Artificial Intelligence (CS111)

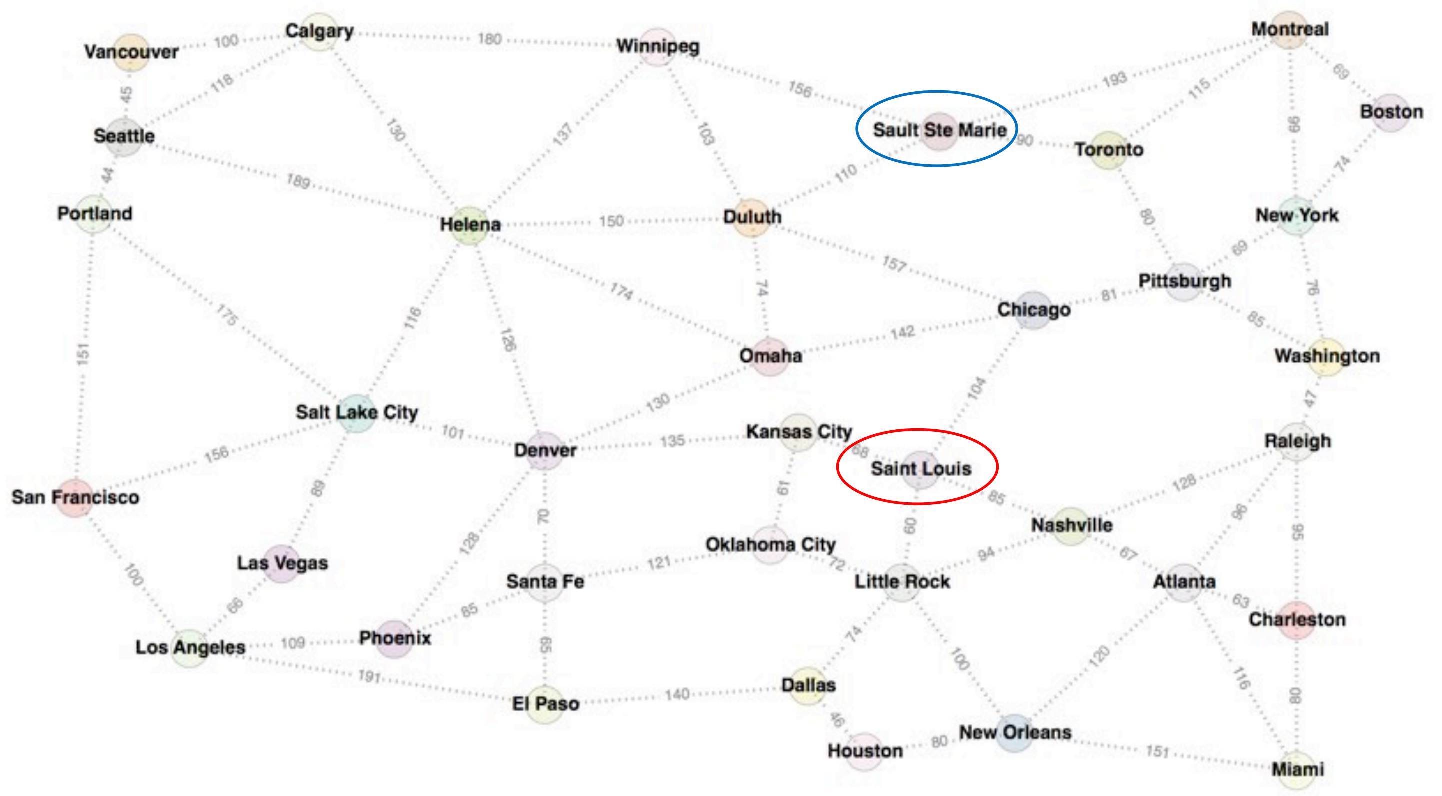
Lecture 4: Informed Search & Local Search

Credit: Ansaf Salleb-Aouissi, and "Artificial Intelligence: A Modern Approach", Stuart Russell and Peter Norvig, and "The Elements of Statistical Learning", Trevor Hastie, Robert Tibshirani, and Jerome Friedman, and "Machine Learning", Tom Mitchell.

Informed search

- **Use domain knowledge!**
  - Are we getting close to the goal?
  - Use a heuristic function that estimates how close a state is to the goal
  - A heuristic does NOT have to be perfect!
  - Example of strategies:
    1. Greedy best-first search
    2. A* search
    3. IDA*

Informed search



The distance is the straight-line distance. The goal is to get to Sault Ste Marie, so all the distances are from each city to Sault Ste Marie.

Atlanta	272
Boston	240
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Heuristic!

Greedy search

- Evaluation function $h(n)$ (heuristic)
- $h(n)$ estimates the cost from n to the goal
- Example: $h_{SLD}(n)$ = straight-line distance from n to Sault Ste Marie
- Greedy search expands the node that **appears** to be closest to goal

Greedy search: Pseudo-code

```
function GREEDY-BEST-FIRST-SEARCH(initialState, goalTest)
    returns SUCCESS or FAILURE :  /* Cost f(n) = h(n) */

    frontier = Heap.new(initialState)
    explored = Set.new()

    while not frontier.isEmpty():
        state = frontier.deleteMin()
        explored.add(state)

        if goalTest(state):
            return SUCCESS(state)

        for neighbor in state.neighbors():
            if neighbor not in frontier ∪ explored:
                frontier.insert(neighbor)
            else if neighbor in frontier:
                frontier.decreaseKey(neighbor)

    return FAILURE
```

Greedy search example

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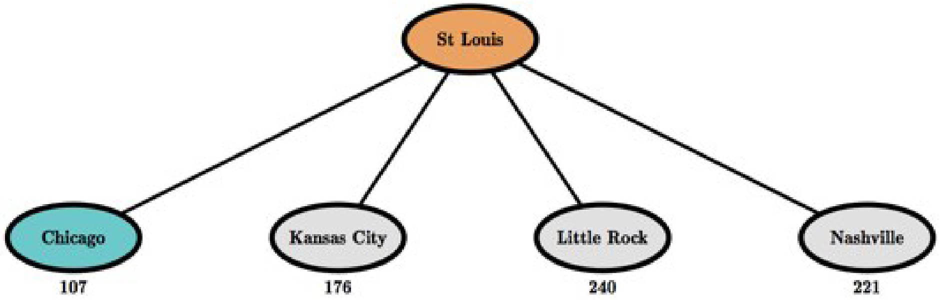
Greedy search example

After expanding St Louis:



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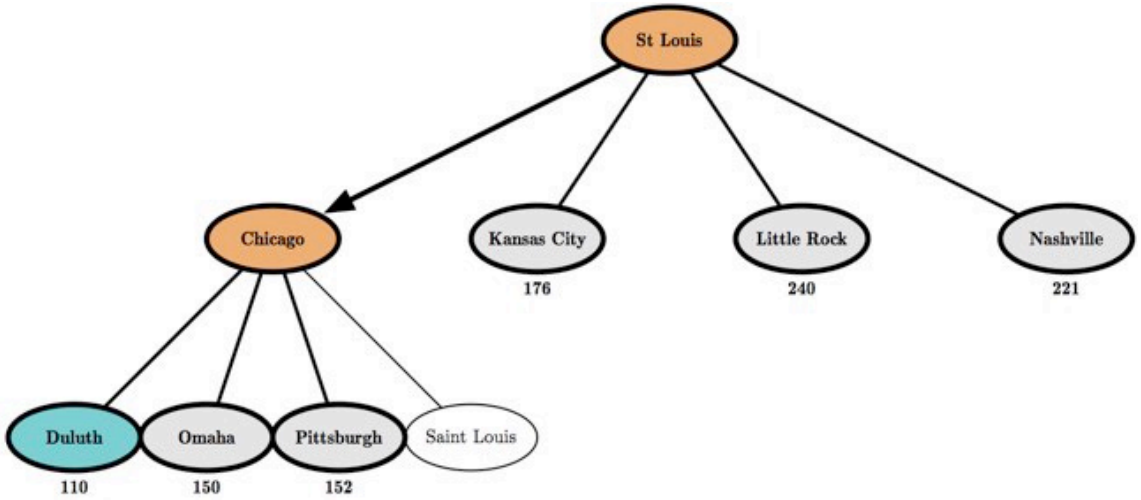
Greedy search example

