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# Momentum

- The momentum of an object is the product of its mass and velocity
- Unit of momentum is $kgms^{-1}$ or Ns
- The Principle of Conservation of Momentum states that for a system of interacting objects the total momentum will remain constant providing no external resultant force acts on the system
- $m_A u_A + m_B u_B = m_A v_A + m_B v_B$

e.g. A bullet of mass 0.01kg is fired from a gun of mass 8kg. The bullet travels at $500ms^{-1}$. Calculate the velocity the gun recoils.

$$m_A u_A + m_B u_B = m_A v_A + m_B v_B$$
$$(0.01)(0) + (8)(0) = (0.01)(500) + (8)(v_B)$$
$$0 = 5 + 8v_B$$
$$8v_B = -5$$
$$v_B = -\frac{5}{8} = \underline{-0.625\ ms^{-1}}$$

## Newton's Laws of Motion

1. An object remains at constant velocity unless acted on by a resultant force
2. The rate of change of momentum of an object is proportional to the resultant force on it
3. When two objects interact they exert equal and opposite forces on each other

- The first law explains how a change in momentum is needed for a change in velocity
- The second law explains the equation: force = mass x acceleration
- The third law explains the normal force when an object is in contact with the ground

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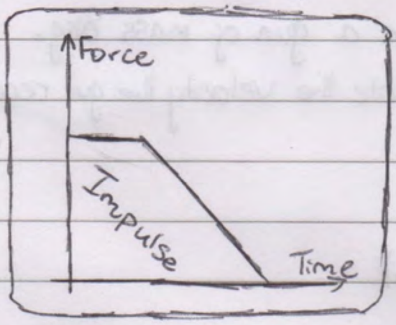
## Elastic and Inelastic Collisions

- An elastic collision is one where kinetic energy is conserved
- A totally inelastic collision is one where the two colliding objects stick together
- Most collisions are somewhere in between
- Momentum is always conserved in all collisions
- $E_K = \frac{1}{2} m v^2$

## Impulse

- Impulse is defined as the product of force and time




- Impulse is also change in momentum
- The unit is Ns. or $Kgms^{-1}$
- Impulse is the area under a force-time graph
- Therefore to reduce the force of an impact the time should be increased

# Circular Motion

## Angular Speed

- The angle an object rotates through per second
- Unit is rad $s^{-1}$, radians per second
- $\omega = \frac{\theta}{t}$
- Even if angular speed, $\omega$, is constant linear speed, $v$, might not be
- $v = \frac{2\pi r}{T} = 2\pi f$
- $v = r\omega$
- $\omega = 2\pi f$ and $\omega = \frac{2\pi}{t}$

## Centripetal Acceleration

- Velocity of object is always changing as direction changes
- Therefore object is always accelerating
- The acceleration is called centripetal acceleration
- Always directed toward centre of circle
- $a = \frac{v^2}{r}$
- $a = \omega^2 r$

## Centripetal Force

- Centripetal acceleration causes centripetal force due to Newton's Second law of motion
- $F = \frac{mv^2}{r} = m\omega^2 r$
- Without centripetal force object would fly off at a tangent
- Acts towards centre of circle
- Examples of centripetal forces are:
    - for a ball on a string the centripetal force will be the tension in the string
    - for a planet orbitting a star the centripetal force will be gravity

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# Simple Harmonic Motion

- Simple harmonic motion (SHM) is an oscillation in which the acceleration of an object is directly proportional to its displacement and is directed to the midpoint
- There is a restoring force pushing or pulling the object back to the centre, and its magnitude is dependent on displacement.

