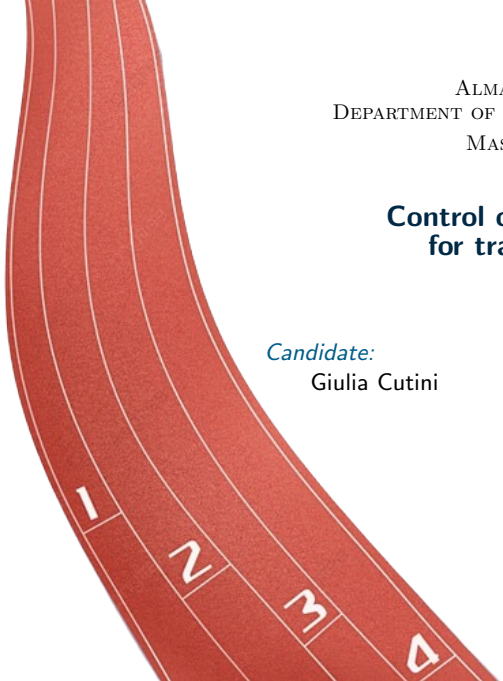

Alma Mater Studiorum Università di Bologna
Department of Electrical, Electronic and Information Engineering
Master's Degree in Automation Engineering

Control of an autonomous aerodynamic airshield for training Olympics 100m sprint athletes

Candidate:
Giulia Cutini

Advisor:
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Co-Advisors:
Prof. Melanie Zeilinger
Dr. Andrea Carron
Ing. Lorenzo Sforni

Bologna, 18 March 2024

ETH Zürich

Introduction

Motivations

In athletics, the **overspeed training** makes possible to enhance the competition performances.
It can be achieved by isolating the runner from the air resistance using an **airshield**.

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- the reliability and the reusability

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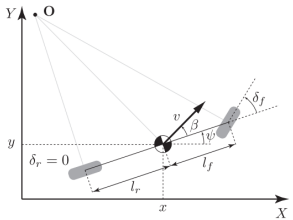
- the safety of the maneuver
- the reliability and the reusability

Contributions

- Model of the system: the autonomous go-kart with the airshield attached
- Design a controller for the system to regulate it with respect to the runner
- Test the controller

Modeling of the go-kart with the aishield attached



Non linear kinematic byclicle model

$$\begin{cases} \dot{x}(t) = v(t)\cos(\psi(t) + \beta(t)) \\ \dot{y}(t) = v(t)\sin(\psi(t) + \beta(t)) \\ \dot{\psi}(t) = \dfrac{v(t)}{l_r}\sin(\beta(t)) \\ \dot{v}(t) = \dfrac{F_x(t)}{m} = \dfrac{1}{m}(C_{m1}a(t) - C_f v(t) - C_d v^2(t) - C_{roll}) \end{cases}$$

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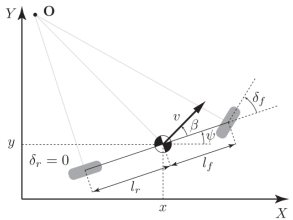

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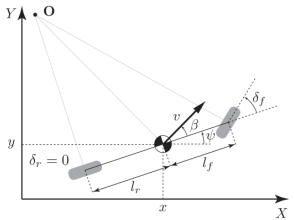

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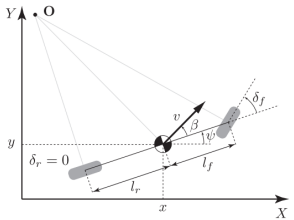

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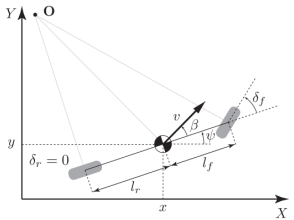

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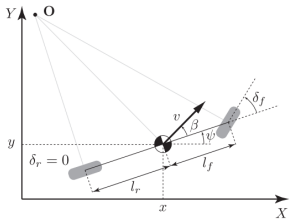

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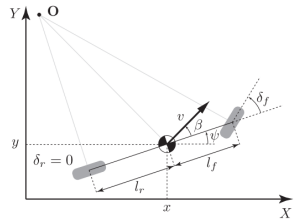

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Linear time-invariant system

