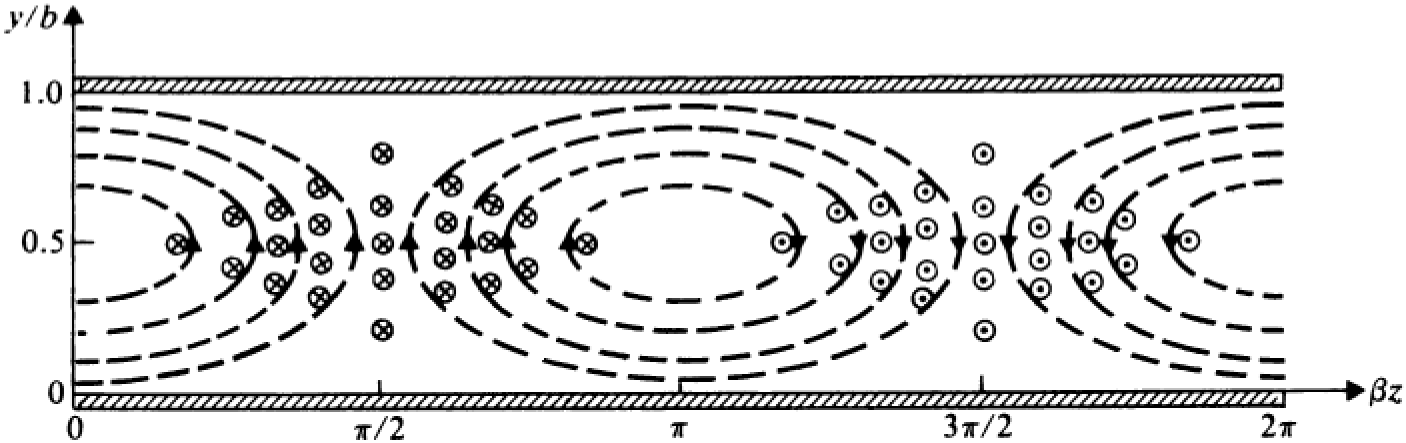
$$H_z(y,z;t) = B_1 \cos\left(\frac{\pi y}{b}\right)\cos(\omega t - \beta z),$$

$$H_y(y,z;t) = -\frac{\beta b}{\pi} B_1 \sin\left(\frac{\pi y}{b}\right)\sin(\omega t - \beta z),$$

$$E_x(y,z;t) = -\frac{\omega \mu b}{\pi} B_1 \sin\left(\frac{\pi y}{b}\right)\sin(\omega t - \beta z),$$

$$E_x(y,z;0) = \frac{\omega \mu b}{\pi} B_1 \sin\left(\frac{\pi y}{b}\right)\sin \beta z.$$

$$t = 0$$



Magnetic field lines,
Electric field lines (*x*-axis into the paper).

Rectangular waveguides

z-direction: propagation direction

TM: H_z=0

$$E_z^0(x,y) = E_0 \sin(\frac{m\pi}{a}x)\sin(\frac{n\pi}{b}y)$$

$$E_x^0(x,y) = -\frac{\gamma}{h^2}(\frac{m\pi}{a})E_0\cos(\frac{m\pi}{a}x)\sin(\frac{n\pi}{b}y)$$

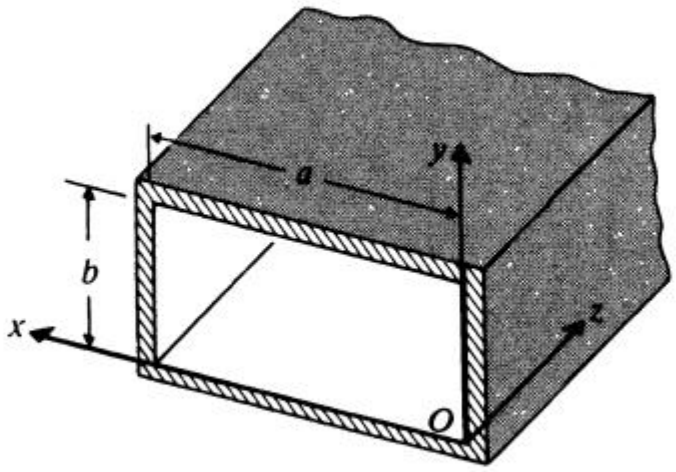
$$E_y^0(x,y) = -\frac{\gamma}{h^2}(\frac{n\pi}{b})E_0\sin(\frac{m\pi}{a}x)\cos(\frac{n\pi}{b}y)$$