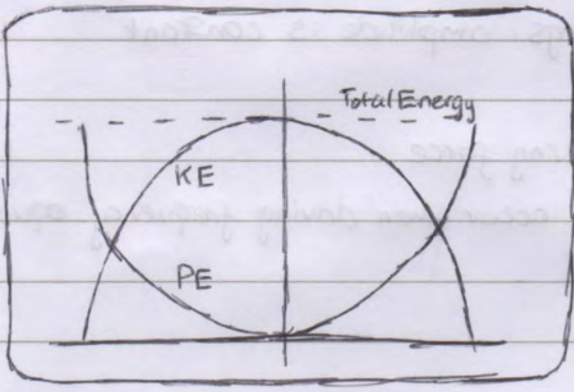
**Formulae**

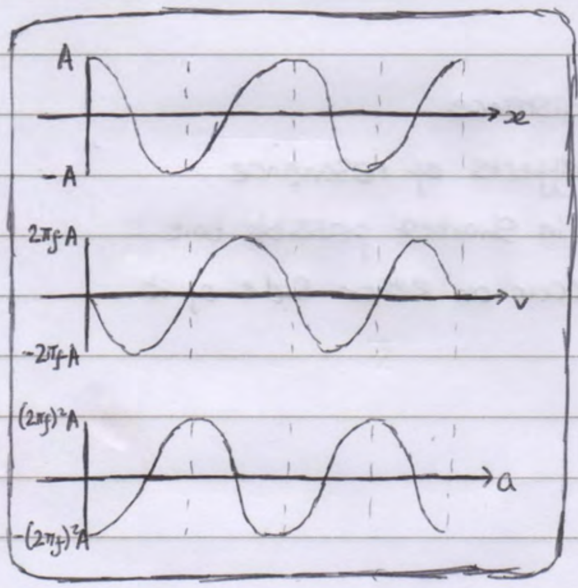
$$a = -(2\pi f)^2 x$$

$$v = \pm 2\pi f\sqrt{A^2 - x^2}$$

$$x = A\cos(2\pi f t)$$

$a$ = acceleration
$f$ = frequency
$x$ = displacement
$v$ = velocity
$A$ = amplitude
$t$ = time




- Gravitational potential energy for pendulums, elastic potential energy for masses on springs
- As the object moves towards midpoint, restoring force does work to transfer $E_p$ to $E_K$
- When moving away, $E_K$ transferred to $E_p$
- At midpoint all energy is kinetic
- At amplitude all energy is potential
- Total energy is mechanical energy and is constant if there is no damping
- Frequency does not depend on amplitude

## Mass on a Spring

- $F = -Kx$
- K is spring constant (stiffness) and unit is $Nm^{-1}$
- $T = 2\pi\sqrt{\frac{m}{K}}$

## Simple Pendulum

- Massless string with dense bob
- Isochronous - constant time period
- $T = 2\pi\sqrt{\frac{L}{g}}$

## Free Vibrations

- Oscillates at natural frequency
- If no energy transfer with surroundings, amplitude is constant
- Doesn't happen in practice
- Forced vibrations have external driving force
- Resonance (rapidly increasing amplitude) occur when driving frequency equals natural frequency

## Damping

- Energy lost to surroundings
- Due to damping forces like air resistance
- Slows oscillations or minimises effects of resonance
- Critical damping reduces amplitude in shortest possible time
- Light damping and overdamping occur on either side of it

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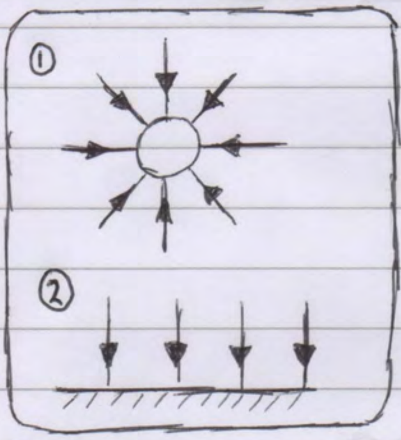
# Gravitational Fields

- Mass in gravitational field experiences attractive force
- Field strength is force per unit mass
- $g = \frac{F}{m}$ (unit is $Nkg^{-1}$ or $ms^{-2}$)
- $g$ is also acceleration due to gravity
- $g = 9.81\,ms^{-2}$ near Earth surface
- Always towards centre of Earth (or centre of mass of object whose field it is)
- In a radial field $g = \frac{-GM}{r^2}$
- $g$ connected to $r$ by inverse square law

## Field Lines




- Describes gravitational lines of force
- Arrows show direction of force
- Closer the lines the stronger the force
- Earth's gravitational field is radial like in image ①
- However field will appear uniform near the surface like image ②

## Newton's Law of Gravitation

- $F = \frac{Gm_1 m_2}{r^2}$
  - $G$ is gravitational constant, $6.67 \times 10^{-11}\,Nm^2 kg^{-2}$
  - $m_1$ and $m_2$ are masses of objects
  - $r$ is distance between centres of masses
- This law assumes the gravitational force between two objects is:
  - always an attractive force
  - proportional to mass of each object
  - proportional to $\frac{1}{r^2}$

