Signposts

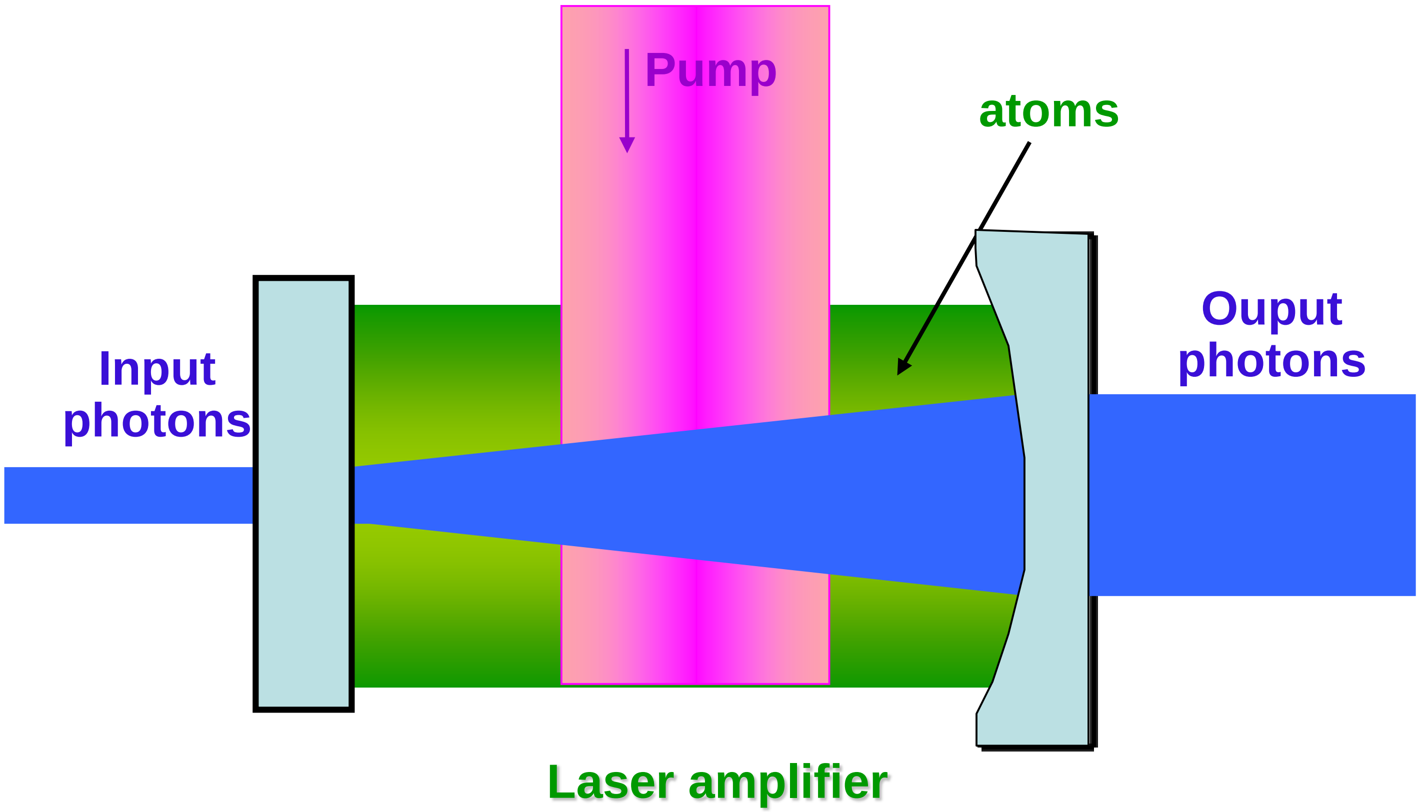
- Beware when Nobel Laureates say something cannot be done!

Beware when Nobel Laureates say something cannot be done!

- Even in the industrial research environment, when exploration of a technology includes a will to explore fundamental basic science, important discoveries can occur

Chapter 4
Laser Amplifiers

Concept of the laser amplifier



$$P_{sp} = \frac{1}{t_{sp}}$$

$$P_{ab} = n\frac{c}{V}\sigma(\nu)$$

$$P_{st} = n\frac{c}{V}\sigma(\nu)$$

$$\sigma(\nu) = \frac{\lambda^2}{8\pi t_{sp}} g(\nu)$$

$$g(\nu) = \frac{\Delta\nu/2\pi}{(\nu-\nu_0)^2 + (\Delta\nu/2)^2}$$

$$\Delta\nu = 1/2\pi\tau$$

$$W_i = P_{ab} = \phi\sigma(\nu)$$

$$W_i = \frac{\lambda^3}{8\pi h t_{sp}} \rho(\nu_0)$$

$$W_i = \frac{\bar{n}}{t_{sp}}$$

$$\begin{cases} P_{sp} = A \\ W_i = B\rho(\nu_0) \end{cases}$$

$$\frac{B}{A} = \frac{\lambda^3}{8\pi h}$$

$$\Delta\nu = \frac{1}{2\pi}\left(\frac{1}{\tau_1} + \frac{1}{\tau_2}\right)$$

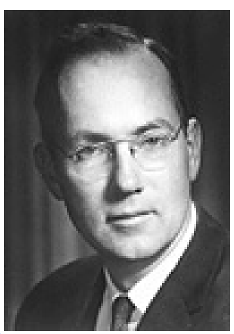
$$\frac{1}{\tau} = \frac{1}{\tau_1} + \frac{1}{\tau_2}$$

$$\tau = \frac{g(\nu_0)}{4}$$

Remember to the inventors of laser amplifier

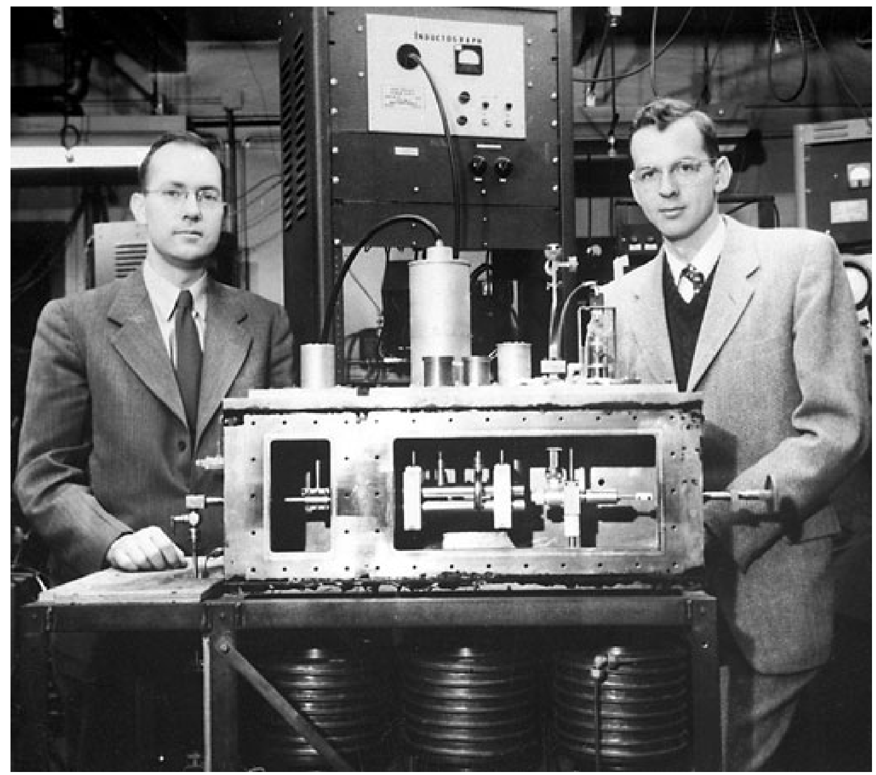
Aleksandr **<u>Prokhorov</u>** *(1916-2002)普罗霍罗夫*

Nikolai **<u>Basov</u>** **<u>(1922-2001)</u>**



Charles Hard **<u>Townes</u>** **<u>(1915-)</u>**

The Nobel Prize in Physics 1964

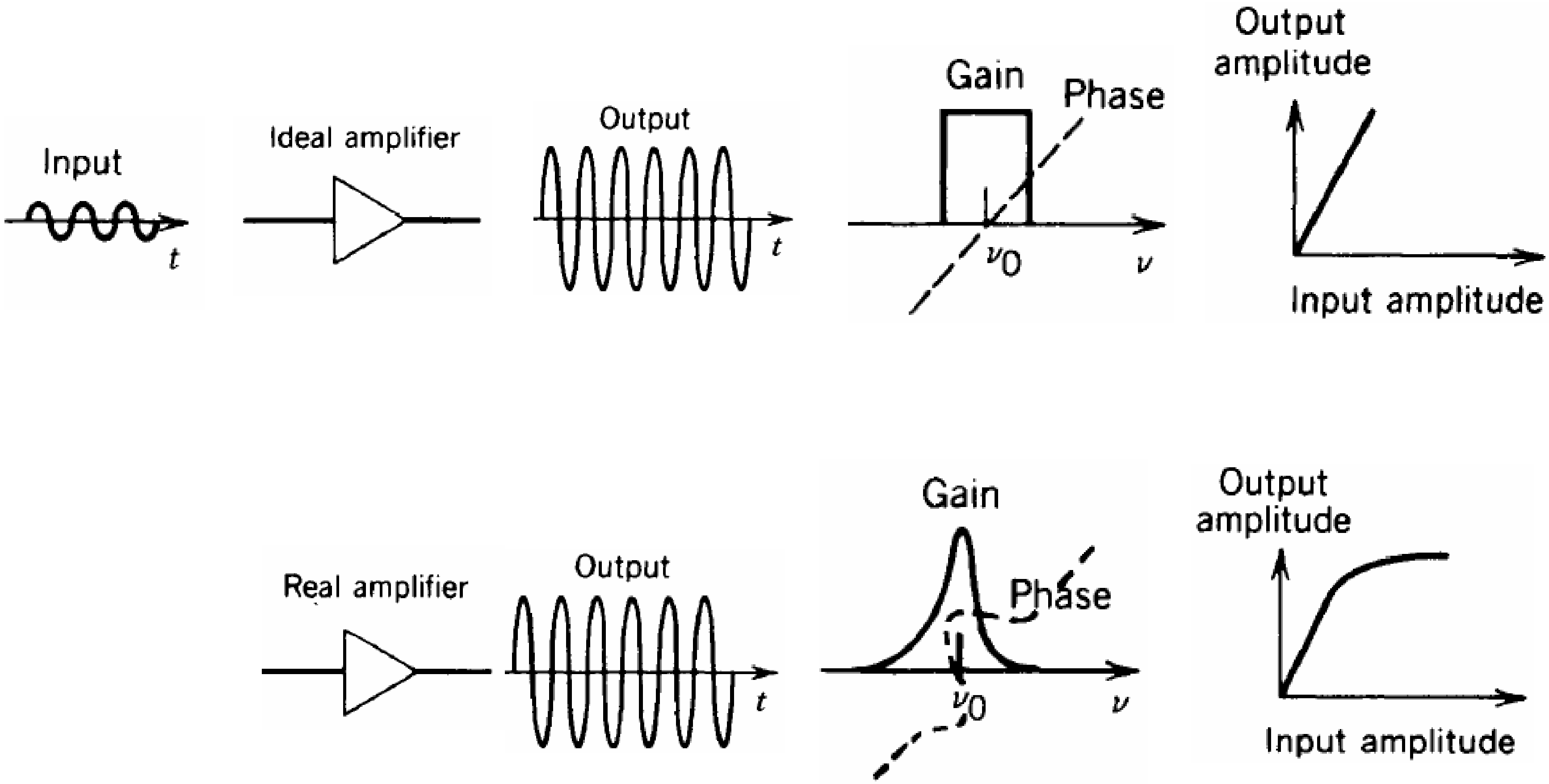


Outline

Optical Regeneration

Ideal analog amplification

- Faithfully reproduces input signal with minimal distortion
- Linear repeater by periodically boosting
- Optical power can be used in nonlinear region as a level clamping amplifier
- Can be used as a multichannel amplifier
- Minimal crosstalk and distortion

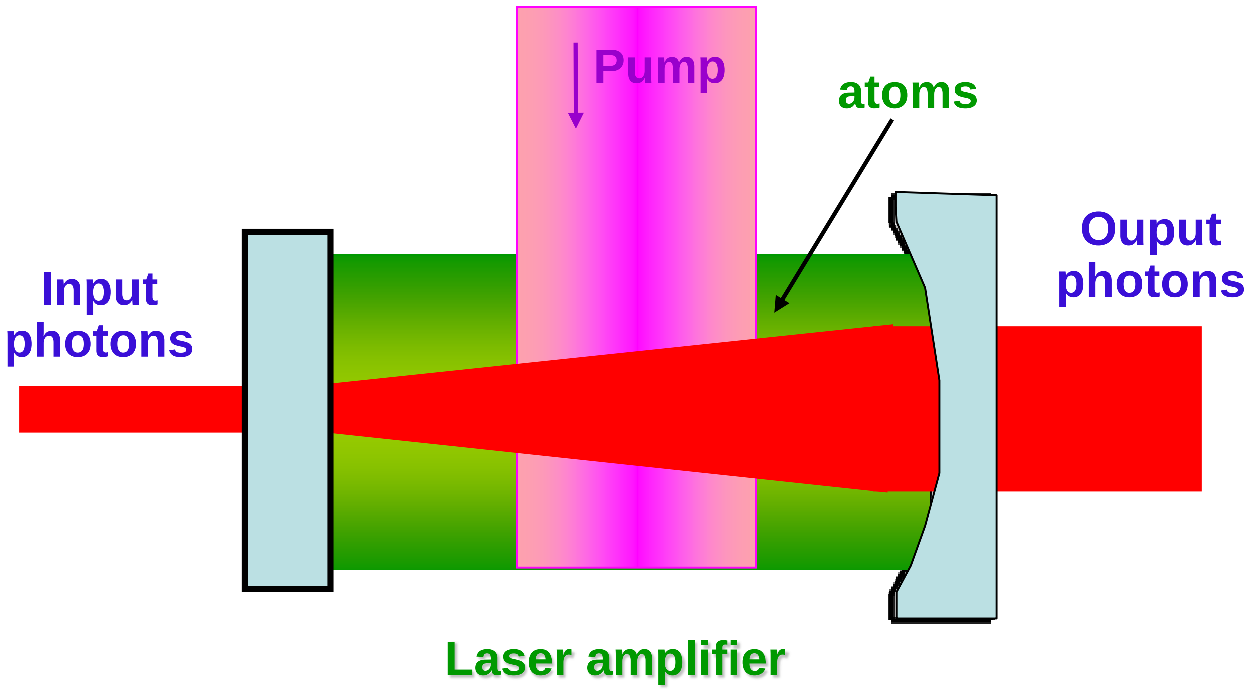


Input
t
Ideal amplifier
Output
t
Gain
Phase
ν_0
ν
Output amplitude
Input amplitude
Real amplifier
Output
t
Gain
Phase
ν_0
ν
Output amplitude
Input amplitude

Real Amplifier

- Gain
- Bandwidth
- Phase shift
- Power source
- Nonlinearity and gain saturation
- Noise

4.1 The laser Amplifier

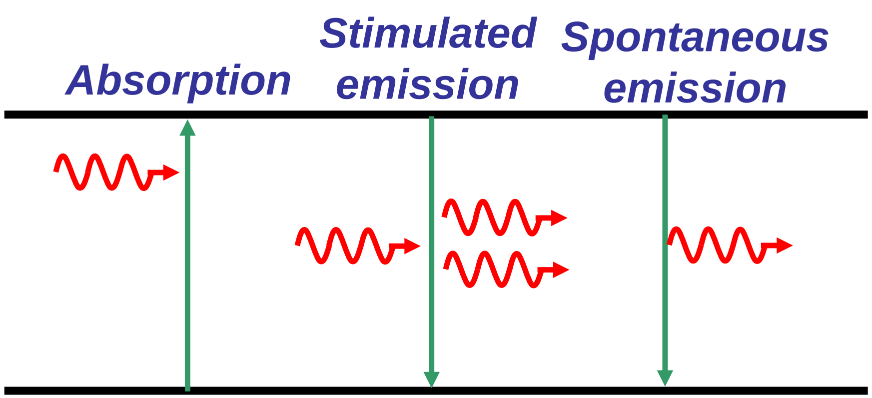


The laser amplifier. An external power source (called a pump) excites the active medium (represented by a collection of atoms), producing a population inversion. Photons interact with the atoms; when stimulated emission is more prevalent than absorption, the medium acts as a coherent amplifier.

Optical Amplifier Physics

An atomic system with two energy levels can

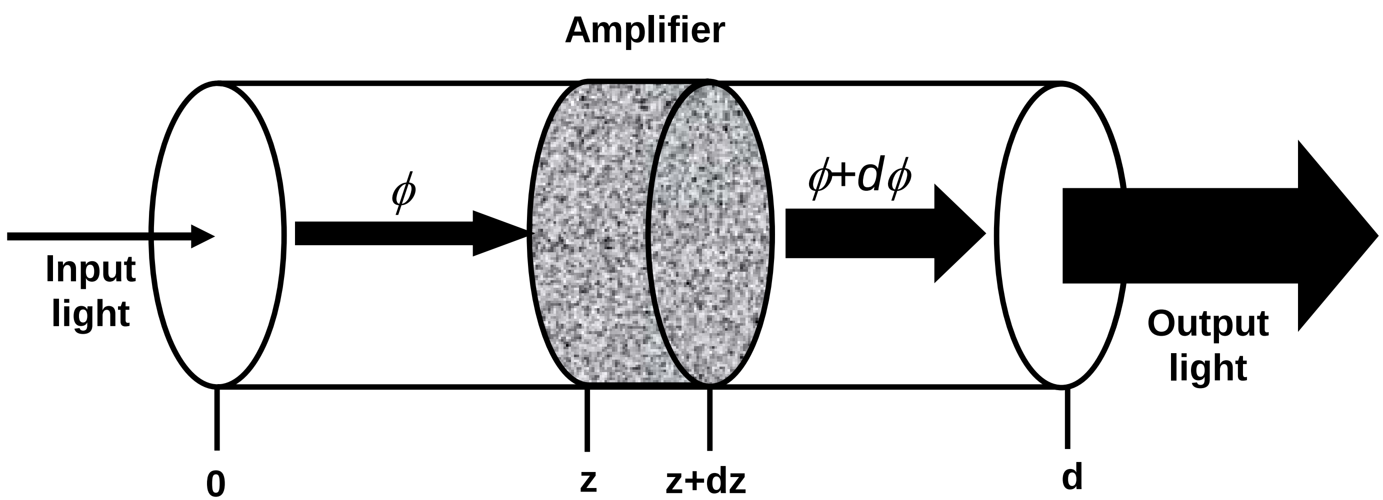
- **absorb light**
- **amplify light**
- **spontaneously emit light**



Stimulated and spontaneous emission are achieved by pumping the amplifier electrically or optically.

A. Laser gain

The photon-flux density ϕ (photons/cm^2-s) entering an incremental cylinder containing excited atoms grows to $\phi+d\phi$ after length *dz*.



The probability density (s^{-1}) W_i for stimulated emission is:

$$W_i = P_{ab} = n\frac{c}{V}\sigma(\nu) = \phi\frac{V}{c}\frac{c}{V}\sigma(\nu) = \phi\sigma(\nu)$$

where $\sigma(\nu) = (\lambda^2/8\pi t_{sp})g(\nu)$

Gain coefficient of laser medium

The change of photon flux is

$$d\phi = NW_i dz$$

where $N=N_2-N_1$ population difference between two states

We can write the photon flux changes as

$$\frac{d\phi(z)}{dz} = \gamma(\nu)\phi(z) = NW_i$$

By define the Gain coefficient of laser medium $\gamma(\nu)$:

$$\gamma(\nu) = N\sigma(\nu) = N\frac{\lambda^2}{8\pi t_{sp}}g(\nu)$$

where $\sigma(\nu) = (\lambda^2/8\pi t_{sp})g(\nu)$

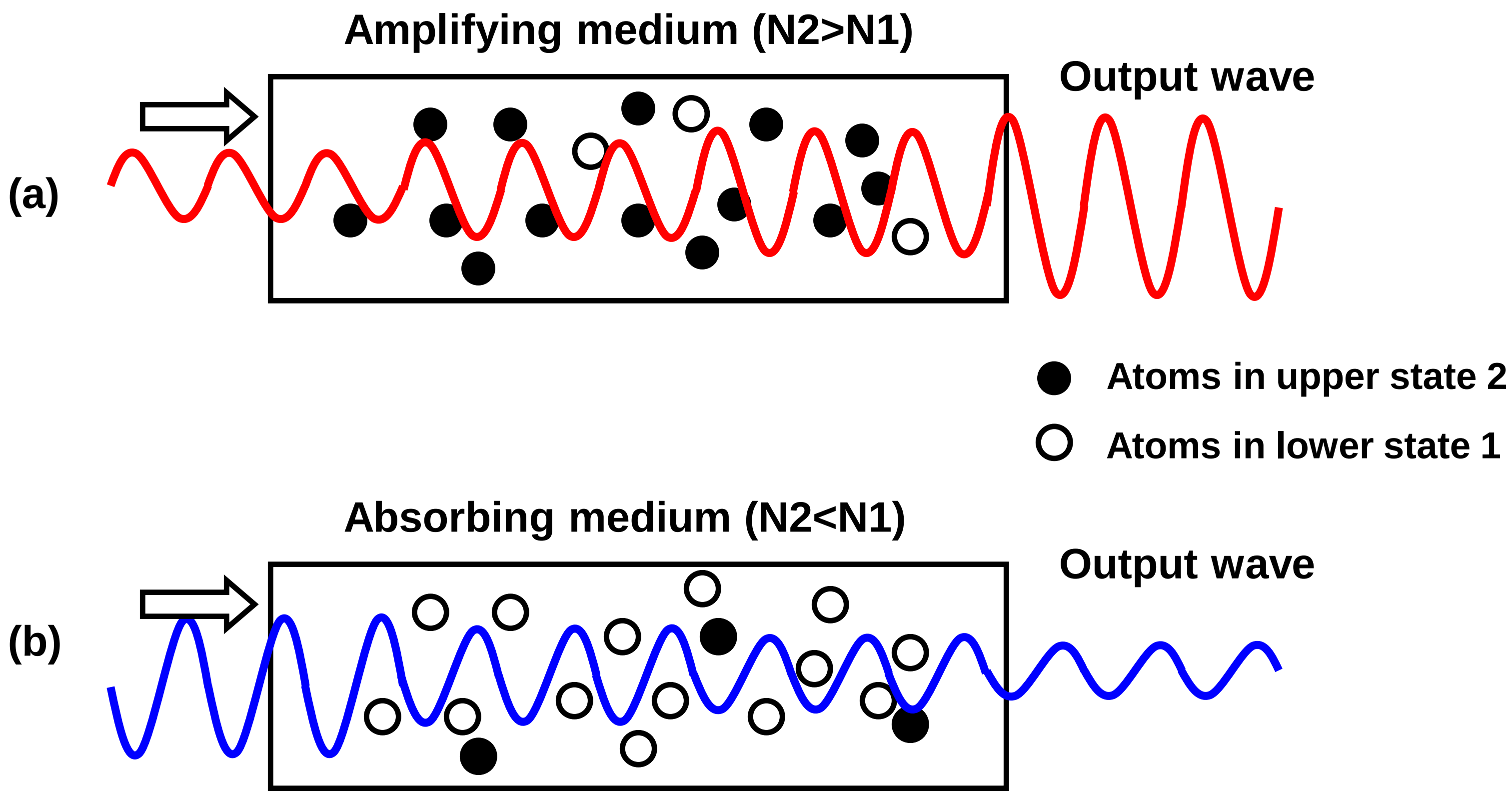
The relation between the population inversion and light Amplification

- Amplifier gain coefficient $\gamma(\nu)$ is seen to be proportional to the population difference $N=N_2 - N_1$.
- If N is positive, it is means population inversion, and gain coefficient is positive, light will be amplified
- If N is negative, it is means absence of population inversion, but because of absorption, the light is diminished

$$\gamma(\nu) = (N_2 - N_1)\frac{c^2}{8\pi n^2 \nu^2 t_{sp}} g(\nu)$$

$$if: \gamma(\nu) > 0 \xrightarrow{N_2 > N_1} \textbf{Amplification}$$

$$if: \gamma(\nu) < 0 \xrightarrow{N_2 < N_1} \textbf{Attenuation}$$



(a) Amplification of a traveling electromagnetic wave in an inverted population (N2>N1), and (b) attenuation in a absorbing medium (N2<N1).

Population Inversion

Negative temperature

At thermal equilibrium $$\frac{N_2}{N_1} = \exp(-h\nu / k_B T)$$

$case1 : N_2 < N_1$ → ***As usual, T>0***

$case2 : N_2 > N_1$ → ***Negative temperature***

Population Inversion

light intensity grows exponentially!!

From the equation

$$\frac{d\phi(z)}{dz} = \gamma(\nu)\phi(z)$$

we have the photon flux as :

$$\phi(z) = \phi(0)\exp[\gamma(\nu)z]$$

We define the **Amplifier Gain** $G(\nu)$ as:

for an interaction region of total length *d* , the overall gain of the laser amplifier *G(v)* is defined as the ratio of the photon-flux density at the output to the photon-flux density at the input,

$$G(\nu) = \phi(d)/\phi(0)$$

Amplifier Gain

$$G(\nu) = \exp[\gamma(\nu)d]$$

Amplification bandwidth

The dependence of the gain coefficient $\gamma(v)$ on the frequency of the incident light v is contained in its proportionality to the lineshape function $g(v)$,

$$\gamma(\nu) = N\sigma(\nu) = N\frac{\lambda^2}{8\pi t_{sp}}g(\nu)$$

The bandwidth Δv centered about the atomic resonance frequency :

$v_0 = (E_2 – E_1)/h$,

where E_2 and E_1 are the atomic energies.

The laser amplifier is a resonant device, with a resonance frequency and bandwidth determined by the line-shape function of the atomic transition.

