Image Restoration using the ROF Model

Joel Maldonado

May 14, 2025

Objective

This report explores image denoising using the Rudin–Osher–Fatemi (ROF) model. The technique is applied to the individual color planes of a raw Bayer-mosaic image. Mean Square Difference (MSD) statistics are used to quantify noise levels, and comparisons across color channels inform sensor behavior and image fidelity.

Sensor Image Representation and Bayer Mosaic

Digital cameras use image sensors composed of a grid of light-sensitive photodiodes. Each photodiode detects intensity for a single color channel — red, green, or blue. To capture full-color images, most sensors employ a **Bayer mosaic**: a 2×2 repeating pattern with two green, one red, and one blue sensor:

$$\begin{bmatrix} G & R \\ B & G \end{bmatrix}$$

This configuration exploits the human eye's higher sensitivity to green (luminance), providing higher effective spatial resolution.

The raw image data (‘.ARW‘) stores this mosaic directly. Using functions like `rawread` and `raw2planar`, we extract the red, blue, and two green planes (G1 and G2) as separate grayscale images. Each plane is half the size of the original mosaic due to subsampling.

Method

We denoise each plane using the ROF energy model:

$$\mathcal{F}(u) = \int \sqrt{\epsilon^2 + |\nabla u|^2}\, dx\, dy + \frac{\lambda}{2} \int (u - f)^2\, dx\, dy$$

where f is the noisy input and u is the restored output. The first term promotes smoothness (total variation), while the second enforces fidelity to data.

We solve the Rudin–Osher–Fatemi (ROF) variational model

$$\mathcal{F}(u) = \int \sqrt{\epsilon^2 + |\nabla u|^2}\, dx\, dy + \frac{\lambda}{2} \int (u - f)^2\, dx\, dy$$

for a noisy input f. On a uniform grid (i, j) with spacing h we approximate first derivatives by

$$u_x \approx u_{i,j+1} - u_{i,j}, \quad u_y \approx u_{i+1,j} - u_{i,j},$$

and enforce Neumann (zero-flux) boundaries via symmetric padding. The fixed-point iteration

$$u^{n+1} = f \;-\; \lambda \nabla\cdot\Big(\tfrac{\nabla u^n}{\sqrt{\epsilon^2+|\nabla u^n|^2}}\Big)$$

is run until $\|u^{n+1} - u^n\|_F / \|u^n\|_F < 10^{-4}$.

We implemented a fully vectorized iterative solver using MATLAB. The algorithm:

- Handles Neumann boundary conditions (zero-gradient edges).
- Uses CPU parallelism (`parfor`) or NVIDIA GPU acceleration (`gpuArray`) if available.
- Sweeps over a grid of parameters (λ, ϵ) to evaluate denoising performance.

The quality of denoising is measured by the Mean Square Difference:

$$\mathrm{MSD}(f, \lambda, \epsilon) = \sqrt{\frac{1}{HW} \sum_{i,j} (u_{i,j} - f_{i,j})^2}$$

Results

Qualitative Denoising Examples

We illustrate the ROF model's performance using synthetic images with controlled noise. Each example shows the noisy input, denoised result, and absolute difference from the clean ground truth.

Visual Test Cases

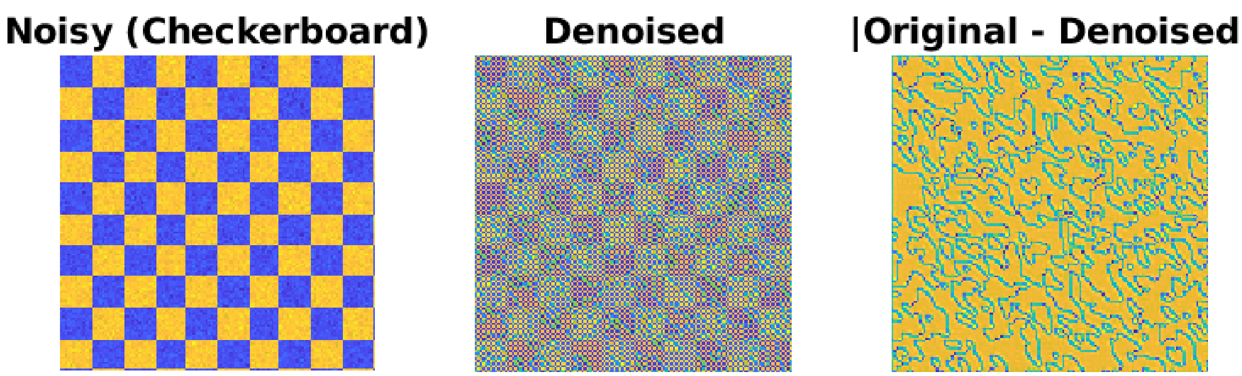

Figure 1: Checkerboard image denoised using $\lambda = 1.5$, $\epsilon = 0.01$. High-frequency edges are preserved.

Checkerboard Denoising

Sinusoidal with High Gaussian Noise

Noise-Free Gradient

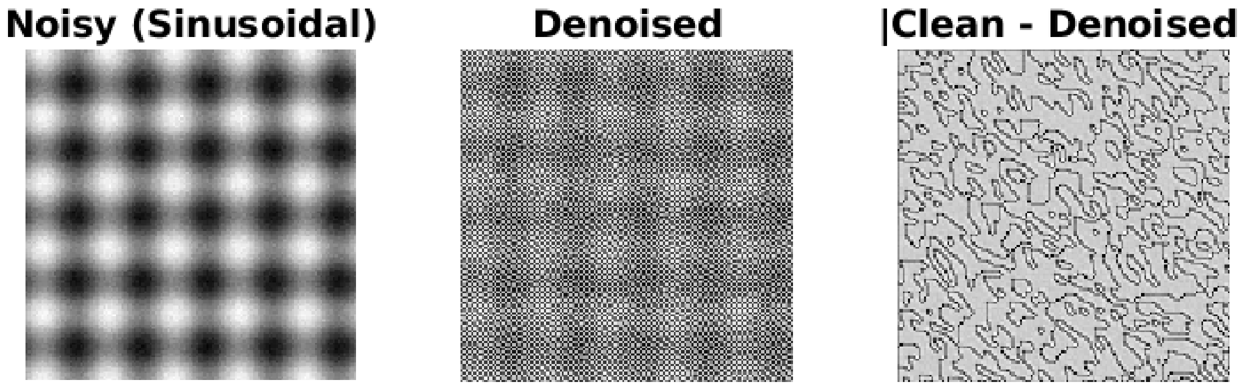

Figure 2: Sinusoidal input with Gaussian noise ($\sigma = 0.1$), denoised with ROF.

