I realize that, no matter how careful I have endeavored to be, occasional errors may still exist. I should be grateful if you would be kind enough to notify me as you discover them either in the book or in this manual.

Sincerely,

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(For the use of instructors only.)

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A note to instructors using FIELD AND WAVE ELECTROMAGNETICS.

Dear Colleague:

As teachers of introductory electromagnetics, we are all aware of two facts: that most students consider the subject matter difficult, and that there are numerous books on the market dealing with this subject. It is understandable that students find electromagnetics difficult. First of all, the subject matter is built upon abstract models that demand a good mathematical background. Second, before the course on electromagnetics, students who have studied circuit theory normally encounter functions of only one independent variable, namely, time; whereas in electromagnetics they are suddenly required to deal with functions of four variables (space and time). This is a big transition, and visualization problems associated with solid geometry add to the difficulty. Finally, students are often confused about the way the subject matter is developed, even after they have completed the course, mainly because most books do not provide a unified and comprehensible approach.

As I point out in the Preface of the book, the inductive approach of beginning with the various experimental laws tends to be fragmented and lacks cohesiveness, whereas the practice of writing the four general Maxwell's equations at the outset without discussing their necessity and sufficiency presents a major stumbling block for learning. Students are often puzzled about the structure of the electromagnetic model. I sincerely believe that the gradual axiomatic approach based on Helmholtz's theorem used in this book provides unity in the gradational development of the electromagnetic model from the very simple model for electrostatics. Although a rigorous mathematical proof of Helmholtz's theorem is relatively involved (not included in the book), the physical concept of specifying both the flow source and the vortex (circulation) source in order to define a vector field is quite simple.

Many review questions are provided at the end of each chapter. They are designed to review and reinforce the essential material in the chapter without the need for a calculator. You may wish to use them as a vehicle for discussion in class.

I have tried to make the problems in each chapter meaningful and to avoid trivial number-plugging types. This solutions manual gives the solutions and answers to all the problems in the book. I hope it proves to be a useful aid in teaching from the book. Answers to odd-numbered problems are included in the back of the book.

Chapter 2

P.2-1 (a) $\bar{a}_A = \frac{\bar{A}}{A} = \frac{\bar{a}_x + \bar{a}_y 2 - \bar{a}_z 3}{\sqrt{1^2 + 2^2 + (-3)^2}} = \frac{1}{\sqrt{14}}(\bar{a}_x + \bar{a}_y 2 - \bar{a}_z 3)$

(b) $|\bar{A} - \bar{B}| = |\bar{a}_x + \bar{a}_y 6 - \bar{a}_z 4| = \sqrt{1^2 + 6^2 + 4^2} = \sqrt{53}$

(c) $\bar{A} \cdot \bar{B} = 0 + 2(-4) + (-3) = -11$

(d) $\theta_{AB} = \cos^{-1}(\bar{A}\cdot\bar{B}/AB) = \cos^{-1}(-11/\sqrt{14}\sqrt{17}) = 135.5°$

(e) $\bar{A} \cdot \bar{a}_C = \bar{A} \cdot \frac{\bar{C}}{C} = \bar{A} \cdot \frac{1}{\sqrt{29}}(\bar{a}_x 5 - \bar{a}_z 2) = \frac{11}{\sqrt{29}}$

(f) $\bar{A} \times \bar{C} = -\bar{a}_x 4 - \bar{a}_y 13 - \bar{a}_z 10$

(g) $\bar{A} \cdot (\bar{B} \times \bar{C}) = (\bar{A} \times \bar{B}) \cdot \bar{C} = -42$

(h) $(\bar{A} \times \bar{B}) \times \bar{C} = \bar{B}(\bar{A}\cdot\bar{C}) - \bar{A}(\bar{C}\cdot\bar{B}) = \bar{a}_x 2 - \bar{a}_y 40 + \bar{a}_z 5$

$\bar{A} \times (\bar{B} \times \bar{C}) = \bar{B}(\bar{A}\cdot\bar{C}) - \bar{C}(\bar{A}\cdot\bar{B}) = \bar{a}_x 55 + \bar{a}_y 44 - \bar{a}_z 11$

P.2-2 Position vectors of the three corners:

$\overrightarrow{OP_1} = \bar{a}_y - \bar{a}_z 2$, $\overrightarrow{OP_2} = \bar{a}_x 4 - \bar{a}_y - \bar{a}_z 3$, $\overrightarrow{OP_3} = \bar{a}_x 6 + \bar{a}_y 2 + \bar{a}_z 5$

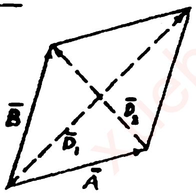
Vectors representing the three sides of the triangle:

$\overrightarrow{P_1P_2} = \overrightarrow{OP_2} - \overrightarrow{OP_1} = \bar{a}_x 4 - \bar{a}_z$, $\overrightarrow{P_2P_3} = \bar{a}_x 2 + \bar{a}_y + \bar{a}_z 8$, $\overrightarrow{P_3P_1} = -\bar{a}_x 6 - \bar{a}_y - \bar{a}$

(a) $\overrightarrow{P_1P_2} \cdot \overrightarrow{P_2P_3} = 0$. $\therefore \Delta P_1P_2P_3$ is a right triangle.

(b) Area of triangle $= \frac{1}{2}|\overrightarrow{P_1P_2} \times \overrightarrow{P_2P_3}| = 17.1$

P.2-3



$\bar{D}_1 = \bar{B} + \bar{A}$, $\bar{D}_2 = \bar{B} - \bar{A}$

$\bar{D}_1 \cdot \bar{D}_2 = (\bar{B} + \bar{A}) \cdot (\bar{B} - \bar{A})$

$= \bar{B}\cdot\bar{B} - \bar{A}\cdot\bar{A} = 0$

$\therefore \bar{D}_1 \perp \bar{D}_2$

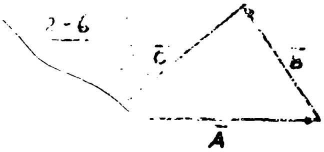
P.2-4 From $\bar{A}\cdot\bar{B} = \bar{A}\cdot\bar{C}$, we have $\bar{A}\cdot(\bar{B} - \bar{C}) = 0$. (1)

From $\bar{A}\times\bar{B} = \bar{A}\times\bar{C}$, we have $\bar{A}\times(\bar{B} - \bar{C}) = 0$. (2)

(1) implies $\bar{A} \perp (\bar{B} - \bar{C})$ and (2) implies $\bar{A} \parallel (\bar{B} - \bar{C})$. Since $\bar{A}$ is not a null vector, (1) and (2) cannot hold at the same time unless $(\bar{B} - \bar{C})$ is a null vector. Thus, $\bar{B} - \bar{C} = 0$ or $\bar{B} = \bar{C}$.

P.2-5 $\bar{a}_A \cdot \bar{a}_B = \cos(\alpha - \beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$.

2-6



$\bar{A} + \bar{B} + \bar{C} = 0$

$\bar{A}\times$: $\bar{A} \times \bar{B} = \bar{C} \times \bar{A}$

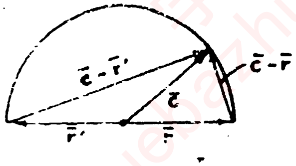
$\bar{C}\times$: $\bar{C} \times \bar{A} = \bar{B} \times \bar{C}$

$\bar{B}\times$: $\bar{B} \times \bar{C} = \bar{A} \times \bar{B}$

Magnitude relations: $AB \sin\theta_{AB} = CA \sin\theta_{CA} = BC \sin\theta_{BC}$

Hence $$\frac{A}{\sin\theta_{BC}} = \frac{B}{\sin\theta_{CA}} = \frac{C}{\sin\theta_{AB}}.$$

2-7



$\bar{r}' = -\bar{r}, \quad r' = r$

$(\bar{c}-\bar{r}')\cdot(\bar{c}-\bar{r}) = (\bar{c}+\bar{r})\cdot(\bar{c}-\bar{r})$
$= 0.$

$\therefore\ (\bar{c}-\bar{r}') \perp (\bar{c}-\bar{r}).$

2-9 Expand $\bar{A}\times(\bar{A}\times\bar{X}) = \bar{A}(\bar{A}\cdot\bar{X}) - \bar{X}(\bar{A}\cdot\bar{A})$

or $\bar{A}\times\bar{P} = p\bar{A} - A^2\bar{X}$

$\therefore\ \bar{X} = \frac{1}{A^2}(p\bar{A} + \bar{P}\times\bar{A}).$

2-10 $\bar{A}_{P_1} = -\bar{a}_y 3 - \bar{a}_z 2$, $\overrightarrow{OP_1} = -\bar{a}_y 2 + \bar{a}_z 3$

$\overrightarrow{OP_2} = \bar{a}_x(r\cos\phi) + \bar{a}_y(r\sin\phi) + \bar{a}_z = \bar{a}_x\frac{\sqrt{3}}{2} - \bar{a}_y\frac{1}{2} + \bar{a}_z$

$\overline{P_1P_2} = \overline{OP_2} - \overline{OP_1} = \bar{a}_x\frac{\sqrt{3}}{2} + \bar{a}_y\frac{1}{2} - \bar{a}_z 2$, $|\overline{P_1P_2}| = \sqrt{5}$

$\bar{A}_{P_1}\cdot\bar{a}_{P_1P_2} = \bar{A}_{P_1}\cdot\frac{\overline{P_1P_2}}{|\overline{P_1P_2}|} = \frac{\sqrt{5}}{2} = 1.12.$

2-11 (a) $x = r\cos\phi = 4\cos(2\pi/3) = -2$

$y = r\sin\phi = 4\sin(2\pi/3) = 2\sqrt{3}$

$z = 3$

(b) $R = (r^2+z^2)^{1/2} = (4^2+3^2)^{1/2} = 5$

$\theta = \tan^{-1}(r/z) = \tan^{-1}(4/3) = 53.1°$

$\phi = 2\pi/3 = 120°.$

2-12 (a) $\bar{E}_p = \bar{a}_R\frac{25}{(-3)^2+4^2+(-5)^2} = \bar{a}_R\frac{1}{2}$

$(E_p)_x = \frac{1}{2}\left(-\frac{3}{\sqrt{50}}\right) = 0.212$

(b) $\bar{a}_R = \frac{1}{\sqrt{50}}(-\bar{a}_x 3 + \bar{a}_y 4 - \bar{a}_z 5)$, $\bar{a}_B = \frac{\bar{B}}{B} = \frac{1}{3}(\bar{a}_x 2 - \bar{a}_y 2 + \bar{a}_z)$

$\theta = \cos^{-1}(\bar{a}_R\cdot\bar{a}_B) = \cos^{-1}\left(-\frac{19}{3\sqrt{50}}\right) = 154°.$

P.2-13 $\bar{a}_R = \bar{a}_x \sin\theta\cos\phi + \bar{a}_y \sin\theta\sin\phi + \bar{a}_z \cos\theta = \dfrac{\bar{a}_x x + \bar{a}_y y + \bar{a}_z z}{\sqrt{x^2+y^2+z^2}}$

$\bar{a}_\theta = \bar{a}_x \cos\theta\cos\phi + \bar{a}_y \cos\theta\sin\phi - \bar{a}_z \sin\theta = \dfrac{\bar{a}_x xz + \bar{a}_y yz - \bar{a}_z(x^2+y^2)}{\sqrt{(x^2+y^2)(x^2+y^2+z^2)}}$

$\bar{a}_\phi = -\bar{a}_x \sin\phi + \bar{a}_y \cos\phi = \dfrac{-\bar{a}_x y + \bar{a}_y x}{\sqrt{x^2+y^2}}$.

P.2-14 $\int_{P_1}^{P_2} \bar{E}\cdot d\bar{\ell} = \int_{P_1}^{P_2} (y\,dx + x\,dy)$.

(a) $x = 2y^2$, $dx = 4y\,dy$; $\int_{P_1}^{P_2} \bar{E}\cdot d\bar{\ell} = \int_1^2 (4y^2dy + 2y^2dy) = 14$

(b) $x = 6y-4$, $dx = 6dy$; $\int_{P_1}^{P_2} \bar{E}\cdot d\bar{\ell} = \int_1^2 [6y\,dy + (6y-4)]dy = 14$.

Equal line integrals along two specific paths do not necessarily imply a conservative field. $\bar{E}$ is a conservative field in this case because $\bar{E} = \bar{\nabla}(xy + C)$.

P.2-15
$$\begin{bmatrix} E_r \\ E_\phi \end{bmatrix} = \begin{bmatrix} \cos\phi & \sin\phi \\ -\sin\phi & \cos\phi \end{bmatrix} \begin{bmatrix} E_x \\ E_y \end{bmatrix} = \begin{bmatrix} \cos\phi & \sin\phi \\ -\sin\phi & \cos\phi \end{bmatrix} \begin{bmatrix} r\sin\phi \\ r\cos\phi \end{bmatrix}$$

$\bar{E} = \bar{a}_r\, r\sin 2\phi + \bar{a}_\phi\, r\cos 2\phi$

$\bar{E}\cdot d\bar{\ell} = r\sin 2\phi\, dr + r^2\cos 2\phi\, d\phi$.

$P_3(3,4,-1) = P_3(5, 53.1°, -1)$; $P_4(4,-3,-1) = P_4(5, -36.9°, -1)$.
There is no change in r ($=5$) from P_3 to P_4.

$\therefore \int_{P_3}^{P_4} \bar{E}\cdot d\bar{\ell} = 5^2 \int_{53.1°}^{-36.9°} \cos 2\phi\, d\phi = -24$.

P.2-16 (a) $\bar{\nabla} V = \left[\bar{a}_x \left(\frac{\pi}{2}\cos\frac{\pi}{2}x\right)\left(\sin\frac{\pi}{3}y\right) + \bar{a}_y \left(\sin\frac{\pi}{2}x\right)\left(\frac{\pi}{3}\cos\frac{\pi}{3}y\right) + \bar{a}_z\left(\sin\frac{\pi}{2}x\right)\left(\sin\frac{\pi}{3}y\right)\right] e^{-z}$.

$(\bar{\nabla} V)_P = -\left(\bar{a}_y \frac{\pi}{6} + \bar{a}_z \frac{\sqrt{3}}{2}\right) e^{-3} = -(\bar{a}_y 0.026 + \bar{a}_z 0.043)$.

(b) $\overrightarrow{PO} = -\bar{a}_x - \bar{a}_y 2 - \bar{a}_z 3$; $\bar{a}_{PO} = -\frac{1}{\sqrt{14}}(\bar{a}_x + \bar{a}_y 2 + \bar{a}_z 3)$.

$\therefore (\bar{\nabla} V)_P \cdot \bar{a}_{PO} = \frac{1}{\sqrt{14}}\left(\frac{\pi}{3} - \frac{3\sqrt{3}}{2}\right) e^{-3} = 0.0485$.

P.2-17 $\oint (\bar{a}_R 3\sin\theta)\cdot(\bar{a}_R 5^2 \sin\theta)\, d\theta\, d\phi = \int_0^{2\pi}\int_0^{\pi} 75 \sin^2\theta\, d\theta\, d\phi = 75\pi^2$

P.2-19 $\oint \bar{A}\cdot d\bar{s} = \left(\int_{\substack{top\\face}} + \int_{\substack{bottom\\face}} + \int_{walls}\right) \bar{A}\cdot d\bar{s}$.

Top face ($z=4$): $\bar{A} = \bar{a}_r r^2 + \bar{a}_z 8$, $d\bar{s} = \bar{a}_z ds$.

$$\int_{\substack{top\\face}} \bar{A}\cdot d\bar{s} = \int_{\substack{top\\face}} 8\, ds = 8(\pi 5^2) = 200\pi.$$

Bottom face ($z=0$): $\bar{A} = \bar{a}_r r^2$, $d\bar{s} = -\bar{a}_z ds$.

$$\int_{\substack{bottom\\face}} \bar{A}\cdot d\bar{s} = 0.$$

Walls ($r=5$): $\bar{A} = \bar{a}_r 25 + \bar{a}_z 2z$, $ds = \bar{a}_r ds$.

$$\int_{walls} \bar{A}\cdot d\bar{s} = 25\int_{wall} ds = 25(2\pi 5\times 4) = 1000\pi.$$

$$\therefore \oint \bar{A}\cdot d\bar{s} = 200\pi + 0 + 1000\pi = 1{,}200\pi.$$

$\bar{\nabla}\cdot\bar{A} = 3r+2$, $\int \bar{\nabla}\cdot\bar{A}\, dv = \int_0^4\int_0^{2\pi}\int_0^5 \bar{\nabla}\cdot\bar{A}\, r\,dr\,d\phi\,dz = 1{,}200\pi.$

<u>P.2-20</u> $\bar{\nabla}\cdot\bar{F} = \frac{1}{r}\frac{\partial}{\partial r}(rF_r) + \frac{\partial}{\partial z}F_z = k_2$, $\int \bar{\nabla}\cdot\bar{F}\, dv = 24\pi k_2$.

Divergence theorem fails because $\bar{F}$ has a singularity at $r=0$.

<u>P.2-21</u> a) $\oint \bar{D}\cdot d\bar{s} = \oint\left(\frac{\cos^2\phi}{R^3}\right)R^2\sin\theta\, d\theta\, d\phi$

$$= \int_0^{2\pi}\int_0^{\pi}\left(\frac{1}{2}-1\right)\cos^2\phi\,\sin\theta\, d\theta\, d\phi = -\pi.$$

b) $\bar{\nabla}\cdot\bar{D} = -(\cos^2\phi)/R^4$. $\int \bar{\nabla}\cdot\bar{D}\, dv = \int_0^{2\pi}\int_0^{\pi}\int_1^2 (\bar{\nabla}\cdot\bar{D})R^2\sin\theta\, dR\, d\theta\, d\phi$
$= -\pi.$

<u>P.2-22</u> $\bar{\nabla}\cdot\bar{F} = \frac{1}{R^2}\frac{\partial}{\partial R}(R^2F_R) = \frac{1}{R^2}\frac{\partial}{\partial R}[R^2 f(R)] = 0.$

$\therefore R^2 f(R) = \text{constant}, C$; i.e., $f(R^2) = \frac{C}{R^2}$.

<u>P.2-24</u> a) $\bar{A}\cdot d\bar{\ell} = 3x^2y^3dx - x^3y^2dy$

$$\oint \bar{A}\cdot d\bar{\ell} = 21 + \frac{56}{3} - 7 = \frac{98}{3} = 32\tfrac{2}{3}.$$

b) $\bar{\nabla}\times\bar{A} = -\bar{a}_z 12x^2y^2$

$$\int(\bar{\nabla}\times\bar{A})\cdot d\bar{s} = \int_1^2\int_y^2(-\bar{a}_z 12x^2y^2)\cdot(-\bar{a}_z dx\,dy) = 32\tfrac{2}{3}.$$

c) No, because $\bar{\nabla}\times\bar{A}\neq 0$.

<u>P.2-25</u> $\bar{\nabla}\times\bar{A} = \frac{1}{R\sin\theta}\left(\bar{a}_R\cos\theta\sin\frac{\phi}{2} - \bar{a}_\theta\sin\theta\sin\frac{\phi}{2}\right)$

$$\int_S(\bar{\nabla}\times\bar{A})\cdot d\bar{s} = \int_0^{2\pi}\int_0^{\pi/2}(\bar{\nabla}\times\bar{A})_{R=b}\cdot(\bar{a}_R b^2\sin\theta\, d\theta\, d\phi) = 4b.$$

$$\oint_C \bar{A}\cdot d\bar{\ell} = \int_0^{2\pi} (\bar{A})_{\substack{R=b\\ \theta=\pi/2}} \cdot (\bar{a}_\phi\, b\, d\phi) = \int_0^{2\pi} b \sin\frac{\phi}{2}\, d\phi = 4b.$$

Chapter 3

P.3-1 a) $\alpha = \tan^{-1}\left(\frac{L-w}{d_2}\right) = \tan^{-1}\left(\frac{mv_0^2}{ewE_d}\right).$

b) $d_1 = \frac{d_0}{20}$, $\quad \frac{eE_d}{2m}\frac{w^2}{v_0^2} = \frac{1}{20}\frac{eE_d}{mv_0^2}w\left(L-\frac{w}{2}\right)$, $\quad \frac{L}{w} = 10.5$

P.3-2 a) Max. voltage V_{max} will make $d_1 = h/2$ at $z = w$.

$$\frac{h}{2} = \frac{e}{2m}\left(\frac{V_{max}}{h}\right)\left(\frac{w}{v_0}\right)^2, \quad \text{or} \quad V_{max} = \frac{m}{e}\left(\frac{v_0 h}{w}\right)^2.$$

b) At the screen, $(d_0)_{max} = D/2$. Hence L must be $\leq L_{max}$ where

$$L_{max} = \frac{1}{2}\left(w + \frac{mv_0^2 Dh}{ewV_{max}}\right).$$

c) Double V_{max} by doubling v_0^2, or doubling the anode accelerating voltage.

P.3-3 $F = \frac{e^2}{4\pi\epsilon_0 R^2} = (9\times10^9)\frac{(1.602\times10^{-19})^2}{(5.28\times10^{-11})^2} = 8.29\times10^{-8}\,(N).$

Attractive force.

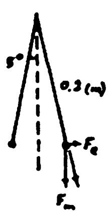
P.3-4 $\overrightarrow{Q_1P} = -\bar{a}_x 2 - \bar{a}_y\,; \quad \overrightarrow{Q_2P} = -\bar{a}_x 3 + \bar{a}_y.$

$$\bar{E}_{P1} = \frac{Q_1}{4\pi\epsilon_0(\sqrt{5})^3}(-\bar{a}_x 2 - \bar{a}_y); \quad \bar{E}_{P2} = \frac{Q_2}{4\pi\epsilon_0(\sqrt{10})^3}(-\bar{a}_x 3 + \bar{a}_y)$$

a) No x-component of $\bar{E}_P$: $-\frac{2Q_1}{(\sqrt{5})^3} - \frac{3Q_2}{(\sqrt{10})^3} = 0$, or $\frac{Q_1}{Q_2} = -\frac{3}{4\sqrt{2}}$

b) No y-component of $\bar{E}_P$: $-\frac{Q_1}{(\sqrt{5})^3} + \frac{Q_2}{(\sqrt{10})^3} = 0$, or $\frac{Q_1}{Q_2} = \frac{1}{2\sqrt{2}}$

P.3-5



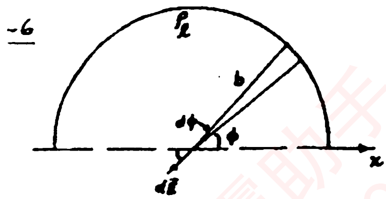
At equilibrium, electric force $\bar{F}_e$ and gravitational force $\bar{F}_m$ must add to give a resultant along the thread.

$$\frac{F_e}{F_m} = \tan 5° = 0.0875.$$

$F_m = mg = 9.80\times10^{-4}$ (N

$F_e = \frac{Q^2}{4\pi\epsilon_0(2\times0.3\sin 5°)^2} = 7.41\times10^{-12}Q^2\,(N).$ $\quad Q = 3.40$

-6

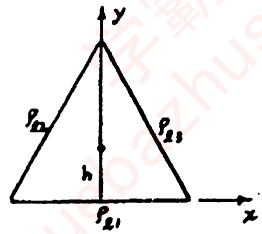


$$dE_y = -\frac{\rho_\ell\,(b\,d\phi)}{4\pi\epsilon_0 b^2}\sin\phi,$$

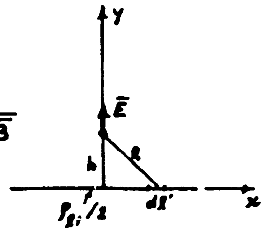
$$\bar{E} = \bar{a}_y E_y = -\bar{a}_y \frac{\rho_\ell}{4\pi\epsilon_0 b}\int_0^\pi \sin\phi\,d\phi$$

$$= -\bar{a}_y \frac{\rho_\ell}{2\pi\epsilon_0 b}.$$

-7



$$h = \frac{L}{2\sqrt{3}}$$



$\bar{E}$ at the center of triangle would be zero if all three line charges were of the same charge density. The problem is equivalent to that of a single line charge of density $\rho_{\ell 1}/2$. By symmetry, there will only be a y-component.

$$\bar{E} = \bar{a}_y E_y = \bar{a}_y \int_{-L/2}^{L/2} \frac{(\rho_{\ell 1}/2)\,d\ell'}{4\pi\epsilon_0 R^2}\left(\frac{h}{R}\right) = \bar{a}_y \int_{-L/2}^{L/2} \frac{\rho_{\ell 1}\,h\,d\ell'}{8\pi\epsilon_0 (h^2+\ell'^2)^{3/2}}$$

$$= \bar{a}_y \frac{3\rho_{\ell 1}}{4\pi\epsilon_0 L} = \bar{a}_y \frac{3\rho_{\ell 1}}{2\pi\epsilon_0 L}.$$

3-8 Use Gauss's law: $\oint \bar{E}\cdot d\bar{s} = Q/\epsilon_0$.

a) $\bar{E}$ is normal to the two faces at $x = \pm 0.05$ (m), where $\bar{E} = \pm\bar{a}_x 5$ and $\bar{a}_n = \pm\bar{a}_x$ respectively.

$$Q = 2\epsilon_0(5\times 0.1^2) = 0.1\epsilon_0 = 8.84\times 10^{-13}\ \text{(C)}.$$

b) $\bar{E} = \bar{a}_r(100x)\cos\phi - \bar{a}_\phi(100x)\sin\phi = \bar{a}_r(100r\cos^2\phi) - \bar{a}_\phi(100r\sin^2\phi)$.

$$\oint \bar{E}\cdot\bar{a}_n\,ds = \int_0^{2\pi}(100\times 0.05\cos^2\phi)(0.1\times 0.05\,d\phi) = 0.025\pi.$$

$$Q = 0.0785\epsilon_0 = 6.94\times 10^{-13}\ \text{(C)}.$$

3-9 Spherical symmetry: $\bar{E} = \bar{a}_R E_R$. Apply Gauss's law.

1) $0 \le R \le b$. $4\pi R^2 E_{R1} = \frac{\rho_0}{\epsilon_0}\int_0^R \left(1-\frac{R^2}{b^2}\right)4\pi R^2\,dR = \frac{4\pi\rho_0}{\epsilon_0}\left(\frac{R^3}{3}-\frac{R^5}{5b^2}\right)$.

$$E_{R1} = \frac{\rho_0}{\epsilon_0}R\left(\frac{1}{3}-\frac{R^2}{5b^2}\right).$$

2) $b \le R < R_i$: $4\pi R^2 E_{R2} = \frac{\rho_0}{\epsilon_0}\int_0^b \left(1-\frac{R^2}{b^2}\right)4\pi R^2 dR = \frac{8\pi\rho_0}{15\epsilon_0} b^3$

$$E_{R2} = \frac{2\rho_0 b^3}{15\epsilon_0 R^2}.$$

3) $R_i \le R \le R_o$. $\quad E_{R3} = 0.$

4) $R > R_o$. $\quad E_{R4} = \frac{2\rho_0 b^3}{15\epsilon_0 R^2}.$

<u>P.3-10</u> Cylindrical symmetry: $\bar{E} = \bar{a}_r E_r$. Apply Gauss's law.

a) $r < a$, $E_r = 0$; $\quad a < r < b$, $E_r = a\rho_{sa}/\epsilon_0 r$;

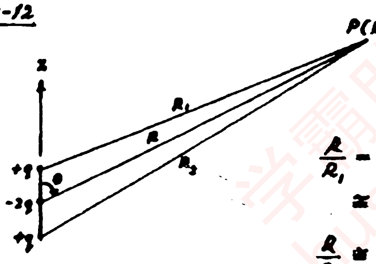
$r > b$, $E_r = (a\rho_{sa} + b\rho_{sb})/\epsilon_0 r$.

b) $b/a = -\rho_{sa}/\rho_{sb}$.

<u>P.3-11</u> Refer to Eq. (3-49) and Fig. 3-14. $\bar{E}$ will have no z-component if $E_R \cos\theta = E_\theta \sin\theta$, or $2\cos^2\theta = \sin^2\theta$

$$\theta = 54.7° \text{ and } 125.3°.$$

<u>P.3-12</u>



$$V = \frac{q}{4\pi\epsilon_0 R}\left(\frac{R}{R_1} + \frac{R}{R_2} - 2\right).$$

$$R_1^2 = R^2 + \left(\frac{d}{2}\right)^2 - Rd\cos\theta,$$

$$\frac{R}{R_1} = \left[1 + \left(\frac{d}{2R}\right)^2 - \frac{d}{R}\cos\theta\right]^{-1/2}$$

$$\cong 1 + \frac{d}{2R}\cos\theta + \frac{d^2}{4R^2}\,\frac{3\cos^2\theta - 1}{2}.$$

$$\frac{R}{R_2} \cong 1 - \frac{d}{2R}\cos\theta + \frac{d^2}{4R^2}\,\frac{3\cos^2\theta - 1}{2}.$$

a) $\quad \therefore V = \frac{q(d/2)^2}{4\pi\epsilon_0 R}(3\cos^2\theta - 1)$, $\quad R^2 >> d^2$.

$$\bar{E} = -\bar{\nabla}V = -\bar{a}_R\frac{\partial V}{\partial R} - \bar{a}_\theta\frac{\partial V}{R\partial\theta} = \frac{q(d/2)^2}{4\pi\epsilon_0 R^4}\left[\bar{a}_R 3(3\cos^2\theta - 1) + \bar{a}_\theta 3\sin 2\theta\right].$$

b) Equation for equipotential surfaces:

$$R = C_V(3\cos^2\theta - 1)^{1/3}.$$

Equation for streamlines:

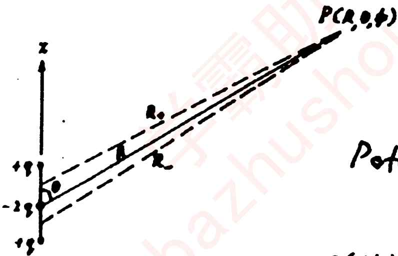
$$\frac{dR}{E_R} = \frac{R\,d\theta}{E_\theta}, \text{ or } \frac{dR}{3\cos^2\theta - 1} = \frac{R\,d\theta}{\sin 2\theta}$$

$$\frac{dR}{R} = \frac{3}{2}\frac{d(\sin\theta)}{\sin\theta} - \frac{d\theta}{\sin 2\theta}$$

$$R = c_E\left(\sin^{3/2}\theta / \sqrt{|\tan\theta|}\right).$$

(Also see next

-12 A simpler approach (not to the same degree of approximation) is to consider the problem as a pair of diplaced dipoles, each with a moment $\bar{p} = q\bar{d}/2$.



From Eq. (3-48), potential due to $\bar{p}$ is

$$V_+ = \frac{q(d/2)\cos\theta}{4\pi\epsilon_0 R_+^2}.$$

Potential due to $-\bar{p}$ is

$$V_- = -\frac{q(d/2)\cos\theta}{4\pi\epsilon_0 R_-^2}.$$

$$V = V_+ + V_- = \frac{q(d/2)\cos\theta}{4\pi\epsilon_0}\left(\frac{1}{R_+^2} - \frac{1}{R_-^2}\right).$$

$$R_+^{-2} \cong R^{-2}\left(1 + \frac{d}{2R}\cos\theta\right), \quad R_-^{-2} \cong R^{-2}\left(1 - \frac{d}{2R}\cos\theta\right).$$

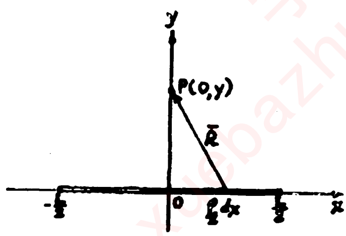
a) $$\therefore\ V = \frac{q(d\cos\theta)^2/2}{4\pi\epsilon_0 R^3}.$$

$$\bar{E} = -\bar{\nabla}V = \frac{qd^2/2}{4\pi\epsilon_0 R^4}\left(\bar{a}_R 3\cos^2\theta + \bar{a}_\theta \sin 2\theta\right).$$

b) Equation for equipotential surfaces: $R = C_V'(\cos\theta)^{2/3}$.

Equation for streamlines: $R = C_S'(\sin\theta)^{1/2}$.

-13



a) $$V = 2\int_0^{L/2} \frac{\rho_\ell\, dx}{4\pi\epsilon_0 R}$$

$$= \frac{\rho_\ell}{2\pi\epsilon_0}\int_0^{L/2} \frac{dx}{\sqrt{x^2+y^2}}$$

$$= \frac{\rho_\ell}{2\pi\epsilon_0}\left\{\ln\left[\sqrt{\left(\frac{L}{2}\right)^2+y^2} - \frac{L}{2}\right] - \ln y\right\}.$$

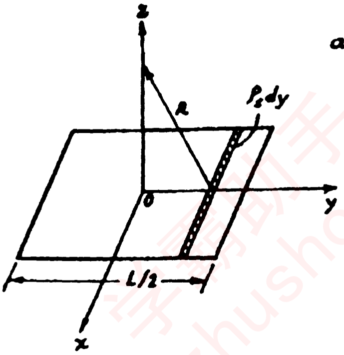
b) From Coulomb's law:

$$\bar{E} = \bar{a}_y E_y = 2\int_0^{L/2} \bar{a}_y \frac{\rho_\ell\, y\, dx}{4\pi\epsilon_0 R^3} = \bar{a}_y \frac{\rho_\ell}{2\pi\epsilon_0 y}\frac{L/2}{\sqrt{(L/2)^2+y^2}}.$$

c) $E = -\bar{\nabla}V$ gives the same answer.

-14 Surface charge density $\rho_s = \frac{Q}{L^2}$.

Use the results of problem P. 3-13 for the coordinate system chosen in the figure on the next page. Replace ρ_ℓ by $\rho_s dy$ and y by $\sqrt{y^2+z^2}$.



a) $$V = 2\cdot\frac{\rho_s}{2\pi\epsilon_0}\int_0^{L/2}\left\{\ln\left[\sqrt{\left(\frac{L}{2}\right)^2+y^2+z^2}+\left(\frac{L}{2}\right)\right] - \ln\sqrt{y^2+z^2}\right\}dy$$

$$= \frac{Q}{\pi\epsilon_0 L^2}\left\{\frac{L}{2}\ln\left[\frac{\sqrt{2\left(\frac{L}{2}\right)^2+z^2}+\frac{L}{2}}{\sqrt{2\left(\frac{L}{2}\right)^2+z^2}-\frac{L}{2}}\right] - z\tan^{-1}\left[\frac{\left(\frac{L}{2}\right)^2}{z\sqrt{2\left(\frac{L}{2}\right)^2+z^2}}\right]\right\}.$$

b) $$\bar{E} = -\bar{\nabla}V = \frac{\bar{a}_z Q}{\pi\epsilon_0 L^2}\tan^{-1}\left[\frac{\left(\frac{L}{2}\right)^2}{z\sqrt{2\left(\frac{L}{2}\right)^2+z^2}}\right].$$

P.3-15 Assume the circular tube sits on the xy-plane with its axis coinciding with the z-axis. The surface charge on the tube wall is $\rho_s = Q/2\pi bh$. First find the potential along the axis at z due to a circular line charge of density ρ_ℓ situated at z'.

$$V = \oint \frac{\rho_\ell\, d\ell}{4\pi\epsilon_0 R} = \int_0^{2\pi}\frac{\rho_\ell\, b\, d\phi}{4\pi\epsilon_0\sqrt{b^2+(z-z')^2}} = \frac{\rho_\ell\, b}{2\epsilon_0\sqrt{b^2+(z-z')^2}}$$

a) The expression above is the contribution dV due to a circular line charge of density $\rho_\ell = \rho_s dz'$.

$$dV = \frac{\rho_s\, b\, dz'}{2\epsilon_0\sqrt{b^2+(z-z')^2}}.$$

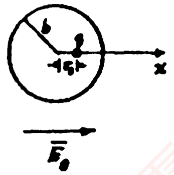
At a point outside the tube:

$$V = \int_{z'=0}^{z'=h} dV = \frac{b\rho_s}{2\epsilon_0}\ln\frac{z+\sqrt{b^2+z^2}}{(z-h)+\sqrt{b^2+(z-h)^2}}$$

$$\bar{E} = -\bar{a}_z\frac{dV}{dz} = \bar{a}_z\frac{b\rho_s}{2\epsilon_0}\left[\frac{1}{\sqrt{b^2+(z-h)^2}} - \frac{1}{\sqrt{b^2+z^2}}\right].$$

b) Same expressions are obtained for V and $\bar{E}$ at a point inside the tube.

3-16



Applied $\bar{E}_0$ causes a displacement r_0.

Force of separation: qE_0;

Restoring force (attraction): qE_x.

E_x at q due to spherical volume of electrons of radius r_0 is (by Gauss's law) $E_x = \frac{\rho r_0}{3\epsilon_0} = -\frac{r_0}{3\epsilon_0}|\rho|$

$$|\rho| = \frac{q}{\frac{4}{3}\pi b^3} = \frac{3N|e|}{4\pi b^3}.$$

At equilibrium: $E_0 = |E_x| = \frac{r_0 N|e|}{4\pi\epsilon_0 b^3}$, or $r_0 = \frac{4\pi\epsilon_0 b^3}{N|e|}E_0$.

3-17 $W = -q\int \bar{E}\cdot d\bar{\ell} = -q\int (y\,dx + x\,dy)$.

a) Along the parabola $x = 2y^2$; $dx = 4y\,dy$

$$W = -q\int_1^2 6y^2 dy = -14q = 28\ (\mu J).$$

b) Along the straight line $x = 6y-4$; $dx = 6\,dy$

$$W = -q\int_1^2 (12y-4)dy = -14q = 28\ (\mu J).$$

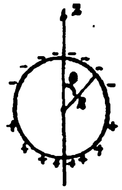
3-18 a) $\rho_{ps} = \bar{P}\cdot\bar{a}_n = P_0\frac{L}{2}$ on all six faces of the cube.

$\rho_p = -\bar{\nabla}\cdot\bar{P} = -3P_0$.

b) $Q_s = 6L^2\rho_{ps} = 3P_0L^3$, $\quad Q_v = L^3\rho_p = -3P_0L^3$.

Total bound charge $= Q_s + Q_v = 0$.

3-19 Assume $\bar{P} = \bar{a}_z P$. Surface charge denity $\rho_{ps} = \bar{P}\cdot\bar{a}_n$
$= (\bar{a}_z P)\cdot(-\bar{a}_R)$
$= -P\cos\theta$.



The z-component of the electric field intensity due to a ring of ρ_{ps} contained in width $Rd\theta$ at θ is

$$dE_z = \frac{P\cos\theta}{4\pi\epsilon_0 R^2}(2\pi R\sin\theta)(Rd\theta)\cos\theta$$
$$= \frac{P}{2\epsilon_0}\cos^2\theta\sin\theta\,d\theta.$$

At the center of the cavity: $\bar{E} = \bar{a}_z E_z = \bar{a}_z\frac{P}{2\epsilon_0}\int_0^\pi \cos^2\theta\sin\theta\,d\theta = \frac{\bar{P}}{3\epsilon_0}$.

P.3-20 a) $V_b = E_{ba}d = 3\times 50 = 150\ (kV)$

b) $V_b = E_{bp}d = 20\times 50 = 1{,}000\ (kV)$

c) $V_b = E_{ba}(d-d_p) + \frac{1}{3}E_{ba}d_p = 3(40+\frac{1}{3}\times 10) = 130\ (kV)$.

P.3-21 At the z=0 plane : $\bar{E}_1 = \bar{a}_x 2y - \bar{a}_y 3x + \bar{a}_z 5$.

$$\bar{E}_{1t}(z=0) = \bar{E}_{2t}(z=0) = \bar{a}_x 2y - \bar{a}_y 3x,$$

$$\bar{D}_{1n}(z=0) = \bar{D}_{2n}(z=0) \longrightarrow 2\bar{E}_{1n}(z=0) = 3\bar{E}_{2n}(z=0)$$

$$\longrightarrow \bar{E}_{2n}(z=0) = \frac{2}{3}(\bar{a}_z 5) = \bar{a}_z\frac{10}{3}.$$

$$\therefore \bar{E}_2(z=0) = \bar{a}_x 2y - \bar{a}_y 3x + \bar{a}_z\frac{10}{3},$$

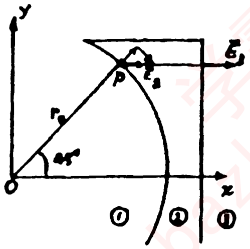
$$\bar{D}_2(z=0) = (\bar{a}_x 6y - \bar{a}_y 9x + \bar{a}_z 10)\epsilon_0.$$

P.3-22 $\bar{P}_{t,n} = \epsilon_0(\epsilon_r - 1)\bar{E}_{t,n}$: $\bar{E}_{t1} = \bar{E}_{t2} \longrightarrow \frac{1}{\epsilon_{r1}-1}P_{1t} = \frac{1}{\epsilon_{r2}-1}P_{2t}$.

$$\epsilon_{r1}\bar{E}_{n1} = \epsilon_{r2}\bar{E}_{n2} \longrightarrow \frac{\epsilon_{r1}}{\epsilon_{r1}-1}P_{1n} = \frac{\epsilon_{r2}}{\epsilon_{r2}-1}P_{2n}.$$

P.3-23 $\epsilon_1\frac{\partial V_1}{\partial n} = \epsilon_2\frac{\partial V_2}{\partial n}$ and $V_1 = V_2$.

P.3-24



Assume $\bar{E}_2 = \bar{a}_r E_{2r} + \bar{a}_\phi E_{2\phi}$

B.C.: $\bar{a}_n\times\bar{E}_1 = \bar{a}_n\times\bar{E}_2 \longrightarrow E_{2\phi} = -$

For $\bar{E}_1$, and hence $\bar{E}_2$, to be parallel to the x-axis,

$E_{2\phi} = -E_{2r} \longrightarrow E_{2r} = 3$.

B.C.: $\bar{a}_n\cdot\bar{D}_1 = \bar{a}_n\cdot\bar{D}_2 \longrightarrow 5 = 3\epsilon$

$\therefore\ \epsilon_{r2} = 5/3$.

P.3-25 $\epsilon = \frac{\epsilon_2-\epsilon_1}{d}y + \epsilon_1$.

Assume Q on plate at y=d. $\bar{E} = -\bar{a}_y\frac{\rho_s}{\epsilon} = -\bar{a}_y\frac{Q}{S(\frac{\epsilon_2-\epsilon_1}{d}y+$

$$V = -\int_{y=0}^{y=d}\bar{E}\cdot d\bar{\ell} = \frac{Qd\ln(\epsilon_2/\epsilon_1)}{S(\epsilon_2-\epsilon_1)},$$

$$C = \frac{Q}{V} = \frac{S(\epsilon_2-\epsilon_1)}{d\ln(\epsilon_2/\epsilon_1)}.$$

P.3-26 a) $C = 4\pi\epsilon_0 R = \frac{1}{9}\times 10^{-9}\times(6.37\times 10^6) = 7.08\times 10^{-4}\ (F)$.

b) $E_b = 3\times 10^6 = \frac{Q_{max}}{4\pi\epsilon_0 R^2}$, $Q_{max} = 1.35\times 10^{10}\ (C)$.

P.3-27 Assume charge Q on conducting sphere.

$b<R<b+d$, $\bar{E}_1 = \bar{a}_R \dfrac{Q}{4\pi\epsilon_0(1+\chi_e)R^2}$.

$R>b+d$, $\bar{E}_2 = \bar{a}_R \dfrac{Q}{4\pi\epsilon_0 R^2}$.

$$V = -\int_\infty^b \bar{E}\cdot d\bar{\ell} = -\int_\infty^{b+d} E_2\,dR - \int_{b+d}^b E_1\,dR = \frac{Q}{4\pi\epsilon_0(1+\chi_e)}\left(\frac{\chi_e}{b+d}+\frac{1}{b}\right).$$

$$C = \frac{Q}{V} = \frac{4\pi\epsilon_0(1+\chi_e)}{\frac{\chi_e}{b+d}+\frac{1}{b}}.$$

P.3-28 Assume charge Q on inner shell, $-Q$ on outer shell.

$R_i<R<R_o$, $\bar{D} = \bar{a}_R \dfrac{Q}{4\pi R^2}$.

$R_i<R<b$, $\bar{E}_1 = \dfrac{\bar{D}}{\epsilon_0\epsilon_r}$; $b<R<R_o$, $\bar{E}_2 = \dfrac{\bar{D}}{2\epsilon_0\epsilon_r}$.

$$V = -\int_{R_o}^{R_i} \bar{E}\cdot d\bar{R} = -\int_b^{R_i} E_1\,dR - \int_{R_o}^b E_2\,dR = \frac{Q}{4\pi\epsilon_0\epsilon_r}\left(\frac{1}{R_i}-\frac{1}{2b}-\frac{1}{2R_o}\right).$$

a) $\bar{D} = \bar{a}_R \dfrac{\epsilon_0\epsilon_r V}{R^2\left(\frac{1}{R_i}-\frac{1}{2b}-\frac{1}{2R_o}\right)}$, $R_i<R<R_o$. $\bar{D}=0, \bar{E}=0$ for $R<R_i$ and $R>R_o$.

$\bar{E}_1 = \bar{a}_R \dfrac{V}{R^2\left(\frac{1}{R_i}-\frac{1}{2b}-\frac{1}{2R_o}\right)}$; $\bar{E}_2 = \bar{a}_R \dfrac{V}{R^2\left(\frac{2}{R_i}-\frac{1}{b}-\frac{1}{R_o}\right)}$.

b) $$C = \frac{Q}{V} = \frac{4\pi\epsilon_0\epsilon_r}{\frac{1}{R_i}-\frac{1}{2b}-\frac{1}{2R_o}}.$$

P.3-29 Let ρ_ℓ be the lineal charge density on the inner conductor.

$\bar{E} = \bar{a}_r \dfrac{\rho_\ell}{2\pi\epsilon r}$, $V_0 = -\int_b^a \bar{E}\cdot d\bar{r} = \dfrac{\rho_\ell}{2\pi\epsilon}\ln\left(\dfrac{b}{a}\right)$.

$$\rho_\ell = \frac{2\pi\epsilon V_0}{\ln(b/a)}.$$

a) $\bar{E}(a) = \bar{a}_r \dfrac{V_0}{a\ln(b/a)}$.

b) Let $x = b/a$, $f(x) = \dfrac{\ln x}{x}$, $\dfrac{\partial f(x)}{\partial x} = 0 \longrightarrow \ln x = 1$,

$x = \dfrac{b}{a} = 2.718$.

c) $C = \dfrac{\rho_\ell}{V_0} = \dfrac{2\pi\epsilon}{\ln(b/a)} = 2\pi\epsilon$ (F/m).