After expanding Chicago:



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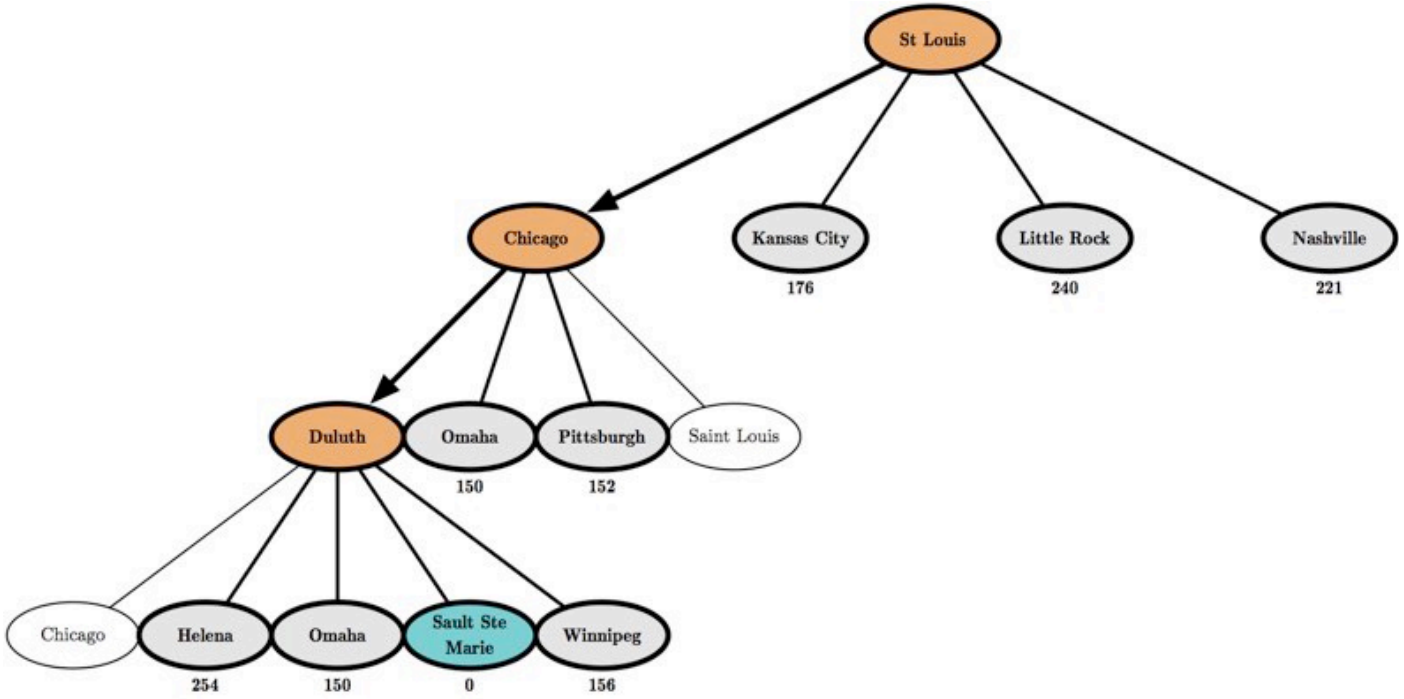
Greedy search example

After expanding Duluth:



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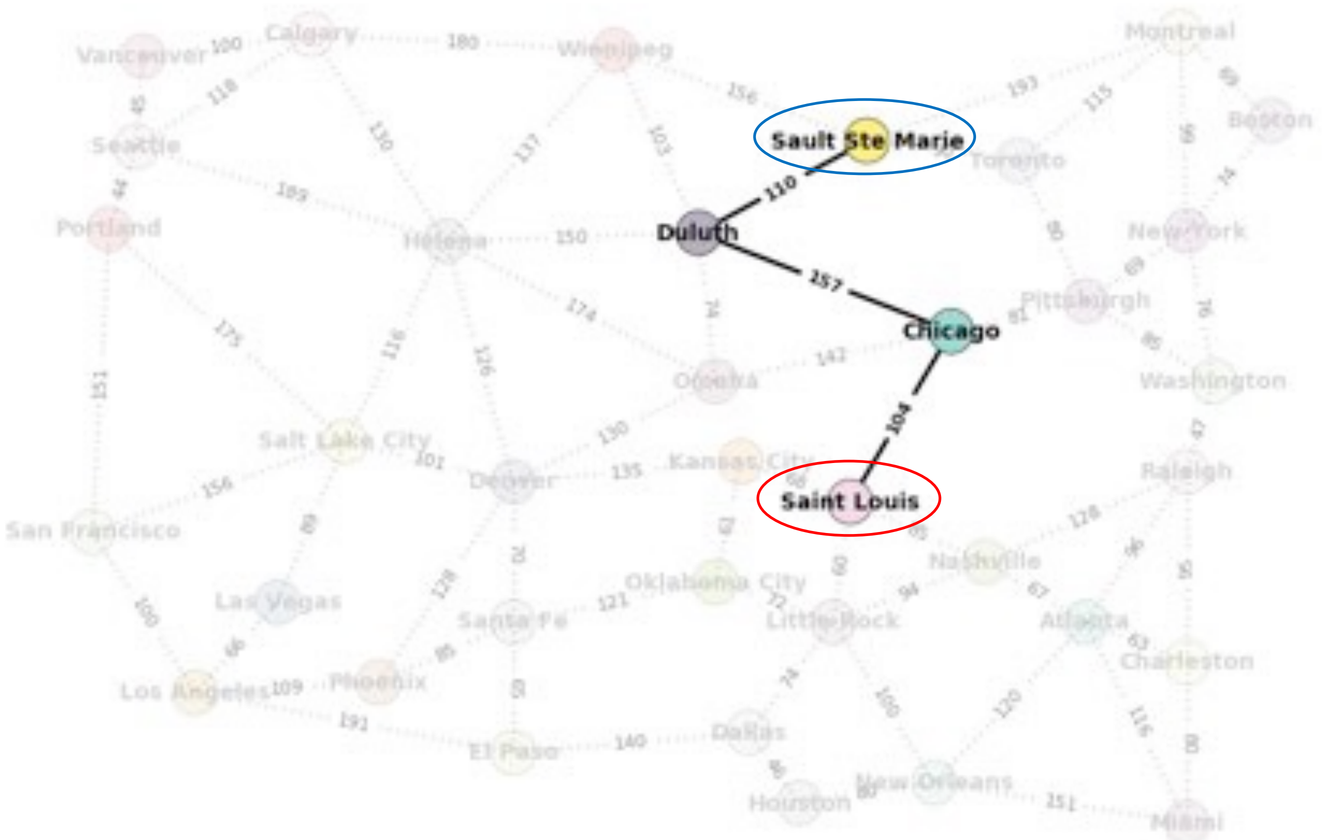
Examples using the map (Greedy search)