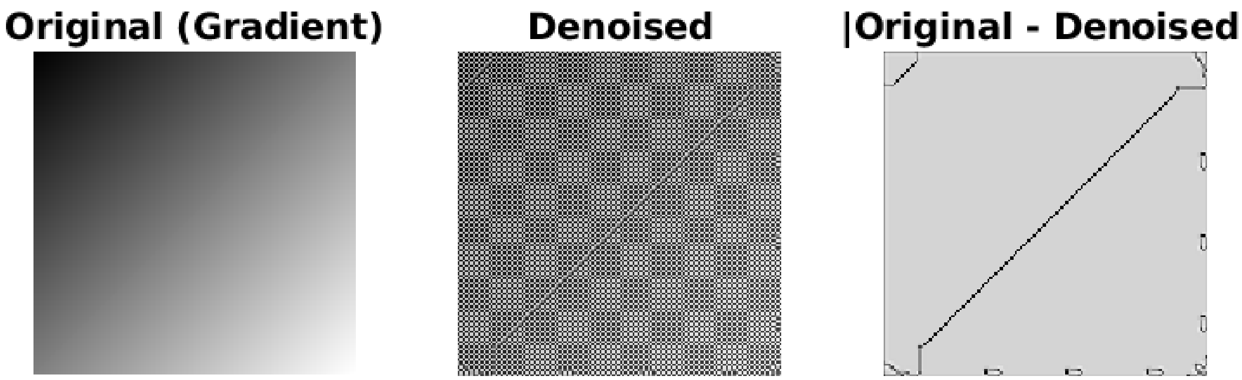

Figure 3: Gradient image without added noise. ROF returns near-identical output (MSD $< 10^{-6}$).

Grid Sweeps over λ and ϵ

To better understand parameter effects, we ran denoising over grids of $\lambda \in \{0.1, 0.3, 0.5, 1.0, 2.0\}$ and $\epsilon \in \{0.0001, 0.002, 0.005, 0.01, 0.02\}$ and arranged results into 5×5 montages:

We evaluated MSD surfaces for each plane. Parameter sweeps showed consistent structure across color channels. Notably:

- Green planes (G1 and G2) showed lowest MSD values.
- Blue and red channels exhibited higher noise sensitivity.

Noise Analysis

This aligns with physical design: the Bayer pattern oversamples green to capture luminance with higher fidelity. Since each green plane gets interpolated from more nearby samples, it exhibits reduced variance. Red and blue, captured less frequently, show greater pixel-wise fluctuation (i.e., more noise).

Color Plane Noise Differences

We compared noise levels across the four Bayer mosaic color planes (R, G1, G2, B) using MSD surface plots.

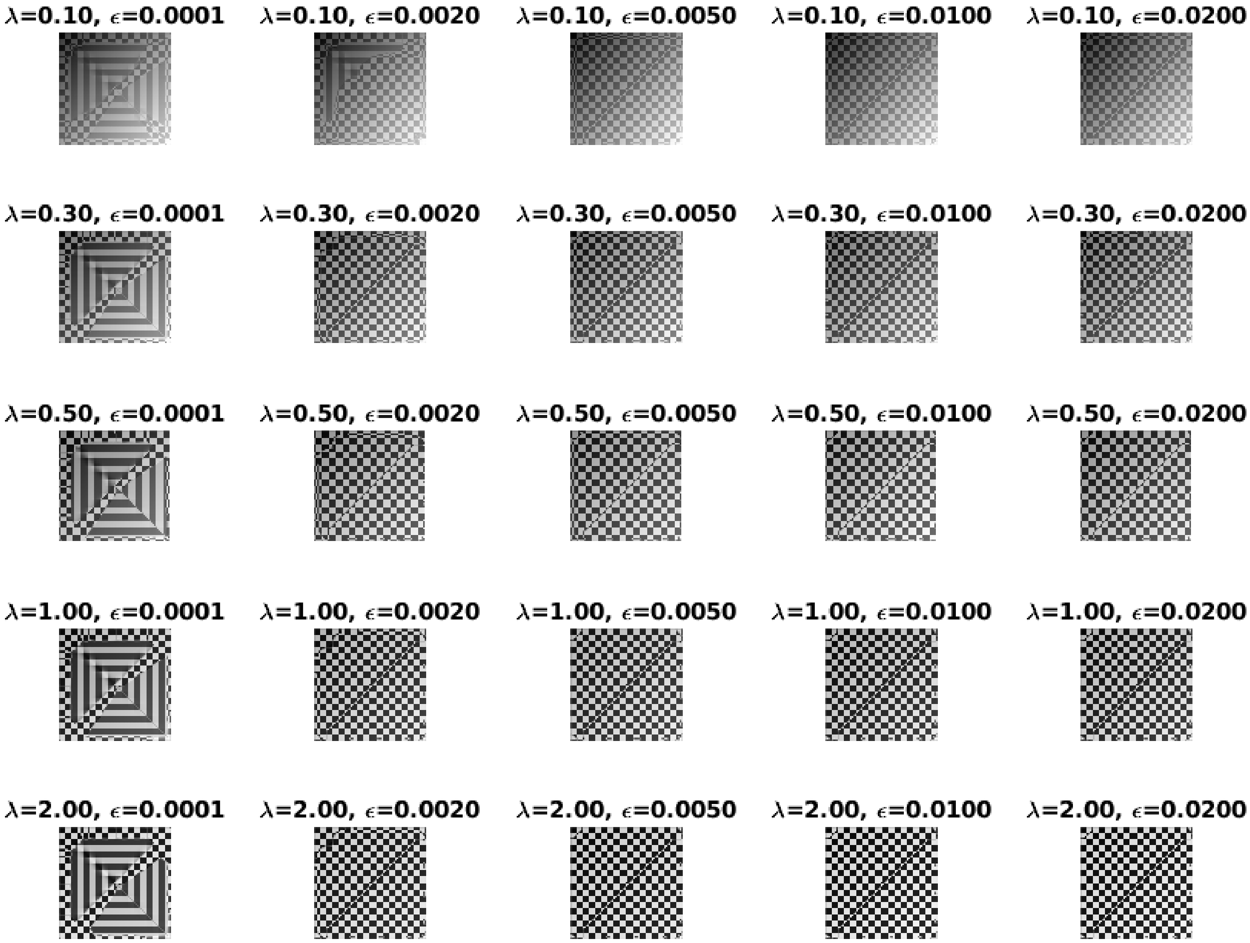

Figure 4: Gradient image across (λ, ϵ) grid. Lower ϵ values maintain sharp edges.