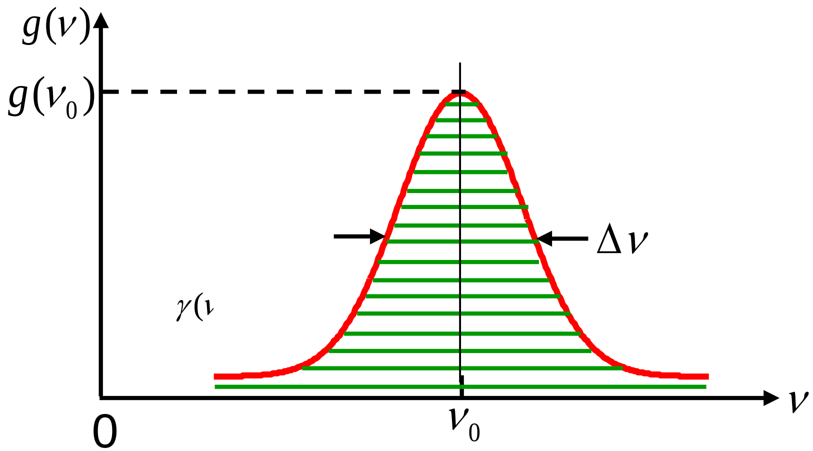
The linewidth $\Delta\nu$ is measured either in units of frequency (Hz) or in units of wavelength

$$\Delta\lambda = |\Delta(c_o/\nu)| = +(c_o/\nu^2)\,\Delta\nu = (\lambda_o^2/c_o)\,\Delta\nu \quad \Rightarrow \quad \Delta\lambda = \left(\frac{\lambda_0^2}{c_0}\right)\Delta\nu$$

Lorentzian lineshape

$$g(\nu)=\frac{\Delta\nu/2\pi}{(\nu-\nu_0)^2+(\Delta\nu/2)^2}=g(\nu_0)\frac{(\Delta\nu/2)^2}{(\nu-\nu_0)^2+(\Delta\nu/2)^2}$$

and $g(\nu_0)=2/\pi\Delta\nu$



The gain coefficient is

$$\gamma(\nu)=N\frac{\lambda^2}{8\pi t_{sp}}g(\nu)$$

then also Lorentzain with the **same width**,

$$\gamma(\nu)=\gamma(\nu_0)\frac{(\Delta\nu/2)^2}{(\nu-\nu_0)^2+(\Delta\nu/2)^2}$$

where

$$\gamma(\nu_0)=N(\lambda^2/4\pi^2 t_{sp}\,\Delta\nu)$$

B. Amplifier phase shift

The optical intensity and field are related by: $I(z) = \left|E(z)\right|^2 / 2\eta$

Since $I(z) = I(0)\exp(\gamma(\nu)z)$

in accordance with the optical field obeys the relation

$$E(z) = E(0)\exp[\frac{1}{2}\gamma(\nu)z]\exp[-i\varphi(\nu)z]$$

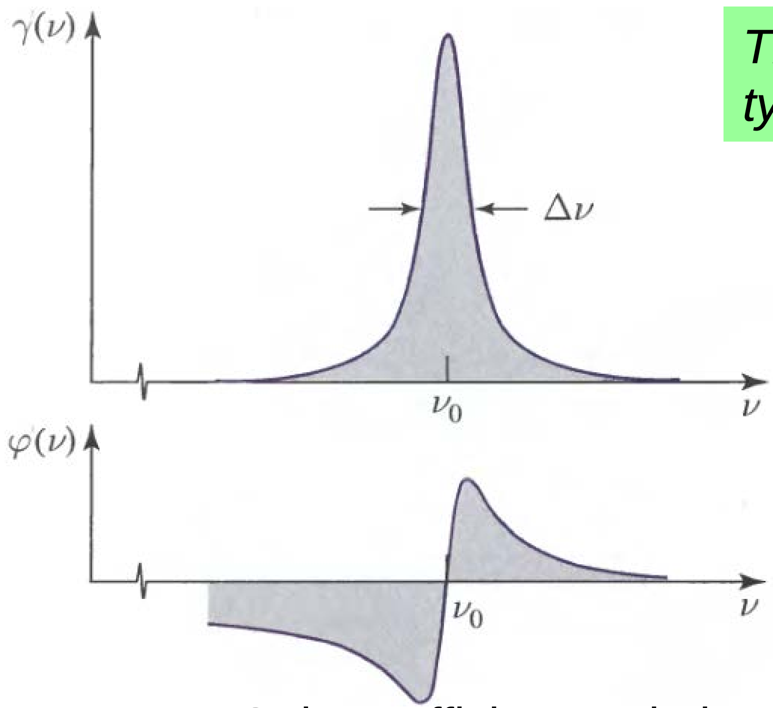
So that

$$E(z+\Delta z) = E(0)\exp[\frac{1}{2}\gamma(\nu)z + \frac{1}{2}\gamma(\nu)\Delta z]\exp[-\{i\varphi(\nu)z + i\varphi(\nu)\Delta z\}]$$

$$\approx E(z)\left[1 + \frac{1}{2}\gamma(\nu)\Delta z - i\varphi(\nu)\Delta z\right]$$

$$\Rightarrow \quad \frac{\Delta E}{\Delta z} = E(z)\left[\frac{1}{2}\gamma(\nu) - i\phi(\nu)\right]$$

The function of a linear causal system are related by the Hilbert transform. It follows that - $\phi(v)$ is the Hilbert transform of $0.5\gamma(v)$.
so that the ***amplifier phase shift function is determined by its gain coefficient***.



The phase shift coefficient (for Lorentzian type line shape) is

$$\varphi(\nu) = \frac{\nu - \nu_0}{\Delta\nu}\gamma(\nu)$$

$$\varphi(\nu) = \frac{\nu - \nu_0}{\Delta\nu}\gamma(\nu_0)\frac{(\Delta\nu/2)^2}{(\nu-\nu_0)^2 + (\Delta\nu/2)^2}$$

Gain coefficient and phase-shift coefficient for a laser amplifier with a Lorentzian line-shape function

4.2 AMPLIFIER POWER SOURCE

A. Rate Equations

Gain coefficient

$$\gamma(\nu) = N\sigma(\nu) = N\frac{\lambda^2}{8\pi t_{sp}} g(\nu)$$

So that the amplification is very concern on N, one can used the changement of N to describe the laser function

Definition of Rate equations:

The equations that describe the rates of change of the population densities N, and N, as a result of pumping, radiative, and nonradiative transitions.

Atomic rate equations

•1. Radiation-atom interaction:

- Stimulated emission
- Absorption

2. Population inversion and laser pumping:

$$N_2 - \frac{g_2}{g_1} N_1 > 0$$

3. Lifetime of atoms in upper energy level:

$$N(t) = N_0 e^{-\frac{t}{\tau}}$$

τ : ***lifetime*** of atoms in the upper energy level.

Either the radiation-atom interaction, laser pumping and energy decay change the population density distribution.

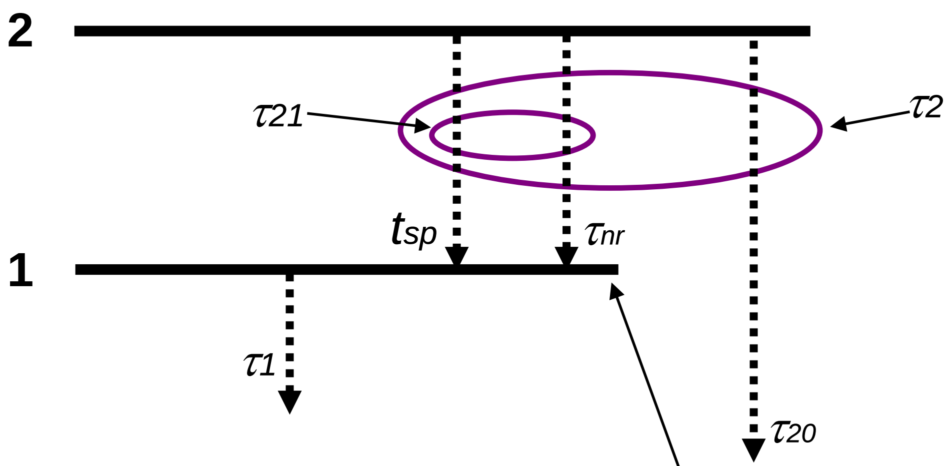
To describe in details the rates of these changes

↓

Rate equations

Two level system transition

The overall lifetimes of level 1 and level 2 are τ_1 *and* τ_2



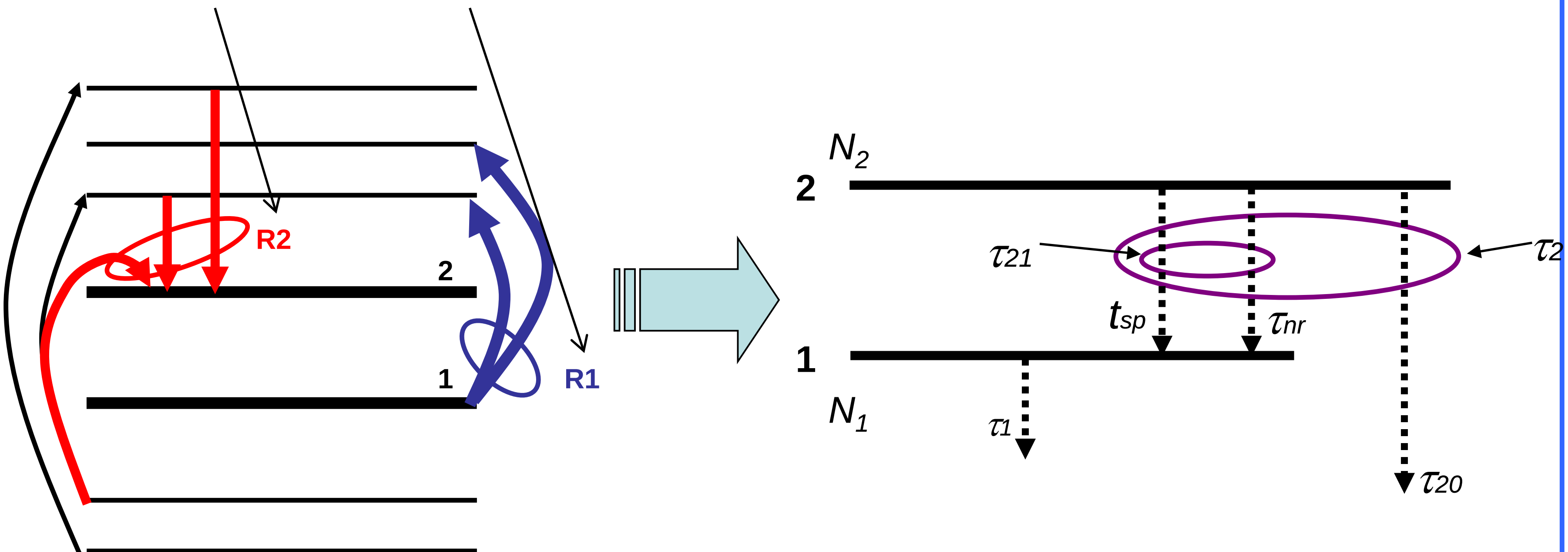
Energy levels 1 and 2 and their decay times.

$$\tau_2^{-1} = \tau_{21}^{-1} + \tau_{20}^{-1}$$

$$\tau_{21}^{-1} = t_{sp}^{-1} + \tau_{nr}^{-1}$$

Non radiation transmission

Pumping and depumping rate

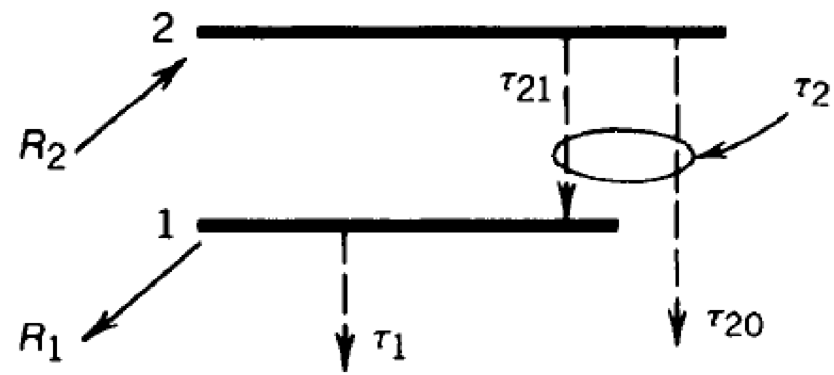


Energy levels 1 and 2, together with surrounding higher and lower energy levels, as there are pumping and transmission from other higher levels

Rate equations in the absence of amplifier radiation

$$\frac{dN_2}{dt} = R_2 - \frac{N_2}{\tau_2}$$

$$\frac{dN_1}{dt} = -R_1 - \frac{N_1}{\tau_1} + \frac{N_2}{\tau_{21}}$$



Energy levels 1 and 2 and their decay times. By means of pumping, the population density of level 2 is increased at the rate R_2 while that of level 1 is decreased at the rate R_1

Steady-state population difference (in absence of amplifier radiation)

$$(dN_1/dt = dN_2/dt = 0)$$

$$N_0 = R_2\tau_2\left(1 - \frac{\tau_1}{\tau_{21}}\right) + R_1\tau_1$$

where the symbol N_0 represents the steady-state population difference $N = N_2 - N_1$ in the absence of amplifier radiation.

the steady-state population difference $N=N_2-N_1$ in the absence of amplifier radiation

$$N_0 = R_2\tau_2(1-\frac{\tau_1}{\tau_{21}}) + R_1\tau_1$$

For large N_0, *needs*

- Large R_1 and R_2
- Long τ_2 (but τ_{sp} which contributes to τ_2 through τ_{21} must be sufficiently long so as to make the radiative transition large)
- Short τ_1, if $R_1<(\tau_2/\tau_{21})R_2$

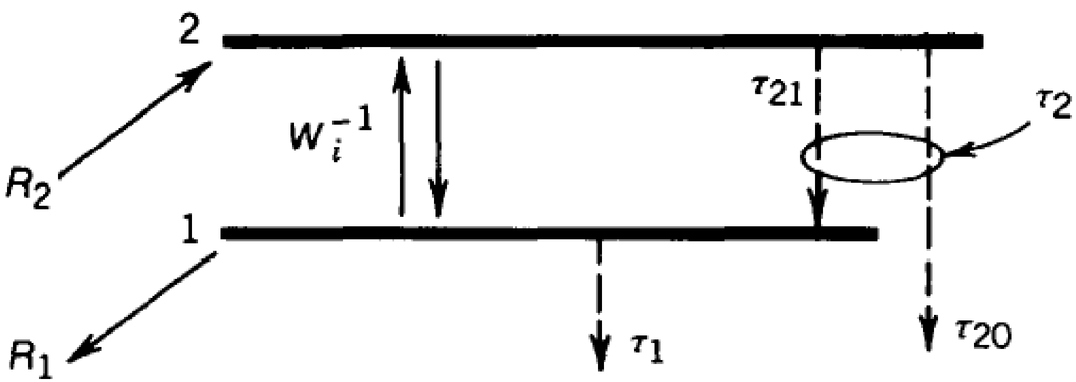
$$\begin{cases} \tau_2^{-1} = \tau_{21}^{-1} + \tau_{20}^{-1} \\ \tau_{21}^{-1} = t_{sp}^{-1} + \tau_{nr}^{-1} \end{cases}$$

$\tau_{21} \approx t_{sp} \ll \tau_{20}$ so that $\tau_2 \approx t_{sp}$ and $\tau_1 \ll t_{sp}$ $\Rightarrow$ $N_0 \approx R_2 t_{sp} + R_1\tau_1$

In case of absent of depumping $(R_1 = 0)$, or when $R_1 \ll (t_{sp}/\tau_1)R_2$

$$N_0 \approx R_2 t_{sp}$$

Rate equations in the presence of amplifier radiation



$$\begin{cases} \dfrac{dN_2}{dt} = R_2 - \dfrac{N_2}{\tau_2} - N_2 W_i + N_1 W_i \\[2ex] \dfrac{dN_1}{dt} = -R_1 - \dfrac{N_1}{\tau_1} + \dfrac{N_2}{\tau_{21}} + N_2 W_i - N_1 W_i \end{cases}$$

The population densities $N_1$and N_2 (cm^{-3}-s^{-1}) of atoms in energy levels 1 and 2 are determined by three processes: decay (at the rates $1/\tau_1$ and $1/\tau_2$ respectively, which includes the effects of spontaneous emission), pumping (at the rates – R_1 and R_2 respectively), the absorption and stimulated emission (at the rate w_i)

For steady state, there are $(dN_1/dt = dN_2/dt = 0)$

$$\begin{cases} N = \dfrac{N_0}{1+\tau_s W_i} \\[2ex] \tau_s = \tau_2 + \tau_1\left(1-\dfrac{\tau_2}{\tau_{21}}\right) \end{cases}$$

Steady-state population difference
(**in present of amplifier radiation**)

Saturation time constant

where N_0 is the steady-state population difference in the absence of amplifier radiation,

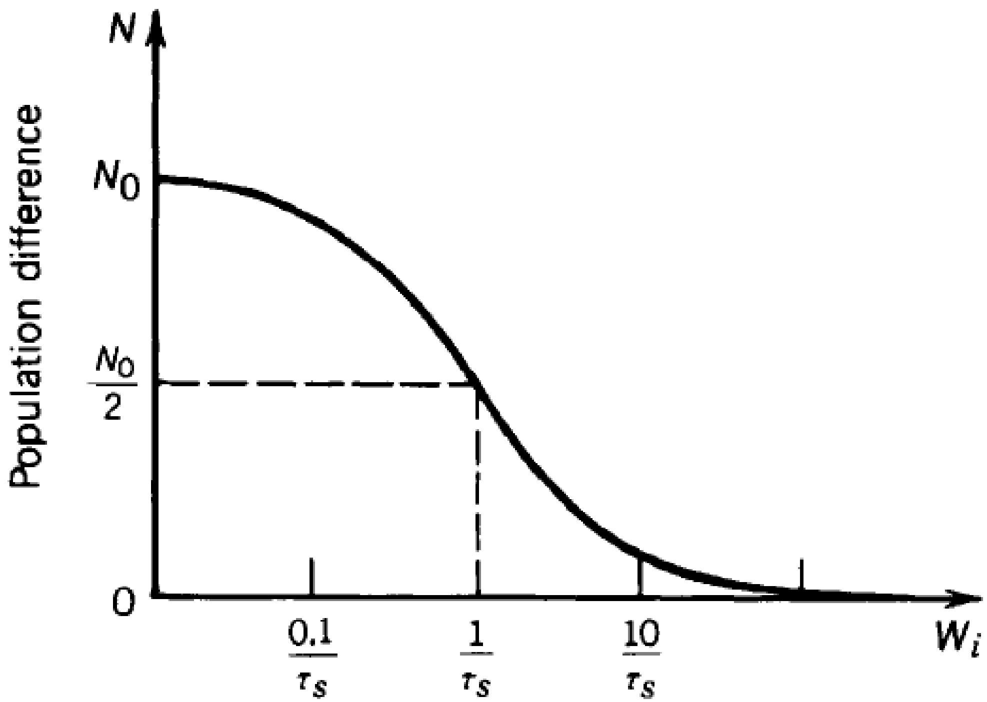
The quantity τ_s plays the role of a ***saturation time constant***,

$$N = \frac{N_0}{1+\tau_s W_i} \quad \textit{where} \quad \tau_s = \tau_2 + \tau_1 (1 - \frac{\tau_2}{\tau_{21}})$$

- In case of absence of amplification, $W_i=0$, so that $N=N_0$.
- Because τ_s is positive, the steady-state population difference in the presence of radiation always has a smaller absolute value than in the absence of radiation.
- If the radiation is sufficiently weak (the small-signal approximation) $\tau_s W_i \ll 1$, we may take $N = N_0$.
- As the radiation becomes stronger, W_i increases, and N approaches zero regardless of the initial sign of N_0.

➢ τ_s is the ***saturation time constant***

τ_s Saturation time constant

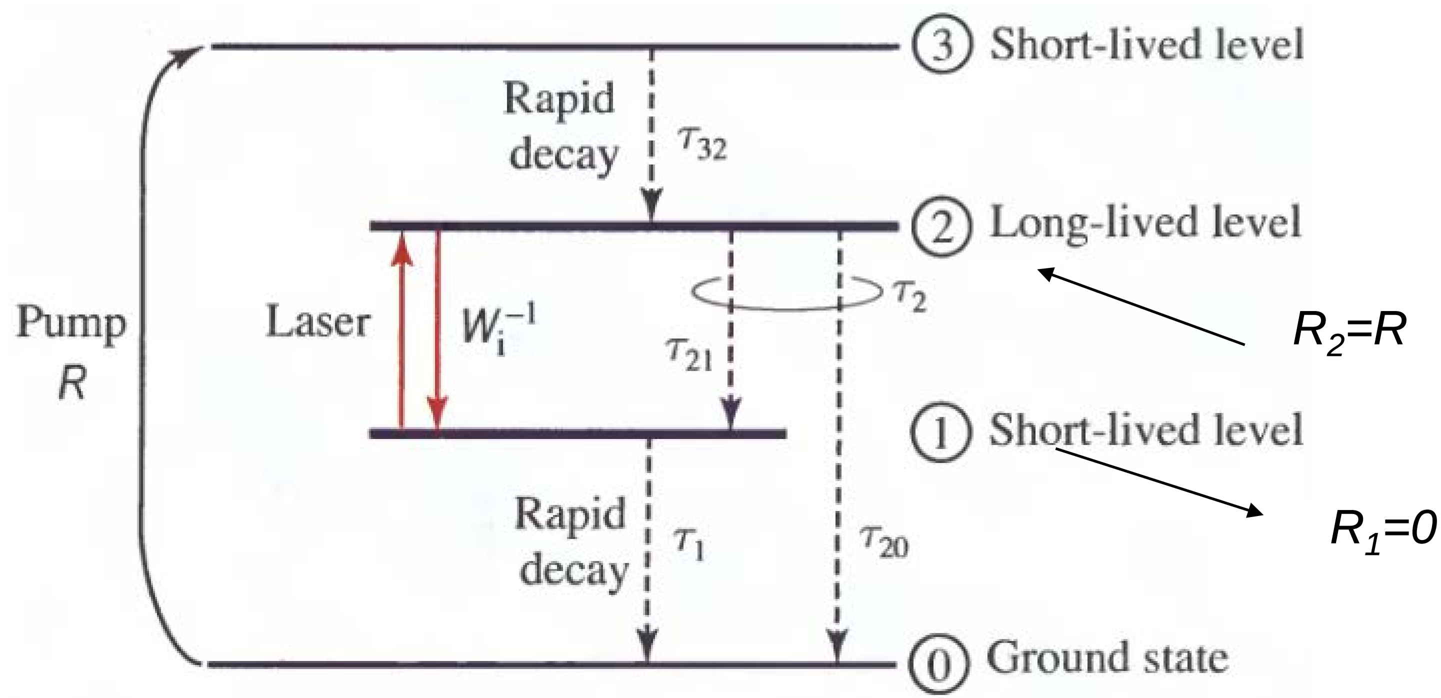


Depletion of the steady-state population difference $N = N_2 - N_1$ as the rate of absorption and stimulated emission W_i increases. When $W_i = 1/\tau_s$, N is reduced by a factor of 2 from its value when $W_i = 0$.

B. Four- and Three-Level Pumping Schemes

Derivation of atomic rate equations

1. Four-level pumping schemes



Energy levels and decay rates for a four-level system.

In case of the **absence of amplifier radiation**

We have steady-state $W_i=\phi=0$ *and* $R_1=0$

from $N_0 = R_2\tau_2(1-\frac{\tau_1}{\tau_{21}}) + R_1\tau_1$ $\Rightarrow$ $N_0 = R\tau_2(1-\frac{\tau_1}{\tau_{21}})$

Typically as $t_{sp} << \tau_{nr}$ *and* $\tau_{20} >> t_{sp} >> \tau_1$

because $\begin{cases} \tau_2^{-1} = \tau_{21}^{-1} + \tau_{20}^{-1} \\ \tau_{21}^{-1} = t_{sp}^{-1} + \tau_{nr}^{-1} \end{cases}$ $\Rightarrow$ $\tau_2 \approx \tau_{21} \approx t_{sp}$, $\tau_1 << t_{sp}$

$N_0 \approx Rt_{sp}$

In case of **amplifier radiation**

as $\tau_s = \tau_2 + \tau_1(1-\frac{\tau_2}{\tau_{21}})$ $\Rightarrow$ $\tau_s \approx t_{sp}$

So that $N = \frac{N_0}{1+\tau_s W_i}$ *Change to* $N \approx Rt_{sp} /(1+t_{sp}W_i)$

If considering the total atomic density in the system N_a is a constant

$$N_g + N_1 + N_2 + N_3 = N_a \quad \text{and} \quad N_1 \approx N_3 \approx 0$$

Then $$N_g \approx N_a - N_2 \approx N_a - N$$

the pump rate R is a linearly decreasing function of population difference, not independent of it.

$$R \approx (N_a - N)W$$

Where W is the pumping transition probability between ground state and level 3

So that

$$N \approx Rt_{sp} / (1 + t_{sp} W_i)$$

becomes

$$N \approx \frac{t_{sp} N_a W}{1 + t_{sp} W_i + t_{sp} W}$$

Comparing with two expressions

$$N = \frac{N_0}{1+\tau_s W_i} \quad \Rightarrow \quad N \approx \frac{t_{sp} N_a W}{1+t_{sp} W_i + t_{sp} W}$$

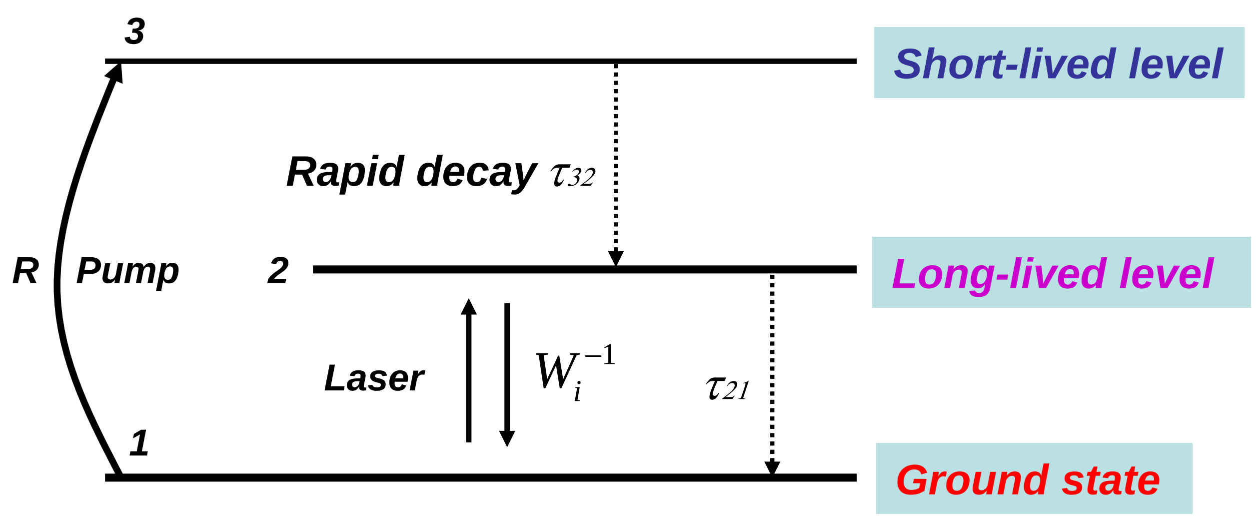
We have

$$\begin{cases} N_0 \approx \dfrac{t_{sp} N_a W}{1+t_{sp} W} \\ \tau_s \approx \dfrac{t_{sp}}{1+t_{sp} W} \end{cases}$$

Under conditions of weak pumping ($W_i << 1/t_{sp}$), $N_0 \approx t_{sp} N_a W_i$ is proportional to W_i(the pumping transition probability density), and $\boldsymbol{\tau_s \approx t_{sp}}$ giving rise to the results obtained previously. However, as the pumping increases N_0 saturates and τ_S decreases.

Derivation of atomic rate equations

2. Three-level pumping scheme



Energy levels and decay rates for a three-level system.

$$R_1 = R_2 = R, \qquad \tau_1 = \infty, \qquad \tau_2 = \tau_{21}.$$

In the steady state, from $\frac{dN_2}{dt} = R_2 - \frac{N_2}{\tau_2} - N_2 W_i + N_1 W_i$, we have

$$\frac{dN_2}{dt} = 0 = R - \frac{N_2}{\tau_{21}} - N_2 W_i + N_1 W_i$$

Note $N_1 + N_2 = N_a$

$$N_0 = (N_2 - N_1)_{\max} = 2(N_2)_{\max} - N_a = 2R\tau_{21} - N_a$$

$$N_0 = 2R\tau_{21} - N_a \approx 2Rt_{sp} - N_a$$

$$\tau_s = 2\tau_{21} \approx 2t_{sp}$$

Different from 4 level system where $\tau_s = t_{sp}$

From $R \approx (N_1 - N_3)W, N_3 \approx 0$

because $N_1 = 0.5(N_a - N)$

We have $R \approx 0.5(N_a - N)W$

because $N_0 = 2R\tau_{21} - N_a \approx 2Rt_{sp} - N_a$ *and* $N = \frac{N_0}{1 + \tau_s W_i}$

So $N_0 = 2R\tau_{21} - N_a \approx 2Rt_{sp} - N_a = N(1 + 2t_{sp}W_i)$

Change to $N = (2Rt_{sp} - N_a)/(1 + 2t_{sp}W_i)$

Finally ,we have for three level system:

$$N = \frac{N_0}{1 + \tau_s W_i}; \qquad N_0 = \frac{N_a(t_{sp}W - 1)}{1 + t_{sp}W} \qquad \tau_s = \frac{2t_{sp}}{1 + t_{sp}W}$$

To summarize:

$$N = \frac{N_0}{1+\tau_s W_i} \qquad \tau_s = \tau_2 + \tau_1\left(1-\frac{\tau_2}{\tau_{21}}\right)$$

We have

For a three level system

$$N = \frac{2Rt_{sp} - N_a}{1+2t_{sp}W_i}$$

$$N_0 = \frac{N_a(t_{sp}W - 1)}{1+t_{sp}W}$$

$$\tau_s = \frac{2t_{sp}}{1+t_{sp}W}$$

For a four level system

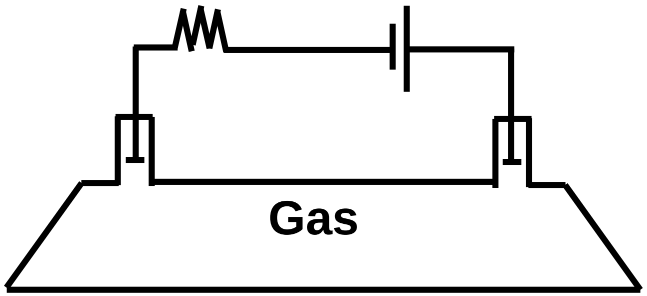
$$N \approx \frac{t_{sp}N_a W}{1+t_{sp}W_i + t_{sp}W}$$

$$N_0 \approx \frac{t_{sp}N_a W}{1+t_{sp}W}$$

$$\tau_s \approx \frac{t_{sp}}{1+t_{sp}W}$$

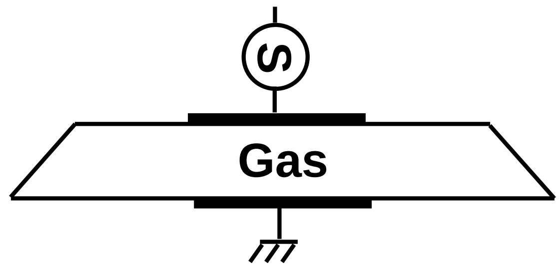
C. Different kinds of Laser Amplifiers

Case A:



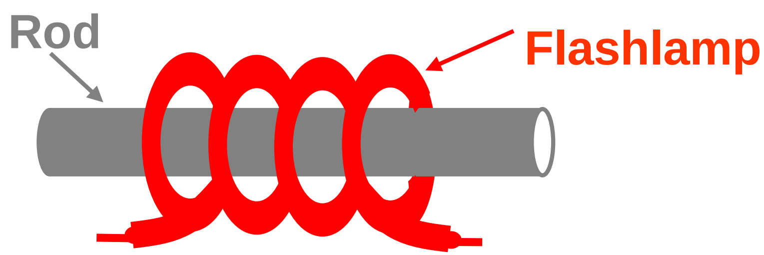
Direct current (dc) is often used to pump gas lasers. The current may be passed either along the laser axis, to give a longitudinal discharge, or transverse to it. The latter configuration is often used for high-pressure pulsed lasers, such as the transvesely excited atmospheric (TEA) CO2 lasers.