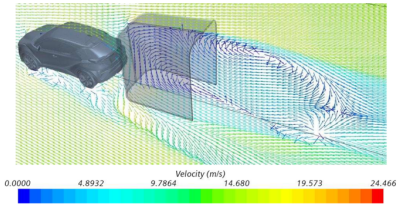
$$x_{k,t+1} = \begin{bmatrix} p_{k,t+1} \\ v_{k,t+1} \end{bmatrix} = \begin{bmatrix} 1 & dt \\ 0 & 1 - dt\frac{C_f}{m} \end{bmatrix} x_{k,t} + \begin{bmatrix} 0 \\ dt\frac{C_{m1}}{m} \end{bmatrix} u_t$$
$$= A\, x_{k,t} + B\, u_t$$

Application setup

Aerodynamics studies [1] have shown that, by running in the slipstream of a shield:

- Runner results to be isolated from the drag resistance
- A pushing force from behind enhancing the speed
- Reference distance: $d_{\text{des}} = 2.5$ meters

[1] Italian National Olympic Committee (CONI) Institute of Sports Science "Aerodynamic Shield – new training support technologies"



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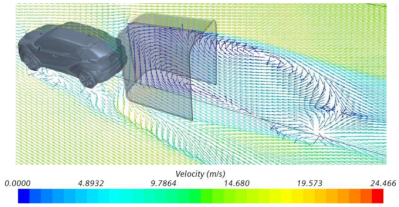

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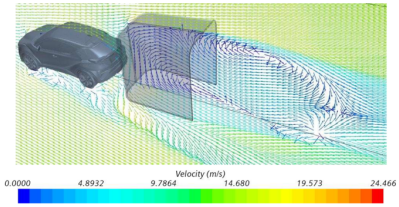

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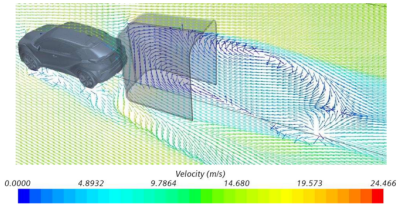

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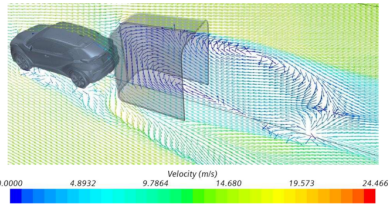

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Wheel Encoders
Absolute kart velocity v_k

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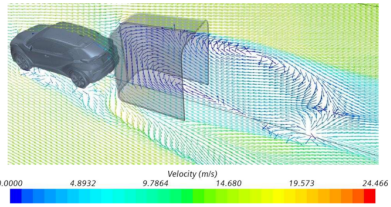

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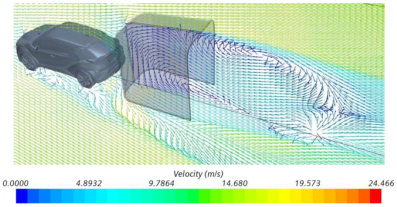

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Estimated:

- Relative velocity $\Delta v = v_k - v_r$
- Absolute runner acceleration a_r

Gain Scheduling Linear Quadratic Regulator

$$h_t = \begin{bmatrix} \Delta p_t \\ \Delta v_t \end{bmatrix} = \begin{bmatrix} p_{k,t} - p_{r,t} \\ v_{k,t} - v_{r,t} \end{bmatrix}$$

$$h_{\text{des}} = \begin{bmatrix} d_{\text{des}} \\ 0 \end{bmatrix}$$

- Compute the Linear Quadratic Regulator: K
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Catch-up maneuver K_{catch}

Cruise control maneuver K_{cruise}

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Catch-up maneuver K_{catch}

In the first couple of seconds the runner has an acceleration exceeding the maximum go-kart capabilities.

- Ask the runner to start the sprint at $\Delta p_{\text{init}} = d_{\text{des}} + \bar{d}$
- Control using $u^* = -K_{\text{catch}}(h_t - h_{\text{des}})$
- Match the velocities with a safe distance bigger than d_{des}

Cruise control maneuver K_{cruise}

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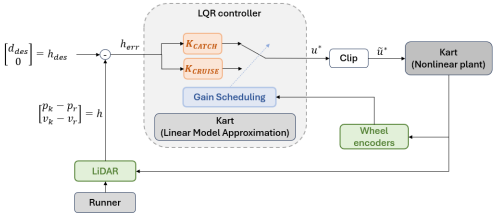
Cruise control maneuver K_{cruise}