Start: Saint Louis

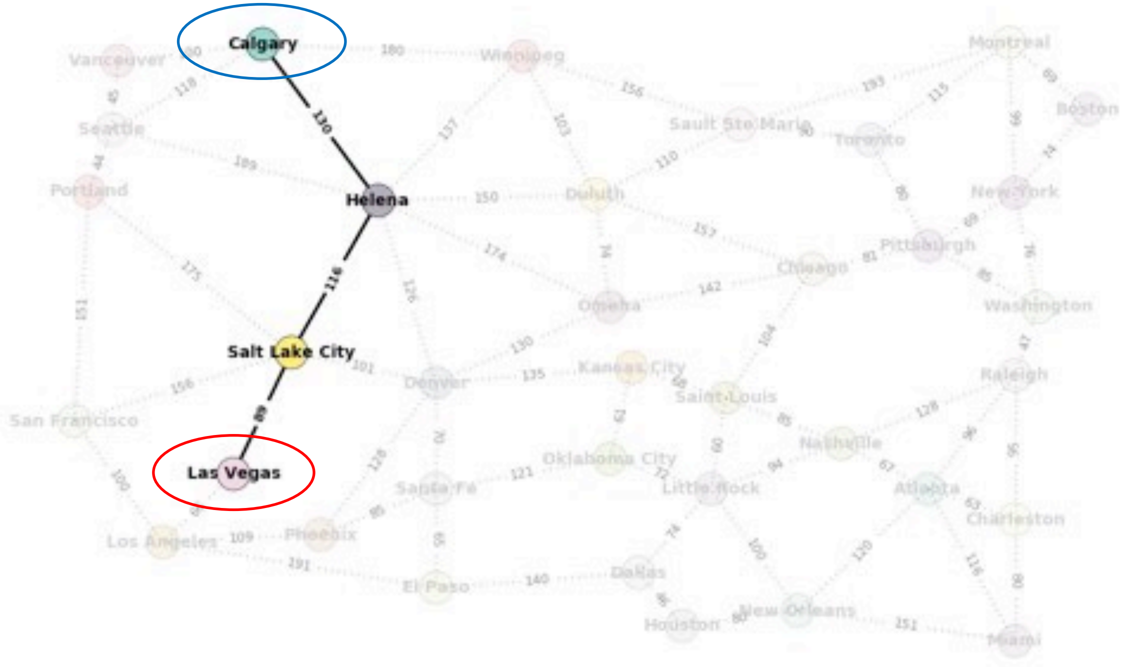
Goal: Sault Ste Marie



Examples using the map (Greedy search)

Start: Las Vegas

Goal: Calgary



A* search

- Minimize the total estimated solution cost
- Combines:
  - $g(n)$: cost to reach node n
  - $h(n)$: cost to get from n to the goal
  - $f(n) = g(n)+h(n)$

$f(n)$ is the estimated cost of the cheapest solution through n

A* search: Pseudo-code

```
function A-STAR-SEARCH(initialState, goalTest)
    returns SUCCESS or FAILURE :  /* Cost f(n) = g(n) + h(n) */

    frontier = Heap.new(initialState)
    explored = Set.new()

    while not frontier.isEmpty():
        state = frontier.deleteMin()
        explored.add(state)

        if goalTest(state):
            return SUCCESS(state)

        for neighbor in state.neighbors():
            if neighbor not in frontier ∪ explored:
                frontier.insert(neighbor)
            else if neighbor in frontier:
                frontier.decreaseKey(neighbor)

    return FAILURE
```

A* search example

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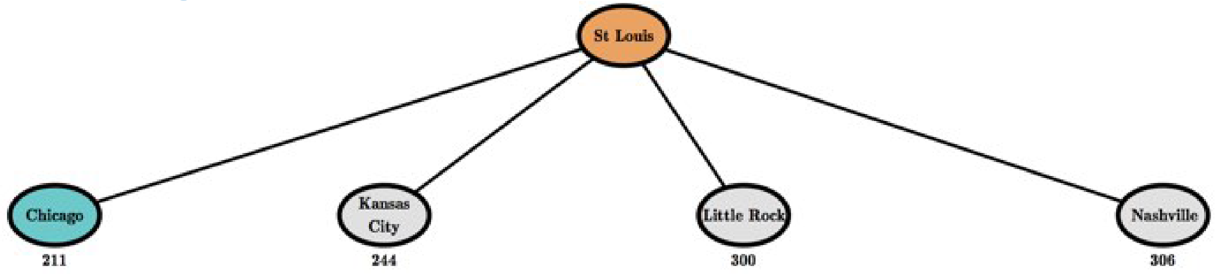
A* search example

After expanding St Louis:



$g(n)$=104
$h(n)$=107
$f(n)$=211

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A* search example

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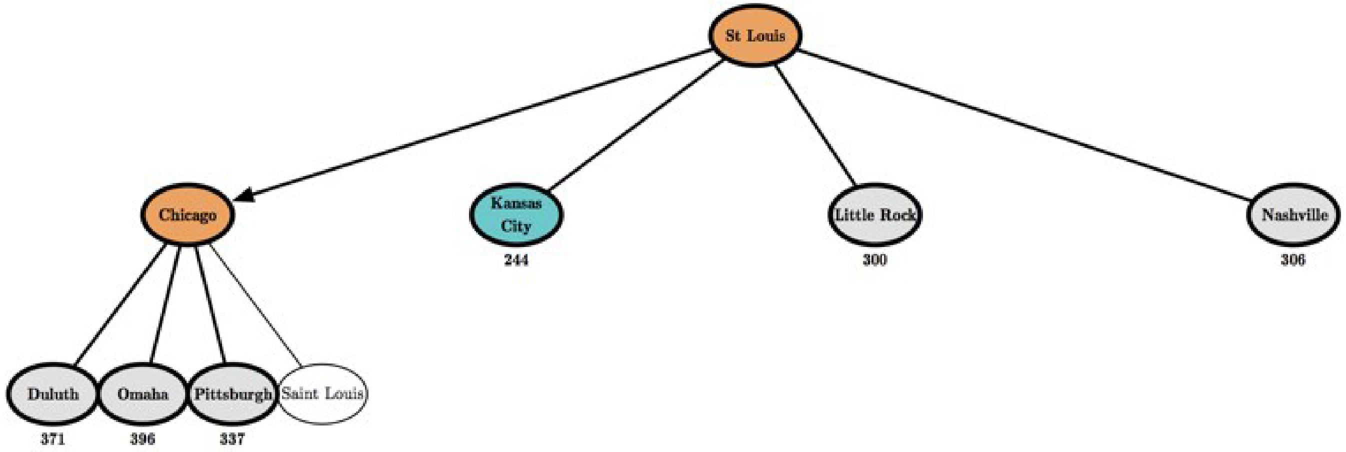

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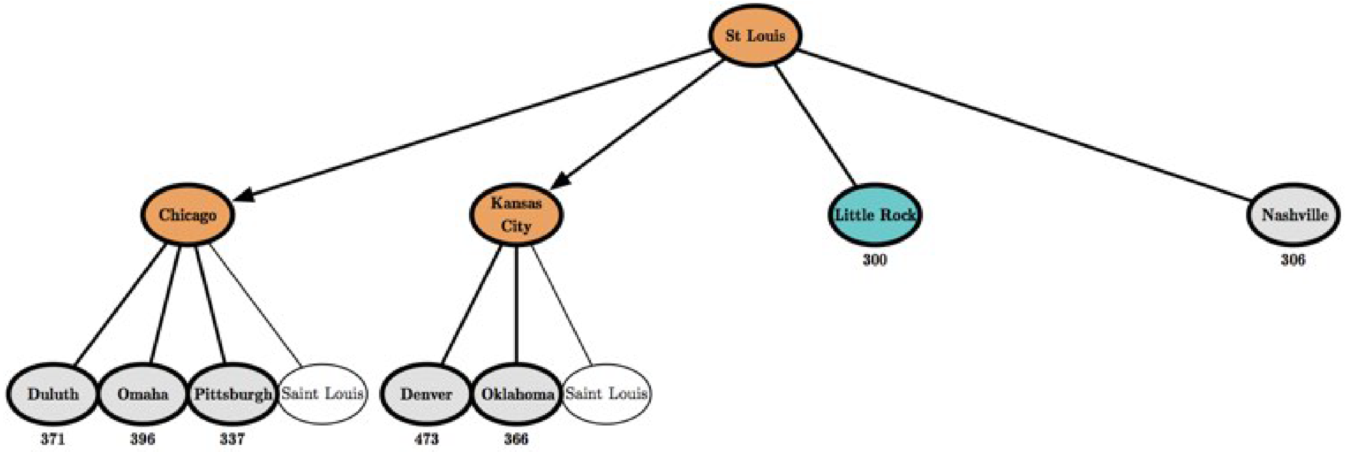
A* search example

