Supplementary

- 光强的定义
- Snell law讨论
  - p/s两种情况
  - normal/oblique两种情况
  - 光密到光疏和光疏到光密两种情况
- 透反射Transmittance T and Reflectance R
- 全反射和倏逝波
- 光波段金属表面的反射

S0: **Light Intensity**

Consider only (1) nonmagnetic media with $\mu = \mu_0$ (2) Optical region

$$\boxed{\mathscr{P}_{av} = \frac{1}{2}\mathscr{R}e(\mathbf{E} \times \mathbf{H}^*) \qquad (\mathrm{W/m^2}),} \qquad (8\text{–}96)^\dagger$$

which is a general formula for computing the average power density in a propagating wave.

$$I = \frac{P}{S} = \frac{1}{2}\sqrt{\frac{\varepsilon}{\mu}}E_0^{\ 2}$$

intensity is the power transferred per unit area, where the area is an imagined surface that is perpendicular to the direction of propagation of the energy

$$I = \bar{S} = \frac{1}{T}\int_0^T S dt = \frac{1}{T}\int_0^T \upsilon\varepsilon E^2 dt = \upsilon\varepsilon A^2 \frac{1}{T}\int_0^T \cos^2(kr - \omega t) dt = \frac{1}{2}\upsilon\varepsilon A^2$$

光强/功率密度

S1: Snell's Law

Perpendicular polarization (s-polarization)

Reflection Coefficient

$$\boxed{\Gamma_s = r_s = \frac{E_{r0}}{E_{i0}} = -\frac{\sin(\theta_i - \theta_t)}{\sin(\theta_i + \theta_t)} = \frac{n_1 \cos\theta_i - n_2 \cos\theta_t}{n_1 \cos\theta_i + n_2 \cos\theta_t}}$$

$$E_{i0} + E_{r0} = E_{t0}$$

Transmission Coefficient

$$\boxed{\tau_s = t_s = \frac{E_{t0}}{E_{i0}} = 1 + \Gamma_s = \frac{2\sin\theta_t \cos\theta_i}{\sin(\theta_i + \theta_t)} = \frac{2n_1 \cos\theta_i}{n_1 \cos\theta_i + n_2 \cos\theta_t}}$$

Parallel polarization (p-polarization)

Reflection Coefficient $\Gamma_p = r_p = \dfrac{tg(\theta_i - \theta_t)}{tg(\theta_i + \theta_t)} = \dfrac{n_2 \cos\theta_i - n_1 \cos\theta_t}{n_2 \cos\theta_i + n_1 \cos\theta_t}$

Transmission Coefficient $\tau_p = t_p = \dfrac{2 \sin\theta_t \cos\theta_i}{\sin(\theta_i + \theta_t)\cos(\theta_i - \theta_t)} = \dfrac{2 n_1 \cos\theta_i}{n_2 \cos\theta_i + n_1 \cos\theta_t}$

Normal incidence (正入射)

