CS 2050 Exam 1 Practice Exam

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Preface: Here is a sample exam to help you prepare for the upcoming Exam 1. This is meant to take the same amount of time as a full exam (70 minutes). Also, note that this exam may not be comprehensive of the full topic list. We recommend you take this practice exam in the same way you would approach the actual exam, avoiding any notes or external resources and managing your time wisely. Good luck!

Here is a small cheatsheet of exactly and only the laws you can use.

- $p \land T \equiv p$ **Identity**
- $p \lor F \equiv p$

- $p \lor T \equiv T$ **Domination**
- $p \land F \equiv F$

- $p \land p \equiv p$ **Idempotent**
- $p \lor p \equiv p$

- $\neg\neg p \equiv p$ **Double Negation**

- $p \land q \equiv q \land p$ **Communitivity**
- $p \lor q \equiv q \lor p$

- $(p \land q) \land r \equiv p \land (q \land r)$ **Associativity**
- $(p \lor q) \lor r \equiv p \lor (q \lor r)$

- $p \land (q \lor r) \equiv (p \land q) \lor (p \land r)$ **Distributive Laws**
- $p \lor (q \land r) \equiv (p \lor q) \land (p \lor r)$

- $\neg(p \land q) \equiv (\neg p \lor \neg q)$ **DeMorgan's Laws**
- $\neg(p \lor q) \equiv (\neg p \land \neg q)$

- $p \lor (p \land q) \equiv p$ **Absorption**
- $p \land (p \lor q) \equiv p$

- $p \lor \neg p \equiv T$ **Negation**
- $p \land \neg p \equiv F$

Implication

- $p \implies q \equiv \neg q \implies \neg p$ contrapositive
- $p \implies q \equiv \neg p \lor q$ conditional disjunction equivalence

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- $p \iff q \equiv (p \implies q) \land (q \implies p)$ **Biconditional**

Rule of Inference	***Name***
$\begin{array}{rl} & p \\ & p \to q \\ \hline \therefore & q \end{array}$	Modus ponens
$\begin{array}{rl} & \neg q \\ & p \to q \\ \hline \therefore & \neg p \end{array}$	Modus tollens
$\begin{array}{rl} & p \to q \\ & q \to r \\ \hline \therefore & p \to r \end{array}$	Hypothetical syllogism
$\begin{array}{rl} & p \vee q \\ & \neg p \\ \hline \therefore & q \end{array}$	Disjunctive syllogism
$\begin{array}{rl} & p \\ \hline \therefore & p \vee q \end{array}$	Addition
$\begin{array}{rl} & p \wedge q \\ \hline \therefore & p \end{array}$	Simplification
$\begin{array}{rl} & p \\ & q \\ \hline \therefore & p \wedge q \end{array}$	Conjunction
$\begin{array}{rl} & p \vee q \\ & \neg p \vee r \\ \hline \therefore & q \vee r \end{array}$	Resolution

Rule of Inference	***Name***
$\begin{array}{rl} & \forall x P(x) \\ \hline \therefore & P(c) \end{array}$	Universal instantiation
$\begin{array}{rl} & P(c) \text{ for an arbitrary } c \\ \hline \therefore & \forall x P(x) \end{array}$	Universal generalization
$\begin{array}{rl} & \exists x P(x) \\ \hline \therefore & P(c) \text{ for some element } c \end{array}$	Existential instantiation
$\begin{array}{rl} & P(c) \text{ for some element } c \\ \hline \therefore & \exists x P(x) \end{array}$	Existential generalization

1. Determine if this statement is true or false and explain your reasoning.

$$\text{If } \forall x\, P(x) \text{ is false, then } \exists x\, P(x) \text{ must be true.}$$

2. Determine if this statement is true or false and explain your reasoning.

$$\text{If } \forall x\, \exists y\, P(x,y) \text{ is true, then } \exists y\, \forall x\, P(x,y) \text{ must be true.}$$

3. Select the truth value of the statement

$$(\mathrm{P} \;\vee\; \mathrm{Q}) \;\wedge \neg(\mathrm{P} \;\vee\; \mathrm{Q})$$

   (a) Tautology
   (b) Contradiction
   (c) Both of the above
   (d) None of the above (Contingency)

4. Select the truth value of the statement

$$((\mathrm{P} \;\wedge\; \mathrm{Q}) \;\rightarrow\; \mathrm{P}) \;\wedge\; ((\mathrm{P} \;\wedge\; \mathrm{Q}) \;\rightarrow\; \mathrm{Q})$$

   (a) Tautology
   (b) Contradiction
   (c) Both of the above
   (d) None of the above (Contingency)

5. Select the truth value of the statement

$$(\mathrm{P} \;\vee\; \mathrm{Q}) \;\rightarrow\; (\mathrm{P} \;\rightarrow\; \mathrm{Q})$$

   (a) Tautology
   (b) Contradiction
   (c) Both of the above
   (d) None of the above (Contingency)

Use the following quantifiers for following questions:

(a) $A(x)$: x is an airbender

(b) $M(x)$: x is a Master of their element.