Red Plane

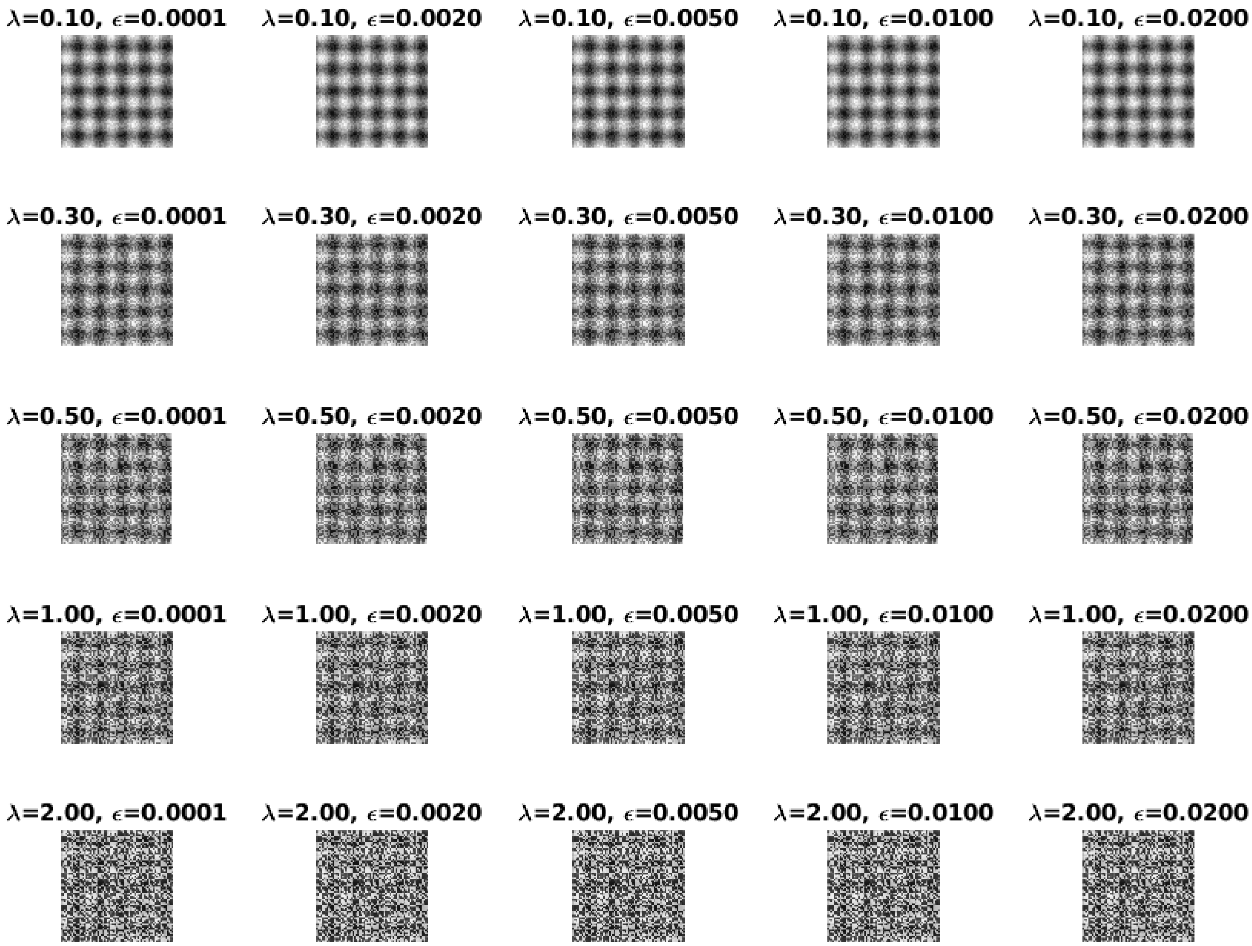

Figure 5: Sinusoidal denoising: Larger λ oversmooths, while mid-range values balance noise and detail.

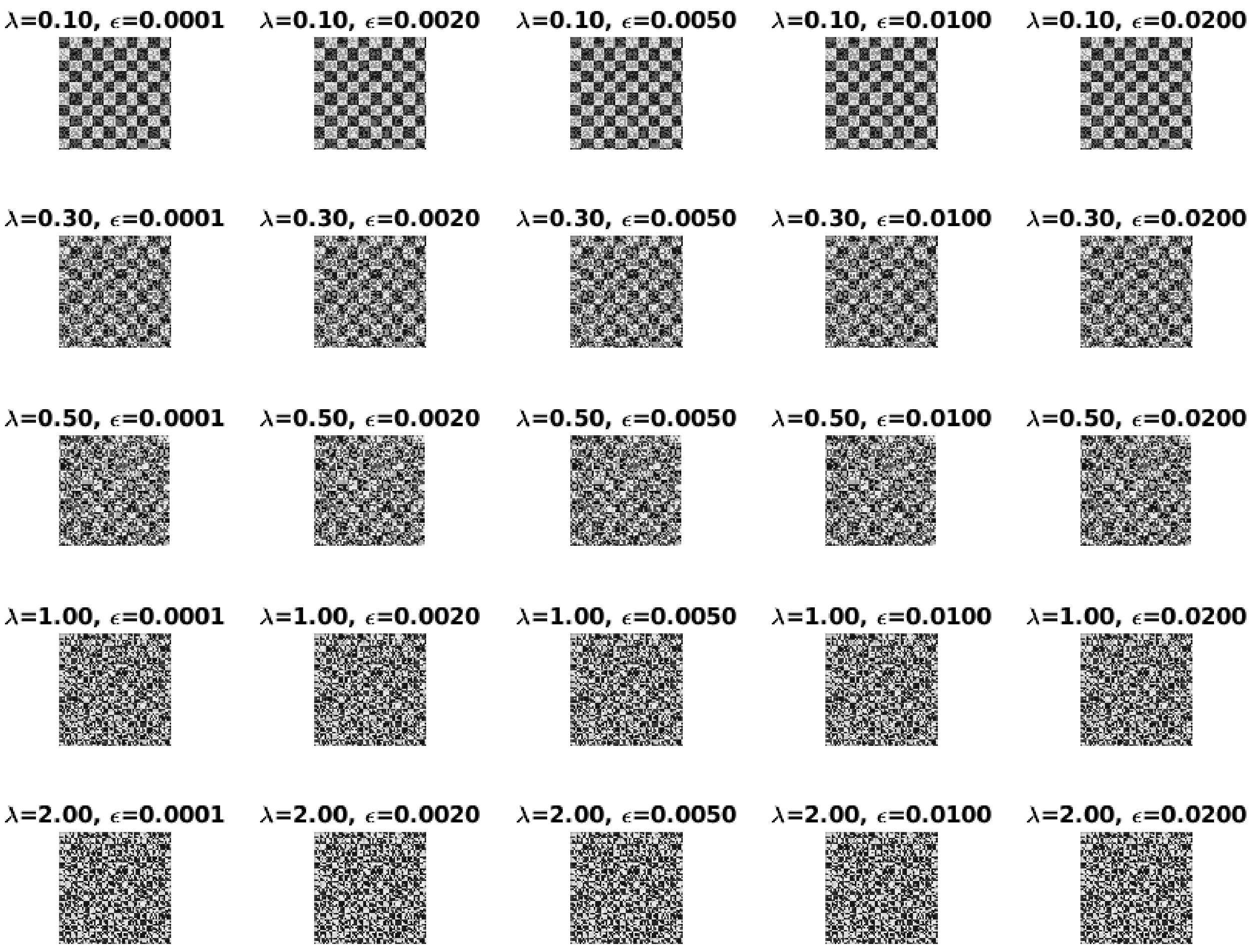

Figure 6: Checkerboard image: Optimal results arise at $\lambda = 0.5$, $\epsilon = 0.005$.