相对折射率

$$n = \frac{n_2}{n_1}$$

$$r_s = -\frac{n-1}{n+1} \qquad t_s = \frac{2}{n+1}$$

$$r_p = \frac{n-1}{n+1} \qquad t_p = \frac{2}{n+1}$$

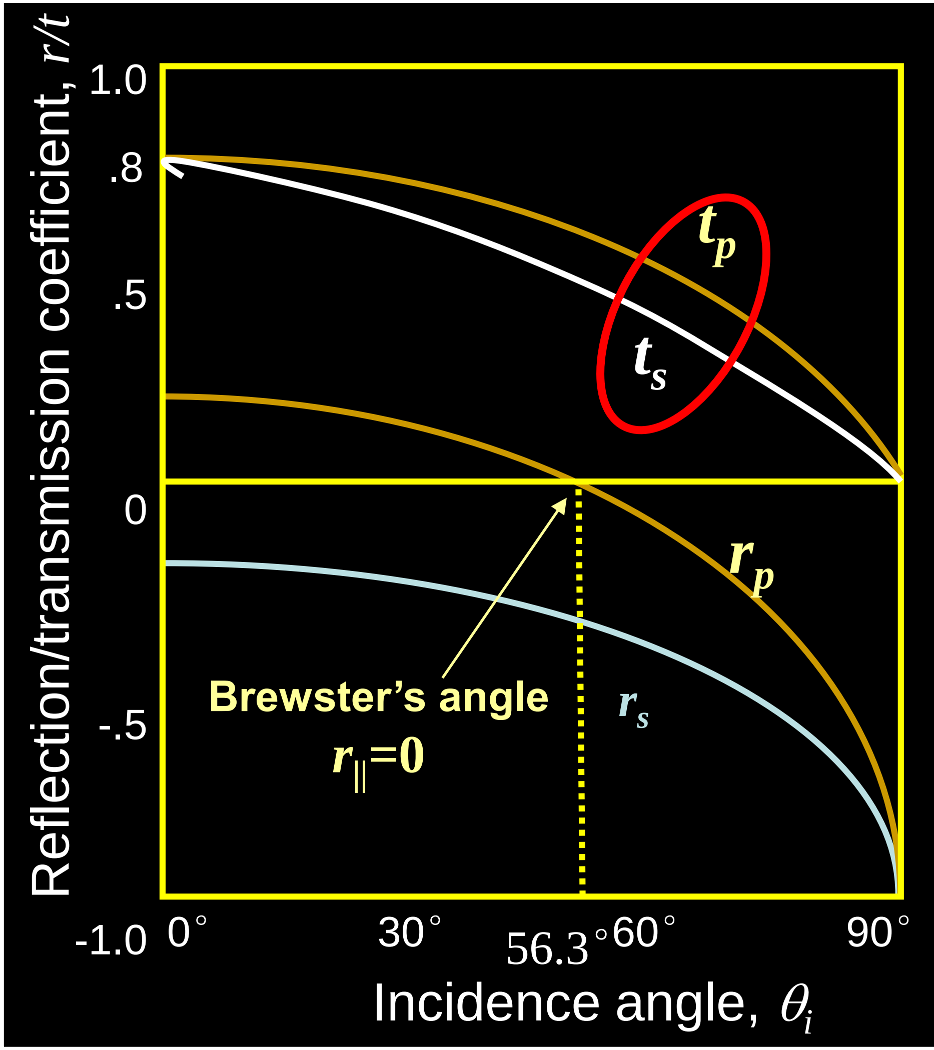
Case 1: $n_i < n_t$

Transmitted Field

$$t_s = \frac{2\sin\theta_t\cos\theta_i}{\sin(\theta_i+\theta_t)} > 0$$

$$t_p = \frac{2\sin\theta_t\cos\theta_i}{\sin(\theta_i+\theta_t)\cos(\theta_i-\theta_t)} > 0$$

t_s、t_p **decrease with θ_i**



$n_1=1.0,\ n_2=1.5$

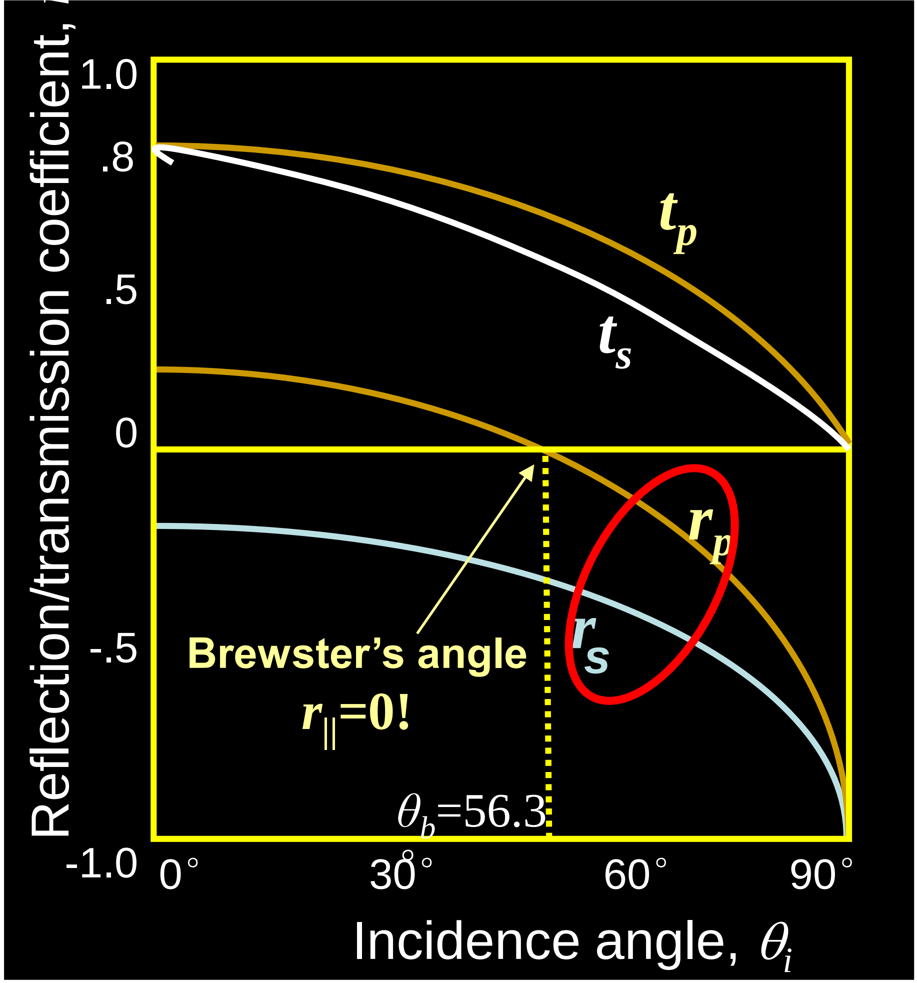
transmitted and incident field: in-phase

折射光与入射光同相位

Case 1: $n_i < n_t$

Reflected Field

$$r_s = -\frac{\sin(\theta_i - \theta_t)}{\sin(\theta_i + \theta_t)} \qquad r_p = \frac{tg(\theta_i - \theta_t)}{tg(\theta_i + \theta_t)}$$



$|r_s|$ **increases with θ_i until 1**

$|r_p| = 0$ **at** $\theta_i = \theta_B = 90^\circ - \theta_t$

No p component in reflection field

total polarization (全偏振)

