Kinematics

Suvat Equations

$v^2 = u^2 + 2as$ — s = displacement

$s = ut + \frac{1}{2}at^2$ — u = initial velocity

$s = \frac{1}{2}(v+u)t$ — v = final velocity

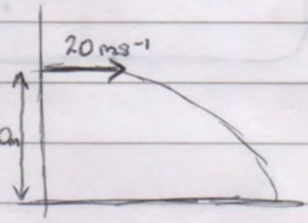
$v = u + at$ — a = acceleration

$s = vt - \frac{1}{2}at^2$ — t = time

can only be used when acceleration is constant

- Horizontal and vertical components should be calculated seperately for a projectile (time will be the same)
- Horizontal is usually constant so use distance = speed × time
- Vertical acceleration, $g = 9.8\ ms^{-2}$

e.g. A ball is thrown horizontally at $20 ms^{-1}$, 30m above ground. How far does it travel?



vertical, $s = 30 \quad u = 0 \quad a = 9.8 \quad t = ?$

$s = ut + \frac{1}{2}at^2$

$30 = 4.9t^2$

$t = \sqrt{\frac{30}{4.9}}$

$= 2.5\ s$

distance $= 20 ms^{-1} \times 2.5 s$

$= \underline{50m}$

- If thrown up, at the highest point there will be no velocity

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- If acceleration changes then calculus must be used
- With respect to time

$$\text{Displacement} \xrightarrow{\text{Differentiate}} \text{Velocity} \xrightarrow{\text{Differentiate}} \text{Acceleration}$$

$$\underline{x} \text{ or } \underline{r} \xleftarrow{\text{Integrate}} \underline{v} \xleftarrow{\text{Integrate}} \underline{a}$$

e.g. The displacement of a particle at t seconds is given by $\underline{r} = t^3 - 2t^2 + 8t - 3$. Find the acceleration when $t = 3$.

$\underline{r} = t^3 - 2t^2 + 8t - 3$

$\underline{v} = 3t^2 - 4t + 8$

$\underline{a} = 6t - 4$

when $t = 3$

$\underline{a} = 6(3) - 4$

$= \underline{\underline{14 \text{ ms}^{-2}}}$

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Centres of Mass

- To find the centre of mass of a set of particles, deal with x and y seperately
- $m_1 x_1 + m_2 x_2 + m_3 x_3 + \dots = m_{total}\, x$
- $m_1 y_1 + m_2 y_2 + m_3 y_3 + \dots = m_{total}\, y$

e.g. Find the coordinates of the centre of mass of the following system of particles: 2kg at (1, 2), 3kg at (3, 1), 5kg at (4, 3)

$m_{total} = 10$

$\bar{x} = \frac{(2\times1) + (3\times3) + (5\times4)}{10}$

$= \frac{31}{10}$

$= 3.1$

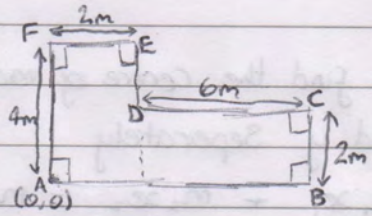
$\bar{y} = \frac{(2\times2) + (3\times1) + (5\times3)}{10}$

$= \frac{22}{10}$

$= 2.2$

- A lamina is a two-dimensional model of an object
- A lamina is uniform if its mass is spread evenly throughout its area
- There are certain lamina shapes where the centre of mass should be remembered:
  - Circular Lamina: at centre of circle
  - Rectangular Lamina: at centre of rectangle, half x and y
  - Triangular Lamina: at point $\left(\frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3}\right)$
  - Sector of Circle: on axis of symetry, $\frac{2r\sin\alpha}{3\alpha}$ from centre where r is radius and α is angle with axis of symmetry
- The centre of mass of a composite shape can be found by finding the centres of mass for individual shapes and then treating those as particles.

e.g. Find the distance of the centre of mass of the lamina shown from AF.



	area	$\bar{x}$	$\bar{y}$
AFED □	8	1	2
DCB. □	12	5	1
Total	20	$\bar{x}$	$\bar{y}$

$$\bar{x} = \frac{(8\times 1) + (12\times 5)}{20}$$

$$= \frac{68}{20}$$

$$= 3.4$$

$\therefore$ distance from AF = 3.4m

- For a framework the centres of mass can be found the same, except for a circular arc where the centre of mass is $\frac{r\sin\alpha}{\alpha}$
- When a lamina is suspended from a fixed point the centre of mass will rest below the point
- When a lamina rests on an incline, the line of action of the centre of mass must be through the side in contact with the plane or it will topple

Work, Energy and Power

- Work Done = Force × Distance in direction of force
- Work Done against Gravity = mgh
- Work Done is measured in joules (J)
- Kinetic Energy, $E_K = \frac{1}{2}mv^2$
- Potential Energy, $E_P = mgh$
- Work Done = Change in Kinetic Energy
- The principle of conservation of mechanical energy states that when no external forces do work on a particle the sum of its kinetic and potential energy must remain constant
- The work-energy principle states the change in total energy is equal to work done
- Power is the rate of doing work, measured in Watts (W)
- Power = $\frac{\text{Energy}}{\text{Time}}$
- Power = Driving Force × Velocity
- Force = Mass × Acceleration

e.g. a van of mass 1250kg travels along a road and its engine works at 24kW. The constant resistance to motion is 600N. Find the maximum speed of the van.

