Image Restoration using the ROF Model

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Objective

This report explores image denoising using the Rudin–Osher–Fatemi (ROF) model. The technique is applied to the individual color planes of a raw Bayer-mosaic image. Mean Square Difference (MSD) statistics are used to quantify noise levels, and comparisons across color channels inform sensor behavior and image fidelity.

Sensor Image Representation and Bayer Mosaic

Digital cameras use image sensors composed of a grid of light-sensitive photodiodes. Each photodiode detects intensity for a single color channel — red, green, or blue. To capture full-color images, most sensors employ a **Bayer mosaic**: a 2×2 repeating pattern with two green, one red, and one blue sensor:

$$\begin{bmatrix} G & R \\ B & G \end{bmatrix}$$

This configuration exploits the human eye's higher sensitivity to green (luminance), providing higher effective spatial resolution.

The raw image data ('.ARW') stores this mosaic directly. Using functions like `rawread` and `raw2planar`, we extract the red, blue, and two green planes (G1 and G2) as separate grayscale images. Each plane is half the size of the original mosaic due to subsampling.

Method

We denoise each plane using the ROF energy model:

$$\mathcal{F}(u) = \int \sqrt{\epsilon^2 + |\nabla u|^2}\, dx\, dy + \frac{\lambda}{2} \int (u - f)^2\, dx\, dy$$

where f is the noisy input and u is the restored output. The first term promotes smoothness (total variation), while the second enforces fidelity to data.

We implemented a fully vectorized iterative solver using MATLAB. The algorithm:

- Handles Neumann boundary conditions (zero-gradient edges).
- Uses CPU parallelism (`parfor`) or NVIDIA GPU acceleration (`gpuArray`) if available.
- Sweeps over a grid of parameters (λ, ϵ) to evaluate denoising performance.

The quality of denoising is measured by the Mean Square Difference:

$$\mathrm{MSD}(f, \lambda, \epsilon) = \sqrt{\frac{1}{HW} \sum_{i,j} (u_{i,j} - f_{i,j})^2}$$

Results

report/msd_surfaces.png

Figure 1: MSD surfaces for R, G1, G2, and B planes, across parameter grid (λ, ϵ). Vertical offsets applied for comparison.

We evaluated MSD surfaces for each plane. Parameter sweeps showed consistent structure across color channels. Notably:

- Green planes (G1 and G2) showed lowest MSD values.
- Blue and red channels exhibited higher noise sensitivity.

report/rof_grid_planes_combined.png

Figure 2: MSD comparison across color planes. Each surface was computed using coarse-to-fine parameter sweep.

Noise Analysis

This aligns with physical design: the Bayer pattern oversamples green to capture luminance with higher fidelity. Since each green plane gets interpolated from more nearby samples, it exhibits reduced variance. Red and blue, captured less frequently, show greater pixel-wise fluctuation (i.e., more noise).

Discussion

The ROF model demonstrates strong noise suppression while preserving edges. We observed that:

- Increasing λ increases smoothing — reduces MSD but can oversmooth.
- Smaller ϵ preserves edges better but is more sensitive to noise.
- A mid-range (λ, ϵ) provides a good trade-off.

Bayer Design Insight

Two green pixels per 2×2 block boost horizontal and vertical luminance sampling — helping demosaicing and noise rejection in green-dominant vision. Our MSD trends empirically support this hypothesis.

Validation and Testing

We built a robust test suite with over a dozen tests:

Automated Tests

- **Zero noise recovery:** flat input returns MSD = 0.
- **Monotonicity:** MSD increases with λ.
- **Boundary conditions:** edge gradients remain zero.
- **GPU fallback:** system detects GPU and falls back to CPU if needed.
- **Output shape and batching:** 4D arrays handled correctly.
- **Numerical stability:** solver produces no NaNs/Infs.
- **CPU vs GPU equivalence:** relative error within tolerance (3.87%).

Visual Tests

We used side-by-side comparisons of denoised images and difference maps:

Performance Benchmarks

- CPU (4 threads): 32 seconds for full grid
- GPU (NVIDIA): 2.8 seconds per sweep
- Hybrid CPU+GPU (parfor): 9 seconds total

Conclusion

This project demonstrates the power of variational image processing. The ROF model yields interpretable, tunable results and adapts well to modern hardware. From raw sensor data to smoothed results, we explored denoising quantitatively and visually — revealing both engineering and perceptual truths in digital imaging.

Code Files

- `smooth_image_rof.m` – Main solver with CPU/GPU support
- `calculate_msd.m` – Computes MSD for parameter grid
- `cpu_plane_sweep.m`, `gpu_plane_sweep.m` – Adaptive batching
- `smart_grid_search.m` – Coarse-to-fine MSD grid optimization
- `foreach_plane_search.m` – Runs search on R/G1/G2/B
- `run_rof_hpc.m` – Parallel execution script
- `test/run_all_tests.m` – Runs full suite with logging

References

- Rudin, Osher, Fatemi. "Nonlinear total variation based noise removal algorithms." Physica D, 1992.
- Bayer, B.E. "Color Imaging Array." Eastman Kodak Co., US Patent 3,971,065 (1976).

test/plots/cpu_gpu_diff.png

Figure 3: CPU vs GPU output: Left: CPU, Center: GPU, Right: Difference (amplified).