P.3-30 $\bar{D} = \bar{a}_r \dfrac{\rho_\ell}{2\pi r}$. $\bar{E}_1 = \bar{a}_r \dfrac{\rho_\ell}{2\pi\epsilon_0\epsilon_{r1} r}$, $r_i<r<b$; $\bar{E}_2 = \bar{a}_r \dfrac{\rho_\ell}{2\pi\epsilon_0\epsilon_{r2} r}$, $b<r<r_o$

$$V = -\int_{r_o}^{r_i} \bar{E}\cdot d\bar{r} = \frac{\rho_\ell}{2\pi\epsilon_0}\left[\frac{1}{\epsilon_{r1}}\ln\left(\frac{b}{r_i}\right) + \frac{1}{\epsilon_{r2}}\ln\left(\frac{r_o}{b}\right)\right].$$

$$C = \frac{\rho_\ell}{V} = \frac{2\pi\epsilon_0}{\frac{1}{\epsilon_{r1}}\ln\left(\frac{b}{r_i}\right)+\frac{1}{\epsilon_{r2}}\ln\left(\frac{r_o}{b}\right)} \quad \text{(F/m)}.$$

P.3-31 From Gauss's law, $\oint \bar{D}\cdot d\bar{s} = \rho_\ell L$.

$\bar{E}_1 = \bar{E}_2 = \bar{a}_r E_r$. $\quad \pi r L(\epsilon_0\epsilon_{r1} + \epsilon_0\epsilon_{r2})E_r = \rho_\ell L$.

$$E_r = \frac{\rho_\ell}{\pi r \epsilon_0(\epsilon_{r1}+\epsilon_{r2})} \; ; \; V = -\int_{r_o}^{r_i} E_r\,dr = \frac{\rho_\ell}{\pi\epsilon_0(\epsilon_{r1}+\epsilon_{r2})}\ln\left(\frac{r_o}{r_i}\right).$$

$$\therefore\; C = \frac{\rho_\ell L}{V} = \frac{\pi\epsilon_0(\epsilon_{r1}+\epsilon_{r2})L}{\ln(r_o/r_i)}.$$

P.3-32 $$\bar{E} = \bar{a}_r\frac{\rho_\ell}{2\pi\epsilon r} = \bar{a}_r\frac{\rho_\ell}{2\pi\epsilon_0(2+\frac{4}{r})r} = \bar{a}_r\frac{\rho_\ell}{4\pi\epsilon_0(r+2)}.$$

$$V = -\int_{r_o}^{r_i}\bar{E}\cdot d\bar{r} = \frac{\rho_\ell}{4\pi\epsilon_0}\ln(r+2)\Big|_5^7 = \frac{\rho_\ell}{4\pi\epsilon_0}\ln\left(\frac{9}{7}\right).$$

$$C = \frac{\rho_\ell L}{V} = \frac{4\pi L\epsilon_0}{\ln(9/7)} = 1500\,\epsilon_0 = 13.26\ (\mu F).$$

P.3-33 $\bar{D} = \bar{a}_R\frac{R}{3}\rho$, $R<b$; $\quad \bar{D} = \bar{a}_R\frac{b^3\rho}{3R^2}$, $R>b$; $\quad \bar{E} = \frac{1}{\epsilon_0}\bar{D}$.

a) $$W_i = \frac{1}{2}\int_V \bar{D}\cdot\bar{E}\,dv = \frac{1}{2}\int_0^b \frac{1}{\epsilon_0}\left(\frac{R}{3}\rho\right)^2 4\pi R^2\,dR = \frac{2\pi b^5\rho^2}{45\,\epsilon_0}$$

b) $$W_o = \frac{1}{2}\int_b^\infty \frac{1}{\epsilon_0}\left(\frac{b^3\rho}{3R^2}\right)^2 4\pi R^2\,dR = \frac{2\pi b^5\rho^2}{9\,\epsilon_0}.$$

Total $$W = W_i + W_o = \frac{4\pi b^5\rho^2}{15\,\epsilon_0}$$

P.3-34 $$\bar{E} = \frac{p}{4\pi\epsilon_0 R^3}(\bar{a}_R\,2\cos\theta + \bar{a}_\theta\sin\theta).$$

$$W = \frac{1}{2}\epsilon_0\int_V E^2\,dv = \frac{\epsilon_0}{2}\left(\frac{p}{4\pi\epsilon_0}\right)^2\int_0^{2\pi}d\phi\int_0^{\pi}d\theta\int_b^\infty \frac{1}{R^6}(4\cos^2\theta+\sin^2\theta)R^2\sin\theta\,dR$$
$$= \frac{p^2}{12\pi\epsilon_0 b^3}.$$

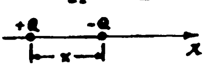
P.3-35 Two conductors at potentials V_1 and V_2 carrying charges $+Q$ and $-Q$:
$$W_e = \frac{1}{2}V_1\int_{S_1}\rho_s\,ds + \frac{1}{2}V_2\int_{S_2}\rho_s\,ds = \frac{1}{2}Q(V_1-V_2)$$
$$= \frac{1}{2}CV^2,\quad V = V_1 - V_2.$$

P.3-36 a) Region 1 — dielectric; region 2 — air.

$\bar{E}_1 = -\bar{a}_y\frac{V_0}{d}$, $\quad \bar{D}_1 = -\bar{a}_y\epsilon_0\epsilon_r\frac{V_0}{d}$, $\quad \rho_{s1} = \epsilon_0\epsilon_r\frac{V_0}{d}$ (top plate)

$\bar{E}_2 = -\bar{a}_y\frac{V_0}{d}$, $\quad \bar{D}_2 = -\bar{a}_y\epsilon_0\frac{V_0}{d}$, $\quad \rho_{s2} = \epsilon_0\frac{V_0}{d}$ (top plate).

$$\frac{W_{e1}}{W_{e2}} = \frac{\epsilon_r x}{L-x} = 1 \longrightarrow x = \frac{L}{\epsilon_r+1}.$$

P.3-37 

From Eqs. (3-136) and (3-137), $W_e = -\cdot$

$x = x_- - x_+$ $\quad$ Force on $+Q$, $\bar{F}_{Q+} = -\bar{a}_x\frac{\partial W_e}{\partial x_+} = \bar{a}_x\frac{Q^2}{4\pi\epsilon_0 x^2} = -\bar{F}$

P.3-38 a) $V = V_0 = \text{constant}.$ $\quad C = \dfrac{\epsilon x + \epsilon_0(L-x)}{d} w,$

$$W_e = \frac{1}{2} C V^2, \quad \bar{F}_v = \bar{\nabla} W_e = \bar{a}_x \frac{V_0^2 w}{2d}(\epsilon - \epsilon_0).$$

b) $Q = \text{constant} = C V_0.$

$$W_e = \frac{Q^2}{2C}, \quad \bar{F}_e = -\bar{\nabla} W_e = \frac{Q^2 d}{2} \frac{\bar{a}_x (\epsilon - \epsilon_0) w}{[\epsilon x + \epsilon_0 (L-x)]^2}$$

$$= \bar{a}_x \frac{V_0^2 w}{2d}(\epsilon - \epsilon_0).$$

Chapter 4

P.4-1 $\bar{\nabla}^2 V = 0 \longrightarrow V_d = c_1 y + c_2,\ \bar{E}_d = -\bar{a}_y c_1,\ \bar{D}_d = -\bar{a}_y \epsilon_0 \epsilon_r c_1.$

$V_a = c_3 y + c_4,\ \bar{E}_a = -\bar{a}_y c_3,\ \bar{D}_a = -\bar{a}_y \epsilon_0 c_3.$

B.C.: $V_d = 0$ at $y = 0$; $\quad V_a = V_0$ at $y = d$;

$V_d = V_a$ at $y = 0.8d$; $\quad \bar{D}_d = \bar{D}_a$ at $y = 0.8d$.

Solving: $c_1 = \dfrac{V_0}{(0.8+0.2\epsilon_r)d},\ c_2 = 0,\ c_3 = \dfrac{\epsilon_r V_0}{(0.8+0.2\epsilon_r)d},\ c_4 = \dfrac{(1-\epsilon_r)V_0}{1+0.25\epsilon_r}.$

a) $V_d = \dfrac{5yV_0}{(4+\epsilon_r)d}, \quad \bar{E}_d = -\bar{a}_y \dfrac{5V_0}{(4+\epsilon_r)d}.$

b) $V_a = \dfrac{5\epsilon_r y - 4(\epsilon_r - 1)d}{(4+\epsilon_r)d} V_0, \quad \bar{E}_a = -\bar{a}_y \dfrac{5\epsilon_r V_0}{(4+\epsilon_r)d}.$

c) $(\rho_s)_{y=d} = -(D_a)_{y=d} = \dfrac{5\epsilon_0 \epsilon_r V_0}{(4+\epsilon_r)d}.$

$(\rho_s)_{y=0} = (D_d)_{y=0} = -\dfrac{5\epsilon_0\epsilon_r V_0}{(4+\epsilon_r)d}.$

P.4-3 At a point where V is a maximum (minimum) the second partial derivatives of V with respect to x, y and z would all be negative (positive); their sum could not vanish, as required by Laplace's equation.

P.4-6 $\bar{\nabla}^2 V = -\dfrac{A}{\epsilon r} \longrightarrow \dfrac{1}{r}\dfrac{\partial}{\partial r}\left(r \dfrac{\partial V}{\partial r}\right) = -\dfrac{A}{\epsilon r}.$

$V = -\dfrac{A}{\epsilon} r + c_1 \ln r + c_2$

$$\begin{cases} V_0 = -\dfrac{A}{\epsilon} a + c_1 \ln a + c_2 \\ 0 = -\dfrac{A}{\epsilon} b + c_1 \ln b + c_2 \end{cases}$$

$c_1 = \dfrac{\frac{A}{\epsilon}(b-a) - V_0}{\ln(b/a)},$

$c_2 = \dfrac{V_0 \ln b + \frac{A}{\epsilon}(a \ln b - b \ln a)}{\ln(b/a)}.$

P.4-7

$\bar{E}\Big|_{y=0} = -\bar{a}_y \dfrac{Q}{4\pi\epsilon R^2} 2\sin\theta = -\bar{a}_y \dfrac{Qd}{2\pi\epsilon(d^2+r^2)^{3/2}}.$

a) $\rho_s = \bar{a}_y \cdot \epsilon \bar{E}\Big|_{y=0} = -\dfrac{Qd}{2\pi(d^2+r^2)^{3/2}}.$

b) $\displaystyle\int_0^\infty \rho_s 2\pi r\, dr = -Q.$

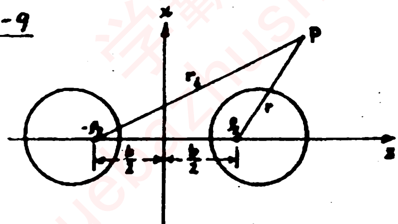
P.4-8 a) Original point charge Q at $y = d/3$ and images

Q at $y = (1+6n)d/3$, $n = \pm 1, \pm 2, \cdots$

$-Q$ at $y = (5+6n)d/3$, $n = 0, \pm 1, \pm 2, \cdots$

b) Original line charge ρ_ℓ at $\theta = 30°$ and image line charges ρ_ℓ at $\theta = 150°$ and $-90°$; $-\rho_\ell$ at $\theta = 90°, -30°$, and $-150°$

P.4-9



From Eq. (4-40), $V = \frac{\rho_\ell}{2\pi\epsilon_0} \ln \frac{r_i}{r}$

$$r = [x^2 + (z - \tfrac{b}{2})^2]^{1/2}$$

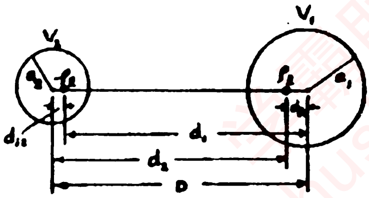
$$r_i = [x^2 + (z + \tfrac{b}{2})^2]^{1/2}$$

For equipotential surfaces,

$$\frac{r_i}{r} = \sqrt{k} \longrightarrow x^2 + \left[z - \left(\frac{k+1}{k-1}\right)\frac{b}{2}\right]^2 = \frac{b^2 k}{(k-1)^2}.$$

P.4-10

$$V_1 = -\frac{\rho_\ell}{2\pi\epsilon_0} \ln \frac{a_1}{d_1}, \quad V_2 = +\frac{\rho_\ell}{2\pi\epsilon_0} \ln \frac{a_2}{d_2}$$



Capacitance per unit length

$$C = \frac{\rho_\ell}{V_1 - V_2} = \frac{2\pi\epsilon_0}{\ln \frac{d_1 d_2}{a_1 a_2}}$$

Four equations:

$$a_1^2 = d_{i1} d_1, \qquad a_2^2 = d_{i2} d_2$$

$$d_1 + d_{i2} = D, \qquad d_2 + d_{i1} = D.$$

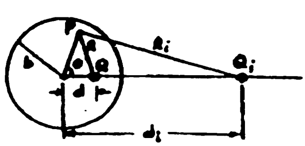
We obtain

$$\frac{d_1 d_2}{a_1 a_2} = \frac{a_1 a_2}{d_{i1} d_{i2}} \quad \text{and} \quad a_1^2 + a_2^2 + d_1 d_2 + d_{i1} d_{i2} = D,$$

$$\frac{d_1 d_2}{a_1 a_2} = \frac{D^2}{2a_1 a_2} - \frac{a_1}{2a_2} - \frac{a_2}{2a_1} + \sqrt{\left(\frac{D^2}{2a_1 a_2} - \frac{a_1}{2a_2} - \frac{a_2}{2a_1}\right)^2 - 1}.$$

$$\therefore C = \frac{2\pi\epsilon_0}{\ln\left[\frac{1}{2}\left(\frac{D^2}{a_1 a_2} - \frac{a_1}{a_2} - \frac{a_2}{a_1}\right) - \sqrt{\frac{1}{4}\left(\frac{D^2}{a_1 a_2} - \frac{a_1}{a_2} - \frac{a_2}{a_1}\right)^2 - 1}\right]} = \frac{2\pi\epsilon_0}{\cosh^{-1}\left[\frac{1}{2}\left(\frac{D^2}{a_1 a_2} - \frac{a_1}{a_2} - \frac{a_2}{a_1}\right)\right]}$$

4-12



$$Q_i = -\frac{b}{d} Q, \quad d_i = \frac{b^2}{d}$$

a) $V = \dfrac{Q}{4\pi\epsilon_0}\left(\dfrac{1}{R} - \dfrac{b}{dR_i}\right).$

b) $\rho_s = -\epsilon_0 \dfrac{\partial V}{\partial R}\Big|_{R=b}$

$$= -\frac{Q(b^2-d}{4\pi b(b^2+d^2}$$

P.4-13 At the plane boundary $x=0$: $V_1=V_2$ and $\epsilon_1\frac{\partial V_1}{\partial x}=\epsilon_2\frac{\partial V_2}{\partial x}$.

Two equations: $\dfrac{Q-Q_1}{\epsilon_1}=\dfrac{Q_2+Q}{\epsilon_2}$ and $Q+Q_1=Q_2+Q$.

$$\longrightarrow\ Q_1=Q_2=\frac{\epsilon_2-\epsilon_1}{\epsilon_2+\epsilon_1}Q.$$

P.4-14 The solution $V(x,y)=V_0$ satisfies $\bar{\nabla}^2V=0$ and all boundary conditions. $\longrightarrow$ Unique solution.

P.4-16 $V(x,y)=\sum_n \sin\frac{n\pi}{a}x\left[A_n\sinh\frac{n\pi}{a}y+B_n\cosh\frac{n\pi}{a}y\right]$.

At $y=0$, $V(x,0)=V_2=\sum_n B_n\sin\frac{n\pi}{a}x \longrightarrow B_n=\begin{cases}\frac{4V_2}{n\pi}, & n \text{ odd}\\ 0, & n \text{ even.}\end{cases}$

At $y=b$, $V(x,b)=V_1=\sum_n \sin\frac{n\pi}{a}x\left[A_n\sinh\frac{n\pi b}{a}+B_n\cosh\frac{n\pi b}{a}\right]$

$$\longrightarrow\ A_n\sinh\frac{n\pi b}{a}+B_n\cosh\frac{n\pi b}{a}=\begin{cases}\frac{4V_1}{n\pi}, & n \text{ odd}\\ 0, & n \text{ even.}\end{cases}$$

$$\therefore\ A_n=\begin{cases}\dfrac{4}{n\pi\sinh\frac{n\pi b}{a}}\left(V_1-V_2\cosh\frac{n\pi b}{a}\right), & n \text{ odd}\\ 0, & n \text{ even.}\end{cases}$$

P.4-18 The solution is the superposition of that for Example 4-9 and that for Fig. 4-12 rotated 90° in the clockwise direction. (In both cases V_0 should be replaced by $V_0/2$.)

Inside: $V(r,\phi)=\dfrac{2V_0}{\pi}\sum_{n=odd}\frac{1}{n}\left(\frac{r}{b}\right)^n\left[\sin n\phi+\sin n\left(\phi+\frac{\pi}{2}\right)\right]$, $r<b$

Outside: $V(r,\phi)=\dfrac{2V_0}{\pi}\sum_{n=odd}\frac{1}{n}\left(\frac{b}{r}\right)^n\left[\sin n\phi+\sin n\left(\phi+\frac{\pi}{2}\right)\right]$, $r>b$.

P.4-19 $V(r,\phi)=-E_0r\cos\phi+\sum_{n=1}^{\infty}B_nr^{-n}\cos n\phi$. $\left(\begin{array}{l}\text{At } r\gg b,\\ \bar{E}=\bar{a}_xE_0,\ V=-E_0r\cos\phi\end{array}\right)$

At $r=b$, $V(b,\phi)=-E_0b\cos\phi+\sum_{n=1}^{\infty}B_nb^{-n}\cos n\phi$

$$\longrightarrow\ B_1=E_0b^2;\quad B_n=0 \text{ for } n\neq 1.$$

Outside the cylinder: $r>b$
$$V(r,\phi)=-E_0r\left(1-\frac{b^2}{r^2}\right)\cos\phi,$$
$$\bar{E}(r,\phi)=-\bar{\nabla}V=\bar{a}_rE_0\left(\frac{b^2}{r^2}+1\right)\cos\phi+\bar{a}_\phi E_0\left(\frac{b^2}{r^2}-1\right)\sin\phi.$$

P.4-20 For $r\geq b$, $V_o(r,\phi)=-E_0r\cos\phi+\sum_{n=1}^{\infty}B_nr^{-n}\cos n\phi$,

$r\leq b$, $V_i(r,\phi)=\sum_{n=1}^{\infty}A_nr^n\cos n\phi$.

At $r=b$: $V_o(b,\phi)=V_i(b,\phi)\longrightarrow -E_0b+B_1b^{-1}=A_1b;\ B_nb^{-n}=A_nb^n,\ n\neq1.$

$$-\left.\frac{\partial V_o}{\partial r}\right|_{r=b}=-\epsilon_r\left.\frac{\partial V_i}{\partial r}\right|_{r=b}\longrightarrow E_0+B_1b^{-2}=-\epsilon_rA_1;\ nB_nb^{-(n+1)}=-\epsilon_rnA_nb^{n-1},\ n\neq1.$$

Solving: $\quad A_1 = -\dfrac{2E_0}{\epsilon_r+1}, \quad B_1 = \dfrac{\epsilon_r - 1}{\epsilon_r + 1} b^2 E_0,$

$$A_n = B_n = 0 \quad \text{for } n \neq 1.$$

$$V_o(r,\phi) = -\left(1 - \frac{\epsilon_r - 1}{\epsilon_r + 1}\frac{b^2}{r^2}\right) E_0 r\cos\phi.$$

$$V_i(r,\phi) = -\frac{2}{\epsilon_r + 1} E_0 r\cos\phi.$$

$$\bar{E} = -\bar{\nabla}V = -\bar{a}_r \frac{\partial V}{\partial r} - \bar{a}_\phi \frac{\partial V}{r\partial\phi}.$$

$$\bar{E}_o = \bar{a}_x E_0 - \frac{\epsilon_r - 1}{\epsilon_r + 1}\left(\frac{b}{r}\right)^2 E_0 (\bar{a}_r \cos\phi + \bar{a}_\phi \sin\phi).$$

$$\bar{E}_i = \frac{2}{\epsilon_r + 1}\bar{a}_x E_0 = \frac{2}{\epsilon_r + 1}(\bar{a}_r \cos\phi - \bar{a}_\phi \sin\phi).$$

<u>P.4-21</u> Starting from Eq. (4-134) and applying the boundary condition $V(b,\theta) = V_0$:

$$V_0 = \frac{B_0}{b} + \left(\frac{B_1}{b^2} - E_0 b\right)\cos\theta - \sum_{n=2}^{\infty} B_n b^{-(n+1)} P_n(\cos\theta), \quad R \geq b$$

$$\longrightarrow \quad B_0 = bV_0;\ B_1 = E_0 b^3;\ B_n = 0 \ \text{ for } n \geq 2.$$

$$\therefore\ V(R,\theta) = \frac{b}{R}V_0 - E_0\left[1 - \left(\frac{b}{R}\right)^3\right] R\cos\theta, \quad R \geq b.$$

$$\bar{E}(R,\theta) = \bar{a}_R \left\{\frac{bV_0}{R^2} + E_0\left[1 + 2\left(\frac{b}{R}\right)^3\right]\cos\theta\right\}$$
$$- \bar{a}_\theta E_0\left[1 - \left(\frac{b}{R}\right)^3\right] R\sin\theta, \quad R \geq b.$$

$$\rho_s(\theta) = \epsilon_0 E_R\Big|_{R=b} = \epsilon_0 \frac{V_0}{b} + 3\epsilon_0 E_0 \cos\theta.$$

<u>P.4-22</u> $\quad V_i(R,\theta) = \sum_{n=0}^{\infty} A_n R^n P_n(\cos\theta), \quad R \leq b.$

$$V_o(R,\theta) = \sum_{n=0}^{\infty} \left(B_n R^n + C_n R^{-(n+1)}\right) P_n(\cos\theta), \quad R \geq b.$$

For $R \gg b$, $V_o(R,\theta) = -E_0 z = -E_0 R\cos\theta \longrightarrow B_1 = -E_0;\ B_n = C_n = 0$ for $n \neq 1$.

$$\therefore\ V_o(R,\theta) = -E_0 R\cos\theta + C_1 R^{-2}\cos\theta.$$

B.C.: $V_i(b,\theta) = V_o(b,\theta) \longrightarrow A_1 b = -E_0 b + C_1 b^{-2}$

$\epsilon_r \dfrac{\partial V_i}{\partial R}\Big|_{R=b} = \dfrac{\partial V_o}{\partial R}\Big|_{R=b} \longrightarrow \epsilon_r A_1 = -E_0 - 2C_1 b^{-3}$ $\Big\}\quad A_1 = -\dfrac{3E_0}{\epsilon_r + 2}, \quad C_1 = \dfrac{\epsilon_r - 1}{\epsilon_r + 2} E_0 b^3$

$$\therefore\ V_i(R,\theta) = -\frac{3E_0}{\epsilon_r + 2} R\cos\theta, \qquad V_o(R,\theta) = -E_0 R\cos\theta - \frac{(\epsilon_r - 1)b^3}{(\epsilon_r + 2)R^3} E_0 \cos\theta.$$

$$\bar{E}_i(R,\theta) = -\bar{\nabla}V_i = \frac{3E_0}{\epsilon_r + 2}(\bar{a}_R\cos\theta - \bar{a}_\theta \sin\theta) = \bar{a}_z \frac{3E_0}{\epsilon_r + 2}.$$

$$\bar{E}_o(R,\theta) = -\bar{\nabla}V_o = \bar{a}_R\left[1 + \frac{2(\epsilon_r - 1)b^3}{(\epsilon_r + 2)R^3}\right] E_0 \cos\theta - \bar{a}_\theta\left[1 - \frac{(\epsilon_r - 1)b^3}{(\epsilon_r + 2)R^3}\right] E_0 \sin\theta$$

<u>Chapter 5</u>

R_1 = Resistance per unit length of core $= \dfrac{1}{\sigma S_1} = \dfrac{1}{\sigma \pi a^2}$

R_2 = Resistance per unit length of coating $= \dfrac{1}{0.1\sigma S_2}$

Let b = Thickness of coating.

$$S_2 = \pi(a+b)^2 - \pi a^2 = \pi(2ab+b^2).$$

a) $R_1 = R_2 \longrightarrow b = (\sqrt{11}-1)a = 2.32a$.

b) $I_1 = I_2 = I/2$: $J_1 = \dfrac{I}{2\pi a^2}$, $E_1 = \dfrac{J_1}{\sigma} = \dfrac{I}{2\pi a^2 \sigma}$;

$$J_2 = \frac{I}{2S_2} = \frac{I}{20\pi a^2},\quad E_2 = \frac{J_2}{0.1\sigma} = \frac{I}{2\pi a^2\sigma} = E_1.$$

4 $I_1 = 0.7\,(A)$, $P_{R1} = 0.163\,(W)$; $I_2 = 0.140\,(A)$, $P_{R2} = 0.392\,(W)$;

$I_3 = 0.093\,(A)$, $P_{R3} = 0.261\,(W)$; $I_4 = 0.233\,(A)$, $P_{R4} = 0.436\,(W)$;

$I_5 = 0.467\,(A)$, $P_{R5} = 2.178\,(W)$.

5 $\rho = \rho_0 e^{-(\sigma/\epsilon)t}$, $\rho_0 = \dfrac{Q_0}{(4\pi/3)b^3} = \dfrac{10^{-3}}{(4\pi/3)(0.1)^3} = 0.239\,(C/m^3)$.

a) $R<b$, $\bar{E}_i = \bar{a}_R \dfrac{(4\pi/3)R^3\rho}{4\pi\epsilon R^2} = \bar{a}_R \dfrac{\rho_0 R}{3\epsilon} e^{-(\sigma/\epsilon)t}$

$$= \bar{a}_R\, 7.5\times10^9 R\, e^{-9.42\times10^{11}t}\quad (V/m);$$

$R>b$, $\bar{E}_o = \bar{a}_R \dfrac{Q_0}{4\pi\epsilon_0 R^2} = \bar{a}_R \dfrac{9}{R^2}\times10^6\quad (V/m)$.

b) $R<b$, $\bar{J}_i = \sigma\bar{E}_i = \bar{a}_R\, 7.5\times10^{10} R\, e^{-9.42\times10^{11}t}\quad (A/m^2)$;

$R>b$, $\bar{J}_o = 0$.

6 a) $e^{-(\sigma/\epsilon)t} = \dfrac{\rho}{\rho_0} = 0.01 \longrightarrow t = \dfrac{\ln 100}{(\sigma/\epsilon)} = 4.88\times10^{-12}\,(s)$

$$= 4.88\,(ps).$$

b) $W_i = \dfrac{\epsilon}{2}\displaystyle\int_{V'} E_i^2\, dv' = \dfrac{2\pi\rho_0^2 b^5}{45\epsilon} e^{-2(\sigma/\epsilon)t} = (W_i)_0 \left[e^{-(\sigma/\epsilon)t}\right]^2$.

$\therefore \dfrac{W_i}{(W_i)_0} = \left[e^{-(\sigma/\epsilon)t}\right]^2 = 0.01^2 = 10^{-4}$. Energy dissipated as heat loss.

c) Electrostatic energy stored outside the sphere

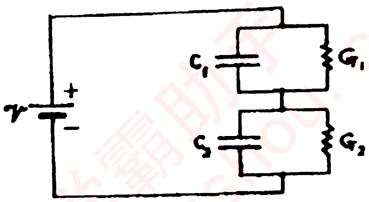
$$W_o = \frac{\epsilon_0}{2}\int_b^\infty E_o^2\, 4\pi R^2 dR = \frac{Q_0^2}{8\pi\epsilon_0 b} = 45\,(kJ).\ \text{Constant.}$$

7 a) $R = \dfrac{\ell}{\sigma S} = \dfrac{V}{I} \longrightarrow \sigma = \dfrac{\ell I}{SV} = 3.537\times10^7\,(S/m)$.

b) $E = \dfrac{V}{\ell} = 6\times10^{-3}\,(V/m)$, or $E = \dfrac{J}{\sigma} = \dfrac{I}{\sigma S}$.

c) $P = VI = 1\,(W)$.

P.5-8



a) $C_1 = \dfrac{\epsilon_1 S}{d_1}, \quad G_1 = \dfrac{\sigma_1 S}{d_1};$

$C_2 = \dfrac{\epsilon_2 S}{d_2}, \quad G_2 = \dfrac{\sigma_2 S}{d_2}.$

b) $P = \mathcal{V}^2 G = \mathcal{V}^2 \dfrac{G_1 G_2}{G_1 + G_2}$

$= \mathcal{V}^2 S \dfrac{\sigma_1 \sigma_2}{\sigma_1 d_2 + \sigma_2 d_1}.$

P.5-9 a) $G_1 = \dfrac{2\pi\sigma_1}{\ln(c/a)}, \quad G_2 = \dfrac{2\pi\sigma_2}{\ln(b/c)}.$

$$I = \mathcal{V} G = \mathcal{V}\frac{G_1 G_2}{G_1 + G_2} = \frac{2\pi\sigma_1\sigma_2\mathcal{V}}{\sigma_1 \ln(b/c) + \sigma_2 \ln(c/a)}.$$

$$J_1 = J_2 = \frac{I}{2\pi r L} = \frac{\sigma_1\sigma_2\mathcal{V}}{rL[\sigma_1 \ln(b/c) + \sigma_2 \ln(c/a)]}.$$

b) $$\rho_{sa} = \epsilon_1 E_1\Big|_{r=a} = \frac{\epsilon_1\sigma_2\mathcal{V}}{aL[\sigma_1 \ln(b/c) + \sigma_2 \ln(c/a)]},$$

$$\rho_{sb} = -\epsilon_2 E_2\Big|_{r=b} = -\frac{\epsilon_2\sigma_1\mathcal{V}}{bL[\sigma_1 \ln(b/c) + \sigma_2 \ln(c/a)]},$$

$$\rho_{sc} = -(\epsilon_1 E_1 - \epsilon_2 E_2)\Big|_{r=c} = \frac{(\epsilon_2\sigma_1 - \epsilon_1\sigma_2)\mathcal{V}}{cL[\sigma_1 \ln(b/c) + \sigma_2 \ln(c/a)]}$$

P.5-10 $\nabla^2 V = 0 \longrightarrow \dfrac{1}{r}\dfrac{\partial}{\partial r}\left(r\dfrac{\partial V}{\partial r}\right) = 0.$

$V(r) = c_1 \ln r + c_2$; B.C.: $V(a) = V_0, \quad V(b) = 0.$

$$V(r) = V_0 \frac{\ln(b/r)}{\ln(b/a)}.$$

$$\bar{E}(r) = -\bar{a}_r \frac{\partial V}{\partial r} = \bar{a}_r \frac{V_0}{r\ln(b/a)}, \qquad \bar{J} = \sigma\bar{E}.$$

$$I = \int_S \bar{J}\cdot d\bar{s} = \int_0^{\pi/2} \bar{J}\cdot(\bar{a}_r h r\, d\phi) = \frac{\pi\sigma h V_0}{2\ln(b/a)}.$$

$$R = \frac{V_0}{I} = \frac{2\ln(b/a)}{\pi\sigma h}.$$

P.5-11 Assume a current I between the spherical surfaces.

$$\bar{J} = \bar{a}_R \frac{I}{4\pi R^2} = \sigma\bar{E}.$$

$$V_0 = -\int_{R_2}^{R_1} \bar{E}\cdot d\bar{R} = \int_{R_1}^{R_2} \frac{I\, dR}{4\pi R^2\sigma} = \frac{I}{4\pi\sigma_0}\int_{R_1}^{R_2} \frac{dR}{R^2(1 + k/R)}$$

$$= \frac{I}{4\pi\sigma_0}\int_{R_1}^{R_2} \frac{1}{k}\left(\frac{1}{R} - \frac{1}{R+k}\right) dR = \frac{I}{4\pi\sigma_0 k} \ln \frac{R}{R+k}\Big|_{R_1}^{R_2}$$

$$R = \frac{V_0}{I} = \frac{1}{4\pi\sigma_0 k} \ln \frac{R_2(R_1+k)}{R_1(R_2+k)}.$$

P. 5-12 Assume I. $\bar{J}(R) = \bar{a}_R \dfrac{I}{S(R)}$.

$$S(R) = \int_0^{2\pi}\int_0^{\theta_0} R^2 \sin\theta \, d\theta \, d\phi = 2\pi R^2 (1-\cos\theta_0).$$

$$\bar{E}(R) = \frac{\bar{J}(R)}{\sigma} = \bar{a}_R \frac{I}{2\pi\sigma R^2(1-\cos\theta_0)}.$$

$$V_0 = -\int_{R_2}^{R_1} E(R)\,dR = \frac{I(R_2-R_1)}{2\pi\sigma R_1 R_2 (1-\cos\theta_0)}.$$

$$\therefore \mathcal{R} = \frac{V_0}{I} = \frac{R_2-R_1}{2\pi\sigma R_1 R_2(1-\cos\theta_0)}.$$

P. 5-13 $\bar{\nabla}\cdot\bar{J} = \bar{\nabla}\cdot(\sigma\bar{E}) = \sigma\bar{\nabla}\cdot\bar{E} + (\bar{\nabla}\sigma)\cdot\bar{E} = 0.$

$$\bar{E} = \bar{a}_R E, \quad \bar{\nabla}\cdot\bar{E} = \frac{1}{R^2}\frac{\partial}{\partial R}(R^2 E); \quad \bar{\nabla}\sigma = \bar{a}_R\frac{\partial\sigma}{\partial R} = -\bar{a}_R\frac{\sigma_0 R_1}{R^2}.$$

$$\longrightarrow R\frac{\partial E}{\partial R} = -E, \quad \bar{E} = \bar{a}_R\frac{c}{R}.$$

$$V_0 = -\int_{R_2}^{R_1}\bar{E}\cdot d\bar{R} = -\int_{R_2}^{R_1}\frac{c}{R}\,dR = c\ln\frac{R_2}{R_1}.$$

$$c = \frac{V_0}{\ln(R_2/R_1)}, \quad \bar{E} = \bar{a}_R\frac{V_0}{R\ln(R_2/R_1)}.$$

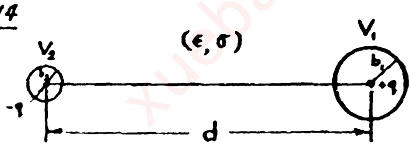
$$I = \int_S \bar{J}\cdot d\bar{s} = \int_S \sigma\bar{E}\cdot d\bar{s}$$

$$= \int_0^{2\pi}\int_0^{\theta_0}\left(\frac{\sigma_0 R_1}{R}\right)\left[\frac{V_0}{R\ln(R_2/R_1)}\right]R^2\sin\theta\,d\theta\,d\phi$$

$$= \frac{2\pi\sigma_0 R_1 V_0(1-\cos\theta_0)}{\ln(R_2/R_1)}.$$

$$\therefore \mathcal{R} = \frac{V_0}{I} = \frac{\ln(R_2/R_1)}{2\pi\sigma_0 R_1(1-\cos\theta_0)}.$$

P. 5-14



Assume charges $+q$ and $-q$ to concentrate at the centers of spheres 1 and 2 respectively.

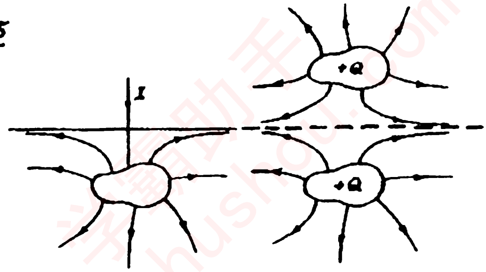
$d \gg b_1$, $d \gg b_2$. $\quad V_1 \cong \dfrac{q}{4\pi\epsilon}\left(\dfrac{1}{b_1} - \dfrac{1}{d-b_1}\right)$,

$$V_2 \cong \frac{q}{4\pi\epsilon}\left(\frac{1}{d-b_2} - \frac{1}{b_2}\right).$$

$$C = \frac{q}{V_1-V_2} \cong \frac{4\pi\epsilon}{\frac{1}{b_1}+\frac{1}{b_2}-\frac{1}{d-b_1}-\frac{1}{d-b_2}} = G\frac{\epsilon}{\sigma} = \frac{\epsilon}{R\sigma},$$

$$R = \frac{1}{4\pi\sigma}\left(\frac{1}{b_1}+\frac{1}{b_2}-\frac{1}{d-b_1}-\frac{1}{d-b_2}\right) = \frac{1}{4\pi\sigma}\left(\frac{1}{b_1}+\frac{1}{b_2}-\frac{2}{d}\right).$$

P.5-15



(a) (b)

The current flow pattern of the lower half of Fig. (b) if both the conductor and its image are fed with the same current is exactly the same as that of Fig. (a). All boundary conditions are satisfied.

The streamlines are similar to the electric field lines of a conductor and its image, both carrying a charge +Q, in the electrostatic case.

P.5-16 According to P.5-15, the current flow pattern would be the same as that of a whole sphere in unbounded earth medium. Hence the current lines are radial. Assume a current I.

$$\bar{J} = \bar{a}_R \frac{I}{2\pi R^2}, \quad \bar{E} = \bar{a}_R \frac{I}{2\pi\sigma R^2},$$

$$V_0 = -\int_\infty^b E\, dR = -\frac{I}{2\pi\sigma}\int_\infty^b \frac{dR}{R^2} = \frac{I}{2\pi\sigma b}.$$

$$R = \frac{V_0}{I} = \frac{1}{2\pi\sigma b} = \frac{1}{2\pi(10^{-6})(25\times10^{-3})} = 6.36\times10^6 (\Omega)$$

P.5-17 The bounday conditions at $y=0$ and $y=b$ require that $Y(y) \sim \cos(\frac{n\pi}{b}y)$; the boundary condition at $x=0$ indicates that $X(x) \sim \sinh(\frac{n\pi}{b}x)$. Thus,

a) $$V(x,y) = \sum_{n=0}^{\infty} C_n \sinh\left(\frac{n\pi}{b}x\right)\cos\left(\frac{n\pi}{b}y\right).$$

B.C. at $x=a$: $$V(a,y) = V_0 = \sum_{n=0}^{\infty} C_n \sinh\left(\frac{n\pi}{b}a\right)\cos\left(\frac{n\pi}{b}y\right)$$
$$= \sum_{n=0}^{\infty} B_n \cos\left(\frac{n\pi}{b}y\right).$$

$\int_0^b [\quad]\cos(\frac{n\pi}{b}y)dy$: $\quad 0 = B_n\left(\frac{b}{2}\right) \longrightarrow B_n = 0$ for $n \neq$

For $n=0$, $V_0 = B_0 \longrightarrow C_0 = \dfrac{V_0}{\sinh(n\pi a/b)}$.

$$V(x,y) = V_0\left[\frac{\sinh(n\pi x/b)}{\sinh(n\pi a/b)}\cos\left(\frac{n\pi}{b}y\right)\right]_{n=0} = \frac{V_0}{a}x.$$

b) $\bar{J} = \sigma \bar{E} = -\sigma \bar{\nabla} V = -\bar{a}_x \frac{\sigma V_0}{a}$.

P.5-18 $\quad V(r,\phi) = \sum_{n=0}^{\infty} (A_n r^n + B_n r^{-n})(C_n \cos n\phi + D_n \sin n\phi)$.

B.C.: $V(r,\phi) = V(r,-\phi) \longrightarrow D_n = 0$.

$r \to \infty$, $V(r,\phi) = -\frac{1}{\sigma} J_0 r \cos\phi \longrightarrow A_n = C_n = 0$ for $n \neq 1$.

Write $\quad V(r,\phi) = (K_1 r + \frac{K_2}{r})\cos\phi$. $\quad K_1 = A_1 C_1$, $K_2 = B_1 C_1$.

$$K_1 = -\frac{J_0}{\sigma}$$

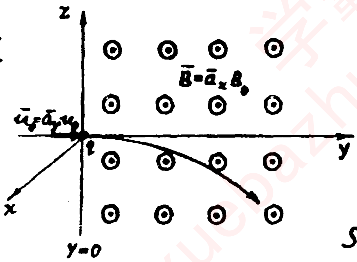
B.C.: $\left.\frac{\partial V}{\partial r}\right|_{r=b} = 0 \longrightarrow K_1 - \frac{K_2}{b^2} = 0$, $\quad K_2 = b^2 K_1 = -\frac{J_0}{\sigma} b^2$.

$$\therefore \quad V(r,\phi) = -\frac{J_0}{\sigma}\left(r + \frac{b^2}{r}\right)\cos\phi.$$

$$\bar{J} = -\sigma \bar{\nabla} V = -\sigma\left(\bar{a}_r \frac{\partial V}{\partial r} + \bar{a}_\phi \frac{\partial V}{r\partial\phi}\right)$$
$$= \bar{a}_r J_0\left(1 - \frac{b^2}{r^2}\right)\cos\phi - \bar{a}_\phi J_0\left(1 + \frac{b^2}{r^2}\right)\sin\phi$$
$$= J_0(\bar{a}_r \cos\phi - \bar{a}_\phi \sin\phi) - \frac{J_0 b^2}{r^2}(\bar{a}_r \cos\phi + \bar{a}_\phi \sin\phi)$$
$$= \bar{a}_x J_0 - \frac{J_0 b^2}{r^2}(\bar{a}_r \cos\phi + \bar{a}_\phi \sin\phi), \quad r > a.$$
$$\bar{J} = 0, \quad r < b.$$

Chapter 6

P.6-1



$\frac{du_y}{dt} = \frac{qB_0}{m} u_z = \omega_0 u_z \quad (1)$

$\frac{du_z}{dt} = -\frac{qB_0}{m} u_y = -\omega_0 u_y \quad (2)$ $\qquad \omega_0 = \frac{qB_0}{m}$

Combining (1) and (2):

$$\frac{d^2 u_z}{dt^2} + \omega_0^2 u_z = 0.$$

Solution: $u_z = A\cos\omega_0 t + B\sin\omega_0 t$.

At $t=0$, $u_z = 0 \longrightarrow A = 0$, $u_z = B\sin\omega_0 t$

Substituting u_z in (2): $\quad u_y = -B\cos\omega_0 t$.

At $t=0$, $u_y = u_0 \longrightarrow B = -u_0$.

$\therefore \quad u_y = u_0 \cos\omega_0 t \longrightarrow y = \frac{u_0}{\omega_0}\sin\omega_0 t \quad (t=0, y=0)$,

$u_z = -u_0 \sin\omega_0 t \longrightarrow z = \frac{u_0}{\omega_0}\cos\omega_0 t + C \quad (t=0, z=0 \text{ — } C = -\frac{u_0}{\omega_0})$

$= -\frac{u_0}{\omega_0}(1 - \cos\omega_0 t)$.

We obtain $\quad y^2 + \left(z + \frac{u_0}{\omega_0}\right)^2 = \left(\frac{u_0}{\omega_0}\right)^2$ --- Eq. of a shifted circle.

P.6-2 $\quad \dfrac{\partial \bar{u}}{\partial t} = -\dfrac{|e|}{m}(\bar{E} + \bar{u}\times\bar{B})$

a) $\bar{E} = \bar{a}_z E_0$, $\bar{B} = \bar{a}_x B_0$.

$$\left.\begin{aligned} \frac{\partial u_x}{\partial t} &= 0 \\ \frac{\partial u_y}{\partial t} &= -\frac{|e|}{m}B_0 u_z \\ \frac{\partial u_z}{\partial t} &= -\frac{|e|}{m}(E_0 - B_0 u_y) \end{aligned}\right. \longrightarrow \left\{\begin{aligned} u_x &= 0 \\ u_y &= \left(u_0 - \frac{E_0}{B_0}\right)\cos\omega_0 t + \frac{E_0}{B_0} \\ u_z &= \left(\frac{E_0}{B_0} - u_0\right)\sin\omega_0 t, \end{aligned}\right.$$

$$\omega_0 = \frac{|e|}{m}B_0$$

If the electron is injected at the origin $(x=y=z=0)$ at $t=0$,

We have $x = 0$, $y = \dfrac{c}{\omega_0}\sin\omega_0 t + \dfrac{E_0}{B_0}t$, $z = -\dfrac{c}{\omega_0}(1-\cos\omega_0 t)$.

$$c = u_0 - \frac{E_0}{B_0}$$

Equation of motion:

$$\left(y - \frac{E_0}{B_0}t\right)^2 + \left(z + \frac{c}{\omega_0}\right)^2 = \left(\frac{c}{\omega_0}\right)^2.$$

If $E_0/B_0 = u_0$, $u_x = u_z = 0$, $u_y = u_0$; $x = z = 0$, $y = u_0 t$.

b) $\bar{E} = -\bar{a}_z E_0$, $\bar{B} = -\bar{a}_z B_0$.

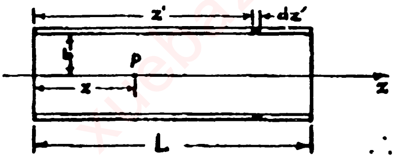
$$\left.\begin{aligned} \frac{\partial u_x}{\partial t} &= \frac{|e|}{m}B_0 u_y = \omega_0 u_y \\ \frac{\partial u_y}{\partial t} &= -\omega_0 u_x \end{aligned}\right\} \text{Circular motion (See P.6-1)}$$

$$\frac{\partial u_z}{\partial t} = \frac{|e|}{m}E_0.$$

} Helical motion with constant acceleration $|e|E_0/m$ in z direction.

P.6-3 Straightforward application of Ampère's circuital law.

P.6-4



From Example 6-6, Eq. (6-38)

$$dB = \frac{\mu_0 I b^2}{2[(z'-z)^2 + b^2]^{3/2}}\left(\frac{N}{L}dz'\right)$$

$$\therefore\ B = \frac{\mu_0 N I b^2}{2L}\int_0^L \frac{dz'}{[(z'-z)^2 + b^2]^{3/2}}$$

or,

$$B = \frac{\mu_0 N I}{2L}\left[\frac{L-z}{\sqrt{(L-z)^2 + b^2}} + \frac{z}{\sqrt{z^2 + b^2}}\right] \longrightarrow \mu_0\left(\frac{N}{L}\right)I \quad \text{as } L \to \infty.$$

Direction of B is determined by the right-hand rule.