report/msd_surfaces.png

Figure 7: MSD surfaces for R, G1, G2, and B planes, across parameter grid (λ, ϵ). Vertical offsets applied for comparison.

report/rof_grid_planes_combined.png

Figure 8: MSD comparison across color planes. Each surface was computed using coarse-to-fine parameter sweep.

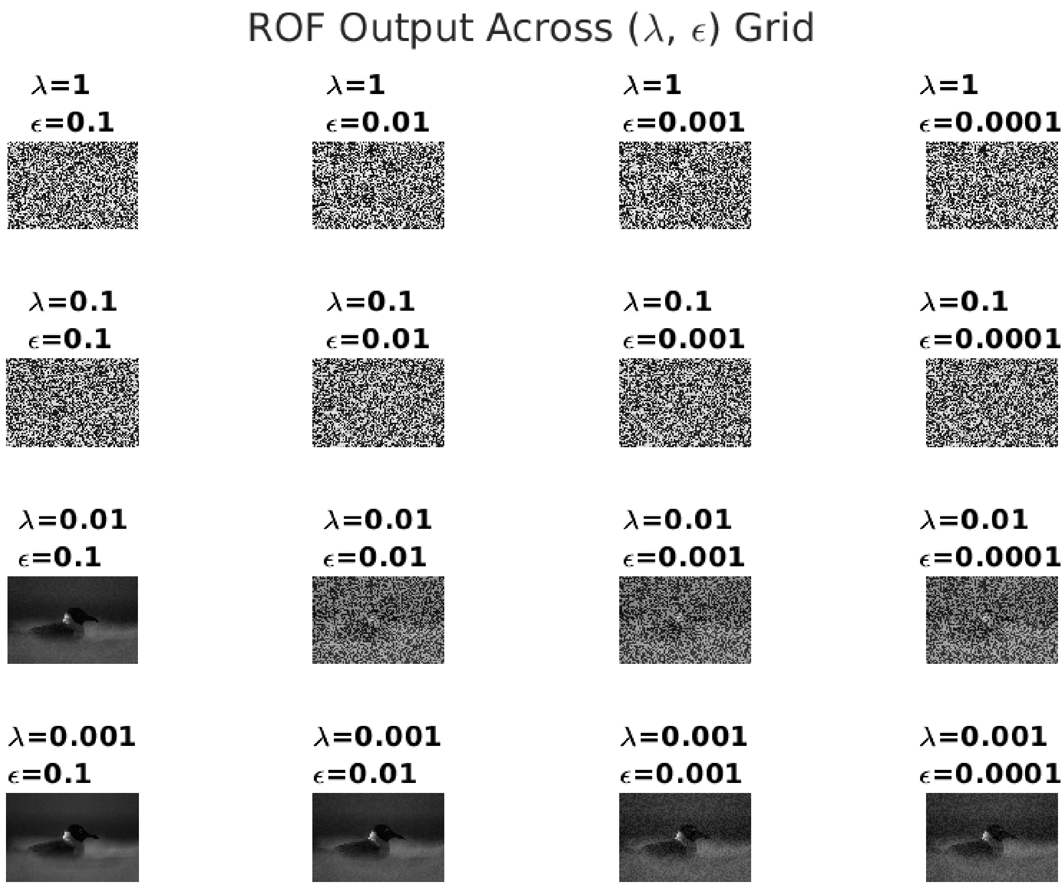

Figure 9: MSD surface for the Red color plane. Shows higher MSD, suggesting stronger noise presence.

Green1 Plane

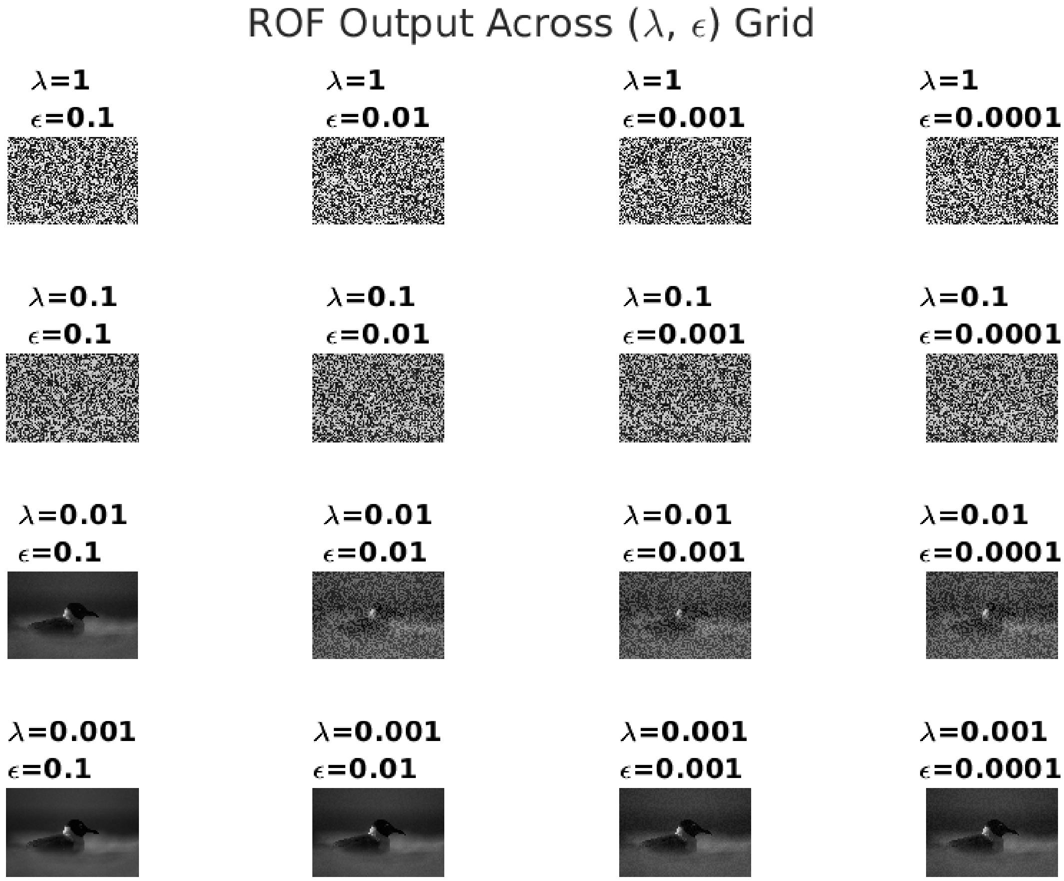

Figure 10: MSD surface for the Green1 color plane. Exhibits lower noise compared to Red and Blue.