Case B:



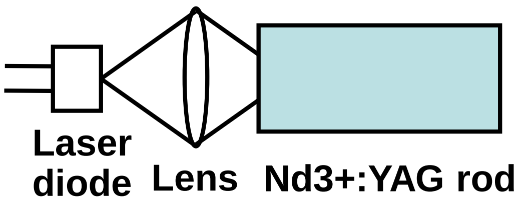
Radio-frequency (RF) discharge currents are also used for pumping gas lasers.

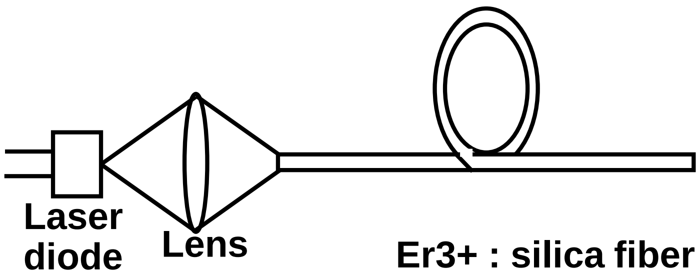
Case C:



Flashlamps are effective for optically pumping ruby and rare-earth solid state lasers

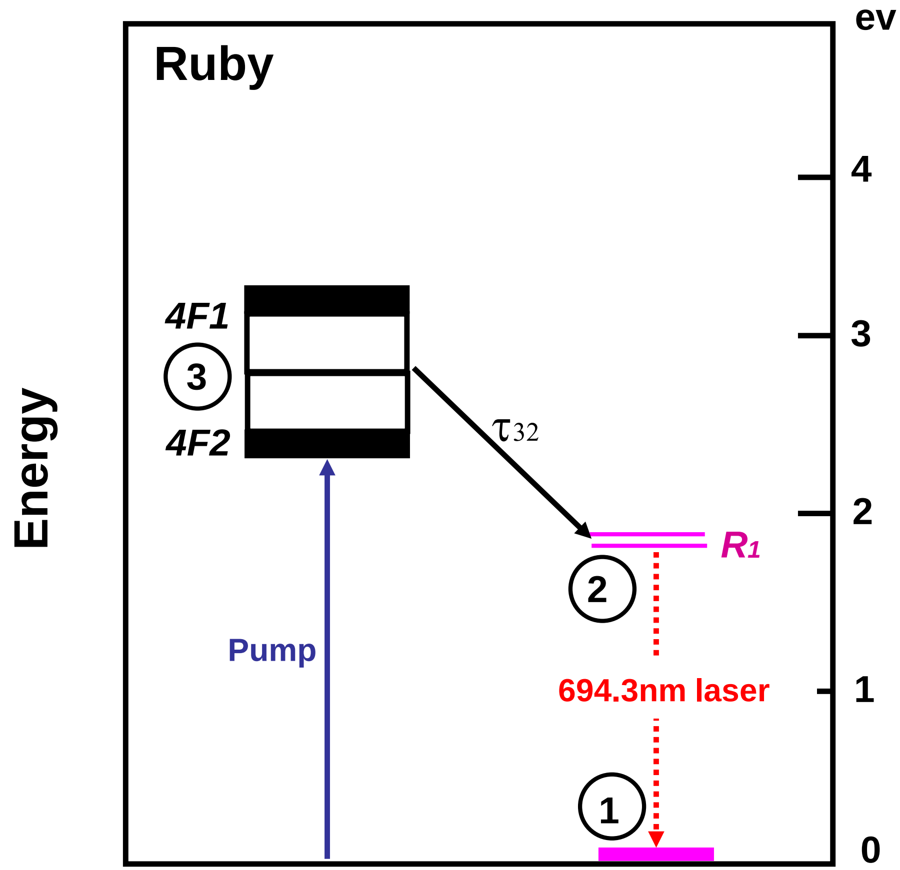
Case D:



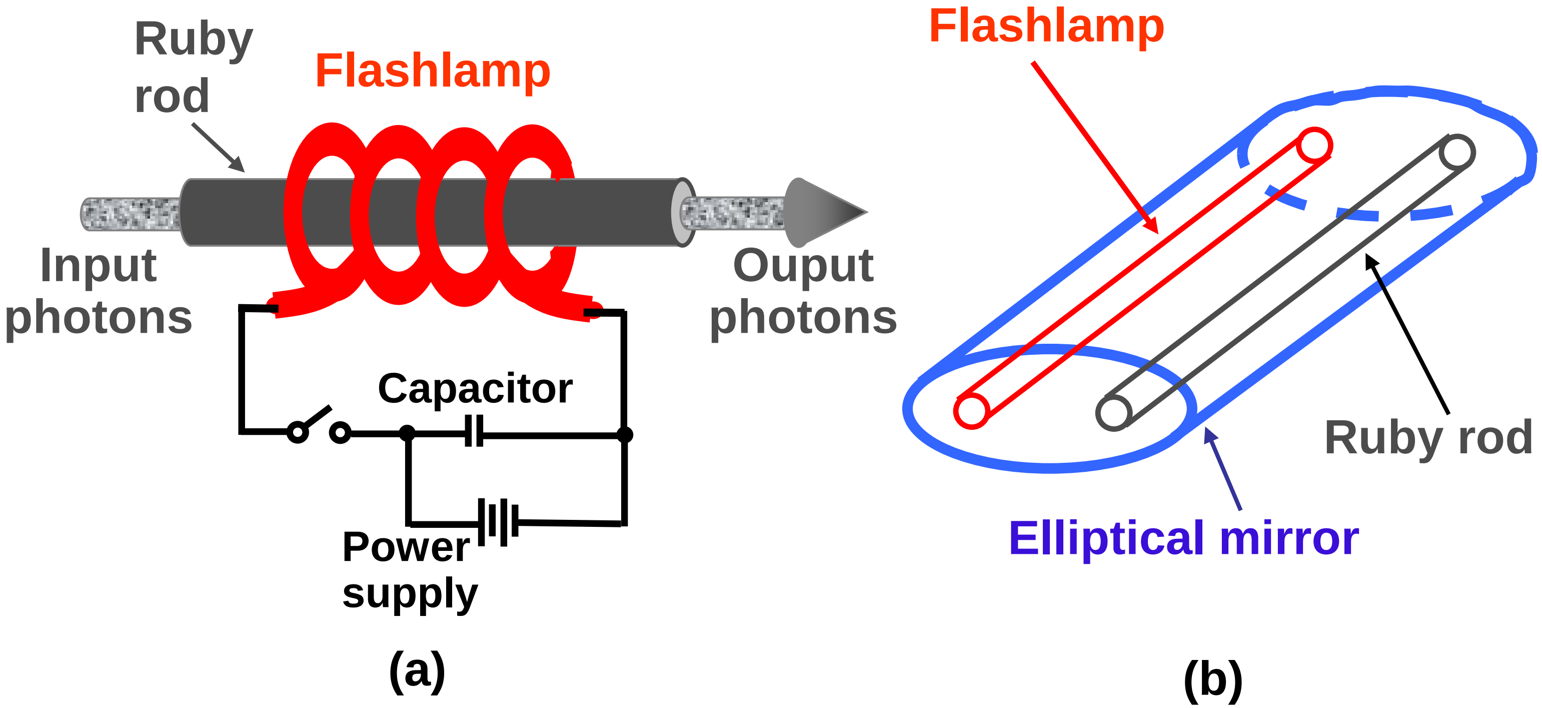


A semiconductor injection laser diode (or array of laser diodes) can be used to optically pump Nd3+:YAG or Er3+:silica fiber laers.

Ruby

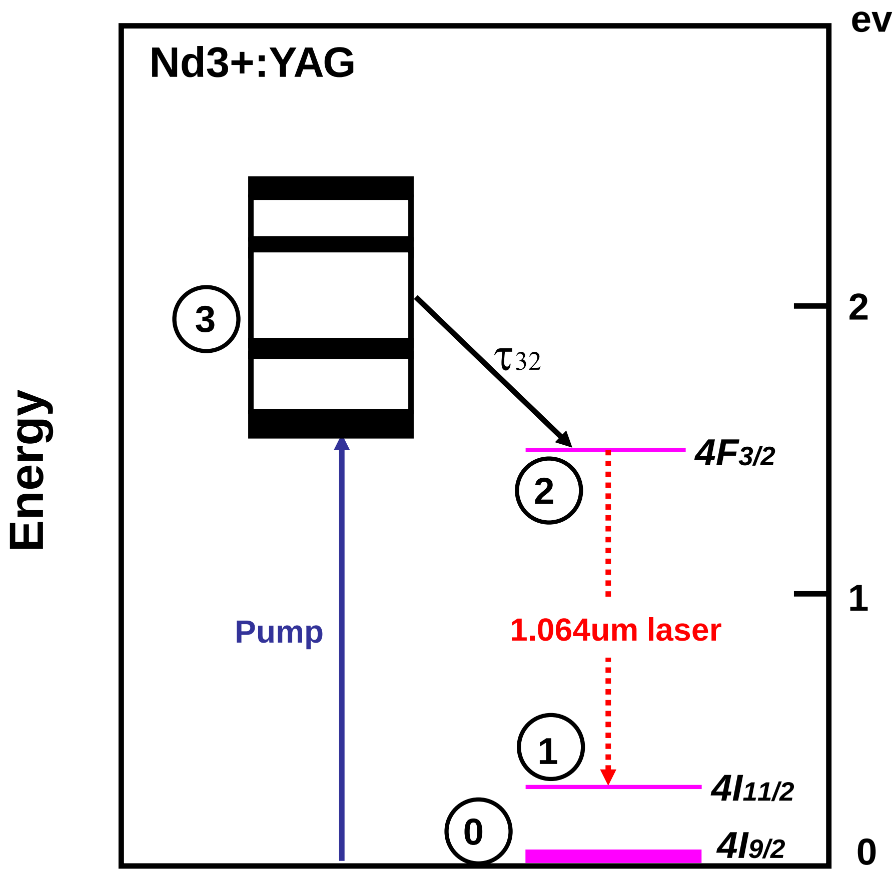


Energy levels pertinent to the 694.3nm red ruby transition. The three interacting levels are indicated in circles.



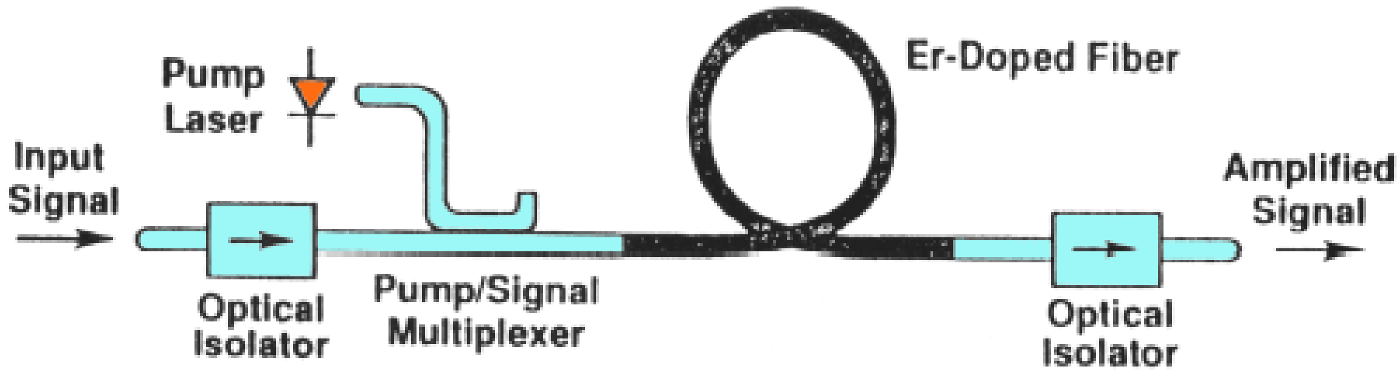
The ruby laser amplifier. (a) Geometry used in the first laser oscillator built by Maiman in 1960. (b) Cross sction of a high-efficiency geometry using a rod-shaped flashlamp and a reflecting elliptical cylinder.

Nd3+:YAG and Nd3+:Glass

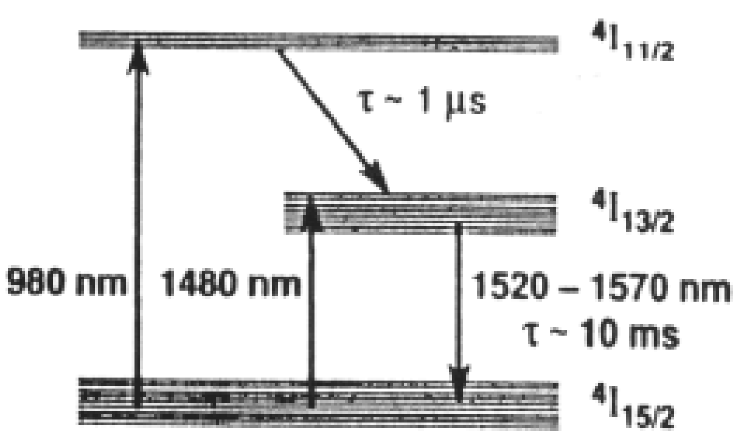


Energy levels pertinent to the 1.064um Nd3+:YAG laser transition. The energy levels for Nd3+:glass are similar but the absorption bands are broader.

Er3+:Silica Fiber



Energy Level Diagram



Absorption and Emission Spectra

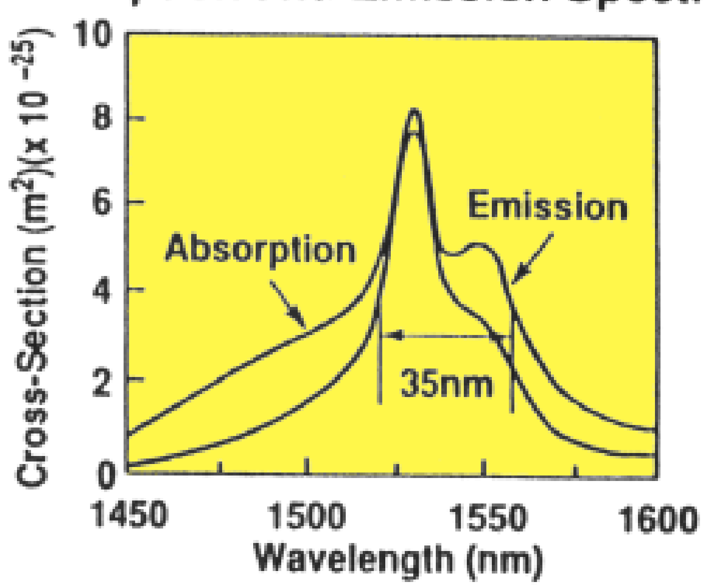


TABLE 3.2-1 Characteristics of a Number of Important Laser Transitions

Laser Medium	Transition Wavelength λ_o (μm)	Transition Cross Section σ_0 (cm^2)	Spontaneous Lifetime t_{sp}	Transition Linewidth[a] $\Delta\nu$		Refractive Index n
He–Ne	0.6328	1×10^{-13}	0.7 μs	1.5 GHz	I	≈ 1
Ruby	0.6943	2×10^{-20}	3.0 ms	60 GHz	H	1.76
Nd^{3+}:YAG	1.064	4×10^{-19}	1.2 ms	120 GHz	H	1.82
Nd^{3+}:glass	1.06	3×10^{-20}	0.3 ms	3 THz	I	1.5
Er^{3+}:silica fiber	1.55	6×10^{-21}	10.0 ms	4 THz	H/I	1.46
Rhodamine-6G dye	0.56–0.64	2×10^{-16}	3.3 ns	5 THz	H/I	1.33
Ti^{3+}:Al_2O_3	0.66–1.18	3×10^{-19}	3.2 μs	100 THz	H	1.76
CO_2	10.6	3×10^{-18}	2.9 s	60 MHz	I	≈ 1
Ar^+	0.515	3×10^{-12}	10.0 ns	3.5 GHz	I	≈ 1

[a] H and I indicate line broadening dominated by homogeneous and inhomogeneous mechanisms, respectively.

4.3 Amplifier nonlinearity and gain saturation

Features of homogeneous broadening:

1. Each atom in the system has a common emitting spectrum width $\Delta\nu$. $g(\nu)$ describes the response of any of the atoms, which are indistinguishable
2. Due most often to the finite interaction lifetime of the absorbing and emitting atoms

Mechanisms of homogeneous broadening:

1. The spontaneous lifetime of the exited state
2. Collision of an atom embedded in a crystal with a phonon
3. Pressure broadening of atoms in a gas

Features of Inhomogeneous Broadening

1. Individual atoms are distinguishable, each having a slightly different frequency.

2. The observed spectrum of spontaneous emission reflects the spread in the individual transition frequencies (not the broadening due to the finite lifetime of the excited state).

Typical Examples:

- The energy levels of ions presents as impurities in a host crystal.
- *Random strain*
- *Crystal imperfection*

Gain coefficient $\gamma(v)$

The gain coefficient $\gamma(v)$ of a laser medium depends on the population difference N; that N depends on the transition rate W_i ; and that W_i , in turn, depends on the radiation photon-flux density ϕ

It follows that **the gain coefficient of a laser medium is dependent on the photon-flux density that is to be amplified**

$$From: \quad N=\frac{N_0}{1+\tau_s W_i}, \quad and \quad W_i=\phi\sigma(\nu)$$

$$Define: \quad N=\frac{N_0}{1+\phi/\phi_s(\nu)}$$

where $1/\tau_s=\sigma(\nu)\phi_s(\nu)$

Then

$$\gamma(\nu) = \frac{\gamma_0(\nu)}{1 + \phi / \phi_s(\nu)}$$

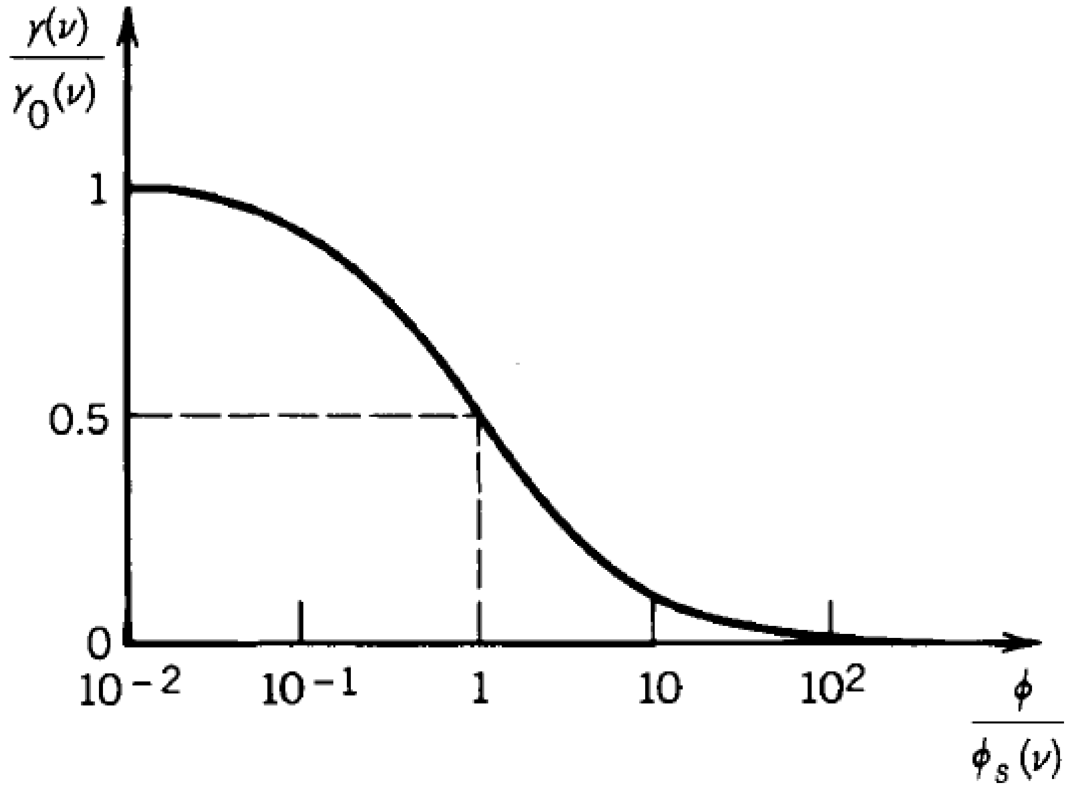
Saturated Gain Coefficient

where

$$\gamma_0(\nu) = N_0 \sigma(\nu) = N_0 \frac{\lambda^2}{8\pi t_{sp}} g(\nu)$$

Small-signal Gain Coefficient

The gain coefficient is a ***decreasing*** function of the photon-flux density ϕ,



Dependence of the normalized saturated gain coefficient on the normalized photon-flux densit. When ϕ equals its saturation value, the gain coefficient is reduced by a factor of 2.

The quantity $\phi_s(\nu) = 1/\tau_s\sigma(\nu)$

represents the photon-flux density at which the gain coefficient decreases to half its maximum value;

it is therefore called the ***saturation photon-flux density***.

When $\tau_s \approx t_{sp}$

the interpretation of $\phi_s(v)$ is straightforward:

Roughly one photon can be emitted during each spontaneous emission time into each transition cross-sectional area:

$$[\sigma(\nu)\phi_s(\nu)t_{sp} = 1]$$

Spectral Broadening of a Saturated Amplifier.

Consider a homogeneously broadened amplifying medium with a Lorentzian lineshape of width $\Delta\nu$. Show that when the photon-flux density is ϕ, the amplifier gain coefficient $\gamma(\nu)$ assumes a Lorentzian lineshape with width:

$$\because \quad \gamma(\nu) = (N_2 - N_1)\frac{c^2}{8\pi n^2 \nu^2 t_{spont}} g(\nu)$$

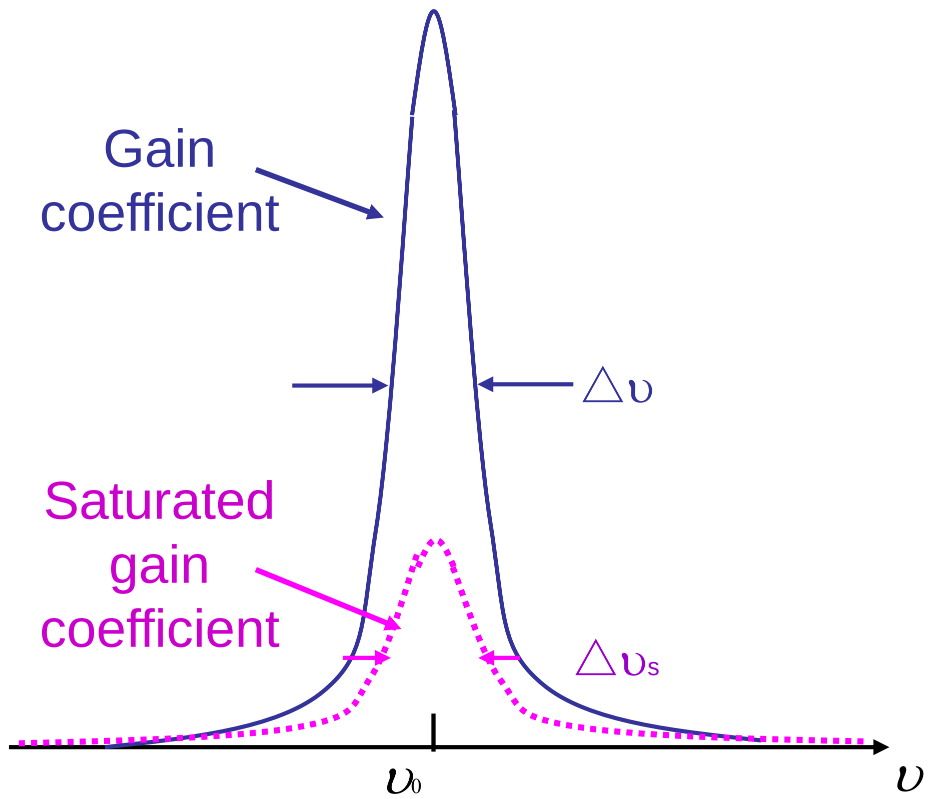
$$\gamma(\nu) = \frac{\gamma_0(\nu)}{1 + \phi/\phi_s(\nu)}$$

$$g(\nu) = \frac{\Delta\nu/2\pi}{(\nu-\nu_0)^2 + (\Delta\nu/2)^2} = g(\nu_0)\frac{(\Delta\nu/2)^2}{(\nu-\nu_0)^2 + (\Delta\nu/2)^2}$$

$$\Delta\nu_s = \Delta\nu\left[1 + \frac{\phi}{\phi_s(\nu_0)}\right]^{1/2}$$

Linewidth of Saturated Amplifier

Gain coefficient reduction and bandwidth *increase* resulting from saturation



Gain coefficient reduction and bandwidth increase resulting from saturation when

$$\phi = 2\phi_s(\nu_0)$$

Gain

If the photon-flux density at position z is $\phi(z)$, then the gain coefficient at that position is also a function of z

$$\frac{d\phi}{dz} = \gamma\phi = \frac{\gamma_0\phi}{1+\phi/\phi_s} \longrightarrow (1/\phi + 1/\phi_s)\,d\phi = \gamma_0\,dz$$

$$\ln\frac{\phi(z)}{\phi(0)} + \frac{\phi(z)-\phi(0)}{\phi_s} = \gamma_0 z$$

$$[\ln(Y)+Y] = [\ln(X)+X] + \gamma_0 d$$

where $X = \phi(0)/\phi_s, and, Y = \phi(d)/\phi_s$

$G = \phi(d)/\phi(0) = Y/X$

The solution for the gain $G = \phi(d)/\phi(0) = Y/X$ can be examined in two limiting cases:

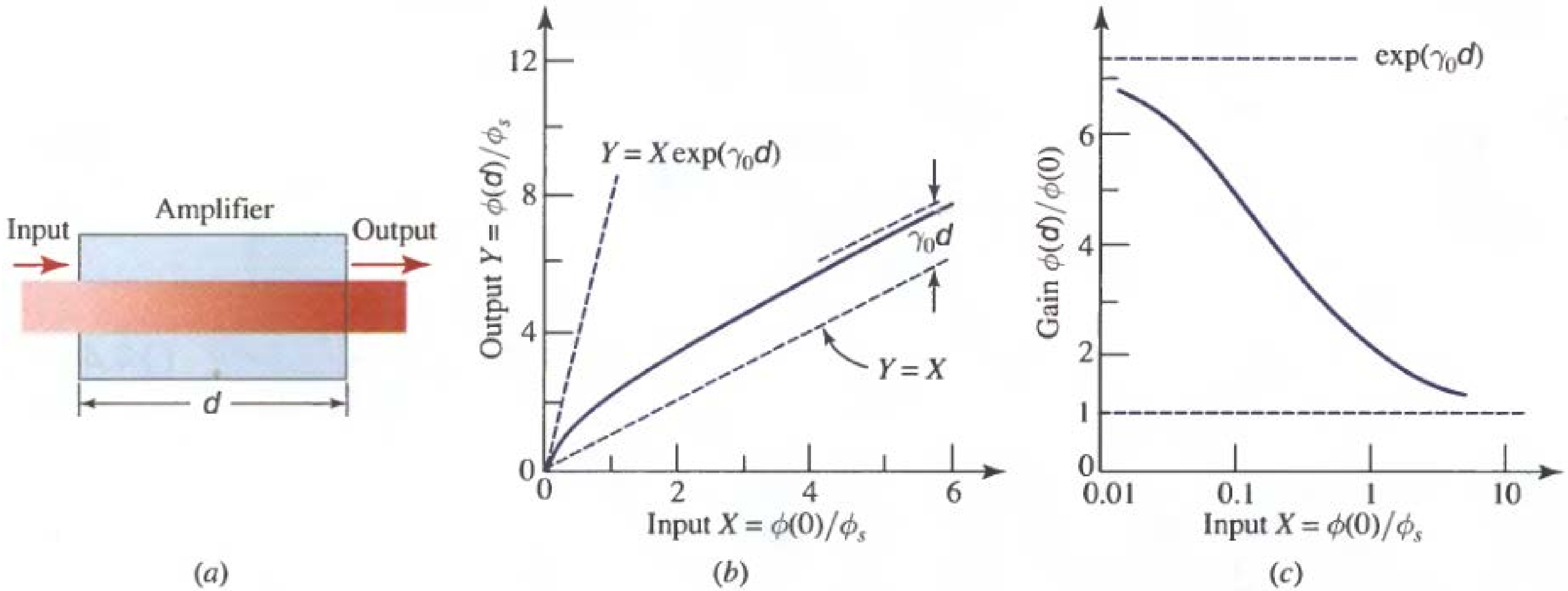
Case 1: If both X and Y are much smaller than unity (i.e., the photon-flux densities are much smaller than the saturation photon-flux density), *small-signal approximation*,

$$[\ln(Y)+Y]=[\ln(X)+X]+\gamma_0 d \quad \Rightarrow \quad \ln(Y) \approx \ln(X) + \gamma_0 d$$

$$Y \approx X \exp(\gamma_0 d)$$

Case 2: When X >>1, we can neglect In(X) in comparison with X, and In(Y) in comparison with Y, *heavily saturated conditions,*

$$\phi(d) \approx \phi(0) + \gamma_0 \phi_s d \approx \phi(0) + \frac{N_0 d}{\tau_s}$$



(a) A nonlinear (saturated) amplifier. (b) Relation between the normalized output photon-flux density Y and the normalized input photon-flux density X. For X<<1, the gain Y/X=exp(γ_0d). For X>>1, the gain Y=X+γ_0d. (c) Gain as a function of the input normalized photon-flux density X in an amplifier of length d when γ_0d=2.