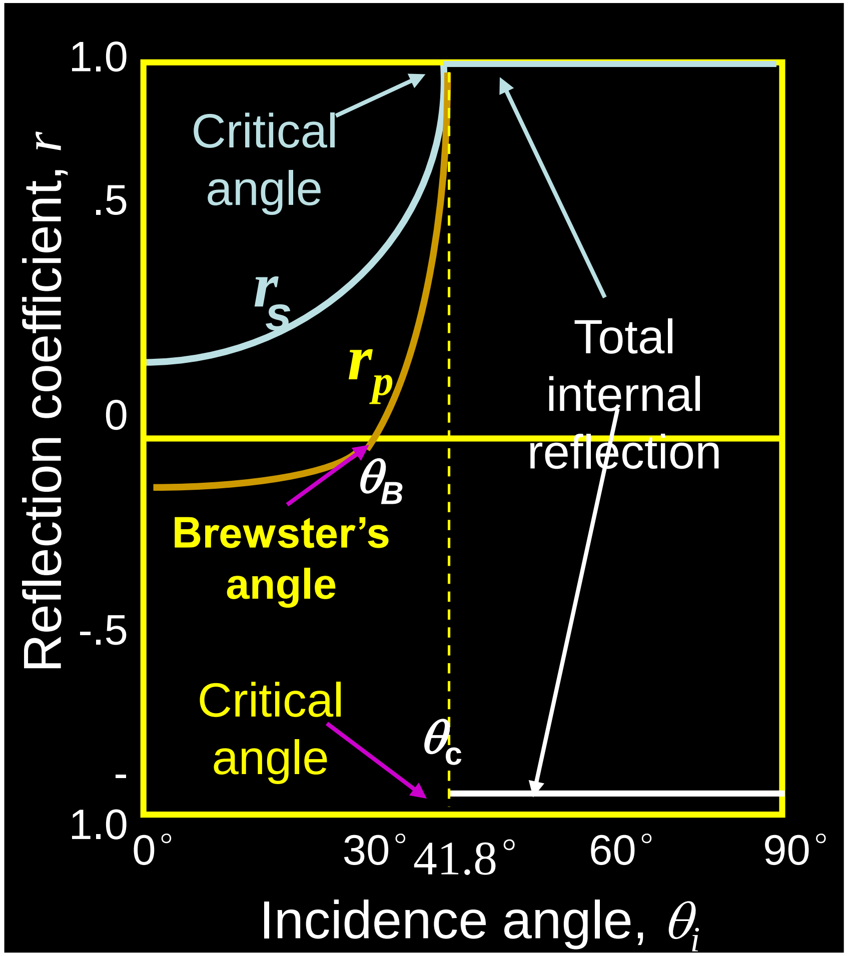
r_s<0，reflected and incident *s*-field: out-of-phase (π 相位突变)

$\theta_i < \theta_B$，r_p >0，reflected and incident *p*-field: in phase

$\theta_i > \theta_B$，r_p<0，reflected and incident *s*-field: out-of-phase (π相位突变)

Case 2: $n_i > n_t$

$n_i = 1.5, n_t = 1.0$



total polarization (全偏振)

-Brewster's angle

total reflection (全反射)

-critical angle(临界角)

$$\theta_{crit} \equiv \arcsin(n_2/n_1)$$

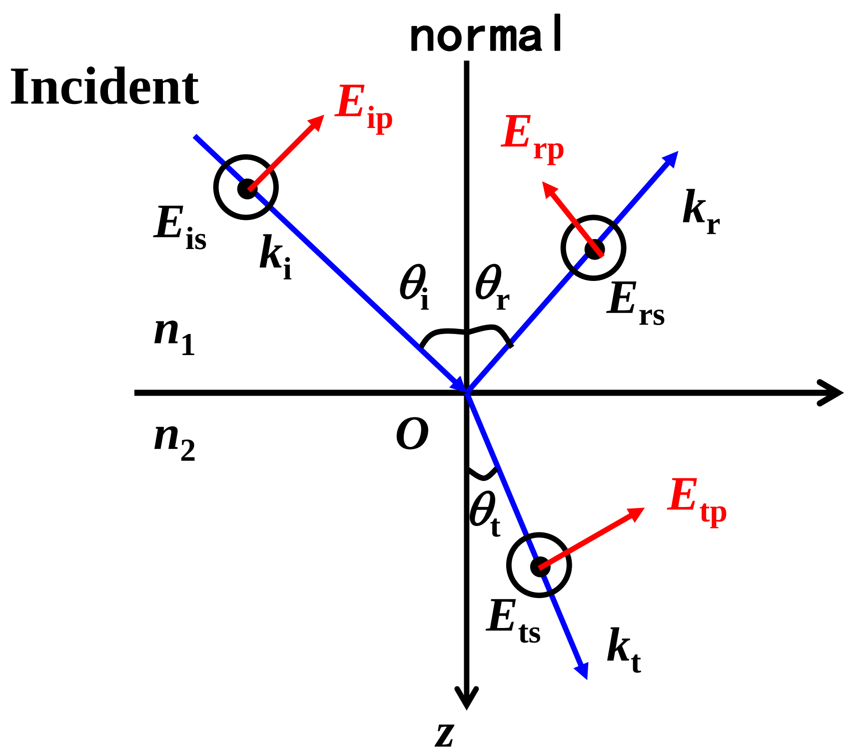
$\underline{\theta_i < \theta_c}$, $r_s > 0$, **reflected and incident *s*-field: in-phase**

$\underline{\theta_C \geq \theta_i \geq \theta_B}$, $r_p > 0$, **reflected and incident *p*-field: in-phase**

$\underline{\theta_i < \theta_B}$, $r_p < 0$, **reflected and incident *p*-field: out-of-phase**