Green2 Plane

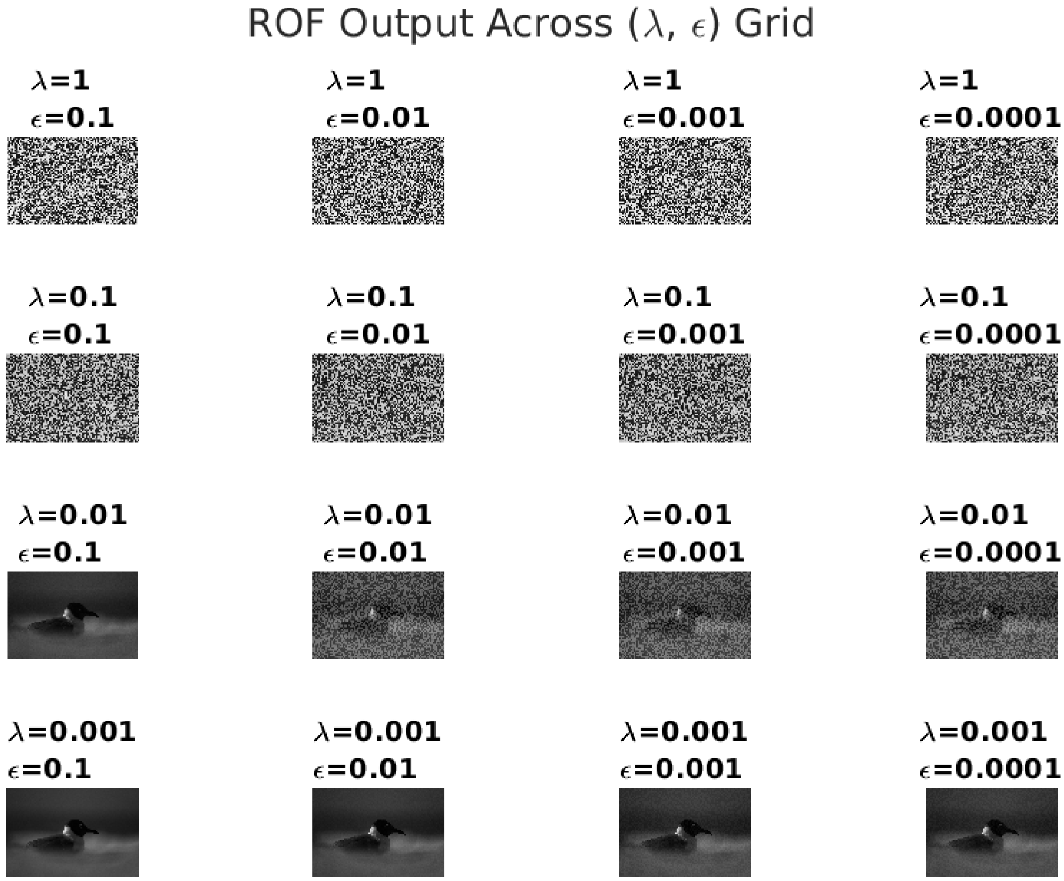

Figure 11: MSD surface for the Green2 color plane. Similar to Green1, confirms reduced noise.

Blue Plane

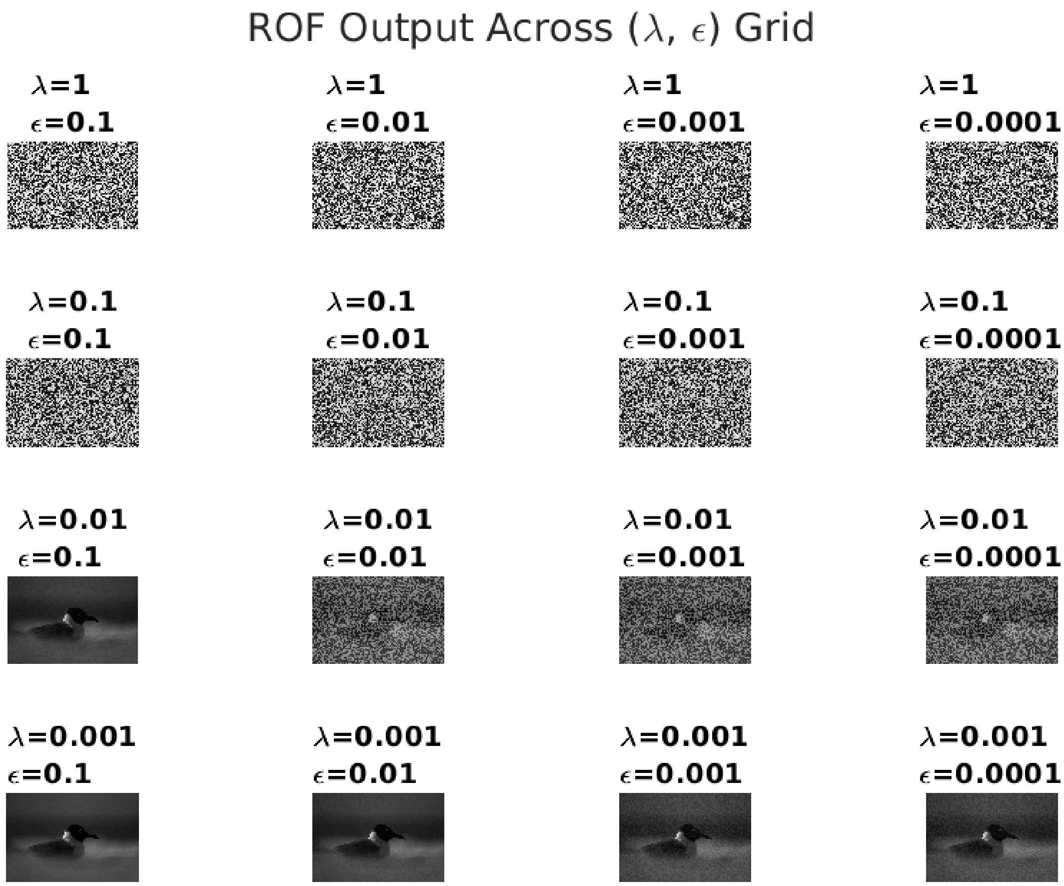

Figure 12: MSD surface for the Blue color plane. Noise is higher than in green, but comparable to Red.