Saturable Absorbers

If the gain coefficient γ_0 is negative, i.e., if the population is normal rather than inverted ($N_0 < 0$), the medium provides attenuation rather than amplification.

The attenuation coefficient: $\alpha(v) = -\gamma(v)$

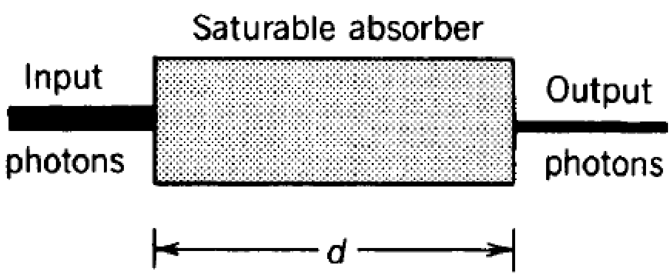
also suffers from saturation, in accordance with the relation:

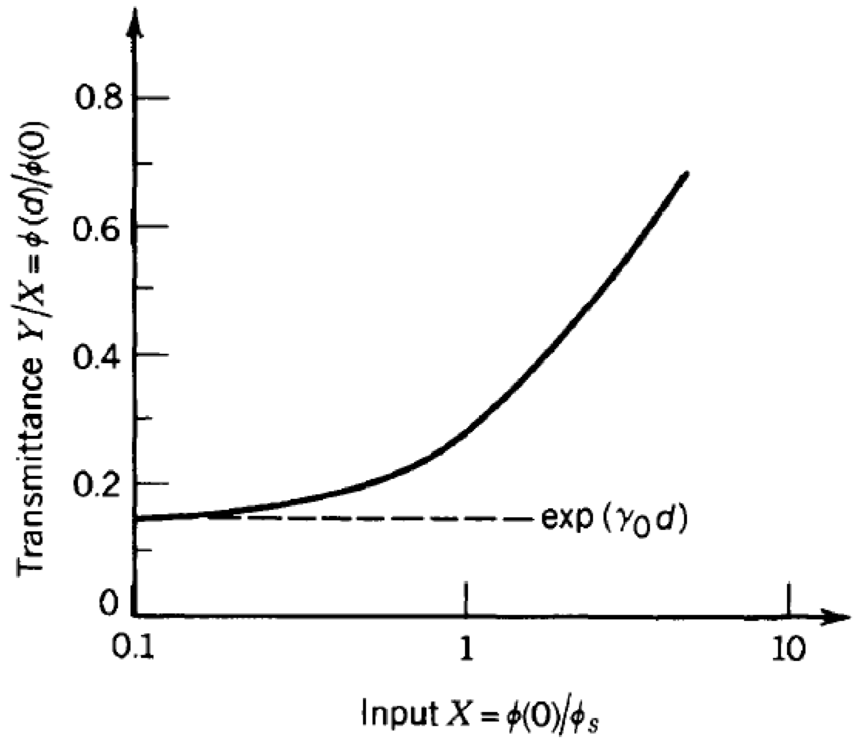
$$\alpha(v) = \alpha_0(v)/[l + \phi/\phi_s(v)]$$

This indicates that ***there is less absorption for large values of the photon-flux density.***

A material exhibiting this property is called a ***saturable absorber***.

Saturable Absorbers





Carbon, graphene, and lots of polymer have presented as saturable absorber. The saturable absorption response of graphene is wavelength independent from UV to IR, mid-IR and even to THz frequencies. In rolled-up graphene sheets (carbon nanotubes), saturable absorption is dependent on diameter and chirality.

C. Gain of Inhomogeneously Broadened Amplifiers

An inhomogeneously broadened medium comprises a collection of atoms with different properties. The subset of atoms labeled p has a homogeneously broadened lineshape function $g_\beta(v)$. The overall inhomogeneous average lineshape function of the medium is described by $g(v) = \langle g^*_\beta(v) \rangle$, where (*) represents an average with respect to β.

$$\overline{\gamma_0}(\nu) = N_0 \frac{\lambda^2}{8\pi t_{sp}} \overline{g}(\nu)$$

average gain coefficient

$$\overline{\gamma}(\nu) = \langle \gamma_\beta(\nu) \rangle$$

where $$\gamma_\beta(\nu) = \frac{\gamma_{0\beta}(\nu)}{1+\phi/\phi_{s\beta}(\nu)} = b\frac{g_\beta(\nu)}{1+\phi a^2 g_\beta(\nu)}$$

with $b = N_0(\lambda^2/8\pi t_{sp})$ and $a^2 = (\lambda^2/8\pi)(\tau_s/t_{sp})$

Doppler-Broadened Medium

Although all of the atoms in a Doppler-broadened medium share a *g(v)* of identical shape, the center frequency of the subset β is shifted by an amount v_β proportional to the velocity v_β of the subset.

$$\gamma_\beta(\nu) = \frac{b(\Delta\nu/2\pi)}{(\nu-\nu_\beta-\nu_0)^2+(\Delta\nu_s/2)^2}$$

where

$$\Delta\nu_s = \Delta\nu[1+\frac{\phi}{\phi_s(\nu_0)}]^{1/2}$$

and

$$\phi_s^{-1}(\nu_0) = \frac{2a^2}{\pi\Delta\nu} = \frac{\lambda^2}{8\pi}\frac{\tau_s}{t_{sp}}\frac{2}{\pi\Delta\nu} = \frac{\lambda^2}{8\pi}\frac{\tau_s}{t_{sp}}g(\nu_0)$$

$$\bar{\gamma} = \int_{-\infty}^{\infty} \gamma_\beta(\nu) p(\nu_\beta) d\nu_\beta$$

where $p(\nu_\beta) = (2\pi\sigma_D^2)^{-1/2} \exp(-\nu_\beta^2 / 2\sigma_D^2)$

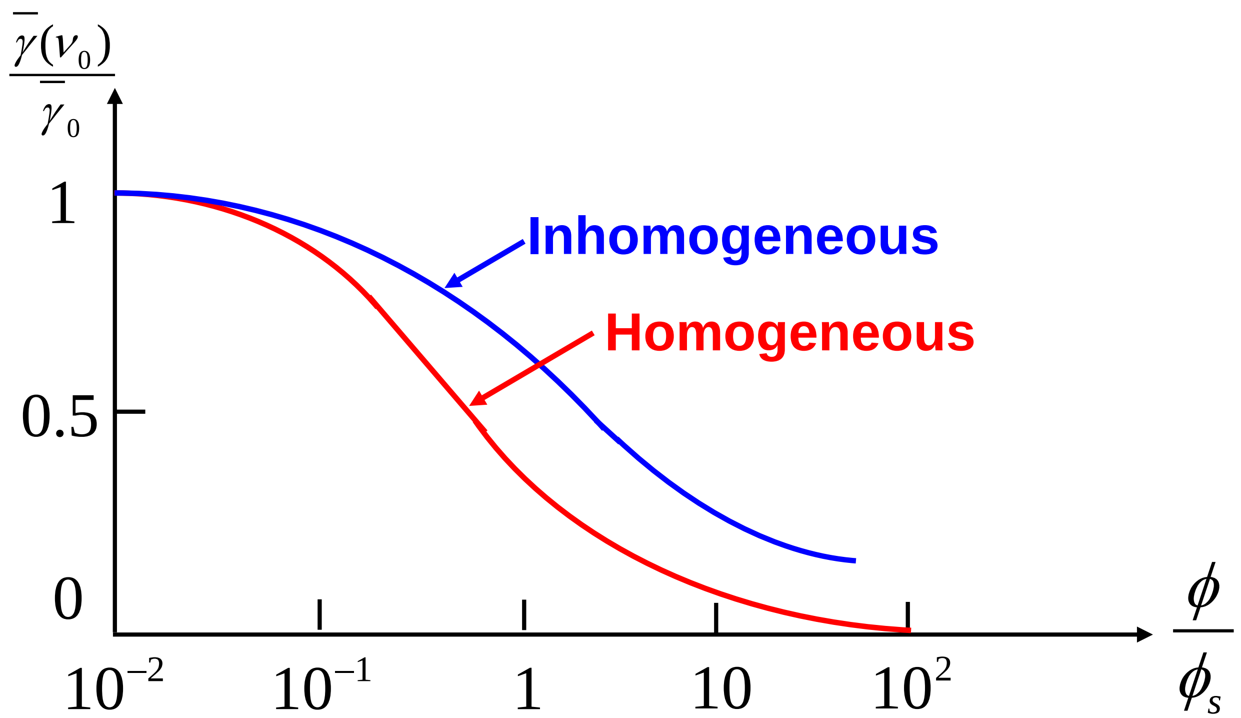
Setting $\nu = \nu_0$ and $\nu_\beta = 0$ in the exponential provides

$$\bar{\gamma}(\nu_0) = \frac{bp(0)}{(1 + 2\phi a^2 / \pi\Delta\nu)^{1/2}} = \frac{\bar{\gamma}_0}{[1 + \phi / \phi_s(\nu_0)]^{1/2}}$$

where the average small-signal gain coefficient

$$\bar{\gamma}_0 = N_0 \frac{\lambda^2}{8\pi t_{sp}} (2\pi\sigma_D^2)^{-1/2}$$

Difference of gain saturation between inhomogeneous and homogeneous media



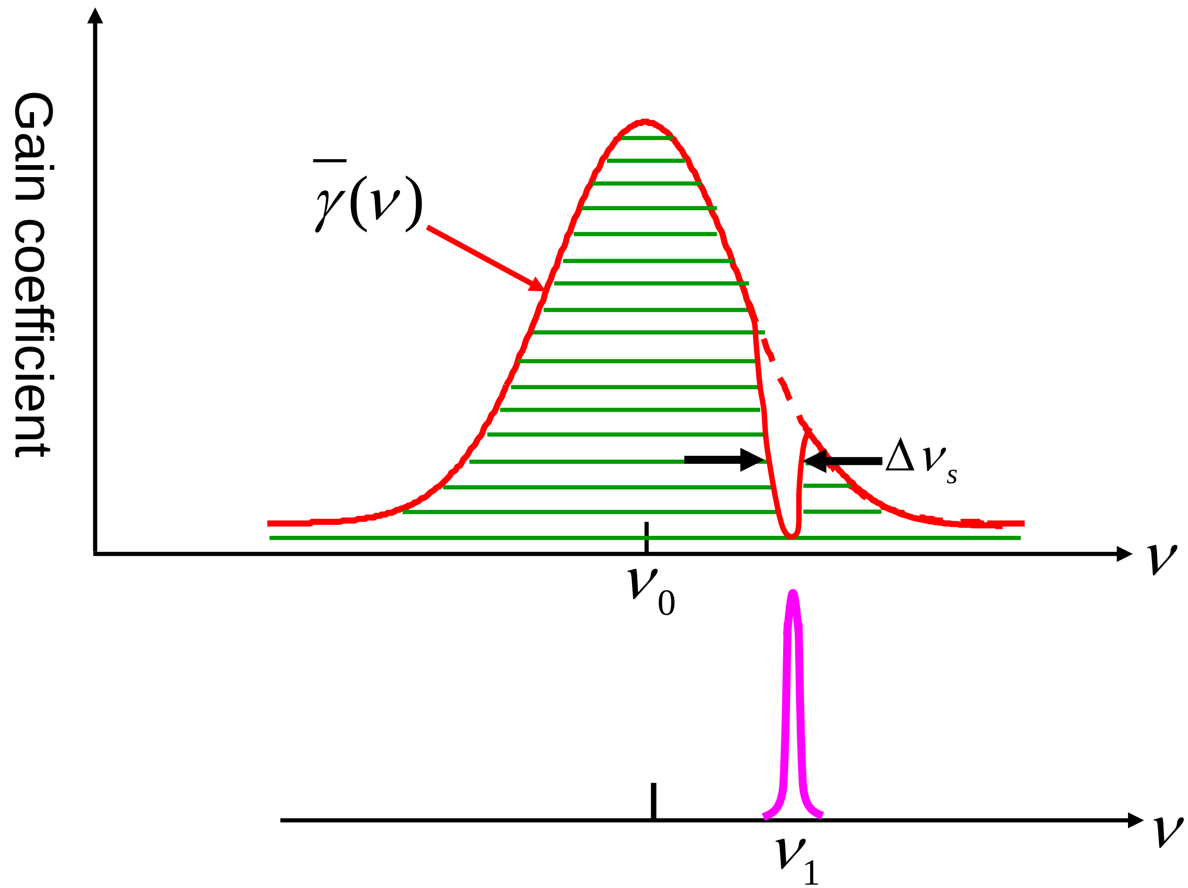
Comparison of gain saturation in homogeneous and inhomogeneous broadened media.

Hole burning

When a large flux density of monochromatic photons at frequency v_1 is applied to an inhomogeneously broadened medium, the gain saturates only for those atoms whose lineshape function overlaps ν_1. Other atoms simply do not interact with the photons and remain unsaturated. When the saturated medium is probed by a weak monochromatic light source of varying frequency v, the profile of the gain coefficient therefore exhibits a hole centered around v_1.

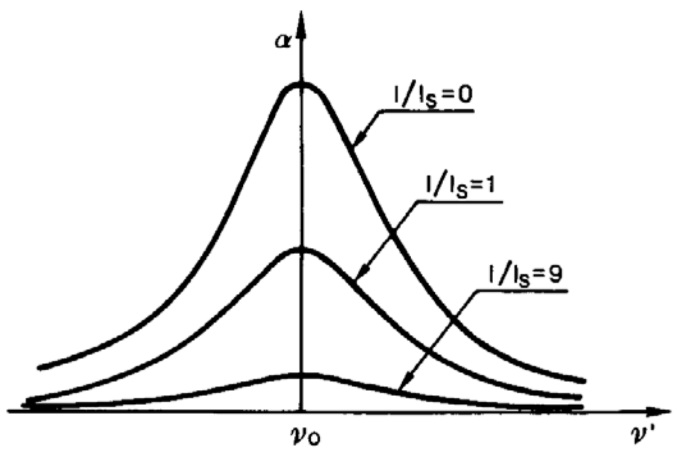
This phenomenon is known as ***hole burning***.

Hole burning

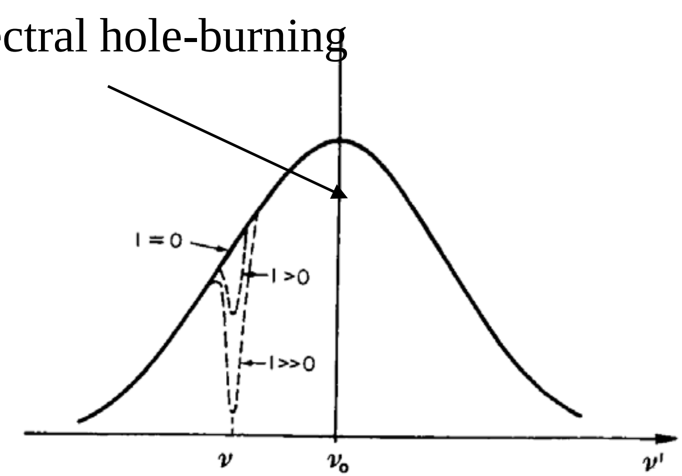


The gain coefficient of an inhomogeneously broadened medium is locally saturated by a large flux density of monochromatic photons at frequency ν_1

Gain saturation in homogeneously and inhomogeneously broadened systems:



(Homogeneous)



(Inhomogeneous)

Laser noise

- *Spontaneous emission will also be amplified via the amplification of the stimulated emission in the laser system, because the later increase higher level population.*
- *This is called Amplified Spontaneous Emission ASE, or the laser noise.*
- *ASE has more broadband, multidirectional, and unpolarized.*

- In the homogeneously broadened lasers, when gain saturation occurs, the entire gain curve saturates proportionally. The stronger the saturation effect, the lower the gain curve (or the smaller the gain coefficients).

- In the inhomogeneously broadened lasers, saturation at one particular frequency causes a reduction in the gain profile only near that frequency. Effectively, a hole is burned in the gain profile at the frequency---- phenomenally it is called spectral hole burning. No effect it will have on the gain at other frequencies!

Home works

A three levels system, E_1 is ground state, pumping light frequency corresponds to the transition between E_1 and E_3,and the transition probability is $W_{13}=W_{31}=W_p$. The lifetime of E_3 is τ_3 quite long, and the lifetime for E_2 is short τ_2, the transition propability from E_3 to E_2 is $1/\tau_{32}$.

Determine the condition of population inversion between E_3 and E_2

The relation of about population inversion with the W_p

For strong pumping, the population inversion between E_3 and E_2

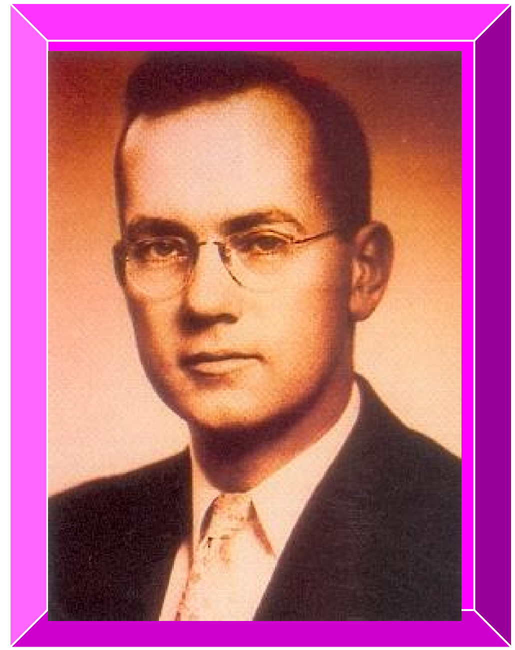
- *一个三能级系统，E1是基态，泵浦光频率与E1和E3之能级跃迁相对应，其跃迁几率W13=W31=Wp。能级E3的寿命较长t3，E2能级寿命较短t2，E3到E2的跃迁几率为1/t32，求：*
- *E3，E2之间形成粒子数反转的条件*
- *E3，E2之间粒子数反转密度与跃迁几率Wp的关系*
- *泵浦极强时，E3，E2之间的粒子数反转密度（E3，E2之间的受激辐射可以忽略）*

P.165,　no:1,3,4,5,6,7,8

思考几个问题:

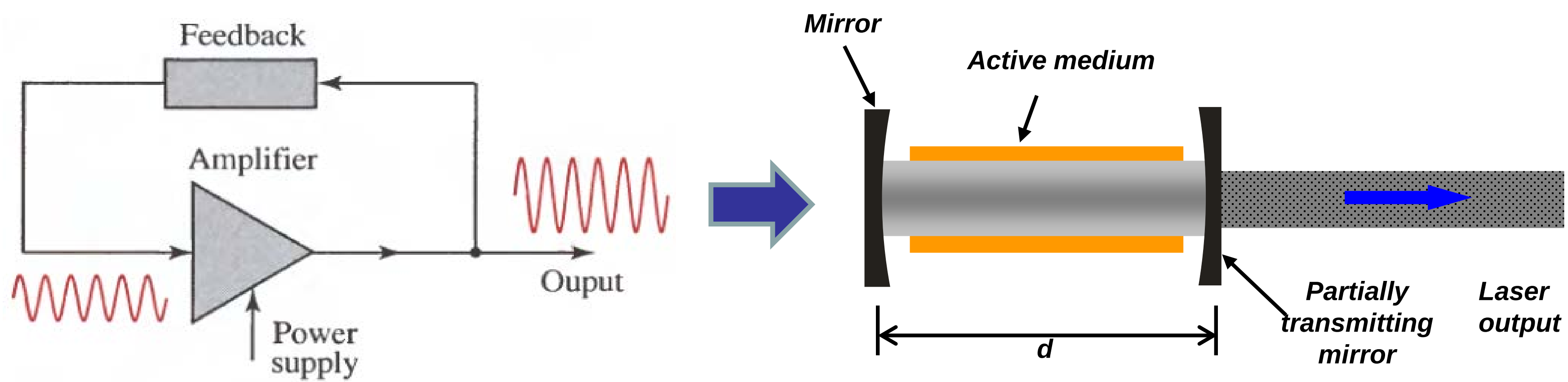
- 如何区分自发辐射与受激辐射?
- 为什么会出现增益饱和?
- 粒子数反转的其他可能方法?
- 我们有了自发辐射寿命，问受激辐射寿命又是怎样?
- 如何利用饱和增益与饱和吸收效应?

Lasers and Laser system



In 1958 Arthur Schawlow, together with Charles Townes, showed how to extend the principle of the maser to the optical region. He shared the 1981 Nobel Prize with Nicolaas Bloembergen. Maiman demonstrated the first successful operation of the ruby laser in 1960.

LASERS

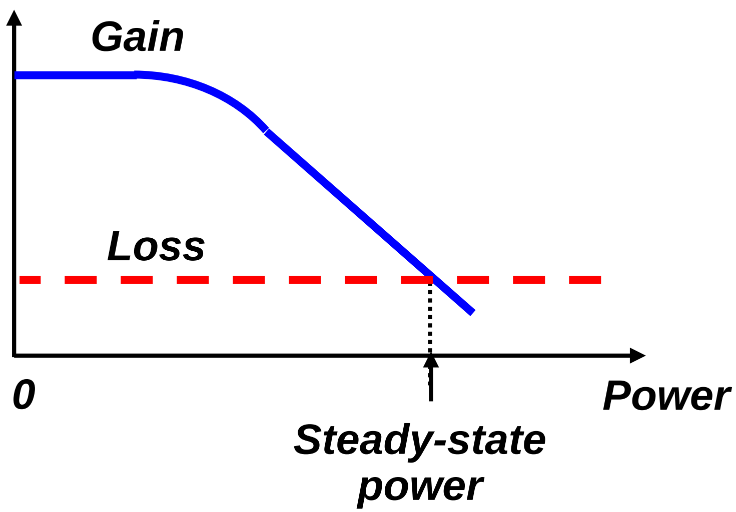


an oscillator is an amplifier with positive feedback

Two conditions for an oscillation:

1. Gain greater than loss: net gain
2. Phase shift in a round trip is a multiple of 2π

Stable condition 2: gain = loss



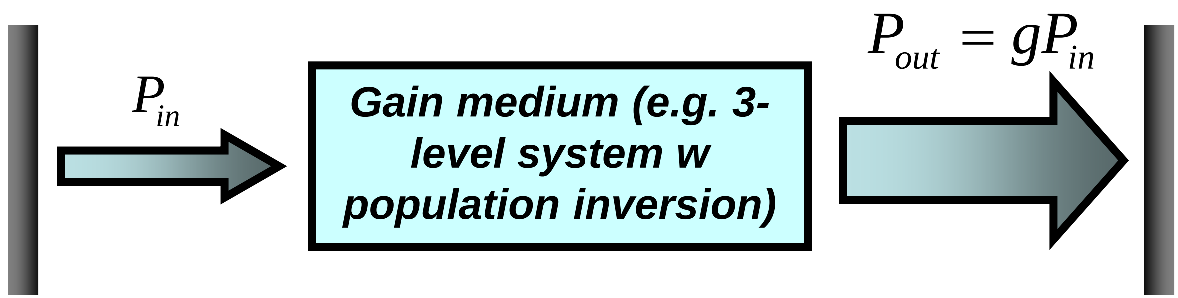
If the initial amplifier gain is greater than the loss, oscillation may initiate. The amplifier then satuates whereupon its gain decreases.