Once the runner and go-kart velocities are matched at 80%

- Reduce the distance up to the reference d_{des}
- Control using $u^* = -K_{\text{cruise}}(h_t - h_{\text{des}})$
- Maintain the desired reference distance
- Maintain to zero the relative velocity

Gain Scheduling Linear Quadratic Regulator



Algorithm 1 LQR implementation on hardware system

catched = false;
Set initial runner position with a proper distance $\bar{d}$ from kart;
Define desired system behaviour $h_{\text{des}} = [d_{\text{des}}, 0]^T$;
for $t = 0, 1, 2, \ldots$ **do**;
    Take LiDAR information Δp_t and estimate Δv_t;
    $h_t = [\Delta p_t, \Delta v_t]^T$
    Take absolute kart velocity $v_{k,t}$ from wheel encoders;
    Evaluate absolute runner velocity $v_{r,t} = v_{k,t} - \Delta v_t$;
    if $v_{k,t} \geq 0.8\, v_{r,t}$ **and** catched = false **then**
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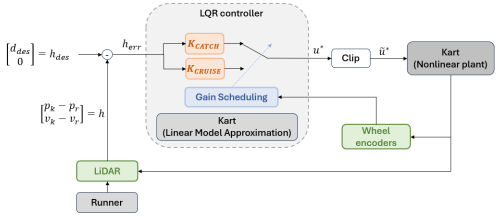

Pros LQR controller

- Reduced computational effort required

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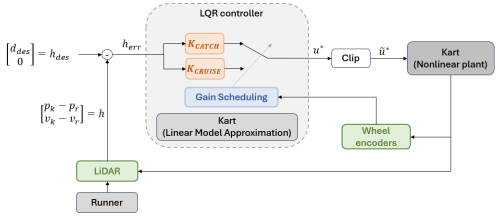

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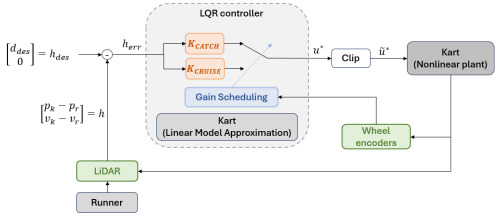

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- Impossibility of directly including constraints
- Clip block required to respect actuator limits $a_{\min} \leq u \leq a_{\max}$

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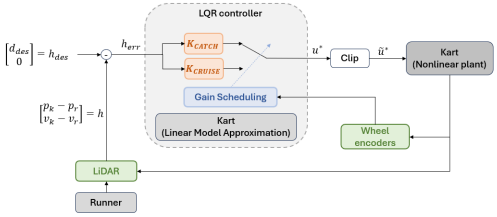
for $t = 0, 1, 2, \ldots$ **do**;
    Take LiDAR information Δp_t and estimate Δv_t;
    $h_t = [\Delta p_t, \Delta v_t]^T$
    Take absolute kart velocity $v_{k,t}$ from wheel encoders;
    Evaluate absolute runner velocity $v_{r,t} = v_{k,t} - \Delta v_t$;
    if $v_{k,t} \geq 0.8\, v_{r,t}$ **and** catched = false **then**
        catched = true;
    end if
    if catched = false **then**
        $u^* = -K_{\text{catch}}(h_t - h_{\text{des}})$;
    else
        $u^* = -K_{\text{cruise}}(h_t - h_{\text{des}})$;
    end if
    Clip input according to limit saturation limits $\tilde{u}^* = \text{sat}_{[a_{\min}, a_{\max}]}(u^*)$;
    Inject $\tilde{u}^*$ in the plant;
end for

Gain Scheduling Linear Quadratic Regulator



Pros LQR controller

- Reduced computational effort required

Cons of LQR controller

- Impossibility of directly including constraints
- Clip block required to respect actuator limits $a_{\min} \leq u \leq a_{\max}$
- Impossibility of including state constraint (i.e. $\Delta p \leq d_{\text{safe}}$)

Algorithm 1 LQR implementation on hardware system

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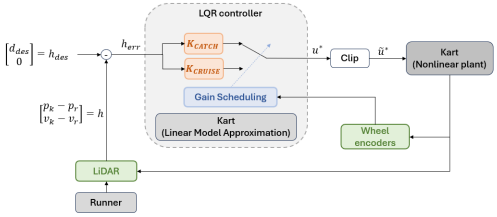
Gain Scheduling Linear Quadratic Regulator