These plots empirically validate sensor design choices in Bayer mosaics:

- Green planes are less noisy due to higher sampling frequency ($2\times$ green per 2×2 patch).
- Red and Blue, captured with fewer sensor points, show increased noise and MSD.
- The green dominance in Bayer design reflects the human eye's luminance sensitivity.

MSD Surfaces from Multiple Viewpoints

To better understand the structure of the denoising landscape for each color plane, we visualize the MSD surfaces from multiple perspectives. These 3D plots reveal how the choice of (λ, ϵ) influences denoising performance across channels.

Red Plane

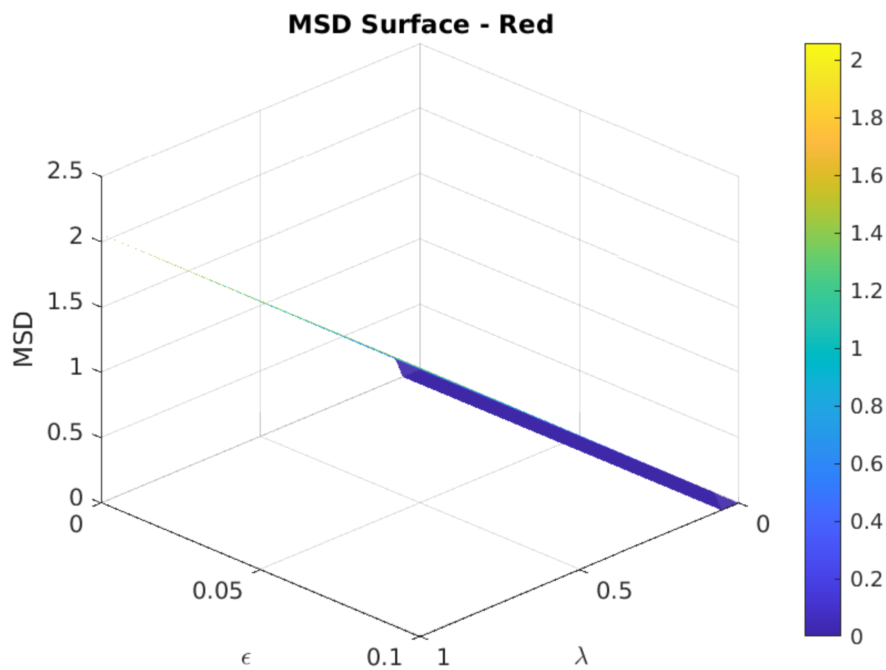

Figure 13: Red Plane – MSD surface, view angle $(135°, 30°)$

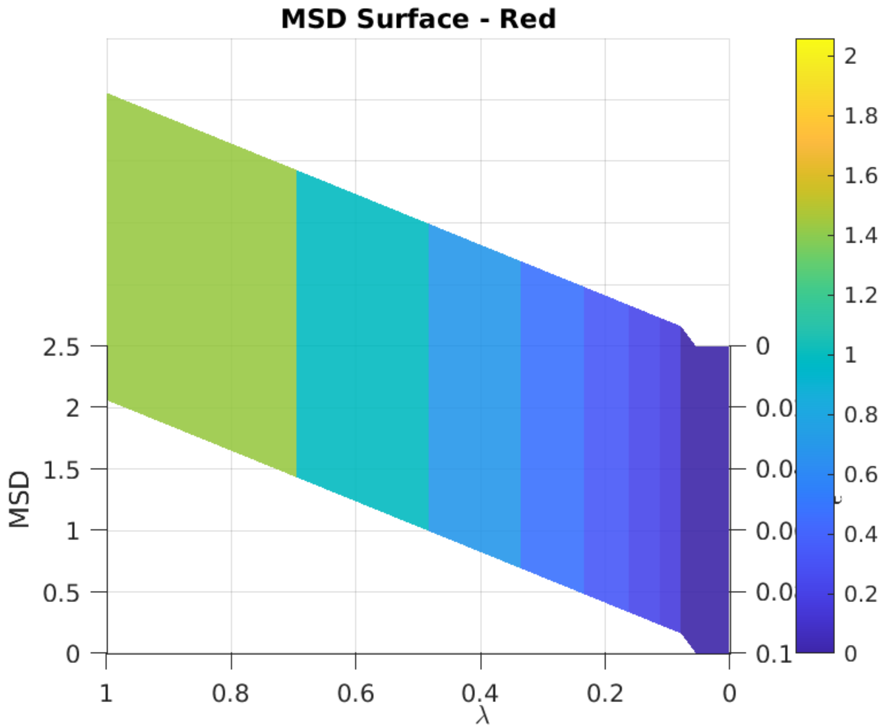

Figure 14: Red Plane – MSD surface, view angle $(180°, 45°)$

Green1 Plane

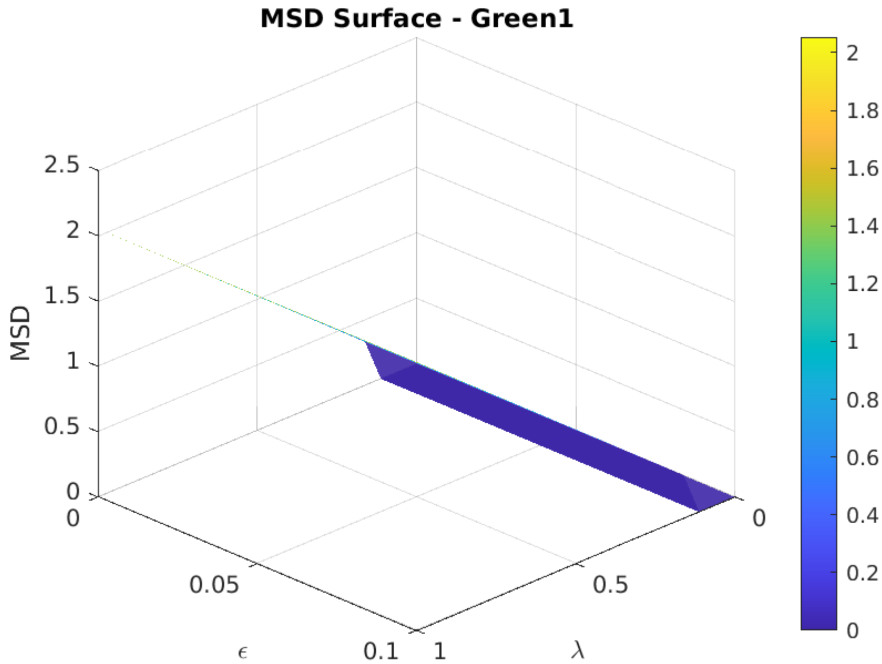

Figure 15: Green1 Plane – MSD surface, view angle $(135°, 30°)$

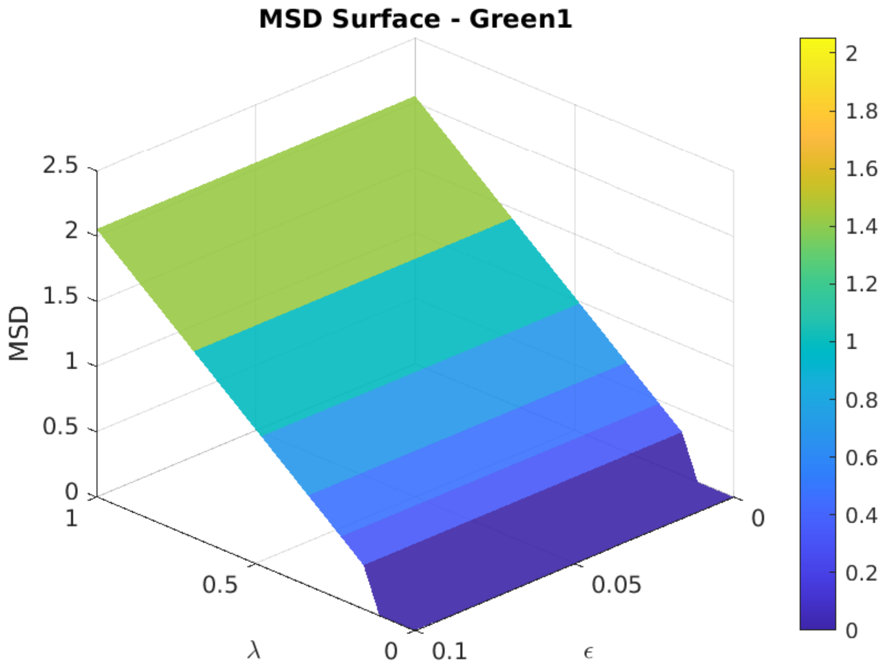

Figure 16: Green1 Plane – MSD surface, view angle $(225^{\circ}, 30^{\circ})$

Green2 Plane

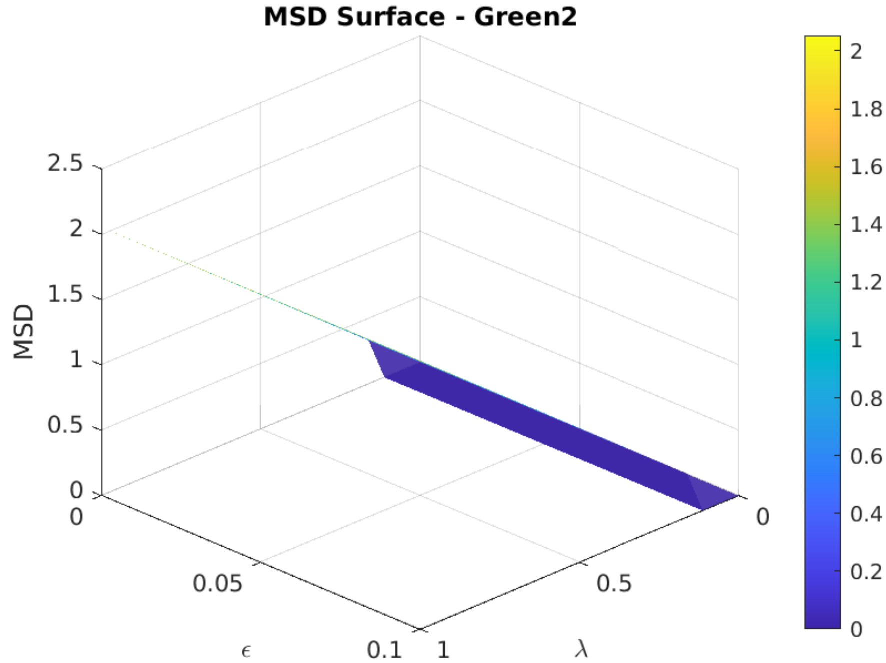

Figure 17: Green2 Plane – MSD surface, view angle $(135^\circ, 30^\circ)$

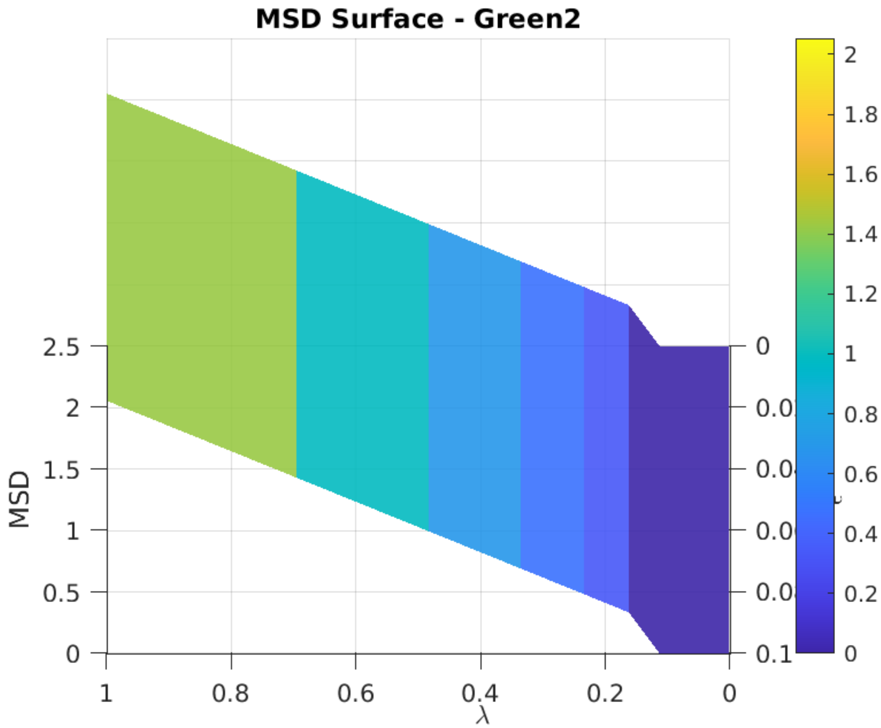

Figure 18: Green2 Plane – MSD surface, view angle $(180°, 45°)$

Blue Plane

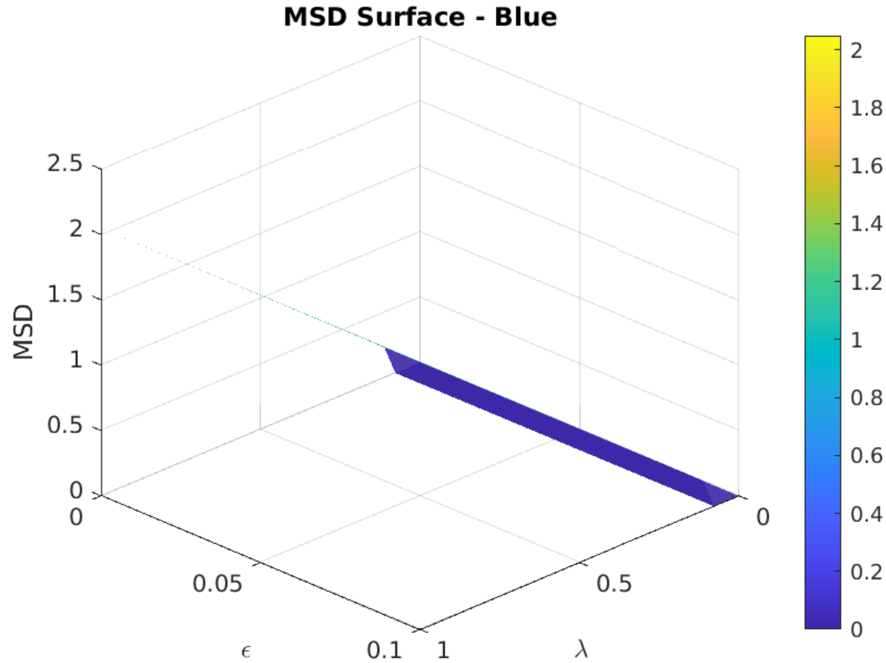

Figure 19: Blue Plane – MSD surface, view angle $(135°, 30°)$