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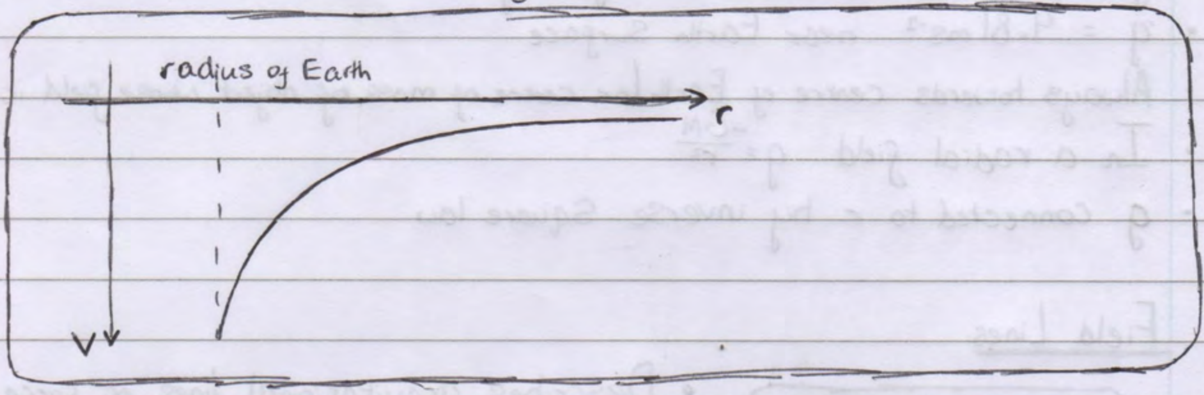
## Gravitational Potential

- The work done per unit mass to move a small object from infinity to that point
- $\Delta W = m \Delta V$ , W is work done, m is mass, V is potential
- $g = -\frac{\Delta V}{\Delta r}$
- $V = -\frac{GM}{r}$ (for a radial field)




## Satellites

- $v^2 = \frac{GM}{r}$ or $v = \sqrt{\frac{GM}{r}}$
- T, orbital period, is $\sqrt{\frac{4\pi^2 r^3}{GM}}$
- Energy of orbitting satellite is constant
- Kept in orbit by gravitational force
- Geosynchronous satellites orbit every 24 hours:
  - Also known as geostationary
  - Directly above equator
  - Always above same point on Earth
  - Same angular speed as Earth
  - Useful for TV and telephone signals

# Electric Fields

- Charge can be positive or negative
- Measured in Coulombs (C)
- Every charged object has an electric field around it
- Like charges repel, unlike charges attract
- An electrical conductor has free electrons not attached to any atoms and they can move about
- An electrical insulator does not have free electrons, they are all attatched to atoms

## Gold Leaf Electroscope

- Used to detect charge
- If a charged object is brought near the metal cap some charge is transferred to the electroscope
- This causes gold leaf to rise as it repels the stem
- The greater the charge the more the leaf rises

## Coulomb's Law

- Coulomb's law states the force between two point charges is proportional to the product of the charges and inversely proportional to the square of the distance between them
- Negative force is attractive
- $F = \frac{1}{4\pi\varepsilon_0} \frac{Q_1 Q_2}{r^2}$
- $\varepsilon_0 = 8.85 \times 10^{-12}\ Fm^{-1}$ and is permittivity of free space

## Electric Field Strength

- The force per unit charge on a small positive charge at that point
- $E = F/Q$
- In a radial field, $E = \frac{1}{4\pi\varepsilon_0} \frac{Q}{r^2}$
- In a uniform field, $E = V/d$
- Electric field strength is constant anywhere on a field

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## Electric Potential

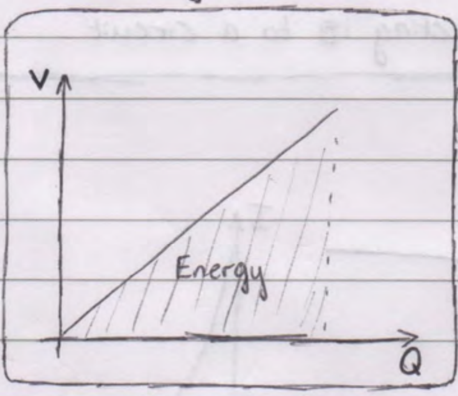
- The work done per unit charge on a small positive charge when it is moved from infinity to that point
- $V = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r}$
- $\Delta W = Q\Delta V$

# Capacitors

- Capacitors store charge
- Capacitance is the amount of charge stored per volt
- $C = \frac{Q}{V}$
- The unit of capacitance (c) in Farad, F

## Energy Stored

- When charge builds up on the plates, energy is stored by the capacitor




- The energy stored is the area under a potential difference against charge graph
- $E = \frac{1}{2} QV$.
- $E = \frac{1}{2} CV^2$
- $E = \frac{1}{2} \frac{Q^2}{C}$

e.g. a 600μF capacitor is charged to a potential difference of 320V. Calculate the energy stored.