Phases between reflected and incident field (with both p and s components)

s、p 分量光电场振动正方向规定



Reference（参考）

Only considering normal incidence ($\theta_i = 0$)

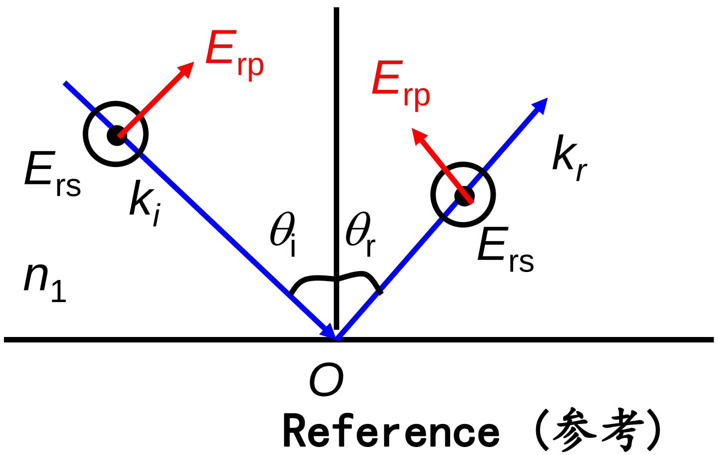
Case 1: $n_1 < n_2$

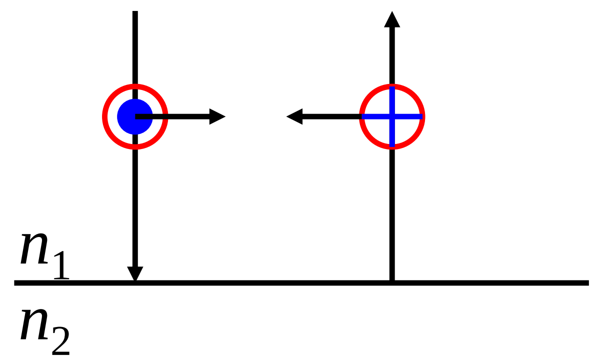
$r_s < 0$，reflected s-component is opposite to that for Reference

反射光中的*s*分量与规定正方向相反

$r_p > 0$，reflected p-component is the same as that for Reference

反射光中的*p*分量与规定正方向相同。



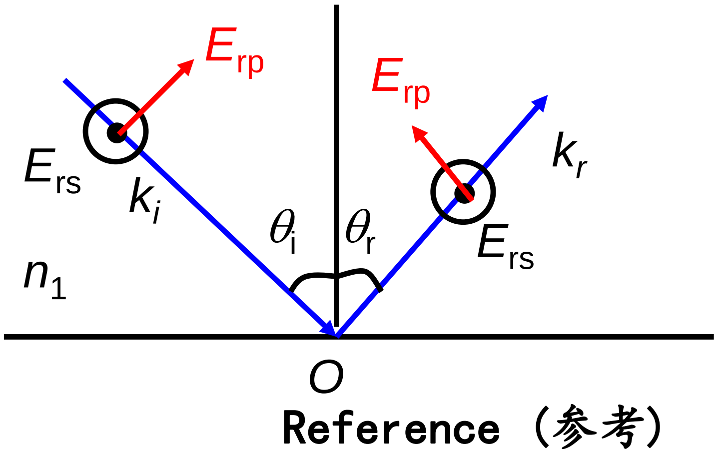


Total reflected field vs total incident field: π phase shift/ half-wavelength loss (相位发生π 突变，或半波损失)。

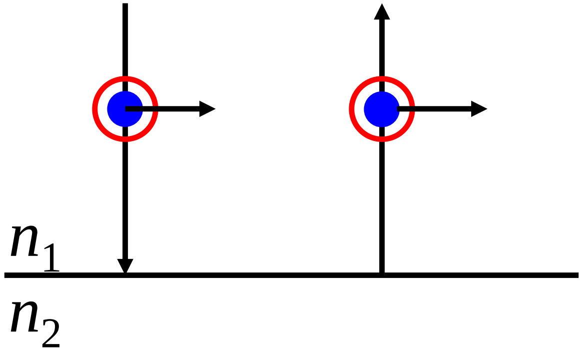
即光从疏到密反射有半波损失.

Only considering normal incidence ($\theta_i = 0$)

Case 2: $n_1 > n_2$



$$r_p < 0, r_s > 0$$



Total reflected field vs total incident field: in-phase

<u>光从密到疏反射无半波损失</u>

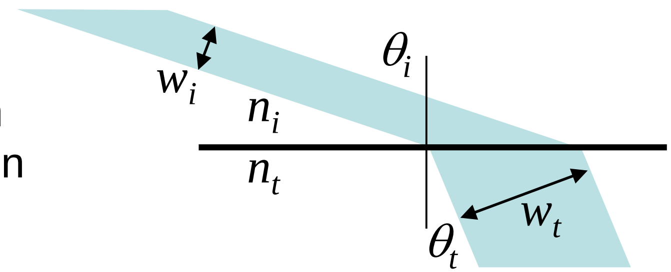
S2 Transmittance (T) and Reflectance (R)

$$I = \left(n \frac{\varepsilon_0 c_0}{2} \right) \left| E_0 \right|^2$$

$T \equiv$ Transmitted Power / Incident Power $= \dfrac{I_t A_t}{I_i A_i}$

A = Area

1D beam expansion



$$\frac{A_t}{A_i} = \frac{w_t}{w_i} = \frac{\cos\theta_t}{\cos\theta_i}$$

The beam expands in one dimension on refraction.

