Applications in Scientific Computing
Assignment 7: Inital-value problems

530.390.13

Due: Wednesday 20 January 2016

Submit all code by committing it to the directory `assignments/assignment5` in your `530.390.13` GitHub repository. For a reminder of how to use Git, refer to the repository file `notes/using-git`.

1. Couette flow, which determines the velocity field of fluid flow between two moving plates, is given by

$$\frac{\partial u}{\partial t} = \mu \frac{\partial^2 u}{\partial y^2}$$

   where $u(t, y)$ is the fluid velocity, and μ is the viscosity. The motion of the walls at velocity $\pm U$ drive the fluid flow: $u(t > 0, -L/2) = -U$ and $u(t > 0, L/2) = U$. Determine the expected steady-state solution (i.e., $\frac{\partial u}{\partial t} = 0$) for Couette flow between walls separated by width L.

2. Numerically solve the time-dependent Couette flow equation above using $L = 1$, $U = 1$, and $\mu = 1$. Using initial condition $u(0, y) = 0$, simulate the first $T = 0.1$ and plot the solution at $t = \{0, T/1, T/2, T/4, T/8, T\}$. Also plot your solution to verify the numerical solution for long time.