$$H_x^0(x,y) = \frac{j\omega\varepsilon}{h^2}(\frac{n\pi}{b})E_0\sin(\frac{m\pi}{a}x)\cos(\frac{n\pi}{b}y)$$

$$H_y^0(x,y) = -\frac{j\omega\varepsilon}{h^2}(\frac{m\pi}{a})E_0\cos(\frac{m\pi}{a}x)\sin(\frac{n\pi}{b}y)$$

$$h^2 = (\frac{m\pi}{a})^2 + (\frac{n\pi}{b})^2$$

$$\gamma = j\beta = j\sqrt{\omega^2\mu\varepsilon - (\frac{m\pi}{a})^2 - (\frac{n\pi}{b})^2}$$



$$(f_c)_{mn} = \frac{1}{2\sqrt{\mu\varepsilon}}\sqrt{(\frac{m}{a})^2 + (\frac{n}{b})^2}$$

$$(\lambda_c)_{mn} = \frac{2}{\sqrt{(\frac{m}{a})^2 + (\frac{n}{b})^2}}$$

Neither m nor n can be zero ??? Why

10-7 (a)Write the instantaneous field expressions for TM$_{11}$ mode in a rectangular waveguide of sides *a* and *b*.
(b) Sketch the electric and magnetic field lines in a typical *xy*-plane and in a typical *yz*-plane.

$$E_x(x,y,z;t)=\frac{\beta}{h^2}(\frac{\pi}{a})E_0\cos(\frac{\pi}{a}x)\sin(\frac{\pi}{b}y)\sin(\omega t-\beta z),$$

$$E_y(x,y,z;t)=\frac{\beta}{h^2}(\frac{\pi}{b})E_0\sin(\frac{\pi}{a}x)\cos(\frac{\pi}{b}y)\sin(\omega t-\beta z),$$

$$\beta=\sqrt{k^2-h^2}=\sqrt{\omega^2\mu\varepsilon-(\frac{\pi}{a})^2-(\frac{\pi}{b})^2}.$$

$$E_z(x,y,z;t)=E_0\sin(\frac{\pi}{a}x)\sin(\frac{\pi}{b}y)\cos(\omega t-\beta z),$$

$$H_x(x,y,z;t)=-\frac{\omega\varepsilon}{h^2}(\frac{\pi}{b})E_0\sin(\frac{\pi}{a}x)\cos(\frac{\pi}{b}y)\sin(\omega t-\beta z),$$

$$H_y(x,y,z;t)=\frac{\omega\varepsilon}{h^2}(\frac{\pi}{a})E_0\cos(\frac{\pi}{a}x)\sin(\frac{\pi}{b}y)\sin(\omega t-\beta z),$$