$$T = \frac{I_t A_t}{I_i A_i} = \frac{n_2 \left| E_{0t} \right|^2}{n_1 \left| E_{0i} \right|^2} \frac{\cos\theta_t}{\cos\theta_i} = \frac{n_2}{n_1} \frac{\cos\theta_t}{\cos\theta_i} t^2$$

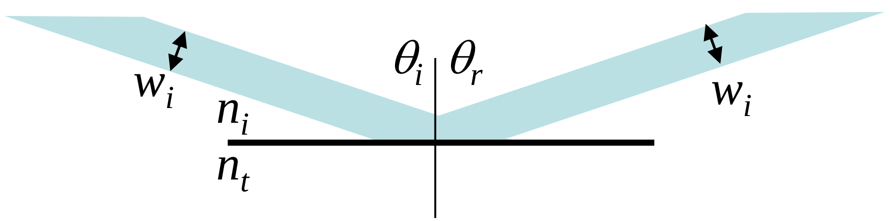
The Transmittance is also called the Transmissivity.

S2 Transmittance (T) and Reflectance (R)

$$I = \left(n \frac{\varepsilon_0 c_0}{2} \right) |E_0|^2$$

$$R \equiv \text{Reflected Power / Incident Power} = \frac{I_r A_r}{I_i A_i}$$

A = Area



Because the angle of incidence = the angle of reflection, the beam area doesn't change on reflection.

Also, n is the same for both incident and reflected beams.

So:

$$\boxed{R = r^2}$$

The Reflectance is also called the Reflectivity.

S2 Transmittance (T) and Reflectance (R)

$$R_s = r_s^2 = \frac{\sin^2(\theta_i - \theta_t)}{\sin^2(\theta_i + \theta_t)} \qquad R_p = r_p^2 = \frac{\tan^2(\theta_i - \theta_t)}{\tan^2(\theta_i + \theta_t)} \qquad \boxed{R_s + T_s = 1}$$

$$T_s = \frac{n_2 \cos\theta_t}{n_1 \cos\theta_i} t_s^2 = 1 - R_s \qquad T_p = \frac{n_2 \cos\theta_t}{n_1 \cos\theta_i} t_p^2 = 1 - R_p \qquad \boxed{R_p + T_p = 1}$$

Reflection at normal incidence

When $\theta_i = 0$,

$$R = \left(\frac{n_2 - n_1}{n_2 + n_1} \right)^2$$

and

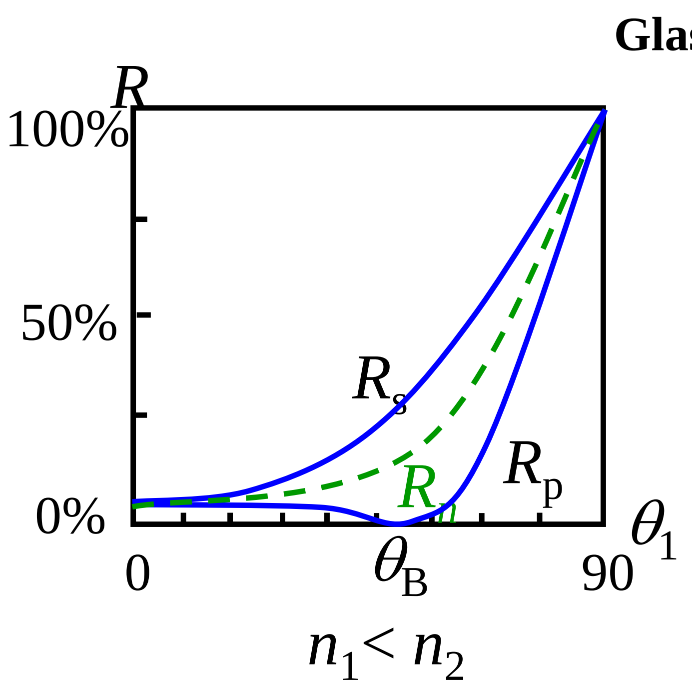
$$T = \frac{4 n_2 n_1}{\left(n_2 + n_1 \right)^2}$$

For an air-glass interface ($n_1 = 1$ and $n_2 = 1.5$),

R = 4% and T = 96%

The values are the same, whichever direction the light travels, from air to glass or from glass to air.

The 4% has big implications for photography lenses.



$\theta_1 < \theta_B$， R increases slowly with θ_1; At θ_1=0, $R_s = R_P = 4\%$
$\theta_1 > \theta_B$， R increases drastically with θ_1 until $R_s = R_p = 1$。

Natural light（自然光）： equal *s*- and *p*-components

$$R_n = \frac{W_1'}{W_1} = \frac{W_s + W_p}{W_1} = \frac{1}{2}(R_s + R_p)$$

S3 Total reflection (全反射)

Reflection Coefficient

$$\sin\theta_t = \frac{n_1}{n_2}\sin\theta_i = \frac{\sin\theta_i}{n} \qquad \cos\theta_t = \pm i\sqrt{\frac{\sin^2\theta_i}{n^2} - 1}$$

$$r_s = \frac{\cos\theta_i - i\sqrt{\sin^2\theta_i - n^2}}{\cos\theta_i + i\sqrt{\sin^2\theta_i - n^2}} \qquad r_p = \frac{n^2\cos\theta_i - i\sqrt{\sin^2\theta_i - n^2}}{n^2\cos\theta_i + i\sqrt{\sin^2\theta_i - n^2}}$$