After expanding Kansas City:



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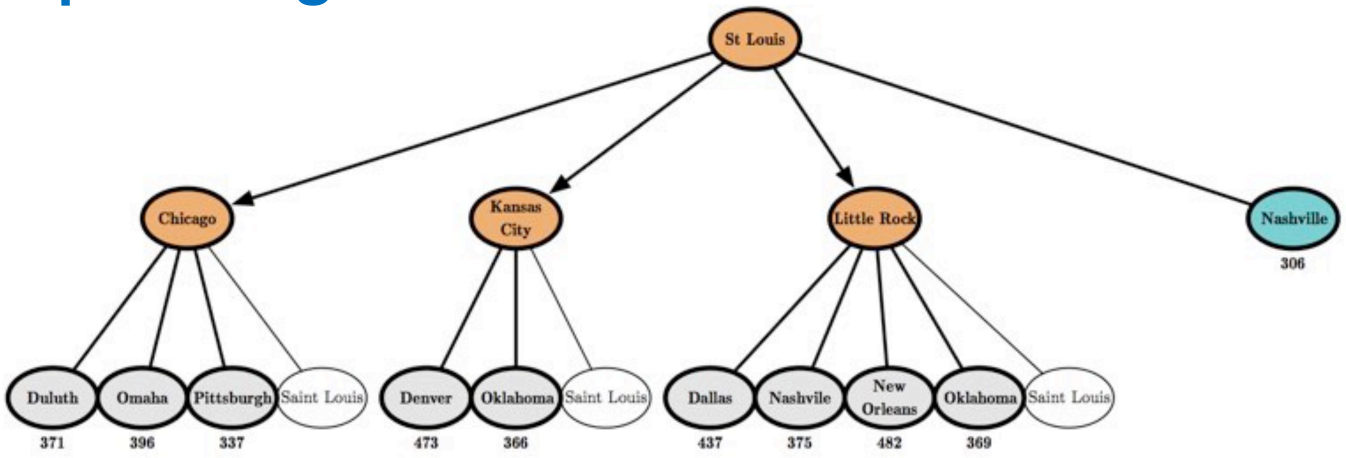
A* search example

After expanding Little Rock:



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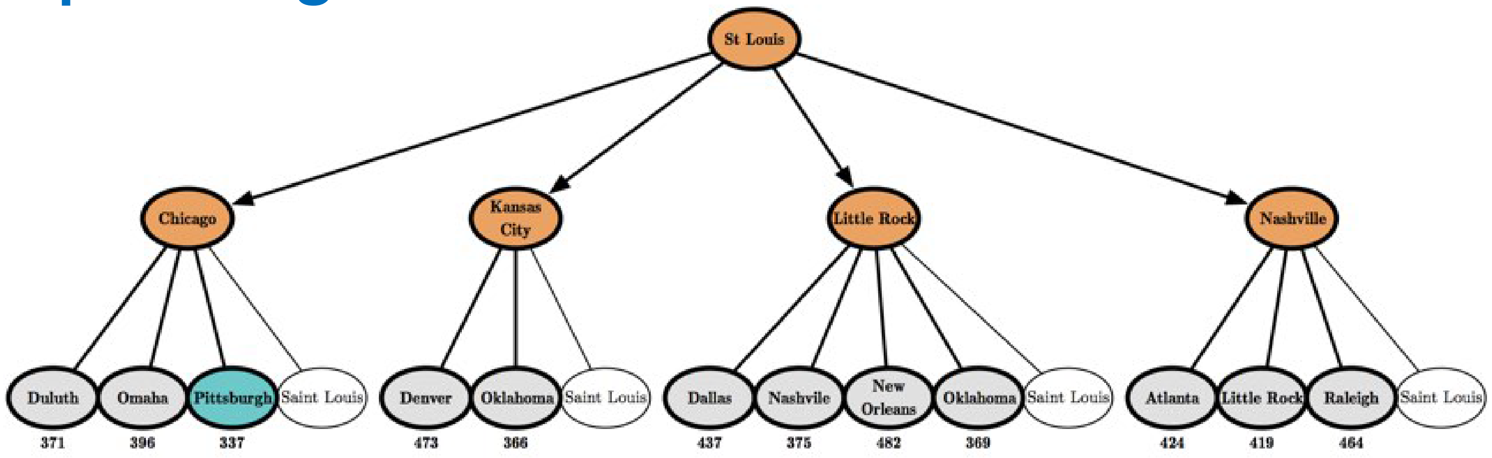
A* search example

After expanding Nashville:



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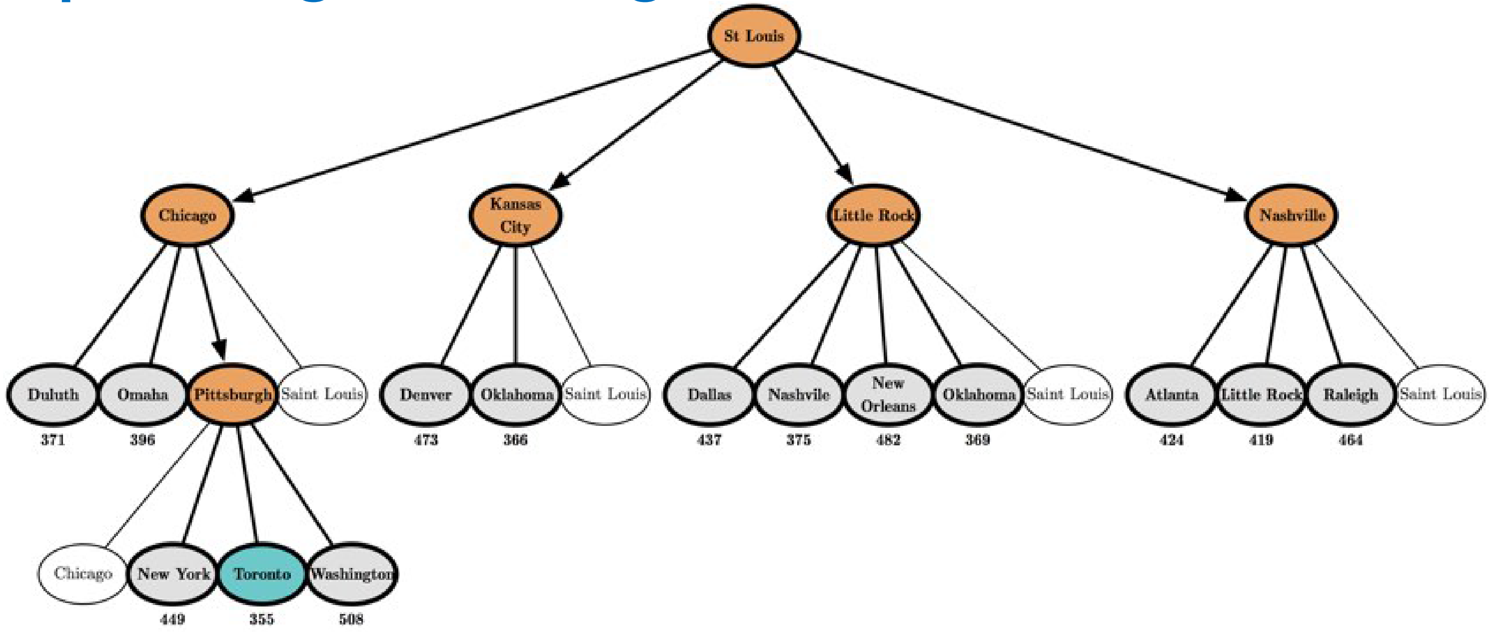
A* search example