$$E = \frac{1}{2} CV^2$$
$$= \frac{1}{2} \times 600 \times 10^{-6} \times 320^2$$
$$= \underline{30.72 \text{ J}}$$

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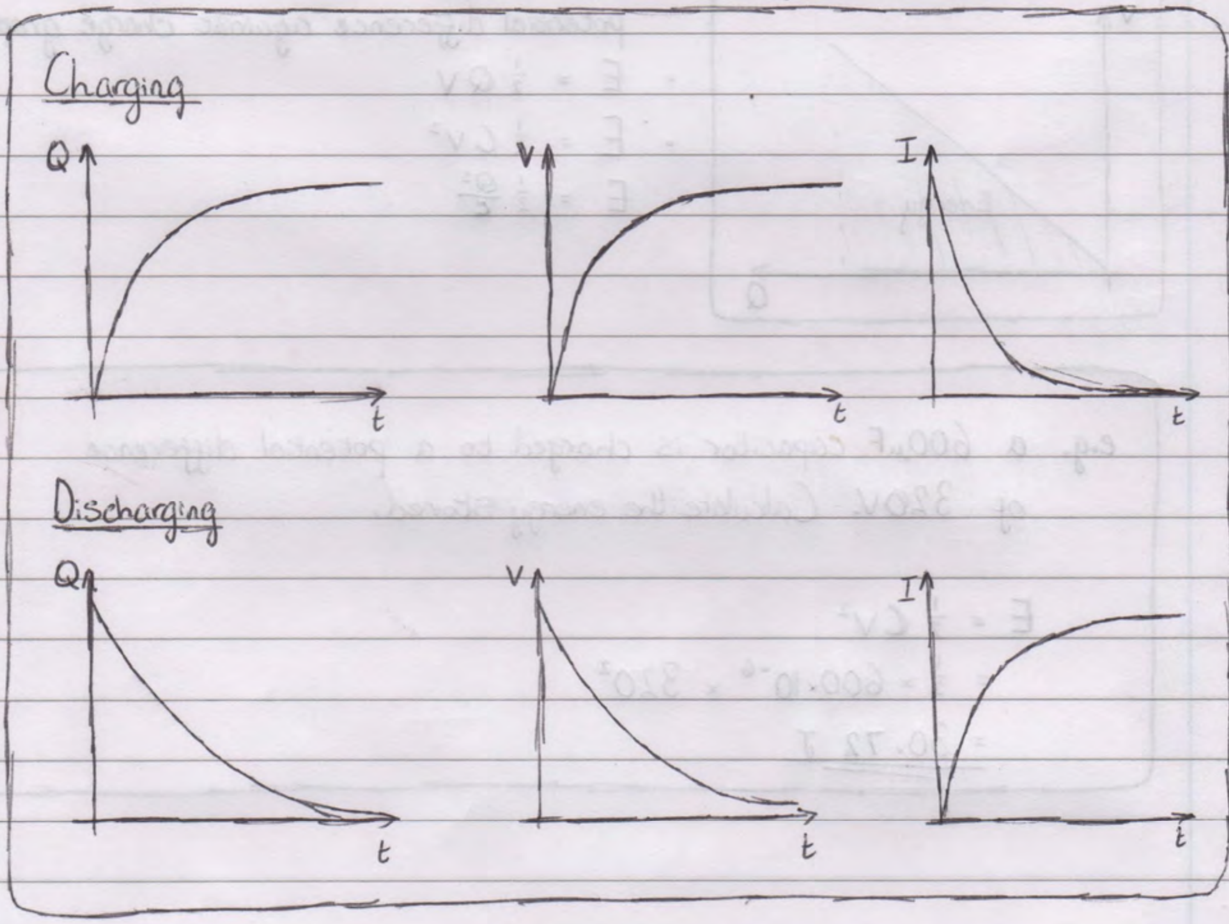
## Charging and Discharging

- When a capacitor is connected to a battery current will flow until the capacitor is charged
- Electrons flow onto the plate connected to the negative terminal
- Electrons on the other plate are repelled to the positive terminal
- This builds up a potential difference
- Charge cannot flow between the plates as there is an insulator
- When potential difference across the capacitor and battery is equal, current stops flowing
- A capacitor can be discharged by connecting it to a circuit