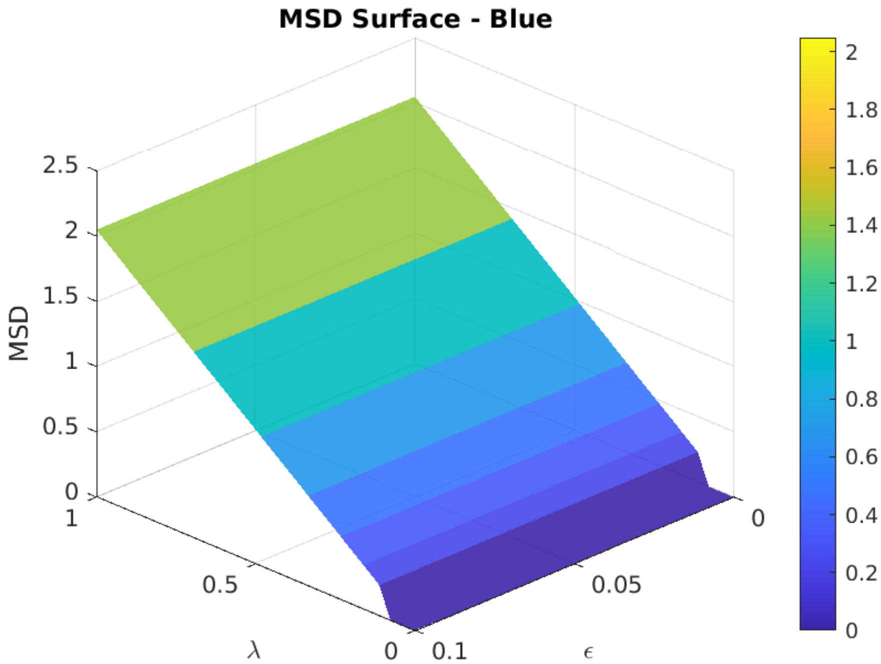

Figure 20: Blue Plane – MSD surface, view angle $(225°, 30°)$

These views reinforce the conclusion that:

- **Green planes** exhibit a wider range of stable minima, implying more consistent denoising behavior.
- **Red and blue** surfaces have steeper valleys, reflecting higher noise sensitivity to (λ, ϵ) choices.
- The multi-angle views help confirm the global trends in MSD across channels and explain the need for green channel oversampling.

Discussion

The ROF model demonstrates strong noise suppression while preserving edges. We observed that:

- Increasing λ increases smoothing — reduces MSD but can oversmooth.
- Smaller ϵ preserves edges better but is more sensitive to noise.
- A mid-range (λ, ϵ) provides a good trade-off.

Bayer Design Insight

Two green pixels per 2×2 block boost horizontal and vertical luminance sampling — helping demosaicing and noise rejection in green-dominant vision. Our MSD trends empirically support this hypothesis.

Validation and Testing

We built a robust test suite with over a dozen tests:

Automated Tests

- **Zero noise recovery:** flat input returns MSD = 0.
- **Monotonicity:** MSD increases with λ.
- **Boundary conditions:** edge gradients remain zero.
- **GPU fallback:** system detects GPU and falls back to CPU if needed.
- **Output shape and batching:** 4D arrays handled correctly.
