Applications in Scientific Computing
Assignment 4: Fast Fourier Transforms

530.390.13

Due: Wednesday 13 January 2016

Submit all code by committing it to the directory `assignments/assignment4` in your `530.390.13` GitHub repository. For a reminder of how to use Git, refer to the repository file `notes/using-git`.

1. We saw in class that the convolution

   $$g * h \equiv \int_{-\infty}^{\infty} g(\tau)\, h(t-\tau)\; d\tau$$

   is equal to the inverse Fourier transform of the product of $G(f)$ and $H(f)$, which are the Fourier transforms of $g(t)$ and $h(t)$, respectively:

   $$g * h = \int_{-\infty}^{\infty} G(f)\, H(f)\, e^{-2\pi i f t}\, df.$$

   In a similar way, show the following relation for the *correlation* between g and h:

   $$\operatorname{corr}(g, h) \equiv \int_{-\infty}^{\infty} g(\tau + t)\, h(\tau)\; d\tau = \int_{-\infty}^{\infty} G(f)\, H(-f)\, e^{-2\pi i f t}\, df.$$

   Why, in the case of purely real-valued g and h, does $H(-f) = [H(f)]^*$, where $[z]^*$ indicates complex conjugation of $z = a + ib$?

2. Beginning with the convolution code that we constructed in class, write a function to compute the correlation.

3. Using two different discretizations N of a signal of length $L = 1$, take the correlation of the following two functions:

   $$g(t) = \begin{cases} 1; & 0 \le x \le 0.1 \\ 0; & 0.1 < x \le 1 \end{cases} \qquad h(t) = \begin{cases} 0; & 0 \le x < 0.4 \\ 1; & 0.4 \le x \le 0.6 \\ 0; & 0.6 < x \le 1 \end{cases}.$$

   How must you normalize g for the maximum value of the correlation function to be equal to one? Why?