A steady-state condition *is reached when the gain just equals the loss.*

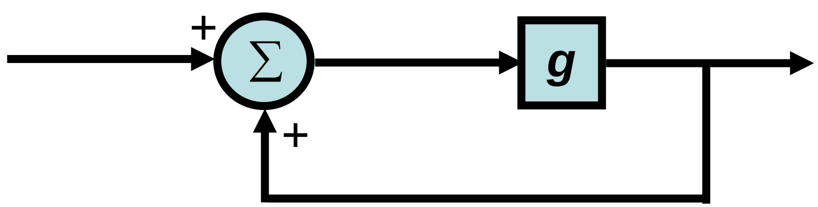
An oscillator comprises:

- ***An amplifier with a gain-saturation mechanism***
- ***A feedback system***
- ***A frequency-selection mechanism***
- ***An output coupling scheme***

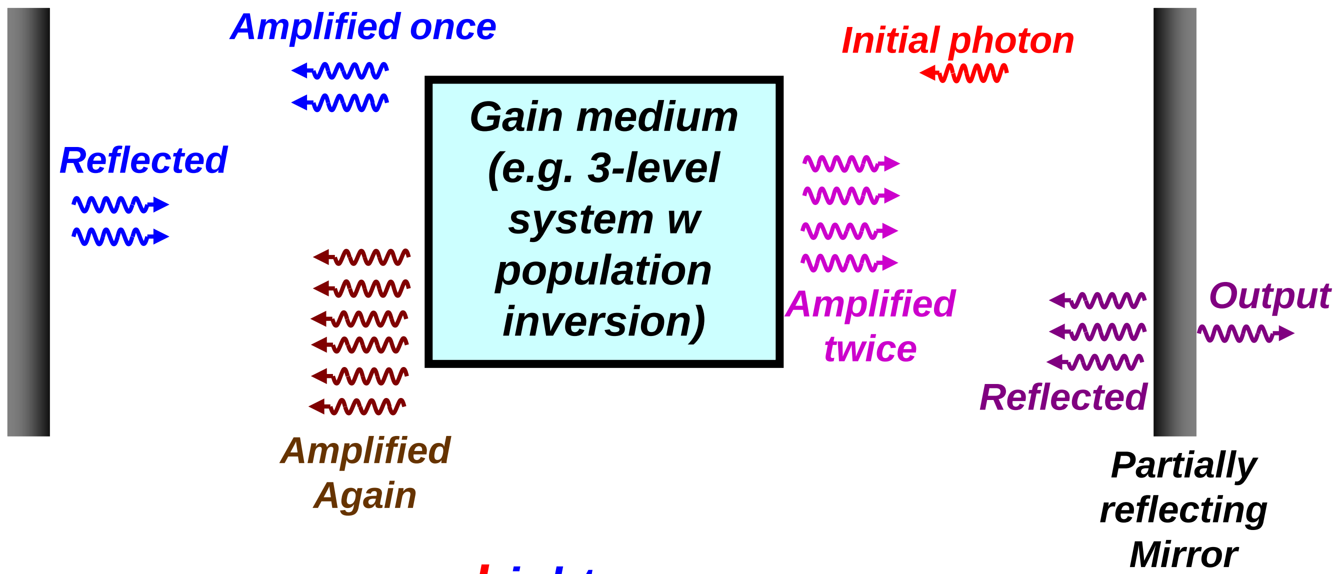
Light amplifier with positive feedback



When the gain exceeds the roundtrip losses, the system goes into oscillation



LASERS



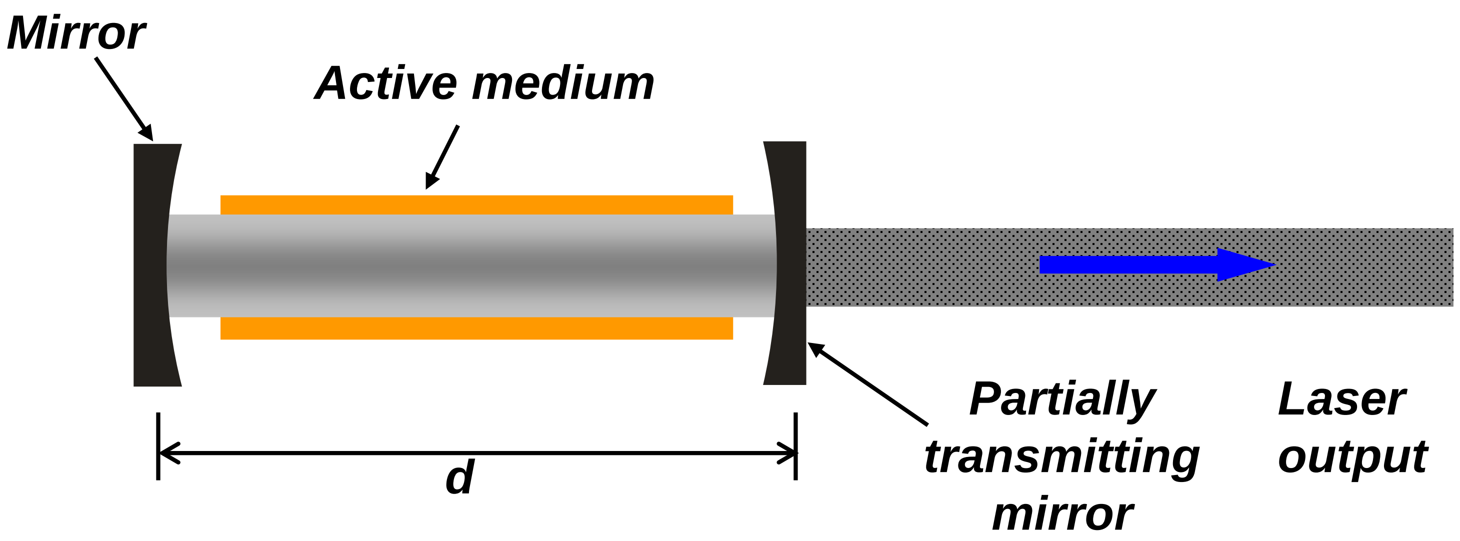
Light

Amplification through

Stimualted

Emission

Radiation



A laser consists of an optical amplifier (employing an active medium) placed within an optical resonator. The output is extracted through a partially transmitting mirror.

Optical amplification and feedback

★ Gain medium

The laser amplifier is a distributed-gain device characterized by its gain coefficient

$$\gamma_0(\nu) = N_0\sigma(\nu) = N_0\frac{\lambda^2}{8\pi t_{sp}}g(\nu)$$

Small signal Gain Coefficient

$$\gamma(\nu) = \frac{\gamma_0(\nu)}{1+\phi/\phi_s(\nu)}$$

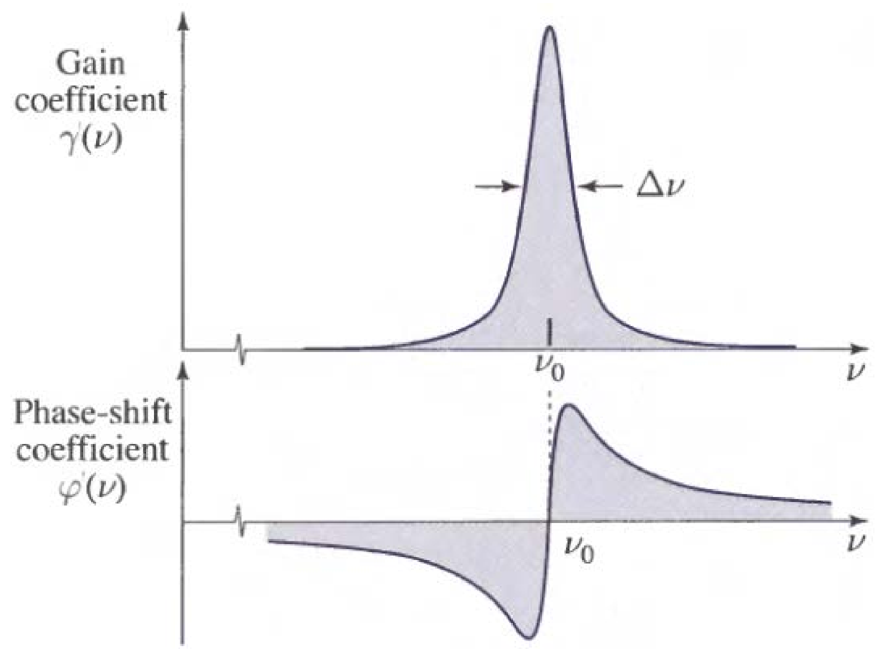
Saturated Gain Coefficient

where $\phi_s(\nu) = [\tau_s\sigma(\nu)]^{-1}$ = saturation photon-flux density

For 4 level system τ_s=t_{sp}, for 3 level system τ_s=2t_{sp}

$$\varphi(\nu) = \frac{\nu - \nu_0}{\Delta\nu} \gamma(\nu)$$

Phase-shift Coefficient (Lorentzian Lineshape)



Spectral dependence of the gain and phase-shift coefficients for an optical amplifier with Lorentzian lineshape function

Optical Feedback-Optical Resonator

Feedback and Loss: The optical resonator

Optical feedback is achived by placing the active medium in an optical resonator. A Fabry-Perot resonator, comprising two mirrors separated by a distance d, contains the medium (refractive index n) in which the active atoms of the amplifier reside. Travel through the medium introduces a phase shift per unit length equal to the wavenumber

$$\boxed{k = \frac{2\pi\nu}{c}}$$

The resonator also contributes to losses in the system. Absorption and scattering of light in the medium introduces a distributed loss characterized by the attenuation coefficient α_s, (loss per unit length). In traveling a round trip through a resonator of length d, the photon-flux density is reduced by the factor $R_1R_2\exp(-2\alpha_s d)$, where R_1 and R_2 are the reflectances of the two mirrors. The overall loss in one round trip can therefore be described by a total effective distributed loss coefficient α_r, where

$$\boxed{\exp(-2\alpha_r d) = R_1 R_2 \exp(-2\alpha_s d)}$$

Loss coefficient

$$\alpha_r = \alpha_s + \alpha_{m1} + \alpha_{m2}$$

$$\alpha_{m1} = \frac{1}{2d} \ln \frac{1}{R_1}$$

$$\alpha_{m2} = \frac{1}{2d} \ln \frac{1}{R_2}$$

$$\alpha_m = \alpha_{m1} + \alpha_{m2} = \frac{1}{2d} \ln \frac{1}{R_1 R_2}$$

Photon lifetime

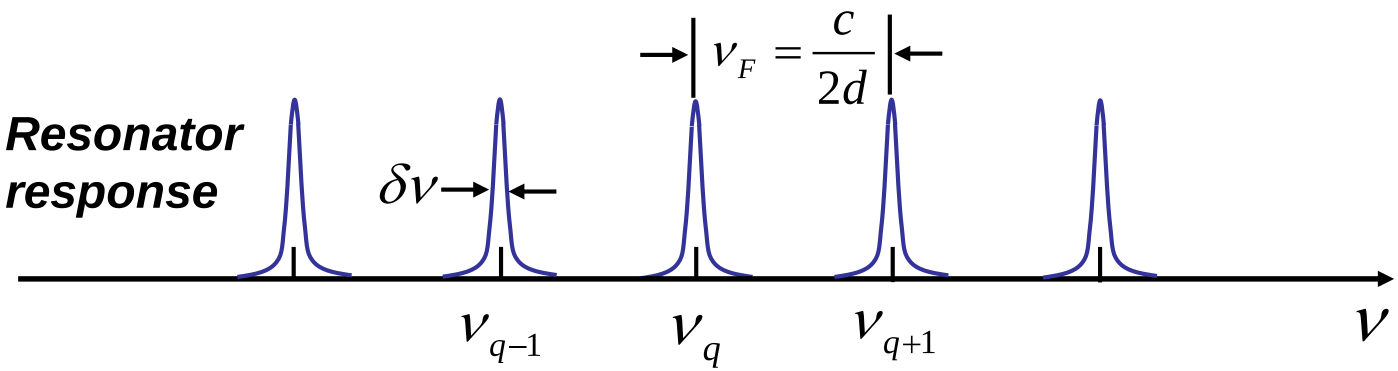
$$\boxed{\tau_p = \frac{1}{\alpha_r c}}$$

α_r represents the total loss of energy (or number of photons) per unit length, $\alpha_r c$ represents the loss of photons per second

$$\nu_q = q\nu_F, q = 1, 2, \ldots,$$

$$\delta\nu \approx \frac{\nu_F}{F}, \nu_F = c/2d$$

$$F \approx \frac{\pi}{\alpha_r d} = 2\pi\tau_p \nu_F$$



Resonator modes are separated by the frequency

$\nu_F = c/2d$ *and have linewidths* $\delta\nu = \nu_F / F = 1/2\pi\tau_p$.

Conditions for laser oscillation

Condition 1: Gain condition, Laser threshold

$$\gamma_0(\nu) > \alpha_r$$

Threshold Gain Condition

because

$$\gamma_0(\nu) = N_0 \sigma(\nu)$$

$$N_0 = \gamma_0(\nu) / \sigma(\nu) > \alpha_r / \sigma(\nu)$$

$$N_0 > N_t$$

where

$$N_t = \frac{\alpha_r}{\sigma(\nu)}$$

or

$$N_t = \frac{1}{c\tau_p \sigma(\nu)}$$

$$N_t = \frac{8\pi}{\lambda^2} \frac{t_{sp}}{\tau_p} \frac{1}{g(\nu)}$$

Threshold Population Difference

For a Lorentzian lineshape function, as $g(\nu_0) = 2/\pi\Delta\nu$

$$N_t = \frac{2\pi}{\lambda^2 c}\frac{2\pi\Delta\nu t_{sp}}{\tau_p}$$

If the transition is limited by lifetime broadening with a decay time t_{sp}

At the center frequency ν_0, ***with*** $g(\nu_0)=2/(\pi\Delta\nu)$, ***then assuming*** $\Delta\nu=1/(2\pi t_{sp})$

$$N_t = \frac{2\pi}{\lambda^2 c\tau_p} = \frac{2\pi\alpha_r}{\lambda^2}$$

As a numerical example, if $\lambda_0=1\ \mu m$, $\tau_p=1$ ***ns, and the refractive index n=1, we obtain*** $N_t=2.1\times10^7\ cm^{-3}$

Conditions for laser oscillation(2)

Condition 2: Phase condition, Laser Frequencies

$$2kd + 2\varphi(\nu)d = 2\pi q, q = 1, 2\cdots\cdots$$

Frequency Pulling

$$\nu + \frac{c}{2\pi}\frac{\nu - \nu_0}{\Delta\nu}\gamma(\nu) = \nu_q$$

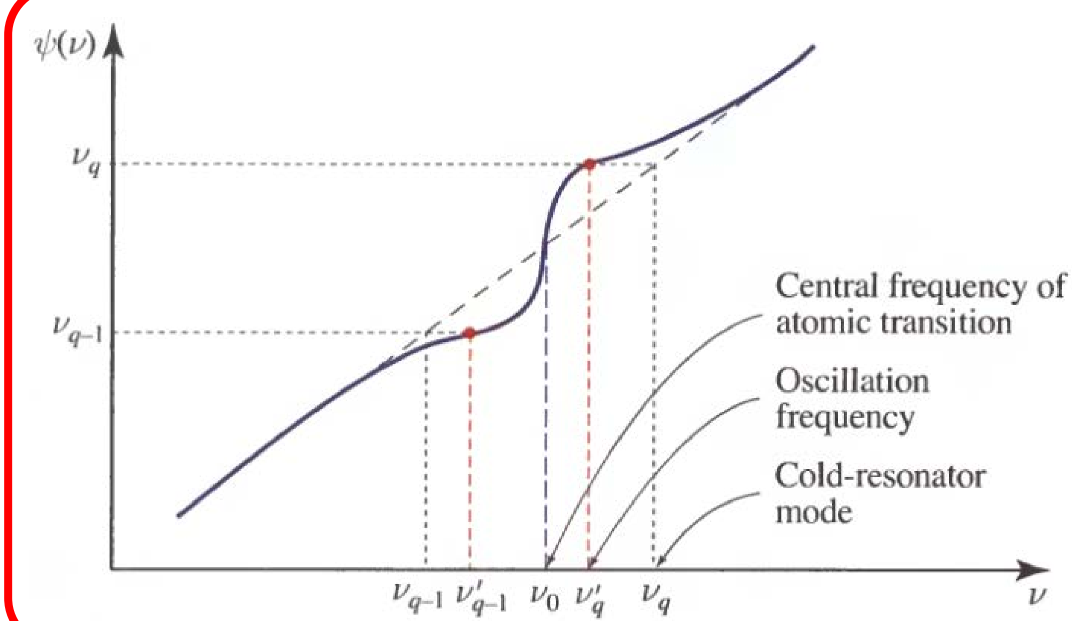
or

$$\nu = \nu_q - \frac{c}{2\pi}\frac{\nu - \nu_0}{\Delta\nu}\gamma(\nu)$$

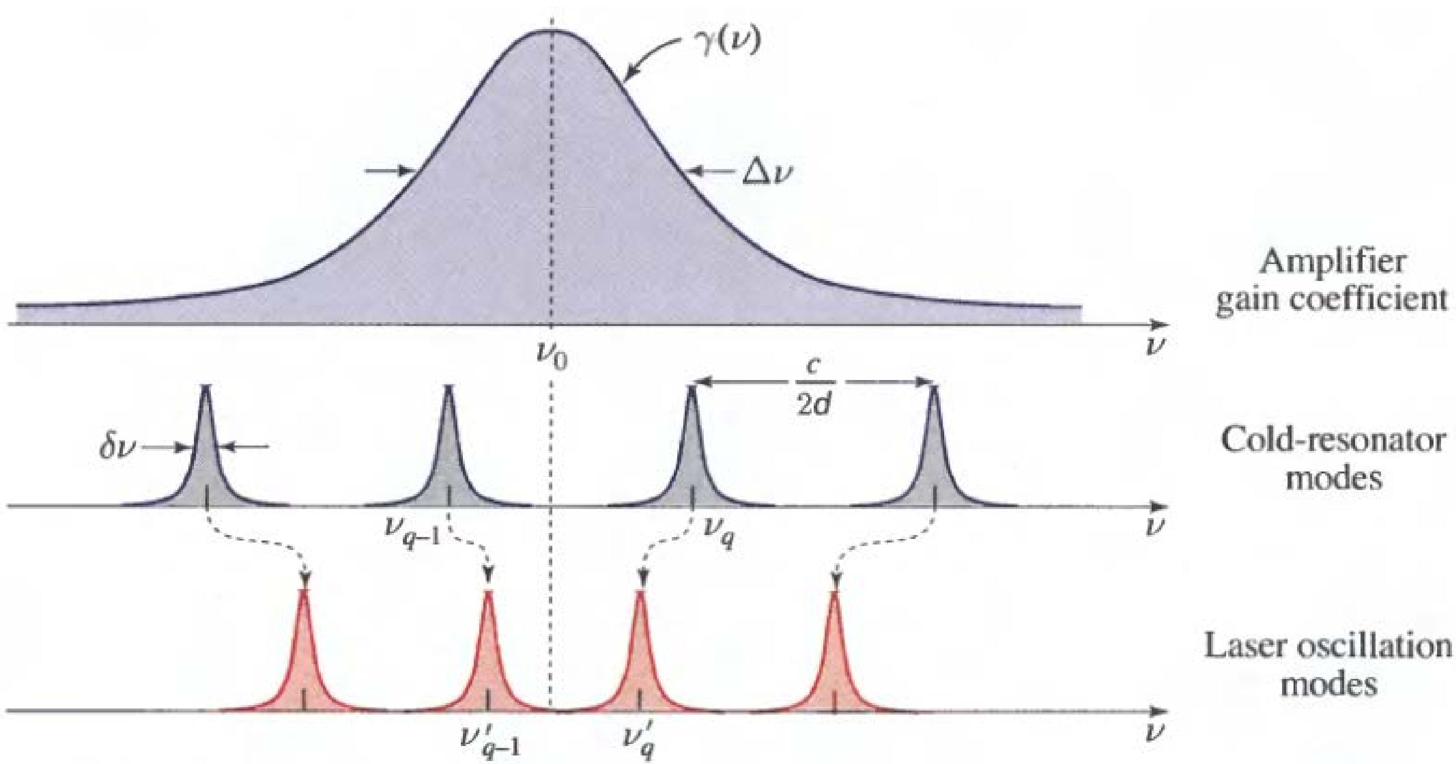
$$\nu = \nu_q' = \nu_q$$

$$\nu_q' = \nu_q - \frac{c}{2\pi}\frac{\nu_q - \nu_0}{\Delta\nu}\gamma(\nu_q)$$

$$\nu_q' = \nu_q - (\nu_q - \nu_0)\frac{\delta\nu}{\Delta\nu}$$ ***Laser Frequencies***



The $\psi(\nu)$, plotted as a function of ν. The frequency ν for which $\psi(\nu) = \nu_q$ is the solution Each "cold" resonator frequency ν_q corresponds to a "hot" resonator frequency ν_q', which is shifted in the direction of the atomic resonance central frequency ν_0.



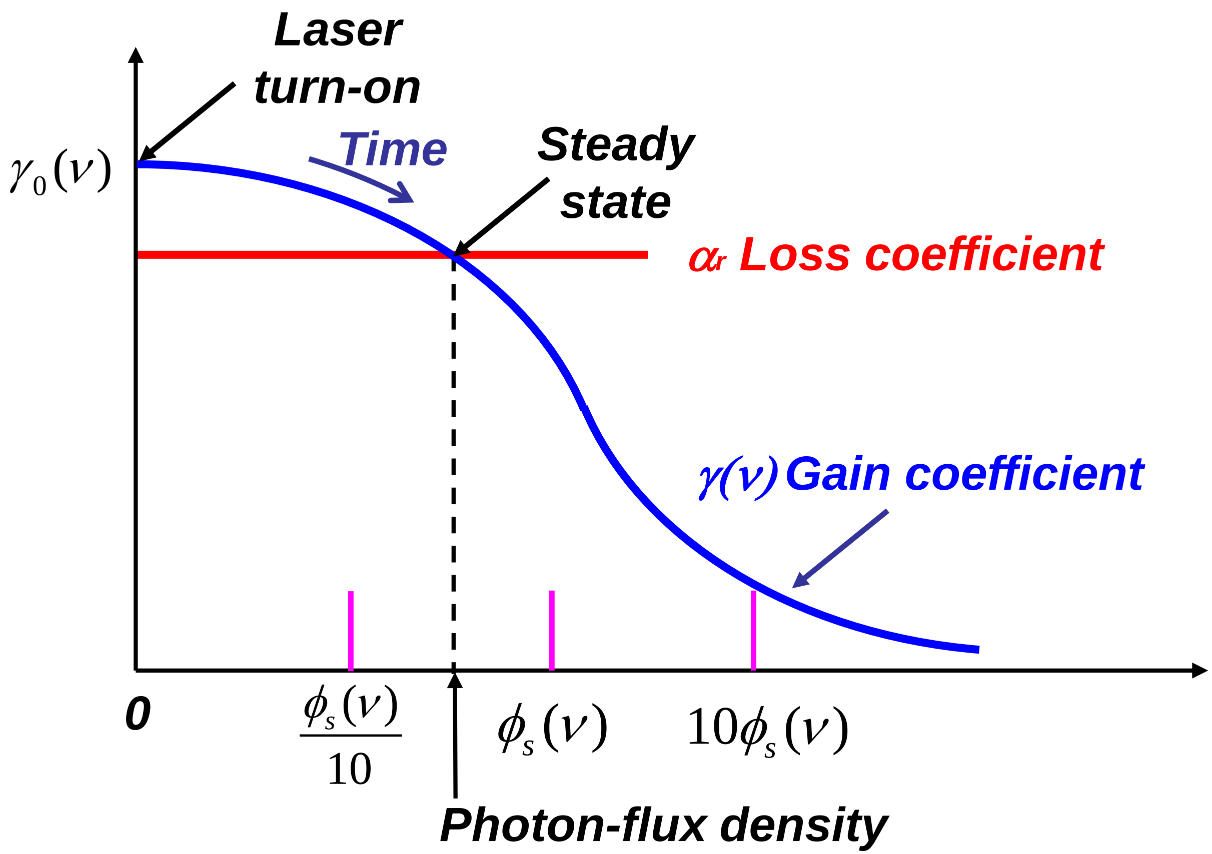
The laser oscillation frequencies fall near the cold-resonator modes; they are pulled slightly toward the atomic resonance central frequency ν_0.

Characteristics of the laser output

Internal Photon-Flux Density

Gain Clamping

$$\gamma_0(\nu)/[1+\phi/\phi_s(\nu)]=\alpha_r$$



Determination of the steady-state laser photon-flux density ϕ. At the time of laser turn on, $\phi=0$ so that $\gamma(\nu)=\gamma_0(\nu)$. As the oscillation builds up in time, the increase in ϕ causes $\gamma(\nu)$ to decrease through gain saturation. When γ reached α_r , the photon-flux density causes its growth and steady-state conditions are achieved. The smaller the loss, the greater the value of ϕ.

Steady Photon Density

$$\phi = \phi_s(\nu)[\frac{\gamma_0(\nu)}{\alpha_r} - 1], \qquad \gamma_0(\nu) > \alpha_r$$

$$\phi = 0, \qquad \gamma_0(\nu) \le \alpha_r$$

$$\phi_s(\nu) = [\tau_s \sigma(\nu)]^{-1}$$

$\tau_s = t_{sp}$ ***For four levels system***

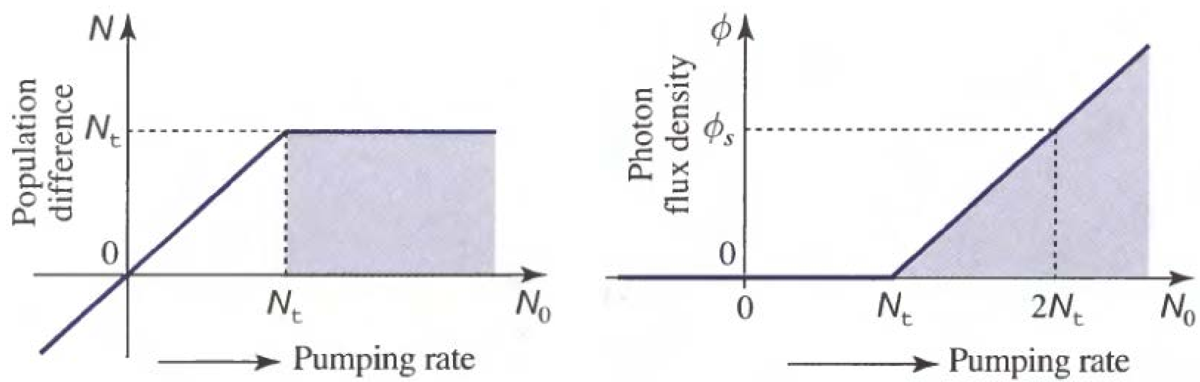
$\tau_s = 2t_{sp}$ ***For three levels system***

Since $\gamma_0(\nu) = N_0 \sigma(\nu)$ ***and*** $\alpha_r = N_t \sigma(\nu)$

$$\phi = \phi_s(\nu)(\frac{N_0}{N_t} - 1), N_0 > N_t$$

$$\phi = 0, N_0 \le N_t$$

Steady-State Laser Internal Photon-Flux Density



Steady-state values of the population difference N, and the laser internal photon-flux density ϕ, as functions of N_0 (the population difference in the absence of radiation; N_0, increases with the pumping rate R). Laser oscillation occurs when N_0 exceeds N_t; the steady-state value of N_t then saturates, clamping at the value N_t, [just as $r_0(v)$ is clamped at α_r]. Above threshold, ϕ is proportional to N_t-N_0.

Output photon-flux density

$$\phi_0 = \frac{T\phi}{2}$$

Optical Intensity of Laser Output

$$I_0 = \frac{h\nu T\phi}{2}$$

Optimization of the output photon-flux density

From
$$\alpha_{m1} = \frac{1}{2d}\ln\frac{1}{R_1} = -\frac{1}{2d}\ln(1-T)$$

We obtain
$$\alpha_r = \alpha_s + \alpha_{m2} - \frac{1}{2d}\ln(1-T)$$

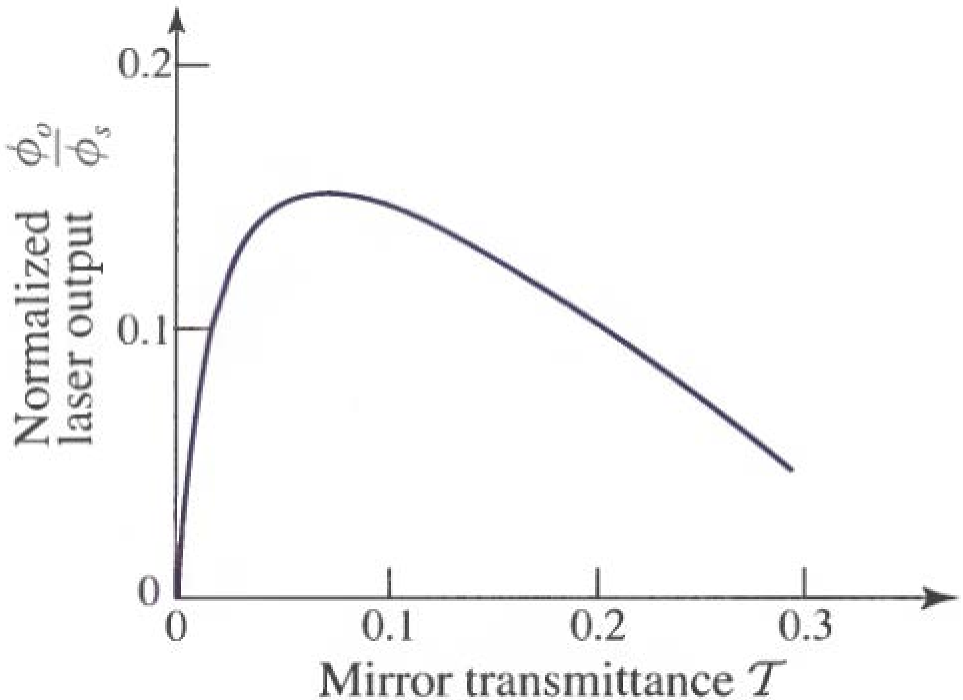
$$\because \quad \phi = \phi_s\left[\frac{\gamma_0}{\alpha_r} - 1\right]$$

$$\phi_0 = \frac{T\phi}{2} \Rightarrow \phi_0 = \frac{1}{2}\phi_s T\left[\frac{g_0}{L-\ln(1-T)} - 1\right], g_0 = 2\gamma_0(\nu)d, L = 2(\alpha_s + \alpha_{m2})d$$

When, $T \ll 1$

use the approximation $\ln(1-T) \approx -T$

Then $T_{op} \approx (g_0 L)^{1/2} - L$



Dependence of the transmitted steady-state photon-flux density ϕ_o on the mirror transmittance $\mathcal{T}$. For the purposes of this illustration, the gain factor $g_0 = 2\gamma_0 d$ has been chosen to be 0.5 and the loss factor $L = 2(\alpha_s + \alpha_{m2})d$ is 0.02 (2%). The optimal transmittance $\mathcal{T}_{op}$ turns out to be 0.08.