P.6-5 Eq. (6-22): $\bar{A} = \dfrac{\mu_0}{4\pi}\displaystyle\int_{V'} \frac{\bar{J}}{R}\,dv'$

$$\bar{B} = \bar{\nabla}\times\bar{A} = \frac{\mu_0}{4\pi}\int_{V'}\bar{\nabla}\times\left(\frac{1}{R}\bar{J}\right)dv' = \frac{\mu_0}{4\pi}\int_{V'}\underbrace{\left[\frac{1}{R}\bar{\nabla}\times\bar{J} + \left(\bar{\nabla}\frac{1}{R}\right)\times\bar{J}\right]}_{\text{From P.2-26}}\ldots$$

$\bar{\nabla}\times\bar{J} = 0$ because the curl operation $\bar{\nabla}\times$ respect to unprimed coordinates at the and $\bar{J}$ is a function of primed (sourc

$$\bar{\nabla}\frac{1}{R} = -\bar{a}_R\frac{1}{R^2}.$$

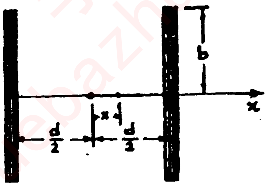
$$\therefore \quad \bar{B} = \frac{\mu_0}{4\pi}\int_{V'}\frac{-\bar{a}_R\times\bar{J}}{R^2}\,dv' = \frac{\mu_0}{4\pi}\int_{V'}\frac{\bar{J}\times\bar{a}_R}{R^2}\,dv'.$$

$$\bar{\nabla}\cdot\bar{B} = \frac{\mu_0}{4\pi}\int_{V'}\bar{\nabla}\cdot\left[\bar{\nabla}\left(\frac{1}{R}\right)\times\bar{J}\right]dv'$$

$$= \frac{\mu_0}{4\pi}\int_{V'}\Big[\bar{J}\cdot\underbrace{\left(\bar{\nabla}\times\bar{\nabla}\frac{1}{R}\right)}_{0,\text{ from Eq.(2-133)}} - \left(\bar{\nabla}\frac{1}{R}\right)\cdot\underbrace{\bar{\nabla}\times\bar{J}}_{0}\Big]dv', \quad \text{from P.2-23.}$$

$$= 0.$$

<u>P.6-6</u>



Use Eq. (6-38).

$$B_x = \frac{N\mu_0 Ib}{2}\left\{\frac{1}{[(\frac{d}{2}+x)^2+b^2]^{3/2}} + \frac{1}{[(\frac{d}{2}-x)^2+b^2]^{3/2}}\right\}.$$

a) At $x=0$, $B_x = \dfrac{N\mu_0 Ib}{[(d/2)^2+b^2]^{3/2}}$.

b) $$\frac{dB_x}{dx} = \frac{N\mu_0 Ib}{2}\left\{-\frac{3}{2}\,\frac{2(\frac{d}{2}+x)}{[(\frac{d}{2}+x)^2+b^2]^{5/2}} + \frac{3}{2}\,\frac{2(\frac{d}{2}-x)}{[(\frac{d}{2}-x)^2+b^2]^{5/2}}\right\}.$$

At the midpoint, $x=0$, $\dfrac{dB_x}{dx} = 0$.

c) $$\frac{d^2B_x}{dx^2} = -\frac{3N\mu_0 Ib}{2}\left\{\frac{1}{[(\frac{d}{2}+x)^2+b^2]^{5/2}} - \frac{5(\frac{d}{2}+x)^2}{[(\frac{d}{2}+x)^2+b^2]^{7/2}} + \frac{1}{[(\frac{d}{2}-x)^2+b^2]^{5/2}} - \frac{5(\frac{d}{2}-x)^2}{[(\frac{d}{2}-x)^2+b^2]^{7/2}}\right\}.$$

At $x=0$, $$\frac{d^2B_x}{dx^2} = -\frac{6N\mu_0 Ib}{2}\left\{\frac{(d/2)^2+b^2-5(d/2)^2}{[(d/2)^2+b^2]^{7/2}}\right\},$$

which vanishes if $b^2-4(d/2)^2=0$, or $b=d$.

<u>P.6-7</u>

Use Eq. (6-35) for a wire of length $2L$. $\bar{B} = \bar{a}_\phi\dfrac{\mu_0 IL}{2\pi r\sqrt{L^2+r^2}}$.

$\alpha = \dfrac{2\pi}{2N} = \dfrac{\pi}{N}$,

$\dfrac{L}{r} = \tan\alpha = \tan\dfrac{\pi}{N}$.

In this problem,
$$\bar{B} = \bar{a}_n N\left(\frac{\mu_0 IL}{2\pi r b}\right) = \bar{a}_n\frac{\mu_0 NI}{2\pi b}\tan\frac{\pi}{N}.$$

When N is very large, $\tan\dfrac{\pi}{N} \approx \dfrac{\pi}{N}$, $\bar{B} \to \bar{a}_n\dfrac{\mu_0 I}{2b}$,

which is the same as Eq. (6-38) with $z=0$.

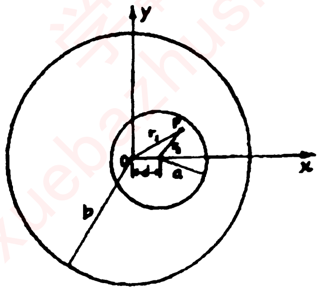
P.6-8 $B_\phi = \frac{\mu_0 N I}{2\pi r}$.

$$\Phi = \int_S B_\phi\, ds = \frac{\mu_0 NI}{2\pi}\int_a^b \frac{1}{r} h\, dr = \frac{\mu_0 NIh}{2\pi}\ln\frac{b}{a}.$$

If B_ϕ at $r = \frac{a+b}{2}$ is used, $\Phi' = \frac{\mu_0 NIh}{\pi}\left(\frac{b-a}{b+a}\right)$.

$$\%\ \text{error} = \frac{\Phi' - \Phi}{\Phi}\times 100\% = \left[\frac{2(b-a)}{(b+a)\ln(b/a)} - 1\right]\times 100\%.$$

P.6-9



$\bar J = \bar a_z J$, $\oint \bar B\cdot d\bar\ell = \mu_0 I$

If there were no hole,

$2\pi r_1 B_{\phi 1} = \mu_0 \pi r_1^2 J$

$$B_{\phi 1} = \frac{\mu_0 r_1}{2} J \qquad \begin{cases} B_{x1} = -\frac{\mu_0 J}{2} y_1 \\ B_{y1} = +\frac{\mu_0 J}{2} x_1. \end{cases}$$

For $-J$ in the hole portion:

$$B_{\phi 2} = -\frac{\mu_0 r_2}{2} J \qquad \begin{cases} B_{x2} = +\frac{\mu_0 J}{2} y_2 \\ B_{y2} = -\frac{\mu_0 J}{2} x_2. \end{cases}$$

Superposing $B_{\phi 1}$ and $B_{\phi 2}$ and noting that $y_1 = y_2$ and $x_1 = x_2 + d$, we have

$$B_x = B_{x1} + B_{x2} = 0 \quad\text{and}\quad B_y = B_{y1} + B_{y2} = \frac{\mu_0 J}{2} d.$$

P.6-11 $\bar B = \bar\nabla\times\bar A$, $\bar B = \bar a_\phi B = \bar a_\phi\left(\frac{\partial A_r}{\partial z} - \frac{\partial A_z}{\partial r}\right) = -\bar a_\phi \frac{\partial A_z}{\partial r}$.

For $0 \le r \le b$, Eq. (6-10) gives $\bar B_1 = \bar a_\phi \frac{\mu_0 I}{2\pi b^2} r$.

For $r \ge b$, Eq. (6-11) gives $\bar B_2 = \bar a_\phi \frac{\mu_0 I}{2\pi}\frac{1}{r}$.

Integrating, $\bar A_1 = \bar a_z\left[-\frac{\mu_0 I}{4\pi}\left(\frac{r}{b}\right)^2 + c_1\right]$, $0 \le r \le b$

$\bar A_2 = \bar a_z\left[-\frac{\mu_0 I}{2\pi}\ln r + c_2\right]$, $r \ge b$

At $r = b$, $\bar A_1 = \bar A_2 \longrightarrow c_2 = -\frac{\mu_0 I}{4\pi} + \frac{\mu_0 I}{2\pi}\ln b + c_1$,

$$\therefore\ \bar A_2 = \bar a_z\left\{-\frac{\mu_0 I}{4\pi}\left[\ln\left(\frac{r}{b}\right)^2 + 1\right] + c_1\right\},\ r \ge b.$$

P.6-12 Eq. (6-34) for one wire: $\bar A = \bar a_z \frac{\mu_0 I}{4\pi}\ln\frac{\sqrt{L^2+r^2}+L}{\sqrt{L^2+r^2}-L}$.

For two wires with equal and opposite currents:

a) $$\bar A = \bar a_z\frac{\mu_0 I}{4\pi}\ln\left[\frac{\sqrt{L^2+r_2^2}+L}{\sqrt{L^2+r_2^2}-L}\ \frac{\sqrt{L^2+r_1^2}-L}{\sqrt{L^2+r_1^2}+L}\right] = \bar a_z\frac{\mu_0 I}{2\pi}\ln\left[\frac{r_1}{r_2}\ \frac{\sqrt{L^2+r_2^2}+L}{\sqrt{L^2+r_1^2}+L}\right].$$

b) For a very long two-wire transmission line, $L \to \infty$:

$$\bar{A} = \bar{a}_z \frac{\mu_0 I}{2\pi} \ln\left(\frac{r_1}{r_2}\right) = \bar{a}_z \frac{\mu_0 I}{4\pi} \ln \frac{(\frac{d}{2}+y)^2+x^2}{(\frac{d}{2}-y)^2+x^2}.$$

c) $\bar{B} = \bar{\nabla}\times\bar{A} = \bar{a}_x \frac{\partial A_z}{\partial y} - \bar{a}_y \frac{\partial A_z}{\partial x}$

$$= \bar{a}_x \frac{\mu_0 I}{2\pi}\left[\frac{\frac{d}{2}+y}{(\frac{d}{2}+y)^2+x^2} - \frac{\frac{d}{2}-y}{(\frac{d}{2}-y)^2+x^2}\right]$$

$$-\bar{a}_y \frac{\mu_0 I}{2\pi}\left[\frac{x}{(\frac{d}{2}+y)^2+x^2} - \frac{x}{(\frac{d}{2}-y)^2+x^2}\right] = \frac{\mu_0 I}{2\pi}\left\{\bar{a}_{\phi 1}\frac{1}{r_1} - \bar{a}_{\phi 2}\frac{1}{r_2}\right\}.$$

P.6-14 Apply divergence theorem to $(\bar{F}\times\bar{C})$:

$$\int_V \bar{\nabla}\cdot(\bar{F}\times\bar{C})\,dv = \oint_S (\bar{F}\times\bar{C})\cdot d\bar{s}.$$

Now $\bar{\nabla}\cdot(\bar{F}\times\bar{C}) = \bar{C}\cdot(\bar{\nabla}\times\bar{F}) - \bar{F}\cdot(\bar{\nabla}\times\bar{C}) = \bar{C}\cdot(\bar{\nabla}\times\bar{F})$

$(\bar{F}\times\bar{C})\cdot d\bar{s} = -(\bar{F}\times d\bar{s})\cdot\bar{C}.$

Thus, $\bar{C}\cdot\int_V(\bar{\nabla}\times\bar{F})\,dv = -\bar{C}\cdot\oint_S \bar{F}\times d\bar{s}$

$\longrightarrow \int_V (\bar{\nabla}\times\bar{F})\,dv = -\oint_S \bar{F}\times d\bar{s}$ because $\bar{C}$ is an arbitrary constant vector.

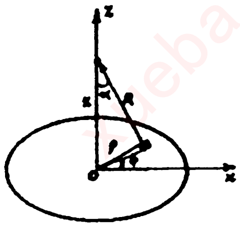
P.6-15 a) $r<a$: $\bar{H} = \bar{a}_z nI$, $\bar{B} = \bar{a}_z \mu nI$ } Eq.(6-13)

$\bar{M} = \frac{\bar{B}}{\mu_0} - \bar{H} = \bar{a}_z\left(\frac{\mu}{\mu_0}-1\right)nI$

$a<r<b$: $\bar{H} = \bar{a}_z nI$, $\bar{B} = \bar{a}_z \mu_0 nI$, $\bar{M} = 0$

b) $\bar{J}_m = \bar{\nabla}\times\bar{M} = 0$; $\bar{J}_{ms} = \bar{M}\times\bar{a}_n = (\bar{a}_z\times\bar{a}_r)\left(\frac{\mu}{\mu_0}-1\right)nI = \bar{a}_\phi\left(\frac{\mu}{\mu_0}-1\right)nI.$

P.6-17



a) $V_m = \frac{I}{4\pi}\int \frac{d\bar{s}\cdot\bar{a}_R}{R^2} = \frac{I}{4\pi}\Omega,$

$d\bar{s}\cdot\bar{a}_R = (\cos\alpha)\rho\,d\rho\,d\phi$

$= \frac{z}{\sqrt{z^2+\rho^2}}\rho\,d\rho\,d\phi,$

$R = \sqrt{z^2+\rho^2}.$

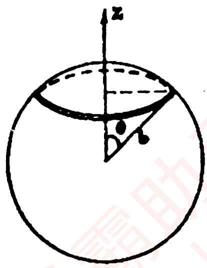
$$\therefore V_m = \frac{I}{4\pi}\int_0^{2\pi}\int_0^b \frac{z}{(z^2+\rho^2)^{3/2}}\rho\,d\rho\,d\phi$$

$$= \frac{I}{2}\left(1 - \frac{z}{\sqrt{z^2+\rho^2}}\right).$$

b) $\bar{B} = -\mu_0\bar{\nabla}V_m = -\bar{a}_z\mu_0\frac{\partial V_m}{\partial z} = \bar{a}_z\frac{\mu_0 I b^2}{2(z^2+b^2)^{3/2}},$

which is the same as Eq.(6-38).

P.6-18



$\bar{M} = \bar{a}_z M_0$

a) $\bar{J}_m = \bar{\nabla} \times \bar{M} = 0.$

$$\bar{J}_{ms} = \bar{M} \times \bar{a}_n = \bar{a}_z M_0 \times \frac{1}{b}(\bar{a}_x x + \bar{a}_y y + \bar{a}_z z)$$
$$= \frac{M_0}{b}(-\bar{a}_x y + \bar{a}_y x) = \bar{a}_\phi \frac{M_0}{b}\sqrt{x^2+y^2}$$
$$= \bar{a}_\phi M_0 \sin\theta .$$

$$\left\{ \text{Or, } \bar{J}_{ms} = (\bar{a}_R \cos\theta - \bar{a}_\theta \sin\theta) M_0 \times \bar{a}_R = \bar{a}_\phi M_0 \sin\theta \right\}$$

b) Apply Eq.(6-38) to a loop of radius $b\sin\theta$ carrying a current $J_{ms} b\, d\theta$:

$$d\bar{B} = \bar{a}_z \frac{\mu_0 (J_{ms} b\, d\theta)(b\sin\theta)^2}{2(b^2)^{3/2}} = \bar{a}_z \frac{\mu_0 M_0}{2} \sin^3\theta .$$

$$\bar{B} = \int d\bar{B} = \bar{a}_z \frac{\mu_0 M_0}{2} \int_0^\pi \sin^3\theta\, d\theta$$
$$= \bar{a}_z \frac{2}{3}\mu_0 M_0 = \frac{2}{3}\mu_0 \bar{M} .$$

P.6-19 a) $$\mathcal{R}_g = \frac{l_g}{\mu_0 S} = \frac{3\times10^{-3}}{4\pi\times10^{-7}\times\pi\times0.025^2} = 1.21\times10^6 \ (H^{-1})$$

$$\mathcal{R}_c = \frac{2\pi\times0.08 - 0.003}{3000\times(4\pi\times10^{-7})\times\pi\times0.025^2} = 6.75\times10^4 \ (H^{-1}).$$

b) $$\bar{B}_g = \bar{B}_c = \bar{a}_\phi \frac{10^{-5}}{\pi\times0.025^2} = \bar{a}_\phi 5.09\times10^{-3} \ (T)$$

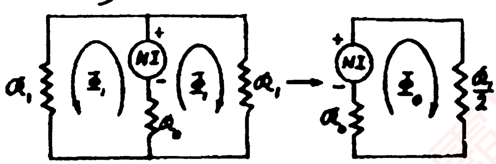
$$\bar{H}_g = \frac{1}{\mu_0}\bar{B}_g = \bar{a}_\phi \frac{5.09\times10^{-3}}{4\pi\times10^{-7}} = \bar{a}_\phi 4.05\times10^3 \ (A/m)$$

$$\bar{H}_c = \frac{1}{\mu_0\mu_r}\bar{B}_c = \bar{a}_\phi \frac{4.05\times10^3}{3000} = \bar{a}_\phi 1.35 \ (A/m).$$

c) $NI = \Phi(\mathcal{R}_c + \mathcal{R}_g)$, $\quad I = \frac{1}{N}\Phi(\mathcal{R}_c + \mathcal{R}_g) = \frac{10^{-5}}{500}\times1.2775\times10^6$

$$= 0.0256 \ (A) = 25.6 \ (mA).$$

P.6-20 Magnetic circuit:



$\Phi_1 = \Phi_0/2$

$S = 10^{-3} \ (m^2)$

$\frac{1}{\mu_0 S} = \frac{1}{(4\pi10^{-7})\times10^{-3}} = 7.95\times1$

Neglecting leakage flux and assuming constant flux densit over S,

$$\mathcal{R}_0 = \frac{1}{\mu_0 S}\left(0.002 - \frac{0.24-0.02}{5000}\right) = 1.60\times10^6 \ (H^{-1}).$$

$$\mathcal{R}_1 = \frac{1}{\mu_0 S}\left(\frac{0.24+2\times0.2}{5000}\right) = 0.102\times10^6 (H^{-1}).$$

$$\Phi_0 = \frac{NI}{\mathcal{R}_0 + \mathcal{R}_1/2} = \frac{200\times 3}{(1.60 + \frac{0.102}{2})\times 10^6} = 3.63\times 10^{-4} \ (Wb).$$

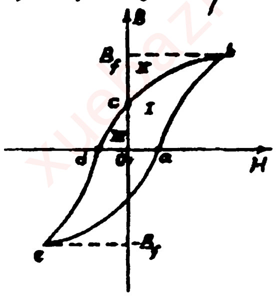
$$\Phi_1 = \frac{\Phi_0}{2} = 1.82\times 10^{-4} \ (Wb).$$

b) $H_1 = \frac{B_1}{\mu_0\mu_r} = \frac{\Phi_1}{\mu_0\mu_r S} = (7.95\times 10^8)\frac{1.82\times 10^{-4}}{5000} = 28.9 \ (A/m)$

$(H_0)_g = \frac{B_0}{\mu_0} = \frac{1}{\mu_0 S}\Phi_0 = (7.95\times 10^8)\times 3.63\times 10^{-4} = 28.9\times 10^4 \ (A/m)$, in air gap.

$(H_0)_c = (H_0)_g/5000 = 57.8 \ (A/m)$, in core.

6-21 a) Work required per unit length in time dt:



$P_1 dt = nI\, d\Phi$.

Work per unit volume in dt:

$dW = \frac{1}{S}P_1 dt = nI\, dB = H dB.$

Thus, $W_1 = \int_0^{B_f} H dB.$

b) Work done per unit volume in changing from 0 to B_f along path ab is W_1, which is represented by areas I and II.

Along path bc, B is decreased, inducing a voltage that tends to maintain the current. Work is done against the source. The work per unit volume W_2 is represented by $-$(area II). In going from c to d, the direction of current is reversed and the work done W_3 is represented by area III. Same amount of work is done in changing B along the path from d to e and back to a as that required in going from a to b through c to d.

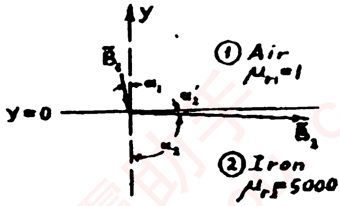
$\therefore$ Work done per unit volume in one cycle $= 2(W_1 + W_2 + W_3)$
$= 2\times$Areas$[(\text{I}+\text{II}) - \text{II} + \text{III}]$ = Area of the hysteresis loop.

6-23 $\bar{H}_1 = -\bar{\nabla} V_{m1}$, $\quad \bar{H}_2 = -\bar{\nabla} V_{m2}$.

Boundary Conditions: $\mu_1 H_{1n} = \mu_2 H_{2n} \longrightarrow \mu_1 \frac{\partial V_{m1}}{\partial n} = \mu_2 \frac{\partial V_{m2}}{\partial n}$.

$H_{1t} - H_{2t} = J_{sn} \longrightarrow \frac{\partial V_{m2}}{\partial t} - \frac{\partial V_{m1}}{\partial t} = J_{sn}$.

P.6-24



a) $\bar{B}_1 = \bar{a}_x 0.5 - \bar{a}_y 10$ (mT)

$\bar{B}_2 = \bar{a}_x B_{2x} + \bar{a}_y B_{2y}$.

$H_{2x} = \frac{B_{2x}}{5000\mu_0} = H_{1x} = \frac{0.5}{\mu_0}$

$\longrightarrow B_{2x} = 2{,}500$ (mT).

$B_{2y} = B_{1y} = -10$ (mT)

$\therefore\ \bar{B}_2 = \bar{a}_x 2500 - \bar{a}_y 10$ (mT).

$\tan\alpha_2 = \frac{\mu_2}{\mu_1}\tan\alpha_1$

$= 5000 \tan 0.05 \cong 250 \longrightarrow \alpha_2 = 89.77°,\ \alpha_2' = 0.23°$.

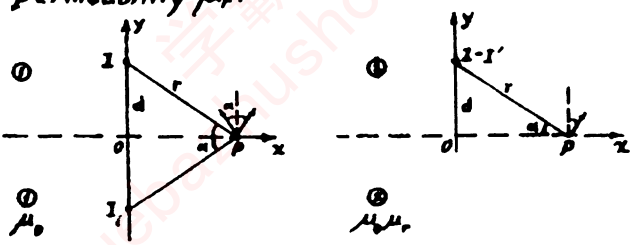
b) If $\bar{B}_2 = \bar{a}_x 10 + \bar{a}_y 0.5$ (mT), $\quad \bar{B}_1 = \bar{a}_x B_{1x} + \bar{a}_y B_{1y}$.

$H_{1x} = \frac{B_{1x}}{\mu_1} = H_{2x} = \frac{B_{2x}}{\mu_2} \longrightarrow B_{1x} = \frac{\mu_{r1}}{\mu_{r2}} B_{2x} = \frac{10}{5000} = 0.002$

$B_{1y} = B_{2y} = 0.5$. $\quad \therefore\ \bar{B}_1 = \bar{a}_x 0.002 + \bar{a}_y 0.5$ (mT).

$\alpha_1 = \tan^{-1}\frac{B_{1x}}{B_{1y}} \cong \frac{0.002}{0.5} = 0.004\,(\text{rad}) = 0.23°$.

P.6-25 a) Consider two situations: ① I and I_i both in air; and ② I and $-I$ both in magnetic medium with relative permeability μ_r.



Find B_{1y} and H_{1x} at P(y=0). $\quad$ Find B_{2y} and H_{2x} at P(y=0).

$B_{1y} = \frac{\mu_0}{2\pi r}(I+I_i)\cos\alpha = \frac{\mu_0\mu_r}{\pi(\mu_r+1)}\frac{x}{r^2}I$; $\quad B_{2y} = \frac{\mu_0\mu_r}{2\pi r}(I-I_i)\cos\alpha = \frac{\mu_0\mu_r}{\pi(\mu_r+1)}\frac{x}{r^2}I$.

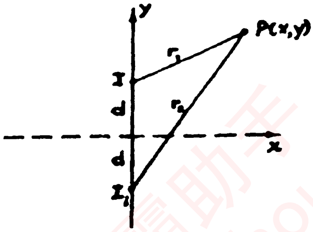
$B_{1x} = \frac{\mu_0}{2\pi r}(I-I_i)\sin\alpha = \frac{\mu_0}{\pi(\mu_r+1)}\frac{d}{r^2}I$; $\quad B_{2x} = \frac{\mu_0\mu_r}{2\pi r}(I-I_i)\sin\alpha = \frac{\mu_0\mu_r}{\pi(\mu_r+1)}\frac{d}{r^2}I$

$H_{1x} = \frac{B_{1x}}{\mu_0} = \frac{I}{\pi(\mu_r+1)}\frac{d}{r^2}$; $\quad H_{2x} = \frac{B_{2x}}{\mu_0\mu_r} = \frac{I}{\pi(\mu_r+1)}\frac{d}{r^2}$.

$\therefore\ B_{1y} = B_{2y}$ and $H_{1x} = H_{2x}$ (Boundary conditions satisfied)

b) For $\mu_r \gg 1$, $I_i = \frac{\mu_r - 1}{\mu_r + 1} I \cong I$.

Refer to following figure.



$$\bar{B}_I = \frac{\mu_0 I}{2\pi r_1}\left(-\bar{a}_x \frac{y-d}{r_1} + \bar{a}_y \frac{x}{r_1}\right).$$

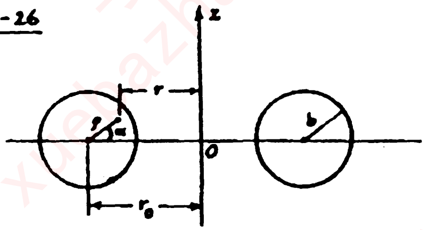
$$\bar{B}_{I_i} = \frac{\mu_0 I}{2\pi r_2}\left(-\bar{a}_x \frac{y+d}{r_2} + \bar{a}_y \frac{x}{r_2}\right).$$

$$\therefore\ \bar{B} = \bar{B}_I + \bar{B}_{I_i}$$

$$= -\bar{a}_x \frac{\mu_0 I}{2\pi}\left[\frac{y-d}{(y-d)^2+x^2} + \frac{y+d}{(y+d)^2+x^2}\right]$$

$$+ \bar{a}_y \frac{\mu_0 I x}{2\pi}\left[\frac{1}{(y-d)^2+x^2} + \frac{1}{(y+d)^2+x^2}\right].$$

P.6-26



$$\bar{B} = \bar{a}_\phi B_\phi = \bar{a}_\phi \frac{\mu_0 N I}{2\pi r}.$$

$$r = r_0 - \rho\cos\alpha.$$

$$\Phi = \frac{\mu_0 N I}{2\pi}\int_0^b \int_0^{2\pi} \frac{\rho\, d\alpha\, d\rho}{r_0 - \rho\cos\alpha}$$

$$= \frac{\mu_0 N I}{2\pi}\int_0^b \left[\frac{2\pi}{\sqrt{r_0^2-\rho^2}}\right]\rho\, d\rho$$

$$= \mu_0 N I\left(r_0 - \sqrt{r_0^2 - b^2}\right).$$

$$\therefore\ L = \frac{N\Phi}{I} = \mu_0 N^2\left(r_0 - \sqrt{r_0^2-b^2}\right).$$

If $r_0 >> b$, $B_\phi \cong \dfrac{\mu_0 N I}{2\pi r_0}$ (constant).

$$\Phi \cong B_\phi S = \frac{\mu_0 N I}{2\pi r_0}\cdot \pi b^2 = \frac{\mu_0 N b^2 I}{2 r_0}.$$

$$L = \frac{N\Phi}{I} \cong \frac{\mu_0 N^2 b^2}{2 r_0}.$$

P.6-27 For $b \le r \le (b+d)$, $\bar{B}_3 = \bar{a}_\phi B_{\phi 3} = \bar{a}_\phi \dfrac{\mu_0 I}{2\pi r}\left[1 - \dfrac{\pi(r^2-b^2)}{\pi(b+d)^2 - \pi b^2}\right]$

$$= \bar{a}_\phi \frac{\mu_0 I}{2\pi r}\left[\frac{(b+d)^2 - r^2}{(b+d)^2 - b^2}\right].$$

Magnetic energy per unit length stored in the outer conductor

$$W'_{m3} = \frac{1}{2\mu_0}\int_b^{b+d} B_{\phi 3}^2\, 2\pi r\, dr$$

$$= \frac{\mu_0 I^2}{4\pi}\left\{\frac{(b+d)^4}{[(b+d)^2-b^2]^2}\ln\left(1+\frac{d}{b}\right) + \frac{b^2 - 3(b+d)^2}{4[(b+d)^2-b^2]}\right\}.$$

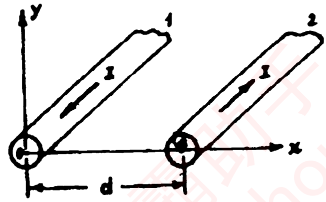
From Eqs. (6-154) – (6-155b) on p.246 we have

$$L' = \frac{2}{I^2}\left(W'_{m1} + W'_{m2} + W'_{m3}\right)$$

$$= \frac{\mu_0}{2\pi}\left\{\frac{1}{4} + \ln\frac{b}{a} - \frac{(b+d)^4}{[(b+d)^2-b^2]^2}\ln\left(1+\frac{d}{b}\right) + \frac{b^2-3(b+d)^2}{4[(b+d)^2-b^2]}\right\}$$

(H/m).

P.6-28



$$\Phi_e' = \int_a^{d-a} (B_{y1} + B_{y2})\, dx$$

$$= \int_a^{d-a} \left[\frac{\mu_0 I}{2\pi x} + \frac{\mu_0 I}{2\pi(d-x)}\right] dx$$

$$= \frac{\mu_0 I}{\pi} \ln\left(\frac{d-a}{a}\right) \cong \frac{\mu_0 I}{\pi} \ln\frac{d}{a}.$$

$$\therefore\ L_e' = \frac{\Phi_e'}{I} \cong \frac{\mu_0}{\pi} \ln\frac{d}{a} \quad (H/m).$$

$$L' = L_i' + L_e' = \frac{\mu_0}{4\pi} + \frac{\mu_0}{\pi} \ln\frac{d}{a} \quad (H/m).$$

P.6-29 For a current I in the long straight wire,

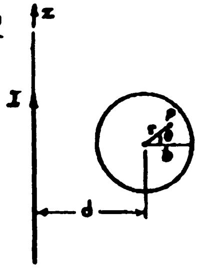
$$\bar{B} = \bar{a}_\phi \frac{\mu_0 I}{2\pi r}.$$

$$\Lambda_{12} = \int_S \bar{B}\cdot d\bar{s} = 2\int B_\phi \frac{1}{\sqrt{3}}(r-d)\, dr = \frac{\mu_0 I}{\pi\sqrt{3}} \int_d^{d+\frac{\sqrt{3}}{2}b} \left(\frac{r-b}{r}\right) dr$$

$$= \frac{\mu_0 I}{\pi\sqrt{3}} \left[\frac{\sqrt{3}}{2} b - d \ln\left(1 + \frac{\sqrt{3}\, b}{2d}\right)\right].$$

$$L_{12} = \frac{\Lambda_{12}}{I} = \frac{\mu_0}{\pi}\left[\frac{b}{2} - \frac{d}{\sqrt{3}} \ln\left(1 + \frac{\sqrt{3}\, b}{2d}\right)\right] \quad (H).$$

P.6-30



Assume a current I.

$\bar{B}$ at $P(r, \theta)$ is $\bar{a}_\phi \dfrac{\mu_0 I}{2\pi(d + r\cos\theta)}$.

$$\Lambda_{12} = \frac{\mu_0 I}{2\pi} \int_0^b \int_0^{2\pi} \frac{r\, dr\, d\theta}{d + r\cos\theta} = \frac{\mu_0 I}{2\pi} \int_0^b \frac{2\pi r\, dr}{\sqrt{d^2 - r^2}}$$

$$= \mu_0 I \left(d - \sqrt{d^2 - b^2}\right).$$

$$L_{12} = \mu_0 \left(d - \sqrt{d^2 - b^2}\right).$$

P.6-31 Since $h_1 >> h_2$, the magnetic flux due to the long loop linking with the small loop can be approximated by that due to two infinitely long wires carrying equal and opposite current I.

$$\Lambda_{12} = \frac{\mu_0 I}{2\pi} \int_0^{w_2} \left(\frac{1}{d+x} - \frac{1}{w_1 + d + x}\right) dx = \frac{\mu_0 I}{2\pi} \ln\left(\frac{w_2 + d}{d} \cdot \frac{w_1 + d}{w_1 + w_2 + d}\right).$$

$$L = \frac{\Lambda_{12}}{I} = \frac{\mu_0}{2\pi} \ln \frac{(w_1 + d)(w_2 + d)}{d(w_1 + w_2 + d)}.$$

P.6-32 Eq. (6-140): $W_2 = \frac{1}{2} L_1 I_1^2 + L_{21} I_1 I_2 + \frac{1}{2} L_2 I_2^2.$

a) $$W_2 = \frac{I_2^2}{2}\left[L_1\left(\frac{I_1}{I_2}\right)^2 + 2L_{21}\left(\frac{I_1}{I_2}\right) + L_2\right]$$

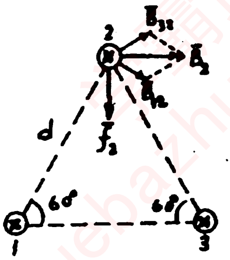
$$= \frac{I_2^2}{2}\left(L_1 x^2 + 2L_{21} x + L_2\right), \quad x = \frac{I_1}{I_2}.$$

$$\frac{dW_2}{dx} = \frac{I_2^2}{2}(2L_1x + 2L_{21}) = 0\ , \qquad \frac{d^2W_2}{dx^2} = I_2^2 L_1 > 0 .$$

$$\therefore\quad x = \frac{I_1}{I_2} = -\frac{L_{21}}{L_1} \quad \text{for minimum } W_2 .$$

b) $$(W_2)_{min} = \frac{I_2^2}{2}\left(L_2 - \frac{L_{21}^2}{L_1}\right) \geq 0 \longrightarrow L_{21} \leq \sqrt{L_1 L_2} .$$

P.6-33



$I_1 = I_2 = I_3 = 25$ (A)

$d = 0.15$ (m)

$$\bar{B}_2 = \bar{a}_x 2B_{12}\cos 30° = \bar{a}_x 2\left(\frac{\mu_0 I}{2\pi d}\right)\frac{\sqrt{3}}{2}$$

$$= \bar{a}_x \frac{\mu_0 I\sqrt{3}}{2\pi d} .$$

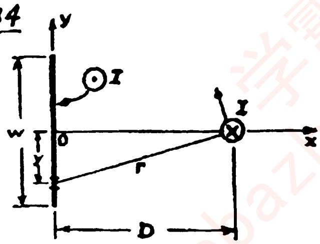
Force per unit length on wire 2:

$$\bar{f}_2 = -\bar{a}_y I B_2 = -\bar{a}_y \frac{\mu_0 I^2\sqrt{3}}{2\pi d}$$

$$= -\bar{a}_y 1150\mu_0 = -\bar{a}_y 1.44\times 10^{-3}\ (\text{N/m}).$$

Forces on all three wires are of equal magnitude and toward the center of the triangle.

P.6-34



Elemental strip dy: $dI = \frac{I}{w}dy$,

$$|d\bar{H}| = \frac{dI}{2\pi r} = \frac{I\,dy}{2\pi w\sqrt{D^2+y^2}} .$$

Symmetry $\longrightarrow$ $\bar{H}$ has only a y-component.

$$\bar{H}\ (\text{at wire}) = \bar{a}_y \int dH\cdot\left(\frac{D}{r}\right)$$

$$= \bar{a}_y 2\int_0^{w/2} \frac{ID}{2\pi w\sqrt{D^2+y^2}}dy$$

$$= \bar{a}_y \frac{I}{\pi w}\tan^{-1}\left(\frac{w}{2D}\right).$$

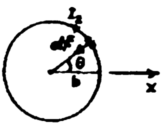
$$\bar{f} = \bar{I}\times\bar{B}$$

$$= (-\bar{a}_z I)\times(\mu_0\bar{H}) = \bar{a}_x \frac{\mu_0 I^2}{\pi w}\tan^{-1}\left(\frac{w}{2D}\right) \quad (\text{N/m}).$$

P.6-35 $\bar{B}$ due I_1 in straight wire in z direction at an elemental arc $b\,d\theta$ on the circular loop:

$$\bar{B} = \bar{a}_\phi \frac{\mu_0 I_1}{2\pi(d+b\cos\theta)} .$$

Vertical component of $d\bar{F}$ on $b\,d\theta$ at θ is cancelled by that on $b\,d\theta$ at $-\theta$



$$\therefore \bar{F}_{\text{on loop}} = -\bar{a}_x 2\int_0^{\pi}(I_2 b\,d\theta)B\cos\theta .$$

$$\bar{F}_{\text{on loop}} = -\bar{a}_x \frac{\mu_0 I_1 I_2 b}{\pi} \int_0^{\pi} \frac{\cos\theta \, d\theta}{d + b\cos\theta}$$

$$= a_x \mu_0 I_1 I_2 \left[\frac{1}{\sqrt{1-(b/d)^2}} - 1 \right] \quad \text{(Repulsive force)}$$

P.6-36 Resolve the circular loop into many small loops, each with a magnetic dipole moment $d\bar{m} = I_2 d\bar{s}$. $d\bar{T} = d\bar{m} \times \bar{B}$.

$$\bar{T} = \int d\bar{T} = I_2 \int d\bar{s} \times \bar{B}$$

$$= -\bar{a}_x I_2 \sin\alpha \int B\, ds = -\bar{a}_x \mu_0 I_1 I_2 (d - \sqrt{d^2 - b^2}) \sin\alpha$$

in the direction of aligning the direction of the flux by I_2 in the loop to that of $\bar{B}$ due to I_1 in the straight wire.

($\int B\, ds$ over the circular loop has been found in problem P. 6-30 as Λ_{12}.)

P.6-37 Magnetic flux density at the center of the large circular turn of wire carrying current I_2 is

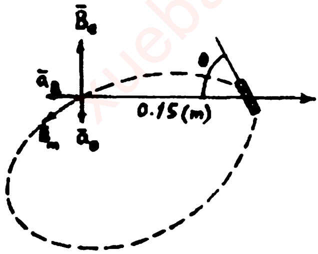
$$\bar{B}_2 = \bar{a}_{z2} \frac{\mu_0 I_2}{2 r_2}. \quad \text{(Set } z=0 \text{ in Eq. (6-38))}$$

Torque on the small circular wire:

$$\bar{T} = \bar{m}_1 \times \bar{B}_2 \cong (\bar{a}_{z1} I_1 \pi r_1^2) \times (\bar{a}_{z2} \frac{\mu_0 I_2}{2 r_2}) = (\bar{a}_{z1} \times \bar{a}_{z2}) \frac{\mu_0 I_1 I_2 \pi r_1^2}{2 r_2}$$

$\longrightarrow$ Magnitude $= \frac{\mu_0 I_1 I_2 \pi r_1^2}{2 r_2} \sin\theta$, in a direction to align the magnetic fluxes produced by I_1 and I_2.

P.6-38



$\bar{B}_m$ (magnetized compass needle)

$$= \frac{\mu_0 m}{4\pi R^3} (\bar{a}_R 2\cos\theta + \bar{a}_\theta \sin\theta)$$

$$= \frac{(4\pi \times 10^{-7}) \times 2}{4\pi (0.15)^3} (\bar{a}_R 2\cos\theta + \bar{a}_\theta \sin\theta)$$

$$= \frac{16}{27} \times 10^{-4} (\bar{a}_R 2\cos\theta + \bar{a}_\theta \sin\theta) \quad (T)$$

$$\bar{B}_e(\text{earth}) = -\bar{a}_\theta 10^{-4} \ (T).$$

Max. deflection when $|B_R/B_\theta|$ is max., or when

$$\left| \frac{B_\theta}{B_R} \right| = \left| \frac{(\frac{16}{27}\sin\theta - 1) \times 10^{-4}}{\frac{32}{27} \times 10^{-4} \cos\theta} \right| \text{ is min.}$$

Set $\frac{d}{d\theta}\left(\frac{1 - \frac{16}{27}\sin\theta}{\frac{32}{27}\cos\theta} \right) = 0 \longrightarrow \sin\theta = \frac{16}{27}$, or $\theta = 36.34°$.

At $\theta = 36.34°$, $|B_R/B_\theta| = 1.471$ and $\alpha = \tan^{-1} 1.471 = 55.8°$.

(If the bar magnet is oriented such that $\bar{B}_m \perp \bar{B}_e$, then $\alpha = 49.8° < 55.8°$)

P.6-39 $F = \frac{\Phi^2}{\mu_0 S} = \frac{(NI)^2}{\mu_0 S\left(\frac{2l_g}{\mu_0 S} - \frac{l_i}{\mu_0\mu_r S}\right)^2} = \frac{(NI)^2\mu_0 S}{\left(2l_g - \frac{l_i}{\mu_r}\right)^2}$

$F = 100\times 9.8 = 980$ (N), $S = 0.01$ (m^2), $l_g = 2\times 10^{-3}$ (m), $l_i = 3$ (m), $\mu_r = 4000$.

Solving: mmf $= NI = 1.326\times 10^3$ (A·t)

P.6-40 $W_m = \frac{1}{2}\int \mu H^2 dv$

Assume a virtual displacement, Δx, of the iron core.

$$W_m(x+\Delta x) = W_m(x) + \frac{1}{2}\int_{S\Delta x}(\mu-\mu_0)H^2 dv$$
$$= W_m(x) + \frac{1}{2}\mu_0(\mu_r-1)n^2I^2S\Delta x$$

$(F_I)_x = \frac{\partial W_m}{\partial x} = \frac{\mu_0}{2}(\mu_r-1)n^2I^2S$, in the direction of increasing x.

Chapter 7

P.7-1 $\mathcal{V} = -\int_S \frac{\partial \bar{B}}{\partial t}\cdot d\bar{s} = -\int_S \frac{\partial}{\partial t}(\bar{\nabla}\times\bar{A})\cdot d\bar{s} = -\oint_C \frac{\partial \bar{A}}{\partial t}\cdot d\bar{\ell}$.

P.7-2 $\bar{B} = \bar{a}_z 3\cos(5\pi 10^7 t - \frac{2}{3}\pi x)\cdot 10^{-6}$ (T)

$$\int_S \bar{B}\cdot d\bar{s} = \int_0^{0.6}\bar{a}_z 3\cos(5\pi 10^7 t - \tfrac{2}{3}\pi x)10^{-6}\cdot(\bar{a}_z 0.2\, dx)$$
$$= -\frac{0.18}{2\pi}\left[\sin(5\pi 10^7 t - 0.4\pi) - \sin 5\pi 10^7 t\right]10^{-6} \text{ (Wb)}$$

$\mathcal{V} = -\frac{d}{dt}\int \bar{B}\cdot d\bar{s} = 45\left[\cos(5\pi 10^7 t - 0.4\pi) - \cos 5\pi 10^7 t\right]$ (V)

$$i = \frac{\mathcal{V}}{2R} = 1.5\left[\cos(5\pi 10^7 t - 0.4\pi) - \cos 5\pi 10^7 t\right]$$
$$= -3\sin(5\pi 10^7 t - 0.2\pi)\sin(-0.2\pi)$$
$$= 1.76\sin(5\pi 10^7 t - 0.2\pi) \text{ (A)}$$

P.7-3 $\bar{B} = \bar{a}_\phi \frac{\mu_0 I \sin\omega t}{2\pi r}$. $\Phi = \int_S \bar{B}\cdot d\bar{s}$, $ds = \bar{a}_\phi 2z\,dr$, $z = \frac{\sqrt{3}}{3}(r-d)$

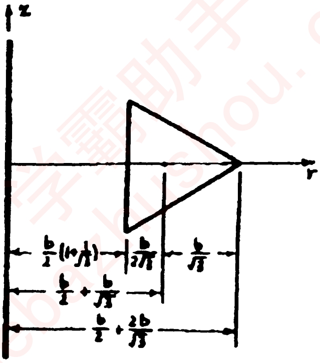
a) $\Phi = \frac{\sqrt{3}}{3}\frac{\mu_0 I\sin\omega t}{\pi}\int_d^{\frac{\sqrt{3}}{2}b+d}\left(1-\frac{d}{r}\right)dr = \frac{\sqrt{3}\mu_0 I\sin\omega t}{3\pi}\left[\frac{\sqrt{3}}{2}b - d\ln\left(\frac{\frac{\sqrt{3}}{2}b+d}{d}\right)\right]$

$d = \frac{b}{2}$, $\mathcal{V} = -\frac{\partial\Phi}{\partial t} = -\frac{\sqrt{3}\mu_0 I\omega b}{3\pi}\left[\frac{\sqrt{3}}{2} - \frac{1}{2}\ln(\sqrt{3}+1)\right]\cos\omega t$
$= V_m\cos\omega t$

$$V_{rms} = \frac{\sqrt{2}}{2}|V_m| = \frac{\sqrt{6}\,\mu_0 I\omega b}{12\pi}\left[\sqrt{3} - \ln(\sqrt{3}+1)\right]$$

$$= 0.0472\,\mu_0 I\omega b \quad (V).$$

b)



$$z = \frac{1}{\sqrt{3}}\left[\frac{b}{2}\left(1+\frac{4}{\sqrt{3}}\right) - r\right].$$

$$\int \bar{B}\cdot d\bar{s} = \frac{\mu_0 I \sin\omega t}{\sqrt{3}\,\pi}\int_{\frac{b}{2}(1+\frac{1}{\sqrt{3}})}^{\frac{b}{2}(1+\frac{4}{\sqrt{3}})}\left[\frac{b}{2}\left(1+\frac{4}{\sqrt{3}}\right)\frac{1}{r} - 1\right]dr$$

$$= \frac{\mu_0 I \sin\omega t}{\sqrt{3}\,\pi}\left[\frac{b}{2}\left(1+\frac{4}{\sqrt{3}}\right)\ln\left(\frac{4+\sqrt{3}}{1+\sqrt{3}}\right) - \frac{\sqrt{3}}{2}b\right]$$

$$V_{rms} = \frac{1}{\sqrt{2}}\,\frac{\mu_0 I\omega}{\sqrt{3}\,\pi}\,\frac{b}{2}\left|\left(1+\frac{4}{\sqrt{3}}\right)\ln\left(\frac{4+\sqrt{3}}{1+\sqrt{3}}\right) - \sqrt{3}\right|$$

$$= 0.0469\,\mu_0 I\omega b \quad (V).$$

P.7-4 From problem P.6-30: $\Phi_{12} = \mu_0 I(\sin\omega t)\left(d - \sqrt{d^2-b^2}\right)$

a)
$$\mathcal{V} = -\frac{d\Phi}{dt} = -\mu_0 I\,\omega(\cos\omega t)\left(d-\sqrt{d^2-b^2}\right)$$

$$= V_m \cos\omega t$$

$$0^{-4} = \frac{|V_m|}{\sqrt{2}R} = \frac{\mu_0 I\omega\left(d-\sqrt{d^2-b^2}\right)}{\sqrt{2}\;R}$$

$$I = \frac{\sqrt{2}R\times 3\times 10^{-4}}{\mu_0\omega\left(d-\sqrt{d^2-b^2}\right)} = \frac{3\sqrt{2}\times 10^{-6}}{4\pi 10^{-7}(2\pi 60)\times 0.0382} = 0.234 \ (A).$$

b) $\alpha = \cos^{-1}\left(\frac{0.2}{0.3}\right) = 48.2°.$

P.7-5 $\Phi(t) = B(t)S(t) = (5\cos\omega t)\times 0.2\,(0.7 - x)$

$$= 0.35\cos\omega t\,(1+\cos\omega t) \quad (mT),$$

$$i = -\frac{1}{R}\frac{d\Phi}{dt} = -\frac{1}{R}\,0.35\,\omega\left(\sin\omega t + \sin 2\omega t\right)$$

$$= -1.75\,\omega\sin\omega t\,(1+2\cos\omega t) \quad (mA).$$

P.7-6
$$i = \frac{\mathcal{V}}{R} = -\frac{1}{R}\frac{d}{dt}\int_S \bar{B}\cdot d\bar{s} = -\frac{1}{R}\frac{d}{dt}\left(B_0 hw\cos\omega t\right)$$

$$= \frac{\omega B_0 hw}{R}\sin\omega t,$$

$$p_d = \mathcal{V} i = \frac{(\omega B_0 hw)^2}{R}\sin^2\omega t \quad \text{(Power dissipated in } R\text{)}$$

On the other hand, for side 1-2: $\bar{F}_{12} = \bar{a}_z\, ihB_0$, $\bar{u}_{12} = -\bar{a}_x\frac{\omega w}{2}\sin\omega t$

for side 4-3: $\bar{F}_{43} = -\bar{a}_z\, ihB_0$, $\bar{u}_{43} = \bar{a}_x\frac{\omega w}{2}\sin\omega t$

Mechanical power required to rotate coil $p_m = -(\bar{F}_{12}\cdot\bar{u}_{12} + \bar{F}_{43}\cdot\bar{u}_{43}) = -(\omega B_0 hwi) = p_d$.

