Logic Q&A

YAO ZHAO

Question for Propositional Logic

- The best combination of the six important players in the volleyball team (1, 3, 4, 6, 9, 12) should follow the following rules:
  - Players 4 and 6 need to play together.
  - Player 3 does not play the game if and only if player 1 does not play
  - Either player 3 or player 6 should appear on the court, and they can not appear at the same time.
  - If players 9 and 12 are on the court, then player 4 must be on the court.

  If players 1 and 12 need to play at the same time in one game, who of the other 4 players should play?

  (1) Please use propositional logic to represent the above statement.

  (2) Convert them into a CNF.

  (3) Apply the DPLL algorithm to derive which players should be on the court.

Answer

(1) Propositional logical expression:

$$((\mathrm{P}_4 \leftrightarrow P_6)) \wedge (\sim \mathrm{P}_3 \leftrightarrow \sim P_1) \wedge ((\sim P_3 \wedge P_6) \vee (P_3 \wedge \sim P_6)) \wedge ((P_9 \wedge P_{12}) \rightarrow P_4) \wedge (P_1 \wedge P_{12})$$

(2) Convert to CNF:

$$(\sim \mathrm{P}_4 \vee P_6) \wedge ((\mathrm{P}_4 \vee \sim P_6) \wedge (\mathrm{P}_3 \vee \sim P_1) \wedge (\sim \mathrm{P}_3 \vee P_1) \wedge (\sim P_3 \vee \sim P_6) \wedge (P_3 \vee P_6) \wedge (\sim P_9 \vee \sim P_{12} \vee P_4) \wedge P_1 \wedge P_{12}$$

(3) Apply DPLL:

1. Split to Clause:

$$(\sim P_4 \vee P_6)\ ((P_4 \vee \sim P_6)\ (P_3 \vee \sim P_1)\ (\sim P_3 \vee P_1)\ (\sim P_3 \vee \sim P_6)\ (P_3 \vee P_6)\ (\sim P_9 \vee \sim P_{12} \vee P_4)\ \ P_1\ \ P_{12}$$

2. Find pure symbol: $\sim P_9$

Model:{P_9:False}

Find Unit Clause:$P_1\ \ P_{12}$

Model:{P_1:True; P_{12}:True;}

3.Unknown Clause: $(\sim P_4 \vee P_6)\ ((P_4 \vee \sim P_6)\ (P_3 \vee \sim P_1)\ \cancel{(\sim P_3 \vee P_1)}\ (\sim P_3 \vee \sim P_6)\ (P_3 \vee P_6)\ (\sim P_9 \vee \sim P_{12} \vee P_4)$

Unit Clause:P_3

Model:{P_1:True; P_{12}:True; P_3: True}

4. Unknown Clause:

$$(\sim P_4 \vee P_6)\ ((P_4 \vee \sim P_6)\ \cancel{(P_3 \vee \sim P_1)}\ \cancel{(\sim P_3 \vee P_1)}\ (\sim P_3 \vee \sim P_6)\ \cancel{(P_3 \vee P_6)}\ (\sim P_9 \vee \sim P_{12} \vee P_4)$$

Unit Clause: $\sim P_6$

Model:{P_1:True; P_{12}:True; P_3: True; P_6: False}

5. Unknown Clause :

$$(\sim P_4 \vee P_6)\ \cancel{(P_4 \vee \sim P_6)}\ \cancel{(P_3 \vee \sim P_1)}\ \cancel{(\sim P_3 \vee P_1)}\ \cancel{(\sim P_3 \vee \sim P_6)}\ \cancel{(P_3 \vee P_6)}\ (\sim P_9 \vee \sim P_{12} \vee P_4)$$

Unit Clause:$\sim P_4$

Model:{P_1:True; P_{12}:True; P_3: True; P_6: False; P_4: False}

6. Unknown Clause :

$$\cancel{(\sim P_4 \vee P_6)}\ \cancel{(P_4 \vee \sim P_6)}\ \cancel{(P_3 \vee \sim P_1)}\ \cancel{(\sim P_3 \vee P_1)}\ \cancel{(\sim P_3 \vee \sim P_6)}\ \cancel{(P_3 \vee P_6)}\ (\sim P_9 \vee \sim P_{12} \vee P_4)$$

Unit Clause: $\sim P_9$

Model:{P_1:True; P_{12}:True; P_3: True; P_6: False; P_4: False; P_9: False}

7. Unknown Clause :

Question First-Order Logic

- R1: Anyone who passes the AI exam and wins a prize is happy.
- R2: Anyone willing to learn or lucky can pass all the exams.
- R3: Any lucky person can win a prize.
- R4: Zhansan is not willing to learn but he is lucky.

Please use First-Order logic to represent the above statements.

Answer

First-Order Logic Expression:

R1: Anyone who passes the AI exam and wins a prize is happy.

$$(\forall x)\left((Pass(x, AIExam)) \wedge Win(x, prize)\right) \rightarrow Happy(x)$$

R2: Anyone willing to learn or lucky can pass all the exams.

$$(\forall x)(\forall y)(Study(x) \vee Lucky(x) \rightarrow Pass(x, y))$$

R3: Any lucky person can win a prize.

$$(\forall x)(Lucky(x) \rightarrow Win(x, prize))$$

R4: Zhansan is not willing to learn but he is lucky.

$$\sim Study(ZhangSan) \wedge Lucky(ZhangSan)$$