

Applications in Scientific Computing

Assignment 4: Fast Fourier Transforms

530.390.13

Due: Wednesday 13 January 2016

Submit all code by committing it to the directory `assignments/assignment3` in your `530.390.13` GitHub repository. For a reminder of how to use Git, refer to the repository file `notes/using-git`.

1. We saw in class that the convolution

$$g * h \equiv \int_{-\infty}^{\infty} g(\tau) h(t - \tau) d\tau$$

is equal to the inverse Fourier transform of the product of $G(f)$ and $H(f)$, which are the Fourier transforms of $g(t)$ and $h(t)$, respectively:

$$g * h = \int_{-\infty}^{\infty} G(f) H(f) e^{-2\pi i f t} df.$$

In a similar way, show the following relation for the *correlation* between g and h :

$$\text{corr}(g, h) \equiv \int_{-\infty}^{\infty} g(\tau + t) h(\tau) d\tau = \int_{-\infty}^{\infty} G(f) H(-f) e^{-2\pi i f t} df.$$

Why, in the case of purely real-valued g and h , does $H(-f) = [H(f)]^*$, where $[z]^*$ indicates complex conjugation of $z = a + ib$?

2. Beginning with the convolution code that we constructed in class, write a function to compute the correlation.
3. Using two different discretizations N of a signal of length $L = 1$, take the correlation of the following two functions:

$$g(t) = \begin{cases} 1; & 0 \leq x \leq 0.1 \\ 0; & 0.1 < x \leq 1 \end{cases} \quad h(t) = \begin{cases} 0; & 0 \leq x < 0.4 \\ 1; & 0.4 \leq x \leq 0.6 \\ 0; & 0.6 < x \leq 1 \end{cases}.$$

How must you normalize g for the maximum value of the correlation function to be equal to one? Why?