Internal Photon-Number Density

The steady-state number of photons per unit volume inside the resonator $n = \frac{\phi}{c}$

The steady-state internal photon-number density $n = n_s(\frac{N_0}{N_t} - 1), \qquad N_0 > N_t$

Where $n_s = \phi_s(\nu)/c$ *is the photon-number density saturation value*

Because: $\alpha_r = 1/c\tau_p \qquad \phi_s(\nu) = [\tau_s \sigma(\nu)]^{-1} \qquad \gamma(\nu) = N\sigma(\nu) = N_t\sigma(\nu)$

We have $n = (N_0 - N_t)\frac{\tau_p}{\tau_s}, \qquad N_0 > N_t$

For 4 level system, there are $\tau_s = t_{sp}$ ***and*** $N_0 \approx Rt_{sp}$ ***so that***

$$\frac{n}{\tau_p} = R = R_{t,} \qquad R > R_t$$

R is pumping rate (s^-1^cm^-3^)

Where $R_t = N_t / t_{sp}$ ***Is the thresh value of pumping rate.***

Output Photon Flux and Efficiency

$$\phi_0 = (R - R_t)V, R > R_t$$

$$\phi_0 = \eta_e (R - R_t)V$$

$$\eta_e = \frac{\alpha_{m1}}{\alpha_r} = \frac{c}{2d}\tau_p \ln\frac{1}{R_1}$$

$$\eta_e \approx \frac{\tau_p}{T_F} T$$ ***where*** $$T_F = 2d/c$$

$$P_o = h\nu\phi_o = \eta_e h\nu(R - R_t)V$$

Spectral Distribution

Determined both by the atomic lineshape and by the resonant modes

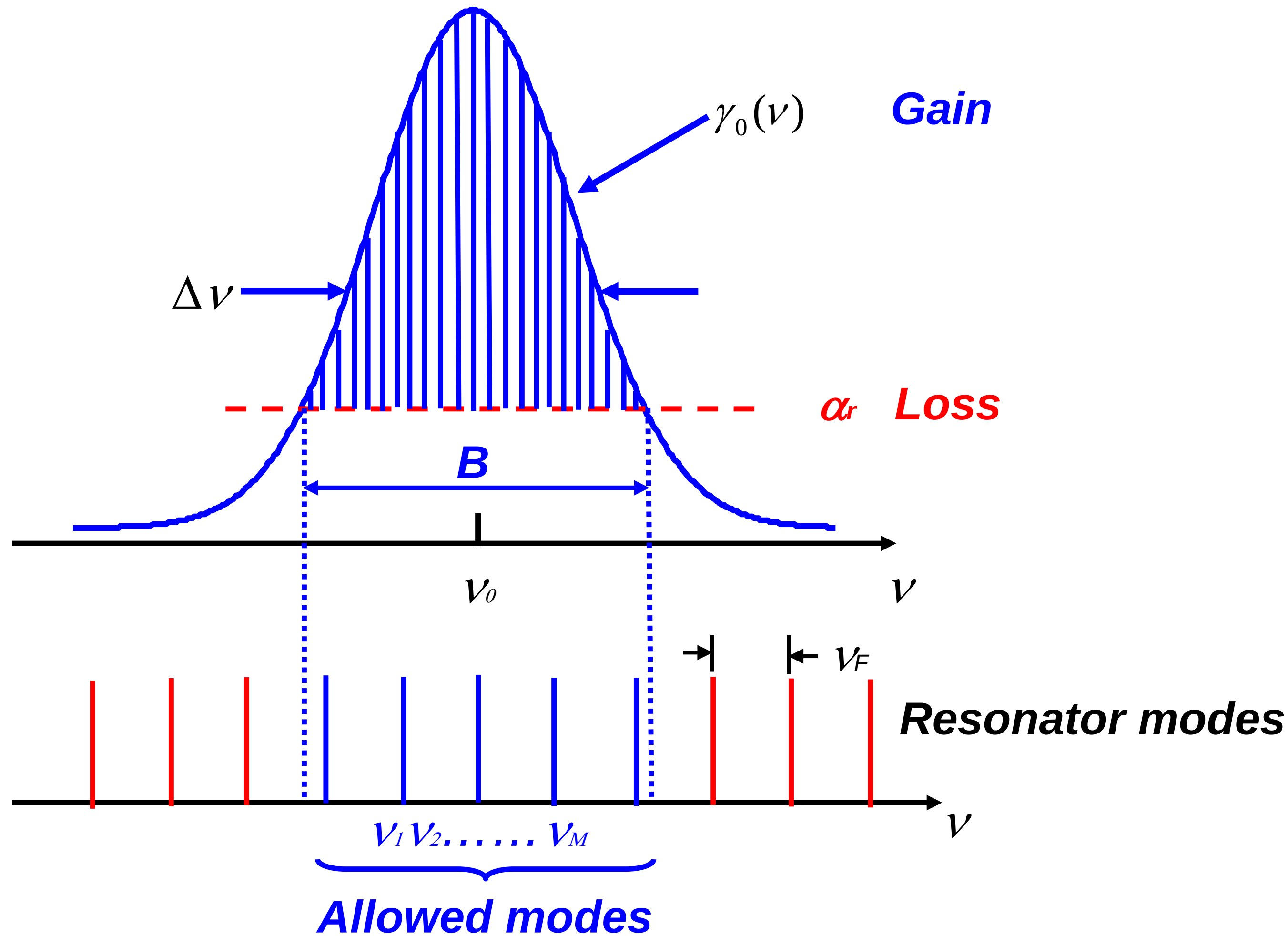
$$M \approx \frac{B}{\nu_F}$$

Number of Possible Laser Modes

Where B is *pectral band of width,* V_F *mode interval*

Linewidth $\approx \delta\nu$ *?*

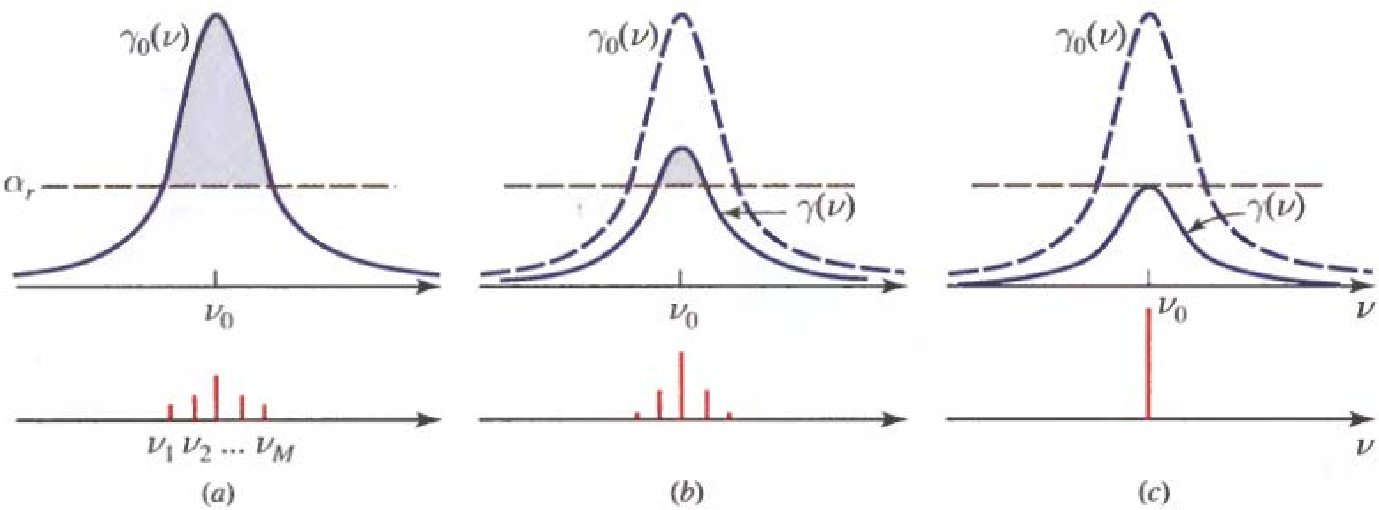
Schawlow-Tones limit



(a) Laser oscillation can occur only at frequencies for which the gain coefficient is greater than the loss coefficient (stippled region). (b) Oscillation can occur only within $\delta\nu$ of the resonator modal frequencies (which are represented as lines for simplicity of illustration).

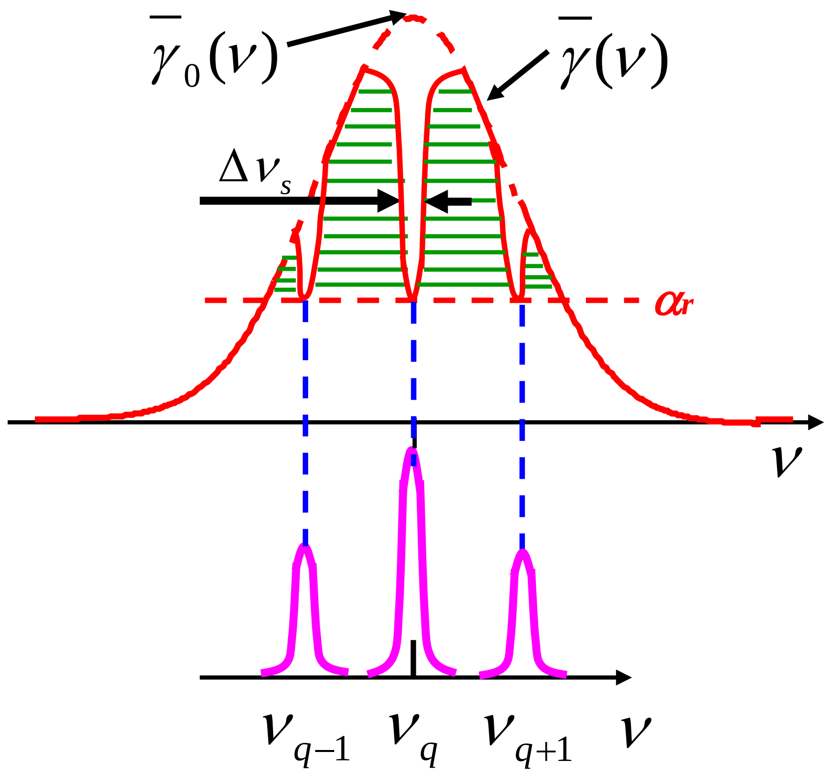
Homogeneously Broadened Medium

$$\gamma(\nu)=\frac{\gamma_0(\nu)}{1+\sum_{i=1}^{M}\phi_j/\phi_s(\nu_j)}$$

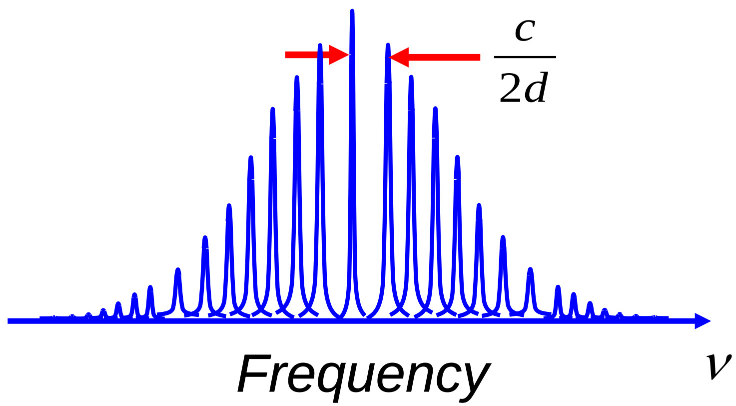


Growth of oscillation in an ideal homogeneously broadened medium. (*a*) Immediately following laser turn-on, all modal frequencies $\nu_1, \nu_2, \dots, \nu_M$, for which the gain coefficient exceeds the loss coefficient, begin to grow, with the central modes growing at the highest rate. (*b*) After a short time the gain saturates so that the central modes continue to grow while the peripheral modes, for which the loss has become greater than the gain, are attenuated and eventually vanish. (*c*) In the absence of spatial hole burning, only a single mode survives.

Inhomogeneously Broadened Medium



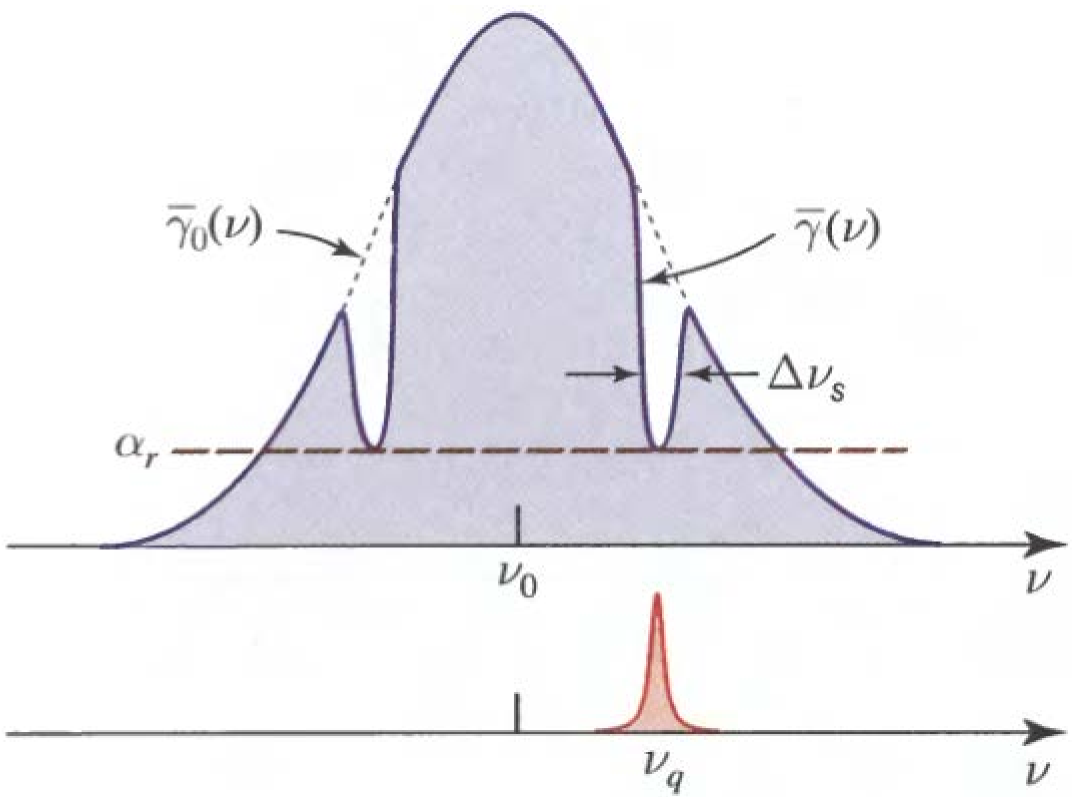
(a)



(b)

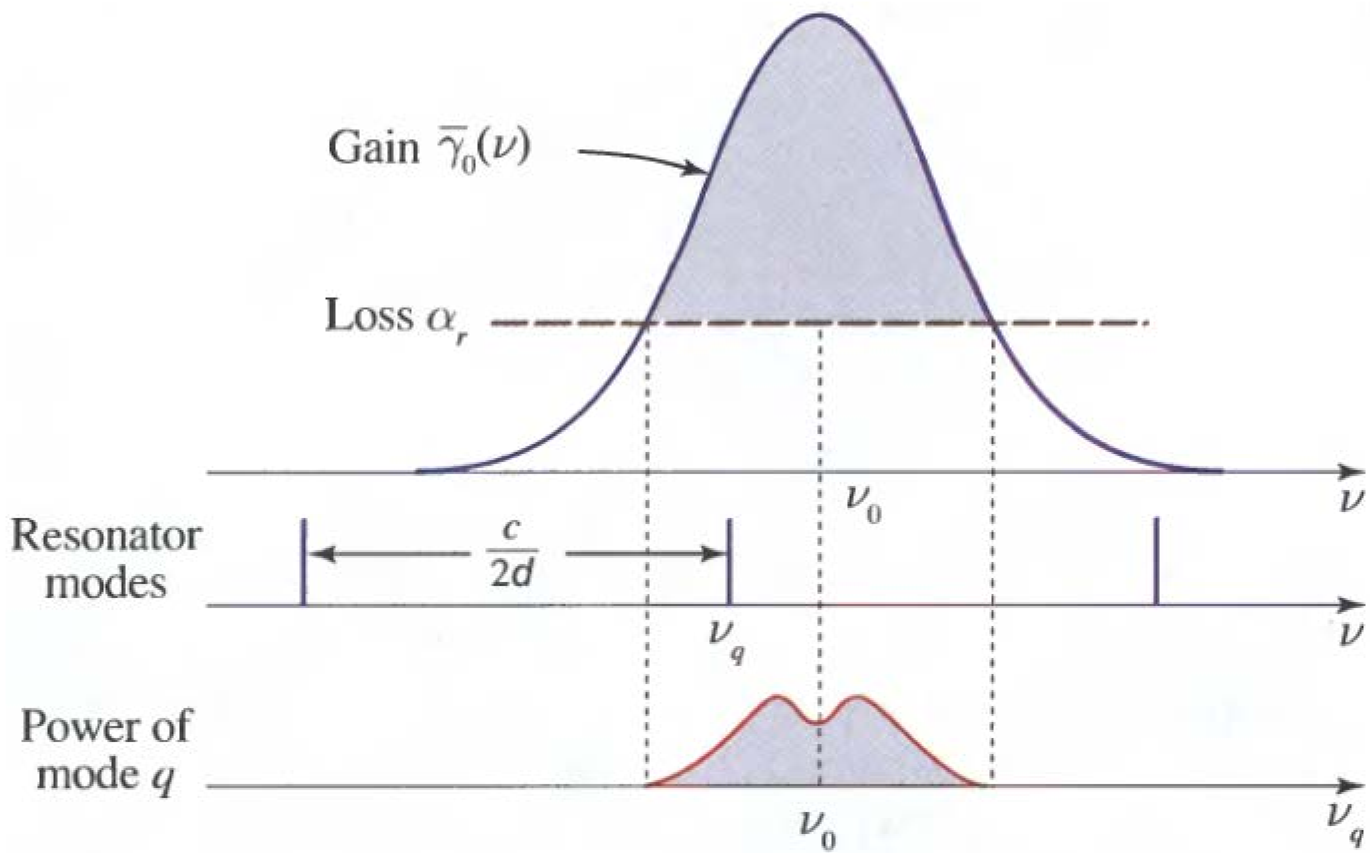
(a) Laser oscillation occurs in an inhomogeneously broadened medium by each mode independently burning a hole in the overall spectral gain profile. The gain provided by the medium to one mode does not influence the gain it provides to other modes. The central modes garner contributions from more atoms, and therefore carry more photons than do the perpheral modes. (b) Spectrum of a typical inhomogeneously broadened multimode gas laser.

Hole burning in a Doppler-broadened medium



Hole burning in a Doppler-broadened medium. A probe wave at frequency ν_q saturates those atomic populations with velocities $v = \pm c(\nu_q/\nu_0 - 1)$ on both sides of the central frequency, burning two holes in the gain profile.

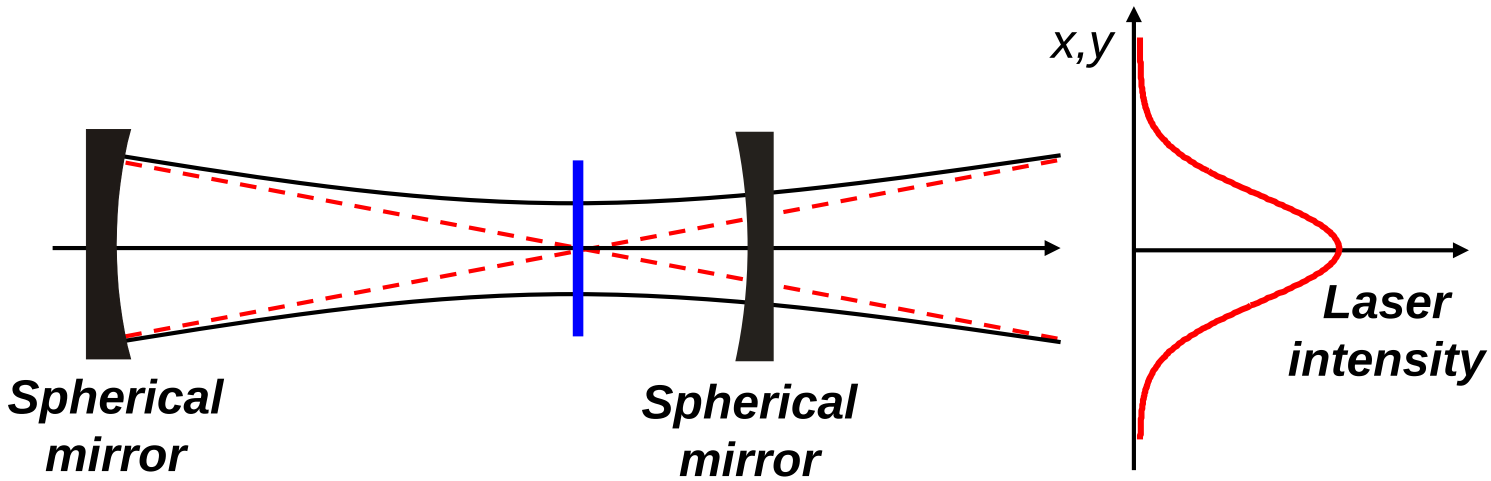
Hole burning in a Doppler-broadened medium



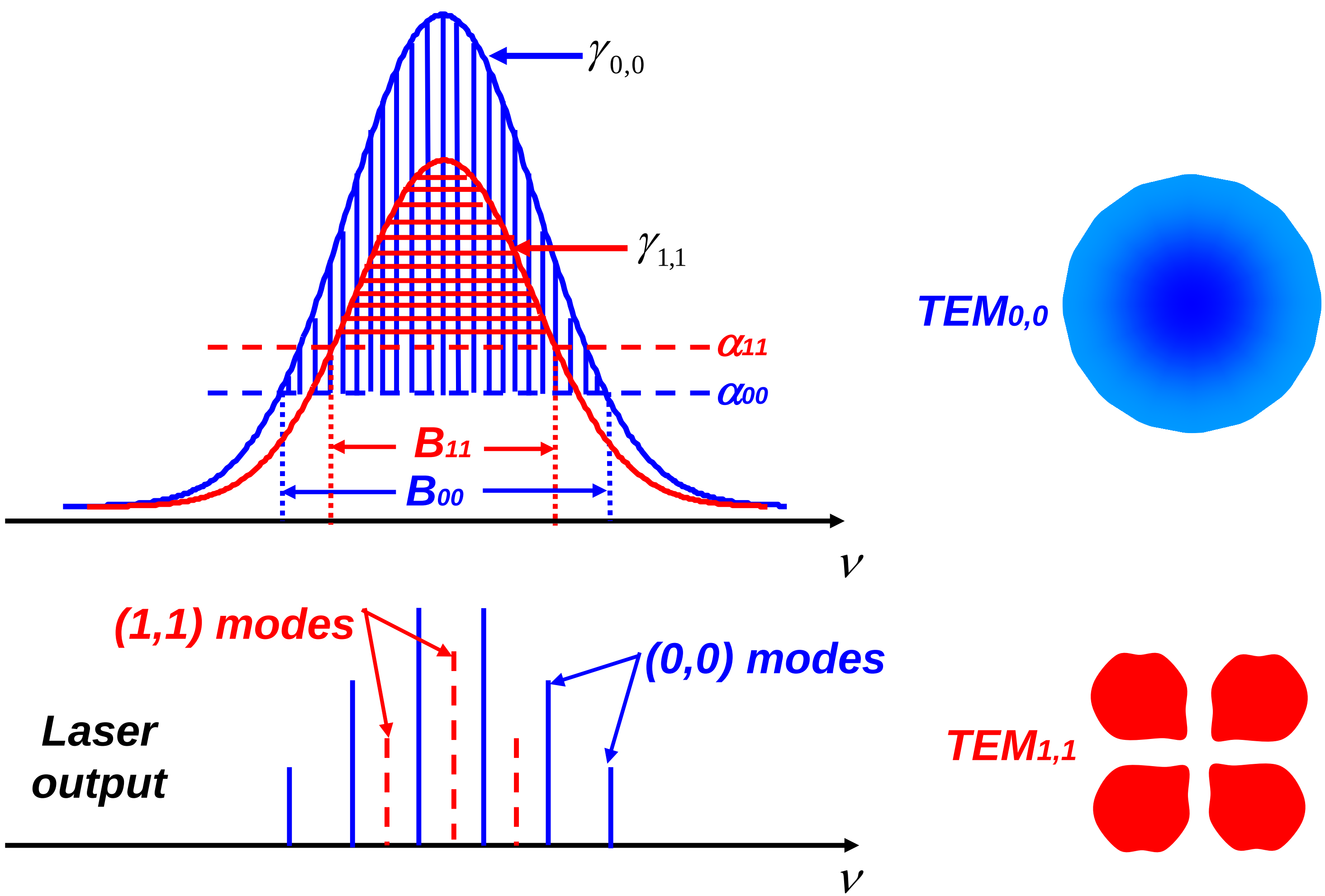
Power in a single laser mode of frequency ν_q in a Doppler-broadened medium whose gain coefficient is centered about ν_0. Rather than providing maximum power at $\nu_q = \nu_0$, it exhibits the Lamb dip.

Spatial distribution and polarization

Spatial distribution



The laser output for the (0,0) transverse mode of a spherical-mirror resonator takes the form of a Gaussian beam.



The gains and losses for two transverse modes, say (0,0) and (1,1), usually differ because of their different spatial distributions. A mode can contribute to the output if it lies in the spectral band (of width B) within the gain coefficient exceeds the loss coefficient. The allowed longitudinal modes associated with each transverse mode are shown.

Two Issues: Polarization, Unstable Resonators

Each (l, m, q) mode has two degrees of freedom, corresponding to two independent orthogonal polarizations. These two polarizations are regarded as two independent modes.