After expanding Pittsburgh:



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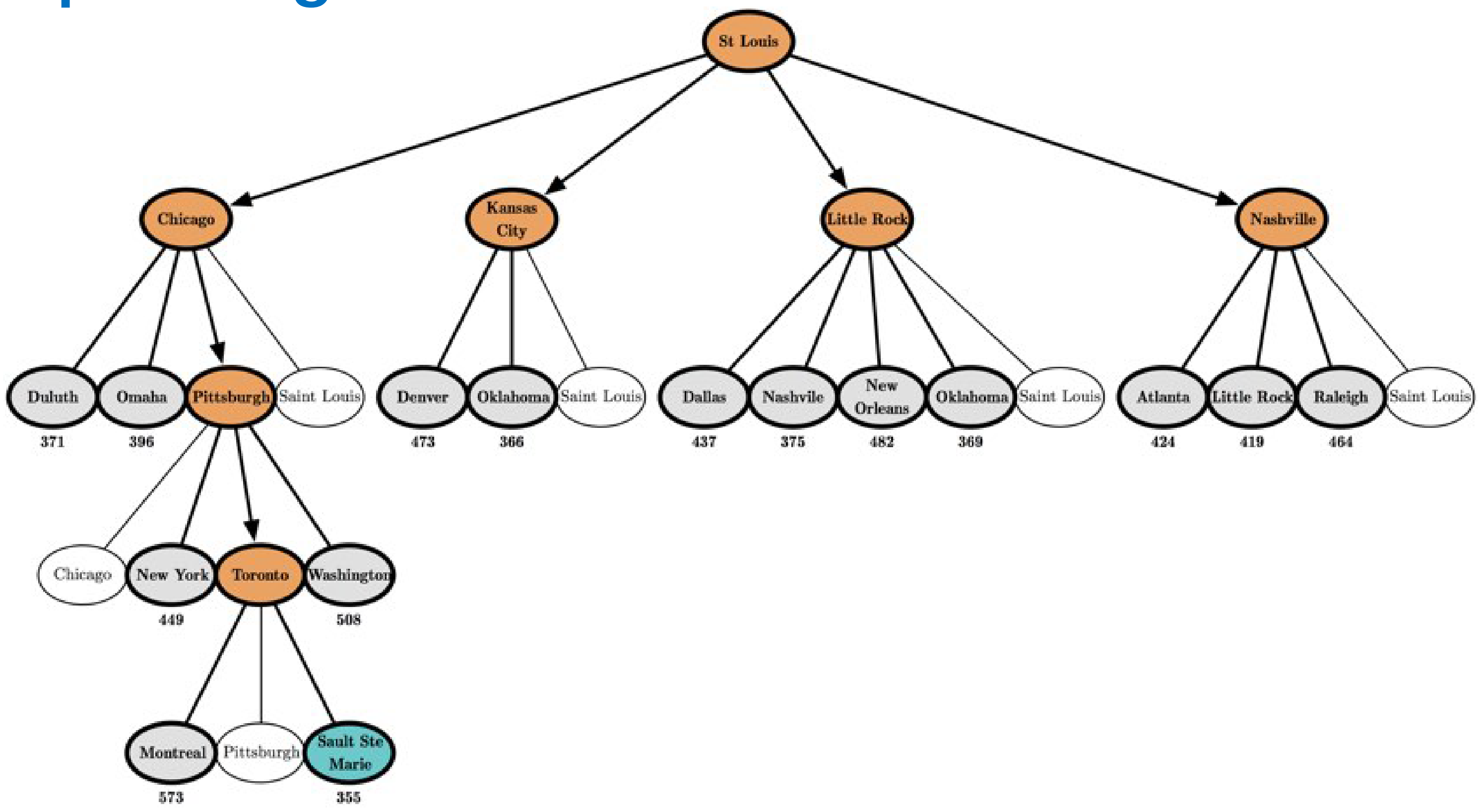
A* search example

After expanding Toronto:



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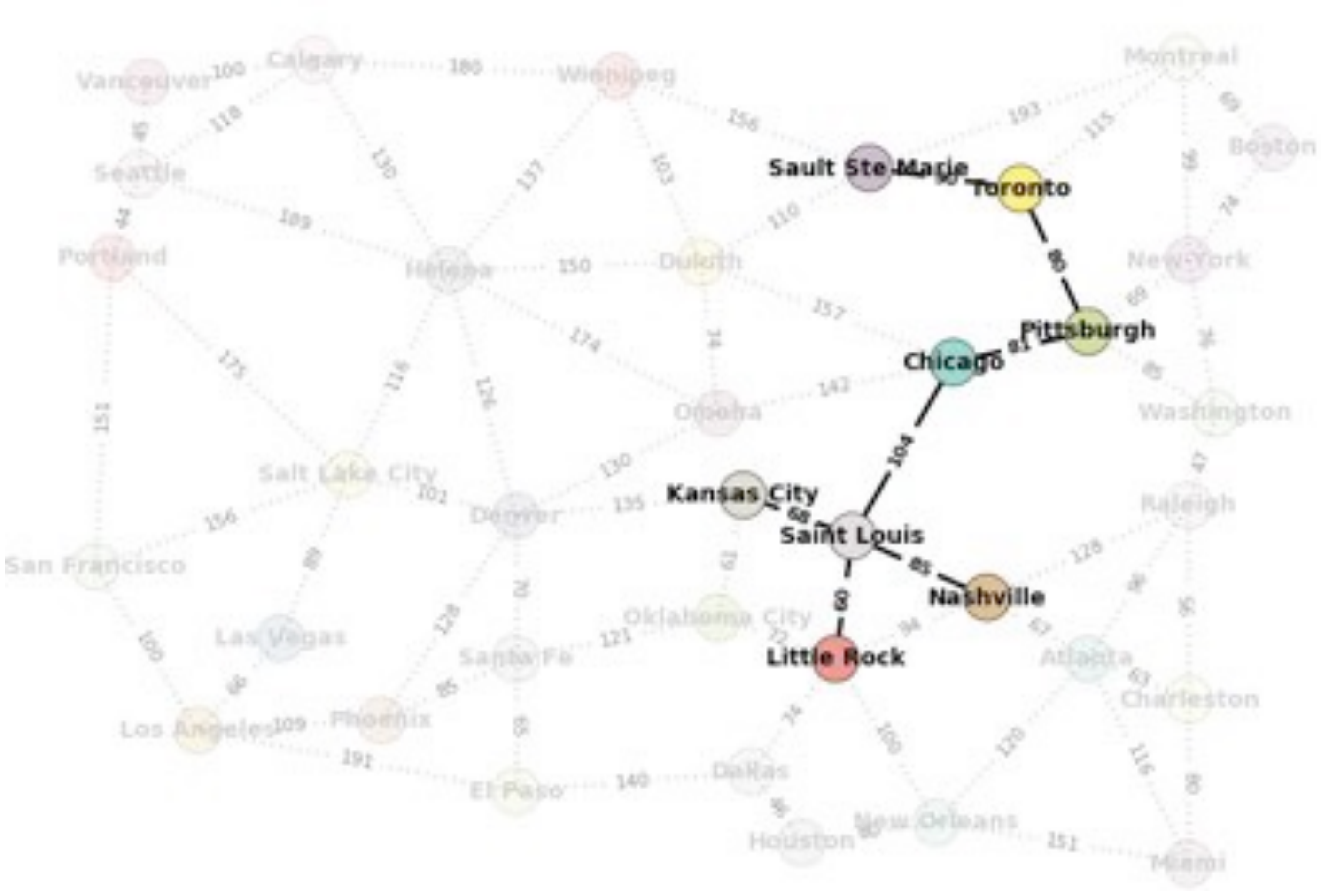
Examples using the map (A* search)