7-7 Take the divergence of Eq. (7-37a):

$$\bar{\nabla}\cdot(\bar{\nabla}\times\bar{E}) = -\frac{\partial}{\partial t}(\bar{\nabla}\cdot\bar{B}) = 0 \quad \text{from Eq. (2-137)}$$

$\longrightarrow$ $\bar{\nabla}\cdot\bar{B} = f(x,y,z)$, which is to hold at all times everywhere whether $\bar{B}$ exists or not; hence $f(x,y,z)$ must vanish and $\bar{\nabla}\cdot\bar{B}=0$.

Similarly, take the divergence of Eq. (7-37b):

$$\bar{\nabla}\cdot(\bar{\nabla}\times\bar{H}) = \bar{\nabla}\cdot\bar{J} + \frac{\partial}{\partial t}(\bar{\nabla}\cdot\bar{D}) = 0$$

$$= -\frac{\partial\rho}{\partial t} + \frac{\partial}{\partial t}(\bar{\nabla}\cdot\bar{D}), \text{ from Eq. (7-32)}$$

$\longrightarrow$ $\bar{\nabla}\cdot\bar{D} = \rho$.

7-8 Eq. (7-46): $\bar{\nabla}\cdot\bar{A} + \mu\epsilon\frac{\partial V}{\partial t} = 0$.

$$\bar{A} = \frac{\mu}{4\pi}\int_{V'}\frac{\bar{J}}{R}\,dv', \qquad V = \frac{1}{4\pi\epsilon}\int_{V'}\frac{\rho}{R}\,dv'.$$

$$\therefore \quad \frac{\mu_0}{4\pi}\left\{\int_{V'}\left[\bar{\nabla}\cdot\left(\frac{\bar{J}}{R}\right) + \frac{1}{R}\frac{\partial\rho}{\partial t}\right]dv'\right\} = 0. \qquad (i)$$

Now, $\bar{\nabla}\cdot\left(\frac{\bar{J}}{R}\right) = \frac{1}{R}\bar{\nabla}\cdot\bar{J} + \bar{J}\cdot\bar{\nabla}\left(\frac{1}{R}\right) = \bar{J}\cdot\bar{\nabla}\left(\frac{1}{R}\right)$

$$= -\bar{J}\cdot\bar{\nabla}'\left(\frac{1}{R}\right). \qquad (ii)$$

Also, $\bar{\nabla}'\cdot\left(\frac{\bar{J}}{R}\right) = \frac{1}{R}\bar{\nabla}'\cdot\bar{J} + \bar{J}\cdot\bar{\nabla}'\left(\frac{1}{R}\right)$. $\qquad$ (iii)

Substituting (iii) in (ii), $\bar{\nabla}\cdot\left(\frac{\bar{J}}{R}\right) = \frac{1}{R}\bar{\nabla}'\cdot\bar{J} - \bar{\nabla}'\cdot\left(\frac{\bar{J}}{R}\right)$. $\qquad$ (iv)

Substituting (iv) in (i), $\int_{V'}\frac{1}{R}\left(\bar{\nabla}'\cdot\bar{J} + \frac{\partial\rho}{\partial t}\right)dv' = \int_{V'}\bar{\nabla}'\cdot\left(\frac{\bar{J}}{R}\right)dv'$

$$= \oint_{S'}\frac{\bar{J}\cdot d\bar{s}'}{R}.$$

Let $R\to\infty$, $S'\to\infty$, $\bar{J}\cdot d\bar{s} = 0$.

$$\therefore \quad \bar{\nabla}'\cdot\bar{J} + \frac{\partial\rho}{\partial t} = 0 \quad \text{(Eq. of continuity).}$$

7-9 Eq. (7-37b): $\bar{\nabla}\times\bar{H} = \bar{J} + \frac{\partial\bar{D}}{\partial t} \longrightarrow \bar{\nabla}\times\left(\frac{\bar{B}}{\mu}\right) = \bar{J} + \epsilon\frac{\partial\bar{E}}{\partial t}$.

From Eqs. (7-39) and (7-41): $\bar{B} = \bar{\nabla}\times\bar{A}$, $\bar{E} = -\bar{\nabla}V - \frac{\partial\bar{A}}{\partial t}$.

$$\mu\bar{\nabla}\times\left(\frac{1}{\mu}\bar{\nabla}\times\bar{A}\right) = \mu\bar{J} - \mu\epsilon\frac{\partial^2\bar{A}}{\partial t^2} - \mu\epsilon\,\bar{\nabla}\left(\frac{\partial V}{\partial t}\right).$$

Use gauge condition for potentials in an inhomogeneous medium: $\bar{\nabla}\cdot(\epsilon\bar{A}) + \mu\epsilon^2\frac{\partial V}{\partial t} = 0$

$\longrightarrow$ Wave equation for vector potential:

$$-\mu\bar{\nabla}\times\left(\frac{1}{\mu}\bar{\nabla}\times\bar{A}\right) + \mu\epsilon\,\bar{\nabla}\left[\frac{1}{\mu\epsilon^2}\bar{\nabla}\cdot(\epsilon\bar{A})\right] - \mu\epsilon\frac{\partial^2\bar{A}}{\partial t^2} = -\mu\bar{J}.$$

From Eq. (7-37c), $\bar{\nabla}\cdot\bar{D} = \rho \longrightarrow \bar{\nabla}\cdot(\epsilon\bar{\nabla}V) + \frac{\partial}{\partial t}\bar{\nabla}\cdot(\epsilon\bar{A}) = -\rho$.

Wave equation for scalar potential: $\frac{1}{\epsilon}\bar{\nabla}\cdot(\epsilon\bar{\nabla}V) - \mu\epsilon\frac{\partial^2 V}{\partial t^2} = -\rho$.

P.7-12 As shown in problem P.7-7:

a) Eq. (7-37d) can be derived from Eq. (7-37a). Hence, the boundary conditions for the normal components of $\bar{B}$, which are obtained from $\bar{\nabla}\cdot\bar{B}=0$, are not independent of the boundary conditions for the tangential components of $\bar{E}$, which are obtained from $\bar{\nabla}\times\bar{E}=-\frac{\partial\bar{B}}{\partial t}$.

b) Similarly, the boundary conditions for the normal components of $\bar{D}$ are not independent of those for the tangential components of $\bar{H}$ in the time-varying case.

P.7-13 Medium 1: Free space; medium 2: $\mu_2\to\infty$.

$\bar{H}_2$ must be zero so that $\bar{B}_2$ is not infinite.

$$\therefore\quad \bar{a}_{n2}\times\bar{H}_1=0\ ;\qquad B_{n1}=B_{n2}$$

$$\bar{a}_{n2}\times(\bar{D}_1-\bar{D}_2)=\rho_s\ ;\qquad E_{t1}=E_{t2}.$$

P.7-14 $\bar{E}_1(z,t)=\bar{a}_x 0.03\sin 10^8\pi(t-\frac{z}{c})=\bar{a}_x Re\left[0.03e^{-j\pi/2}e^{j10^8\pi(t-z/c)}\right]$

$\bar{E}_2(z,t)=\bar{a}_x 0.04\cos\left[10^8\pi(t-\frac{z}{c})-\frac{\pi}{3}\right]=Re\left[0.04e^{-j\pi/3}e^{j10^8\pi(t-z/c)}\right].$

$$\text{Phasors: } \bar{E}=\bar{E}_1+\bar{E}_2=\bar{a}_x\left[0.03e^{-j\pi/2}+0.04e^{-j\pi/3}\right]$$

$$=\bar{a}_x\left[-j0.03+(0.02-j0.02\sqrt{3})\right]$$

$$=\bar{a}_x(0.068e^{-j1.27})=\bar{a}_x E_0 e^{j\theta}.$$

$\therefore\ E_0=0.068,\quad \theta=-1.27$ rad. or $-72.8°$.

P.7-16 $\bar{\nabla}^2V-\mu\epsilon\frac{\partial^2V}{\partial t^2}=-\frac{\rho}{\epsilon}$ with $V(R,t)=\frac{1}{4\pi\epsilon}\int_{V'}\frac{\rho(t-R/u)}{R}dv'$

$$\bar{\nabla}^2V-\frac{1}{u^2}\frac{\partial^2V}{\partial t^2}=-\frac{\rho}{\epsilon}.$$

We need $\bar{\nabla}^2(\frac{\rho}{R})=\frac{1}{R}\bar{\nabla}^2\rho+\rho\bar{\nabla}^2(\frac{1}{R})+2(\bar{\nabla}\rho)\cdot\bar{\nabla}(\frac{1}{R})$.

{Formula: $\bar{\nabla}^2(fg)=\bar{\nabla}\cdot\bar{\nabla}(fg)=g\bar{\nabla}^2f+f\bar{\nabla}^2g+2(\bar{\nabla}f)\cdot(\bar{\nabla}g)$}

Let $\xi=t-\sqrt{\mu\epsilon}R=t-R/u$, $\rho(\xi)=\rho(t-R/u)$

$$\bar{\nabla}^2\rho(\xi)=\frac{1}{u^2}\frac{d^2\rho}{d^2\xi}-\frac{2}{uR}\frac{d\rho}{d\xi},\qquad \bar{\nabla}^2(\frac{1}{R})=-4\pi\delta(R).$$

$$(\bar\nabla\rho)\cdot\left(\bar\nabla\frac{1}{R}\right)=\frac{\partial\rho(\xi)}{\partial R}\left(-\frac{1}{R^2}\right)=\frac{1}{uR^2}\frac{d\rho}{d\xi}.$$

Substituting back, $\bar\nabla^2\left(\frac{\rho}{R}\right)=\frac{1}{u^2R}\frac{d^2\rho}{d\xi^2}-4\pi\rho\,\delta(R).$

$$\bar\nabla^2V=\frac{1}{4\pi\epsilon}\bar\nabla^2\int_{V'}\frac{\rho}{R}dv'=\frac{1}{4\pi\epsilon}\int_{V'}\left[\frac{1}{u^2R}\frac{d^2\rho}{d\xi^2}-4\pi\rho\,\delta(R)\right]dv'$$

$$\frac{\partial^2V}{\partial t^2}=\frac{1}{4\pi\epsilon}\int_{V'}\frac{1}{R}\frac{d^2\rho}{d\xi^2}dv'$$

$$\therefore\ \bar\nabla^2V-\frac{1}{u^2}\frac{\partial^2V}{\partial t^2}=\frac{1}{4\pi\epsilon}\int\left[\frac{1}{u^2R}\frac{d^2\rho}{d\xi^2}-4\pi\rho\,\delta(R)-\frac{1}{u^2R}\frac{d^2\rho}{d\xi^2}\right]dv'$$

$$=-\frac{\rho(R)}{\epsilon}.\qquad \text{Q.E.D.}$$

P.7-17 $\quad\bar\nabla\times\bar E=-\mu\frac{\partial\bar H}{\partial t}$ ① $\qquad\bar\nabla\times\bar H=\bar J+\epsilon\frac{\partial\bar E}{\partial t}$ ②

$\bar\nabla\cdot\bar E=\frac{\rho}{\epsilon}$ ③ $\qquad\bar\nabla\cdot\bar H=0$ ④

$\bar\nabla\times$ ①: $\bar\nabla\times\bar\nabla\times\bar E=-\mu\frac{\partial}{\partial t}\left(\bar J+\epsilon\frac{\partial\bar E}{\partial t}\right).$

$$\bar\nabla(\bar\nabla\cdot\bar E)-\bar\nabla^2\bar E=-\mu\frac{\partial}{\partial t}\bar J+\mu\epsilon\frac{\partial^2\bar E}{\partial t^2}.$$

Wave equation for $\bar E$: $\bar\nabla^2\bar E-\mu\epsilon\frac{\partial^2\bar E}{\partial t^2}=\mu\frac{\partial\bar J}{\partial t}+\frac{1}{\epsilon}\bar\nabla\rho.$

$\bar\nabla\times$ ②: $\bar\nabla\times\bar\nabla\times\bar H=\bar\nabla\times\bar J+\epsilon\frac{\partial}{\partial t}(\bar\nabla\times\bar E).$

$$\bar\nabla(\bar\nabla\cdot\bar H)-\bar\nabla^2\bar H=\bar\nabla\times\bar J-\mu\epsilon\frac{\partial^2\bar H}{\partial t^2}.$$

Wave equation for $\bar H$: $\bar\nabla^2\bar H-\mu\epsilon\frac{\partial^2\bar H}{\partial t^2}=-\bar\nabla\times\bar J.$

For sinusoidal time dependence: $\frac{\partial}{\partial t}\to j\omega,\ \frac{\partial^2}{\partial t^2}\to(-\omega^2).$

Helmholtz's equations: $\bar\nabla^2\bar E+\omega^2\mu\epsilon\bar E=j\omega\mu\bar J+\frac{1}{\epsilon}\bar\nabla\rho$

(for phasors) $\qquad\bar\nabla^2\bar H+\omega^2\mu\epsilon\bar H=-\bar\nabla\times\bar J.$

P.7-18 $\quad\bar E=\bar a_y\,0.1\sin(10\pi x)\cos(6\pi10^9t-\beta z)\quad$ (V/m)

Use phasors:

$$\bar H=-\frac{1}{j\omega\mu_0}\bar\nabla\times\bar E=-\frac{1}{j\omega\mu_0}\begin{vmatrix}\bar a_x & \bar a_y & \bar a_z\\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z}\\ 0 & E_y & 0\end{vmatrix}$$

$$=\frac{j}{\omega\mu_0}\left[\bar a_x j0.1\beta\sin(10\pi x)+\bar a_z0.1(10\pi)\cos(10\pi x)\right]e^{-j\beta z}\quad ①$$

$$\bar E=\frac{1}{j\omega\epsilon_0}\bar\nabla\times\bar H=\bar a_y\frac{0.1}{\omega^2\mu_0\epsilon_0}\left[(10\pi)^2+\beta^2\right]\sin(10\pi x)e^{-j\beta z}\quad ②$$

Phasor form for given $\bar{E}$:

$$\bar{E} = \bar{a}_y\, 0.1 \sin(10\pi x)\, e^{-j\beta z}. \qquad ③$$

Equating ② and ③: $(10\pi)^2 + \beta^2 = \omega^2\mu_0\epsilon_0 = 400\pi^2$.

$$\therefore\ \beta = \sqrt{300}\,\pi = 10\sqrt{3}\,\pi = 54.4 \ \ (\text{rad/m}).$$

From ①, $\bar{H}(x,z;t) = \mathcal{Re}(H e^{j\omega t})$

$$= -\bar{a}_x 2.30\times10^{-4} \sin(10\pi x)\cos(6\pi 10^9 t - 54.4z)$$
$$-\bar{a}_z 1.33\times10^{-4}\cos(10\pi x)\sin(6\pi 10^9 t - 54.4z)$$
$$(\text{A/m})$$

P.7-19 $\bar{H}(x,z;t) = \bar{a}_y 2\cos(15\pi x)\sin(6\pi 10^9 t - \beta z)$ (A/m).

Phasor: $\bar{H} = \bar{a}_y 2\cos(15\pi x)\, e^{-j\beta z}$

$$\beta^2 + (15\pi)^2 = \omega^2\mu_0\epsilon_0 = (6\pi 10^9)^2 \frac{1}{(3\times10^8)^2}$$
$$\beta^2 = 400\pi^2 - 225\pi^2 = 175\pi^2$$
$$\beta = 13.2\pi = 41.6 \ \ (\text{rad/m}).$$

$$\bar{E} = \frac{1}{j\omega\epsilon_0}\bar{\nabla}\times\bar{H} = \frac{1}{j\omega\epsilon_0}\left(-\bar{a}_x\frac{\partial H_y}{\partial z} + \bar{a}_z\frac{\partial H_y}{\partial x}\right)$$
$$= -j6\left[-\bar{a}_x j2\beta\cos(15\pi x) - \bar{a}_z 30\pi\sin(15\pi x)\right]e^{-j\beta z}$$
$$= \left[-\bar{a}_x 158\pi\cos(15\pi x) + \bar{a}_z j180\pi\sin(15\pi x)\right]e^{-j\beta z}.$$

$$\bar{E}(x,z;t) = \mathcal{Im}(\bar{E}e^{j\omega t})$$
$$= \bar{a}_x 496\cos(15\pi x)\sin(6\pi 10^9 t - 41.6z)$$
$$+ \bar{a}_z 565\sin(15\pi x)\cos(6\pi 10^9 t - 41.6z) \quad (\text{V/m}).$$

P.7-20 $\bar{E} = \bar{a}_\theta \frac{E_0}{R}\sin\theta\cos(\omega t - kR)$. ①

$$\bar{\nabla}\times\bar{E} = \bar{a}_\phi \frac{1}{R}\frac{\partial}{\partial R}(RE_\theta) = \bar{a}_\phi\frac{E_0 k}{R}\sin\theta\sin(\omega t - kR)$$
$$= -\mu_0\frac{\partial\bar{H}}{\partial t} \longrightarrow \bar{H} = \bar{a}_\phi\int -\frac{E_0 k}{\mu_0 R}\sin\theta\sin(\omega t - kR)\,dt$$
$$= \bar{a}_\phi\frac{E_0 k}{\omega\mu_0 R}\sin\theta\cos(\omega t - kR). \qquad ②$$

Also, $\bar{\nabla}\times\bar{H} = \epsilon_0\frac{\partial\bar{E}}{\partial t} \longrightarrow \frac{\partial\bar{E}}{\partial t} = \frac{1}{\epsilon_0}\bar{\nabla}\times\bar{H} = -\bar{a}_\theta\frac{1}{R}\frac{\partial}{\partial R}(RH_\phi)$

$$\bar{E} = \bar{a}_\theta\int -\frac{E_0 k^2}{\omega\mu_0\epsilon_0 R}\sin\theta\sin(\omega t - kR)\,dt$$
$$= \bar{a}_\theta\frac{E_0 k^2}{\omega^2\mu_0\epsilon_0 R}\sin\theta\sin(\omega t - kR). \qquad ③$$

Comparing ① and ③,

$$k = \omega\sqrt{\mu_0\epsilon_0}.$$

From ②, $\bar{H} = \bar{a}_\phi\frac{E_0}{R}\sqrt{\frac{\epsilon_0}{\mu_0}}\sin\theta\cos\omega(t - \sqrt{\mu_0\epsilon_0}\,R)$.

P.7-21 Maxwell's curl equations: $\bar{\nabla}\times\bar{E} = -j\omega\mu\bar{H}$ ①

$\bar{\nabla}\times\bar{H} = j\omega\epsilon\bar{E}$ ②

From $\bar{\nabla}\cdot\bar{E}=0$, define $\bar{A}_e$ such that $\bar{E}=\bar{\nabla}\times\bar{A}_e$ ③

From ①, $\bar{H}=\frac{j}{\omega\mu}\bar{\nabla}\times\bar{E}=\frac{j}{\omega\mu}\bar{\nabla}\times\bar{\nabla}\times\bar{A}_e$

$$=\frac{j}{\omega\mu}\left[\bar{\nabla}(\bar{\nabla}\cdot\bar{A}_e)-\bar{\nabla}^2\bar{A}_e\right]. \quad ④$$

From ②, $\bar{\nabla}\times(\bar{H}-j\omega\epsilon\bar{A}_e)=0$, let $\bar{H}-j\omega\epsilon\bar{A}_e=-\bar{\nabla}V_m$. ⑤

Subtracting ⑤ from ④: $\omega\epsilon\bar{A}_e=\frac{1}{\omega\mu}[\bar{\nabla}(\bar{\nabla}\cdot\bar{A}_e)-\bar{\nabla}^2\bar{A}_e]-j\bar{\nabla}V_m$. ⑥

Choose $\bar{\nabla}\cdot\bar{A}_e=j\omega\mu V_m$.

a) Eq. ⑤ becomes $\bar{H}=j\omega\epsilon\bar{A}_e+\frac{j}{\omega\mu}\bar{\nabla}(\bar{\nabla}\cdot\bar{A}_e)$.

b) Eq. ⑥ becomes $\bar{\nabla}^2\bar{A}_e+\omega^2\mu\epsilon\bar{A}_e=0$, a homogeneous Helmholtz's eq.

P.7-22 $\bar{H}=j\omega\epsilon_0\bar{\nabla}\times\bar{\pi}_e$ ①

$\bar{\nabla}\times\bar{E}=-j\omega\mu_0\bar{H}=\omega^2\mu_0\epsilon_0\bar{\nabla}\times\bar{\pi}_e$

$\bar{\nabla}\times(\bar{E}-k_0^2\bar{\pi}_e)=0 \longrightarrow$ Let $\bar{E}-k_0^2\bar{\pi}_e=\bar{\nabla}V_e$ ②

$\bar{\nabla}\times\bar{H}=j\omega\bar{D}=j\omega(\epsilon_0\bar{E}+\bar{P})=j\omega\epsilon_0(\bar{E}+\frac{\bar{P}}{\epsilon_0})$. ③

Substituting ① and ② in ③:

$$j\omega\epsilon_0\bar{\nabla}\times\bar{\nabla}\times\bar{\pi}_e=j\omega\epsilon_0\left(k_0^2\bar{\pi}_e+\bar{\nabla}V_e+\frac{\bar{P}}{\epsilon_0}\right)$$

$$=j\omega\epsilon_0(\bar{\nabla}\bar{\nabla}\cdot\bar{\pi}_e-\bar{\nabla}^2\bar{\pi}_e). \quad ④$$

Choose $\bar{\nabla}\cdot\bar{\pi}_e=V_e$. Eq. ④ becomes

b) $$\bar{\nabla}^2\bar{\pi}_e+k_0^2\bar{\pi}_e=-\frac{\bar{P}}{\epsilon_0}; \quad (7\text{-}95)$$

a) Eq. ② becomes

$$\bar{E}=k_0^2\bar{\pi}_e+\bar{\nabla}\bar{\nabla}\cdot\bar{\pi}_e$$

$$=k_0^2\bar{\pi}_e+(\bar{\nabla}^2\bar{\pi}_e+\bar{\nabla}\times\bar{\nabla}\times\bar{\pi}_e). \quad ⑤$$

Combination of Eqs. (7-95) and ⑤ gives

$$\bar{E}=\bar{\nabla}\times\bar{\nabla}\times\bar{\pi}_e-\frac{\bar{P}}{\epsilon_0}.$$

P.7-23 a) $\left|\frac{\text{Displacement current}}{\text{Conduction current}}\right|=\frac{\omega\epsilon}{\sigma}=\frac{(2\pi 100\times10^6)\times\frac{1}{36\pi}\times10^{-9}}{5.70\times10^7}=9.75\times10^{-8}$.

b) $\bar{\nabla}\times\bar{H}=\sigma\bar{E}$, $\bar{\nabla}\times\bar{E}=-j\omega\mu\bar{H}$

$\bar{\nabla}\times\bar{\nabla}\times\bar{H}=\bar{\nabla}(\bar{\nabla}\cdot\bar{H})-\bar{\nabla}^2\bar{H}=\sigma\bar{\nabla}\times\bar{E}$.

But $\bar{\nabla}\cdot\bar{H}=0$.

$\therefore\quad \bar{\nabla}^2\bar{H}+\sigma\bar{\nabla}\times\bar{E}=0$,

or, $\bar{\nabla}^2\bar{H}-j\omega\mu\sigma\bar{H}=0$.

Chapter 8

P.8-2 (8-4) Harmonic time dependence: $e^{j\omega t}$; $\frac{\partial}{\partial t}\rightarrow j\omega$

Let phasors $\quad \bar{E}=\bar{E}_0 e^{-j\bar{k}\cdot\bar{R}}$

$\bar{H}=\bar{H}_0 e^{-j\bar{k}\cdot\bar{R}}$,

where $\bar{E}_0$ and $\bar{H}_0$ are constant vectors.

Maxwell's equations: $\bar{\nabla}\times\bar{E}=\bar{\nabla}(e^{-j\bar{k}\cdot\bar{R}})\times\bar{E}_0=-j\omega\mu\bar{H}$

$\bar{\nabla}\times\bar{H}=\bar{\nabla}(e^{-j\bar{k}\cdot\bar{R}})\times\bar{H}_0=j\omega\epsilon E$

$\bar{\nabla}\cdot\bar{E}=\bar{\nabla}(e^{-j\bar{k}\cdot\bar{R}})\cdot\bar{E}_0=0$

$\bar{\nabla}\cdot\bar{H}=\bar{\nabla}(e^{-j\bar{k}\cdot\bar{R}})\cdot\bar{H}_0=0$.

But $\bar{\nabla}(e^{-j\bar{k}\cdot\bar{R}})=e^{-j\bar{k}\cdot\bar{R}}\,\bar{\nabla}(-j\bar{k}\cdot\bar{R})=e^{-j\bar{k}\cdot\bar{R}}[-j\bar{\nabla}(k_x x+k_y y+k_z z)]$

$=-j(\bar{a}_x k_x+\bar{a}_y k_y+\bar{a}_z k_z)e^{-j\bar{k}\cdot\bar{R}}=-j\bar{k}e^{-j\bar{k}\cdot\bar{R}}$.

$\therefore$ Maxwell's equations become: $\bar{k}\times\bar{E}=\omega\mu H$, $\quad\bar{k}\times\bar{H}=-\omega\epsilon\bar{E}$

$\bar{k}\cdot\bar{E}=0$, $\quad \bar{k}\cdot\bar{H}=0$.

(8-5)

P.8-3 $\bar{H}=\bar{a}_z 4\times10^{-6}\cos\left(10^7\pi t-k_0 y+\frac{\pi}{4}\right)$ (A/m).

a) $k_0=\omega\sqrt{\mu_0\epsilon_0}=\frac{10^7\pi}{3\times10^8}=\frac{\pi}{30}=0.105$ (rad/m).

At $t=3\times10^{-3}$ (s), we require

$10^7\pi(3\times10^{-3})-\frac{\pi}{30}y+\frac{\pi}{4}=\pm n\pi+\frac{\pi}{2}$, $\quad n=0,1,2,\cdots$

$y=\pm30n-7.5$ (m).

But $\lambda=\frac{2\pi}{k_0}=60$ (m), $\quad\therefore y=22.5\pm n\lambda/2$ (m).

b) $\bar{E}=\eta_0\bar{H}\times\bar{a}_y=-\bar{a}_x 1.508\times10^{-3}\cos(10^7\pi t-\frac{\pi}{30}y+\frac{\pi}{4})$ (V/m)

P.8-4 (8-7) Let $\alpha=\omega t-kz$, $\quad\bar{E}=\bar{a}_x E_{10}\sin\alpha+\bar{a}_y E_{20}\sin(\alpha+\psi)$

$=\bar{a}_x E_x+\bar{a}_y E_y$.

$\frac{E_x}{E_{10}}=\sin\alpha$, $\quad\frac{E_y}{E_{20}}=\sin(\alpha+\psi)=\sin\alpha\cos\psi+\cos\alpha\sin\psi$

$=\frac{E_x}{E_{10}}\cos\psi+\sqrt{1-\left(\frac{E_x}{E_{10}}\right)^2}\sin\psi$.

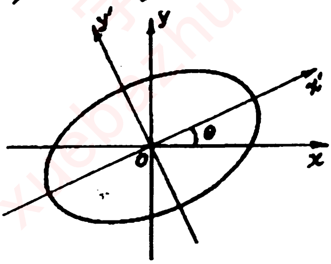
8-7. 续

$$\left(\frac{E_y}{E_{20}}-\frac{E_x}{E_{10}}\cos\psi\right)^2=\left(1-\frac{E_x}{E_{10}}\right)\sin^2\psi$$

$$\left(\frac{E_y}{E_{20}\sin\psi}\right)^2+\left(\frac{E_x}{E_{10}\sin\psi}\right)^2-2\frac{E_xE_y}{E_{10}E_{20}}\frac{\cos\psi}{\sin^2\psi}=1, \quad (1)$$

which is the equation of an ellipse.

In order to find the parameters of the ellipse, rotate the coordinate axes x-y counterclockwise by an angle θ to x'-y'. Assume the equation of the ellipse in terms of the new coordinates to be



$$\left(\frac{E_x'}{a}\right)^2+\left(\frac{E_y'}{b}\right)^2=1, \quad (2)$$

where

$$E_x'=E_x\cos\theta+E_y\sin\theta \quad (3)$$

$$E_y'=-E_x\sin\theta+E_y\cos\theta. \quad (4)$$

Substituting (3) and (4) in (2) and rearranging:

$$E_x^2\left(\frac{\cos^2\theta}{a^2}+\frac{\sin^2\theta}{b^2}\right)+E_y^2\left(\frac{\sin^2\theta}{a^2}+\frac{\cos^2\theta}{b^2}\right)-2E_xE_y\sin\theta\cos\theta\left(\frac{1}{b^2}-\frac{1}{a^2}\right)=1. \quad (5)$$

Comparing (1) and (5), we obtain

$$\begin{cases}\dfrac{\cos^2\theta}{a^2}+\dfrac{\sin^2\theta}{b^2}=\dfrac{1}{E_{10}^2\sin^2\psi} & (6)\\ \dfrac{\sin^2\theta}{a^2}+\dfrac{\cos^2\theta}{b^2}=\dfrac{1}{E_{20}^2\sin^2\psi} & (7)\\ \sin\theta\cos\theta\left(\dfrac{1}{b^2}-\dfrac{1}{a^2}\right)=\dfrac{\cos\psi}{E_{10}E_{20}\sin^2\psi}. & (8)\end{cases}$$

Eqs. (6), (7) and (8) can be solved for three unknowns:

$$\theta=\frac{1}{2}\tan^{-1}\left(\frac{2E_{10}E_{20}\cos\psi}{E_{10}^2-E_{20}^2}\right)$$

$$a=\sqrt{\frac{2}{\frac{1}{E_{10}^2}(1+\sec 2\theta)+\frac{1}{E_{20}^2}(1-\sec 2\theta)}}\,\sin\psi$$

$$b=\sqrt{\frac{2}{\frac{1}{E_{10}^2}(1-\sec 2\theta)+\frac{1}{E_{20}^2}(1+\sec 2\theta)}}\,\sin\psi.$$

In particular, if $E_{10}=E_{20}=E_0$: $\theta=45°$, $a=\sqrt{2}E_0\cos\frac{\psi}{2}$, $b=\sqrt{2}E_0\sin\frac{\psi}{2}$.

P.8-5 Let an elliptically polarized plane wave be represented by the phasor (with propagation factor e^{-jkz} omitted):

a) $$\bar{E} = \bar{a}_x E_1 \pm \bar{a}_y E_2 e^{j\alpha},$$

where E_1, E_2, and α are arbitrary constants.

Right-hand circularly polarized wave: $\bar{E}_{rc} = E_{rc}(\bar{a}_x - j\bar{a}_y)$.

Left-hand circularly polarized wave: $\bar{E}_{\ell c} = E_{\ell c}(\bar{a}_x + j\bar{a}_y)$.

If $$E_{rc} = \frac{1}{2}(E_1 \pm jE_2 e^{j\alpha})$$
$$E_{\ell c} = \frac{1}{2}(E_1 \mp jE_2 e^{j\alpha}),$$
then $$\bar{E} = \bar{E}_{rc} + \bar{E}_{\ell c}.$$

b) Let a right-hand circularly polarized wave be

$$\bar{E}_{rc} = E(\bar{a}_x - j\bar{a}_y)$$
$$= E(\bar{a}_x \tfrac{1}{2} - \bar{a}_y j2) + E(\bar{a}_x \tfrac{1}{2} + \bar{a}_y j)$$
$$= \bar{E}_{e+} + \bar{E}_{e-},$$

where E_{e+} and E_{e-} are right-hand and left-hand elliptically polarized waves respectively.

Similarly, a left-hand circularly polarized wave can be written as

$$\bar{E}_{\ell c} = E(\bar{a}_x + j\bar{a}_y)$$
$$= E(\bar{a}_x \tfrac{1}{2} + \bar{a}_y j2) + E(\bar{a}_x \tfrac{1}{2} - \bar{a}_y j)$$
$$= \bar{E}'_{e-} + \bar{E}'_{e+}.$$

P.8-6 For conducting media,

$$k_c^2 = \omega^2 \mu \epsilon_c = \omega^2 \mu \epsilon \left(1 - j\frac{\sigma}{\omega\epsilon}\right)$$

$$k_c = \beta - j\alpha, \quad k_c^2 = \beta^2 - \alpha^2 - 2j\alpha\beta.$$

$$\therefore \quad \beta^2 - \alpha^2 = Re(k_c^2) = \omega^2\mu\epsilon \qquad ①$$

$$\beta^2 + \alpha^2 = |k_c^2| = \omega^2\mu\epsilon\sqrt{1 + \left(\frac{\sigma}{\omega\epsilon}\right)^2}. \qquad ②$$

From ① and ② we obtain

$$\alpha = \omega\sqrt{\frac{\mu\epsilon}{2}}\left[\sqrt{1+\left(\frac{\sigma}{\omega\epsilon}\right)^2} - 1\right]^{1/2}, \quad \beta = \omega\sqrt{\frac{\mu\epsilon}{2}}\left[\sqrt{1+\left(\frac{\sigma}{\omega\epsilon}\right)^2} + 1\right]^{1/2}.$$

8-7 All are good conductors, $\left(\frac{\sigma}{\omega\epsilon}\right)^2 >> 1$.

$$\alpha = \sqrt{\pi f \mu \sigma}, \qquad \delta = \frac{1}{\alpha}, \qquad \eta_c = (1+j)\frac{\alpha}{\sigma}.$$

a) $f = 60$ (Hz)

	η_c (Ω)	α (Np/m)	α (dB/m)	δ (m)
Copper	$2.02(1+j)\times10^{-6}$	0.117×10^{3}	1.02×10^{3}	8.53×10^{-3}
Silver	$2.08(1+j)\times10^{-6}$	0.121×10^{3}	1.05×10^{3}	8.29×10^{-3}
Brass	$3.86(1+j)\times10^{-6}$	0.061×10^{3}	0.53×10^{3}	16.3×10^{-3}

b) $f = 1$ (MHz)

	η_c (Ω)	α (Np/m)	α (dB/m)	δ (m)
Copper	$2.61(1+j)\times10^{-4}$	1.51×10^{4}	1.31×10^{5}	6.61×10^{-5}
Silver	$2.57(1+j)\times10^{-4}$	1.58×10^{4}	1.35×10^{5}	6.32×10^{-5}
Brass	$4.98(1+j)\times10^{-4}$	0.79×10^{4}	0.69×10^{5}	12.6×10^{-5}

c) $f = 1$ (GHz)

	η_c (Ω)	α (Np/m)	α (dB/m)	δ (m)
Copper	$8.25(1+j)\times10^{-3}$	4.79×10^{5}	4.16×10^{6}	2.09×10^{-6}
Silver	$8.01(1+j)\times10^{-3}$	4.93×10^{5}	4.28×10^{6}	2.03×10^{-6}
Brass	$15.8(1+j)\times10^{-3}$	2.51×10^{5}	2.18×10^{6}	3.99×10^{-6}

~~8-8~~ 8-11 $f = 3\times10^9$ (Hz), $\epsilon_r = 2.5$, $\tan\delta_c = \frac{\sigma}{\omega\epsilon} = 10^{-2}$.

a) $$\alpha = \omega\sqrt{\frac{\mu\epsilon}{2}}\left[\sqrt{1+\left(\frac{\sigma}{\omega\epsilon}\right)^2}-1\right]^{1/2} \cong \omega\sqrt{\frac{\mu\epsilon}{2}}\,\frac{1}{\sqrt{2}}\left(\frac{\sigma}{\omega\epsilon}\right)^2$$
$$= 0.497 \text{ (Np/m)}.$$

$$e^{\alpha x} = \frac{1}{2} \longrightarrow x = \frac{1}{\alpha}\ln 2 = 1.395 \text{ (m)}.$$

b) $$\eta_c = \frac{1}{\sqrt{\epsilon_r}}\sqrt{\frac{\mu_0}{\epsilon_0}}\left(1+j\frac{\sigma}{2\omega\epsilon}\right) = 238\,(1+j0.005) \;(\Omega)$$

$$\beta = \omega\sqrt{\mu\epsilon}\left[1+\frac{1}{8}\left(\frac{\sigma}{\omega\epsilon}\right)^2\right] = 31.6\pi \text{ (rad/m)}$$

$$\lambda = \frac{2\pi}{\beta} = 0.063 \text{ (m)}$$

$$u_p = \frac{\omega}{\beta} = 1.8973\times10^{8} \text{ (m/s)}$$

$$u_g = \frac{1}{\frac{d\beta}{d\omega}} = \frac{c}{\sqrt{\epsilon_r}}\left[1+\frac{1}{8}\left(\frac{\sigma}{\omega\epsilon}\right)^2\right] = 1.8975\times10^{8} \text{ (m/s)}.$$

8-8 cont.

c) At $x=0$, $\bar{E} = \bar{a}_y 50^{j\pi/3}$.

$$\bar{H} = \frac{1}{\eta_c}\bar{a}_x \times \bar{E} = \bar{a}_z 0.210 e^{j(\frac{\pi}{3} - 0.0016\pi)}$$

$$\therefore \bar{H} = \bar{a}_z 0.210 e^{-0.497x} \sin(6\pi 10^9 t - 31.6\pi x + \frac{\pi}{3} - 0.0016\pi). \quad (A/m)$$

P.8-9 a) $\frac{\sigma}{\omega\epsilon} = 0.18$

$$\alpha = \omega\sqrt{\frac{\mu\epsilon}{2}}\left[\sqrt{1+\left(\frac{\sigma}{\omega\epsilon}\right)^2} - 1\right]^{1/2} = 84 \quad (Np/m)$$

$$\beta = \omega\sqrt{\frac{\mu\epsilon}{2}}\left[\sqrt{1+\left(\frac{\sigma}{\omega\epsilon}\right)^2} + 1\right]^{1/2} = 300\pi \quad (rad/m)$$

$$\eta_c = \sqrt{\frac{\mu}{\epsilon_c}} = \frac{120\pi}{\sqrt{\epsilon_r}\left[1+\left(\frac{\sigma}{\omega\epsilon}\right)^2\right]^{1/4}} e^{j\frac{1}{2}\tan^{-1}(\sigma/\omega\epsilon)} = 41.8 e^{j0.0283\pi} \quad (\Omega)$$

$$u_p = \frac{\omega}{\beta} = 33.3 \times 10^6 \quad (m/s)$$

$$\lambda = \frac{2\pi}{\beta} = 6.67 \times 10^{-3} \quad (m)$$

$$\delta = \frac{1}{\alpha} = 11.9 \times 10^{-3} \quad (m).$$

b) $e^{-\alpha y} = \frac{1}{10}$, $\quad y = \frac{1}{\alpha}\ln 10 = 27.4 \times 10^{-3}$ (m).

c) At $y = 0.5$ (m),

$$\bar{H}(y,t) = \bar{a}_x 0.1 e^{-84\times 0.5} \sin(10^{10}\pi t - 300\pi \times 0.5 - \frac{\pi}{3})$$

$$= \bar{a}_x 5.75 \times 10^{-20} \sin(10^{10}\pi t - \frac{\pi}{3}) \quad (A/m)$$

$$\bar{E}(y,t) = \mathcal{I}m\left[\eta_c \bar{H}(y) \times \bar{a}_y\right] e^{j\omega t} = \bar{a}_z 2.41 \times 10^{-18} \sin(10^{10}\pi t - \frac{\pi}{3} + 0.029\pi) \quad (V/m).$$

P.8-10 a) $\delta = \frac{1}{\sqrt{\pi f \mu \sigma}}$

$$\sigma = \frac{1}{\pi f \mu \delta^2} = 0.99 \times 10^5 \quad (S/m).$$

b) At $f = 10^9$ (Hz), $\quad \alpha = \sqrt{\pi f \mu \sigma} = 1.98 \times 10^4 \quad (Np/m)$

$$20 \log_{10} e^{-\alpha z} = -30 \quad (dB)$$

$$z = \frac{1.5}{\alpha \log_{10} e} = 1.75 \times 10^{-4} \quad (m).$$

P.8-11 a) From Eq. (8-52), $u_g = \frac{d\omega}{d\beta} = \frac{d}{d\beta}(\beta u_p) = u_p + \beta\frac{du_p}{d\beta}$.

b) $\lambda = \frac{2\pi}{\beta}$, $\quad \frac{d\lambda}{d\beta} = -\frac{2\pi}{\beta^2} = -\frac{\lambda}{\beta}$.

$$u_g = u_p + \beta\left(\frac{du_p}{d\lambda}\frac{d\lambda}{d\beta}\right) = u_p - \lambda\frac{du_p}{d\lambda}.$$

P.8-12 $\mathscr{P}_{av} = \dfrac{|E|^2}{2\eta_0} = 10^{-2}$ (W/cm²)

a) $|E| = \sqrt{0.02\,\eta_0} = 2.75$ (V/cm) = 275 (V/m)

$|H| = \dfrac{1}{\eta_0}|E| = 7.28\times10^{-3}$ (A/cm) = 0.728 (A/m).

b) $\mathscr{P}_{av} = \dfrac{|E|^2}{2\eta_0} = 1.3\times10^3$ (W/cm²)

$|E| = 990$ (V/cm) $= 9.90\times10^4$ (V/m)

$|H| = 2.63$ (A/cm) = 263 (A/m).

P.8-13 Assume that a circularly polarized plane wave be represented by

$$\bar{E}(z,t) = \bar{a}_x E_0 \cos(\omega t - kz + \phi) + \bar{a}_y E_0 \sin(\omega t - kz + \phi)$$

$$\bar{H}(z,t) = \bar{a}_y \frac{E_0}{\eta}\cos(\omega t - kz + \phi) - \bar{a}_x \frac{E_0}{\eta}\sin(\omega t - kz + \phi).$$

The Poynting vector is

$$\bar{\mathscr{P}} = \bar{E}\times\bar{H} = \bar{a}_z \frac{E_0^2}{\eta}\left[\cos^2(\omega t - kz + \phi) + \sin^2(\omega t - kz + \phi)\right]$$

$$= \bar{a}_z \frac{E_0^2}{\eta}, \quad \text{a constant independent of } t \text{ and } z.$$

P.8-14 $\bar{E} = \bar{a}_\theta E_\theta + \bar{a}_\phi E_\phi$

$$\bar{H} = \frac{1}{\eta}\bar{a}_R \times \bar{E} = \frac{1}{\eta}(\bar{a}_\phi E_\theta - \bar{a}_\theta E_\phi)$$

$$\bar{\mathscr{P}}_{av} = \frac{1}{2}\mathcal{R}e(\bar{E}\times\bar{H}^*) = \bar{a}_R \frac{1}{2\eta}(|E_\theta|^2 + |E_\phi|^2). \quad \text{(W/m}^2\text{)}$$

P.8-15 From Gauss's law, $\bar{E} = \bar{a}_r \dfrac{\rho}{2\pi\epsilon r}$, where ρ is the line charge density on the inner conductor.

$$V_0 = -\int_b^a \bar{E}\cdot d\bar{r} = \frac{\rho}{2\pi\epsilon}\ln\left(\frac{b}{a}\right)$$

$$\therefore \bar{E} = \bar{a}_r \frac{V_0}{r\ln(b/a)}.$$

From Ampère's circultal law, $\bar{H} = \bar{a}_\phi \dfrac{I}{2\pi r}$.

Poynting vector $\bar{\mathscr{P}} = \bar{E}\times\bar{H} = \bar{a}_z \dfrac{V_0 I}{2\pi r^2 \ln(b/a)}$.