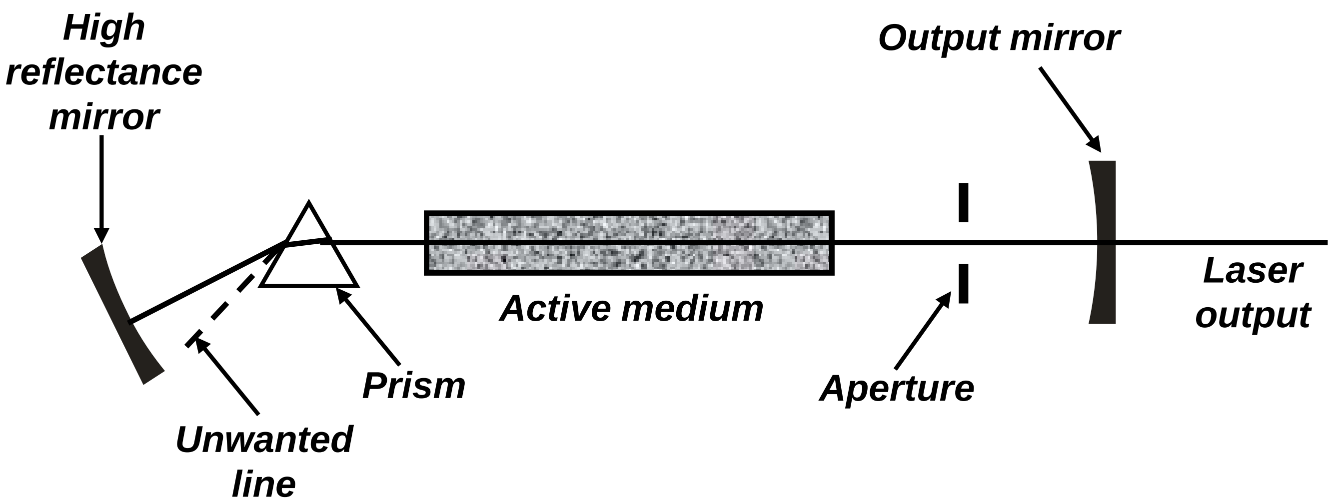
Unstable Resonators

use of unstable resonators offers a number of advantages in the operation of high-power lasers.
(1) a greater portion of the gain medium contributing to the laser output power as a result of the availability of a larger modal volume;
(2) higher output powers attained from operation on the lowest-order transverse mode, rather than on higher-order transverse modes as in the case of stable resonators;
(3) high output power with minimal optical damage to the resonator mirrors, as a result of the use of purely reflective optics that permits the laser light to spill out around the mirror edges

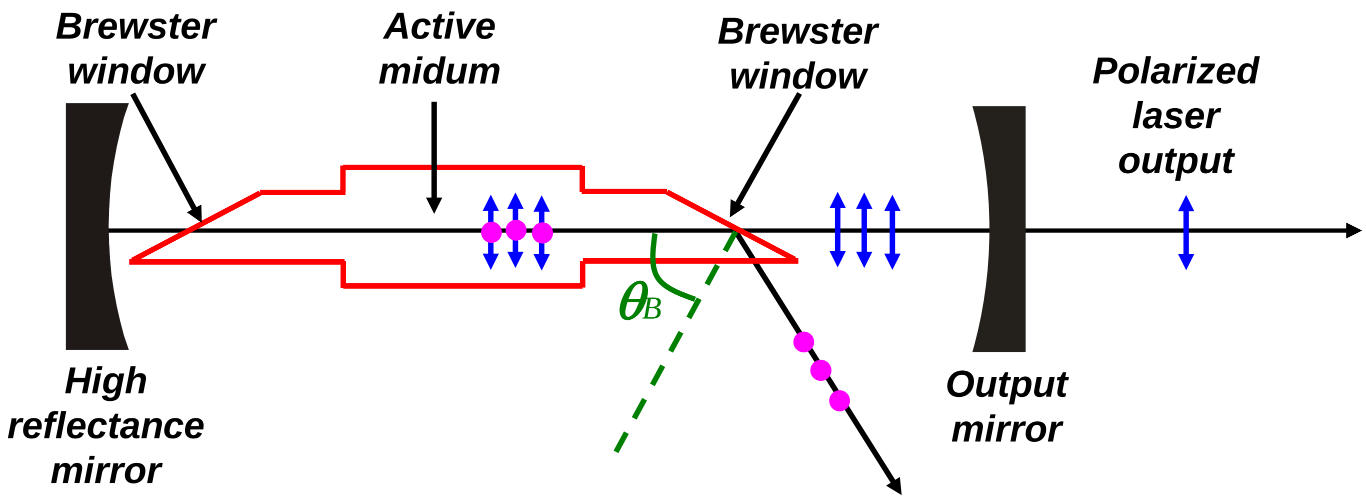
Mode Selection

Selection of

1. ***Laser Line***
2. ***Transverse Mode***
3. ***Polarization***
4. ***Longitudinal Mode***

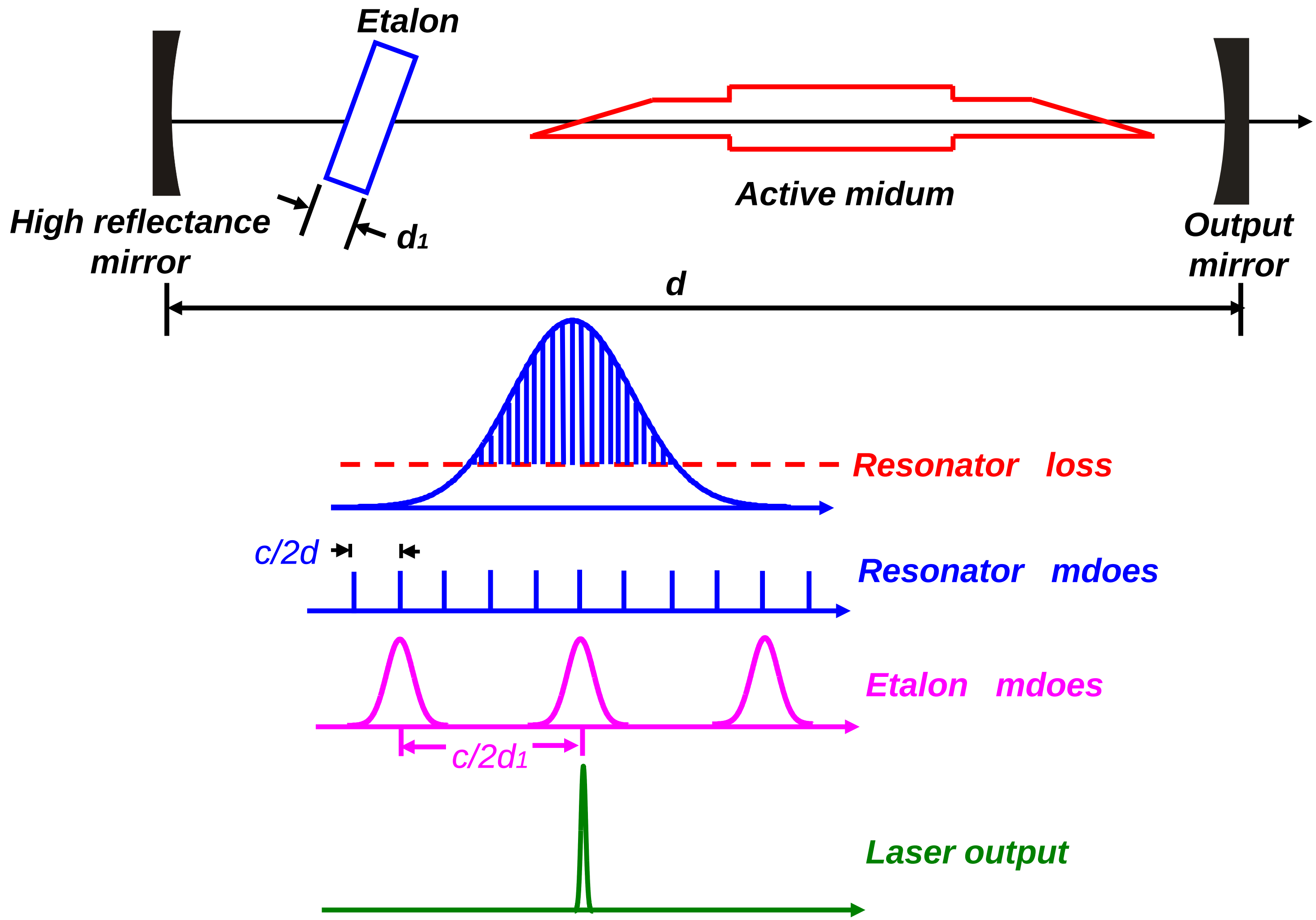


A particular atomic line may be selected by the use of a prism placed inside the resonator. A transverse mode may be selected by means of a spatial aperture of carefully chosen shaped and size.



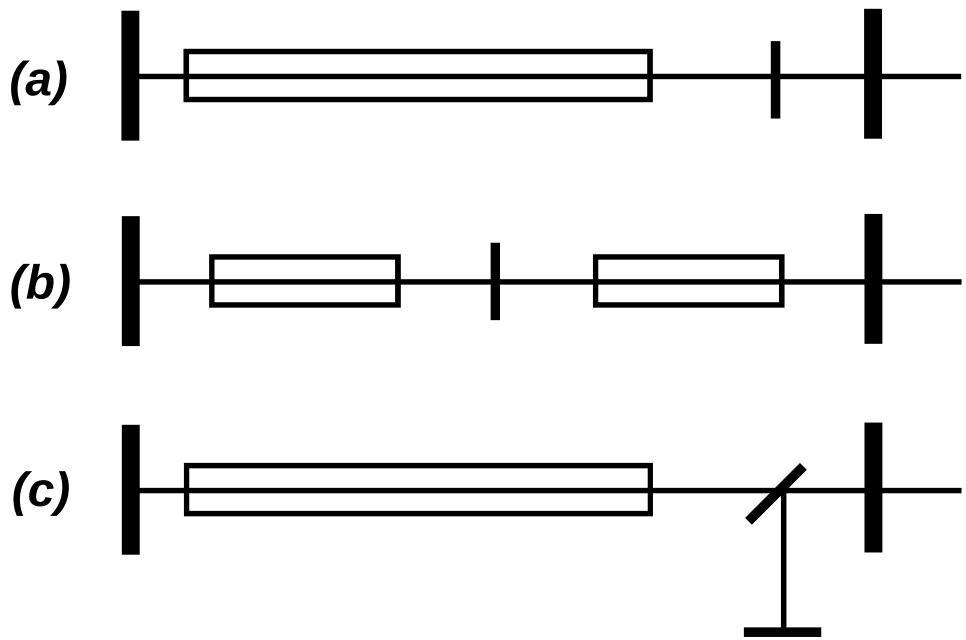
The use of Brewster windows in a gas laser provides a linearly polarized laser beam. Light polarized in the plane of incidence (the TM wave) is transmitted without reflection loss through a window placed at the Brewster angle. The orthogonally polarized (TE) mode suffers reflection loss and therefore does not oscillate.

Selection of Longitudinal Mode



Longitudianl mode selection by the use of an intracavity etalon. Oscillation occurs at frequencies where a mode of the resonator coincides with an etalon mode; both must, of course, lie within the spectral window where the gain of the medium exceeds the loss.

Multiple Mirror Resonators



Longitudinal mode selection by use of (a) two coupled resonators (one passive and one active); (b) two coupled active resonators; (c) a coupled resonator-interferometer.

Characteristics of Common Lasers

Solid State Lasers: Ruby, Nd3+:YAG, Nd3+:Silica, Er3+:Fiber, Yb3+:Fiber

Gas Lasers: He-Ne, Ar+; CO2, CO, KF;

Liquid Lasers: Dye

Plasma X-Ray Lasers

Free Electron Lasers

Laser Medium	Transition Wavelength λ_o	Single Mode (S) or Multimode (M)	CW or Pulsed[b]	Approximate Overall Efficiency $\eta_c(\%)$[c]	Output Power or Energy[d]	Energy-Level Diagram
Ag^{19+} (p)	13.9 nm	M	Pulsed	0.0002	25 μJ	
C^{5+} (p)	18.2 nm	M	Pulsed	0.0005	2 mJ	Fig. 13.1-1
ArF Excimer (g)	193 nm	M	Pulsed	1.	200 mJ	
KrF Excimer (g)	248 nm	M	Pulsed	1.	500 mJ	
Er^{3+}:Silica fiber (s)	1550 nm	S/M	CW	10.	100 W	Fig. 14.3-6
Tm^{3+}:Fluoride fiber (s)	1.8–2.1 μm	S/M	CW	5.	150 W	
He–Ne (g)	3.39 μm	S/M	CW	0.05	20 mW	Fig. 13.1-2
CO_2 (g)	10.6 μm	S/M	CW	10.	500 W	Fig. 13.1-4
H_2O (g)	28 μm	S/M	CW	0.02	100 mW	
FEL at UCSB	60 μm–2.5 mm	M	Pulsed	0.5	5 mJ	
H_2O (g)	118.7 μm	S/M	CW	0.01	50 mW	
CH_3OH (g)	118.9 μm	S/M	CW	0.02	100 mW	
HCN (g)	336.8 μm	S/M	CW	0.01	20 mW	

[a] Gas (g), solid (s), liquid (l), plasma (p).

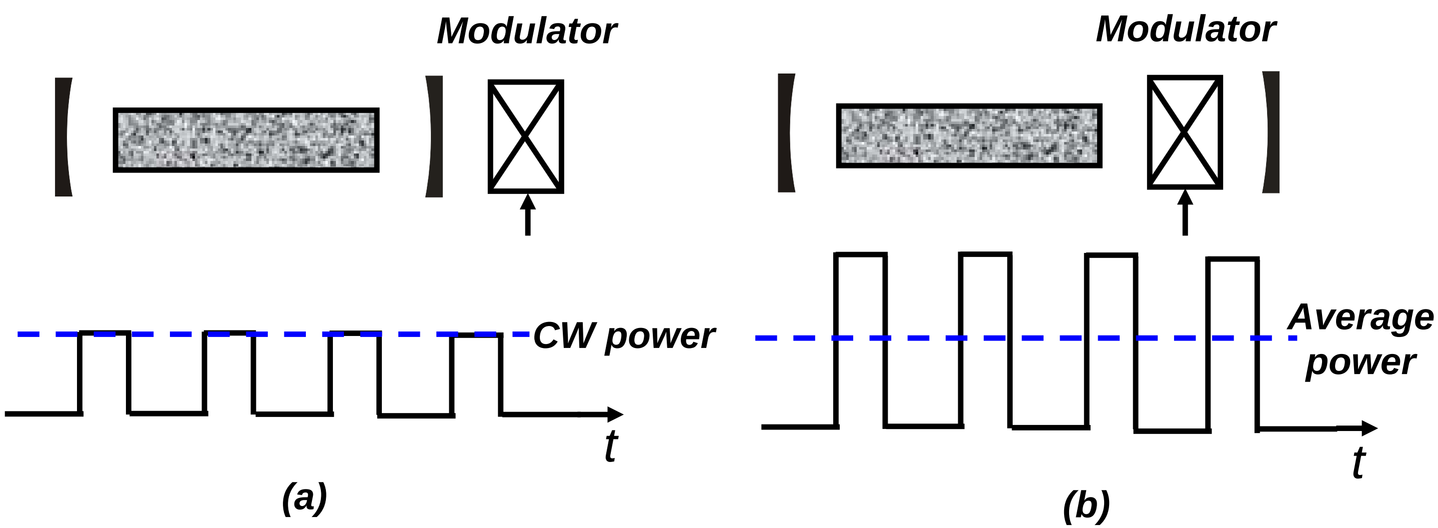
[b] Lasers designated "CW" can, of course, be operated in a pulsed mode; lasers designated "pulsed" are usually operated in that mode.

[c] The power-conversion efficiency η_c (also called the overall efficiency and wall-plug efficiency) is the ratio of output light power to input electrical power (for pulsed lasers, the ratio of output light energy to input electrical energy). Values reported have substantial uncertainty since in some cases they include the electrical power consumed for overhead functions such as cooling and monitoring. Laser diodes exhibit the highest efficiencies, readily exceeding 50%, as discussed in Sec. 17.4C.

[d] The output power (for CW systems) and output energy per pulse (for pulsed systems) vary over a substantial range, in part because of the wide range of pulse durations; representative values are provided.

Pulsed Lasers

Method of pulsing lasers → ***External Modulator or Internal Modulator?***

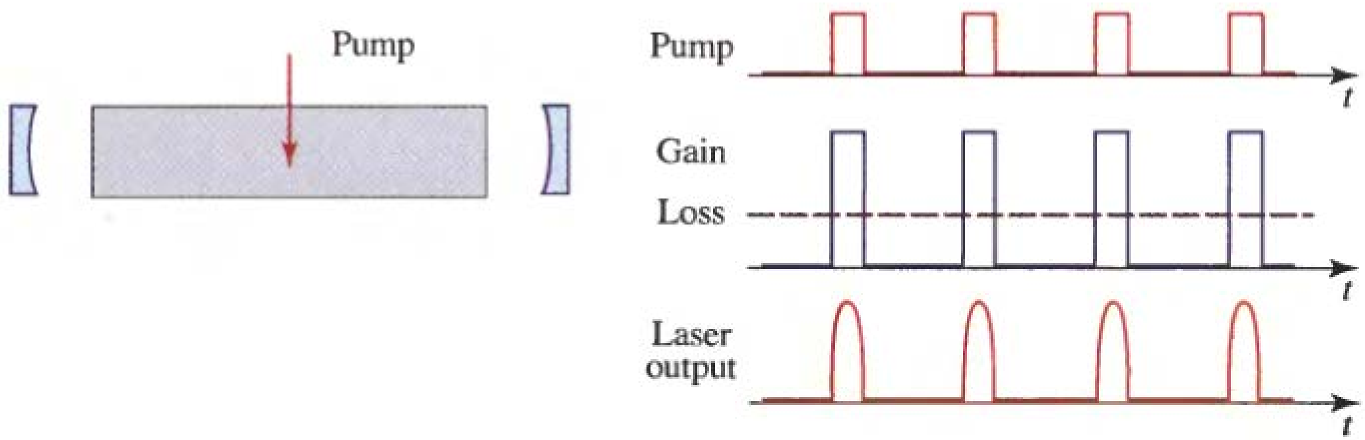


Comparison of pulsed laser outputs achievable with (a) an external modulator, and (b) an internal modulator

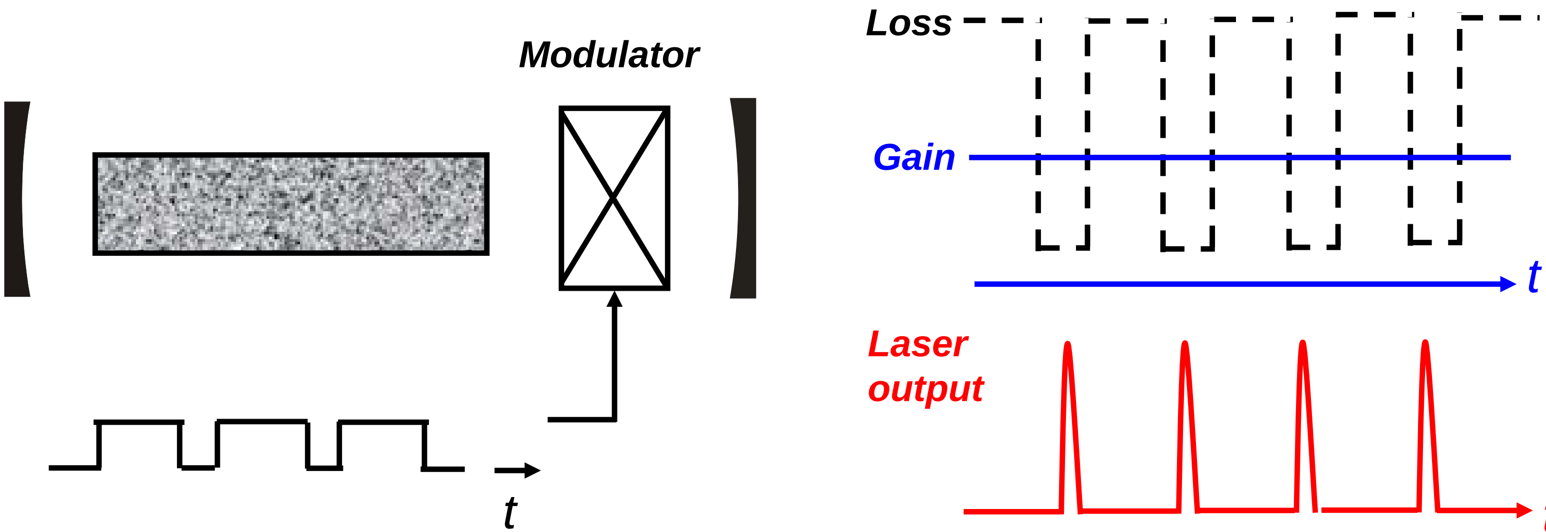
1. Gain switching ***2. Q-Switching***

3. Cavity Dumping ***4.Mode Locking***

Gain Switching

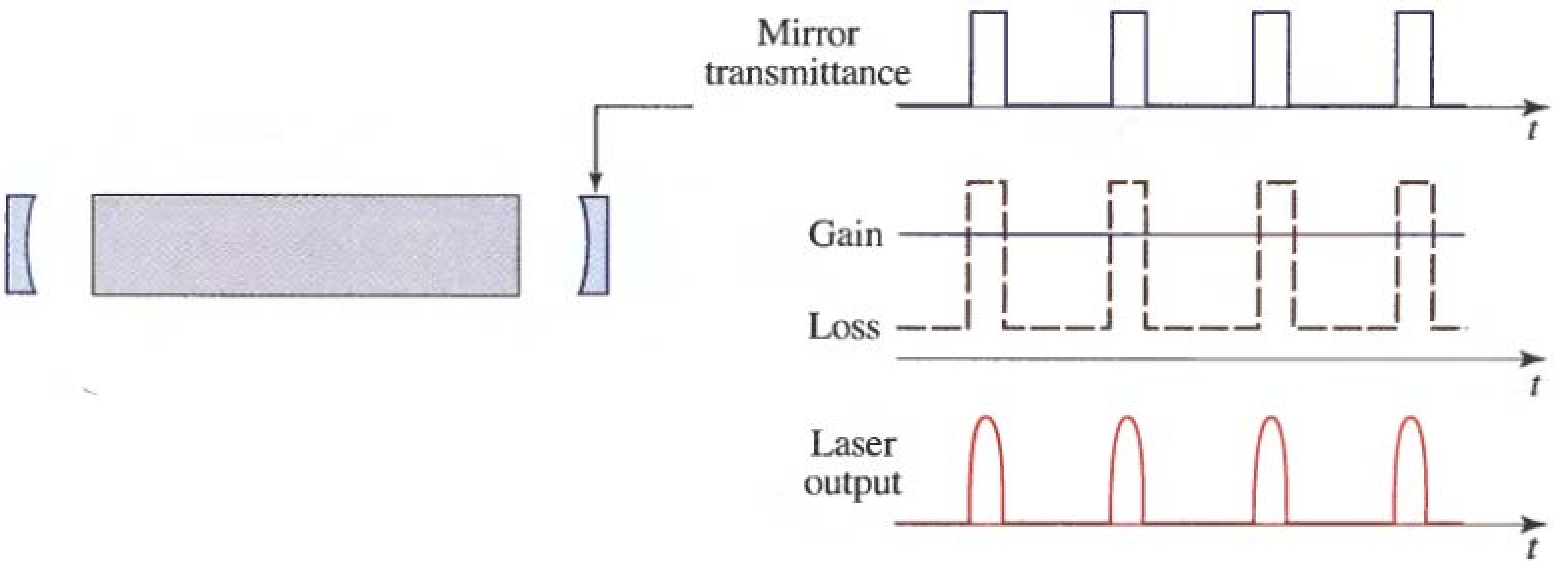


Q- Switching



Q-switching.

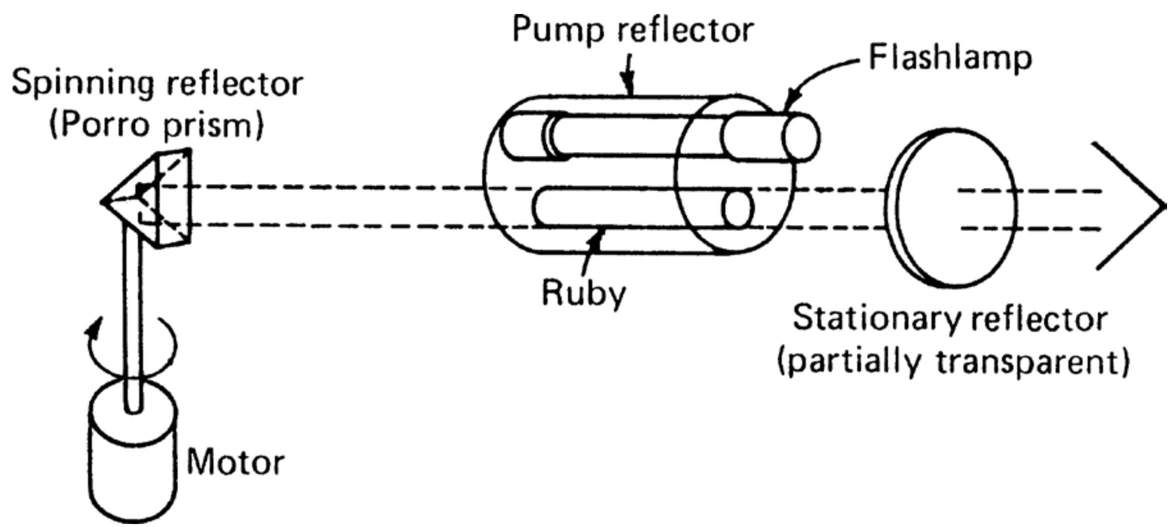
Cavity Dumping



Cavity dumping. One of the mirrors is removed altogether to dump the stored photons as useful light.

Techniques for Q-switching

1. Mechanical rotating mirror method:



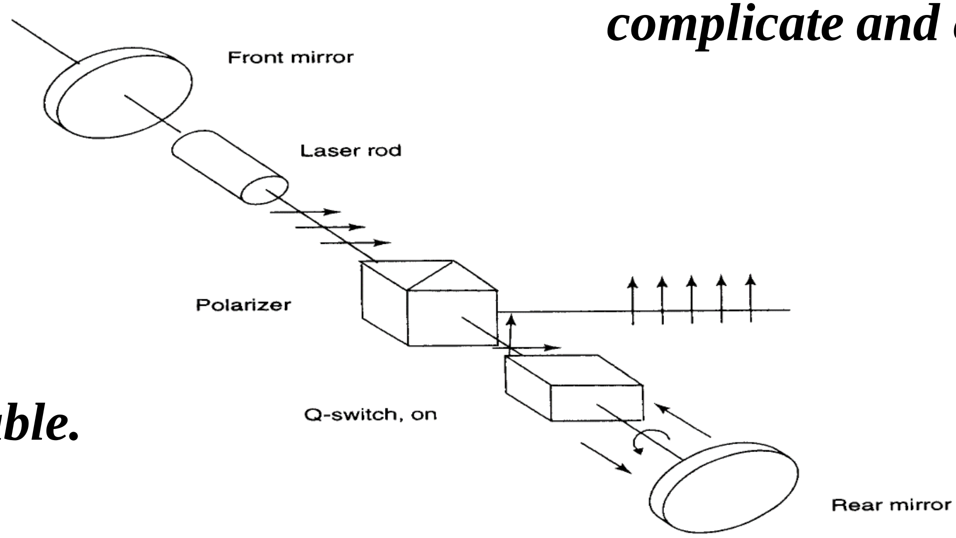
Q-switching principle: rotating the cavity mirror results in the cavity losses high and low, so the Q-switching is obtained.

Advantages: *simple, inexpensive.*
Disadvantages: *very slow, mechanical vibrations.*

2. Electro-optic Q-switching

Disadvantages:

complicate and expensive

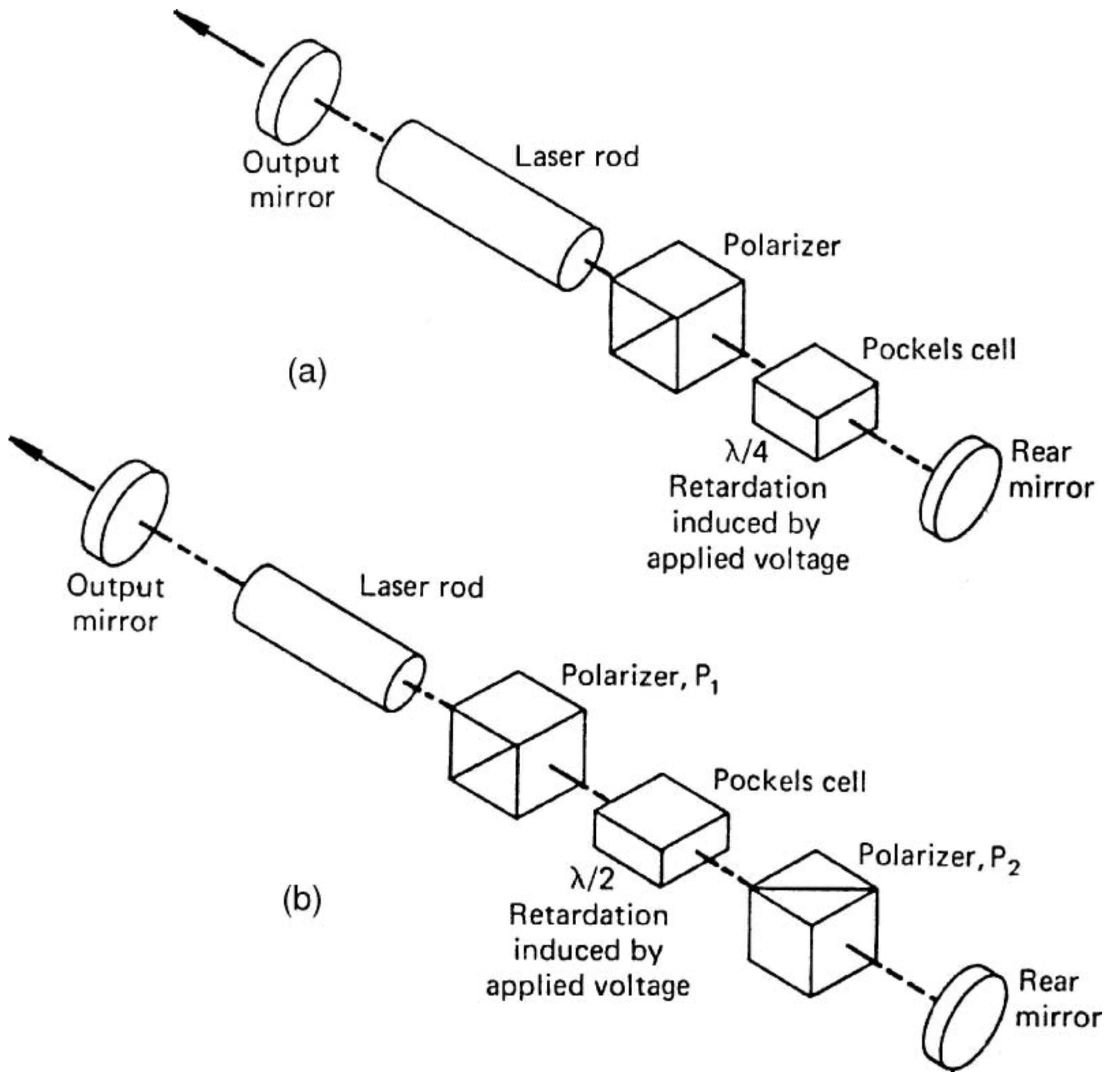


Advantages:

very fast and stable.