Start: Saint Louis

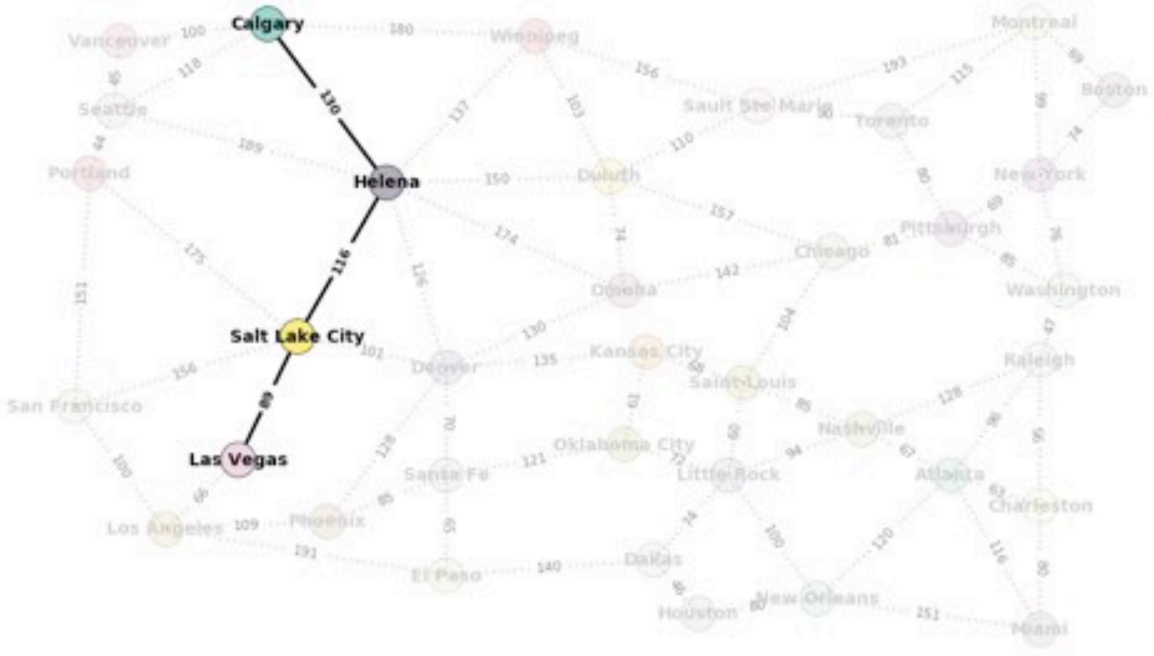
Goal: Sault Ste Marie



Examples using the map (A* search)

Start: Las Vegas

Goal: Calgary



Admissible heuristics

A good heuristic can be powerful.

Only if it is of a "good quality"

A good heuristic must be admissible.

Admissible heuristics

- An **admissible** heuristic never overestimates the cost to reach the goal, that is it is **optimistic**
- A heuristic h is admissible if

$$\forall node\ n,\ \ h(n) \le h^*(n)$$

  where $h*$ is true cost to reach the goal from n.
- h_{SLD} (used as a heuristic in the map example) is admissible because it is by definition the shortest distance (straight line) between two points.

A^* Optimality

If $h(n)$ is admissible, A^* using tree search is optimal.

Rationale:

- Suppose G_o is the optimal goal.
  Suppose G_s is some suboptimal goal.
  Suppose n is on the shortest path to Go.
- $f(G_s) = g(G_s)$ since $h(G_s) = 0$
  $f(G_o) = g(G_o)$ since $h(G_o) = 0$
  $g(G_s) > g(G_o)$ since G_s is suboptimal
  Then $\mathbf{f(G_s) > f(G_o) \ldots (1)}$
- $h(n) \leq h^*(n)$ since h is admissible
  $g(n)+h(n) \leq g(n)+h^*(n) = g(G_o) = f(G_o)$
  Then, $\mathbf{f(n) \leq f(G_o) \ldots (2)}$

From (1) and (2) $f(G_s) > f(n)$, so A^* will never select G_s during the search and hence A^* is optimal.

⬇

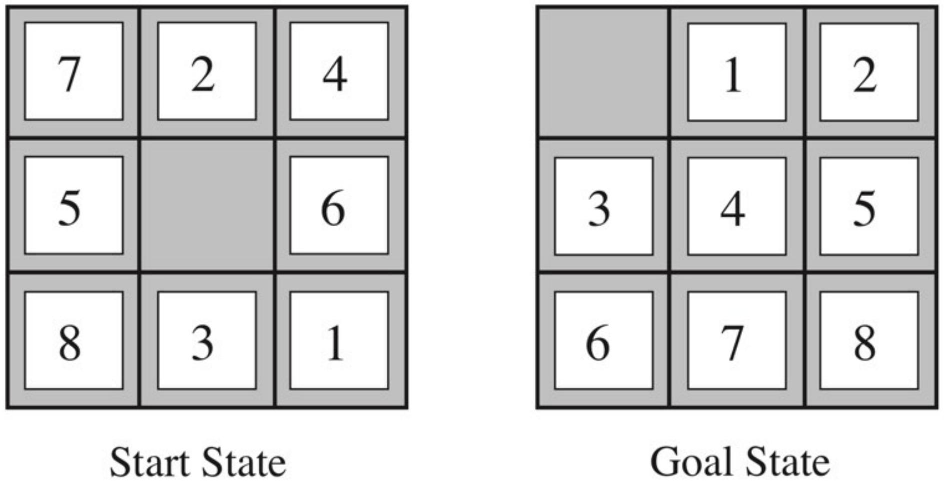
We always will go toward *Go* rather than to go *Gs*.

A* : PF Metrics

- **Complete:** Yes.
- **Time:** exponential
- **Space:** keeps every node in memory, the biggest problem
- **Optimal:** Yes!

Heuristics

- The solution is 26 steps long.
- $h_1(n)$ = number of misplaced tiles
- $h_1(n)$ = 8
- $h_2(n)$ =total Manhattan distance (sum of the horizontal and vertical distances).
- Tiles 1 to 8 in the start state gives: h_2 = 3+1+2+2+2+3+3+2 = 18 which does not overestimate the true solution.



Recap: Search Methods

- **Uniformed search**: Use **no** domain knowledge.
  - BFS, DFS, DLS, IDS, UCS

- **Informed search**: Use a heuristic function that **estimates** how close a state is **to the goal**.
  - Greedy search, A$*$, IDA$*$.

Recap: Search Methods

We can organize the algorithms into pairs where the first proceeds by layers, and the other proceeds by subtrees.

(1) Iterate on Node Depth:

- BFS searches layers of increasing node depth.
- IDS searches subtrees of increasing node depth.

(2) Iterate on Path Cost + Heuristic Function:

- A* searches layers of increasing path cost + heuristic function.
- IDA* searches subtrees of increasing path cost + heuristic function.

Recap: Search Methods

Which cost function?

- UCS searches layers of increasing path cost.
- Greedy best first search searches layers of increasing heuristic function.
- A* search searches layers of increasing path cost + heuristic function.

