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# Algebraic Fractions

- You must be able to:
  - Simplify algebraic fractions
  - Add and subtract algebraic fractions
  - Multiply algebraic fractions
  - The remainder theorem

e.g. Simplify $\frac{3x+2}{x+1} + \frac{9}{2(x+2)}$

$$\frac{3x+2}{x+1} + \frac{9}{2(x+2)} = \frac{2(3x+2)(x+2) + 9(x+1)}{2(x+1)(x+2)}$$

$$= \frac{6x^2 + 16x + 8 + 9x + 9}{2(x+1)(x+2)}$$

$$= \frac{6x^2 + 25x + 17}{2(x+1)(x+2)}$$

e.g. Solve $\frac{x+1}{3-x} > 2$ , $x \neq 3$ , $x < 3$

$$\frac{x+1}{3-x} > 2$$

$$x+1 > (3-x)2$$

$$x+1 > 6-2x$$

$$3x > 5$$

$$x > 5/3$$

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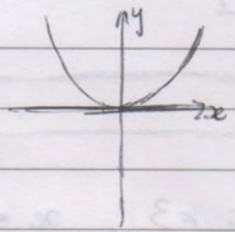
# Functions

- A function is a special mapping such that every element of set A (domain) is mapped to exactly one element of set B (range)
- Domain is all the possible inputs
- Range is all the possible outputs
- A mapping can sometimes be made into a function by restricting the domain
- Functions can be written in two ways:
  - $f(x) =$     e.g. $f(x) = 2x+1$
  - $f: x \rightarrow$     e.g. $f: x \rightarrow 2x+1$

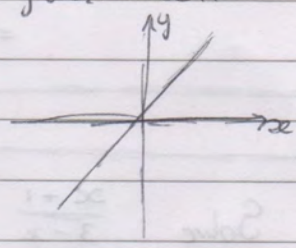
e.g. State the range of the following functions

$f(x) = x^2$




Range: $f(x) \geq 0$

$f(x) = 2x+1$




Range: $f(x) \in \mathbb{R}$

- A composite function is a combination of multiple functions
- For $fg(x)$, $g$ is applied first, then $f$

e.g. Given $f(x) = x^2$ and $g(x) = x-2$ find $fg(4)$

$fg(x) = (x-2)^2$

$fg(4) = 4$

- An inverse function, $f^{-1}(x)$, maps the range back to the domain

e.g. Find the inverse of $f(x) = \frac{3}{x-1}$

$y = \frac{3}{x-1}$

$x = \frac{3+y}{y}$

$\therefore f^{-1}(x) = \frac{3+x}{x}$

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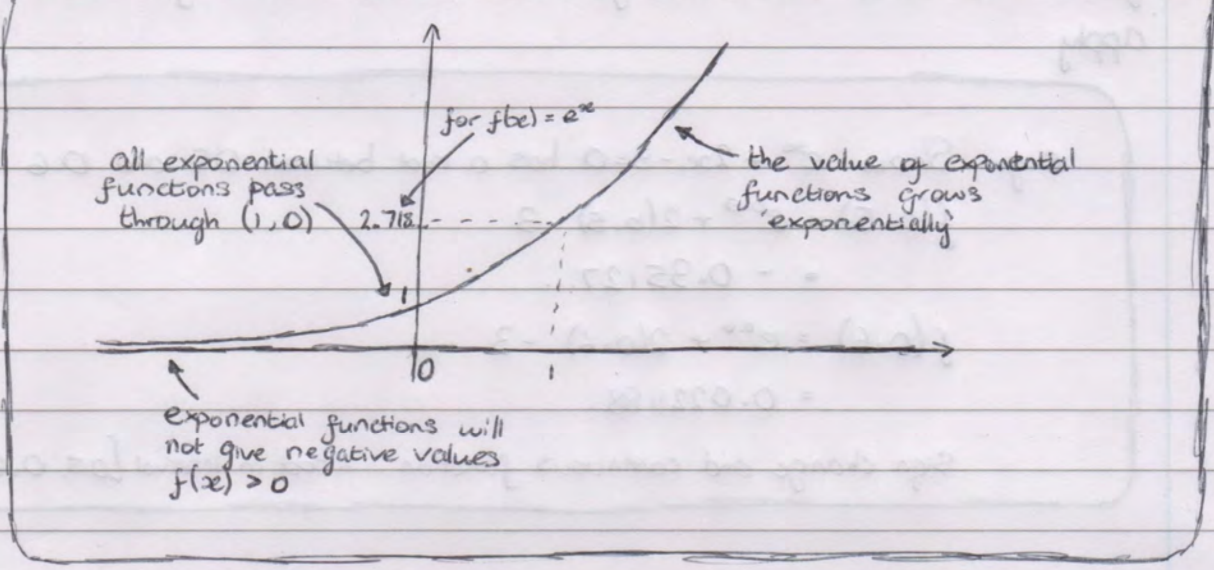
# Exponential Functions