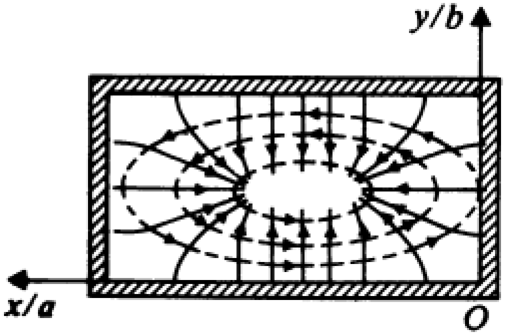
$$H_z(x,y,z;t)=0,$$

$$\omega t-\beta z=0$$

$$(\frac{dy}{dx})_E=\frac{a}{b}\tan(\frac{\pi}{a}x)\cot(\frac{\pi}{b}y),$$

$$(\frac{dy}{dx})_H=-\frac{b}{a}\cot(\frac{\pi}{a}x)\tan(\frac{\pi}{b}y),$$

$$(\frac{dy}{dx})_E(\frac{dy}{dx})_H=-1$$

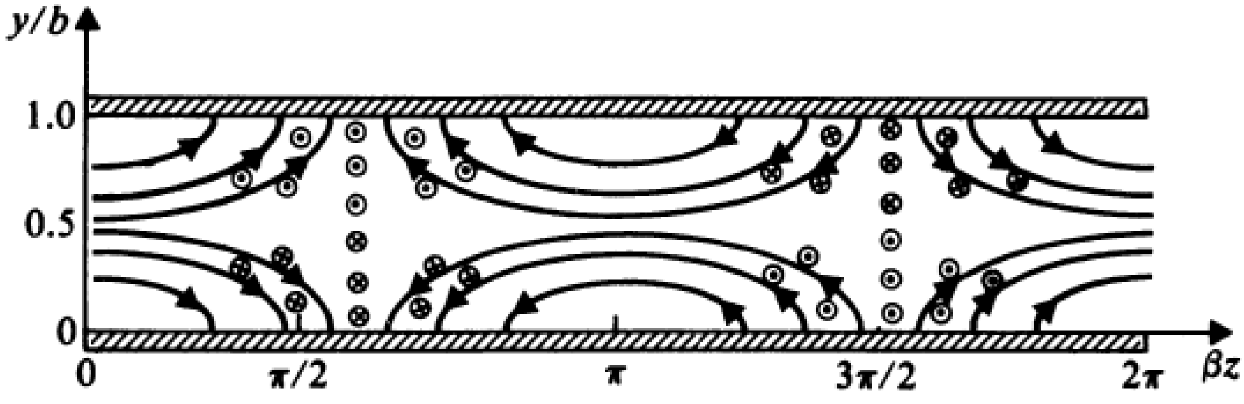


E and H lines are perpendicular to one another

E/H lines normal/parallel to the wall

$$t = 0$$

$$x = \frac{a}{2} \quad \Rightarrow \quad \sin(\frac{\pi}{a}x) = 1, \quad \cos(\frac{\pi}{a}x) = 0 \quad \Rightarrow \quad E_y,\ E_z,\ H_x$$



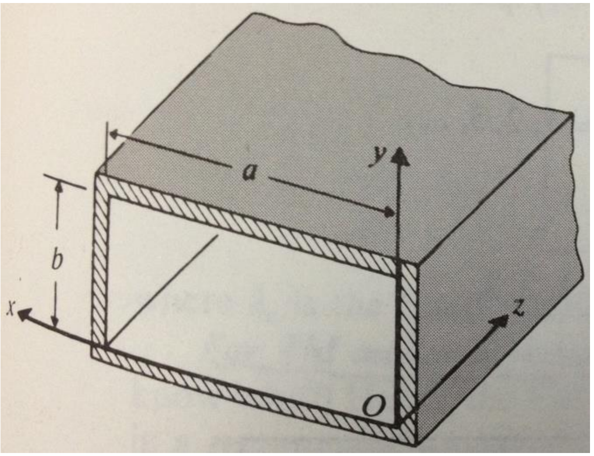
$$x = 0?$$

***xz*-plane?**

Rectangular waveguides

z-direction: propagation direction

TE: E_z=0



$$H_z^{\,0}(x,y) = H_0 \cos(\frac{m\pi}{a}x)\cos(\frac{n\pi}{b}y)$$

$$E_x^{\,0}(x,y) = \frac{j\omega\mu}{h^2}(\frac{n\pi}{b})H_0 \cos(\frac{m\pi}{a}x)\sin(\frac{n\pi}{b}y)$$

$$E_y^{\,0}(x,y) = -\frac{j\omega\mu}{h^2}(\frac{m\pi}{a})H_0 \sin(\frac{m\pi}{a}x)\cos(\frac{n\pi}{b}y)$$

$$H_x^{\,0}(x,y) = \frac{\gamma}{h^2}(\frac{m\pi}{a})H_0 \sin(\frac{m\pi}{a}x)\cos(\frac{n\pi}{b}y)$$

$$H_y^{\,0}(x,y) = \frac{\gamma}{h^2}(\frac{n\pi}{b})H_0 \cos(\frac{m\pi}{a}x)\sin(\frac{n\pi}{b}y)$$

$$h^2 = (\frac{m\pi}{a})^2 + (\frac{n\pi}{b})^2$$

$$\gamma = j\beta = j\sqrt{\omega^2\mu\varepsilon - (\frac{m\pi}{a})^2 - (\frac{n\pi}{b})^2}$$

$$(f_c)_{mn} = \frac{1}{2\sqrt{\mu\varepsilon}}\sqrt{(\frac{m}{a})^2 + (\frac{n}{b})^2}$$

$$(\lambda_c)_{mn} = \frac{2}{\sqrt{(\frac{m}{a})^2 + (\frac{n}{b})^2}}$$

Either m or n can be zero (not both)

The lowest mode TE_{10} if a>b

EXAMPLE 10–8 (a) Write the instantaneous field expressions for the TE_{10} mode in a rectangular waveguide having sides a and b. (b) Sketch the electric and magnetic field lines in typical xy-, yz-, and xz-planes. (c) Sketch the surface currents on the guide walls.