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- Clip block required to respect actuator limits $a_{\min} \leq u \leq a_{\max}$
- Impossibility of including state constraint (i.e. $\Delta p \leq d_{\text{safe}}$)
- Maintain to zero the relative velocity

Algorithm 1 LQR implementation on hardware system

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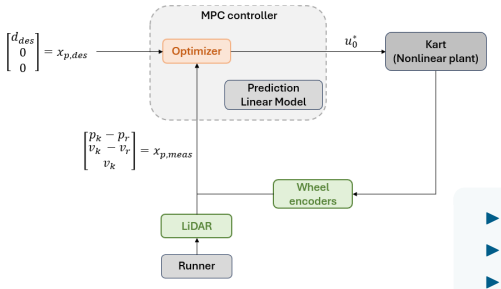
Model Predictive Control

$$\begin{bmatrix}\Delta p_{t+1}\\ \Delta v_{t+1}\\ v_{k,t+1}\end{bmatrix} = \begin{bmatrix}1 & dt & 0\\ 0 & 1 & -dt\frac{C_f}{m}\\ 0 & 0 & 1-dt\frac{C_f}{m}\end{bmatrix} x_{p,t} + \begin{bmatrix}0\\ dt\frac{C_{m1}}{m}\\ dt\frac{C_{m1}}{m}\end{bmatrix} u_t + \begin{bmatrix}0\\ -dta_{r,t}\\ 0\end{bmatrix}$$
$$= A_p\, x_{p,t} + B_p\, u_t + a_t$$



$$\min_{\boldsymbol{x},\boldsymbol{u}} \quad \sum_{i=t}^{t+N-1} \|(x_{p,i|t} - x_{p,\text{des}})\|_Q + \|u_{i|t}\|_R$$

subj. to
$$x_{p,i+1|t} = A_p\, x_{p,i|t} + B_p\, u_{i|t} + a_t$$
$$a_{\min} \le u_{i|t} \le a_{\max}$$
$$d_{\text{safe}} \le \Delta p_{i|t}$$
$$x_{p,t|t} = x_{p,\text{meas}}$$

- Possibility of explicitly including constraints in the optimization
- Possibility of having safety guarantees
- Possibility of including a runner model in the prediction model

Model Predictive Control

$$\begin{bmatrix}\Delta p_{t+1}\\ \Delta v_{t+1}\\ v_{k,t+1}\end{bmatrix} = \begin{bmatrix}1 & dt & 0\\ 0 & 1 & -dt\frac{C_f}{m}\\ 0 & 0 & 1-dt\frac{C_f}{m}\end{bmatrix} x_{p,t} + \begin{bmatrix}0\\ dt\frac{C_{m1}}{m}\\ dt\frac{C_{m1}}{m}\end{bmatrix} u_t + \begin{bmatrix}0\\ -dta_{r,t}\\ 0\end{bmatrix}$$
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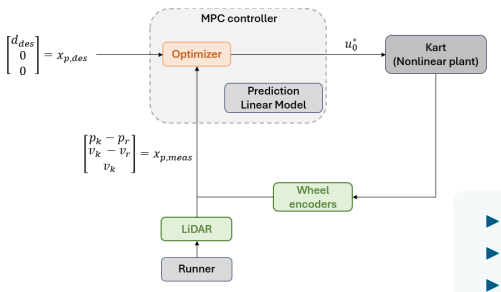
- The prediction model is augmented with the kart velocity state



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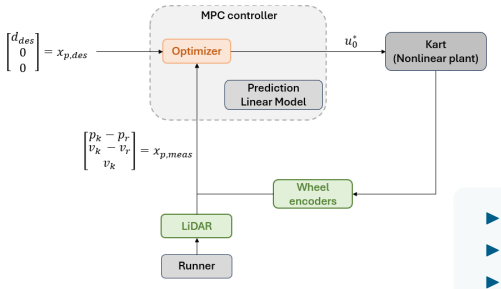
$$= A_p\, x_{p,t} + B_p\, u_t + a_t$$

- The prediction model is augmented with the kart velocity state
- Runner acceleration considered in the predictions



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Offset-free Model Predictive Control

Cio

Simulation tests and Controllers comparison

Hardware-in-the-loop tests

Focus on the catch-up maneuver

Hardware-in-the-loop tests

Focus on maintaining a constant distance with the runner having an almost constant velocity