FIN 971: Corporate Finance
Fall 2016, University of Wisconsin
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Problem set #7 - Due 11/30/16

This problem is designed to have you compute the dynamic contracting problem in DeMarzo and Fishman (2007,RFS). The environment and associated programming problem are given in the Handout on Demarzo and Fishman on my website. As in that handout, the programming problem solves choose $\{d_i(a) \geq 0, p_i(a) \in [0,1], a_i'(a) \in [\underline{a}, \overline{a}]\}_{i \in \{H,L\}}$ to

$$\begin{aligned} b(a) &= \max \{\pi \cdot [\widehat{y}_H(a) - d_H(a) + p_H(a) \cdot L] + (1-\pi) \cdot [\widehat{y}_L(a) - d_L(a) + p_L(a) \cdot L]\} \\ &\quad + \beta \{\pi \cdot (1 - p_H(a)) \cdot b(a_H'(a)) + (1-\pi) \cdot (1 - p_L(a)) \cdot b(a_L'(a))\} \end{aligned} \tag{1}$$

s.t.

$$\begin{aligned} a &= \pi \cdot \left[\lambda \left(Y^H - \widehat{y}_H(a)\right) + d_H(a) + p_H(a) \cdot R\right] \\ &\quad + (1-\pi) \cdot \left[\lambda \left(Y^L - \widehat{y}_L(a)\right) + d_L(a) + p_L(a) R\right] \\ &\quad + \delta \{\pi \cdot (1 - p_H(a)) a_H'(a) + (1-\pi) \cdot (1 - p_L(a)) a_L'(a)\} \end{aligned} \tag{2}$$

$$\begin{aligned} & \lambda \cdot \left(Y^i - \widehat{y}_i(a)\right) + d_i(a) + (1 - p_i(a)) \cdot \delta \cdot a_i'(a) + p_i(a) \cdot R \\ \geq \; & \lambda \cdot (Y^i - y_j') + d_j(a) + (1 - p_j(a)) \cdot \delta \cdot a_j'(a) + p_j(a) \cdot R, \forall i, j \in \{H, L\}, i \neq j \end{aligned} \tag{3}$$

$$p_i(a) \cdot R + (1 - p_i(a)) \cdot \delta \cdot a_i'(a) \geq R \tag{4}$$

$$Y^i \geq \widehat{y}_i(a), \forall i \tag{5}$$

$$a \in [\underline{a}, \overline{a}] \tag{6}$$

where

- $\overline{a} = \left[\pi Y^H + (1-\pi) Y^L\right] / (1-\beta)$ (agent gets everything every period, using principal's rate)
- $\underline{a} = \max\{\pi\lambda(Y^H - Y^L) + R, \delta^{-1} R\}$ (agent can always quit).

The algorithm you will use is:

1. Initialize the value function $b^i(a)$, with $i = 0$, to the first best value.

2. Given $b^i(a)$, for each $a \in [\underline{a}, \overline{a}]$, solve (1)-(5) using the fmincon function in matlab to get decision rules which induces $b^{i+1}(a)$. fmincon minimizes an objective subject to a constraint set. On my website is an example program (fmincon_example.m) which minimizes an objective (objf.m) subject to a constraint (const.m).

3. If $\sup |b^{i+1} - b^i|$ is greater than your tolerance level, then return to step 2 otherwise stop.

The parameters for your problem come from DeMarzo and Fishman. Set $\beta = \exp(-0.0953)$, $\delta = \exp(-0.0998)$, $L = 75$, $R = 0$, and $\lambda = 1$. For the cash flow let $Y^L = 0$ with probability $1 - \pi = 1/2$ and $Y^H = 20$ with probability $\pi = 1/2$.

After solving the problem, graph the value function $b(a)$ and decision rules $d_i(a), p_i(a), a'_i(a)$. Interpret.