Expressed in complex

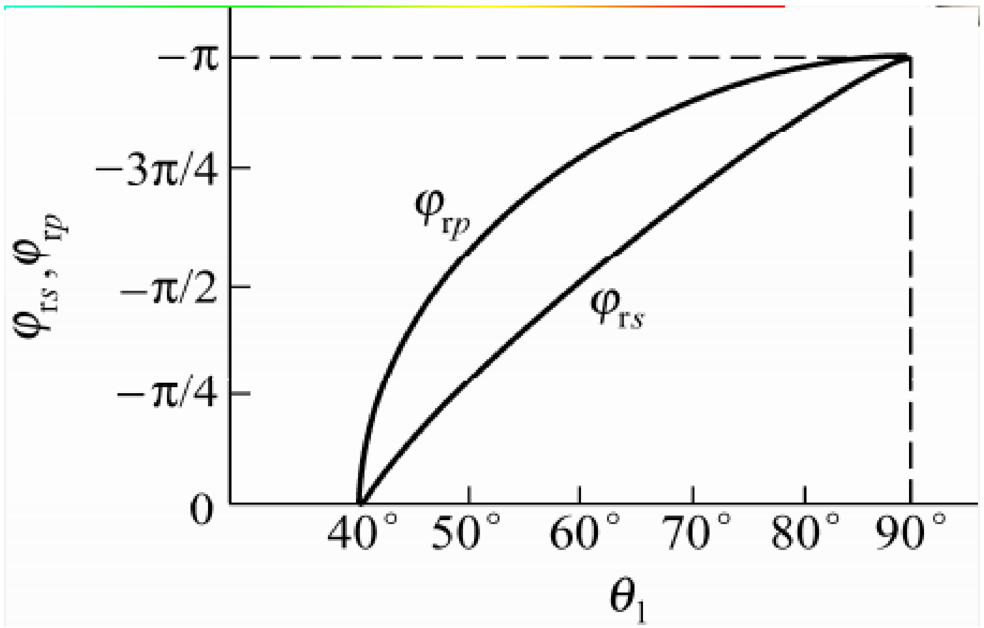
$$r_s = |r_s| e^{i\delta s} \qquad r_p = |r_p| e^{i\delta p}$$

$$tg\frac{\delta s}{2} = -\frac{\sqrt{\sin^2\theta_i - n^2}}{\cos\theta_i} \qquad tg\frac{\delta p}{2} = -\frac{\sqrt{\sin^2\theta_i - n^2}}{n^2\cos\theta_i}$$

Reflected field: *s*-, *p*- *components*

Phase shift is different

S3 Total reflection (全反射)



Reflected field
***s*-, *p*- components**
Phase shift is different

$\theta_i = \theta_c$, **if incident is linearly polarized, then reflected is also linearly polarized.**

$\theta_i > \theta_c$, **if incident is linearly polarized with both s- and p-components, then reflected is elliptically polarized.**

S3 Total reflection (全反射)

Evanescent Wave (倏逝波)

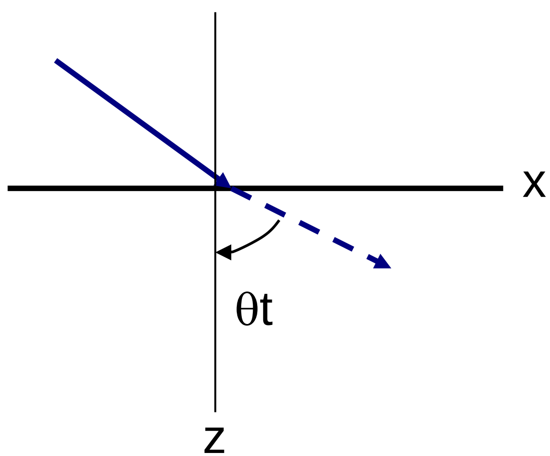
Though the entire incident wave is reflected back into the originating medium, there is some penetration into the second medium at the boundary. The evanescent wave appears to travel along the boundary between the two materials, leading to the Goos-Hänchen shift.

Transmitted field：

xz plane

$$E_t = A_t \exp[i(\vec{k}\cdot\vec{r} - \omega t)]$$

$$E_t = A_t \exp[i(k_{tx}x + k_{tz}z - \omega t)]$$



$$k_{tx} = k_t \sin\theta_t = k_t \frac{\sin\theta_i}{n}$$

$$\boldsymbol{n} = \frac{\boldsymbol{n}_1}{\boldsymbol{n}_2}$$

$$k_{tz} = k_t \cos\theta_t = \pm i k_t \sqrt{\frac{\sin^2\theta_i}{n^2} - 1} = \pm i\kappa$$

