Image Restoration using the ROF Model

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1

Objective This report explores image denoising using the Rudin–Osher–Fatemi (ROF) model. We apply the model to color planes of a Bayer-mosaic image and analyze noise levels through Mean Square Difference (MSD) statistics.

2

Method We solve the ROF model:

$$\mathcal{F}(u) = \int \sqrt{\epsilon^2 + |\nabla u|^2} + \frac{\lambda}{2} \int (u - f)^2 \, dx \, dy$$

using a vectorized iterative scheme implemented in MATLAB. The solver adapts to CPU or GPU hardware and computes the solution over a parameter grid (λ, ϵ).

MSD is defined as:

$$\mathrm{MSD}(f, \lambda, \epsilon) = \sqrt{\frac{1}{HW} \sum_{i,j} (u_{i,j} - f_{i,j})^2}$$

where u is the denoised output.

3

Results The following figure shows MSD surfaces for R, G1, G2, and B planes.

3.1

Observations

- Green planes (G1, G2) consistently had lower MSD, indicating less noise.
- Blue showed the highest noise, followed by red.
- This aligns with the Bayer mosaic design: two green sensors increase spatial luminance resolution.

4 Visualization of MSD Surfaces

We computed $MSD(f, \lambda, \epsilon)$*forall4planesandplottedtheresultingsurfacesusingMATLAB′s*`surf`*function.Thep* `results/msd_surfaces/msd_surface_combined_offset.png`

Figure 1: MSD surfaces across $\$(\lambda, \epsilon)\$ foreachcolorplane.Offsetswereappliedtoaidvisualization.$

5 Visual Test Images

We tested the ROF algorithm on synthetic images with known structure and noise to assess its visual performance.

5.1 Gradient Image (No Noise)

5.2 Sinusoidal Image (High Noise)

5.3 Checkerboard Image (Moderate Noise)

6 Interpretation and Discussion

The ROF model effectively attenuates noise while preserving edges. From our results:

- Green planes are least noisy. This aligns with the Bayer pattern having two green sensors per 2x2 pixel block — a design optimized for human vision sensitivity to green.

- Higher $\$\lambda\$valuesresultinoverlysmoothimages, reducingMSDbutalsopossiblyremovingfinedetail.Low\$\epsilon\$values$
  This analysis informs future image denoising workflows: a balanced regularization (mid $\$\lambda, \epsilon\$)yieldsminimalno$

7 Conclusion

This project demonstrates the power of numerical PDE methods (like ROF) for image processing. The implementation highlights how vectorization, GPU acceleration, and parallelism can scale analysis over hyperparameter grids effectively.

8 Code Files Overview

- `smooth_image_rof.m` – Vectorized ROF solver supporting GPU/CPU. Computes smoothed image over $\$(\lambda, \epsilon)\$grid.$`calculate_msd.m`$--ComputesMeanSquareDifferencebetweenoriginalandsmooth$

- `cpu_plane_sweep.m` / `gpu_plane_sweep.m` – Batch computation for ROF over CPU or GPU.

- `smart_grid_search.m` – Coarse-to-fine parameter optimization over MSD values.

- `foreach_plane_search.m` – Applies grid search to each color plane (R, G1, G2, B).

- `run_rof_hpc.m` – Main driver script to orchestrate parallel parameter sweeps and save results.

9 Validation and Testing

To verify correctness and robustness, we implemented an extensive suite of automated and visual tests.

9.1 Automated Tests

- **Zero noise recovery:** confirms perfect input returns zero error.

- **High noise recovery:** verifies robustness under strong degradation.

- **Monotonicity:** ensures MSD trends with $\$\lambda\$and\$\epsilon\$followexpectedbehavior.$**Output shape:**$confirmscorr$

- **Boundary conditions:** ensures Neumann conditions are respected.

- **Numerical stability:** checks for NaNs and Infs.

- **GPU fallback:** confirms CPU fallback path on non-NVIDIA hardware.
- **Batch speedup:** compares vectorized vs loop timings.

9.2 Visual Tests

We include visual inspection of denoised images and 3D surface plots of MSD across parameter grids.

9.3 GPU vs CPU Equivalence

A relative error of approximately $3.87

$language = matlab, caption = Alltestsrunner]disp('=== ROFTestLog ===');test\_zero\_noise;test\_high\_noise\_$

A Performance and Benchmarks

- CPU-only (4 threads): 32s for a full grid sweep.
- GPU (NVIDIA RTX): 2.8s per sweep, 12x speedup.
- CPU+GPU hybrid (parfor): 9s total with full parallelism.

results/msd_surfaces/msd_surface_red.png

results/msd_surfaces/msd_surface_green1.png results/test_images/zero_noise_gradient.png

Figure 3: Test on a smooth gradient image with no noise. ROF recovers the input with minimal error.

results/test_images/high_noise_sinusoidal.png

Figure 4: Test on a sinusoidal image with high Gaussian noise. ROF significantly denoises while preserving shape.

results/test_images/checkerboard_denoising.png

Figure 5: Checkerboard pattern denoising. Fine edges are mostly preserved while noise is reduced.