Power transmitted of cross-sectional area:

$$P = \int_s \bar{\mathscr{P}}\cdot d\bar{s} = \frac{V_0 I}{2\pi\ln(b/a)}\int_0^{2\pi}\int_a^b \left(\frac{1}{r^2}\right) r\,dr\,d\phi = V_0 I.$$

P.8-16 Given $\bar{E}_i = E_0(\bar{a}_x - j\bar{a}_y)e^{-j\beta z}$.

a) Assume the reflected electric field intensity to be

$$\bar{E}_r(z) = (\bar{a}_x E_{rx} + \bar{a}_y E_{ry})e^{j\beta z}.$$

Boundary condition at $z=0$:

$$[\bar{E}_i(z) + \bar{E}_r(z)]_{z=0} = 0.$$

$\therefore\ \bar{E}_r(z) = E_0(-\bar{a}_x + j\bar{a}_y)e^{j\beta z}$, a left-hand circularly polarized wave in $-z$ direction.

b) $\bar{a}_{n2}\times(\bar{H}_1 - \bar{H}_2) = \bar{J}_s \longrightarrow -\bar{a}_z\times[\bar{H}_i(0) + \bar{H}_r(0)] = \bar{J}_s$

$\bar{H}_i(0) = \frac{1}{\eta_0}\bar{a}_z\times\bar{E}_i(0) = \frac{E_0}{\eta_0}(j\bar{a}_x + \bar{a}_y)$ $\qquad \bar{H}_2 = 0$ in perfect conductor

$\bar{H}_r(0) = \frac{1}{\eta_0}(-\bar{a}_z)\times\bar{E}_r(0) = \frac{E_0}{\eta_0}(j\bar{a}_x + \bar{a}_y)$

$\bar{H}_1(0) = \bar{H}_i(0) + \bar{H}_r(0) = \frac{2E_0}{\eta_0}(j\bar{a}_x + \bar{a}_y)$

$\therefore\ \bar{J}_s = -\bar{a}_z\times\bar{H}_1(0) = \frac{2E_0}{\eta_0}(\bar{a}_x - j\bar{a}_y)$.

c) $\bar{E}_1(z,t) = \mathcal{R}e[E_i(z) + E_r(z)]e^{j\omega t}$

$= \mathcal{R}e\, E_0[(\bar{a}_x - j\bar{a}_y)e^{-j\beta z} + (-\bar{a}_x + j\bar{a}_y)e^{j\beta z}]e^{j\omega t}$

$= \mathcal{R}e\, E_0[-2j(\bar{a}_x - j\bar{a}_y)\sin\beta z]e^{j\omega t}$

$= 2E_0\sin\beta z(\bar{a}_x\sin\omega t - \bar{a}_y\cos\omega t)$.

P.8-17 Given $\bar{E}_i(x,z) = \bar{a}_y 10 e^{-j(6x+8z)}$ (V/m).

8-22 a) $k_x = 6$, $k_z = 8 \longrightarrow k = \beta = \sqrt{k_x^2 + k_z^2} = 10$ (rad/m)

$\lambda = \frac{2\pi}{k} = \frac{2\pi}{10} = 0.628$ (m)

$f = \frac{c}{\lambda} = 4.78\times10^8$ (Hz); $\omega = 2\pi f = kc = 3\times10^9$ (rad/s)

b) $\bar{E}_i(x,z;t) = \bar{a}_y 10\cos(3\times10^9 t - 6x - 8z)$ (V/m)

$\bar{H}_i(x,z) = \frac{1}{\eta_0}\bar{a}_{ni}\times\bar{E}_i$ $\qquad \bar{a}_{ni} = \frac{\bar{k}}{k} = \bar{a}_x 0.6 + \bar{a}_z 0.8$

$= \frac{1}{120\pi}(\bar{a}_x 0.6 + \bar{a}_z 0.8)\times\bar{a}_y 10 e^{-j(6x+8z)}$

$= \left(-\bar{a}_x\frac{1}{15\pi} + \bar{a}_z\frac{1}{20\pi}\right)e^{-j(6x+8z)}$

$\bar{H}_i(x,z;t) = \left(-\bar{a}_x\frac{1}{15\pi} + \bar{a}_z\frac{1}{20\pi}\right)\cos(3\times10^9 t - 6x - 8z)$ (A/m).

c) $\cos\theta_i = \bar{a}_{ni}\cdot\bar{a}_z = \left(\frac{\bar{k}}{k}\right)\cdot\bar{a}_z = (\bar{a}_x 0.6 + \bar{a}_z 0.8)\cdot\bar{a}_z = 0.8$.

cont. 8-22

$\theta_i = \cos^{-1} 0.8 = 36.9°$.

d) $\bar{E}_i(x,0) + \bar{E}_r(x,0) = 0 \longrightarrow \bar{E}_r(x,z) = -\bar{a}_y 10 e^{-j(6x-8z)}$ (V/m).

$\bar{H}_r(x,z) = \frac{1}{\eta_0} \bar{a}_{nr} \times \bar{E}_r(x,z)$ $\qquad \bar{a}_{nr} = \bar{a}_x 0.6 + \bar{a}_z 0.8$.

$= -\left(\bar{a}_x \frac{1}{15\pi} - \bar{a}_z \frac{1}{20\pi}\right) e^{-j(6x-8z)}$ (A/m).

e) $\bar{E}_1(x,z) = \bar{E}_i(x,z) + \bar{E}_r(x,z) = \bar{a}_y 10 (e^{-j8z} - e^{j8z}) e^{-j6x}$

$= -\bar{a}_y j20 e^{-j6x} \sin 8z$ (V/m).

$\bar{H}_1(x,z) = \bar{H}_i(x,z) + \bar{H}_r(x,z)$

$= -\left(\bar{a}_x \frac{2}{15\pi} \cos 8z - \bar{a}_z \frac{j}{10\pi} \sin 8z\right) e^{-j6x}$ (A/m).

8-18 Given $\bar{E}_i(y,z) = 5(\bar{a}_y + \bar{a}_z\sqrt{3}) e^{j6(\sqrt{3}y-z)}$ (V/m):

a) $k_y = -6\sqrt{3}$, $\quad k_z = 6 \longrightarrow k = \sqrt{k_y^2 + k_z^2} = 12$ (rad/m).

$\lambda = \frac{2\pi}{k} = \frac{2\pi}{12} = \frac{\pi}{6} = 0.524$ (m),

$f = \frac{c}{\lambda} = 5.73 \times 10^8$ (Hz); $\quad \omega = 2\pi f = kc = 3.6 \times 10^9$ (rad/s).

b) $\bar{E}_i(y,z;t) = 5(\bar{a}_y + \bar{a}_z\sqrt{3}) \cos(3.6\times 10^9 t + 6\sqrt{3}y - 6z)$ (V/m).

$\bar{H}_i(y,z) = \frac{1}{\eta_0} \bar{a}_{ni} \times \bar{E}_i = \frac{1}{120\pi}\left(-\bar{a}_y \frac{\sqrt{3}}{2} + \bar{a}_z \frac{1}{2}\right) \times 5(\bar{a}_y + \bar{a}_z\sqrt{3}) e^{j6(\sqrt{3}y-z)}$

$= \bar{a}_x \left(-\frac{1}{12\pi}\right) e^{j6(\sqrt{3}y-z)}$.

$\bar{H}_i(y,z;t) = \bar{a}_x \left(-\frac{1}{12\pi}\right) \cos(3.6\times 10^9 t + 6\sqrt{3}y - 6z)$ (A/m).

c) $\cos\theta_i = \bar{a}_{ni} \cdot \bar{a}_z = \frac{1}{2} \longrightarrow \theta_i = \cos^{-1}\left(\frac{1}{2}\right) = 60°$.

d) $\bar{a}_{nr} \times \bar{E}_r(y,z) = 0$ and $E_{iy}(y,0) + E_{ry}(y,0) = 0$ lead to:

$\bar{E}_r(y,z) = 5(-\bar{a}_y + \bar{a}_z\sqrt{3}) e^{j6(\sqrt{3}y+z)}$ (V/m),

$\bar{H}_r(y,z) = \frac{1}{\eta_0} \bar{a}_{nr} \times \bar{E}_r(y,z)$

$= \frac{1}{120\pi}\left(-\bar{a}_y \frac{\sqrt{3}}{2} - \bar{a}_z \frac{1}{2}\right) \times 5(-\bar{a}_y + \bar{a}_z\sqrt{3}) e^{j6(\sqrt{3}y+z)}$

$= \bar{a}_x \left(-\frac{1}{12\pi}\right) e^{j6(\sqrt{3}y+z)}$ (A/m).

e) $\bar{E}_1(y,z) = \bar{E}_i(y,z) + \bar{E}_r(y,z)$

$= \left(\bar{a}_y(-10j)\sin 6z + \bar{a}_z 10\sqrt{3}\cos 6z\right) e^{j6\sqrt{3}y}$.

$\bar{H}_1(y,z) = \bar{H}_i(y,z) + \bar{H}_r(y,z)$

$= \bar{a}_x \left(-\frac{1}{6\pi}\right) \cos 6z \cdot e^{j6\sqrt{3}y}$ (A/m).

P.8-19 a) From Eqs. (8-80a) and (8-80b):

$$\bar{E}_1(x,z;t) = \bar{a}_y 2E_{i0} \sin(\beta_1 z\cos\theta_i)\sin(\omega t - \beta_1 x\sin\theta_i),$$

$$\bar{H}_1(x,z;t) = \bar{a}_x\left(-2\frac{E_{i0}}{\eta_1}\right)\cos\theta_i \cos(\beta_1 z\cos\theta_i)\cos(\omega t-\beta_1 x\sin\theta_i)$$

$$+\bar{a}_z\left(2\frac{E_{i0}}{\eta_1}\right)\sin\theta_i \sin(\beta_1 z\cos\theta_i)\sin(\omega t-\beta_1 x\sin\theta_i).$$

b) $$\bar{\mathcal{P}}_{av} = \frac{1}{2}\mathcal{R}e(\bar{E}\times\bar{H}^*) = \bar{a}_x\frac{2E_{i0}^2}{\eta_1}\sin\theta_i \sin^2(\beta_1 z\cos\theta_i).$$

P.8-20 a) From Eqs. (8-86a) and (8-86b):

$$\bar{E}_1(x,z;t) = -2E_{i0}\left[\bar{a}_x\cos\theta_i\sin(\beta_1 z\cos\theta_i)\cos(\omega t-\beta_1 x\sin\theta_i)\right.$$

$$\left.+\bar{a}_z\sin\theta_i\cos(\beta_1 z\cos\theta_i)\sin(\omega t-\beta_1 x\sin\theta_i)\right],$$

$$\bar{H}_1(x,z;t) = +\bar{a}_y\frac{2E_{i0}}{\eta_1}\cos(\beta_1 z\cos\theta_i)\sin(\omega t-\beta_1 x\sin\theta_i).$$

b) $$\bar{\mathcal{P}}_{av} = \frac{1}{2}\mathcal{R}e(\bar{E}\times\bar{H}^*) = \bar{a}_x\frac{2E_{i0}^2}{\eta_1}\sin\theta_i\cos^2(\beta_1 z\cos\theta_i).$$

P.8-21 $1+\Gamma=\tau, \quad |\Gamma|\le 1.$

$$|\tau| = |\Gamma| \longrightarrow \Gamma<0 \longrightarrow \eta_1-\eta_2 = 2\eta_2.$$

$$\longrightarrow \eta_1 = 3\eta_2 \longrightarrow |\Gamma| = \frac{1}{2}.$$

$$S = \frac{1+|\Gamma|}{1-|\Gamma|} = 3, \qquad S_{dB} = 20\log_{10}3 = 9.54 \text{ (dB)}.$$

P.8-22 a) In the lossy medium (medium 2):

$$\bar{E}_t = \bar{a}_x E_{t0} e^{-\alpha_2 z}e^{-j\beta_2 z},$$

where $\alpha_2 = \sqrt{\frac{\mu_2\epsilon_2}{2}}\left[\sqrt{1+\left(\frac{\sigma_2}{\omega\epsilon_2}\right)^2}-1\right]^{\frac{1}{2}}, \quad \beta_2 = \sqrt{\frac{\mu_2\epsilon_2}{2}}\left[\sqrt{1+\left(\frac{\sigma_2}{\omega\epsilon_2}\right)^2}+1\right]^{\frac{1}{2}}.$

In air, $\beta_1 = 6$ (rad/m), $\omega = \beta_1 c = 1.8\times10^9$ (rad/s).

$$\tan\delta_c = \frac{\sigma_2}{\omega\epsilon_2} = 0.5 \longrightarrow \alpha_2 = 2.30 \text{ (Np/m)},\ \beta_2 = 9.76 \text{ (rad}$$

$$\eta_2 = \sqrt{\frac{\mu_2}{\epsilon_2}} = \frac{120\pi}{\sqrt{\epsilon_{r2}}(1+\tan^2\delta_c)^{1/4}} = 225e^{j13.3^\circ}$$

$$\therefore\ \bar{E}_t = \bar{a}_x E_{t0}e^{-2.30z}e^{-j9.76z}$$

$$\bar{H}_t = \bar{a}_z\times\frac{\bar{E}_t}{\eta_2} = \bar{a}_y\frac{E_{t0}}{225}e^{-j13.3^\circ}e^{-2.3z}e^{-j9.76z}.$$

We also have $\bar{H}_i = \bar{a}_y \frac{10}{120\pi} e^{-j6z}$.

Let $\bar{E}_r = \bar{a}_x E_{r0} e^{j6z} \longrightarrow \bar{H}_r = -\bar{a}_y \frac{E_{r0}}{120\pi} e^{j6z}$.

Boundary conditions for $\bar{E}$ and $\bar{H}$ at $z = 0$:

$$\begin{cases} 10 + E_{r0} = E_{t0} \\ 10 - E_{r0} = E_{t0}\sqrt{\epsilon_{r2}}\,(1+\tan^2\delta_c)^{1/4} e^{-j13.3°} \end{cases}$$

$$E_{r0} = 2.77\,e^{j157°}, \qquad E_{t0} = 7.53\,e^{-j172°}.$$

$$\bar{E}_r(z,t) = \bar{a}_x 2.77 \cos(1.8\times10^9 t + 6z + 157°) \quad (V/m)$$

$$\bar{H}_r(z,t) = -\bar{a}_y 0.073 \cos(1.8\times10^9 t + 6z + 157°) \quad (A/m)$$

$$\bar{E}_t(z,t) = \bar{a}_x 7.53\, e^{-2.30z} \cos(1.8\times10^9 t - 9.76z - 172°) \quad (V/m)$$

$$\bar{H}_t(z,t) = \bar{a}_y 0.033\, e^{-2.30z} \cos(1.8\times10^9 t - 9.76z + 175°) \quad (A/m).$$

b) $(\bar{\mathcal{P}}_{av})_1 = \bar{a}_z \left(\frac{10^2}{2\times120\pi} - \frac{2.77^2}{2\times120\pi}\right) = \bar{a}_z 0.122 \quad (W/m^2)$,

$$(\bar{\mathcal{P}}_{av})_2 = \bar{a}_z \frac{7.53^2}{2\times225}(\cos 13.3°)e^{-4.61z}$$

$$= \bar{a}_z 0.122\, e^{-4.61z} \quad (W/m^2).$$

P.8-23 From Eqs. (8-107a), (8-107b), (8-108a), (8-108b), (8-109a), and (8-109b) we have

$$\bar{E}_1 = \bar{a}_x (E_{i0} e^{-j\beta_0 z} + E_{r0} e^{j\beta_0 z}), \qquad \bar{H}_1 = \bar{a}_y \frac{1}{\eta_0}(E_{i0} e^{-j\beta_0 z} - E_{r0} e^{j\beta_0 z})$$

$$\bar{E}_2 = \bar{a}_x (E_2^+ e^{-j\beta_2 z} + E_2^- e^{j\beta_2 z}), \qquad \bar{H}_2 = \bar{a}_y \frac{1}{\eta_2}(E_2^+ e^{-j\beta_2 z} - E_2^- e^{j\beta_2 z})$$

$$\bar{E}_t = \bar{a}_x E_{t0} e^{-j\beta_0 z}, \qquad \bar{H}_t = \bar{a}_y \frac{1}{\eta_0} E_{t0} e^{-j\beta_0 z}.$$

Boundary conditions at $z=0$: $\bar{E}_1(0) = \bar{E}_2(0)$, $\bar{H}_1(0) = \bar{H}_2(0)$.

at $z=d$: $\bar{E}_2(d) = \bar{E}_t(d)$, $\bar{H}_2(d) = \bar{H}_t(d)$.

Four equations to solve four unknowns E_2^+, E_2^-, E_{r0}, and E_{t0} in terms of E_{i0}:

a) $$E_{r0} = -\frac{j(\eta_0^2 - \eta_2^2)\tan\beta_2 d}{\eta_0\eta_2 + j(\eta_0^2+\eta_2^2)\tan\beta_2 d} E_{i0}$$

$$E_2^+ = \frac{\eta_2(\eta_0+\eta_2)\,e^{+j\beta_2 d}}{\eta_0\eta_2\cos\beta_2 d + j(\eta_0^2+\eta_2^2)\sin\beta_2 d} E_{i0}.$$

$$E_2^- = \frac{\eta_2(\eta_0-\eta_2)\,e^{-j\beta_2 d}}{\eta_0\eta_2\cos\beta_2 d + j(\eta_0^2+\eta_2^2)\sin\beta_2 d}\,E_{i0}$$

$$E_{t0} = \frac{2\eta_0\eta_2\,e^{j\beta_0 d}}{\eta_0\eta_2\cos\beta_2 d + j(\eta_0^2+\eta_2^2)\sin\beta_2 d}\,E_{i0}.$$

where $\eta_0 = \sqrt{\mu_0/\epsilon_0} = 120\pi$, $\eta_2 = \sqrt{\mu_2/\epsilon_2}$.

$\beta_0 = \omega/c$, $\beta_2 = \omega\sqrt{\mu_2\epsilon_2}$.

b) If $d = \lambda_2/4$, $\beta_2 d = \pi/2$, $E_{r0} = -\dfrac{\eta_0^2-\eta_2^2}{\eta_0^2+\eta_2^2}E_{i0}$.

$\therefore\ \Gamma = -\dfrac{\eta_0^2-\eta_2^2}{\eta_0^2+\eta_2^2} \neq 0$ unless $\eta_2 = \eta_0$.

($\Gamma = 0$ when $d = n\lambda_2/2$, $n = 1, 2, 3, \ldots$)

P.8-24 a) From Example 8-10: $\eta_2 = \sqrt{\eta_1\eta_3} \longrightarrow \epsilon_{2r} = \sqrt{\epsilon_{r1}\epsilon_{r3}} = 2$.

Wavelength of red light in dielectric coating:

$$\lambda_2 = 0.75\frac{u_{p2}}{c} = \frac{0.75}{\sqrt{\epsilon_{2r}}} = \frac{0.75}{\sqrt{2}} = 0.530\ (\mu m)$$

$$d = \lambda_2/4 = 0.133\ (\mu m).$$

b) For violet light, $\lambda_2' = \dfrac{0.42}{\sqrt{2}} = 0.297\ (\mu m)$.

$$\frac{d}{\lambda_2'} = 0.447 \longrightarrow \beta_2 d = 0.894\pi.$$

From Eq. (8-116) and using impedances normalized with respect to $\eta_1 = \eta_0$:

$$Z_2(0) = \eta_2\frac{\eta_3 + j\eta_2\tan\beta_2 d}{\eta_2 + j\eta_3\tan\beta_2 d} = \frac{1}{\sqrt{2}}\,\frac{\frac{1}{2} + j\frac{1}{\sqrt{2}}\tan\beta_2 d}{\frac{1}{\sqrt{2}} + j\frac{1}{2}\tan\beta_2 d}$$

$$= \frac{0.5 - j0.247}{1 - j0.247}.$$

$$\Gamma = \frac{Z_2(0)-1}{Z_2(0)+1} = 0.316\,e^{j198^\circ}.$$

Percentage of incident reflected $= |\Gamma|^2 \times 100\%$

$= (0.316)^2 \times 100\% = 10\%$.

P.8-25 $\Gamma_0 = \dfrac{Z_2(0)-\eta_1}{Z_2(0)+\eta_1}$, $Z_2(0) = \eta_2\dfrac{\eta_3 + j\eta_2\tan\beta_2 d}{\eta_2 + j\eta_3\tan\beta_2 d}$.

$$\Gamma_{12} = \frac{\eta_2-\eta_1}{\eta_2+\eta_1} \longrightarrow \frac{\eta_1}{\eta_2} = \frac{1-\Gamma_{12}}{1+\Gamma_{12}}.$$

$$\Gamma_{23} = \frac{\eta_3 - \eta_2}{\eta_3 + \eta_2} \longrightarrow \frac{\eta_2}{\eta_3} = \frac{1-\Gamma_{23}}{1+\Gamma_{23}}.$$

$$\therefore \Gamma_0 = \frac{1 + j\frac{\eta_2}{\eta_3}\tan\beta_2 d - \frac{\eta_1}{\eta_2}\left(\frac{\eta_2}{\eta_3} + j\tan\beta_2 d\right)}{1 + j\frac{\eta_2}{\eta_3}\tan\beta_2 d + \frac{\eta_1}{\eta_2}\left(\frac{\eta_2}{\eta_3} + j\tan\beta_2 d\right)}$$

$$= \frac{1 + j\frac{1-\Gamma_{23}}{1+\Gamma_{23}}\tan\beta_2 d - \frac{1-\Gamma_{12}}{1+\Gamma_{12}}\left(\frac{1-\Gamma_{23}}{1+\Gamma_{23}} + j\tan\beta_2 d\right)}{1 + j\frac{1-\Gamma_{23}}{1+\Gamma_{23}}\tan\beta_2 d + \frac{1-\Gamma_{12}}{1+\Gamma_{12}}\left(\frac{1-\Gamma_{23}}{1+\Gamma_{23}} + j\tan\beta_2 d\right)}$$

$$= \frac{(\Gamma_{12} + \Gamma_{23}) + j(\Gamma_{12} - \Gamma_{23})\tan\beta_2 d}{(1 + \Gamma_{12}\Gamma_{23}) + j(1 - \Gamma_{12}\Gamma_{23})\tan\beta_2 d}.$$

P.8-26

$$\bar{E}_1 = \bar{a}_x\left(E_{i0}e^{-j\beta_1 z} + E_{r0}e^{j\beta_1 z}\right)$$

$$\bar{H}_1 = \bar{a}_y\frac{1}{\eta_1}\left(E_{i0}e^{-j\beta_1 z} - E_{r0}e^{j\beta_1 z}\right)$$

$$\bar{E}_2 = \bar{a}_x\left(E_2^+ e^{-j\beta_2 z} + E_2^- e^{j\beta_2 z}\right)$$

$$\bar{H}_2 = \bar{a}_y\frac{1}{\eta_2}\left(E_2^+ e^{-j\beta_2 z} - E_2^- e^{j\beta_2 z}\right).$$

At $z = d$, $\bar{E}_2 = 0 \longrightarrow E_2^- = -E_2^+ e^{-j2\beta_2 d}$.

$$\bar{E}_2 = \bar{a}_x E_2^+\left[e^{-j\beta_2 z} - e^{j\beta_2(z-2d)}\right]$$

$$\bar{H}_2 = \bar{a}_y\frac{E_2^+}{\eta_2}\left[e^{-j\beta_2 z} + e^{j\beta_2(z-2d)}\right].$$

Boundary conditions at $z=0$: $E_1(0) = E_2(0) \longrightarrow E_{i0} + E_{r0} = E_2^+(1 - e^{-j2\beta_2 d})$,
$H_1(0) = H_2(0) \longrightarrow E_{i0} - E_{r0} = E_2^+\frac{\eta_0}{\eta_2}(1 + e^{-j2\beta_2 d})$.

$$E_2^+ = \frac{2\eta_2 E_{i0}}{(\eta_0 + \eta_2) + (\eta_0 - \eta_2)e^{-j2\beta_2 d}}$$

$$E_{r0} = -\left(\frac{\eta_0 - j\eta_2\tan\beta_2 d}{\eta_0 + j\eta_2\tan\beta_2 d}\right)E_{i0}.$$

a) $\bar{E}_r(z,t) = \bar{a}_x E_{i0}\cos\left[\omega\left(t - \frac{z}{u_p}\right) + \theta\right]$, $\theta = \pi - 2\tan^{-1}\left(\frac{\eta_2}{\eta_0}\tan\beta_2 d\right)$.

b) $\bar{E}_1(z,t) = \bar{a}_x E_{i0}\left\{\cos\omega\left(t - \frac{z}{u_p}\right) + \cos\left[\omega\left(t - \frac{z}{u_p}\right) + \theta\right]\right\}$.

c) $$\bar{E}_2(z,t) = \bar{a}_x\frac{2\eta_2 E_{i0}}{\sqrt{2\left[(\eta_0^2 + \eta_2^2) + (\eta_0^2 - \eta_2^2)\cos 2\beta_2 d\right]}}\left\{\cos\left[\omega\left(t - \frac{z}{u_{p2}}\right) + \psi\right] - \cos\left[\omega\left(t + \frac{z}{u_{p2}}\right) - \frac{2\omega d}{u_{p2}} + \psi\right]\right\},$$

$$\psi = \tan^{-1}\left[\frac{(\eta_0 - \eta_2)\sin 2\beta_2 d}{(\eta_0 + \eta_2) + (\eta_0 - \eta_2)\cos 2\beta_2 d}\right].$$

d) $(\bar{\mathcal{P}}_{av})_1 = \frac{1}{2}\mathcal{R}e(\bar{E}_1 \times \bar{H}_1^*) = 0.$

e) $(\bar{\mathcal{P}}_{av})_2 = 0.$

f) Let $E_{r0} = -E_{i0} \longrightarrow \tan\beta_2 d = 0 \longrightarrow d = n\lambda_2/2,\ n = 0, 1, 2, \cdots$

P.8-27 $\quad k_2 = \beta_2 - j\alpha_2 = (1-j)\frac{1}{\delta}, \quad \alpha_2 = \beta_2 = \frac{1}{\delta} = \sqrt{\pi f \mu_2 \sigma_2}.$

$\eta_2 = (1+j)\frac{\alpha_2}{\sigma_2} \ll \eta_0 \quad$ at 10 (MHz).

a) From Problem P.8-23,

$$E_2^+ = \eta_2 H_2^+ \cong -j\left(\frac{\eta_2}{\eta_0}\right)\frac{e^{\alpha_2 d}e^{j\beta_2 d}E_{i0}}{\sin(\beta_2 - j\alpha_2)d}.$$

b) $$E_2^- = -\eta_2 H_2^- \cong -j\left(\frac{\eta_2}{\eta_0}\right)\frac{e^{-\alpha_2 d}e^{-j\beta_2 d}E_{i0}}{\sin(\beta_2 - j\alpha_2)d}.$$

c) $$E_{30} = E_{t0} = \eta_0 H_{30} \cong -j\left(\frac{\eta_2}{\eta_0}\right)\frac{2e^{j\beta_0 d}E_{i0}}{\sin(\beta_2 - j\alpha_2)d}.$$

d) $$E_{r0} \cong -\frac{E_{i0}}{1 - j\frac{\eta_2}{\eta_0}\cot(\beta_2 - j\alpha_2)d}$$

$$\cong -\left(1 + j\frac{\eta_2}{\eta_0}\,\frac{1 + j\tan\beta_2 d\tanh\alpha_2 d}{\tan\beta_2 d - j\tanh\alpha_2 d}\right)E_{i0}.$$

$$(\bar{\mathcal{P}}_{av})_1 = \frac{1}{2}\mathcal{R}e\left[(\bar{E}_{i0}\times\bar{H}_{i0}^*) - (\bar{E}_{r0}\times\bar{H}_{r0}^*)\right]$$

$$= \bar{a}_z\frac{\alpha}{\eta_0^2\sigma}(A+B),$$

where $$\frac{1 + j\tan\beta_2 d\tanh\alpha_2 d}{\tan\beta_2 d - j\tanh\alpha_2 d} = A + jB.$$

$$(\mathcal{P}_{av})_1 = \frac{\alpha}{\eta_0^2\sigma}\,\frac{\sin\beta_2 d\cos\beta_2 d + \sinh\alpha_2 d\cosh\alpha_2 d}{(\sin\beta_2 d\cosh\alpha_2 d)^2 + (\cos\beta_2 d\sinh\alpha_2 d)^2}.$$

$$(\mathcal{P}_{av})_3 = \frac{1}{2\eta_0}|E_{30}|^2 = \frac{1}{\eta_0^3}\left(\frac{\alpha}{\sigma}\right)^2\frac{4E_{i0}^2}{(\sin\beta_2 d\cosh\alpha_2 d)^2 + (\cos\beta_2 d\sinh\alpha_2 d)^2}.$$

$$\therefore \quad \frac{(\mathcal{P}_{av})_3}{(\mathcal{P}_{av})_1} = \frac{4}{\eta_0}\left(\frac{\alpha}{\sigma}\right)\frac{1}{\sin\beta_2 d\cos\beta_2 d + \sinh\alpha_2 d\cosh\alpha_2 d}.$$

$$(\mathcal{P}_{av})_i = \frac{1}{2\eta_0}E_{i0}^2.$$

$$\frac{(\mathcal{P}_{av})_3}{(\mathcal{P}_{av})_i} = \frac{8}{\eta_0^2}\left(\frac{\alpha}{\sigma}\right)^2\frac{1}{(\sin\beta_2 d\cosh\alpha_2 d)^2 + (\cos\beta_2 d\sinh\alpha_2 d)^2}.$$

At $f = 10^7$ (Hz), $\sigma = 5.80\times10^7$ (S/m), $\alpha_2 = \beta_2 = 4.785\times10^4$, $d = \delta = \frac{1}{\alpha_2}$

$$\frac{(\mathcal{P}_{av})_3}{(\mathcal{P}_{av})_i} = 1.839\times10^{-11}.$$

P.8-28 $\quad k_{2x}^2 + k_{2z}^2 = k_2^2 = \omega^2\mu_0\epsilon_2 - j\omega\mu_0\sigma_2$. $\quad$ ①

Continuity conditions at $z=0$ for all x and y require:

$$k_{2x} = k_{1x} = \omega\sqrt{\mu_0\epsilon_0}\sin\theta_i = \beta_x = 2.09\times10^{-4} \quad ②$$

$$k_{2z} = \beta_{2z} - j\alpha_{2z}. \quad ③$$

Combining ①, ② and ③, we can solve α_{2z} and β_{2z} in terms of $\omega, \mu_0, \epsilon_2, \sigma_2$, and β_x. But, since

$$\beta_x^2 \ll \omega^2\mu_0\epsilon_2,$$

we have $\alpha_2 = \alpha_{2z} \cong \beta_{2z} \cong \frac{1}{\delta} = \sqrt{\pi f \mu_0 \sigma_2} = 0.3974\ (m^{-1})$.

a) $\theta_t = \tan^{-1}\frac{\beta_x}{\beta_{2z}} \cong \tan^{-1}\frac{2.09}{0.3974}\times10^{-4} \cong 5.26\times10^{-4}$ (rad) $= 0.03°$.

b) $\Gamma_\parallel = \dfrac{2\eta_2\cos\theta_i}{\eta_2\cos\theta_t + \eta_0\cos\theta_i}$ $\qquad \eta_2 = \frac{\alpha_2}{\sigma_2}(1+j) = 0.0993(1+j)$.

$= \dfrac{2\times0.0993(1+j)}{0.0993(1+j)+377\cos 88°}$ $\qquad \cos\theta_t = \cos 0.03° \cong 1$.

$\cong 0.0151(1+j) = 0.0214\, e^{j\pi/4}$.

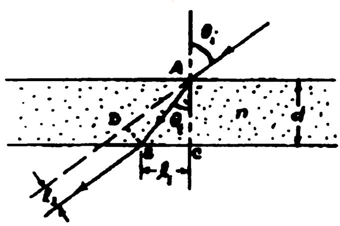
c) $(\mathscr{P}_{av})_i = \dfrac{E_{i0}^2}{2\eta_0}$.

$E_{t0} \cong 2E_{i0}\frac{\eta_2}{\eta_0}$, $\quad H_{t0} \cong \frac{2E_{i0}}{\eta_0} \longrightarrow (\mathscr{P}_{av})_t = 2\frac{E_{i0}^2\alpha_2}{\eta_0^2\sigma_2}e^{-2\alpha_2 z}$.

$\therefore \dfrac{(\mathscr{P}_{av})_t}{(\mathscr{P}_{av})_i} = \dfrac{4\alpha_2}{\eta_0\sigma_2}e^{-2\alpha_2 z} = 1.054\times10^{-3}e^{-0.795z}$.

d) $20\log_{10}e^{-\alpha_2 z} = -30. \longrightarrow z = \dfrac{1.5}{\alpha_2\log_{10}e} = 8.69$ (m).

P.8-29



a) Snell's law:

$$\frac{\sin\theta_t}{\sin\theta_i} = \frac{1}{n},$$

$$\theta_t = \sin^{-1}\left(\frac{1}{n}\sin\theta_i\right).$$

b) $\cos\theta_t = \sqrt{1-\left(\frac{1}{n}\sin\theta_i\right)^2}$.

$$\ell_1 = \overline{BC} = \overline{AC}\tan\theta_t = d\frac{\sin\theta_t}{\cos\theta_t} = \frac{d\sin\theta_i}{\sqrt{n^2-\sin^2\theta_i}}.$$

c) $\ell_2 = \overline{BD} = \overline{AC}\sin(\theta_i-\theta_t) = \dfrac{d}{\cos\theta_t}(\sin\theta_i\cos\theta_t - \cos\theta_i\sin\theta_t)$

$= d\sin\theta_i\left[1 - \dfrac{\cos\theta_i}{\sqrt{n^2-\sin^2\theta_i}}\right]$.

P. 8-30 a) $\sin\theta_c = \sqrt{\frac{\epsilon_2}{\epsilon_1}} \longrightarrow \sin\theta_t = \sqrt{\frac{\epsilon_1}{\epsilon_2}}\sin\theta_i > 1$ for $\theta_i > \theta_c$.

$$\cos\theta_t = -j\sqrt{\left(\frac{\epsilon_1}{\epsilon_2}\right)\sin^2\theta_i - 1}$$

From Eqs. (8-135a) and (8-135b):

$$\bar{E}_t(x,z) = \bar{a}_y E_{t0}\, e^{-\alpha_2 z} e^{-j\beta_{2x}x},$$

$$\bar{H}_t(x,z) = \frac{E_{t0}}{\eta_2}\left(\bar{a}_x j\alpha_2 + \bar{a}_z\sqrt{\frac{\epsilon_1}{\epsilon_2}}\sin\theta_i\right)e^{-\alpha_2 z} e^{-j\beta_{2x}x},$$

where $\beta_{2x} = \beta_2\sin\theta_t = \beta_2\sqrt{\frac{\epsilon_1}{\epsilon_2}}\sin\theta_i$,

$$\alpha_2 = \beta_2\sqrt{\left(\frac{\epsilon_1}{\epsilon_2}\right)\sin^2\theta_i - 1},$$

$$E_{t0} = \frac{2\eta_2\cos\theta_i\, E_{i0}}{\eta_2\cos\theta_i - j\eta_1\sqrt{\left(\frac{\epsilon_1}{\epsilon_2}\right)\sin^2\theta_i - 1}} \quad \text{from Eq. (8-139)}$$

b) $(\mathcal{P}_{av})_{2z} = \frac{1}{2}\mathcal{R}e\,(E_{ty}H_{tx}^*) = 0.$

P. 8-31 a) $\theta_c = \sin^{-1}\sqrt{\epsilon_{r2}/\epsilon_{r1}} = \sin^{-1}\sqrt{1/81} = 6.38°.$

b) $\theta_i = 20° > \theta_c$. $\quad \sin\theta_t = \sqrt{\frac{\epsilon_1}{\epsilon_2}}\sin\theta_i = 3.08$

$$\cos\theta_t = -j\sqrt{\left(\frac{\epsilon_1}{\epsilon_2}\right)\sin^2\theta_i - 1} = -j2.91$$

$$\Gamma_\perp = \frac{\sqrt{\epsilon_{r1}}\cos\theta_i - \cos\theta_t}{\sqrt{\epsilon_{r1}}\cos\theta_i + \cos\theta_t} = e^{j38°} = e^{j0.66}$$

c) $$\tau_\perp = \frac{2\sqrt{\epsilon_{r1}}\cos\theta_i}{\sqrt{\epsilon_{r1}}\cos\theta_i + \cos\theta_t} = 1.89\, e^{j19°} = 1.89\, e^{j0.33}$$

d) The transmitted wave in air varies as

$$e^{-\alpha_2 z} e^{-j\beta_{2x}x},$$

where $$\alpha_2 = \beta_2\sqrt{\left(\frac{\epsilon_1}{\epsilon_2}\right)\sin^2\theta_i - 1} = \frac{2\pi}{\lambda_0}(2.91).$$

Attenuation in air for each wavelength

$$= 20\log_{10} e^{-\alpha_2\lambda_0} = 159 \text{ (dB)}.$$

P. 8-32 When the incident light first strikes the hypotenuse surface, $\theta_i = \theta_t = 0$, $\tau_1 = \frac{2\eta_2}{\eta_2 + \eta_0}$.

$$\frac{(\mathcal{P}_{av})_{t1}}{(\mathcal{P}_{av})_i} = \frac{\eta_0}{\eta_2}\tau_1^2 = \frac{4\eta_0\eta_2}{(\eta_2+\eta_0)^2}.$$

Total reflections occur inside the prism at both slanting surfaces because

$$\theta_i = 45° > \theta_c = \sin^{-1}\left(\frac{1}{2}\right) = 30°.$$

On exit from the prism, $\tau_2 = \dfrac{2\eta_0}{\eta_2+\eta_0}$.

$$\frac{(\mathcal{P}_{av})_o}{(\mathcal{P}_{av})_{t1}} = \frac{\eta_2}{\eta_0}\tau_2^2 = \frac{4\eta_0\eta_2}{(\eta_2+\eta_0)^2}.$$

$$\therefore \frac{(\mathcal{P}_{av})_o}{(\mathcal{P}_{av})_i} = \left[\frac{4\eta_0\eta_2}{(\eta_2+\eta_0)^2}\right]^2 = \left[\frac{4\sqrt{\epsilon_r}}{(1+\sqrt{\epsilon_r})^2}\right]^2 = 0.79.$$

P.8-33 a) For perpendicular polarization and $\mu_1 \neq \mu_2$:

$$\sin\theta_{B\perp} = \frac{1}{\sqrt{1+\left(\frac{\mu_1}{\mu_2}\right)}}.$$

Under condition of no reflection:

$$\cos\theta_t = \sqrt{1-\frac{\eta_1^2}{\eta_2^2}\sin^2\theta_{B\perp}} = \frac{1}{\sqrt{1+\left(\frac{\mu_1}{\mu_2}\right)}}$$

$$= \sin\theta_{B\perp} \longrightarrow \theta_t + \theta_{B\perp} = \pi/2.$$

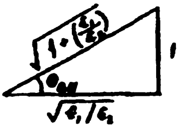
b) For parallel polarization and $\epsilon_1 \neq \epsilon_2$:

$$\sin\theta_{B\parallel} = \frac{1}{\sqrt{1+\left(\frac{\epsilon_1}{\epsilon_2}\right)}}.$$

$$\cos\theta_t = \sqrt{1-\frac{\eta_1^2}{\eta_2^2}\sin^2\theta_{B\parallel}} = \frac{1}{\sqrt{1+\left(\frac{\epsilon_1}{\epsilon_2}\right)}}$$

$$= \sin\theta_{B\parallel} \longrightarrow \theta_t + \theta_{B\parallel} = \pi/2.$$

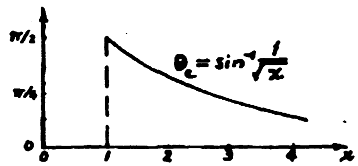
P.8-34 a) $\sin\theta_c = \sqrt{\dfrac{\epsilon_2}{\epsilon_1}}$; $\sin\theta_{B\parallel} = \dfrac{1}{\sqrt{1+\left(\frac{\epsilon_1}{\epsilon_2}\right)}}$

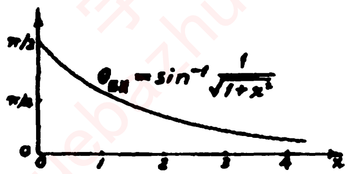


$$\longrightarrow \tan\theta_{B\parallel} = \sqrt{\frac{\epsilon_2}{\epsilon_1}}.$$

$$\therefore \sin\theta_c = \tan\theta_{B\parallel}. \quad (\theta_c > \theta_{B\parallel})$$

b) Let $\epsilon_1/\epsilon_2 = x$.





P.8-35 a) For perpendicular polarization:

$$\Gamma_\perp = \frac{\sqrt{\epsilon_{r1}}\cos\theta_i - \sqrt{\epsilon_{r2}}\cos\theta_t}{\sqrt{\epsilon_{r1}}\cos\theta_i + \sqrt{\epsilon_{r2}}\cos\theta_t}.$$

$$\sin\theta_t = \sqrt{\frac{\epsilon_{r1}}{\epsilon_{r2}}}\sin\theta_i, \quad \cos\theta_t = \sqrt{1-\left(\frac{\epsilon_{r1}}{\epsilon_{r2}}\right)\sin^2\theta_i}.$$

$$\Gamma_\perp = \frac{\sqrt{\frac{\epsilon_{r1}}{\epsilon_{r2}}}\cos\theta_i - \sqrt{1-\frac{\epsilon_{r1}}{\epsilon_{r2}}\sin^2\theta_i}}{\sqrt{\frac{\epsilon_{r1}}{\epsilon_{r2}}}\cos\theta_i + \sqrt{1-\frac{\epsilon_{r1}}{\epsilon_{r2}}\sin^2\theta_i}}$$

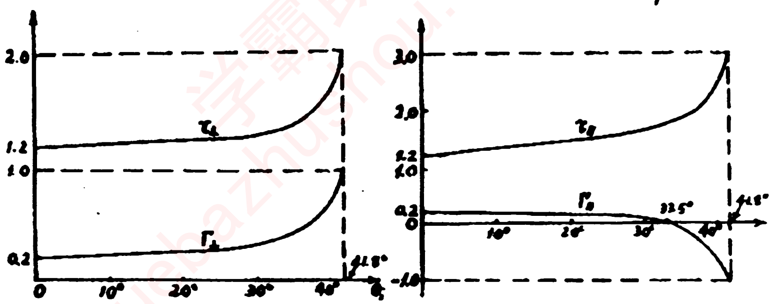
$$\tau_\perp = \frac{2\eta_2\cos\theta_i}{\eta_2\cos\theta_i + \eta_1\cos\theta_t} = \frac{2\sqrt{\frac{\epsilon_{r1}}{\epsilon_{r2}}}\cos\theta_i}{\sqrt{\frac{\epsilon_{r1}}{\epsilon_{r2}}}\cos\theta_i + \sqrt{1-\frac{\epsilon_{r1}}{\epsilon_{r2}}\sin^2\theta_i}}.$$

For parallel polarization:

$$\Gamma_\parallel = \frac{\sqrt{\frac{\epsilon_{r1}}{\epsilon_{r2}}}\sqrt{1-\left(\frac{\epsilon_{r1}}{\epsilon_{r2}}\right)\sin^2\theta_i} - \cos\theta_i}{\sqrt{\frac{\epsilon_{r1}}{\epsilon_{r2}}}\sqrt{1-\left(\frac{\epsilon_{r1}}{\epsilon_{r2}}\right)\sin^2\theta_i} + \cos\theta_i}$$

$$\tau_\parallel = \frac{2\sqrt{\frac{\epsilon_{r1}}{\epsilon_{r2}}}\cos\theta_i}{\sqrt{\frac{\epsilon_{r1}}{\epsilon_{r2}}}\sqrt{1-\left(\frac{\epsilon_{r1}}{\epsilon_{r2}}\right)\sin^2\theta_i} + \cos\theta_i}.$$

b) $\epsilon_{r1}/\epsilon_{r2} = 2.25$, $\sqrt{\epsilon_{r1}/\epsilon_{r2}} = 1.5 \longrightarrow \theta_c = \sin^{-1}\sqrt{\frac{\epsilon_2}{\epsilon_1}} = 41.8°$.



P.8-36 a) $$\Gamma'_\parallel = \left.\frac{(E_r)_{tan}}{(E_i)_{tan}}\right|_{z=0} = \frac{E_{r0}\cos\theta_i}{E_{i0}\cos\theta_i} = \frac{E_{r0}}{E_{i0}} = \Gamma_\parallel = \frac{\eta_2\cos\theta_t - \eta_1\cos\theta_i}{\eta_2\cos\theta_t + \eta_1\cos\theta_i}.$$

$$\tau'_\parallel = \left.\frac{(E_t)_{tan}}{(E_i)_{tan}}\right|_{z=0} = \frac{E_{t0}\cos\theta_t}{E_{i0}\cos\theta_i} = \tau_\parallel\left(\frac{\cos\theta_t}{\cos\theta_i}\right) = \frac{2\eta_2\cos\theta_t}{\eta_2\cos\theta_t + \eta_1\cos\theta_i}.$$

We have $$1 + \Gamma'_\parallel = \tau'_\parallel.$$

which compares with Eq. (8-151):

$$1 + \Gamma_\parallel = \tau_\parallel.$$

Chapter 9

P.9-1

$$\bar{\nabla}\times\bar{H} = \begin{vmatrix} \bar{a}_x & \bar{a}_y & \bar{a}_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & -j\beta \\ H_x & 0 & 0 \end{vmatrix} = \bar{a}_y j\omega\epsilon E_y \longrightarrow \frac{\partial H_x}{\partial y} = 0.$$

$$\bar{\nabla}\times\bar{E} = \begin{vmatrix} \bar{a}_x & \bar{a}_y & \bar{a}_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & -j\beta \\ 0 & E_y & 0 \end{vmatrix} = -\bar{a}_x j\omega\mu H_x \longrightarrow \frac{\partial E_y}{\partial x} = 0.$$

P.9-2 a) $\bar{\nabla}\times(\bar{a}_x E_x + \bar{a}_y E_y) = -j\omega\mu(\bar{a}_x H_x + \bar{a}_y H_y)$

$$\longrightarrow \begin{cases} \beta E_y = -\omega\mu H_x & (1) \\ \beta E_x = \omega\mu H_y & (2) \\ \frac{\partial E_y}{\partial x} = \frac{\partial E_x}{\partial y} & (3) \end{cases}$$

$\nabla\times(a_x H_x + a_y H_y) = j\omega\epsilon(a_x E_x + a_y E_y)$

$$\longrightarrow \begin{cases} \beta H_y = \omega\epsilon E_x & (4) \\ \beta H_x = -\omega\epsilon E_y & (5) \\ \frac{\partial H_y}{\partial x} = \frac{\partial H_x}{\partial y} & (6) \end{cases}$$

From (1) and (5): $\beta = \omega\sqrt{\mu\epsilon}$ (7)

From (2) or (4): $\frac{E_x}{H_y} = \sqrt{\frac{\mu}{\epsilon}} = \eta$ (8)

From (1) or (5): $\frac{E_y}{H_x} = -\sqrt{\frac{\mu}{\epsilon}} = -\eta$ (9)

b) From (3): $\frac{\partial^2 E_y}{\partial y\,\partial x} = \frac{\partial^2 E_x}{\partial y^2}$ (10)

From (4), (5), and (6): $\frac{\partial E_x}{\partial x} = -\frac{\partial E_y}{\partial y} \longrightarrow \frac{\partial^2 E_x}{\partial x^2} = -\frac{\partial^2 E_y}{\partial x\,\partial y}$ (11)

Combining (10) and (11), we have $\frac{\partial^2 E_x}{\partial x^2} + \frac{\partial^2 E_x}{\partial y^2} = 0.$

Similarly, $\frac{\partial^2 H_x}{\partial x^2} + \frac{\partial^2 H_x}{\partial y^2} = 0.$

P.9-3 Eq. (9-20): $Z_0 = \frac{d}{w}\sqrt{\frac{\mu}{\epsilon}}$.

a) $Z_0 = \frac{d'}{w}\sqrt{\frac{\mu}{2\epsilon}} = \frac{d}{w}\sqrt{\frac{\mu}{\epsilon}} \longrightarrow d' = \sqrt{2}\,d.$

b) $Z_0 = \frac{d}{w'}\sqrt{\frac{\mu}{2\epsilon}} = \frac{d}{w}\sqrt{\frac{\mu}{\epsilon}} \longrightarrow w' = \frac{1}{\sqrt{2}}w.$

c) $Z_0 = \frac{2d}{w'}\sqrt{\frac{\mu}{\epsilon}} = \frac{d}{w}\sqrt{\frac{\mu}{\epsilon}} \longrightarrow w' = 2w.$

d) $u_p = \frac{1}{\sqrt{\mu\epsilon}} \longrightarrow$ $u_{pa} = u_p/\sqrt{2}$ for case a.
$u_{pb} = u_p/\sqrt{2}$ for case b.
$u_{pc} = u_p$ for case c.