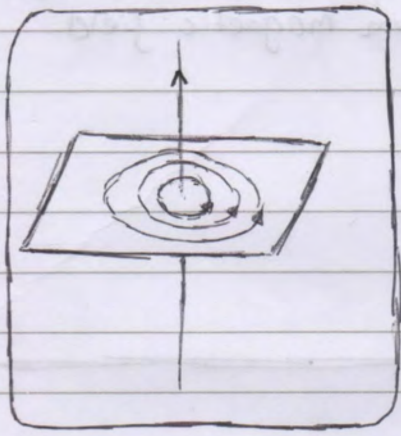



- $Q = Q_0 e^{-\frac{t}{RC}}$ (discharging) or $Q = Q_0 - Q_0 e^{-\frac{t}{RC}}$ (charging)
- $V = V_0 e^{-\frac{t}{RC}}$ (discharging) or $V = V_0 - V_0 e^{-\frac{t}{RC}}$ (charging)
- The time constant ($T = RC$) is the time for charge to fall to 37% of $Q_0$ on a discharging capacitor and rise to 63% on a charging capacitor
- In practice it takes 5RC to fully charge or discharge

# Magnetic Fields

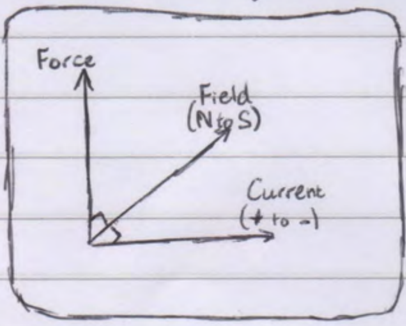
- A magnetic field is a region where a force is exerted on magnetic materials
- Field can be represented by field lines
- Field lines go from north to south

## Magnetic Field of a Wire carrying a Current



- When a current flows in a wire, a magnetic field is induced around it
- Direction of field is shown by right-hand rule
- The field lines are concentric circles centred on the wire

## Wire in a Magnetic Field




- When a current-carrying wire is placed in an external magnetic field the two fields interact
- This creates a force on the wire
- Current must not be parallel to field lines for force to act
- Direction of force can be shown by Fleming's left hand rule
- $F = BIl$

  F = force, B = magnetic flux density, I = current, l = length of wire in field
- Magnetic flux density, strength of field, is the force on one metre of wire carrying a current of one amp at right angles to the magnetic field
- Flux density is measured in teslas, T or $Wbm^{-2}$ or $Nm^{-1}A^{-1}$

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## Charged Particles in Magnetic Fields

- Forces act on charged particles in magnetic fields
- $F = Bqv$
- Force always perpendicular to direction of travel
- This causes circular motion for charged particle
- Used in cyclotrons to accelerate particles to high energies

e.g. What is the force acting on an electron travelling at $2 \times 10^4\ ms^{-1}$ through a uniform magnetic field of strength 2T?

$$F = B\ q\ v$$
$$= 2 \times 1.6 \times 10^{-19} \times 2 \times 10^4$$
$$= \underline{6.4 \times 10^{-15}\ N}$$

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# Electromagnetic Induction

## Faraday's Law and Magnetic Flux

- Faraday's Law states the induced e.m.f is directly proportional to the rate of change of flux linkage
- $E.M.F = \frac{\text{Flux Change}}{\text{Time}} = N\frac{\Delta\phi}{\Delta t}$
- Magnetic flux density, B, is strength of field per unit area
- Magnetic flux, $\phi = BA$
- Unit is Wb
- Flux linkage, $\Phi = N\phi = BAN$
- When conductor moved through magnetic field, force on electrons causes charge and e.m.f to build

## Lenz's Law

- Lenz's law states the induced e.m.f is always in such a direction as to oppose the change that is caused
- $E.M.F = -N\frac{\Delta\phi}{\Delta t}$
- Opposes motion of conductor

## Transformers

- Use electromagnetic induction to change voltage of alternating current
- Alternating current through primary coil produces magnetic flux
- Magnetic field passed through iron core to secondary coil where it induces alternating voltage of same frequency
- $\frac{V_p}{V_s} = \frac{N_p}{N_s}$
- efficiency $= \frac{V_s I_s}{V_p I_p}$
- National grid tries to transfer at lowest possible current to minimise loss, so uses high voltage

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## Generator

- Convert kinetic energy to electrical energy by rotating coil in magnetic field
- Output voltage and current changes direction every half turn, creating alternating current
- $\Phi = BAN\cos\theta$
- $\Phi = BAN\cos\omega t$
- $\varepsilon = BAN\omega\sin\omega t$
- Flux linkage and induced voltage $\frac{\pi}{2}$ out of phase