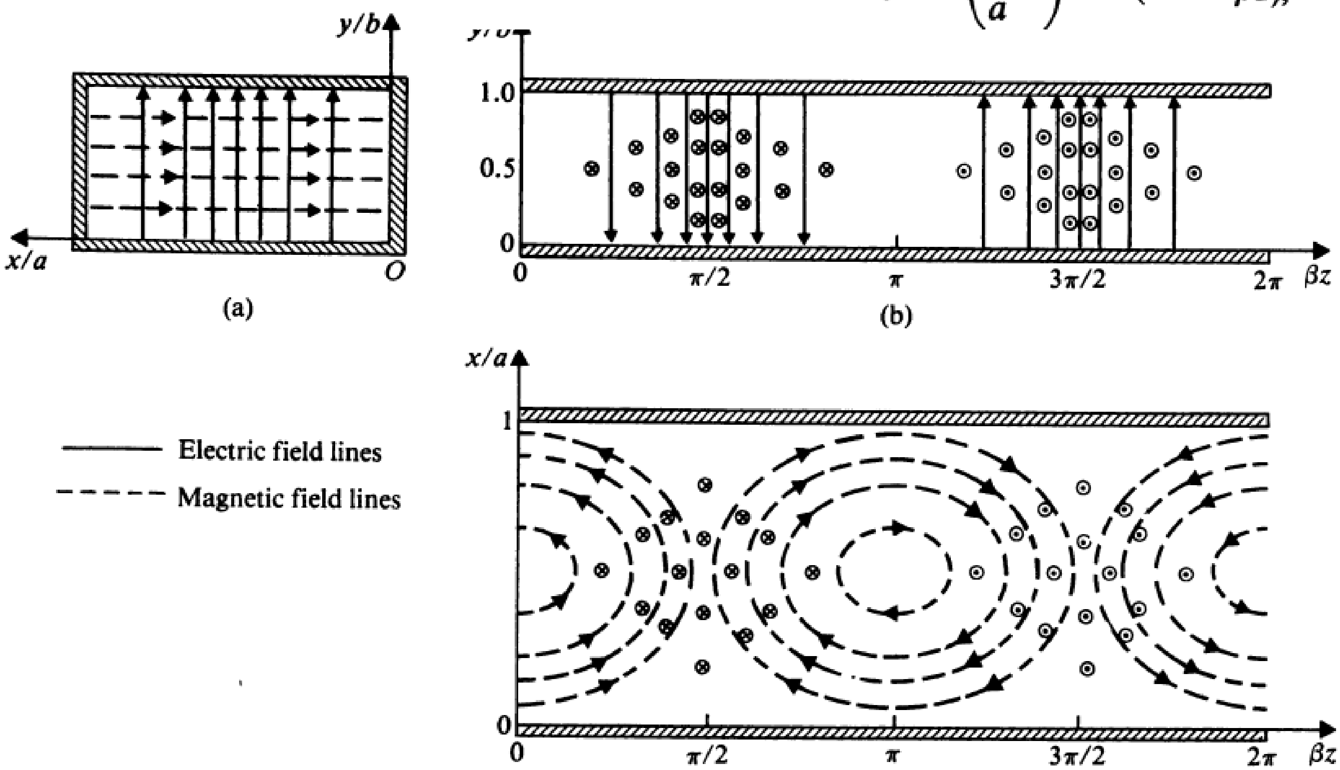
$$E_y(x, y, z; t) = \frac{\omega\mu}{h^2}\left(\frac{\pi}{a}\right)H_0 \sin\left(\frac{\pi}{a}x\right)\sin(\omega t - \beta z),$$

$$E_z(x, y, z; t) = 0,$$

$$H_x(x, y, z; t) = -\frac{\beta}{h^2}\left(\frac{\pi}{a}\right)H_0 \sin\left(\frac{\pi}{a}x\right)\sin(\omega t - \beta z),$$

$$H_y(x, y, z; t) = 0,$$

$$H_z(x, y, z; t) = H_0 \cos\left(\frac{\pi}{a}x\right)\cos(\omega t - \beta z),$$



