Homework 4: Ben-Porath Models

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In this homework you will solve a Ben-Porath style model of human capital dynamics.

1 Data Assignment

In this data assignment you will plot moments from the earnings distribution over the life-cycle simmilar to Huggett, Ventura, and Yaron [2011]. Using your PSID data from 1978 to 1997 create the following graphs using the approach of Huggett, Ventura, and Yaron [2011] to recover the age effects on the followign statistics:

1. The standard deviation of log earnings as a function of age.
2. The skewness of log earnings as a function of age.
3. The kurtosis of log earnings as a function of age.

Note you can use either the time-effects approach or the cohort effects approach, be clear on the choice that you make.

2 Model Overview

Consider a simplied form of the model from Huggett, Ventura, and Yaron [2011]. There are overlapping generations of households who live and for T periods (i..e, there is no retirement). Agents are heterogeneous in the human capital h, and assets k (there is a common level of ability). In each period, agents make a consumption savings decision and a decision about how much time to invest in their human cpaital, which follows a Ben-Porath structure.

A recursive representation of the economy is given by,

$$V_t(h,k) = \max_{(k,s)} \; u(c) + \beta \mathbb{E}\left[V_{t+1}(h^{'}, k^{'})\right]$$

$$V_{T+1} = 0$$

subject to the budget constraint,

$$c + k^{'} \leq R_t hl + k(1 + r)$$

the law of motion for human capital,

$$h^{'} = exp(z^{'})H(h, s)$$

the time constraint,

$$l + s = 1$$

and finally the borrowing constraint,

$$k^{'} \geq \underline{K}$$

2.1 Assignment

To complete the assignment, please do the following:

1. Solve the model above and simulate a panel of indiviudals from the model using the suggested parameters below and report the following.

   (a) Plot the average path of earnings in the model as well as the standard deviation, skewness and kurtosis of earnings by age. How do these graphs compare data estimates you created in Part (1) and those presented in Huggett, Ventura, and Yaron [2011].

   (b) Plot the policy function for investing in human capital as a function a function of (1) assets and (2) human capital for workers of different ages.

   (c) How do these policy functions change if you increase the variance of shocks to human capital? Why do you think you see this pattern?

   (d) Create a measure of lifetime earnings based upon Guvenen et al. [2017]. If you increase the initial dispersion of human capital, how does your measure of lifetime inequality change? How does the path of the standard deviation of earnings by age compare to your graph from part (a)?

2.2 Calibration

- The period is one year, and assume individuals live for $T = 30$ years.
- Set the rental rate on labor for an age t worker to $R_t = (1.0019)^{t-1}$.
- Assume z follows a normal distribution with mean $\mu = -.029$ and variance $\sigma^2 = 0.11$.
- Assume the human capital technology is $H(h, s, a) = h + (hs)^{\alpha}$ with $\alpha = 0.70$.

- Set a discrete grid on human capital between 0 and 10 with 100 evenly spaced grid points. Have workers draw their initial human capital from a normal distribution with mean 2 and variance 0.5.
- Set the annualized risk free rate to 4%, and set $\beta = .99$.
- Individual preferences over non-durable consumption are given by:

$$u(c) = \frac{c^{1-\sigma} - 1}{1 - \sigma}$$

  Set the risk aversion parameter to a standard value, $\sigma = 2$.
- Set the borrowing limit to zero, and assume individuals are born with zero assets.

2.3 Suggested solution method

- Create a grid on assets k, and human capital h. Setup the grid so that assets are the row variable and human capital is the column variable.
- Start in the terminal period. In the terminal period, agents will not save or invest in human capital. From the budget constraint you can find consumption and calculate the value for each combination of state variables.
- Define a grid of choices for the share of the period to invest in human capital. Suppose there are n choices.
  - Create a markov transition matrix for human capital for each of the n choices of human capital investment s.
- Fix a set of state variables (k, h) in period $T - 1$. Solve for the agents value at each combination of (k', s) choices using the transition matrices you made in the step above and the values in the terminal period. Pick the max continuation value. Store the choices of (k', s) that delivered this value. Repeat for each spot in the state space (k, h)
- Repeat for periods $T - 2$ to 1.
- Using the policy functions from above, simulate a panel of earnings from your model.

References

Fatih Guvenen, Greg Kaplan, Jae Song, and Justin Weidner. Lifetime incomes in the united states over six decades. Technical report, National Bureau of Economic Research, 2017.

Mark Huggett, Gustavo Ventura, and Amir Yaron. Sources of lifetime inequality. *American Economic Review*, 101(7):2923–54, 2011.