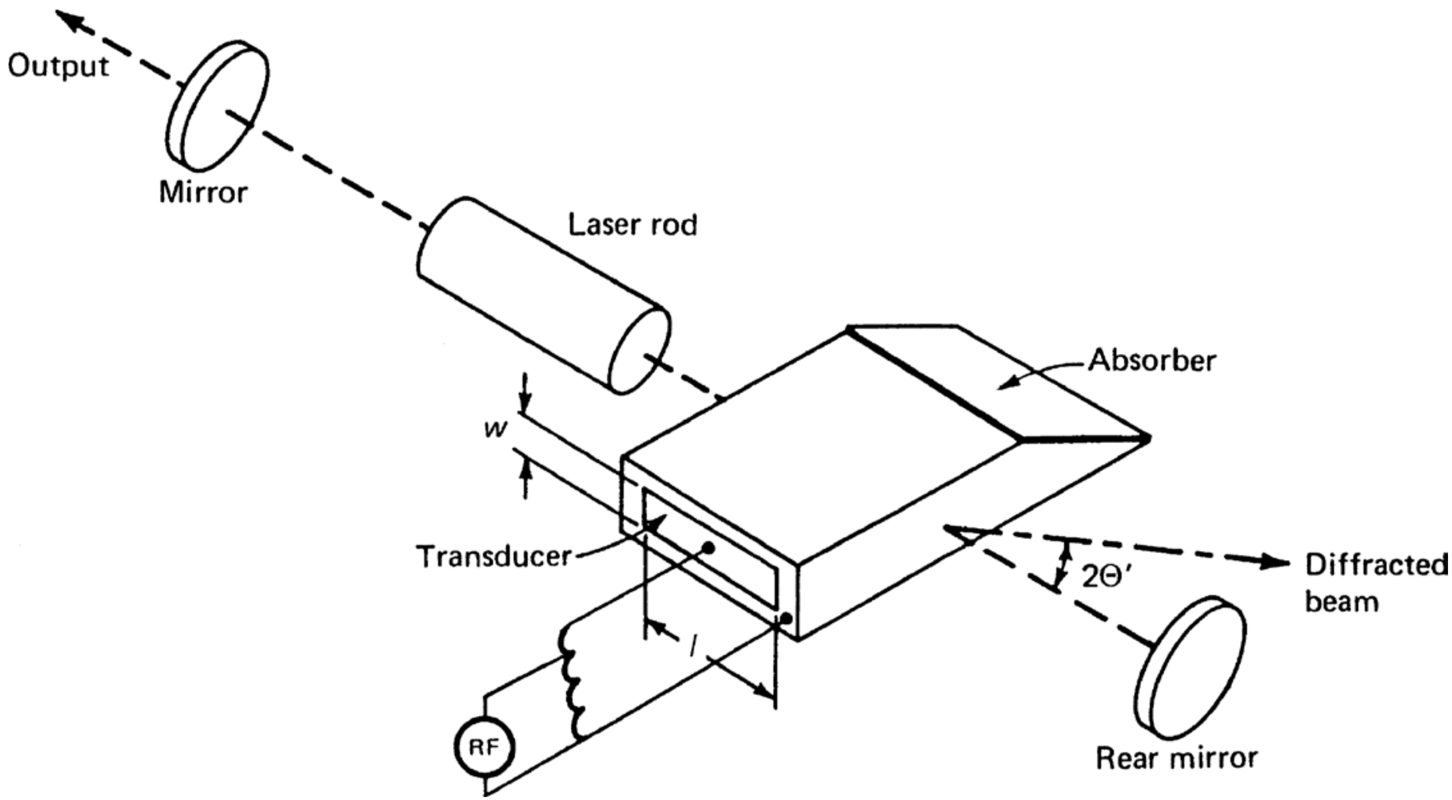
Pockels effect*: applying electrical field in a uniaxial crystal results in additional birefringence, which changes the polarization of light when passing through it.*

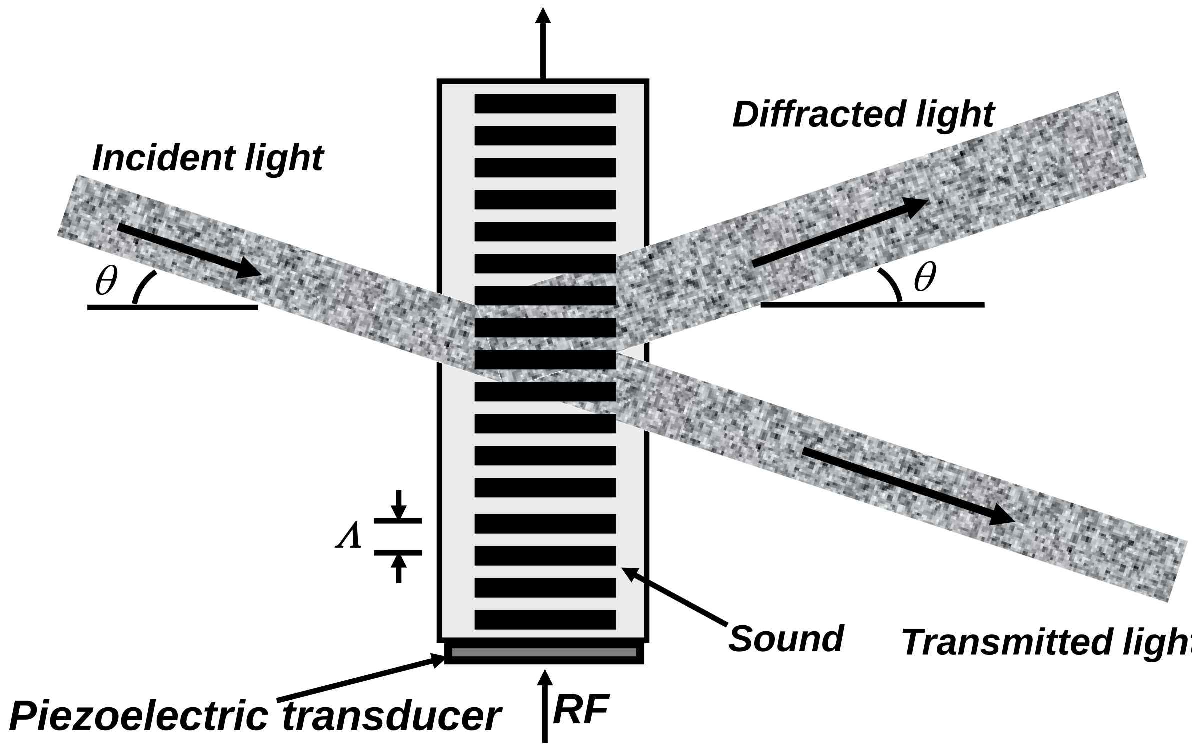
Q-switching principle*: placing an electro-optic crystal between crossed polarizers comprises a Pockels switch. Turning on and off the electrical field results in high and low cavity losses.*



Electro-optic Q-switch operated at (a) quarter-wave and (b) half-wave retardation voltage

3. Acousto-optic Q-switching





Bragg scattering: *due to existence of the acoustic wave, light changes its propagation direction.*

Q-switching principle: *through switching on and off of the acoustic wave the cavity losses is modulated.*

Advantages: *works even for long wavelength lasers.*
Disadvantages: *low modulation depth and slow.*

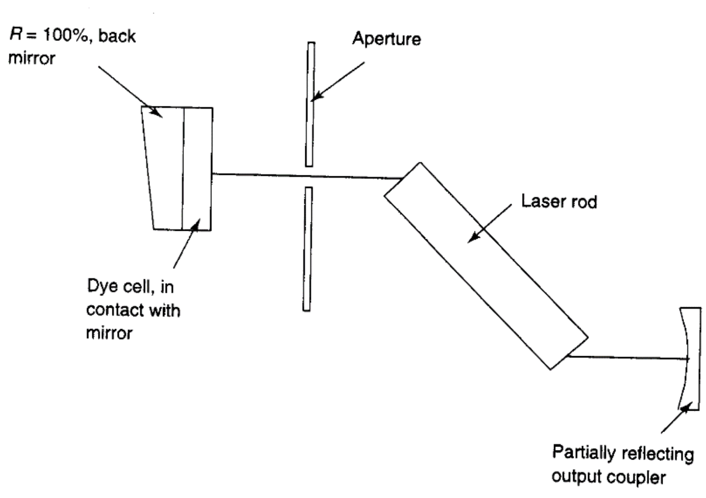
4. Saturable absorber Q-switching

What's a saturable absorber?

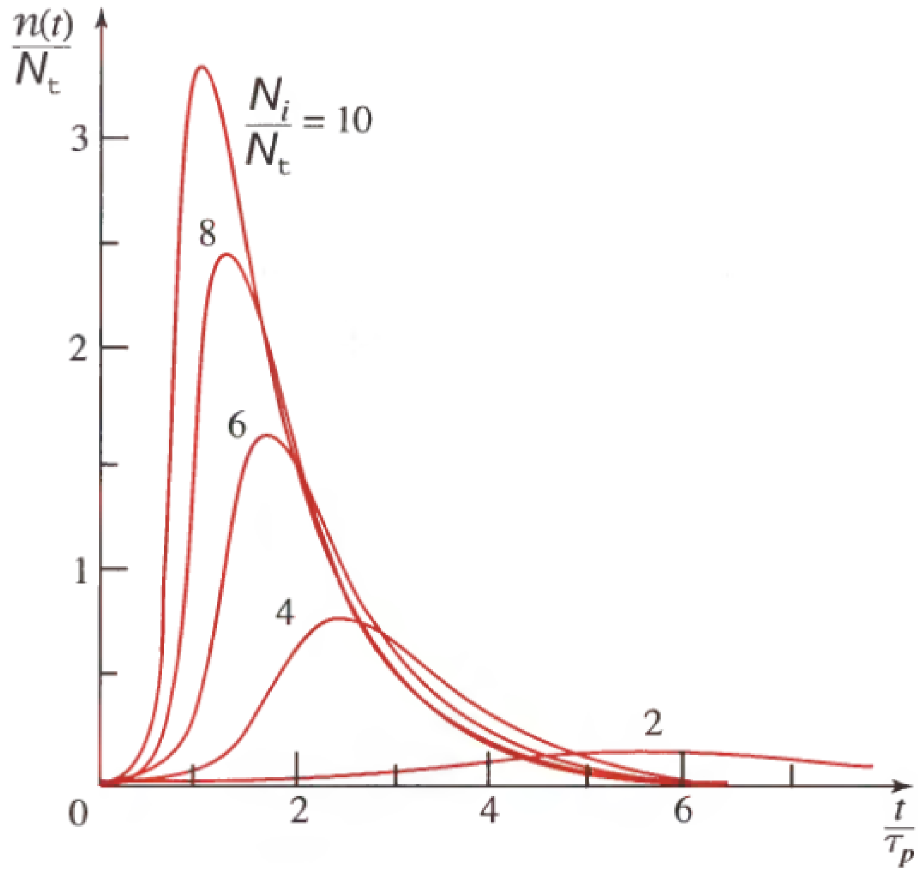
$$\alpha = \frac{\alpha_0}{1 + \frac{I}{I_s}}$$

Absorption coefficient of the material is reversely proportional to the light intensity. I_s: saturation intensity.

Saturable absorber Q-switching:



Insertion a saturable absorber in the laser cavity, the Q-switching will be automatically obtained.



Typical Q-switched pulse shapes obtained from numerical integration of the approximate rate equations. The photon-number density $n(t)$ is normalized to the threshold population difference $N_t = N_{th}$ and the time t is normalized to the photon lifetime τ_p. The pulse narrows and achieves a higher peak value as the ratio N_i/N_t increases. In the limit $N_i/N_t \gg 1$, the peak value of $n(t)$ approaches $\frac{1}{2}N_i$.

General characteristics of laser Q-switching

- **Pulsed laser output:**
  - Pulse duration – related to the photon lifetime.
  - Pulse energy - related to the upper level lifetime.
- **Laser operation mode:**
  - Single or multi-longitudinal modes.
- **Active verses passive Q-switching methods:**
  - Passive: simple, economic, pulse jitter and intensity fluctuations.
  - Active: stable pulse energy and repetition, expensive.
- **Comparison with chopped laser beams:**
  - Energy concentration in time axis.
- **Function of gain medium**
  - Energy storage

Laser mode-locking

Aims:

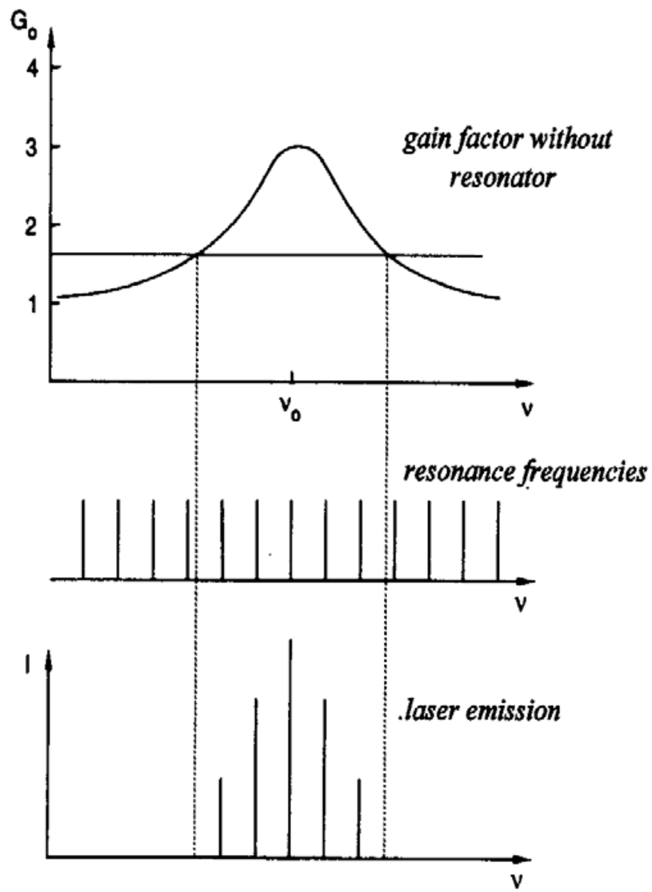
1. *Familiarize with the principle of laser mode-locking.*
2. *Familiarize with different techniques of achieving laser Mode-locking.*

Outlines:

1. *Principle of laser mode-locking.*
2. *Methods of laser mode-locking.*
3. *Active mode-locking.*
4. *Passive mode-locking.*
5. *Transform-limited pulses.*

Principle of laser mode-locking

1. ***Lasing in inhomogeneously broadened lasers:***



i) Laser gain and spectral hole-burning.

ii) Cavity longitudinal mode frequencies.

iii) Multi-longitudinal mode operation.

2. ***Laser multimode operation***:

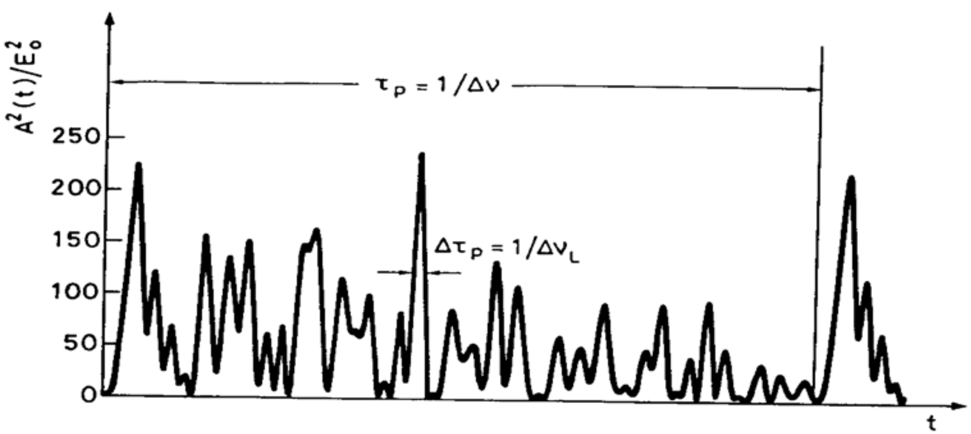
Single mode lasers: $E(t) = E_0 \cos[\omega_0 t + \varphi(t)]$

Multimode lasers: $E(t) = \sum_{i=1}^{M} E_i \cos[\omega_i t + \phi_i(t)]$

Mode-frequency separations: $\sim \dfrac{\pi c}{nd}$

Phase relation between modes: ***Random and independent!***

Total laser intensity fluctuates with time !



The mean intensity of a multimode laser remains constant, however, its instant intensity varies with time.

3. Effect of mode-locking:

(i) Supposing that the phases of all modes are locked together:

$$\varphi_i(t) = \varphi_0 = 0$$

(ii) Supposing that all modes have the same amplitude:

$$E_i = E_0$$

purely for the convenience of the mathematical analysis

(iii) Under the above two conditions, the total electric field of the multimode laser is:

$$E(t) = \mathrm{Re}\left[\sum_{i=1}^{N} E_i e^{j\omega_i t}\right] \qquad \textit{where}$$

$$\omega_i = \omega_0 + \left[i - \frac{M+1}{2}\right]\Delta\omega_c \quad \Delta\omega_c = \frac{\pi c}{d}$$

ω_0 is the frequency of the central mode, M is the number of modes in the laser, $\Delta\omega_c$ is the mode frequency separation. ω_i is the frequency of the i-th mode.

Calculating the summation yields:

$$E(t) = E_0 \frac{\sin\left(M \frac{\Delta\omega_c t}{2}\right)}{\sin\left(\frac{\Delta\omega_c t}{2}\right)} \cos \omega_0 t$$

Note this is the optical field of the total laser Emission !

The optical filed can be thought to consist of a carrier wave of frequency ω_0 that amplitude modulated by the function

$$A_M(x) = \frac{\sin(Mx)}{\sin(x)}$$

The field of one modes: $$U(z,t)=\sum_q A_q \exp[j2\pi v_q(t-z/c)]$$

where $$v_q = v_0 + q v_F, \qquad v_F = c/2d$$

The sum of all the modes: $$U(z,t)=A(t)=\sum_q A_q \exp[\frac{jq2\pi t}{T_F}]$$

where $$T_F = \frac{1}{v_F} = \frac{2d}{c}$$

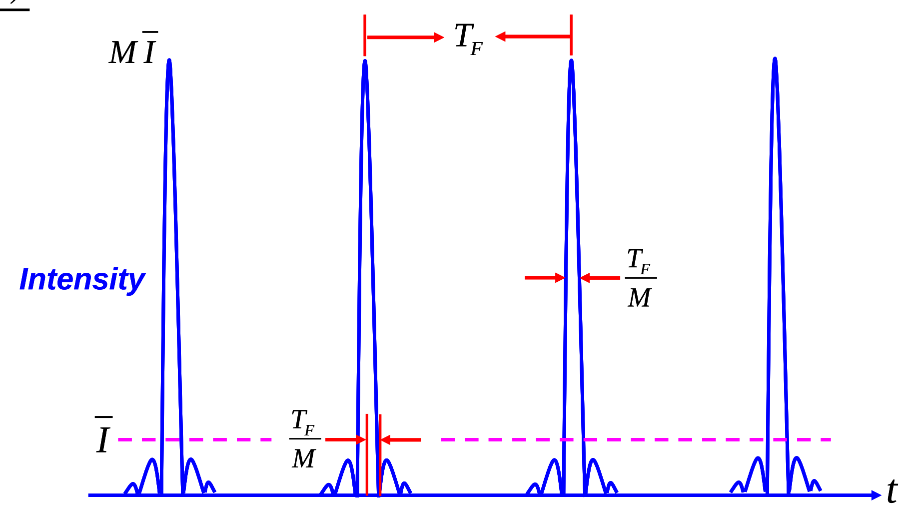
If all the mode have same phase

Then we have $$A(t)=A\frac{\sin\left(M\pi t/T_F\right)}{\sin\left(\pi t/T_F\right)}$$

Where M is mode number

intensity

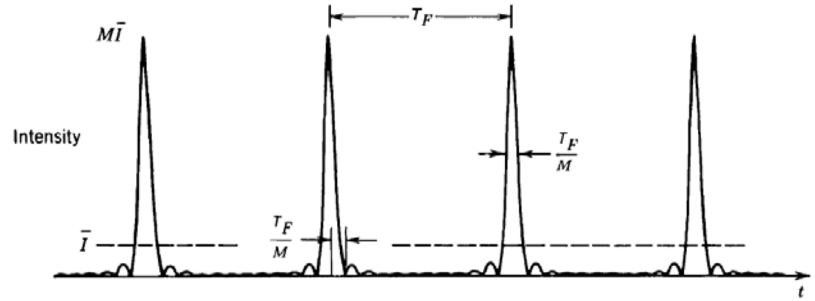
$$I(t,z)=\left|A\right|^2\frac{\sin^2[M\pi(t-z/c)/T_F]}{\sin^2[\pi(t-z/c)/T_F]}$$



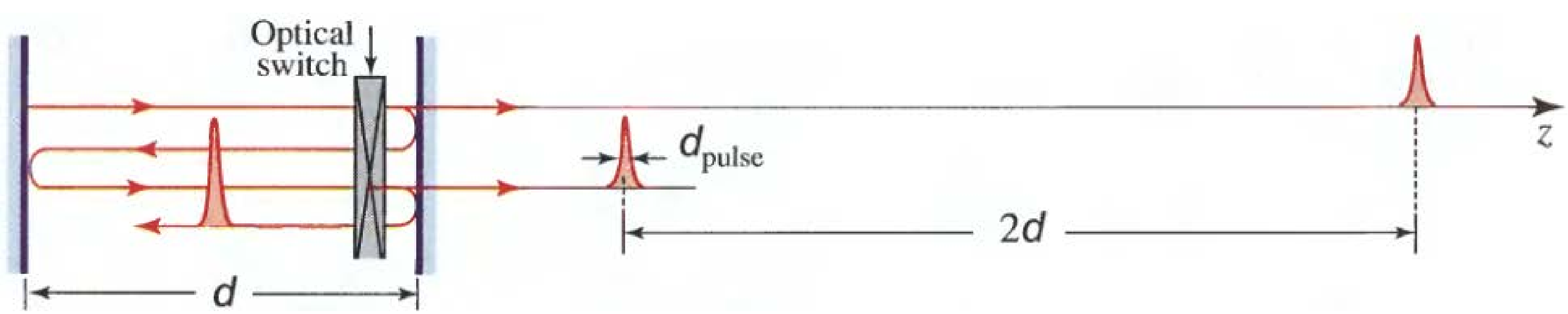
4. *Characteristics of the mode-locked lasers:*

The intensity of the laser field is:

$$I(t,z)=|A|^2\frac{\sin^2[M\pi(t-z/c)/T_F]}{\sin^2[\pi(t-z/c)/T_F]}$$



The output of a mode-locked laser consists of a series of pulses. The time separation between two pulses is determined by τ_{RT} and the pulse width of each pulse is Δt_p.



5. ***Properties of mode-locked pulses:***

i) The pulse separation τ_{RT}:

$$\mathrm{Sin}^2\left(\frac{\Delta\omega_c t}{2}\right) = 0 \Rightarrow \Delta\omega_c t = 2\pi \qquad \Delta\omega_c = 2\pi \nu_F$$

$$\tau_{RT} = \frac{2\pi}{\Delta\omega_c} = \frac{2d}{c} = T_F \Rightarrow$$ ***The round-trip time of the cavity!***

ii) The peak power:

Average power

$$\bar{I} = M\ |A|^2$$

$$\Rightarrow \quad I_{pulse} = M\bar{I} = M^2|A|^2$$

M times of the average power. M: number of modes.

The more the modes the higher the peak power of the Mode-locked pulses.

iii)The individual pulse width:

$$\sin\left(M\frac{\Delta\omega_c t}{2}\right)=0 \quad\Longrightarrow\quad \Delta t_p=\frac{2\pi}{M\Delta\omega_c}$$

$$M\approx\frac{\Delta\omega_a}{\Delta\omega_c} \quad\Longrightarrow\quad \Delta t_p\approx\frac{2\pi}{\Delta\omega_a}=\frac{1}{\Delta\nu_a}$$

$\Delta\nu_a$: bandwidth of the gain profile.

Narrower as M increases.

$$\Delta t_p\approx\frac{\tau_{RT}}{M}=\frac{T_F}{M}$$

The mode locked pulse width is reversely proportional to the gain band width, so the broader the gain profile, the shorter are the mode locked pulses.

summy

Temporal period	$T_F = \dfrac{2d}{c}$
Pulse width	$\tau_{\text{pulse}} = \dfrac{T_F}{M} = \dfrac{1}{M\nu_F}$
Spatial period	$2d$
Pulse length	$d_{\text{pulse}} = c\tau_{\text{pulse}} = \dfrac{2d}{M}$
Mean intensity	$\bar{I} = M\lvert A\rvert^2$
Peak intensity	$I_p = M^2\lvert A\rvert^2 = M\bar{I}$

Techniques of laser mode-locking

Active mode-locking:

Actively modulating the gain or loss of a laser cavity in a periodic way, usually at the cavity repetition frequency c/2nL to achieve mode-locking.

Amplitude modulation:

A modulator with a transmission function of

$$T = \left[1 - \delta\left(1 + \cos\left(\frac{2\pi t}{\tau_{RT}}\right)\right)\right]$$

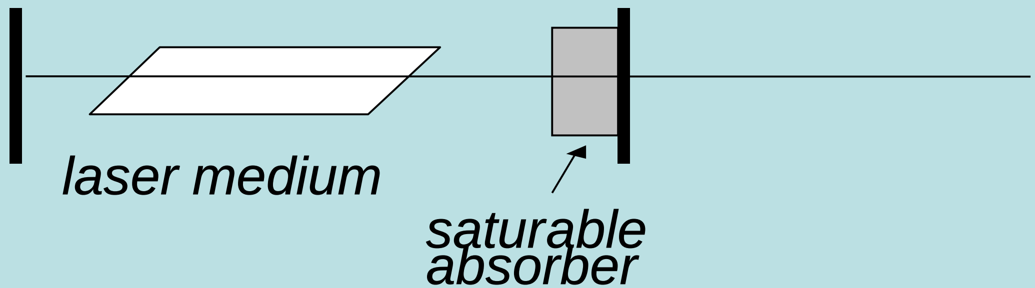
is inserted in the laser cavity to modulate the light. Where δ is the modulation strength and $\delta < 0.5$. Under the influence of the modulation phases of the lasing modes become synchronized and as a consequence become mode-locked.

Operation mechanism of the technique:

Passive mode-locking:

Inserting an appropriately selected saturable absorber inside the laser cavity. Through the mutual interaction between light, saturable absorber and gain medium to automatically achieve mode locking.

A typical passive mode locking laser configuration:



Mechanism of the mode-locking:

i) Interaction between saturable absorber and laser gain:
Survival takes all!

ii)Balance between the pulse shortening and pulse broadening:
Final pulse width.

Home work

1,3,5,7,8,10,12