Local search

- Search algorithms seen so far are designed to **explore search spaces systematically.**
- Problems: observable, deterministic, known environments
- where the solution is a sequence of actions.
- Real-World problems are more complex.
- When a goal is found, the path to that goal constitutes a solution to the problem. But, depending on the applications, **the path may or may not matter**.
- If the path does not matter/systematic search is not possible, then consider another class of algorithms.

Local search

- In such cases, we can use iterative improvement algorithms, **Local search**.
- Also useful in pure **optimization problems** where the goal is to find the best state according to an **optimization function**.
- **Examples:**
  - Integrated circuit design, telecommunications network optimization, etc.
  - 8-queen: what matters is the final configuration of the puzzle, not the intermediary steps to reach it.

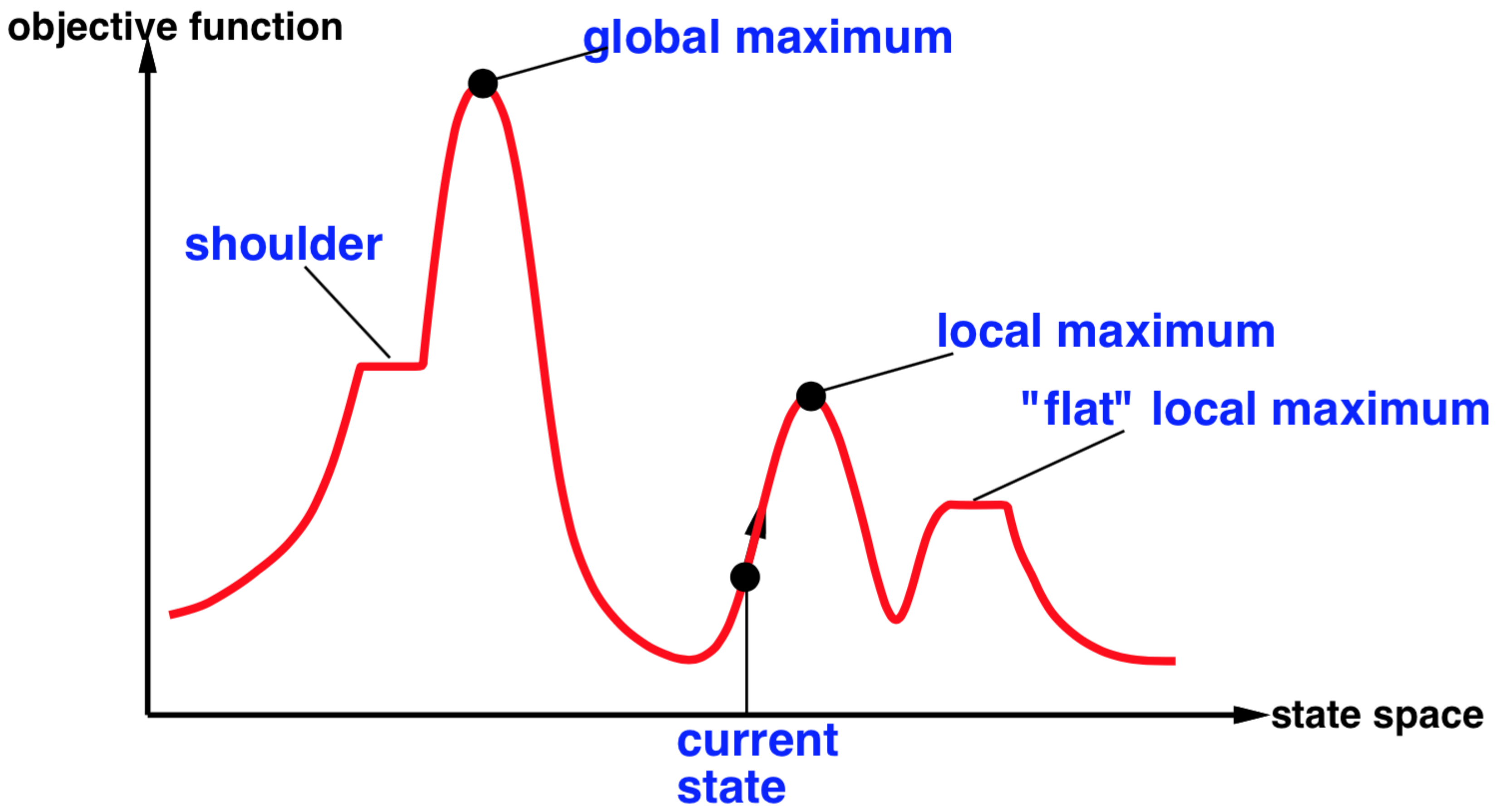
Local search

- **Idea**: keep a single "current" state, and try to improve it.
- Move only to neighbors of that node.
- **Advantages**:
  1. No need to maintain a search tree.
  2. Use very little memory.
  3. Can often find good enough solutions in continuous or large state spaces.

Local search

- **Local Search Algorithms:**
  - Hill climbing (steepest ascent/descent).
  - Simulated Annealing: inspired by statistical physics.
  - Local beam search.
  - Genetic algorithms: inspired by evolutionary biology.

Local search



State space landscape

Hill climbing

- Also called **greedy local search**.
- Looks only to immediate good neighbors and not beyond.
- Search moves uphill: moves in the direction of increasing elevation/value to find the top of the mountain.
- Terminates when it reaches a **peak**.
- Can terminate with a local maximum, global maximum or can get stuck and no progress is possible.
- A **node is a state and a value**.

Hill climbing: Pseudo-code

```
function HILL-CLIMBING(initialState)
    returns State that is a local maximum

    initialize current with initialState

    loop do
        neighbor = a highest-valued successor of current

        if neighbor.value ≤ current.value:
            return current.state

        current = neighbor
```

Hill climbing

Other variants of hill climbing include

- **Sideways moves** escape from a plateau where best successor has same value as the current state.
- **Random-restart** hill climbing overcomes local maxima: keep trying! (either find a goal or get several possible solution and pick the max).
- **Stochastic** hill climbing chooses at random among the uphill moves.

Hill climbing

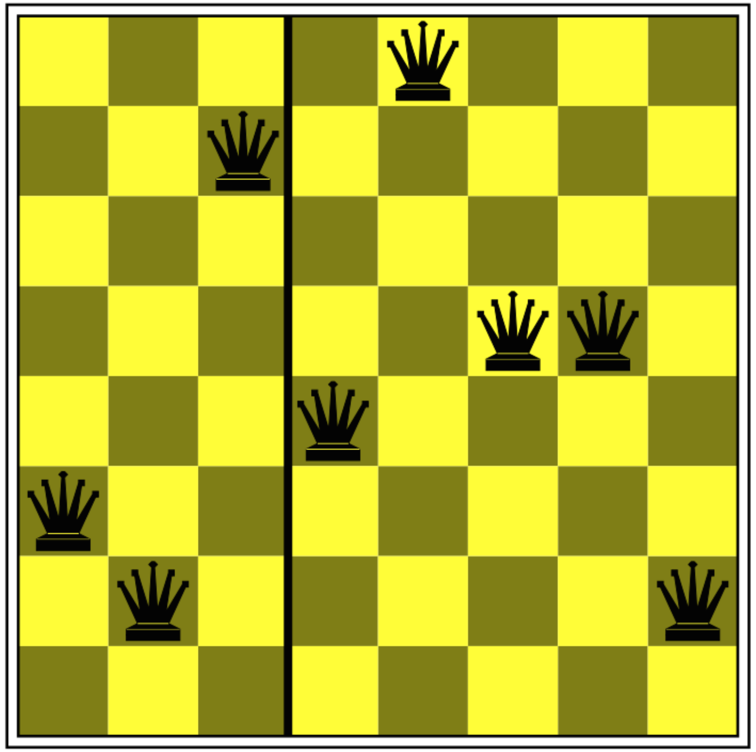
- **Hill climbing** effective in general but depends on shape of the landscape. Successful in many real-problems after a reasonable number of restarts.
- **Local beam search** maintains *k* states instead of one state. Select the *k* best successor, and useful information is passed among the states.
- **Stochastic beam search** choose *k* successors are random. Helps alleviate the problem of the states agglomerating around the same part of the state space.

