Applications in Scientific Computing
Assignment 5: Numerical integration

530.390.13

Due: Friday 15 January 2016

Submit all code by committing it to the directory `assignments/assignment5` in your `530.390.13` GitHub repository. For a reminder of how to use Git, refer to the repository file `notes/using-git`.

1. We saw in class the definition of the *scalar product*:

$$(f, g) \equiv \int_a^b f(x)\, g(x)\ dx.$$

Using the code that we constructed in class, numerically compute the following scalar products in the interval $0 \leq x \leq 2\pi$:

$$(\sin(x), \sin(x))$$
$$(\sin(x), \sin(2x))$$
$$(\sin(2x), \sin(8x))$$
$$(\sin(8x), \sin(8x))$$

What do you find? As we discussed in class, the what you have seen here is an example of the *orthonormality* of the sine functions, where

$$(\sin(mx), \sin(nx)) = \begin{cases} 1; & m = n \\ 0; & m \neq n \end{cases} .$$

2. In the interval $0 \leq x \leq 1$, integrate the following functions:

$$y = x$$
$$y = x^2$$
$$y = x^{11}$$
$$y = x^{12}$$
$$y = e^x$$

Integrate by using the trapezoidal rule with $N = \{10, 100, 1000\}$ and using Gauss-Legendre quadrature for the orders of $n = \{1, 2, 6\}$. Complete the following table and comment on the results. Do you expect the exponential to integrate exactly? Why or why not?

		Trapezoidal rule			Gauss-Legendre quadrature		
Function	Exact result	$N = 10$	$N = 100$	$N = 1000$	$n = 1$	$n = 2$	$n = 6$
$y = x$							
$y = x^2$							
$y = x^{11}$							
$y = x^{12}$							
$y = e^x$							