- Exponential functions are of the form $a^x$
- $a$ is a constant
- The main exponential function is $e^x$ $(e = 2.718...)$




- The inverse of $f(x) = a^x$ is $f^{-1}(x) = \log_a x$
- $\log_e x$ is often written as $\ln x$
- $\ln x$ only accepts positive $x$ values
- $e^x$ and $\ln x$ can be used to solve a variety of problems

e.g. The population of a herd of elephants is given by $N = 150 - 80e^{-\frac{t}{40}}$ where $N$ is number of elephants and $t$ is years after 2003. Find the year population reaches 100.

$$100 = 150 - 80e^{-\frac{t}{40}}$$
$$80e^{-\frac{t}{40}} = 50$$
$$e^{-\frac{t}{40}} = \frac{50}{80}$$
$$-\frac{t}{40} = \ln\frac{50}{80}$$
$$t = -40\ln\frac{50}{80}$$
$$= 18.8001...$$

$\therefore$ year will be 2022

# Numerical Methods

## Interval of a Root

- If the value of $f(x)$ changes sign between two values of $x$ then there must be a root of $f(x) = 0$ in that interval
- $f(x)$ must be a continuous function in the interval for this to apply

e.g. Show $e^x + 2x - 3 = 0$ has a root between 0.5 and 0.6

$f(0.5) = e^{0.5} + 2(0.5) - 3$
$= -0.35127...$

$f(0.6) = e^{0.6} + 2(0.6) - 3$
$= 0.0221188...$

Sign change and continuous function $\therefore$ root in interval $[0.5, 0.6]$

## Iteration

- Iterative formulae can sometimes be used to find a root
- If an answer is repeated to a degree of accuracy the root has been found to that degree of accuracy

e.g. Use $x_{n+1} = 4 - \frac{1}{x_n}$ to find a root to 2dp. $x_0 = 3$

$x_0 = 3$
$x_1 = 3.6666666...$
$x_2 = 3.727272...$
$x_3 = 3.73170...$
$x_4 = 3.732026...$
$\therefore$ root $= 3.73$ (2dp)

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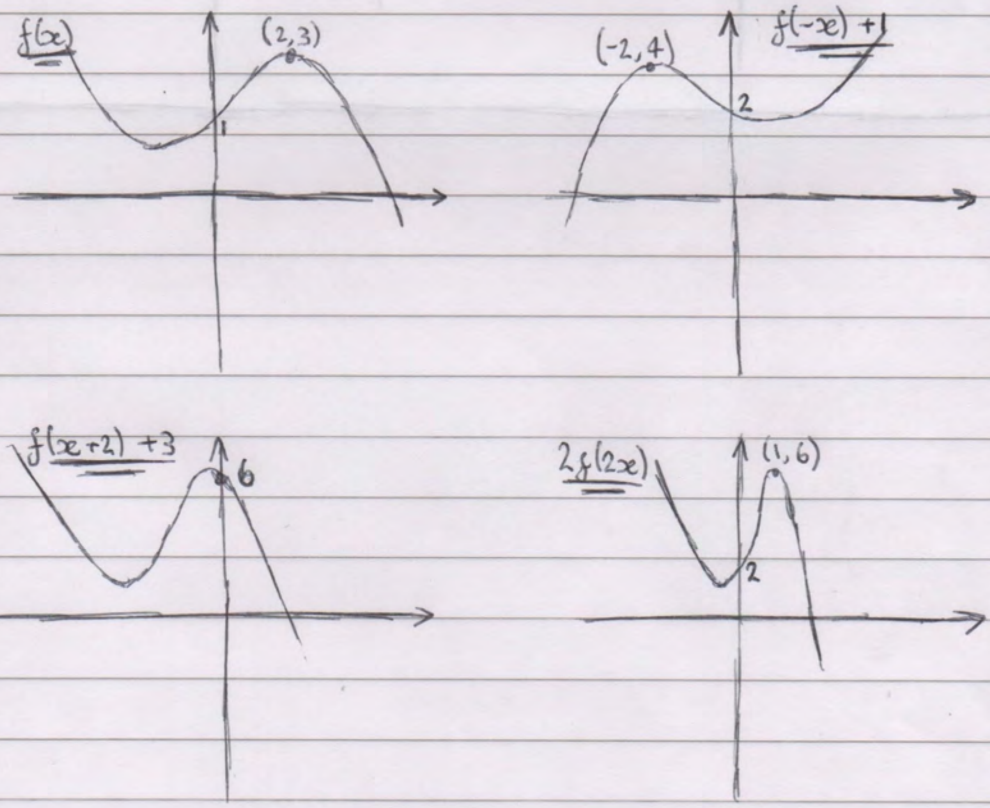
# Transformations