Genetic algorithms

- **Genetic algorithm (GA)** is a variant of stochastic beam search.
- Successor states are generated by combining two parents rather by modifying a single state.
- The process is inspired by **natural selection**.
- Starts with *k* **randomly generated states**, called **population**. Each state is an **individual**.
- An individual is usually represented by a **string** of 0's and 1's, or digits, a finite set.
- The objective function is called **fitness function**: better states have high values of fitness function.

Genetic algorithms

- In the 8-queen problem, an individual can be represented by a **string** digits 1 to 8, that represents the position of the 8 queens in the 8 columns.



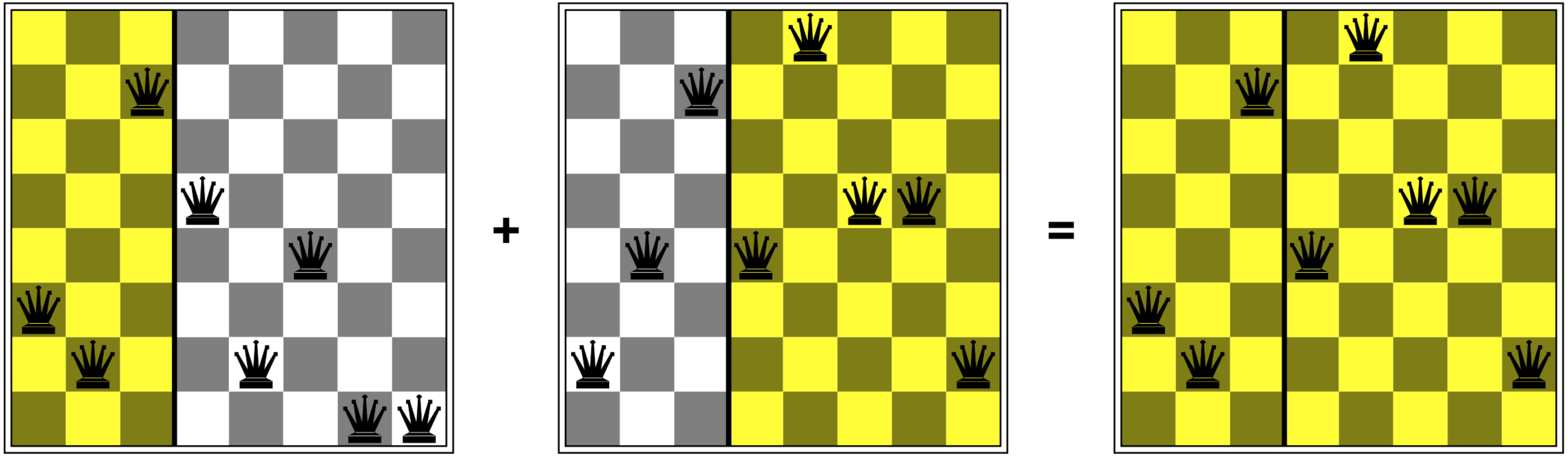
Genetic algorithms

- The objective function is called **fitness function**: better states have high values of fitness function.
- Possible fitness function is the **number of non-attacking pairs of queens.**
- Fitness function of the solution: 28.

Genetic algorithms

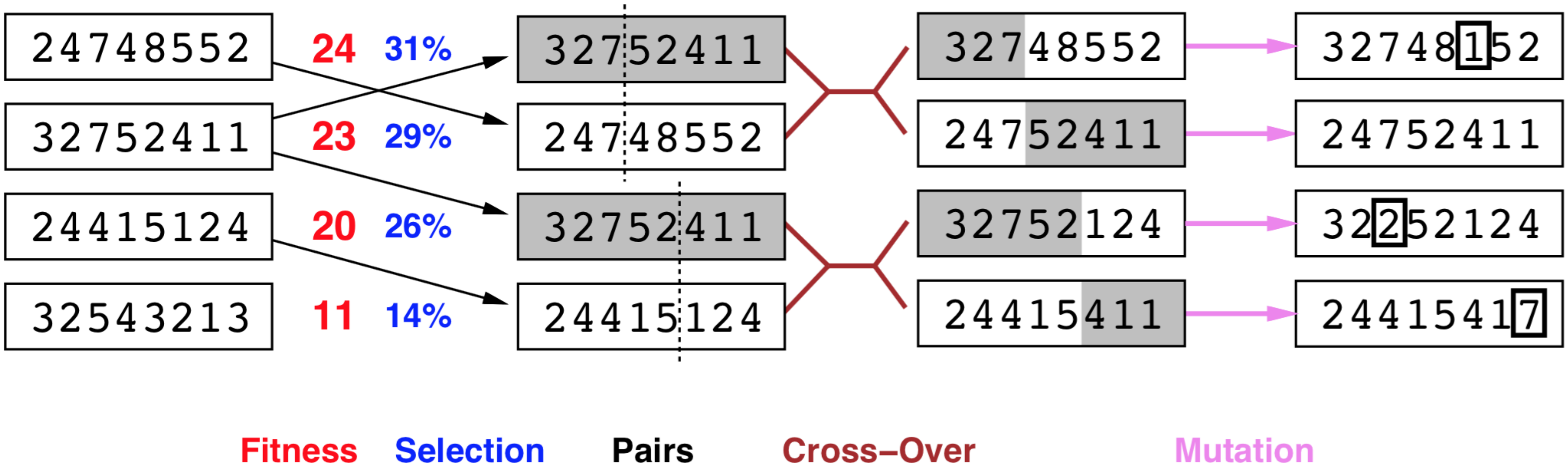
- Pairs of individuals are selected at random for **reproduction** w.r.t. some probabilities.
- A **crossover** point is chosen randomly in the string.
- **Offspring** are created by crossing the parents at the crossover point.
- Each element in the string is also subject to some **mutation** with a small probability.

Genetic algorithms



Genetic algorithms

Generate successors from pairs of states.



Genetic algorithms: Pseudo-code

```
function GENETIC-ALGORITHM(population, fitness-function)
    returns an individual

    repeat

        initialize new-population with ∅

        for i=1 to size(population) do

            x = random-select(population,fitness-function)
            x = random-select(population,fitness-function)
            child = cross-over(x,y)
            mutate (child) with a small random probability
            add child to new-population

        population = new-population

    until some individual is fit enough or enough time has elapsed

    return the best individual in population w.r.t. fitness-function
```

To be continued

Simulated Annealing

- Hill Climbing → Simulated Annealing

function HILL-CLIMBING(*problem*) **returns** a state that is a local maximum
  current ← *problem*.INITIAL
  while *true* **do**
    neighbor ← a highest-valued successor state of *current*
    if VALUE(*neighbor*) ≤ VALUE(*current*) **then return** *current*
    current ← *neighbor*

Simulated Annealing

function SIMULATED-ANNEALING($problem$, $schedule$) **returns** a solution state
    $current \leftarrow problem$.INITIAL
    for t = 1 **to** ∞ **do**
        $T \leftarrow schedule(t)$
        if T = 0 **then return** $current$
        $next \leftarrow$ a randomly selected successor of $current$
        $\Delta E \leftarrow$ VALUE($current$) – VALUE($next$)
        if $\Delta E < 0$ **then** $current \leftarrow next$ //for maximization
        else $current \leftarrow next$ only with probability $e^{-\Delta E/T}$