P.9-7 $\gamma = j\omega\sqrt{LC}\left(1-j\frac{R}{\omega L}\right)^{1/2}\left(1-j\frac{G}{\omega C}\right)^{1/2}$

$$\cong j\omega\sqrt{LC}\left[1-j\frac{R}{2\omega L}+\frac{1}{8}\left(\frac{R}{\omega L}\right)^2+j\frac{R^3}{16\omega^3L^3}\right]$$

$$\times\left[1-j\frac{G}{2\omega C}+\frac{1}{8}\left(\frac{G}{\omega C}\right)^2+j\frac{G^3}{16\omega^3C^3}\right]=\alpha+j\beta$$

$$\longrightarrow \quad \alpha=\frac{\sqrt{LC}}{2}\left(\frac{R}{L}+\frac{G}{C}\right)\left[1-\frac{1}{8\omega^2}\left(\frac{R}{L}-\frac{G}{C}\right)^2\right]$$

$$\beta=\omega\sqrt{LC}\left[1+\frac{1}{8\omega^2}\left(\frac{R}{L}-\frac{G}{C}\right)^2\right].$$

$$Z_0=\sqrt{\frac{L}{C}}\left(1-j\frac{R}{\omega L}\right)^{1/2}\left(1-j\frac{G}{\omega C}\right)^{-1/2}$$

$$\cong\sqrt{\frac{L}{C}}\left[1-j\frac{R}{2\omega L}+\frac{1}{8}\left(\frac{R}{\omega L}\right)^2\right]\left[1-j\frac{G}{2\omega C}+\frac{1}{8}\left(\frac{G}{\omega C}\right)^2\right]=R_0+jX_0$$

$$\longrightarrow \quad R_0=\sqrt{\frac{L}{C}}\left[1+\frac{1}{8\omega^2}\left(\frac{R}{L}-\frac{G}{C}\right)\left(\frac{R}{L}+\frac{3G}{C}\right)\right]$$

$$X_0=-\frac{1}{2\omega}\sqrt{\frac{L}{C}}\left(\frac{R}{L}-\frac{G}{C}\right).$$

$$u_p=\frac{\omega}{\beta}\cong\frac{1}{\sqrt{LC}}\left[1-\frac{1}{8\omega^2}\left(\frac{R}{L}-\frac{G}{C}\right)^2\right].$$

P.9-8 $\gamma=\sqrt{(R+j\omega L)(G+j\omega C)}=\sqrt{RG}\left(1+\frac{j\omega L}{R}\right)^{1/2}\left(1-\frac{j\omega C}{G}\right)^{1/2}=\alpha+j\beta$

$$\longrightarrow \quad \alpha\cong\sqrt{RG}\left[1+\frac{1}{8}\left(\frac{\omega L}{R}-\frac{\omega C}{G}\right)^2\right],\quad \beta\cong\frac{\omega}{2}\sqrt{RG}\left(\frac{L}{R}+\frac{C}{G}\right).$$

$$Z_0=\sqrt{\frac{R+j\omega L}{G+j\omega C}}=\sqrt{\frac{R}{G}}\left(1+j\frac{\omega L}{R}\right)^{1/2}\left(1+j\frac{\omega C}{G}\right)^{-1/2}=R_0+jX_0$$

$$\longrightarrow \quad R_0\cong\sqrt{\frac{R}{G}}\left\{1+\frac{\omega^2}{8}\left[\left(\frac{L}{R}\right)^2+\frac{2LC}{RG}-3\left(\frac{C}{G}\right)^2\right]\right\}$$

$$X_0=\frac{\omega}{2}\sqrt{\frac{R}{G}}\left(\frac{L}{R}-\frac{C}{G}\right).$$

P.9-9 $Z_0=\sqrt{\frac{R+j\omega L}{G+j\omega C}}$. $\quad \mathrm{Im}(Z_0)=0 \longrightarrow \frac{R}{L}=\frac{G}{C}=k$ ①

From Eqs. (9-49a), (9-49b), and (9-51a): $\alpha=R\sqrt{\frac{C}{L}}$ ②

Given: $Z_0=50+j0\ (\Omega)$ $\quad \beta=\omega\sqrt{LC}$ ③

$\alpha=0.01\ (\mathrm{dB/m})=0.00115\ (\mathrm{Np/m})$ $\quad Z_0=\sqrt{\frac{L}{C}}$ ④

$\beta=0.8\pi\ (\mathrm{rad/m})$

$f=10^8\ (\mathrm{Hz})$

$R=\alpha Z_0=0.0576\ (\Omega/\mathrm{m})$, $\quad L=\frac{\beta Z_0}{2\pi f}=0.20\ (\mu\mathrm{H/m})$

$G=\frac{\alpha^2}{R}=23.\ (\mu\mathrm{S/m})$, $\quad C=\frac{L}{Z_0^2}=80\ (\mathrm{pF/m})$.

P.9-10 a) For two-wire transmission line:

$$Z_0=\sqrt{\frac{L}{C}}=\frac{1}{\pi}\sqrt{\frac{\mu}{\epsilon}}\cosh^{-1}\left(\frac{D}{2a}\right)=\frac{120}{\sqrt{\epsilon_r}}\ln\left[\frac{D}{2a}+\sqrt{\left(\frac{D}{2a}\right)^2-1}\right]=300.$$

$$\frac{D}{2a} = 21.27 \longrightarrow D = 25.5 \times 10^{-3} \text{ (m)}.$$

b) For coaxial transmission line:

$$Z_0 = \frac{1}{2\pi}\sqrt{\frac{\mu}{\epsilon}} \ln\left(\frac{b}{a}\right) = \frac{40}{\sqrt{\epsilon_r}} \ln\left(\frac{b}{a}\right) = 75$$

$$\frac{b}{a} = 6.52 \longrightarrow b = 3.91 \times 10^{-3} \text{ (m)}.$$

P.9-11 $(P_{av})_L = (P_{av})_i = \frac{1}{2} \mathcal{R}e[V_i I_i^*]$ $\qquad V_i = \frac{Z_i}{Z_g + Z_i} V_g$

$$= \frac{|V_g|^2 R_i}{(R_g+R_i)^2 + (X_g+X_i)^2} \qquad I_i = \frac{V_g}{Z_g+Z_i}$$

To maximize $(P_{av})_L$, set $\frac{\partial (P_{av})_L}{\partial R_i} = 0$ and $\frac{\partial (P_{av})_L}{\partial X_i} = 0$ $\Big\}$ $R_i = R_g,\ X_i = -X_g$ or $Z_i = Z_g^*$.

$$\text{Max. } (P_{av})_L = \frac{|V_g|^2}{4R_g} = (P_{av})_{Z_g}.$$

$\longrightarrow$ Max. power-transfer efficiency $= 50\%$.

P.9-12 $V(z) = V_0^+ e^{-\gamma z} + V_0^- e^{\gamma z}$

$I(z) = I_0^+ e^{-\gamma z} + I_0^- e^{\gamma z}$

At $z=0$; $V(0) = V_i = V_0^+ + V_0^-$, $\quad I(0) = I_i = I_0^+ + I_0^- = \frac{1}{Z_0}(V_0^+ - V_0^-)$

$$\longrightarrow V_0^+ = \frac{1}{2}(V_i + I_i Z_0), \quad V_0^- = \frac{1}{2}(V_i - I_i Z_0).$$

a) $V(z) = \frac{1}{2}(V_i + I_i Z_0) e^{-\gamma z} + \frac{1}{2}(V_i - I_i Z_0) e^{\gamma z}$

$I(z) = \frac{1}{2Z_0}(V_i + I_i Z_0) e^{-\gamma z} - \frac{1}{2Z_0}(V_i - I_i Z_0) e^{\gamma z}$.

b) $V(z) = V_i \cosh \gamma z - I_i Z_0 \sinh \gamma z$

$I(z) = I_i \cosh \gamma z - \frac{V_i}{Z_0} \sinh \gamma z$.

P.9-13 a) $-\frac{dV}{dz} = RI, \quad -\frac{dI}{dz} = GV$

$$\begin{cases} \frac{d^2V}{dz^2} = RGV \\ \frac{d^2I}{dz^2} = RGI \end{cases}$$

b) $V(z) = V_0^+ e^{-\alpha z} + V_0^- e^{\alpha z}$

$I(z) = I_0^+ e^{-\alpha z} + I_0^- e^{\alpha z}, \quad \alpha = \sqrt{RG}$

$$\frac{V_0^+}{I_0^+} = -\frac{V_0^-}{I_0^-} = R_0 = \sqrt{\frac{R}{G}}.$$

We have $V(z) = \frac{1}{2}(V_i + I_i R_0)e^{-\alpha z} + \frac{1}{2}(V_i - I_i R_0)e^{\alpha z}$

$$I(z) = \frac{1}{2}\left(\frac{V_i}{R_0} + I_i\right)e^{-\alpha z} - \frac{1}{2}\left(\frac{V_i}{R_0} - I_i\right)e^{\alpha z},$$

where $V_i = \frac{R_i}{R_g + R_i} V_g$ and $I_i = \frac{V_g}{R_g + R_i}$.

c) For an infinite line, $R_i = R_0$:

$$V(z) = \frac{R_0}{R_g + R_0} V_g e^{-\alpha z}, \qquad I(z) = \frac{V_g}{R_g + R_0} e^{-\alpha z}.$$

d) For a finite line of length ℓ terminated in R_L:

$$R_i = R_0 \frac{R_L + R_0 \tanh \alpha\ell}{R_0 + R_L \tanh \alpha\ell}.$$

P.9-14 Distortionless line: $R_0 = \sqrt{\frac{L}{C}} = 50\ (\Omega)$, $R = 0.5\ (\Omega/m)$

$$\tan\left(\frac{\sigma_d}{\omega\epsilon}\right) = \tan\left(\frac{G}{\omega C}\right) = 0.0018$$

$\longrightarrow \frac{G}{\omega C} = 1.79999 \times 10^{-3}$ (many digits necessary to obtain accurate answer for phase shift in part b)

$$\frac{G}{C} = 8000\pi(1.79999 \times 10^{-3}) = 45.24 = \frac{R}{L}$$

$$L = \frac{R}{G/C} = 0.1105\ (H/m), \quad C = \frac{L}{R_0^2} = 4.421\ (\mu F/m)$$

$$\alpha = \frac{R}{R_0} = 0.01\ (Np/m), \quad \beta = \omega\sqrt{LC} = 5.5555643\ (rad/m)$$

a) $V(z) = \frac{V_{g0} R_0}{R_0 + Z_g} e^{-\alpha z} e^{-j\beta z} = \frac{50}{9 + j3} e^{-0.01z} e^{-j\beta z}$

$I(z) = V(z)/50$

$\therefore\ V(z,t) = 5.27\, e^{-0.01z} \sin(8000\pi t - 5.555643z - 0.1024\pi)$ (V)

$I(z,t) = 0.105\, e^{-0.01z} \sin(8000\pi t - 5.555643z - 0.1024\pi)$ (A)

b) At $z = 5 \times 10^4$ (m), we obtain $V(5\times10^4, t)$ and $I(5\times10^4, t)$.

c) $(P_{av})_L = \frac{1}{2} \mathcal{R}e\left[V_L I_L^*\right] \longrightarrow$ Very very small.

{Note: The given line length 50 (km) in the problem is a misprint. It should have been 50 (m), which would make the numbers more meaningful.}

P.9-15 a) From Eq. (9-97): $Z_{is} = Z_0 \tanh \gamma \ell \cong \gamma \ell$.

From Eqs. (9-37) and (9-41): $\gamma = \sqrt{(R+j\omega L)(G+j\omega C)}$

$$Z_0 = \sqrt{\frac{R+j\omega L}{G+j\omega C}}$$

$$\therefore \quad Z_{is} = (R+j\omega L)\ell.$$

b) From Eq. (9-96): $Z_{io} = Z_0 \coth \gamma \ell \cong \dfrac{Z_0}{\gamma \ell} = \dfrac{G - j\omega C}{[G^2 + (\omega C)^2]}$.

P.9-16 $\beta \ell = \dfrac{2\pi f}{c}\ell = \dfrac{8\pi}{3} = 480°$.

$\tan \beta \ell = \tan 480° = -1.732$.

$$Z_i = Z_0 \frac{Z_L + jZ_0 \tan \beta \ell}{Z_0 + jZ_L \tan \beta \ell} = 50 \frac{(40+j30)+j50(-1.732)}{50+j(40+j30)(-1.732)}$$

$$= 26.3 - j9.87 \quad (\Omega).$$

P.9-17 Given: $Z_{io} = Z_0 \coth \gamma \ell = 250\angle{-50°} \quad (\Omega)$

$Z_{is} = Z_0 \tan \gamma \ell = 360\angle{20°} \quad (\Omega)$.

a) $Z_0 = \sqrt{Z_{io} Z_{is}} = 300\angle{-15°} = 289.8 - j77.6 \quad (\Omega)$.

$\tanh \gamma \ell = \sqrt{\dfrac{Z_{is}}{Z_{io}}} = 1.2\angle{35°} = 0.983 + j.688 = \tanh(\alpha \ell + j\beta \ell)$

$\ell = 4 \text{ (m)} \longrightarrow \quad \alpha = 0.139 \quad (Np/m)$

$\beta = 0.235 + \dfrac{n\pi}{4} \quad (rad/m)$.

($n = 0$ in order to ensure $+R, L, G$ & C)

b) $Z_0 = \sqrt{\dfrac{R+j\omega L}{G+j\omega C}}, \quad \gamma = \sqrt{(R+j\omega L)(G+j\omega C)}$

$\longrightarrow \quad R + j\omega L = \gamma Z_0; \quad G + j\omega C = \dfrac{\gamma}{Z_0}$.

$\omega = \beta c = 0.235 \times 3 \times 10^8 = 0.705 \times 10^8 \quad (rad/m)$.

We obtain $\quad R = 58.6 \, (\Omega), \qquad L = 0.812 \quad (\mu H/m)$

$G = 0.246 \, (mS/m), \quad C = 12.4 \quad (pF/m)$.

P.9-18 a) For a lossless quarter-wave line section:

$$Z_i = \frac{R_0^2}{Z_L} = \frac{R_0^2}{R_L + jX_L} = \frac{R_0^2 R_L}{R_L^2 + X_L^2} - j\frac{R_0^2 X_L}{R_L^2 + X_L^2} = R_i' + jX_i'$$

$$\longrightarrow R_i' = \frac{R_0^2 R_L}{R_L^2 + X_L^2} \quad ① ; \quad X_i' = -\frac{R_0^2 X_L}{R_L^2 + X_L^2} \quad ②$$

(Resistance R_i' and capacitive reactance X_i' in series.)

Input impedance Z_i can also be expressed in terms of a resistance R_i and a capacitive reactance X_i in parallel:

$$Z_i = \frac{jX_iR_i}{R_i+jX_i} = \frac{R_iX_i^2}{R_i^2+X_i^2} + j\frac{R_i^2X_i}{R_i^2+X_i^2} = R_i' + jX_i'. \quad (3)$$

Combining Eqs. (1), (2), and (3), we find

$$R_i = \frac{R_0^2}{R_L} \quad \text{and} \quad X_i = -\frac{R_0^2}{X_L},$$

both of which are reminiscent of Eq. (9-94).

b) From Eq. (9-80a), $V(z') = I_L(Z_L\cos\beta z' + R_0\sin\beta z')$.

At the input, $z' = \lambda/4$, $\beta z' = \pi/2$, we have

$$V_i = V(\lambda/4) = I_L R_0.$$

At the load, $z'=0$, $\beta z'=0$, and $V_L = V(0) = I_L Z_L$.

$$\therefore \frac{|V_i|}{|V_L|} = \frac{R_0}{|Z_L|} = \frac{R_0}{\sqrt{R_L^2+X_L^2}}.$$

P.9-19 a) $$|\Gamma| = \frac{S-1}{S+1} = \frac{\left|\frac{Z_L}{Z_0}-1\right|}{\left|\frac{Z_L}{Z_0}+1\right|} = \frac{\sqrt{(r_L-1)^2+x_L^2}}{\sqrt{(r_L+1)^2+x_L^2}},$$

where $r_L = R_L/Z_0$ and $x_L = X_L/Z_0$.

$$\longrightarrow x_L = \pm\left[\frac{\left(\frac{S-1}{S+1}\right)^2(r_L+1)^2-(r_L-1)^2}{1-\left(\frac{S-1}{S+1}\right)^2}\right]^{1/2}.$$

When $S=3$, $x_L = \pm\sqrt{(10r_L-3r_L^2-3)/3}$.

b) $S=3$ and $r_L = 150/75 = 2 \longrightarrow x_L = \pm\sqrt{5/3}$.

$$X_L = x_L Z_0 = \pm 96.8\ (\Omega).$$

c) From Eq. (9-114): $r_L + jx_L = \dfrac{r_m+jt}{1+jr_mt}$, where $r_m = \dfrac{R_m}{Z_0}$ and $t = \tan\beta\ell_m$.

$$\longrightarrow r_m = \frac{(1+r_L^2+x_L^2)\pm\sqrt{(1+r_L^2+x_L^2)^2-4r_L^2}}{2r_L}$$

$$= 3 \text{ or } \frac{1}{3}, \text{ for } r_L=2 \text{ and } x_L^2=5/3.$$

Also, $x_L = \dfrac{(1-r_m^2)t}{1+r_m^2t^2} \longrightarrow t = \dfrac{1}{2x_Lr_m^2}\left[(1-r_m^2)\pm\sqrt{(1-r_m^2)^2-4x_L^2r_m^2}\right]$

For $r=\frac{1}{3}$, $t = \begin{cases} 3\sqrt{3/5} \longrightarrow \ell_m = 0.1865\lambda \\ \text{or } \sqrt{15} \longrightarrow \ell_m = 0.2098\lambda \end{cases}$

Use $\ell_m = 0.2098\lambda$ to obtain V_{min} nearest to the load at $(0.5-0.2098)\lambda = 0.2902\lambda$.

P.9-20 a) $|\Gamma|^2 = \left|\frac{(R_L - Z_0) + jX_L}{(R_L + Z_0) + jX_L}\right|^2 = \frac{(R_L - Z_0)^2 + X_L^2}{(R_L + Z_0)^2 + X_L^2}$

$$\frac{\partial|\Gamma|^2}{\partial Z_0} = 0 \longrightarrow Z_0 = \sqrt{R_L^2 + X_L^2}.$$

If $Z_L = 40 + j30\ (\Omega)$, $Z_0 = 50\ (\Omega)$.

b) $\text{Min. } |\Gamma| = \sqrt{\frac{Z_0 - R_L}{Z_0 + R_L}} = \sqrt{\frac{50-40}{50+40}} = \frac{1}{3}.$

$\text{Min. } S = \frac{1+\frac{1}{3}}{1-\frac{1}{3}} = 2.$

c) From Eq. (9-114): $r_i + jx_i = \frac{r_m + jt}{1 + jr_m t} = 0.8 + j0.6.$

$\longrightarrow t = \frac{1}{2x_i r_m^2}\left[(1 - r_m^2) \pm \sqrt{(1-r_m^2)^2 - 4x_i^2 r_m^2}\right]$ (See problem P.9-19.)

At voltage minimum, $r_m = \frac{1}{S} = \frac{1}{2}.$

$t = 1$ (Use negative sign.)

$\tan\beta\ell_m = \tan(2\pi\ell_m/\lambda) = 1 \longrightarrow \ell_m = \frac{\lambda}{8}.$

$\therefore$ Voltage minimum nearest to the load is $\left(\frac{\lambda}{2} - \frac{\lambda}{8}\right)$ or $3\lambda/8$ from the load.

P.9-21 a) From Eqs. (9-100a) and (9-101):

$$V(z') = \frac{I_L}{2}(Z_L + Z_0)e^{\gamma z'}\left[1 + |\Gamma| e^{-2\alpha z'} e^{j\phi}\right].$$

where

$$\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0} = |\Gamma| e^{j\theta_\Gamma}, \qquad \phi = \theta_\Gamma - 2\beta z'.$$

$\text{Max. } |V(z')| = \left|\frac{I_L}{2}(Z_L + Z_0)e^{\alpha z'}\right|\left[1 + |\Gamma| e^{-2\alpha z'}\right]$ for $\phi = 0$.

$\text{min. } |V(z')| = \left|\frac{I_L}{2}(Z_L + Z_0)e^{\alpha z'}\right|\left[1 - |\Gamma| e^{-2\alpha z'}\right]$ for $\phi = \pi$.

$S(z') = \frac{\text{Max. } |V(z')|}{\text{min. } |V(z')|} = \frac{1 + |\Gamma| e^{-2\alpha z'}}{1 - |\Gamma| e^{-2\alpha z'}}.$ (There is a slight ambiguity in z' here, which is insignificant when α is small.)

b) From Eq. (9-132): $Z_i(z') = \frac{1 + |\Gamma| e^{-2\alpha z'} e^{j\phi}}{1 - |\Gamma| e^{-2\alpha z'} e^{j\phi}} Z_0$

At a voltage max., $\phi = 0$, $Z_i(z') = S(z')Z_0$.

c) At a voltage min., $\phi = \pi$, $Z_i(z') = \frac{Z_0}{S(z')}.$

P.9-22 $Z_i = R_0' \dfrac{Z_L + jR_0't}{R_0' + jZ_Lt} \longrightarrow Z_L = R_0' \dfrac{Z_i - jR_0't}{R_0' - jZ_it}$.

With $Z_i = 50\ (\Omega)$ and $Z_L = 40 + j10\ (\Omega)$, we have

$$40 + j10 = R_0' \frac{50 - jR_0't}{R_0' - j50t} \longrightarrow \begin{cases} 40R_0' + 500t = 50R_0' \\ 10R_0' - 2000t = -R_0'^2 t \end{cases}$$

$\therefore\ R_0' = 38.73\ (\Omega)$.

$t = \tan\beta\ell' = 0.7746 \longrightarrow \ell' = 0.1049\lambda$.

P.9-23 a) $|\Gamma| = \dfrac{S-1}{S+1} = \dfrac{2-1}{2+1} = \dfrac{1}{3}$.

From Eqs.(9-100a) and (9-101):

$V(z') = \dfrac{I_L}{2}(Z_L + Z_0)e^{j\beta z'}[1 + |\Gamma|e^{j\phi}]$;

$\Gamma = \dfrac{Z_L - Z_0}{Z_L + Z_0} = |\Gamma|e^{j\theta_\Gamma}$, $\quad \phi = \theta_\Gamma - 2\beta z'$.

$V(z')$ is a minimum when $\phi = \pm\pi \longrightarrow \theta_\Gamma = 2\left(\dfrac{2\pi}{\lambda}\right)\times 0.3\lambda - \pi$
$= 0.2\pi$.

$\therefore\ \Gamma = \dfrac{1}{3}e^{j0.2\pi}$.

b) $Z_L = Z_0\left(\dfrac{1+\Gamma}{1-\Gamma}\right) = 466 + j206\ (\Omega)$.

c) Terminating resistance $R_m = \dfrac{R_0}{S} = \dfrac{300}{2} = 150\ (\Omega)$,

$\ell_m = \dfrac{\lambda}{2} - z_m' = (0.5 - 0.3)\lambda = 0.2\lambda$.

P.9-24 Eq.(9-114): $R_i + jX_i = R_0 \dfrac{R_m + jR_0\tan\beta\ell_m}{R_0 + jR_m\tan\beta\ell_m}$

Let $r_i = \dfrac{R_i}{R_0}$, $x_i = \dfrac{X_i}{R_0}$, $r_m = \dfrac{R_m}{R_0}$, and $t = \tan\beta\ell_m$.

$$r_i + jx_i = \frac{r_m + jt}{1 + jr_mt} \longrightarrow \begin{cases} r_m(1 + x_it) = r_i \\ t(1 - r_mr_i) = x_i \end{cases}$$

We have

$$r_m = \frac{1}{2r_i}\left[(1 + r_i^2 + x_i^2) \pm \sqrt{(1 + r_i^2 + x_i^2)^2 - 4r_i^2}\right],$$

$$t = \frac{1}{2x_i}\left\{-[1 - (r_i^2 + x_i^2)] \pm \sqrt{[1 - (r_i^2 + x_i^2)]^2 + 4x_i^2}\right\},$$

$$\ell_m = \frac{\lambda}{2\pi}\tan^{-1}t.$$

P.9-25 $Z_L = Z_0 \frac{1+\Gamma}{1-\Gamma}$

$\Gamma = |\Gamma| e^{j\theta_\Gamma}$, $|\Gamma| = \frac{S-1}{S+1}$, $\theta_\Gamma = \frac{4\pi}{\lambda} z'_m \pm \pi$.

$$\therefore Z_L = Z_0 \frac{(S+1)-(S-1)e^{j(4\pi z'_m/\lambda)}}{(S+1)+(S-1)e^{j(4\pi z'_m/\lambda)}}$$

$$= Z_0 \frac{(S+1)e^{-j(2\pi z'_m/\lambda)}-(S-1)e^{j(2\pi z'_m/\lambda)}}{(S+1)e^{-j(2\pi z'_m/\lambda)}+(S-1)e^{j(2\pi z'_m/\lambda)}}$$

$$= Z_0 \frac{1-jS\tan(2\pi z'_m/\lambda)}{S-j\tan(2\pi z'_m/\lambda)}.$$

P.9-26 a) Given: $V_g = 0.1\angle 0^\circ$ (V), $Z_g = Z_0 = 50\ (\Omega)$, $R_L = 25\ (\Omega) = 0.5 Z_0$

$$V_i = \frac{Z_i}{Z_0+Z_i} V_g, \quad I_i = \frac{V_g}{Z_0+Z_i},$$

where $Z_i = Z_0 \frac{0.5Z_0 + jZ_0\tan\beta\ell}{Z_0 + j0.5Z_0\tan\beta\ell} = Z_0 \frac{1+j2\tan\beta\ell}{2+j\tan\beta\ell}$.

$$\therefore V_i = \frac{1+j2\tan\beta\ell}{3(1+j\tan\beta\ell)} V_g = \frac{1}{30}\left(\frac{1+j2\tan\beta\ell}{1+j\tan\beta\ell}\right) \quad (V)$$

$$I_i = \frac{2+j\tan\beta\ell}{3Z_0(1+j\tan\beta\ell)} V_g = \frac{2}{3}\left(\frac{2+j\tan\beta\ell}{1+j\tan\beta\ell}\right) \quad (mA).$$

Setting $Z_g = Z_0$ and $\Gamma_g = 0$ in Eqs. (9-120a) and (9-120b), we have $V_L = V(z'=0) = \frac{V_g Z_0}{Z_0+Z_g} e^{-j\beta\ell}(1+\Gamma)$; $\Gamma = \frac{R_L - Z_0}{R_L+Z_0} = -\frac{1}{3}$

$$= \frac{1}{30} e^{-j\beta\ell} \quad (V)$$

$$I_L = I(z'=0) = \frac{V_g}{Z_0+Z_g} e^{-j\beta\ell}(1-\Gamma) = \frac{4}{3} e^{-j\beta\ell} \quad (mA).$$

b) $S = \frac{1+|\Gamma|}{1-|\Gamma|} = 2$.

c) $(P_{av})_L = \frac{1}{2}\mathcal{Re}(V_L I_L^*) = \frac{1}{2}\left(\frac{1}{30}\right)\left(\frac{4}{3}\times 10^{-3}\right) = 2.22\times 10^{-5}$ (W) $= 0.0222$ (mW).

If $R_L = 50\ (\Omega) = Z_0$, $V_L = \frac{V_g}{2} e^{-j\beta\ell}$, $I_L = \frac{V_g}{2Z_0} e^{-j\beta\ell}$

$$\longrightarrow \text{Max.}(P_{av})_L = \frac{V_g^2}{8Z_0} = 2.50\times 10^{-5} \ (W).$$

P.9-27 $\Gamma_g = 0$, $\Gamma = \frac{Z_L - R_0}{Z_L+R_0} = \frac{j-1}{j+1} = j$, $z = \ell - z'$, $\beta = \omega\sqrt{LC}$

$\ell = \lambda/4$, $e^{-j\beta z} = e^{-j\beta(\lambda/4 - z')} = e^{-j(\pi/2 - \beta z')} = -je^{j\beta z'}$

From Eqs. (9-120a) and (9-120b):

a) $V(z') = -j\frac{V_g}{2} e^{j\beta z'}(1+je^{-j2\beta z'}) = 55(e^{-j\beta z'} - je^{j\beta z'})$ (V)

$$I(z') = -j\frac{V_0}{2Z_0}e^{j\beta z'}(1-je^{-j2\beta z'}) = -1.1\,(e^{-j\beta z'}+je^{j\beta z'}) \quad (A).$$

b) $v(z',t) = \mathcal{I}m\,[V(z')e^{j\omega t}] = 55\,[\sin(\omega t-\beta z')-\cos(\omega t+\beta z')] \ (V)$

$i\,(z',t) = -1.1\,[\sin(\omega t-\beta z')+\cos(\omega t+\beta z')] \ (A).$

c) At the load, $z'=0$,

$$p_L(t) = v(0,t)\,i(0,t)$$
$$= 60.5\,(\cos^2\omega t-\sin^2\omega t) = 60.5\,\cos(2\omega t) \quad (W).$$

$$V_L = \frac{V_0}{2}(1-j), \qquad I_L = -\frac{V_0}{2Z_0}(1+j).$$

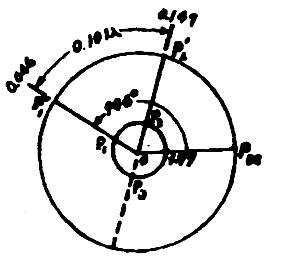
$$(P_{av})_L = \frac{1}{2}\mathcal{R}e\,(V_L I_L^*) = \frac{V_0^2}{4Z_0}\mathcal{R}e(2j) = 0.$$

P.9-28 $\quad f = 2\times10^8\ (Hz), \quad \lambda = \frac{c}{f} = 1.5\ (m)$

a) Open-circuited line, $\ell = 1\ (m)$, $\ell/\lambda = 0.667$.
Smith chart: Start from P_{oc} on the extreme right, rotate clockwise one complete revolution ($\Delta z' = \lambda/2$) and continue on for an additional 0.167λ to 0.417λ on the "wavelength toward generator" scale. Read
$x = -j0.575 \longrightarrow Z_i = 75\times(-j0.575) = -j43.1 \ (\Omega)$.
Draw a straight line from the $(0-j0.575)$ point through the center and intersect at $(0+j1.74)$ on the opposite side of the chart. $\longrightarrow Y_i = \frac{1}{75}\times(j1.74) = j0.0232 \ (S)$.

b) Short-circuited line, $\ell = 0.8\ (m)$, $\ell/\lambda = 0.533$.
Start from the extreme left point P_{sc}, rotate clockwise one complete revolution and continue on for an additional 0.033λ to read $x = j0.21 \longrightarrow Z_i = 75\times j0.21 = j15.8\ (\Omega)$.
Draw a straight line from the $(0+j0.21)$ point through the center and intersect at $(0-j4.75)$ on the opposite side of the chart. $\longrightarrow Y_i = \frac{1}{75}\times(-j4.75) = -j0.063\ (S)$.

P.9-29



$$z_L = \frac{1}{50}(30+j10) = 0.6+j0.2$$

a) 1. Locate $z_L = 0.6+j0.2$ on Smith chart (Point P_1).
2. With center at O draw a $|\Gamma|$-circle through P_1, intersecting OP_{oc} at 1.77. $\longrightarrow S = 1.77$.

b) $\Gamma = \frac{1.77-1}{1.77+1} e^{j146°} = 0.28 e^{j146°}$.

c) 1. Draw line OP_1, intersecting the periphery at P_1'. Read 0.046 on "wavelengths toward generator" scale.

2. Move clockwise by 0.101λ to 0.147 (Point P_2').

3. Join O and P_2', intersecting the $|\Gamma|$-circle at P_2.

4. Read $z_i = 1 + j0.59$ at P_2.

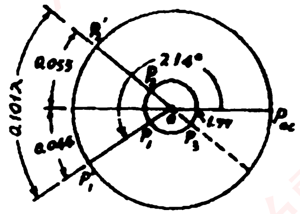
$$Z_i = 50 z_i = 50 + j29.5 \; (\Omega).$$

d) Extend line $P_2'P_2O$ to P_3. Read $y_i = 0.75 - j0.43$.

$$Y_i = \frac{1}{50} y_i = 0.015 - j0.009 \; (S).$$

e) There is no voltage minimum on the line, but $V_L < V_i$.

P.9-30



$z_L = \frac{1}{50}(30 - j10) = 0.6 - j0.2$

a) Locate $z_L = 0.6 - j0.2$ on Smith chart (Point P_1). With center at O draw a $|\Gamma|$-circle through P_1, intersecting line OP_{oc} at 1.77. $\longrightarrow S = 1.77$.

b) $\Gamma = 0.28 e^{j214°}$

c) 1. Draw line OP_1, intersecting the periphery at P_1'. Read 0.454 on "wavelengths toward generator" scale.

2. Move clockwise by 0.101λ to 0.055 (Point P_2').

3. Join O and P_2', intersecting the $|\Gamma|$-circle at P_2.

4. Read $z_i = 0.61 + j0.23$ at P_2.

$$Z_i = 50 z_i = 30.5 + j11.5 \; (\Omega).$$

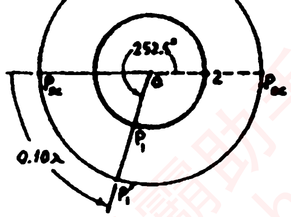
d) Extend line $P_2'P_2O$ to P_3. Read $y_i = 1.42 - j0.54$.

$$Y_i = \frac{1}{50} y_i = 0.0284 - j0.0108 \; (S).$$

e) There is a voltage minimum at $z_m' = 0.046\lambda$.

P.9-31 $\lambda/2 = 25$, $\lambda = 50$ (cm)

First voltage minimum occurs at $z_m' = \frac{5}{50} = 0.1\lambda$.



a) 1. Start from P_{sc} and rotate counterclockwise 0.10λ toward the load to P_1'.

2. Draw the $|\Gamma|$-circle, intersecting line OP_{oc} at 2 $(S=2)$.

3. Join OP_1', intersecting the $|\Gamma|$-circle at P_1.

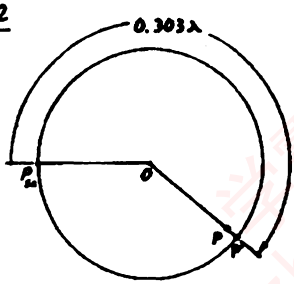
4. Read $z_L = 0.675 - j0.475$.

$$Z_L = 50 z_L = 33.75 - j23.75 \ (\Omega).$$

b) $\Gamma = \frac{2-1}{2+1} e^{j\theta_\Gamma} = \frac{1}{3} e^{j252.5°}$.

c) If $Z_L = 0$, the first voltage minimum would be at $z_m' = \lambda/2 = 25$ (cm) from the short-circuit.

P. 9-32



a) $z_i = \frac{1}{100}(40 - j280)$
$= 0.40 - j2.80$.

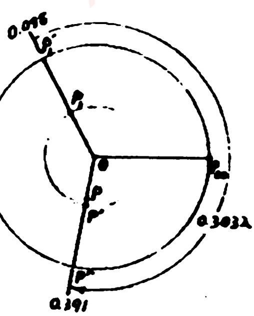
1. Enter z_i on Smith chart (Point P).

2. Join O and P, and extend to P'.

3. Read on "wavelengths toward generator" scale: 0.303.

$$\beta\ell = 0.606\pi, \quad \ell = 1.5 \text{ (m)} \longrightarrow \beta = 1.269 \text{ (rad/m)}.$$

$$\frac{\overline{OP}}{\overline{OP'}} = 0.915 \longrightarrow \alpha = \frac{1}{2\ell} \ln \frac{1}{0.915} = 0.0297 \text{ (Np/m)}.$$



b) 1. Enter $z_L = 0.5 + j0.5$ on Smith chart (Point P_1).

2. Draw line from O through P_1 to P_1'. Read on "wavelengths toward generator" scale: 0.088.

3. Move clockwise by 0.303λ to 0.391 (Point P'').

4. Join OP'', intersecting the $|\Gamma|$- circle through P_1 at P'.

5. Mark point P on line OP' such that $\frac{\overline{OP}}{\overline{OP'}} = 0.915$.

6. Read at P : $z_i = 0.625 - j0.59 \longrightarrow Z_i = 62.5 - j59.0$ (Ω)

c) 1. Move clockwise from P_{sc} on "wavelengths toward generator" scale to 0.15, say P'.

2. Join OP'.

3. Mark point P on line OP' such that
$\overline{OP} = e^{-2\alpha \ell'}\, \overline{OP}' = 0.957\, \overline{OP}'$.

4. Read at P: $z_i = 0.065 + j1.38 \longrightarrow Z_i = 6.5 + j138$ (Ω).

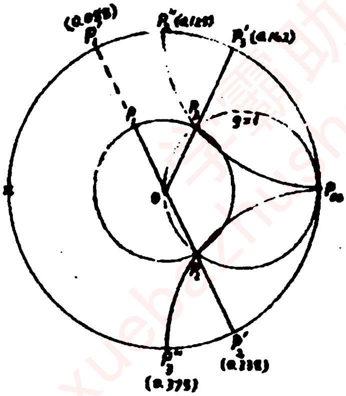
P.9-33 $f = 2\times 10^8$ (Hz), $\lambda = 1.5$ (m) $\longrightarrow \ell = \frac{\lambda}{4} = 0.375$ (m).

$Z_0 = \sqrt{73\times 100} = 148$ (Ω).

For two-wire transmission line: $Z_0 = 120 \cosh^{-1}\left(\frac{D}{2a}\right)$.

$D = 2$ (cm) $\longrightarrow a = 0.54$ (cm).

P.9-34



$z_L = 0.5 + j0.5$

$y_L = 1 - j$

a) See construction.

P_1 : $z_L = 0.5 + j0.5$

P_2 : $y_L = 1 - j1 = y_2 \longrightarrow d_2 = 0$

P_3 : $y_3 = 1 + j1$

$\longrightarrow d_3 = 0.162\lambda + (0.5 - 0.338)\lambda$
$= 0.324\lambda$.

P_2'' : $b_2 = j1 \longrightarrow \ell_2 = (0.5 + 0.125)\lambda$
$= 0.375\lambda$

P_3'' : $b_3 = -j1 \longrightarrow \ell_3 = (0.375 - 0.25)\lambda$
$= 0.125\lambda$.

b) For $Z_0' = 75 = 1.5 Z_0$, $Y_0' = 0.667 Y_0$.

The required normalized stub admittances are $b_2' = -b_3' = \frac{j}{0.667} = j1.5$

	$(Z_0)_{Stub} = (Z_0)_{Line}$	$(Z_0)_{Stub} = 1.5(Z_0)_{Line}$
$z_L = 0.5 + j0.5$	$d_2 = 0$, $\ell_2 = 0.375\lambda$	$d_2' = 0$, $\ell_2' = 0.406\lambda$
$y_L = 1 - j1$	$d_3 = 0.324\lambda$, $\ell_3 = 0.125\lambda$	$d_3' = 0.324\lambda$, $\ell_3' = 0.0936\lambda$

P. 9-35 $\quad z_L = 0.5 + j0.5$

Use Smith chart as an impedance chart. Same construction as that in problem P. 9-34 except P_{sc} would be on the extreme left (marked by a x), and $g=1$ circle becomes $r=1$ circle.

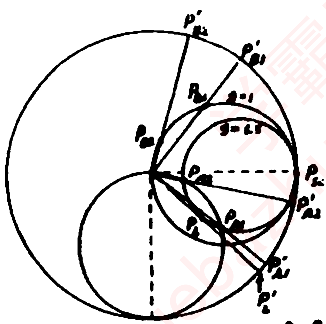
P_1: $z_L = 0.5 + j0.5$; P_2: $z_{i2} = 1 + j1$ with $d_2 = (0.162 - 0.088)\lambda = 0.074\lambda$.

P_3: $z_{i3} = 1 - j1$ with $d_3 = (0.338 - 0.088)\lambda = 0.250\lambda$.

To achieve a match with a series stub having $R_0' = \frac{35}{50}R_0$, we need a normalized stub susceptance $-j\frac{50}{35} = -j1.43$ for solution corresponding to P_2. From Smith chart we obtain the required stub length $\ell_2 = 0.347\lambda$.

Similarly for solution corresponding to P_3, a stub with a normalized susceptance $+j1.43$ is needed, which requires a stub length $\ell_3 = 0.153\lambda$.

P. 9-36



$z_L = 0.33 + j0.33$

P_L: $y_L = 1.50 - j1.50$ $(0.306\lambda$ at $P_L')$

P_{A1}: $y_{A1} = 1.50 - j1.80$ $(0.304\lambda$ at $P_{A1}')$

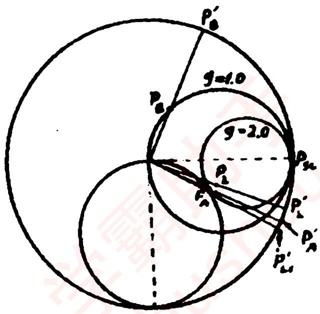
P_{A2}: $y_{A2} = 1.50 - j0.14$ $(0.269\lambda$ at $P_{A2}')$

P_{B1}: $y_{B1} = 1.00 + j1.60$ $(0.179\lambda$ at $P_{B1}')$

P_{B2}: $y_{B2} = 1.00 + j0.40$ $(0.144\lambda$ at $P_{B2}')$

	a) Short-circuited stubs	b) Open-circuited stubs
$(y_{sA})_1 = y_{A1} - y_L = -j0.30$	$\ell_{A1} = 0.203\lambda$	$\ell_{A1} = 0.453\lambda$
$(y_{sA})_2 = y_{A2} - y_L = j1.36$	$\ell_{A2} = 0.399\lambda$	$\ell_{A2} = 0.149\lambda$
$(y_{sB})_1 = -j1.60$	$\ell_{B1} = 0.089\lambda$	$\ell_{B1} = 0.339\lambda$
$(y_{sB})_2 = -j0.40$	$\ell_{B2} = 0.189\lambda$	$\ell_{B2} = 0.439\lambda$

P.9-37



$y_L = \dfrac{300}{100+j50} = 2.4 - j1.2$

Point P_L on Smith chart. (0.280λ at P_L')

Since the rotated $g=1.0$ circle is tangent to the $g=2.0$ circle, an added line length d_L is needed to convert g_L (2.4) to 2.0, moving from P_L along the $|\Gamma|$-circle to P_{L1} (not shown) on the $g=2.0$ circle (0.291λ at P_{L1}'). Note that P_{L1} is different from P_A, the point of tangency between the $g=2.0$ and rotated $g=1.0$ circles.

a) Min. $d_L = 0.291\lambda - 0.280\lambda = 0.011\lambda$.

b) P_A : $y_A = 2 - j1$ (0.287λ at P_A').

P_B : $y_B = 1 + j1$ (0.162λ at P_B').

$y_{SA} = y_A - y_{L1} = (2-j1) - (2-j1.35) = j0.35 \longrightarrow \ell_A = 0.304\lambda$

$y_{SB} = -j1 \longrightarrow \ell_B = 0.125\lambda$.

P.9-38 Let $\theta = \beta d_o = \dfrac{2\pi}{\lambda} d_o$.

Require: $g_L \le \dfrac{1}{\sin^2\theta}$ (Analytic solution†)

d_o	θ	g_L
$\lambda/16$	$22.5°$	≤ 6.83
$\lambda/8$	$45°$	≤ 2.0
$\lambda/4$	$90°$	≤ 1.0
$3\lambda/8$	$135°$	≤ 2.0
$7\lambda/16$	$157.5°$	≤ 6.83

† See D.K. Cheng and C.H. Liang, "Computer Solution of Double-Stub Impedance-Matching Problems," IEEE Transactions on Education, vol. E-25, pp. 120-123, November 1982.

Chapter 10

P.10-1

From $\bar{\nabla}\times\bar{E}=-j\omega\mu\bar{H}$

$$\frac{\partial E_z^0}{r\partial\phi}+\gamma E_\phi^0=j\omega\mu H_r^0$$
$$-\gamma E_r^0-\frac{\partial E_z^0}{\partial r}=-j\omega\mu H_\phi^0$$
$$\frac{\partial(rE_\phi^0)}{r\partial r}-\frac{\partial E_r^0}{r\partial\phi}=-j\omega\mu H_z^0$$

From $\bar{\nabla}\times\bar{H}=j\omega\epsilon\bar{E}$

$$\frac{\partial H_z^0}{r\partial\phi}+\gamma H_\phi^0=j\omega\epsilon E_r^0$$
$$-\gamma H_r^0-\frac{\partial H_z^0}{\partial r}=j\omega\epsilon E_\phi^0$$
$$\frac{\partial(rH_\phi^0)}{r\partial r}-\frac{\partial H_r^0}{r\partial\phi}=j\omega\epsilon E_z^0$$

Combining:

$$E_r^0=-\frac{1}{h^2}\left(\gamma\frac{\partial E_z^0}{\partial r}+j\omega\mu\frac{\partial H_z^0}{r\partial\phi}\right)$$
$$E_\phi^0=-\frac{1}{h^2}\left(\gamma\frac{\partial E_z^0}{r\partial\phi}-j\omega\mu\frac{\partial H_z^0}{\partial r}\right)$$
$$H_r^0=-\frac{1}{h^2}\left(\gamma\frac{\partial H_z^0}{\partial r}-j\omega\epsilon\frac{\partial E_z^0}{r\partial\phi}\right)$$
$$H_\phi^0=-\frac{1}{h^2}\left(\gamma\frac{\partial H_z^0}{r\partial\phi}+j\omega\epsilon\frac{\partial E_z^0}{\partial r}\right),\quad h^2=\gamma^2+\omega^2\mu\epsilon.$$

$E_z=E_z^0e^{-\gamma z}$, where E_z^0 satisfies the following homogeneous Helmholtz's equation: $\nabla_{r\phi}^2E_z^0+h^2E_z^0=0$, or

$$\frac{\partial^2E_z^0}{\partial r^2}+\frac{1}{r}\frac{\partial E_z^0}{\partial r}+\frac{1}{r^2}\frac{\partial^2E_z^0}{\partial\phi^2}+h^2E_z^0=0.$$

Similarly for $H_z=H_z^0e^{-\gamma z}$.