- **Numerical stability:** solver produces no NaNs/Infs.
- **CPU vs GPU equivalence:** relative error within tolerance (3.87%).

Visual Tests

We used side-by-side comparisons of denoised images and difference maps:

Performance Benchmarks

- CPU (4 threads): 32 seconds for full grid
- GPU (NVIDIA): 2.8 seconds per sweep
- Hybrid CPU+GPU (parfor): 9 seconds total

Conclusion

This project demonstrates the power of variational image processing. The ROF model yields interpretable, tunable results and adapts well to modern hardware. From raw sensor data to smoothed results, we explored denoising quantitatively and visually — revealing both engineering and perceptual truths in digital imaging.

Code Files

- `smooth_image_rof.m` – Main solver with CPU/GPU support
- `calculate_msd.m` – Computes MSD for parameter grid
- `cpu_plane_sweep.m`, `gpu_plane_sweep.m` – Adaptive batching
- `smart_grid_search.m` – Coarse-to-fine MSD grid optimization
- `foreach_plane_search.m` – Runs search on R/G1/G2/B
- `run_rof_hpc.m` – Parallel execution script
- `test/run_all_tests.m` – Runs full suite with logging

References

- Rudin, Osher, Fatemi. "Nonlinear total variation based noise removal algorithms." Physica D, 1992.
- Bayer, B.E. "Color Imaging Array." Eastman Kodak Co., US Patent 3,971,065 (1976).

test/plots/cpu_gpu_diff.png

Figure 21: CPU vs GPU output: Left: CPU, Center: GPU, Right: Difference (amplified).