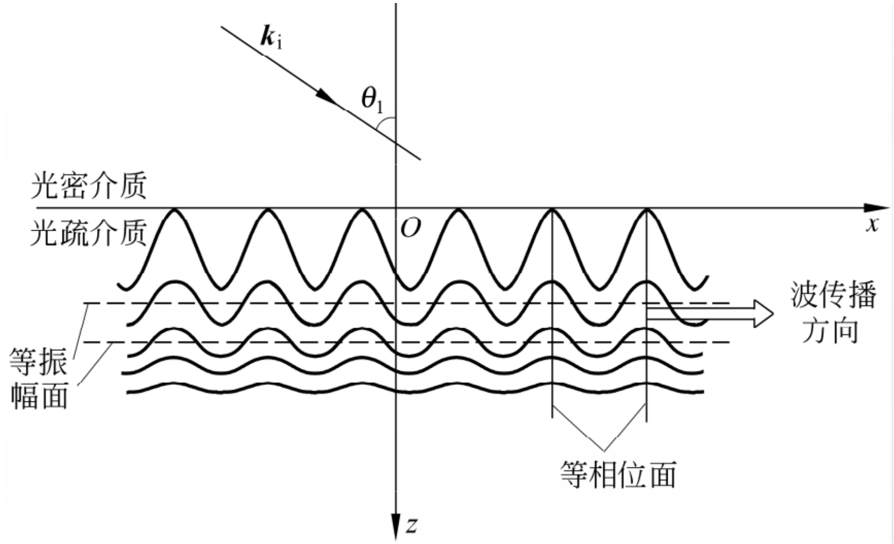
S3 Total reflection (全反射)

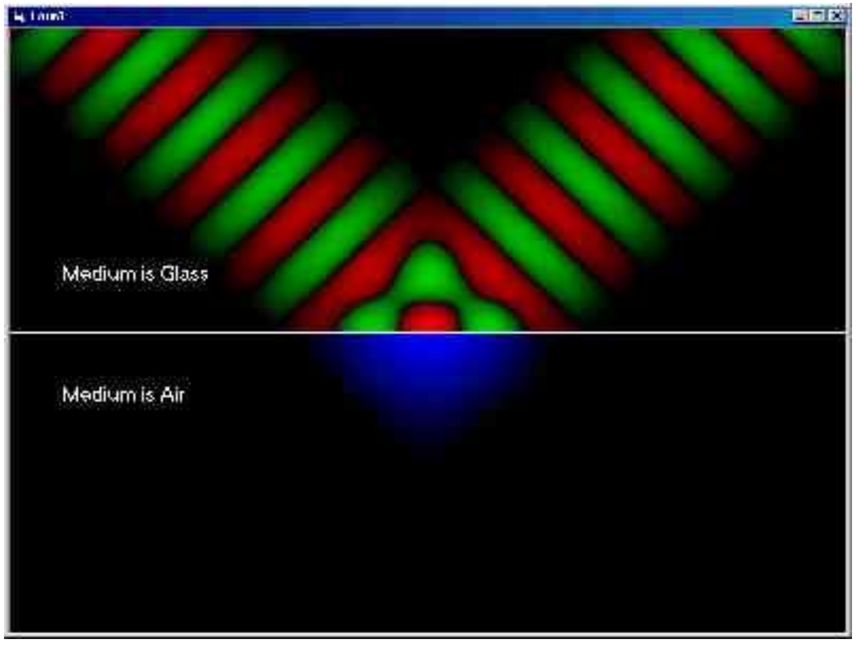
Evanescent Wave (倏逝波)

Transmitted field: $E_t = A_t \exp(\mp\kappa z)\exp[i(k_{tx}x - \omega t)]$

Propagate: x-direction; Attenuate: z-direction

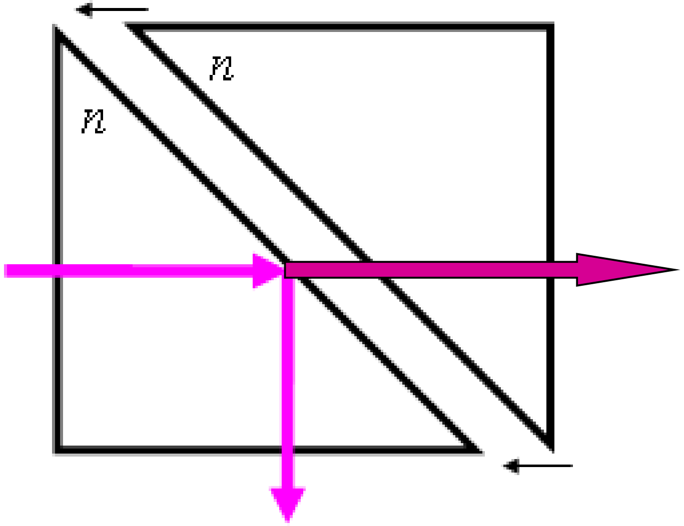
Penetration depth (穿透深度) $z_0 = \frac{1}{\kappa} = \frac{n}{k_t\sqrt{\sin^2\theta_i - n^2}}$