(c) $F(x)$: x is a firebender

(d) $B(x)$: x is a bender.

6. Translate this statement: All Master Airbenders also bend fire.

7. Translate this statement: All airbenders are master firebenders, but not all firebenders are airbender.

8. Translate this statement: Some benders master their element.

9. Translate this statement: All fire benders are masters of their element and no other elements.

10. Translate the following statement to words:

$$\exists x(F(x) \land M(x)) \lor \forall y(A(y) \to M(x))$$

11. Translate the following statement into words:

$$\forall x(A(x) \land M(x) \to B(x))$$

12. Which of the following is/are true (multiselect)

    (a) $(p \wedge q) \to p \equiv \neg p \vee \neg q \vee p$
    (b) $p \vee \neg q \equiv \neg(\neg p \wedge \neg q)$
    (c) $p \to (\neg q \vee p) \equiv p \to \neg q$
    (d) None of the above

13. Match each term with the correct statement by writing the corresponding letter above each blank. Some letters will not be used. **Original Statement:** If you're an airbender, then you're not a water bender

    Converse: ________

    Inverse: ________

    Contrapositive: ________

    (a) If you're not a water bender, then you're an airbender
    (b) If you're not an airbender, then you're not a water bender
    (c) If you're not an airbender, then you're a water bender
    (d) If you're a water bender, then you're not an airbender
    (e) If you're the avatar, then you're an airbender

14. Given the following premises, what can be concluded about Aang and which rule(s) of inference is/are used to conclude this?

    Aang is the best air bender or Kora is the best water bender

    Kora is not the best water bender and Katara is the best water bender

    Conclusion: ________________

    Rule(s): ________________

15. Find a counterexample to the following or write that no counterexample exists. **Assume x is a real number**. Write your answer in the blank.

    $$\neg \exists x((x > 0) \wedge (x^2 \leq x))$$

    Answer: ________________

16. **Part 1:** In the world of the benders, airbenders always tell the truth and firebenders always lie. You try to find an airbending ally and run into Kavya, Anthony, and Aidan.

- **Kavya says** "Aidan is different from me and Anthony is a firebender"
- **Anthony says** "You can trust Aidan"
- **Aidan says** "Kavya is a firebender, don't trust her!"

Based on their statements, use a truth table to determine the roles of all three people. Who can you trust?

Part 2: After calculating for a few minutes, you finally select Kavya as your ally.

- **Anthony says** "Pick me too, Kavya and I are the same!"

Was you choice of Kavya correct? Explain in 1-2 sentences why or why not?

17. Using rules of inference and/or logical equivalences, show that the hypotheses below conclude with $\neg b$. Give the reason for each step as you show that $\neg b$ is concluded. Each reason should be the name of a rule of inference and include which numbered steps are involved. For example, a reason for a step might be "modus ponens using steps 2 and 3". Remember, it is possible that you will use all premises, but it is also possible that some are not needed. If $\neg b$ cannot be concluded, provide a brief explanation as to why this is the case.

$a \wedge e$

$(\neg f \wedge a) \rightarrow \neg b$

$a \rightarrow (b \vee h)$

$f \rightarrow \neg h$

$\neg b \vee h$

$e \vee (\neg a \wedge h)$

18. Demonstrate the following logical equivalence. Please only use the logical equivalence rules contained at the beginning of this exam.

$$(p \wedge q) \leftrightarrow (q \vee p) \equiv (q \vee p) \rightarrow (p \wedge q)$$

19. Katara picks some even integer and adds 2. Then, she multiplies the result by the original integer she chose. Prove that her result will be divisible by 4. Recall that an integer a is divisible by an integer k if it can be expressed as $a = kb$ for some integer b.

20. Use a direct proof to show that every even perfect square is a multiple of 4. Note the following:

    (a) A number is a perfect square if and only if it can be represented as k^2 for some integer k.

    (b) If an integer a can be expressed as bc for some integers b, c, we say that b divides a (or that a is a multiple of b).

    (c) k^2 is even if and only if k is even