P.10-2

$$\bar{\nabla}\times\bar{E}=-j\omega\mu\bar{H}\qquad (1)$$
$$\bar{\nabla}\times\bar{H}=j\omega\epsilon\bar{E}\qquad (2)$$

From (1), $(\bar{\nabla}_T+\bar{a}_z\frac{\partial}{\partial z})\times(\bar{E}_T+\bar{a}_zE_z)=-j\omega\mu(\bar{H}_T+\bar{a}_zH_z)$

$\longrightarrow$ $\bar{\nabla}_T\times(\bar{a}_zE_z)+\gamma\bar{E}_T\times\bar{a}_z=-j\omega\mu\bar{H}_T$

$\longrightarrow$ $\bar{\nabla}_TE_z\times\bar{a}_z+\gamma\bar{E}_T\times\bar{a}_z=-j\omega\mu\bar{H}_T$ (3)

$(\because\ \bar{\nabla}_T\times(\bar{a}_zE_z)=\bar{\nabla}_TE_z\times\bar{a}_z)$

Similarly from (2) we obtain

$$\bar{\nabla}_TH_z\times\bar{a}_z+\gamma\bar{H}_T\times\bar{a}_z=j\omega\epsilon\bar{E}_T\qquad (4)$$

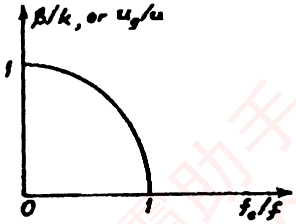
Combining (3) and (4), we have

$$\omega^2\mu\epsilon\bar{E}_T=\gamma[\bar{\nabla}_TE_z\times\bar{a}_z+\gamma\bar{E}_T\times\bar{a}_z]\times\bar{a}_z-j\omega\mu\bar{\nabla}_TH_z\times\bar{a}_z$$

$\longrightarrow$ $\bar{E}_T=-\frac{1}{h^2}(\gamma\bar{\nabla}_TE_z-\bar{a}_zj\omega\mu\times\bar{\nabla}_TH_z)$, $h^2=\gamma^2+\omega^2\mu\epsilon$

Similarly, $\bar{H}_T=-\frac{1}{h^2}(\gamma\bar{\nabla}_TH_z+\bar{a}_zj\omega\epsilon\times\bar{\nabla}_TE_z)$.

P.10-3 (a)



From Eq. (10-33):

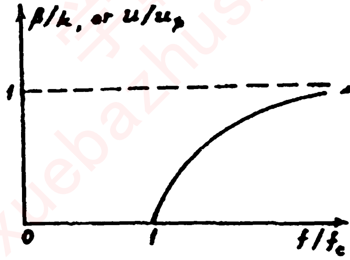
$$\left(\frac{\beta}{k}\right)^2 + \left(\frac{f_c}{f}\right)^2 = 1$$

From Eq. (10-37):

$$\left(\frac{u_g}{u}\right)^2 + \left(\frac{f_c}{f}\right)^2 = 1.$$

Both are equations of a unit circle.

b)

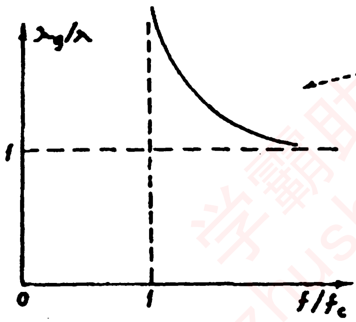


From Eq. (10-33):

$$\left(\frac{\beta}{k}\right)^2 = 1 - \frac{1}{(f/f_c)^2}$$

From Eq. (10-36):

$$\left(\frac{u}{u_p}\right)^2 = 1 - \frac{1}{(f/f_c)^2}.$$



From Eq. (10-34):

$$\left(\frac{\lambda_g}{\lambda}\right)^2 = \frac{(f/f_c)^2}{(f/f_c)^2 - 1}$$

c) At $f/f_c = 1.25$:

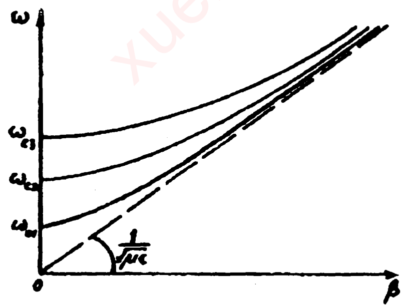
$u_p/u = 1.67$

$u_g/u = 0.60$

$\beta/k = 0.60$

$\lambda_g/\lambda = 1.67.$

P.10-4 a) For parallel-plate waveguides:



$$\omega^2\mu\epsilon - \beta^2 = \left(\frac{n\pi}{b}\right)^2.$$

$$\omega_{c1} = \frac{\pi}{b\sqrt{\mu\epsilon}}$$

$$\omega_{c2} = \frac{2\pi}{b\sqrt{\mu\epsilon}}$$

$$\omega_{c3} = \frac{3\pi}{b\sqrt{\mu\epsilon}}$$

a) Constitutive parameters ϵ and μ affect both ω_c and the slope of the ω-β curves; b affects ω_c but not the slope at high frequencies. b) Yes.

P.10-5 Field expressions for TM_n modes, from Eqs. (10-54a, b, & c):

$$E_z^0(y) = A_n \sin(n\pi y/b)$$

$$H_x^0(y) = \frac{j\omega\epsilon}{h} A_n \cos(n\pi y/b)$$

$$E_y^0(y) = -\frac{\gamma}{h} A_n \cos(n\pi y/b).$$

Surface charge densities:

$$\rho_{s\ell} = \bar{a}_n \cdot \bar{D}\Big|_{y=0} = \epsilon E_y^0(0) = -\frac{\gamma\epsilon}{h} A_n$$

$$\rho_{su} = \bar{a}_n \cdot \bar{D}\Big|_{y=b} = -\epsilon E^0(b) = (-1)^n \frac{\gamma\epsilon}{h} A_n$$

Surface current densities:

$$\bar{J}_{s\ell} = \bar{a}_n \times \bar{H}\Big|_{y=0} = \bar{a}_y \times \bar{H}(0) = -\bar{a}_z \frac{j\omega\epsilon}{h} A_n$$

$$\bar{J}_{su} = \bar{a}_n \times \bar{H}\Big|_{y=b} = -\bar{a}_y \times \bar{H}(b) = \bar{a}_z (-1)^n \frac{j\omega\epsilon}{h} A_n = \begin{cases} \bar{J}_{s\ell} & \text{for } n \text{ odd} \\ -\bar{J}_{s\ell} & \text{for } n \text{ even.} \end{cases}$$

P.10-6 Field expressions for TE_n modes, from Eqs. (10-68a, b, & c):

$$H_z^0(y) = B_n \cos(n\pi y/b)$$

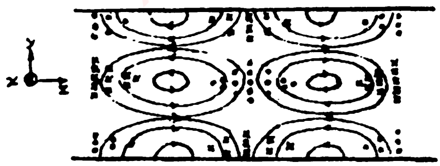
$$H_y^0(y) = \frac{\gamma}{h} B_n \sin(n\pi y/b)$$

$$E_x^0(y) = \frac{j\omega\mu}{h} B_n \sin(n\pi y/b)$$

$$\bar{J}_{s\ell} = \bar{a}_y \times \bar{H}(0) = \bar{a}_x B_n$$

$$\bar{J}_{su} = -\bar{a}_y \times \bar{H}(b) = \bar{a}_x (-1)^{n+1} B_n = \begin{cases} \bar{J}_{s\ell} & \text{for } n \text{ odd} \\ -\bar{J}_{s\ell} & \text{for } n \text{ even.} \end{cases}$$

P.10-7 a) Set $n=2$ in the field expressions in problem P.10-5.

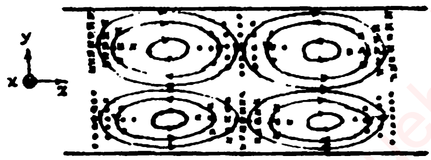


—— Electric field lines

× • Magnectic field lines

Equation for electric field lines: $\cos\beta z = \frac{\cos(2\pi y_0/b)}{\cos(2\pi y/b)}$.

b) Set $n=2$ in the field expressions in problem P.10-6.



× • Electric field lines

—— Magnetic field lines

Equation for magnetic field lines: $\cos\beta z = \frac{\sin(2\pi y_0/b)}{\sin(2\pi y/b)}$.

P.10-8 Given: $\sigma_c = 5.80\times10^7$ (S/m), $\epsilon_r = 2.25$, $\mu_r = 1$
$\sigma = 10^{-10}$ (S/m), $b = 5\times10^{-2}$ (m), $f = 10^{10}$ (Hz)

a) TEM mode

$$\beta = \omega\sqrt{\mu\epsilon} = 314.2 \text{ (rad/m)}$$
$$\alpha_d = \frac{\sigma}{2}\sqrt{\frac{\mu}{\epsilon}} = 1.257\times10^{-8} \text{ (Np/m)}$$
$$\alpha_c = \frac{1}{b}\sqrt{\frac{\pi f\epsilon}{\sigma_c}} = 2.076\times10^{-3} \text{ (Np/m)}$$
$$u_p = u_g = u = \frac{1}{\sqrt{\mu\epsilon}} = 2\times10^8 \text{ (m/s)}$$
$$\lambda_g = \lambda = \frac{u}{f} = 2\times10^{-2} \text{ (m)}.$$

b) TM$_1$ mode — $(f_c)_{TM_1} = \dfrac{1}{2b\sqrt{\mu\epsilon}} = 2\times10^9$ (Hz) $< f$.

$$F_1 = \sqrt{1-(f_c/f)^2} = 0.9798.$$
$$\beta = \omega\sqrt{\mu\epsilon}\cdot F_1 = 307.8 \text{ (rad/m)}$$
$$\alpha_d = \frac{\sigma\eta}{2F_1} = 1.283\times10^{-8} \text{ (Np/m)}$$
$$\alpha_c = \frac{2R_s}{\eta b F_1} = \frac{2}{bF_1}\sqrt{\frac{\pi f\epsilon}{\sigma_c}} = 4.238\times10^{-3} \text{ (Np/m)}$$
$$u_p = u/F_1 = 2.041\times10^8 \text{ (m/s)}$$
$$u_g = u\cdot F_1 = 1.960\times10^8 \text{ (m/s)}$$
$$\lambda_g = \lambda/F_1 = 2.041\times10^{-2} \text{ (m)}.$$

c) TM$_2$ mode — $(f_c)_{TM_2} = \dfrac{1}{b\sqrt{\mu\epsilon}} = 4\times10^9$ (Hz) $< f$.

$$F_2 = \sqrt{1-(f_c/f)^2} = 0.9165.$$
$$\beta = \omega\sqrt{\mu\epsilon}\cdot F_2 = 287.9 \text{ (rad/m)}$$
$$\alpha_d = \frac{\sigma\eta}{2F_2} = 1.371\times10^{-8} \text{ (Np/m)}$$
$$\alpha_c = \frac{2}{bF_2}\sqrt{\frac{\pi f\epsilon}{\sigma_c}} = 4.530\times10^{-3} \text{ (Np/m)}$$
$$u_p = u/F_2 = 2.182\times10^8 \text{ (m/s)}$$
$$u_g = u\cdot F_2 = 1.833\times10^8 \text{ (m/s)}$$
$$\lambda_g = \lambda/F_2 = 2.182\times10^{-2} \text{ (m)}.$$

P.10-9 a) TE$_1$ mode — $(f_c)_{TE_1} = (f_c)_{TM_1} = 2\times10^9$ (Hz) $< f$.

All required quantities are the same as those for the TM$_1$ mode in problem P. 10-8 (b), except α_c. Using Eq. (10-83), we have

$$\alpha_c = \frac{2}{bF_1}\sqrt{\frac{\pi f\epsilon}{\sigma_c}}\left(\frac{f_c}{f}\right)^2 = 1.695\times10^{-4} \text{ (Np/m)}.$$

b) TE$_2$ mode ——— $(f_c)_{TE_2} = (f_c)_{TM_2} = 4\times10^9$ (Hz) $< f$.

All required quantities are the same as those for the TM$_2$ mode in problem P.10-8 (c), except α_c.

$$\alpha_c = \frac{2}{bF_2}\sqrt{\frac{\pi f\epsilon}{\sigma_c}}\left(\frac{f_c}{f}\right)^2 = 7.249\times10^{-4}\ (Np/m).$$

P.10-10 For TM$_n$ modes in a parallel-plate waveguide,

$$\alpha_c = \frac{2}{\eta b}\sqrt{\frac{\pi\mu_c f_c}{\sigma_c}}\ \frac{1}{\sqrt{(f_c/f)[1-(f_c/f)^2]}}$$

$$= \frac{2}{\eta b}\sqrt{\frac{\pi\mu_c f_c}{\sigma_c}}\ \frac{1}{\sqrt{F(x)}},$$

where $F(x) = x - x^3$, $x = f_c/f$.

a) To find minimum α_c, set

$$\frac{\partial F(x)}{\partial x} = 1 - 3x^2 = 0 \longrightarrow x = \frac{1}{\sqrt{3}}.$$

$$\therefore\ f = \sqrt{3}\, f_c.$$

b) At $f_c/f = 1/\sqrt{3}$, $\frac{1}{\sqrt{F(x)}} = 1.612$,

and

$$\min.\ \alpha_c = \frac{3.224}{\eta b}\sqrt{\frac{\pi\mu_c f_c}{\sigma_c}}.$$

c) For $\sigma_c = 5.80\times10^7$ (S/m), $b = 5\times10^{-2}$ (m), $\eta = 120\pi$ (Ω), and $\mu_0 = 4\pi\times10^{-7}$ (H/m),

$$(f_c)_{TM_1} = \frac{1}{2b\sqrt{\mu_0\epsilon_0}} = 3\times10^9\ (Hz)$$

$$\min.\ \alpha_c = 2.444\times10^{-3}\ (Np/m).$$

P.10-11 Parallel-plate waveguide: $b = 0.03$ (m), $f = 10^{10}$ (Hz).

a) TEM mode

From Eqs. (9-1a) and (9-1b):

$$\begin{cases} E_y^0 = E_0 \\ H_x^0 = -\dfrac{E_0}{\eta_0} \end{cases}$$

$$P_{av} = \frac{w}{2}\int_0^b -E_y^0 H_x^0\, dy = \frac{wb}{2\eta_0}E_0^2.$$

Dielectric strength of air: Max. $E_0 = 3\times10^6$ (V/m).

$$\text{Max.}\left(\frac{P_{av}}{w}\right) = \frac{b}{2\eta_0}(3\times10^6)^2 = 358\times10^6\ (W/m) = 358\ (MW/m).$$

b) TM$_1$ mode

From Eqs. (10-54b) and (10-54c):

$$\begin{cases} E_y^0(y) = E_0 \cos\left(\frac{\pi y}{b}\right) \\ H_x^0(y) = -\frac{E_0}{\eta_0\sqrt{1-(f_c/f)^2}}\cos\left(\frac{\pi y}{b}\right) \end{cases}$$

$$f_c = \frac{1}{2b\sqrt{\mu_0\epsilon_0}} = 5\times 10^9 \ (Hz)$$

$$P_{av} = \frac{w}{2}\int_0^b -E_y^0(y)H_x^0(y)\,dy = \frac{wbE_0^2}{4\eta_0\sqrt{1-(f_c/f)^2}}$$

$$Max.\left(\frac{P_{av}}{w}\right) = \frac{b(3\times10^6)^2}{4\eta_0\sqrt{1-(f_c/f)^2}} = 2.07\times10^8 (W/m) = 207 (MW/m).$$

c) TE$_1$ mode

From Eqs. (10-68b) and (10-68c):

$$\begin{cases} E_x^0(y) = E_0 \sin\left(\frac{\pi y}{b}\right) \\ H_y^0(y) = \frac{E_0}{\eta_0}\sqrt{1-\left(\frac{f_c}{f}\right)^2}\sin\left(\frac{\pi y}{b}\right) \end{cases}$$

$$P_{av} = \frac{w}{2}\int_0^b E_x^0(y)H_y^0(y)\,dy = \frac{wbE_0^2}{4\eta_0}\sqrt{1-(f_c/f)^2}$$

$$Max.\left(\frac{P_{av}}{w}\right) = \frac{b(3\times10^6)^2}{4\eta_0}\sqrt{1-(f_c/f)^2} = 1.55\times10^8 (W/m) = 155 (MW/m).$$

P.10-12 $\quad f = \frac{u}{\lambda}, \qquad f_c = \frac{u}{\lambda_c}.$

$$\lambda_g = \frac{\lambda}{\sqrt{1-(f_c/f)^2}} = \frac{\lambda}{\sqrt{1-(\lambda/\lambda_c)^2}}$$

$$\longrightarrow \quad \frac{1}{\lambda_g^2} = \frac{1}{\lambda^2} - \frac{1}{\lambda_c^2}.$$

P.10-13 Equations (10-94a) through (10-94d) for TM$_{11}$ mode:

$$E_x^0(x,y) = \frac{-j\beta_{11}}{h^2}\left(\frac{\pi}{a}\right)E_0\cos\left(\frac{\pi x}{a}\right)\sin\left(\frac{\pi y}{b}\right)$$

$$E_y^0(x,y) = \frac{-j\beta_{11}}{h^2}\left(\frac{\pi}{b}\right)E_0\sin\left(\frac{\pi x}{a}\right)\cos\left(\frac{\pi y}{b}\right)$$

$$E_z^0(x,y) = E_0\sin\left(\frac{\pi x}{a}\right)\sin\left(\frac{\pi y}{b}\right)$$

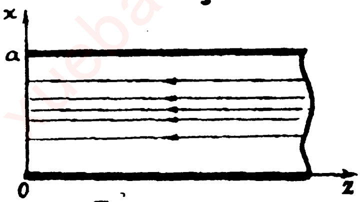
$$H_x^0(x,y) = \frac{j\omega\epsilon}{h^2}\left(\frac{\pi}{b}\right)E_0\sin\left(\frac{\pi x}{a}\right)\cos\left(\frac{\pi y}{b}\right)$$

$$H_y^0(x,y) = \frac{-j\omega\epsilon}{h^2}\left(\frac{\pi}{a}\right)E_0\cos\left(\frac{\pi x}{a}\right)\sin\left(\frac{\pi y}{b}\right).$$

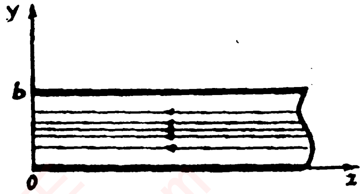
a) Surface current densities:

$$\bar{J}_s(y=0) = \bar{a}_n \times \bar{H}\Big|_{y=0} = \bar{a}_y \times [\bar{a}_x H_x^0(x,0) + \bar{a}_y H_y^0(x,0)]$$
$$= -\bar{a}_z H_x^0(x,0) = -\bar{a}_z \frac{j\omega\epsilon}{h^2}\left(\frac{\pi}{b}\right)E_0 \sin\left(\frac{\pi x}{a}\right)e^{-j\beta_{11}z}$$
$$= \bar{J}_s(y=b).$$

$$\bar{J}_s(x=0) = \bar{a}_n \times \bar{H}\Big|_{x=0} = \bar{a}_x \times [\bar{a}_x H_x^0(0,y) + \bar{a}_y H_y^0(0,y)]$$
$$= \bar{a}_z H_y^0(0,y) = -\bar{a}_z \frac{j\omega\epsilon}{h^2}\left(\frac{\pi}{a}\right)E_0 \sin\left(\frac{\pi y}{b}\right)e^{-j\beta_{11}z}$$
$$= \bar{J}_s(x=a).$$



$\bar{J}_s$ at $y=b$



$\bar{J}_s$ at $x=0$.

10-14 $\quad (f_c)_{mn} = \dfrac{1}{2\sqrt{\mu\epsilon}}\sqrt{\left(\dfrac{m}{a}\right)^2+\left(\dfrac{n}{b}\right)^2} = \dfrac{1}{2a\sqrt{\mu\epsilon}}F(m,n).$

a) $a=2b$, $F(m,n)=\sqrt{m^2+4n^2}$

Modes	$F(m,n)$
TE_{10}	1
TE_{01}, TE_{20}	2
TE_{11}, TM_{11}	$\sqrt{5}$
TE_{02}	4
TM_{12}	$\sqrt{17}$
TM_{22}	$\sqrt{20}$

b) $a=b$, $F(m,n)=\sqrt{m^2+n^2}$.

Modes	$F(m,n)$
TE_{10}, TE_{01}	1
TE_{11}, TM_{11}	$\sqrt{2}$
TE_{02}, TE_{20}	2
TM_{12}	$\sqrt{5}$
TM_{22}	$2\sqrt{2}$

10-15 $\quad f = 3\times10^9$ (Hz), $\lambda = c/f = 0.1$ (m).

Let $a = kb$, $1<k<2$. $\quad (f_c)_{mn} = \dfrac{3\times10^8}{2a}\sqrt{m^2+k^2n^2}$.

a) $(f_c)_{10} = \dfrac{1.5\times10^8}{a}$ for the dominant TE_{10} mode.

For $f > 1.2\,(f_c)_{10}$: $\quad a > 0.06$ (m).

The next higher-order mode is TE_{01} with $(f_c)_{01} = \dfrac{1.5\times10^8}{b}$.

For $f < 0.8\,(f_c)_{01}$: $\quad b < 0.04$ (m).

We choose $a = 6.5$ (cm) and $b = 3.5$ (cm).

b) $u_p = \dfrac{c}{\sqrt{1-(\lambda/2a)^2}} = 4.70 \times 10^8 \ (m/s)$

$\lambda_g = \dfrac{\lambda}{\sqrt{1-(\lambda/2a)^2}} = 0.157 \ (m) = 15.7 \ (cm)$

$\beta = \dfrac{2\pi}{\lambda_g} = 40.1 \ (rad/m)$

$(Z_{TE})_{10} = \dfrac{\eta_0}{\sqrt{1-(\lambda/2a)^2}} = 590 \ (\Omega).$

P.10-16 Given: $a = 2.5 \times 10^{-2} \ (m)$, $b = 1.5 \times 10^{-2} (m)$, $f = 7.5 \times 10^9 (Hz)$

a) $\lambda = \dfrac{c}{f} = \dfrac{3 \times 10^8}{7.5 \times 10^9} = 0.04 \ (m)$

$F_1 = \sqrt{1-(\lambda/2a)^2} = 0.60$

$\lambda_g = \lambda/F_1 = 0.0667 \ (m) = 6.67 \ (cm)$

$\beta = 2\pi/\lambda_g = 94.2 \ (rad/m)$

$u_p = c/F_1 = 5 \times 10^8 \ (m/s)$

$u_g = c \cdot F_1 = 1.8 \times 10^8 \ (m/s)$

$(Z_{TE})_{10} = \eta_0/F_1 = 200\pi = 628 \ (\Omega).$

b) $\lambda' = \dfrac{u}{f} = \dfrac{\lambda}{\sqrt{2}} = 0.0283 \ (m)$

$F_2 = \sqrt{1-(\lambda'/2a)^2} = 0.825$

$\lambda_g' = \lambda'/F_2 = 0.0343 \ (m) = 3.43 \ (cm)$

$\beta' = 2\pi/\lambda_g' = 183.2 \ (rad/m)$

$u_p' = u/F_2 = 2.57 \times 10^8 \ (m/s)$

$u_g' = u \cdot F_2 = 1.75 \times 10^8 \ (m/s)$

$(Z_{TE})_{10} = \dfrac{\eta_0}{\sqrt{2}\, F_2} = 323 \ (\Omega).$

P.10-17 Given: $a = 7.20 \times 10^{-2} (m)$, $b = 3.40 \times 10^{-2} (m)$, $f = 3 \times 10^9 \ (Hz)$

a) $\lambda_c = 2a = 14.40 \times 10^{-2} (m)$

$f_c = \dfrac{c}{\lambda_c} = 2.08 \times 10^9 \ (Hz).$

b) $\lambda = \dfrac{c}{f} = 0.1 \ (m)$, $\sqrt{1-\left(\frac{\lambda}{2a}\right)^2} = 0.720$

$\lambda_g = \dfrac{\lambda}{\sqrt{1-\left(\frac{\lambda}{2a}\right)^2}} = 0.139 \ (m).$

c) $R_s = \sqrt{\dfrac{\pi f \mu}{\sigma}} = 1.429 \times 10^{-2} \ (\Omega) \longrightarrow (\alpha_c)_{TE_{10}} = \dfrac{R_s\left[1+\frac{2b}{a}\left(\frac{f_c}{f}\right)^2\right]}{\eta_0 b\sqrt{1-(f_c/f)^2}} = 2.26 \times 10^{-3} (Np$

d) $e^{-\alpha_c z} = \dfrac{1}{2} \longrightarrow z = \dfrac{1}{\alpha_c} \ln 2 = 307 \ (m).$

P.10-18 Given: $a=2.25\times10^{-2}$ (m), $b=1.00\times10^{-2}$ (m), $f=10^{10}$ (Hz).

a) $\lambda=\frac{c}{f}=3\times10^{-2}$ (m), $\lambda_c=2a=4.50\times10^{-2}$ (m)

$$\sqrt{1-(f_c/f)^2}=\sqrt{1-(\lambda/\lambda_c)^2}=0.745$$

Eq. (10-119): $$(\alpha_c)_{TE_{10}}=\frac{1}{\eta_0 b}\sqrt{\frac{\pi f\mu_c}{\sigma_c[1-(f_c/f)^2]}}\left[1+\frac{2b}{a}\left(\frac{f_c}{f}\right)^2\right]$$

$$=1.295\times10^{-2}\ (\text{Np/m}).$$

b) From Eqs. (10-104a), (10-104b), and (10-103):

$$E_y^0=E_0\sin\left(\frac{\pi x}{a}\right)$$

$$H_x^0=-\frac{E_0}{\eta_0}\sqrt{1-\left(\frac{f_c}{f}\right)^2}\sin\left(\frac{\pi x}{a}\right)$$

$$H_z^0=j\left(\frac{f_c}{f}\right)\frac{E_0}{\eta_0}\cos\left(\frac{\pi x}{a}\right)$$

$$P_{av}=\frac{1}{2}\int_0^b\int_0^a(-E_y^0H_x^0)\,dx\,dy=\frac{E_0^2ab}{4\eta_0}\sqrt{1-\left(\frac{f_c}{f}\right)^2}.$$

For $P_{av}=10^3$ (W) at the load (antenna), assuming under matched conditions:

$|E_y^0|=E_0=94{,}800$ (V/m), $|H_x^0|=187.4$ (A/m), $|H_z^0|=167.6$ (A/m)

The waveguide is 1(m) long. $\longrightarrow$ The field intensities are higher at the sending end by a factor of $e^{\alpha_c\cdot1}=1.138$.

$\therefore$ Max. $|E_y^0|=10{,}788$ (V/m)

Max. $|H_x^0|=213.3$ (A/m)

Max. $|H_z^0|=190.7$ (A/m)

c) $$\bar{J}(x=0)=\bar{a}_x\times(\bar{a}_xH_x^0+\bar{a}_zH_z^0)\Big|_{x=0}=-\bar{a}_yH_z^0(0,y)=-\bar{a}_yj\left(\frac{f_c}{f}\right)\frac{E^0}{\eta_0}$$

$$|\bar{J}(x=0)|=|H_z^0|=167.6\ (\text{A/m})$$

$$\bar{J}(y=0)=\bar{a}_y\times(\bar{a}_xH_x^0+\bar{a}_zH_z^0)\Big|_{y=0}=-a_zH_x^0(x,0)+a_xH_z^0(x,0)$$

$$|\bar{J}(y=0)|=\left[(H_x^0)^2+(H_z^0)^2\right]^{1/2}=\frac{E_0}{\eta_0}\left\{\left(\frac{f_c}{f}\right)^2+\left[1-2\left(\frac{f_c}{f}\right)^2\sin^2\left(\frac{\pi x}{a}\right)\right]\right\}^{1/2},$$

which is maximum at $x=a/2$.

At the sending end: Max. $|\bar{J}|=\frac{E_0}{\eta_0}\sqrt{1-\left(\frac{f_c}{f}\right)^2}\times1.138=213.3$ (A/m).

d) Total amount of average power dissipated in 1 (m) of waveguide:

$$P_d=1000\,(e^{2\alpha_c}-1)=1000\,(e^{0.0259}-1)=26.2\ (\text{W}).$$

P.10-19 From problem P.10-18, we have

$$P_{av} = \frac{E_0^2 ab}{4\eta_0}\sqrt{1-\left(\frac{f_c}{f}\right)^2}, \quad \sqrt{1-\left(\frac{f_c}{f}\right)^2} = 0.745$$

$$\therefore \text{Max. } P_{av} = \frac{(3\times10^6)^2\times(2.25\times10^{-4})}{4\times120\pi}\times0.745 = 10^6\,(W) = 1\,(MW).$$

P.10-20 Let $A = \frac{1}{\eta_0 b}\sqrt{\frac{\pi f_c \mu_c}{\sigma_c}}$ and $x = \frac{f_c}{f}$ in Eq.(10-119).

We write $(\alpha_c)_{TE_{10}} = AF(x)$, where $F(x) = \frac{1+\frac{2b}{a}x^2}{\sqrt{x(1-x^2)}}$.

For min. $(\alpha)_{TE_{10}}$, set $\frac{dF(x)}{dx} = 0$.

$$\longrightarrow x = \frac{f_c}{f} = \sqrt{\frac{3}{2}}\left[\left(1+\frac{a}{2b}\right)-\sqrt{\left(1+\frac{a}{2b}\right)^2-\frac{2a}{9b}}\right]^{1/2}.$$

P.10-21 Field expressions for TM_{11} mode from Eqs. (10-92) and (10-94):

$$E_x^0(x,y) = -\frac{j\beta_{11}}{h^2}\left(\frac{\pi}{a}\right)E_0\cos\left(\frac{\pi x}{a}\right)\sin\left(\frac{\pi y}{b}\right)$$

$$E_y^0(x,y) = -\frac{j\beta_{11}}{h^2}\left(\frac{\pi}{b}\right)E_0\sin\left(\frac{\pi x}{a}\right)\cos\left(\frac{\pi y}{b}\right)$$

$$E_z^0(x,y) = E_0\sin\left(\frac{\pi x}{a}\right)\sin\left(\frac{\pi y}{b}\right)$$

$$H_x^0(x,y) = \frac{j\omega\epsilon}{h^2}\left(\frac{\pi}{b}\right)E_0\sin\left(\frac{\pi x}{a}\right)\cos\left(\frac{\pi y}{b}\right)$$

$$H_y^0(x,y) = -\frac{j\omega\epsilon}{h^2}\left(\frac{\pi}{a}\right)E_0\cos\left(\frac{\pi x}{a}\right)\sin\left(\frac{\pi y}{b}\right)$$

Calculate α_c from Eq. (10-77): $\alpha_c = \frac{P_L(z)}{2P(z)}$.

$$P(z) = \frac{1}{2}\int_0^b\int_0^a\left[E_x^0H_y^0 - E_y^0H_x^0\right]dx\,dy = \frac{\omega\epsilon\beta E_0^2 ab}{8\left[\left(\frac{\pi}{a}\right)^2+\left(\frac{\pi}{b}\right)^2\right]}$$

From problem P.10-13:

$$\bar{J}_s(y=0) = \bar{J}_s(y=b) = -\bar{a}_z\frac{j\omega\epsilon}{h^2}\left(\frac{\pi}{b}\right)E_0\sin\left(\frac{\pi x}{a}\right)e^{-j\beta_{11}z}$$

$$\bar{J}_s(x=0) = \bar{J}_s(x=a) = -\bar{a}_z\frac{j\omega\epsilon}{h^2}\left(\frac{\pi}{a}\right)E_0\sin\left(\frac{\pi x}{a}\right)e^{-j\beta_{11}z}.$$

$$P_L(z) = 2\left[P_L(z)\right]_{x=0} + 2\left[P_L(z)\right]_{y=0}$$

$$\left[P_L(z)\right]_{x=0} = \frac{1}{2}\int_0^b\left|\bar{J}_s(x=0)\right|^2R_s\,dy = \frac{(\omega\epsilon)^2R_s}{4h^4}\left(\frac{\pi}{a}\right)^2E_0^2b$$

$$\left[P_L(z)\right]_{y=0} = \frac{1}{2}\int_0^a\left|\bar{J}_s(y=0)\right|^2R_s\,dx = \frac{(\omega\epsilon)^2R_s}{4h^4}\left(\frac{\pi}{b}\right)^2E_0^2a.$$

$$P_L(z) = \frac{(\omega\epsilon)^2 R_s E_0^2}{2\left[\left(\frac{\pi}{a}\right)^2+\left(\frac{\pi}{b}\right)^2\right]}\left[\left(\frac{\pi}{a}\right)^2 b + \left(\frac{\pi}{b}\right)^2 a\right]$$

$$\therefore \quad (\alpha_c)_{TM_{11}} = \frac{2R_s(b/a^2+a/b^2)}{\pi ab\sqrt{1-(f_c/f)^2}\,(1/a^2+1/b^2)}.$$

P.10-22 From Eqs. (10-124) and (10-126):

Inside the slab: $\beta^2=\omega^2\mu_d\epsilon_d-k_y^2 < \omega^2\mu_d\epsilon_d$

Outside the slab: $\beta^2=\omega^2\mu_0\epsilon_0+\alpha^2 > \omega^2\mu_0\epsilon_0$

$$\therefore \quad \omega\sqrt{\mu_0\epsilon_0} < \beta < \omega\sqrt{\mu_d\epsilon_d}$$

and
$$\frac{1}{\sqrt{\mu_0\epsilon_0}} > u_p = \frac{\omega}{\beta} > \frac{1}{\sqrt{\mu_d\epsilon_d}}.$$

P.10-23 From Eqs. (10-131a) and (10-130):

$$\left(\frac{\alpha d}{2}\right)^2 + \left(\frac{k_y d}{2}\right)^2 = \left(\frac{k_0 d}{2}\right)^2\left(\frac{\mu_d\epsilon_d}{\mu_0\epsilon_0}-1\right) \qquad ①$$

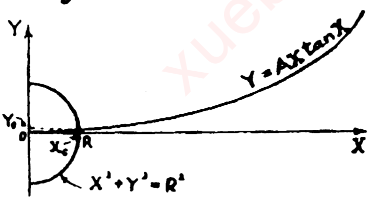
$$\frac{\epsilon_d}{\epsilon_0}\left(\frac{\alpha d}{2}\right) = \left(\frac{k_y d}{2}\right)\tan\left(\frac{k_y d}{2}\right) \qquad ②$$

Let $X = k_y d/2$, $Y = \alpha d/2$, $A = \epsilon_0/\epsilon_d$, and $R = \frac{k_0 d}{2}\sqrt{\frac{\mu_d\epsilon_d}{\mu_0\epsilon_0}-1}$.

Eqs. ① and ② become
$$\begin{cases} X^2+Y^2=R^2 & ③ \\ Y = AX\tan X. & ④ \end{cases}$$

a) $f=2\times10^8$ (Hz), $\lambda = c/f = 1.5$ (m).

$k_0 d/2 = \pi d/\lambda = 0.0209$, $A = \epsilon_0/\epsilon_d = 0.308$, $R = 0.0314$.



Graphical solution:

$X_0 \cong 0.0314$, $Y_0 \cong 3.038\times10^{-4}$

$\alpha = 2Y_0/d = 0.061$ (Np/m)

$k_y = 2X_0/d = 6.28$ (rad/m)

From Eq. (10-124): $\beta = \sqrt{\omega^2\mu_d\epsilon_d - k_y^2} = 4.19$ (rad/m)

b) $f = 5\times10^8$ (Hz), $\lambda = c/f = 0.60$ (m), $k_0 d/2 = 0.0524$

$A = 0.308$, $R = 0.0785$

$X_0 = 0.0785$, $Y_0 = 1.901\times10^{-3}$

We obtain $\alpha = 0.380$ (Np/m)

$\beta = 10.48$ (rad/m).

P.10-24 From Eq. (10-135):

$$\left(\frac{\alpha d}{2}\right) = -\frac{\epsilon_0}{\epsilon_d}\left(\frac{k_y d}{2}\right)\cot\left(\frac{k_y d}{2}\right) \qquad ②'$$

Using the notations in problem P.10-23, we obtain two equations from ① in P.10-23 and ②' above:

$$\begin{cases} X^2 + Y^2 = R^2 & ③' \\ Y = -AX\cot X & ④' \end{cases}$$

a) $f = 2\times10^8$ (Hz), $\lambda = 1.5$ (m)	b) $f = 5\times10^8$ (Hz), $\lambda = 0.60$ (m)
$A = 0.308$	$A = 0.308$
$R = 0.0314$	$R = 0.075$

There are no intersections for curves representing Eqs. ③' and ④'; hence even TM modes do not exist at the given frequencies.

P.10-25 Use Eqs. (10-23d) and (10-23a):

$$E_y^0 = -\frac{j\beta}{h^2}\frac{\partial E_z^0}{\partial y}, \qquad H_x^0 = \frac{j\omega\epsilon}{h^2}\frac{\partial E_z^0}{\partial y}.$$

$$\bar{E}(y,z;t) = \mathcal{Re}\left[\bar{E}^0(y)\,e^{j(\omega t-\beta z)}\right]$$

$$\bar{H}(y,z;t) = \mathcal{Re}\left[\bar{H}^0(y)\,e^{j(\omega t-\beta z)}\right].$$

$|y| \le d/2$:

$$E_z^0(y) = E_e\cos k_y y \longrightarrow E_z(y,z;t) = E_e\cos k_y y\cos(\omega t-\beta z)$$

$$E_y^0(y) = \frac{j\beta}{k_y}E_e\sin k_y y \longrightarrow E_y(y,z;t) = -\frac{\beta}{k_y}E_e\sin k_y y\sin(\omega t-\beta z)$$

$$H_x^0(y) = -\frac{j\omega\epsilon_d}{k_y}E_e\sin k_y y \longrightarrow H_x(y,z;t) = \frac{\omega\epsilon_d}{k_y}E_e\sin k_y y\sin(\omega t-\beta z)$$

$y \ge d/2$:

$$E_z^0(y) = E_e\cos\left(\frac{k_y d}{2}\right)e^{-\alpha(y-\frac{d}{2})} \longrightarrow E_z(y,z;t) = E_e\cos\left(\frac{k_y d}{2}\right)e^{-\alpha(y-\frac{d}{2})}\cos(\omega t-\beta z)$$

$$E_y^0(y) = -\frac{j\beta}{\alpha}E_e\cos\left(\frac{k_y d}{2}\right)e^{-\alpha(y-\frac{d}{2})} \longrightarrow E_y(y,z;t) = \frac{\beta}{\alpha}E_e\cos\left(\frac{k_y d}{2}\right)e^{-\alpha(y-\frac{d}{2})}\sin(\omega t-\beta z)$$

$$H_x^0(y) = \frac{j\omega\epsilon_0}{\alpha}E_e\cos\left(\frac{k_y d}{2}\right)e^{-\alpha(y-\frac{d}{2})} \longrightarrow H_x(y,z;t) = -\frac{\omega\epsilon_0}{\alpha}E_e\cos\left(\frac{k_y d}{2}\right)e^{-\alpha(y-\frac{d}{2})}\sin(\omega t-\beta z)$$

$y \le -d/2$:

$$E_z^0(y) = E_e\cos\left(\frac{k_y d}{2}\right)e^{\alpha(y+\frac{d}{2})} \longrightarrow E_z(y,z;t) = E_e\cos\left(\frac{k_y d}{2}\right)e^{\alpha(y+\frac{d}{2})}\cos(\omega t-\beta z)$$

$$E_y^0(y) = \frac{j\beta}{\alpha}E_e\cos\left(\frac{k_y d}{2}\right)e^{\alpha(y+\frac{d}{2})} \longrightarrow E_y(y,z;t) = -\frac{\beta}{\alpha}E_e\cos\left(\frac{k_y d}{2}\right)e^{\alpha(y+\frac{d}{2})}\sin(\omega t-\beta z)$$

$$H_x^0(y) = -\frac{j\omega\epsilon_0}{\alpha}E_e\cos\left(\frac{k_y d}{2}\right)e^{\alpha(y+\frac{d}{2})} \longrightarrow H_x(y,z;t) = \frac{\omega\epsilon_0}{\alpha}E_e\cos\left(\frac{k_y d}{2}\right)e^{\alpha(y+\frac{d}{2})}\sin(\omega t-\beta z)$$

P.10-26 a) From Table 10-2 on p.485 it is seen that f_{co} 0 for TE_1 mode, which is the dominant mode.

From Eq. (10-142):

$$\alpha = \frac{\mu_0}{\mu_d} k_y \tan\frac{k_y d}{2} \cong \frac{\mu_0 d}{2\mu_d} k_y^2 \text{ , for } k_y d \ll 1.$$

Neglecting the α^2 term in Eq. (10-126):

$$\beta^2 - \omega^2\mu_0\epsilon_0 = \alpha^2 \approx 0 \longrightarrow \beta \cong \omega\sqrt{\mu_0\epsilon_0} = k_0.$$

From Eq. (10-124): $k_y^2 = \omega^2\mu_d\epsilon_d - \beta^2 \cong k_d^2 - k_0^2$

$$\therefore \alpha \cong \frac{\mu_0 d}{2\mu_d}(k_d^2 - k_0^2).$$

b) $d = 5\times10^{-3}$ (m), $\epsilon_d = 3\epsilon_0$, $\mu_d = \mu_0$, $f = 3\times10^8$ (Hz), $k_0 = 2\pi$.

$$\alpha = \frac{d}{2}k_0^2(\epsilon_r - 1) = 0.197 \quad \text{(Np/m)}.$$

$$e^{-\alpha(y-\frac{d}{2})} = 0.368, \quad \alpha\left(y - \frac{d}{2}\right) = 1.$$

$$\longrightarrow \left(y - \frac{d}{2}\right) = 5.066 \text{ (m)}.$$

P.10-27 Use Eqs. (10-42b) and (10-42c):

$$H_y^0 = -\frac{j\beta}{h^2}\frac{\partial H_z^0}{\partial y}, \qquad E_x^0 = -\frac{j\omega\mu}{h^2}\frac{\partial H_z^0}{\partial y}.$$

$$\bar{H}(y,z;t) = \mathcal{R}e\left[\bar{H}^0(y)e^{j(\omega t-\beta z)}\right]$$

$$\bar{E}(y,z;t) = \mathcal{R}e\left[\bar{E}^0(y)e^{j(\omega t-\beta z)}\right].$$

$|y| \le d/2$:

$H_z^0(y) = H_e\cos k_y y \longrightarrow H_z(y,z;t) = H_e\cos k_y y\cos(\omega t-\beta z)$

$H_y^0(y) = \frac{j\beta}{k_y}H_e\sin k_y y \longrightarrow H_y(x,z;t) = -\frac{\beta}{k_y}H_e\sin k_y y\sin(\omega t-\beta z)$

$E_x^0(y) = \frac{j\omega\mu_d}{k_y}H_e\sin k_y y \longrightarrow E_x(x,z;t) = -\frac{\omega\mu_d}{k_y}H_e\sin k_y y\sin(\omega t-\beta z)$

$y \ge d/2$:

$H_z^0(y) = H_e\cos\left(\frac{k_y d}{2}\right)e^{-\alpha(y-\frac{d}{2})} \longrightarrow H_z(y,z;t) = H_e\cos\left(\frac{k_y d}{2}\right)e^{-\alpha(y-\frac{d}{2})}\cos(\omega t-\beta z)$

$H_y^0(y) = -\frac{j\beta}{\alpha}H_e\cos\left(\frac{k_y d}{2}\right)e^{-\alpha(y-\frac{d}{2})} \longrightarrow H_y(y,z;t) = \frac{\beta}{\alpha}H_e\cos\left(\frac{k_y d}{2}\right)e^{-\alpha(y-\frac{d}{2})}\sin(\omega t-\beta z)$

$E_x^0(y) = -\frac{j\omega\mu_0}{\alpha}H_e\cos\left(\frac{k_y d}{2}\right)e^{-\alpha(y-\frac{d}{2})} \longrightarrow E_x(y,z;t) = \frac{\omega\mu_0}{\alpha}H_e\cos\left(\frac{k_y d}{2}\right)e^{-\alpha(y-\frac{d}{2})}\sin(\omega t-\beta z)$

$y \le -d/2$:

$H_z^0(y) = H_e\cos\left(\frac{k_y d}{2}\right)e^{\alpha(y+\frac{d}{2})} \longrightarrow H_z(y,z;t) = H_e\cos\left(\frac{k_y d}{2}\right)e^{\alpha(y+\frac{d}{2})}\cos(\omega t-\beta z)$

$H_y^0(y) = \frac{j\beta}{\alpha}H_e\cos\left(\frac{k_y d}{2}\right)e^{\alpha(y+\frac{d}{2})} \longrightarrow H_y(y,z;t) = -\frac{\beta}{\alpha}H_e\cos\left(\frac{k_y d}{2}\right)e^{\alpha(y+\frac{d}{2})}\sin(\omega t-\beta z)$

$E_x^0(y) = \frac{j\omega\mu_0}{\alpha}H_e\cos\left(\frac{k_y d}{2}\right)e^{\alpha(y+\frac{d}{2})} \longrightarrow E_x(y,z;t) = -\frac{\omega\mu_0}{\alpha}H_e\cos\left(\frac{k_y d}{2}\right)e^{\alpha(y+\frac{d}{2})}\sin(\omega t-\beta z).$

Setting $y = d/2$ in $E_x^o(y) = \frac{j\omega\mu_d}{k_y} H_o \sin k_y y$ and

in $E_x^o(y) = -\frac{j\omega\mu_o}{\alpha} H_o \cos\left(\frac{k_y d}{2}\right) e^{-\alpha(y-\frac{d}{2})}$

and equating, we obtain

$$E_x^o\left(\tfrac{d}{2}\right) = \frac{j\omega\mu_d}{k_y} H_o \sin\left(\frac{k_y d}{2}\right) = -\frac{j\omega\mu_o}{\alpha} H_o \cos\left(\frac{k_y d}{2}\right)$$

$$\longrightarrow \quad \frac{\alpha}{k_y} = -\frac{\mu_o}{\mu_d} \cot\left(\frac{k_y d}{2}\right).$$

P.10-28 a) Odd TM and even TE modes are the propagating modes. Using 2d for d in the formulas in Table 10-2, p.485, we have

$$f_{co} = \frac{n-1}{2d\sqrt{\mu_d\epsilon_d - \mu_o\epsilon_o}} \quad \text{for odd TM modes}$$

$$f_{ce} = \frac{n-\frac{1}{2}}{2d\sqrt{\mu_d\epsilon_d - \mu_o\epsilon_o}} \quad \text{for even TE modes.}$$

b) For odd TM modes — From Eqs. (10-127b and c):

$|y| \le d/2$. $\quad E_y^o(y) = -\frac{j\beta}{k_y} E_o \cos k_y y$

$H_x^o(y) = \frac{j\omega\epsilon_d}{k_y} E_o \cos k_y y$.