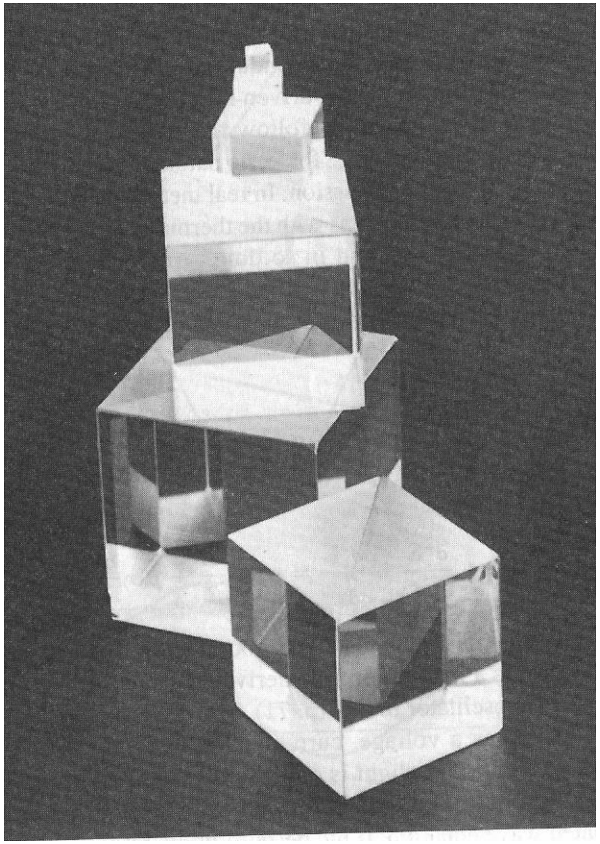




Application

Beamsplitters using TIR





Splitting ratio depends on air gap

S4 Optical reflection on metal surfaces

$$\tilde{\varepsilon} = \varepsilon - i\frac{\sigma}{\omega} = \varepsilon_0 \tilde{\varepsilon}_r$$

$$\tilde{n} = \sqrt{\tilde{\varepsilon}} = n + ik$$

For an air-metal interface, take

$$R = |r|^2 = \left(\frac{\tilde{n}-1}{\tilde{n}+1}\right)\left(\frac{\tilde{n}-1}{\tilde{n}+1}\right)^* = \frac{(n-1)^2 + k^2}{(n+1)^2 + k^2}$$

Metal	***n***	***k***
Ag	0.18	3.64
Au	0.37	2.82
Al	1.44	5.23
Cu	0.64	2.62

at λ=589 nm

If σ is large, k is large and $k >> n$ and $R \to 1$.

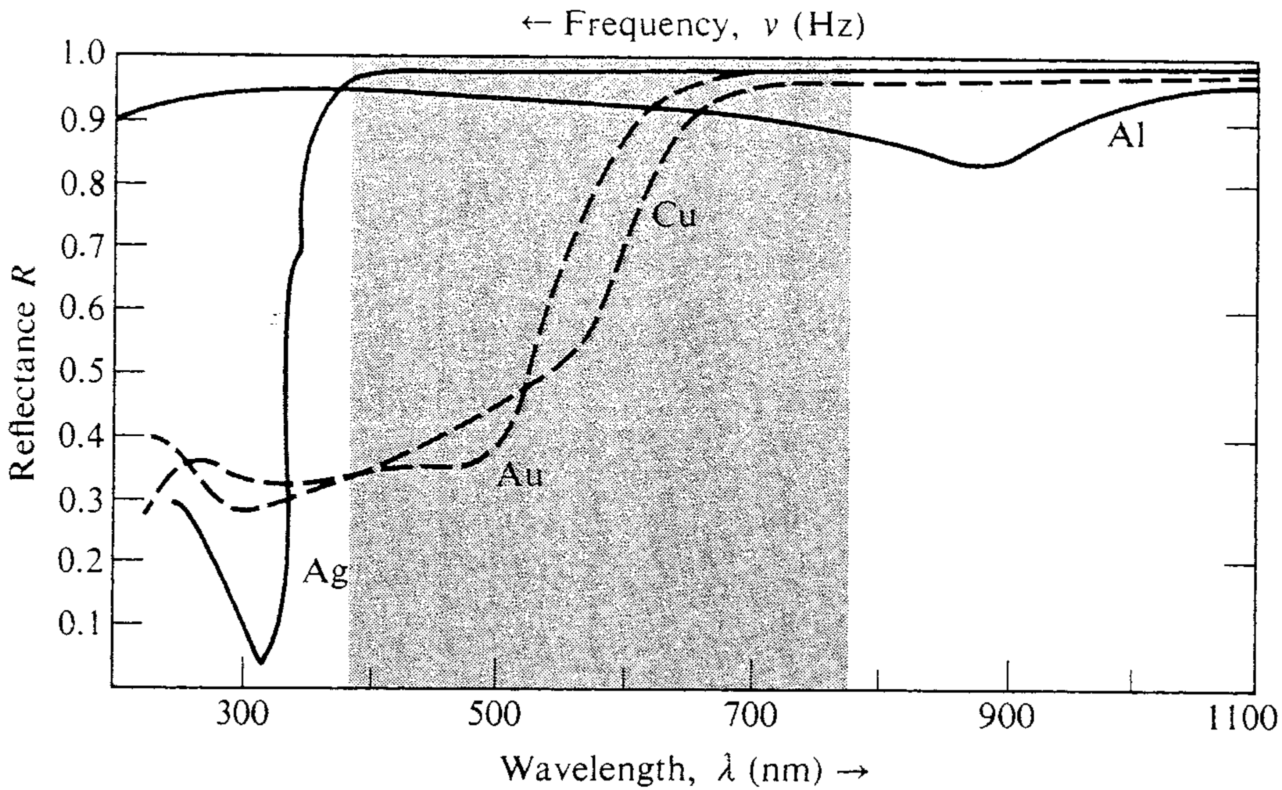


Figure 4.59 Reflectance versus wavelength for silver, gold, copper, and aluminum.