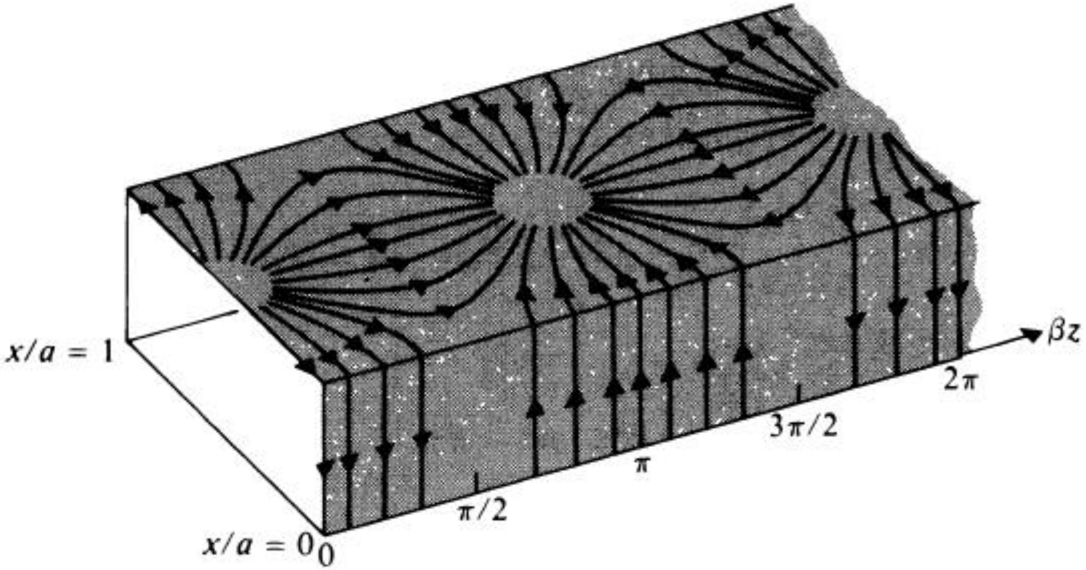
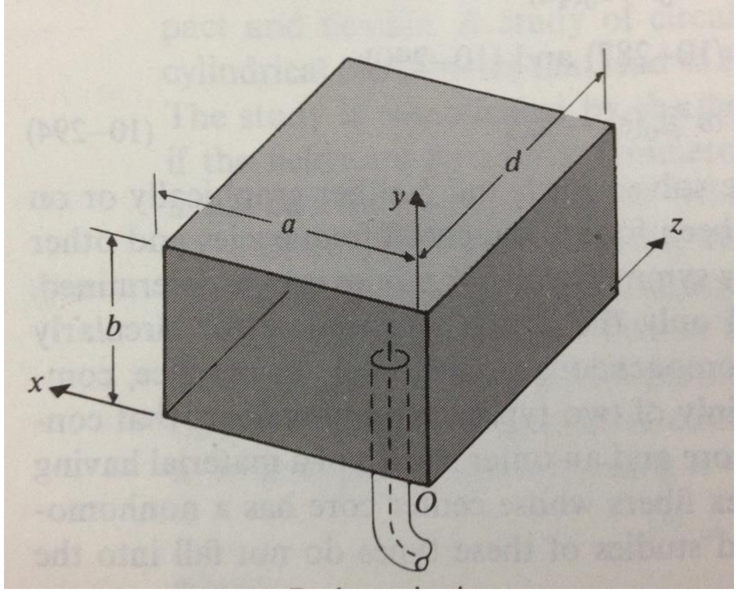


FIGURE 10–13
Surface currents on guide walls for TE_{10} mode in rectangular waveguide.

Cavity resonators

z-direction: propagation direction



$$f_{mnp} = \frac{1}{2\sqrt{\mu\varepsilon}}\sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2 + \left(\frac{p}{d}\right)^2}$$

TM: Neither m nor n can be zero
p can be zero

The lowest mode TM_{110}

TE: Either m or n can be zero (not both)
p can not be zero

The lowest mode TE_{101} or TE_{011}

Quality factor: Q

$$\boxed{Q = 2\pi\,\frac{\text{Time-average energy stored at a resonant frequency}}{\text{Energy dissipated in one period of this frequency}}.}$$

(Dimensionless)

10-15 Determine the dominant modes and their frequencies in an air-filled rectangular cavity resonator for(a) *a>b>d*, (b)*a>d>b*, and (c) *a=b=d*, where *a*, *b*, and *d* are the dimensions in the *x*-, *y*-, and *z*-directions, respectively.

the modes of the lowest orders are

$$TM_{110}, \qquad TE_{011}, \qquad TE_{101}.$$

(a) For *a>b>d*: the lowest resonant frequency is

$$f_{110} = \frac{c}{2}\sqrt{\frac{1}{a^2} + \frac{1}{b^2}}.$$

TM_{110} is the dominant mode.

(b) For *a>d>b*: the lowest resonant frequency is

$$f_{101} = \frac{c}{2}\sqrt{\frac{1}{a^2} + \frac{1}{d^2}}.$$

TE_{101} is the dominant mode.

(c) For *a=d=b*, all three of the lowest-order modes (namely, TM_{110} , TE_{011}, and TE_{101}) have the same field patterns. The resonant frequency of these degenerate modes is

$$f_{110} = \frac{c}{\sqrt{2}a}.$$