$$\text{Speed} = \frac{\text{Power}}{\text{force}} = \frac{24000}{600} = 40$$

$\therefore$ maximum speed is $40\,ms^{-1}$

e.g. Find the acceleration at $6\,ms^{-1}$

$$\text{Driving Force} = \frac{\text{Power}}{\text{velocity}} = 4000$$

$$\text{Acceleration} = \frac{\text{force}}{\text{mass}} = \frac{4000-600}{1250} = 2.72\,ms^{-2}$$

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e.g. a Skier weighing 55kg begins to go downhill at $6ms^{-1}$. After travelling 1400m they have lost a height of 25m and are only travelling at $4ms^{-1}$. The resistance to motion is constant at 12N. Find the work done by the skier

$$\text{Loss of } E_K = \tfrac{1}{2}mv_1^2 - \tfrac{1}{2}mv_2^2$$
$$= \tfrac{1}{2} \times 55 \times (6^2 - 4^2)$$
$$= 550 \text{ J}$$

$$\text{Loss of } E_P = mgh_1 - mgh_2$$
$$= 55 \times 9.8 \times 25$$
$$= 13475 \text{ J}$$

$$\therefore \text{total energy loss} = 550 + 13475$$
$$= 14025 \text{ J}$$

$$\text{Work Done against Resistance} = \text{Force} \times \text{Distance}$$
$$= 12 \times 1400$$
$$= 16800 \text{ J}$$

$$\text{Work Done By Skier} = 16800 - 14025$$
$$= 2775 \text{ J}$$

Collisions

- Momentum is mass × velocity, measured in $Kgms^{-1}$ or Ns
- Impulse - Momentum Principle
  - Impulse of a force is equal to change in momentum produced
  - $I = mv_1 - mv_2$
  - Impulse is also force × time
- Principle of Conservation of Momentum
  - Total momentum before and after impact are equal
  - $m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$

e.g. A ball weighing 0.1kg is moving with velocity $(16\underline{i} + 40\underline{j})\,ms^{-1}$ when it hits a wall. After it moves with a velocity of $(6\underline{i} - 25\underline{j})\,ms^{-1}$. Find the impulse exerted by the wall on the ball.

$$\begin{aligned}\text{Impulse} &= mv_1 - mv_2\\ &= 0.1\left[(16\underline{i}+40\underline{j}) - (6\underline{i} - 25\underline{j})\right]\\ &= 0.1\left[10\underline{i} + 65\underline{j}\right]\\ &= (\underline{i} + 6.5\underline{j})\ Ns\end{aligned}$$

- The coefficient of restitution, e, of two particles defines how fast they seperate after a collision, $e = \frac{\text{speed of seperation}}{\text{speed of approach}}$
- $0 \leq e \leq 1$
- When $e = 1$ it is an elastic collision
- When $e = 0$ it is an inelastic collision

e.g. A moves at $16ms^{-1}$ at B which is at rest. After the collision A is at rest and B moves at $2ms^{-1}$. Find e.

Speed of approach $= 16 - 0 = 16\ ms^{-1}$

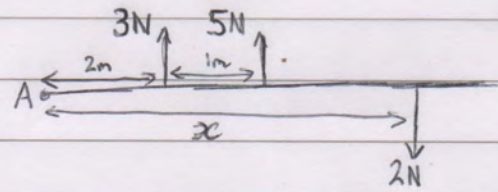
Speed of seperation $= 2 - 0 = 2\ ms^{-1}$

$\therefore e = \frac{2}{16} = 0.125$

Statics of Rigid Bodies

- The moment of a force from point P is the distance from P multiplied by the perpendicular force
- Whether it turns clockwise or anticlockwise must be taken into account
- If a body is in equilibrium:
  - There is no resultant force in any direction
  - The sum of moments about any point is zero

e.g.

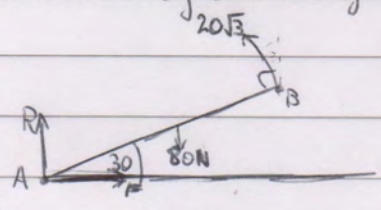


Given the body is in equilibrium, find the value of x.

$$(3)(2) + (5)(3) = (2)(x)$$
$$6 + 15 = 2x$$
$$21 = 2x$$
$$x = 10.5\text{m}$$

- If a body is in limiting equilibrium it is on the point of moving
- Friction, $F \leq \mu R$
- In limiting equilibrium $F = \mu R$

e.g. Given the body is in limiting equilibrium find the value of μ



$$R = 80 - (20\sqrt{3})(\cos 30)$$
$$= 50$$
$$F = (20\sqrt{3})(\sin 30)$$
$$= 10\sqrt{3}$$
$$\mu = \frac{F}{R}$$
$$= \frac{\sqrt{3}}{5}$$