Surf. current density on conductor $\bar{J}_s = \bar{a}_n \times \bar{H}\Big|_{y=0}$

$\bar{J}_s = -\bar{a}_z H_x^o(0) = -\bar{a}_z \frac{j\omega\epsilon_d}{k_y} E_o$.

Surf. charge density on conductor $\rho_s = \bar{a}_n \cdot \bar{D}\Big|_{y=0}$

$\rho_s = \epsilon_d E^o(0) = -\frac{j\beta\epsilon_d}{k_y} E_o$.

For even TE modes — From problem P.10-27:

$|y| \le d$. $\quad H_y^o(y) = \frac{j\beta}{k_y} H_o \sin k_y y$

$E_x^o(y) = \frac{j\omega\mu_d}{k_y} H_o \sin k_y y$

$H_z^o(y) = H_o \cos k_y y$.

$\therefore \bar{J}_s = \bar{a}_y \times [\bar{a}_y H_y^o(0) + \bar{a}_z H_z^o(0)] = \bar{a}_x H_o$

$\rho_s = \bar{a}_y \cdot \epsilon_d \bar{E}(0) = 0$.

P.10-29 From Eq. (10-150): $f_{mnp} = \frac{u}{2}\sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2 + \left(\frac{p}{d}\right)^2}$.

$f_{mnp} = 1.5\times 10^{10} F(m,n,p)$, $\quad F(m,n,p) = \sqrt{\left(\frac{m}{8}\right)^2 + \left(\frac{n}{6}\right)^2 + \left(\frac{p}{5}\right)^2}$.

Lowest-order modes and resonant frequencies:

Modes	$F(m,n,p)$	$(f_c)_{mnp}$ in (Hz)
TM_{110}	0.2083	3.125×10^9
TE_{101}	0.2358	3.538×10^9
TE_{011}	0.2603	3.905×10^9
TE_{111}, TM_{111}	0.2888	4.332×10^9
TM_{210}	0.3005	4.507×10^9
TE_{201}	0.3202	4.802×10^9
TM_{120}	0.3560	5.340×10^9
TE_{211}, TM_{211}	0.3609	5.414×10^9
TE_{021}	0.3887	5.831×10^9
TE_{121}, TM_{121}	0.4083	6.125×10^9

P.10-30 a) Since $d>a>b$, the lowest-order resonant mode is TE_{101} mode.

$$f_{101}=\frac{c}{2}\sqrt{\frac{1}{a^2}+\frac{1}{d^2}}=4.802\times10^9\ (Hz).$$

b) From Eq. (10-161):

$$Q_{101}=\frac{\pi f_{101}\mu_0 abd(a^2+d^2)}{R_s[2b(a^3+d^3)+ad(a^2+d^2)]}\qquad \left(R_s=\sqrt{\frac{\pi f_{101}\mu_0}{\sigma}}\right)$$

$$=\frac{\sqrt{\pi f_{101}\mu_0\sigma}\ abd(a^2+d^2)}{2b(a^3+d^3)+ad(a^2+d^2)}=6869.$$

From Eqs. (10-156a) and (10-156b):

$$W_e=\frac{1}{4}\epsilon_0\mu_0^2a^3bdf_{101}^2H_0^2=0.07728\times10^{-12}\ (J)$$

$$W_m=\frac{\mu_0}{16}abd\left(\frac{a^2}{d^2}+1\right)H_0^2=0.07728\times10^{-12}(J)=W_e.$$

P.10-31 a) $(f_{101})_{\epsilon_d}=\frac{u}{2}\sqrt{\frac{1}{a^2}+\frac{1}{d^2}}=\frac{1}{\sqrt{\epsilon_r}}(f_{101})_{\epsilon_0}=3.037\times10^9\ (Hz).$

b) $(Q_{101})_{\epsilon_d}=\frac{1}{(\epsilon_r)^{1/4}}(Q_{101})_{\epsilon_0}=5462.$

c) $(W_e)_{\epsilon_d}=(W_e)_{\epsilon_0}=0.07728\times10^{-12}(J)=0.07728\ (pJ)$

$=(W_m)_{\epsilon_d}.$

P.10-32 a) $Q_{101} = \dfrac{\sqrt{\pi f_{101}\mu_0\sigma}\ abd(a^2+d^2)}{2b(a^3+d^3)+ad(a^2+d^2)}$.

For $a=d=1.8b$, $f_{101}=\dfrac{1}{2\sqrt{\mu_0\epsilon_0}}\sqrt{\dfrac{1}{a^2}+\dfrac{1}{d^2}} = 1.179\times10^8\left(\dfrac{1}{b}\right)$

$Q_{101}=10.22\sqrt{\sigma b}$.

b) For $Q'_{101}=1.20\,Q_{101}$, $b'=1.20^2 b=1.44b$.

P.10-33 (I) From the field configurations in the cavity we see that the TM_{110} mode with respect to z is the same as the TE_{101} mode with respect to y. Thus, $(Q_{110})_{TM}$ can be obtained from $(Q_{101})_{TE}$ in Eq.(10-161) by changing b to d and d to b.

or, (II) Q for the TM_{110} mode can be derived from the field expressions in Eqs. (10-149a, d, and e) by setting $m=n=1$, and using Eq. (10-155).

$W=2W_m=\dfrac{\mu_0}{8}\left(\dfrac{\omega^2\epsilon_0^2}{h^2}\right)abdE_0^2$ at f_{110}.

$$P_L=\oint\tfrac{1}{2}|\bar{J}_s|^2R_s\,ds=\oint\tfrac{1}{2}|\bar{H}_t|^2R_s\,ds$$
$$=R_s\left\{\int_0^d\!\!\int_0^b|H_y(x=0)|^2dy\,dz+\int_0^d\!\!\int_0^a|H_x(y=0)|^2dx\,dz+\int_0^b\!\!\int_0^a\left[|H_x(z=0)|^2+|H_y(z=0)|^2\right]dx\,dy\right\}$$
$$=\frac{R_s}{2}\left(\frac{\omega^2\epsilon_0^2}{h^2}\right)E_0^2\left\{\frac{1}{h^2}\left(\frac{\pi}{a}\right)^2bd+\frac{1}{h^2}\left(\frac{\pi}{b}\right)^2ad+\frac{1}{2}ab\right\},$$
$$h^2=\left(\frac{\pi}{a}\right)^2+\left(\frac{\pi}{b}\right)^2.$$

$$Q_{110}=\frac{\omega_{110}W}{P_L}=\frac{\pi f_{110}\mu_0\,abd(a^2+b^2)}{R_s\left[2d(a^3+b^3)+ab(a^2+b^2)\right]},\quad R_s=\sqrt{\frac{\pi f_{110}\mu_0}{\sigma}}.$$

P.10-34 $C=\dfrac{\epsilon S}{d}=\dfrac{\epsilon\pi a^2}{d}$

$L=\dfrac{\mu h}{2\pi}\ln\left(\dfrac{b}{a}\right)$

a) $f=\dfrac{1}{2\pi\sqrt{LC}}=\dfrac{1}{\pi a\sqrt{\mu\epsilon}\sqrt{\frac{2h}{d}\ln\left(\frac{b}{a}\right)}}$.

b) $\lambda=\dfrac{1}{f\sqrt{\mu\epsilon}}=\pi a\sqrt{\dfrac{2h}{d}\ln\left(\dfrac{b}{a}\right)}$.

Chapter 11

P.11-1 Maxwell's equations for simple media:

$$\bar{\nabla}\times\bar{E} = -\mu\frac{\partial\bar{H}}{\partial t} \qquad ①$$
$$\bar{\nabla}\times\bar{H} = \bar{J} + \epsilon\frac{\partial\bar{E}}{\partial t} \qquad ②$$
$$\bar{\nabla}\cdot\bar{E} = \frac{\rho}{\epsilon} \qquad ③$$
$$\bar{\nabla}\cdot\bar{H} = 0 \qquad ④$$

a) $\bar{\nabla}\times ①$: $\bar{\nabla}\times\bar{\nabla}\times\bar{E} = -\mu\frac{\partial}{\partial t}(\bar{\nabla}\times\bar{H})$

$$= -\mu\frac{\partial\bar{J}}{\partial t} - \mu\epsilon\frac{\partial^2\bar{E}}{\partial t^2} \qquad ⑤$$

But $\bar{\nabla}\times\bar{\nabla}\times\bar{E} = \bar{\nabla}(\bar{\nabla}\cdot\bar{E}) - \bar{\nabla}^2\bar{E}$

$$= \frac{1}{\epsilon}\bar{\nabla}\rho - \bar{\nabla}^2\bar{E} \qquad ⑥$$

Combining ⑤ and ⑥, we obtain

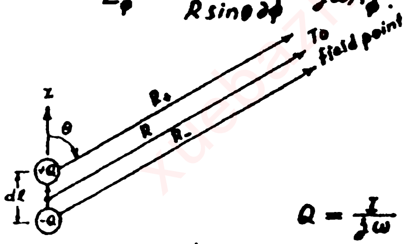
$$\bar{\nabla}^2\bar{E} - \mu\epsilon\frac{\partial^2\bar{E}}{\partial t^2} = \frac{1}{\epsilon}\bar{\nabla}\rho + \mu\frac{\partial\bar{J}}{\partial t}.$$

b) Similarly, we have $\bar{\nabla}^2\bar{H} - \mu\epsilon\frac{\partial^2\bar{H}}{\partial t^2} = -\bar{\nabla}\times\bar{J}$.

P.11-2 Eq. (11-2): $\bar{E} = -\bar{\nabla}V - j\omega\bar{A} = \bar{a}_R E_R + \bar{a}_\theta E_\theta + \bar{a}_\phi E_\phi$.

$$E_R = -\frac{\partial V}{\partial R} - j\omega A_R$$
$$E_\theta = -\frac{\partial V}{R\,\partial\theta} - j\omega A_\theta$$
$$E_\phi = -\frac{\partial V}{R\sin\theta\,\partial\phi} - j\omega A_\phi.$$

The expressions of A_R, A_θ, and A_ϕ are given in Eqs. (11-14 a, b, and c).



$$V = \frac{Q}{4\pi\epsilon_0}\left[\frac{e^{-j\beta R_+}}{R_+} - \frac{e^{-j\beta R_-}}{R_-}\right]$$

$$R_+ \cong R - \frac{1}{2}d\ell\cos\theta$$
$$R_- \cong R + \frac{1}{2}d\ell\cos\theta$$

$$Q = \frac{I}{j\omega}, \qquad (d\ell)^2 \ll R^2.$$

$$V \cong \frac{I e^{-j\beta R}}{4\pi\epsilon_0 j\omega}\frac{1}{R^2}\left[\left(R+\frac{d\ell}{2}\cos\theta\right)e^{j\beta(d\ell\cos\theta)/2} - \left(R-\frac{d\ell}{2}\cos\theta\right)e^{-j\beta(d\ell\cos\theta)/2}\right]$$

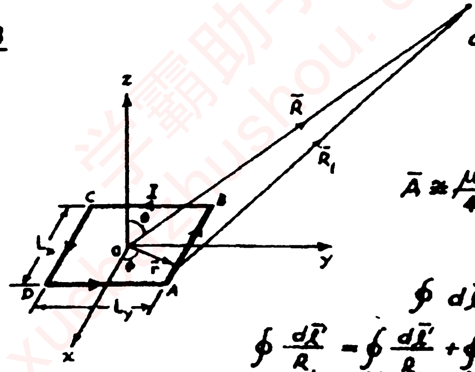
$$= \frac{I e^{-j\beta R}}{4\pi\epsilon_0 j\omega R^2}\left[2jR\sin\left(\frac{\beta d\ell\cos\theta}{2}\right) + 2\left(\frac{d\ell}{2}\cos\theta\right)\cos\left(\frac{\beta d\ell\cos\theta}{2}\right)\right]$$

$$\cong \frac{I e^{-j\beta R}}{4\pi\epsilon_0 j\omega R^2}\left[2jR\left(\frac{\beta d\ell\cos\theta}{2}\right) + d\ell\cos\theta\right]$$

$$= \frac{I\,d\ell\cos\theta}{4\pi R^2}\eta_0\left(R + \frac{1}{j\beta}\right)e^{-j\beta R}.$$

Using A_R, A_θ, A_ϕ, and V in E_R, E_θ, and E_ϕ, we obtain the same results as given in Eqs. (11-16a,b,&c).

P.11-3



a) $$\bar{A} = \frac{\mu_0 I}{4\pi}\oint \frac{e^{-j\beta R_1}}{R_1}\,d\bar{\ell}'$$
$$= \frac{\mu_0 I}{4\pi} e^{-j\beta R}\oint \frac{e^{-j\beta(R_1-R)}}{R_1}\,d\bar{\ell}'$$

$$\bar{A} \cong \frac{\mu_0 I}{4\pi} e^{-j\beta R}\left[(1+j\beta R)\oint \frac{d\bar{\ell}'}{R_1} - j\beta R\oint d\bar{\ell}'\right]$$

$$\oint d\bar{\ell}' = 0.$$

$$\oint \frac{d\bar{\ell}'}{R_1} = \int_{AB}\frac{d\bar{\ell}'}{R_1} + \int_{CD}\frac{d\bar{\ell}'}{R_1} + \int_{BC}\frac{d\bar{\ell}'}{R_1} + \int_{DA}\frac{d\bar{\ell}'}{R_1}$$

$$R_1^2 = R^2 + r^2 - 2\bar{R}\cdot\bar{r}$$

$$\bar{R} = \bar{a}_x R\sin\theta\cos\phi + \bar{a}_y R\sin\theta\sin\phi + \bar{a}_z R\cos\theta$$

$$\bar{r} = \bar{a}_x x + \bar{a}_y \frac{L_y}{2}, \quad \bar{R}\cdot\bar{r} = Rx\sin\theta\cos\phi + R\frac{L_y}{2}\sin\theta\sin\phi$$

$$\frac{1}{R_1} \cong \frac{1}{R}\left[1 + \frac{\bar{R}\cdot\bar{r}}{R^2}\right] = \frac{1}{R}\left(1 + \frac{x}{R}\sin\theta\cos\phi + \frac{L_y}{2R}\sin\theta\sin\phi\right)$$

$$\frac{\mu_0 I}{4\pi}e^{-j\beta R}(1+j\beta R)\int_{AB}\frac{d\bar{\ell}'}{R_1} = \bar{a}_x\frac{\mu_0 I}{4\pi}e^{-j\beta R}(1+j\beta R)\frac{1}{R}\int_{L_x/2}^{-L_x/2}\left(1+\frac{x}{R}\sin\theta\cos\phi + \frac{L_y}{2R}\sin\theta\sin\phi\right)dx$$
$$= \bar{a}_x\frac{\mu_0 I}{4\pi}e^{-j\beta R}(1+j\beta R)\frac{1}{R}\left(-L_x - \frac{L_x L_y}{2R}\sin\theta\sin\phi\right)$$

In the same manner, we have

$$\frac{\mu_0 I}{4\pi}e^{-j\beta R}(1+j\beta R)\int_{CD}\frac{d\bar{\ell}'}{R_1} = \bar{a}_x\frac{\mu_0 I}{4\pi}e^{-j\beta R}(1+j\beta R)\frac{1}{R}\left(L_x - \frac{L_x L_y}{2R}\sin\theta\sin\phi\right)$$

$$\therefore \frac{\mu_0 I}{4\pi}e^{-j\beta R}(1+j\beta R)\int_{AB+CD}\frac{d\bar{\ell}'}{R_1} = -\bar{a}_x\frac{\mu_0 I}{4\pi R^2}e^{-j\beta R}(1+j\beta R)L_x L_y\sin\theta\sin\phi$$

and

$$\frac{\mu_0 I}{4\pi}e^{-j\beta R}(1+j\beta R)\int_{BC+DA}\frac{d\bar{\ell}'}{R_1} = \bar{a}_y\frac{\mu_0 I}{4\pi R^2}e^{-j\beta R}(1+j\beta R)L_x L_y\sin\theta\cos\phi$$

Let $m = IL_xL_y = IS$.

$$\bar{A} = \frac{\mu_0 m}{4\pi R^2}e^{-j\beta R}(1+j\beta R)\sin\theta(-\bar{a}_x\sin\phi + \bar{a}_y\cos\phi)$$
$$= \bar{a}_\phi\frac{\mu_0 m}{4\pi R^2}e^{-j\beta R}(1+j\beta R)\sin\theta.$$

c) $\bar{H} = \frac{1}{\mu_0}\bar{\nabla}\times\bar{A} = \bar{a}_R H_R + \bar{a}_\theta H_\theta$.

b) $\bar{E} = \frac{1}{j\omega\epsilon_0}\bar{\nabla}\times\bar{H} = \bar{a}_\phi E_\phi$.

Expressions for H_R, H_θ, and E_ϕ same as those given in Eqs. (11-26a,b,&c).

In the far zone, $\beta R \gg 1$, $1/(j\beta R)^2$ and $1/(j\beta R)^3$ terms can be neglected. We have the following instantaneous expressions; assuming $i(t) = I\cos\omega t$:

$$\bar{A}(R,\theta;t) = -\bar{a}_\phi \frac{\mu_0 m}{4\pi R}\beta\sin\theta\sin(\omega t - \beta R)$$

$$\bar{E}(R,\theta;t) = \bar{a}_\phi \frac{\omega\mu_0 m}{4\pi R}\beta\sin\theta\cos(\omega t - \beta R)$$

$$\bar{H}(R,\theta;t) = -\bar{a}_\theta \frac{m}{4\pi R}\beta^2\sin\theta\cos(\omega t - \beta R).$$

P.11-4 Far-zone electric field of elemental electric dipole:

$$E_\theta(R) = j\frac{I_0 L}{4\pi}\left(\frac{e^{-j\beta R}}{R}\right)\eta_0\beta\sin\theta \longrightarrow E_\theta(R,t) = -\frac{I_0\eta_0\beta\sin\theta}{4\pi R}(L)\sin(\omega t - \beta R)$$

For the elemental magnetic dipole:

$$E_\phi(R) = \frac{\omega\mu_0 m}{4\pi}\left(\frac{e^{-j\beta R}}{R}\right)\beta\sin\theta \longrightarrow E_\phi(R,t) = \frac{I_0\eta_0\beta\sin\theta}{4\pi R}\left(\frac{2\pi S}{\lambda}\right)\cos(\omega t - \beta R).$$

a) Thus,
$$\frac{E_\theta^2(R,t)}{\left(\frac{I_0\eta_0\beta\sin\theta}{4\pi R}\right)^2(L)^2} + \frac{E_\phi^2(R,t)}{\left(\frac{I_0\eta_0\beta\sin\theta}{4\pi R}\right)^2\left(\frac{2\pi S}{\lambda}\right)^2} = 1$$

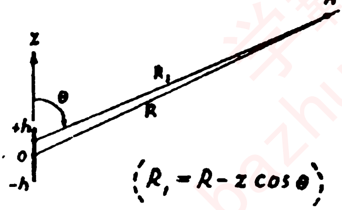
———— Elliptic polarization.

7 b) Circular polarization if $L = 2\pi S/\lambda$.

P.11-5 a)
$$E_\theta = j\frac{I_0\eta_0\beta\sin\theta}{4\pi R}e^{-j\beta R}\int_{-h}^{h}\left(1 - \frac{|z|}{h}\right)e^{j\beta z\cos\theta}dz$$

$$= j\frac{I_0\eta_0\beta\sin\theta}{2\pi R}e^{-j\beta R}\int_0^h\left(1 - \frac{z}{h}\right)\cos(\beta z\cos\theta)dz$$

$$= \frac{j60I_0}{(\beta h)R}e^{-j\beta R}F(\theta)$$



$(R_1 = R - z\cos\theta)$

$$H_\phi = \frac{E_\theta}{\eta_0} = \frac{jI_0}{(\beta h)2\pi R}e^{-j\beta R}F(\theta),$$

$$F(\theta) = \frac{\sin\theta[1 - \cos(\beta h\cos\theta)]}{\cos^2\theta}.$$

In case $\beta h \ll 1$, $\cos(\beta h\cos\theta) \cong 1 - \frac{1}{2!}(\beta h\cos\theta)^2$, and

$$F(\theta) \cong \frac{1}{2}(\beta h)^2\sin\theta.$$

$$\therefore\ E_\theta = \frac{j60I_0}{R}e^{-j\beta R}\left(\tfrac{1}{2}\beta h\sin\theta\right) = \frac{j30\beta h}{R}I_0e^{-j\beta R}\sin\theta$$

$$H_\phi = \frac{jI_0}{2\pi R}e^{-j\beta R}\left(\tfrac{1}{2}\beta h\sin\theta\right) = \frac{j\beta h}{4\pi R}I_0e^{-j\beta R}\sin\theta.$$

b) $W_r = \frac{1}{2}\int_0^{2\pi}\int_0^{\pi}E_\theta H_\phi^* R^2\sin\theta\,d\theta\,d\phi = \frac{I_0^2}{2}\left[80\pi^2\left(\frac{h}{\lambda}\right)^2\right]$

$R_r = W_r/\left(\frac{1}{2}I_0^2\right) = 20\pi^2\left(\frac{2h}{\lambda}\right)^2.$

c) $D = \dfrac{4\pi|E_{max}|^2}{\int_0^{2\pi}\int_0^{\pi}|E_\theta(\theta)|^2\sin\theta\,d\theta\,d\phi} = \dfrac{2}{\int_0^{\pi}\sin^3\theta\,d\theta} = 1.5 \longrightarrow 10\log_{10}D = 1.76\ (\text{dB}).$

5

P.11-6 $2h_e(\theta) = \sin\theta \int_{-h}^{h} \sin\beta(h-|z|)\, e^{j\beta z\cos\theta} dz$

$$= \frac{2[\cos(\beta h\cos\theta) - \cos\beta h]}{\beta\sin\theta}.$$

a) For half-wave dipole, $h = \lambda/4$, $\beta h = \pi/2$.

$$2h_e(\theta) = \frac{2\cos(\frac{\pi}{2}\cos\theta)}{\beta\sin\theta}.$$

b) For maximum $2h_e(\theta)$, set $\frac{d}{d\theta}h_e(\theta) = 0 \longrightarrow \theta = 90°$ or 2

Max. $2h_e(\theta) = 2h_e(90° \text{ or } 270°) = \frac{\lambda}{\pi} = \left(\frac{2}{\pi}\right)\frac{\lambda}{2} = 0.637(2h$

P.11-7 $W_r = \oint \bar{\mathcal{P}}_{av}\cdot d\bar{s} = \frac{1}{2}\mathcal{R}e \int_0^{2\pi}\int_0^{\pi} E_\theta H_\phi^* R^2 \sin\theta\, d\theta\, d\phi$

$$= \frac{(I\,d\ell)^2}{16\pi}\beta^4 R^2 \eta_0 \mathcal{R}e\left\{\left[\frac{1}{j\beta R} + \frac{1}{(j\beta R)^2}\right]\left[-\frac{1}{j\beta R} + \frac{1}{(j\beta R)^2} - \frac{1}{(j\beta R)^3}\right]\right\} \cdot \int_0^{\pi}\sin^3\theta\, d\theta$$

$$= \frac{I^2}{2}\left[80\pi^2\left(\frac{d\ell}{\lambda}\right)^2\right], \text{ same as Eq. (11-37).}$$

9 **P.11-8** From Eqs. (11-27a) and (11-27b):

$$E_\phi = \frac{\omega\mu_0 m}{4\pi}\left(\frac{e^{-j\beta R}}{R}\right)\beta\sin\theta$$

$$H_\theta = -\frac{\omega\mu_0 m}{4\pi\eta_0}\left(\frac{e^{-j\beta R}}{R}\right)\beta\sin\theta$$

$$W_r = \frac{1}{2}\mathcal{R}e\int_0^{2\pi}\int_0^{\pi}(-E_\phi H_\theta^*)R^2\sin\theta\, d\theta\, d\phi = \left(\frac{I^2}{2}\right)320\pi^4\left(\frac{S}{\lambda^2}\right)^2$$

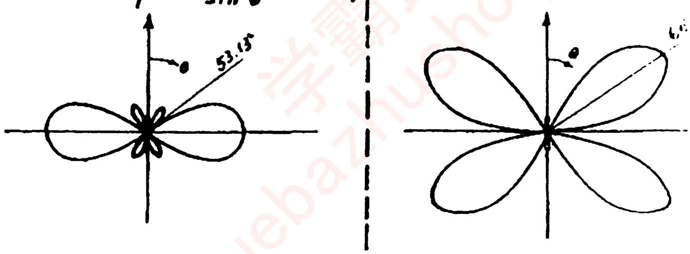
$$\therefore\ R_r = \frac{W_r}{(I^2/2)} = 320\pi^4\left(\frac{S}{\lambda^2}\right)^2.$$

a) Circular loop of radius b: $R_r = 320\pi^6\left(\frac{b}{\lambda}\right)^4 = 20\pi^2\left(\frac{2\pi}{\lambda}\right.$

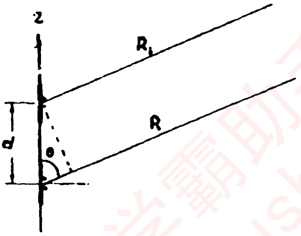
b) Rectangular loop of side L_x and L_y: $R_r = 320\pi^4\left(\frac{L_x L_y}{\lambda^2}\right)^2.$

12 **P.11-9** $F(\theta) = \frac{\cos(\beta h\cos\theta) - \cos\beta h}{\sin\theta}$

a) $2h = 1.25\lambda$, $|F(\theta)| = \left|\frac{\cos(1.25\pi\cos\theta) - \cos(1.25\pi)}{\sin\theta}\right|$ | b) $2h = 2\lambda$, $|F(\theta)| = \left|\frac{\cos(2\pi\cos\theta) - 1}{\sin\theta}\right|$



P.11-10



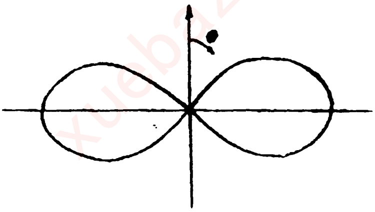
a) $E_\theta = E_{\theta 1} + E_{\theta 2} = \dfrac{j2Ih}{4\pi R}\eta_0 \beta \sin\theta\, e^{-j\beta R} \cdot (1 + e^{j\beta d\cos\theta})$

$$= \frac{j60Ih}{R} 2\beta e^{-j\beta(R - \frac{d}{2}\cos\theta)} F(\theta),$$

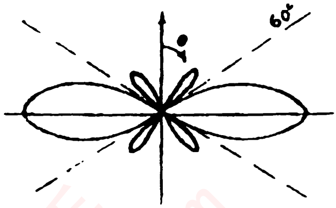
where

$$F(\theta) = \sin\theta \cos\left(\frac{\beta d}{2}\cos\theta\right).$$

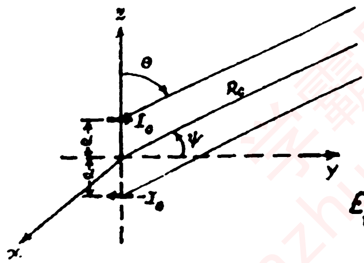
b) $d = \frac{\lambda}{2}$, $|F(\theta)| = |\sin\theta\cos(\frac{\pi}{2}\cos\theta)|$



c) $d = \lambda$, $|F(\theta)| = |\sin\theta\cos(\pi\cos\theta)|$



P.11-11



From Eq. (11-19b):

$$E_{\psi_1} = \frac{jI_0 d\ell\, \eta_0 \beta}{4\pi R_0} e^{-j\beta(R_0 - d\cos\theta)} \sin\psi$$

$$E_{\psi_2} = -\frac{jI_0 d\ell\, \eta_0 \beta}{4\pi R_0} e^{-j\beta(R_0 + d\cos\theta)} \sin\psi.$$

$$E_\psi = E_{\psi_1} + E_{\psi_2} = \frac{jI_0 d\ell\, \eta_0 \beta}{4\pi R_0} e^{-j\beta R_0}\, 2\sin(\beta d\cos\theta)\sin\psi$$

$$E_\psi = j\frac{I_0 d\ell}{2\pi}\left(\frac{e^{-j\beta R_0}}{R_0}\right)\eta_0 \beta \sin(\beta d\cos\theta)\sqrt{1 - \sin^2\phi\sin^2\theta}$$

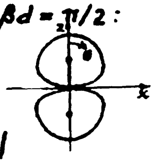
$$\bar{E}_\psi = \bar{a}_\theta E_\theta + \bar{a}_\phi E_\phi = -\frac{E_\psi}{\sqrt{1 - \sin^2\phi\sin^2\theta}}(\bar{a}_\theta \cos\theta\sin\phi + \bar{a}_\phi \cos\phi)$$

a) In the xy-plane, $\theta = 90°$, $F_{xy}(\theta, \phi) = 0$.

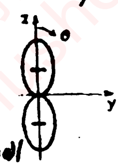
b) In the xz-plane, $\phi = 0°$, $E_\psi = -E_\phi$, $|F_{xz}(\theta)| = |\sin(\beta d\cos\theta)|$

c) In the yz-plane, $\phi = 90°$, $E_\psi = -E_\theta$, $|F_{yz}(\theta)| = |\cos\theta\sin(\beta d\cos\theta)|$.

d) $d = \lambda/4$, $\beta d = \pi/2$:



$|F_{xz}| = |\sin(\frac{\pi}{2}\cos\theta)|$



$|F_{yz}| = |\cos\theta\sin(\frac{\pi}{2}\cos\theta)|$

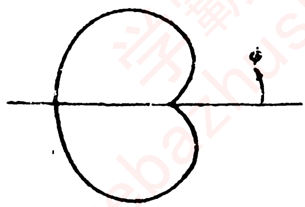
P.11-12 From Eq. (11-57), $|E| = \frac{2E_m}{R}|F(\theta,\phi)|\,|\cos\frac{\psi}{2}|$, where

$$\psi = \beta d \sin\theta\cos\phi + \xi .$$

In the H-plane of a dipole, $\theta = \pi/2$, $F(\frac{\pi}{2},\phi) = 1$.

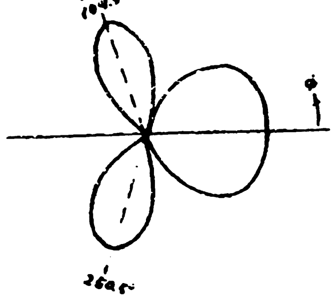
a) $d = \frac{\lambda}{4},\ \xi = \frac{\pi}{2}$

$|f(\phi)| = \left|\cos\left[\frac{\pi}{4}(1+\cos\phi)\right]\right|$



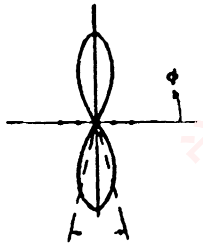
b) $d = \frac{3\lambda}{4},\ \xi = \frac{\pi}{2}$

$|f(\phi)| = \left|\cos\left(\frac{3\pi}{4}\cos\phi + \frac{\pi}{4}\right)\right|$



P.11-13 a) Relative excitation amplitudes: 1:4:6:4:1.

b) Array factor: $|A(\phi)| = \left|\cos\left(\frac{\pi}{2}\cos\phi\right)\right|^4$.



c) $\cos\left(\frac{\pi}{2}\cos\phi\right) = (\sqrt{2})^{-1/4}$

$\longrightarrow \phi = 74.86°$.

Half-power beamwidth

$= 2(90° - 74.86°)$

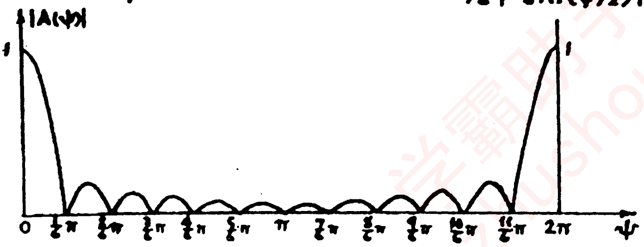
$= 30.28°$.

For uniform array, from Eq. (11-62):

$\frac{1}{5}\left|\frac{\sin\left(\frac{5\pi}{2}\cos\phi\right)}{\sin\left(\frac{\pi}{2}\cos\phi\right)}\right| = \frac{1}{\sqrt{2}} \longrightarrow \phi = 79.61°$

Half-power beamwidth for 5-element uniform array with $\lambda/2$ spacing $= 2(90° - 79.61°) = 20.78°$.

P.11-14 a) From Eq. (11-62) for $N=12$: $|A(\psi)| = \frac{1}{12}\left|\frac{\sin 6\psi}{\sin(\psi/2)}\right|$.



b) <u>Broadside Operation.</u> $\psi = \beta d \cos\phi$.

$$|A(\psi)| = \frac{1}{N}\left|\frac{\sin(N\psi/2)}{\sin(\psi/2)}\right| \cong \left|\frac{\sin X}{X}\right| \quad \text{for } \psi \ll 1,$$

where $X = N\psi/2$.

At half-power points: $\left|\frac{\sin X}{X}\right| = \frac{1}{\sqrt{2}} \longrightarrow X = 1.391$

(for both broadside & endfire operations)

For broadside operation, the half-power beamwidth is

$$(2\Delta\phi)_{1/2} = 0.886\left(\frac{\lambda}{Nd}\right) \quad \text{(rad.)}$$

$$= 50.75\left(\frac{\lambda}{Nd}\right) \quad \text{(deg.)}$$

For $N=12$, $(2\Delta\phi)_{1/2} = 4.23\left(\frac{\lambda}{d}\right)$ (deg.)

From Eq. (11-65): $(2\Delta\phi)_0 = 9.55\left(\frac{\lambda}{d}\right)$ (deg.)

c) <u>Endfire Operation.</u> $\psi = \beta d(\cos\phi - 1)$

$$(2\Delta\phi)_{1/2} = 1.882\sqrt{\frac{\lambda}{Nd}} \text{ (rad.)} = 107.8\sqrt{\frac{\lambda}{Nd}} \text{ (deg.)}$$

For $N=12$, $(2\Delta\phi)_{1/2} = 31.13\sqrt{\frac{\lambda}{d}}$ (deg.)

From Eq. (11-66): $(2\Delta\phi)_0 = 46.78\sqrt{\frac{\lambda}{d}}$ (deg.)

<u>P.11-15</u> $|A(\psi)| = \frac{1}{N}\left|\frac{\sin(N\psi/2)}{\sin(\psi/2)}\right| \cong \left|\frac{\sin X}{X}\right|$, where $X = \frac{-N\psi}{2}$.

Assume broadside operation: $\psi = \beta d \cos\theta$.

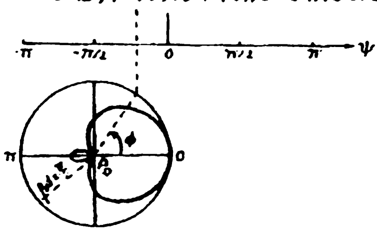
$$D = \frac{4\pi|A(\psi)_{max}|^2}{\int_0^{2\pi}\int_0^{\pi}\left|\frac{\sin X}{X}\right|^2 \sin\theta \, d\theta \, d\phi},$$

$$|A(\psi)_{max}| = 1,$$

$$\int_0^{\pi}\left|\frac{\sin X}{X}\right|^2 \sin\theta \, d\theta \cong \frac{4}{N\beta d}\int_0^{\infty}\left|\frac{\sin X}{X}\right|^2 dX = \frac{4}{N\beta d}\left(\frac{\pi}{2}\right) = \frac{\lambda}{Nd}.$$

$\therefore D = \frac{2Nd}{\lambda} \cong \frac{2L}{\lambda}$, where $L =$ array length.

<u>P.11-16</u> Construction follows the steps outlined on pp. 525-526.



P_0 is at $\xi = -\frac{\pi}{2}$.

Radius of circle is $\beta d = \frac{2\pi}{\lambda}\left(\frac{\lambda}{4}\right) = \frac{\pi}{2}$.

P.11-17 From Eq. (11-43):

$$E_\theta = \frac{j60 I_m N_1 N_2}{R} e^{-j\beta R}\left[\frac{\cos(\frac{\pi}{2}\cos\theta)}{\sin\theta}\right]\left|A_x(\psi_x)A_y(\psi_y)\right| ,$$

where

$$|A_x| = \frac{1}{N_1}\left|\frac{\sin(N_1\psi_x/2)}{\sin(\psi_x/2)}\right| , \quad \psi_x = \frac{\beta d_1}{2}\sin\theta\cos\phi ;$$

$$|A_y| = \frac{1}{N_2}\left|\frac{\sin(N_2\psi_y/2)}{\sin(\psi_y/2)}\right| , \quad \psi_y = \frac{\beta d_2}{2}\sin\theta\cos\phi .$$

$$|F(\theta,\phi)| = \frac{1}{N_1N_2}\left[\frac{\cos(\frac{\pi}{2}\cos\theta)}{\sin\theta}\right]\left|\frac{\sin(\frac{N_1\psi_x}{2})\sin(\frac{N_2\psi_y}{2})}{\sin(\frac{\psi_x}{2})\sin(\frac{\psi_y}{2})}\right| .$$

P.11-18 From Eq. (11-77): $P_L = A_e \mathcal{P}_{av}$. ①

Consider an elemental electric (Hertzian) dipole of length $d\ell$ in the field of an incident plane wave with an electric intensity E_i. $\quad \mathcal{P}_{av} = \frac{|E_i|^2}{2\eta_0}$. ②

Maximum power is absorbed by the load if $Z_L = Z_g^*$.

$$P_L = \frac{1}{2}|I|^2 R = \frac{1}{2}\left(\frac{E_i d\ell}{Z_g + Z_g^*}\right)^2 R = \frac{(E_i d\ell)^2}{8R} \quad ③$$

Combining ①, ②, and ③, we have

$$\left.\begin{aligned} A_e &= \frac{\eta_0}{4R}(d\ell)^2 = \frac{30\pi}{R}(d\ell)^2. \\ \text{From Eq. (11-38): } R &= 80\pi^2\left(\frac{d\ell}{\lambda}\right)^2 \end{aligned}\right\} \longrightarrow A_e = \frac{3}{8\pi}\lambda^2.$$

From p.511, $D = G_D(\frac{\pi}{2},\phi) = \frac{3}{2} \longrightarrow \frac{A_e}{D} = \frac{\lambda^2}{4\pi}$.

P.11-19 From Eq. (11-89): $\quad \frac{P_L}{P_t} = \left(\frac{\lambda}{4\pi r}\right)^2 G_{D1}G_{D2}$.

a) For half-wave dipoles: $G_{D1} = G_{D2} = 1.64$

$f = 3\times10^8$ (Hz), $\lambda = c/f = 1$ (m), $r = 1.5\times10^3$ (m).

$P_L = 7.57\times10^{-9}P_t = 7.57\times10^{-7}$ (W) $= 0.757$ (μW).

b) For Hertzian dipoles: $G_{D1} = G_{D2} = 1.5$

$P_L = 0.633$ (μW).

P.11-20 Let P_T = Power intercepted by the target.

a) $\frac{P_T}{P_t} = \frac{A_T G_D}{4\pi r^2}$, $\quad \frac{P_L}{P_T} = \frac{A_D G_T}{4\pi r^2}$; $\quad G_T = \frac{4\pi}{\lambda^2}$, $A_D = \frac{\lambda^2}{4\pi}G_D$.

$$\therefore \quad \frac{P_L}{P_t} = \left(\frac{P_L}{P_T}\right)\left(\frac{P_T}{P_t}\right) = \frac{A_T^2 G_D^2}{(4\pi r^2)^2}.$$

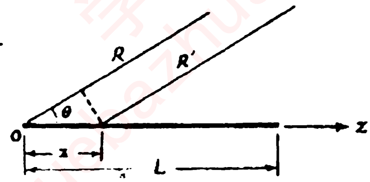
b) Incident power density at the target, $\mathcal{P}_i = \frac{P_t}{4\pi r^2} G_D$.

Power scattered by the target in the direction of the antenna, $P_{sc} = \mathcal{P}_i A_T G_T = \mathcal{P}_i \frac{4\pi}{\lambda^2} A_T^2$

$$\therefore \quad S_r = \frac{P_{sc}}{\mathcal{P}_i} = \frac{4\pi}{\lambda^2} A_T^2 .$$

From the result of part a): $\frac{P_L}{P_t} = \frac{S_r G_D^2 \lambda^2}{(4\pi)^3 r^4}$.

P.11-21



$I(z) = I_0 e^{-j\beta z}$

a) $\bar{A} = \bar{a}_z \frac{\mu}{4\pi} \int_0^L \frac{I_0 e^{-j\beta z} e^{-j\beta R'}}{R'} dz$

In the far-zone, $R' \cong R - z\cos\theta$.

$$\bar{A}(R,\theta) = \bar{a}_z \frac{\mu I_0 e^{-j\beta R}}{4\pi R} \int_0^L e^{-j\beta z(1-\cos\theta)} dz$$

$$= \bar{a}_z \frac{\mu I_0}{2\pi R\beta} e^{-j\beta R} e^{-j\beta \frac{L}{2}(1-\cos\theta)} F(\theta),$$

where $F(\theta) = \frac{\sin\left[\frac{\beta L}{2}(1-\cos\theta)\right]}{1-\cos\theta}$.

b) $A_R = A_z \cos\theta$, $A_\theta = -A_z \sin\theta$, $A_\phi = 0$.

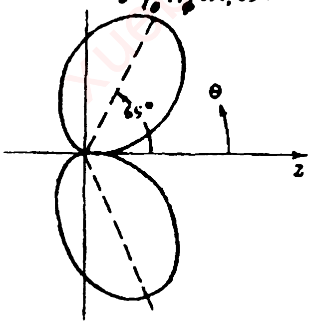
$\bar{H} = \frac{1}{\mu}\bar{\nabla}\times\bar{A} = \bar{a}_\phi \frac{1}{\mu R}\left[\frac{\partial}{\partial R}(RA_\theta) - \frac{\partial A_R}{\partial\theta}\right]$.

In the far-zone, $\frac{1}{R}\frac{\partial A_R}{\partial\theta} \propto \frac{1}{R^2} \longrightarrow \bar{H} = \bar{a}_\phi \frac{1}{\mu R}\frac{\partial}{\partial R}(RA_\theta)$

$\bar{H}(R,\theta) = \bar{a}_\phi \frac{jI_0}{2\pi R} e^{-j\beta[R + L(1-\cos\theta)/2]} F(\theta)\sin\theta$.

$\bar{E}(R,\theta) = \bar{a}_\theta \eta_0 H_\phi(R,\theta)$.

c)



Radiation pattern for $L = \lambda/2$: Plot of $|F(\theta)\sin\theta|$.

P.11-22 From Eq. (11-97b):

$$F(\theta,\phi)=\iint_{aper.} f(x',y')\,e^{j\beta\sin\theta(x'\cos\phi+y'\sin\phi)}dx'dy'.$$

a) In the xz-plane, $\phi=0°$:

$$F_{xz}(\theta)=b\int_{-a/2}^{a/2} f(x')\,e^{j\beta x'\sin\theta}\,dx'$$

$$=2b\int_0^{a/2}\left(1-\frac{2}{a}x'\right)\cos(\beta x'\sin\theta)\,dx'$$

$$=ab\,\frac{1-\cos\left(\frac{\beta a}{2}\sin\theta\right)}{\left(\frac{\beta a}{2}\sin\theta\right)^2}. \qquad \text{Let } \psi=\frac{\beta a}{2}\sin\theta=\frac{\pi a}{\lambda}\sin\theta.$$

$$F_{xz}(\theta)=\frac{ab}{2}\left[\frac{\sin(\psi/2)}{(\psi/2)}\right]^2.$$

b) Set $\left[\frac{\sin(\psi/2)}{(\psi/2)}\right]^2=\frac{1}{\sqrt{2}} \longrightarrow \frac{\psi}{2}=1.005.$

Half-power beamwidth $(2\Delta\theta)_{1/2}=2\sin^{-1}\left(0.640\frac{\lambda}{a}\right)$.

For $\lambda/a \ll 1$, $(2\Delta\theta)_{1/2}\cong 1.280\frac{\lambda}{a}$ (rad)

$=73.3\frac{\lambda}{a}$ (deg).

c) Set $\frac{\psi}{2}=\pi \longrightarrow \theta_{n1}=\sin^{-1}\left(\frac{2\lambda}{a}\right)\cong\frac{2\lambda}{a}$ (rad)

$=114.6\frac{\lambda}{a}$ (deg).

d) First-sidelobe level: $L_1=-20\log_{10}\left(\frac{1}{3\pi/2}\right)^2=26.9$ (dB).

	Uniform Distr.	Triangular Distr.
Pattern function	$ab\left(\frac{\sin\psi}{\psi}\right)$	$\frac{ab}{2}\left(\frac{\sin\frac{\psi}{2}}{\frac{\psi}{2}}\right)^2$
Half-power beamwidth	$50\frac{\lambda}{a}$ (deg)	$73.3\frac{\lambda}{a}$ (deg)
Location of first null	$57.3\frac{\lambda}{a}$ (deg)	$114.6\frac{\lambda}{a}$ (deg)
First-sidelobe level	13.3 (dB)	26.9 (dB)

P.11-23 a) In the xz-plane, $\phi=0°$:

$$F_{xz}(\theta)=2b\int_0^{a/2}\cos\left(\frac{\pi x'}{a}\right)\cos(\beta x'\sin\theta)\,dx'$$

$$=\frac{2ab}{\pi}\left[\frac{(\pi/2)^2\cos\psi}{(\pi/2)^2-\psi^2}\right], \qquad \psi=\frac{\beta a}{2}\sin\theta=\frac{\pi a}{\lambda}\sin\theta.$$