| Function | Transformation |
|---|---|
| $f(x+a)$ | horizontal transformation of $-a$ |
| $f(x)+a$ | vertical transformation of $a$ |
| $f(ax)$ | horizontal stretch of scale factor $\frac{1}{a}$ |
| $af(x)$ | vertical stretch of scale factor $a$ |
| $f(-x)$ | reflection in y-axis |
| $-f(x)$ | reflection in x-axis |
| $f(\lvert x\rvert)$ | reflect $x \geq 0$ in y-axis |
| $\lvert f(x)\rvert$ | reflect in y-axis for all $f(x) < 0$ |

e.g. transformations of $f(x)$



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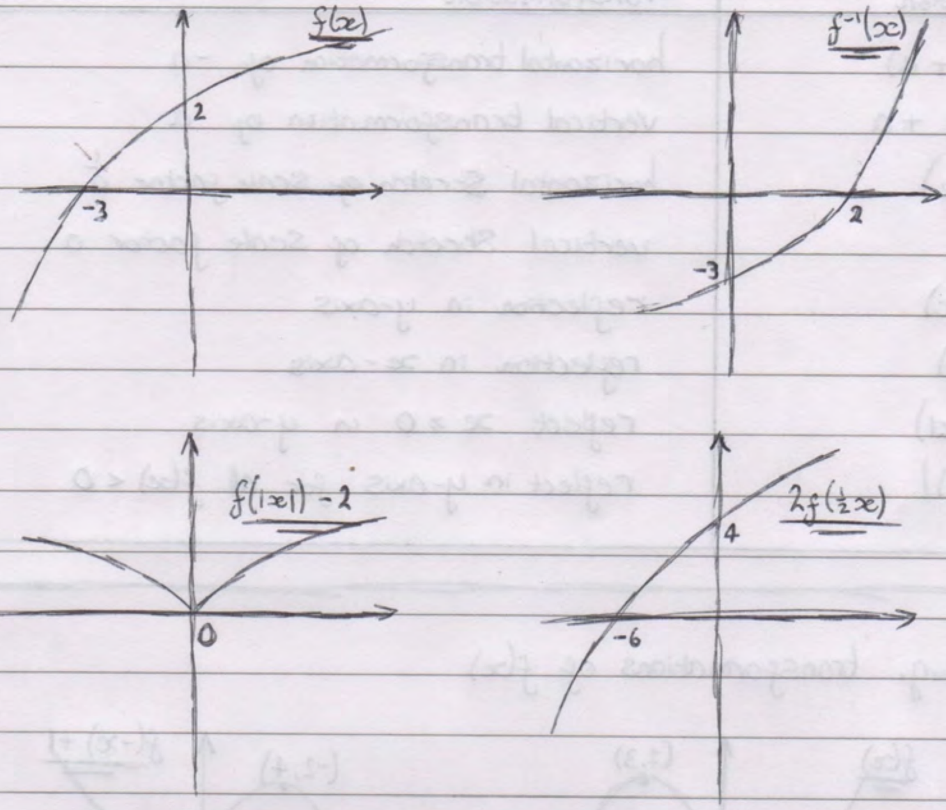
e.g. transformations of $f(x)$



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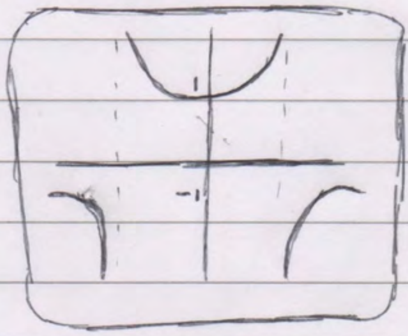
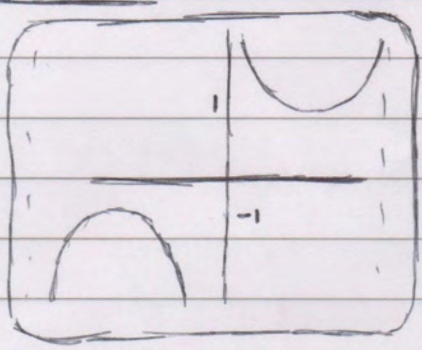
# Trigonometry

## Secant



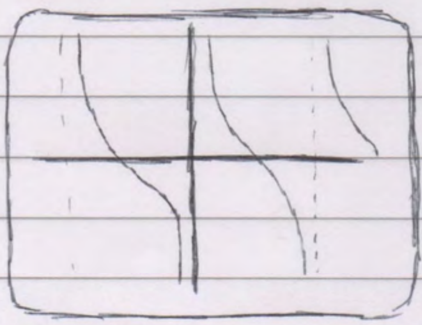

- $\sec\theta = \frac{1}{\cos\theta}$
- undefined for values where $\cos\theta = 0$
- no $f(x)$ values between -1 and 1

## Cosecant




- $\mathrm{cosec}\,\theta = \frac{1}{\sin\theta}$
- undefined for values where $\sin\theta = 0$
- no $f(x)$ values between -1 and 1

## Cotangent



- $\cot\theta = \frac{1}{\tan\theta}$
- undefined for values where $\tan\theta = 0$
- asymptotes every 180° ($\pi$ radians)

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# Trigonometric Identities

- $\tan\theta = \frac{\sin\theta}{\cos\theta}$
- $\sin^2\theta + \cos^2\theta = 1$
- $\sec\theta = \frac{1}{\cos\theta}$
- $\operatorname{cosec}\theta = \frac{1}{\sin\theta}$
- $\cot\theta = \frac{1}{\tan\theta}$
- $1 + \tan^2\theta = \sec^2\theta$
- $1 + \cot^2\theta = \operatorname{cosec}^2\theta$
- $\sin(A \pm B) = \sin A\cos B \pm \sin B\cos A$
- $\cos(A \pm B) = \cos A\cos B \mp \sin A\sin B$
- $\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A\tan B}$
- $\sin 2\theta = 2\sin\theta\cos\theta$
- $\cos 2\theta = \cos^2\theta - \sin^2\theta$
- $\cos 2\theta = 2\cos^2\theta - 1$
- $\cos 2\theta = 1 - 2\sin^2\theta$
- $\tan 2\theta = \frac{2\tan\theta}{1-\tan^2\theta}$
- $\sin A \pm \sin B = 2\sin\frac{A \pm B}{2}\cos\frac{A \mp B}{2}$
- $\cos A + \cos B = 2\cos\frac{A+B}{2}\cos\frac{A-B}{2}$
- $\cos A - \cos B = -2\sin\frac{A+B}{2}\sin\frac{A-B}{2}$

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# Differentiation

## Chain Rule

- If $y = [f(x)]^n$ then $\frac{dy}{dx} = n f'(x)[f(x)]^{n-1}$
- Multiply by power and differential of function in brackets, then lower power by 1

e.g. Differentiate $y = (5x^3 + 2x)^4$

$f(x) = 5x^3 + 2x$

$f'(x) = 15x^2 + 2$

$\therefore \frac{dy}{dx} = 4(15x^2 + 2)(5x^3 + 2x)^3$

## Product Rule

- If $y = uv$ then $\frac{dy}{dx} = uv' + u'v$
- Differentiate the first function and multiply by the second, differentiate the second function and multiply by the first, add two results together

e.g. Differentiate $y = e^{3x}\sin x$

$u = e^{3x}$ $\quad v = \sin x$

$u' = 3e^{3x}$ $\quad v' = \cos x$

$\frac{dy}{dx} = e^{3x}\cos x + 3e^{3x}\sin x$

$= e^{3x}(\cos x + 3\sin x)$

## Quotient Rule

- If $y = \frac{u}{v}$ and $\frac{dy}{dx} = \frac{vu' - uv'}{v^2}$

e.g. Differentiate $y = \frac{x}{2x+5}$

$u = x$ $\quad v = 2x + 5$

$u' = 1$ $\quad v' = 2$

$\frac{dy}{dx} = \frac{(2x+5)(1) - (x)(2)}{(2x+5)^2}$

$= \frac{5}{(2x+5)^2}$

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## Differentiating the Exponential Function

- If $y = e^{f(x)}$ then $\frac{dy}{dx} = f'(x)\, e^{f(x)}$
- Therefore the differential of $e^x$ is $e^x$

e.g. Differentiate $e^{2x+3}$

$f(x) = 2x+3$

$f'(x) = 2$

$\therefore \frac{dy}{dx} = 2e^{2x+3}$

## Differentiating Natural Logs

- If $y = \ln[f(x)]$ then $\frac{dy}{dx} = \frac{f'(x)}{f(x)}$

e.g. Differentiate $y = \ln(6x-1)$

$f(x) = 6x-1$

$f'(x) = 6$

$\therefore \frac{dy}{dx} = \frac{6}{6x-1}$

## Trigonometric Differentials

- $y = \sin x$ $\quad \frac{dy}{dx} = \cos x$
- $y = \cos x$ $\quad \frac{dy}{dx} = -\sin x$
- $y = \tan x$ $\quad \frac{dy}{dx} = \sec^2 x$
- $y = \operatorname{cosec} x$ $\quad \frac{dy}{dx} = -\operatorname{cosec} x \cot x$
- $y = \sec x$ $\quad \frac{dy}{dx} = \sec x \tan x$
- $y = \cot x$ $\quad \frac{dy}{dx} = -\operatorname{cosec}^2 x$