Advection Tests

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Abstract

Volume of Fluid method is validated by chosing three test cases from [**?**],

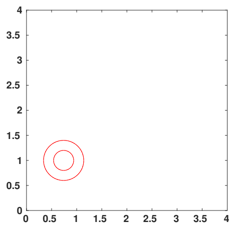
1. Circle in translational flow
2. Solid body rotation of slotted circle
3. Circle in shear flow

1 Motivation for a new method

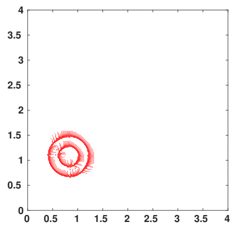
A need of new method arises when conventional methods fail. For multiphase flow, we can try to advect a material property say density ρ in time, which follows the following equation

$$\frac{\partial \rho}{\partial t} + (\overrightarrow{u}.\overrightarrow{\nabla})\rho = 0 \tag{1}$$

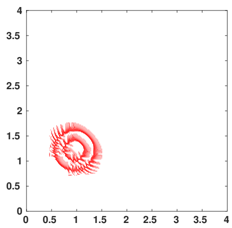
if we use finite difference method to solve above equation to advect the density, the interface does not remain sharp it smears, due to the fact that the density is a step function but finite differencing tries to smooth it over the time. The conventional numerical methods do not retain the discontinuos property of the function. Hence there is need for methods which conserve the discontinuos material properties in multiphase flows.



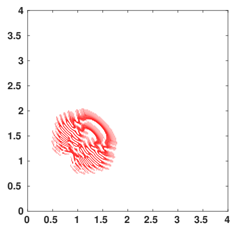
(a) Initial Condition for density field



(b) After advecting density for 20 time steps



(c) After advecting density for 50 time steps

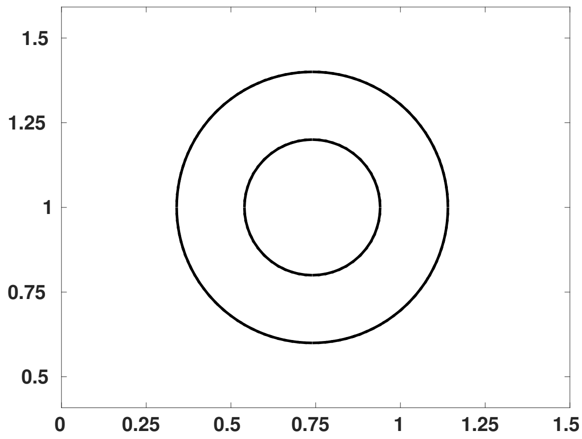


(d) After advecting density for 100 time steps

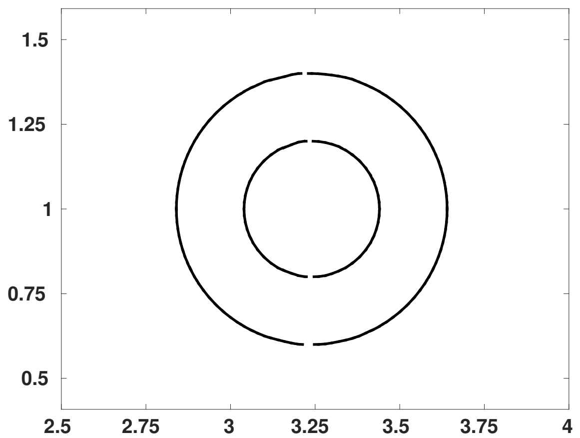
Figure 1: Advection density using finite difference method

2 Advection of circle in translational flow

The algorithm is tested for the simplest case of unidirectional velocity field. Two concentric circles are used as the initial condition for translational test, with center at (0.75,1) and diameter of inner and outer circle is 0.4 and 0.8 respectively (Fig. 1). The volume fraction thus generated by the configuration of concentric circles is of a hollow circle. The volume fraction scalar field is advected by two velocity fields U(1,0) and U(2,1). The refinement of the domain which is $4.0X4.0$ is $200X200$. The time step is 0.005 and advection proceeds for 500 and 504 steps for case 1 and case 2 respectively.

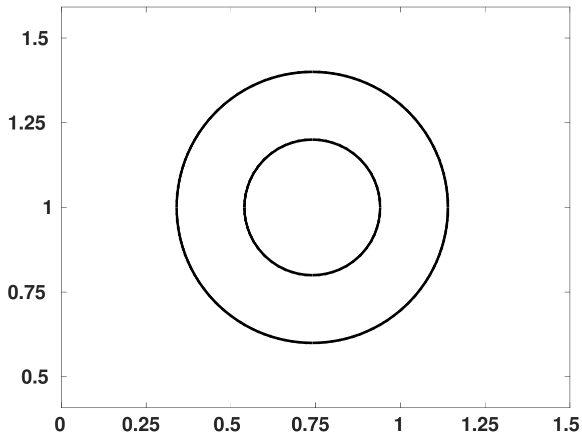


(a) Initial Condition for translational test

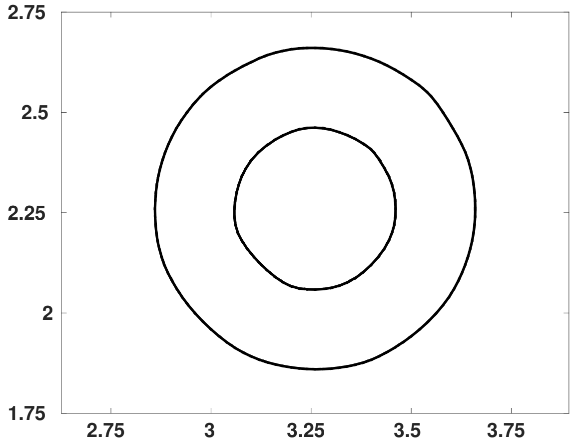


(b) After advecting 500 steps

Figure 2: Advection test for velocity field U(1,0)



(a) Initial Condition for translational test

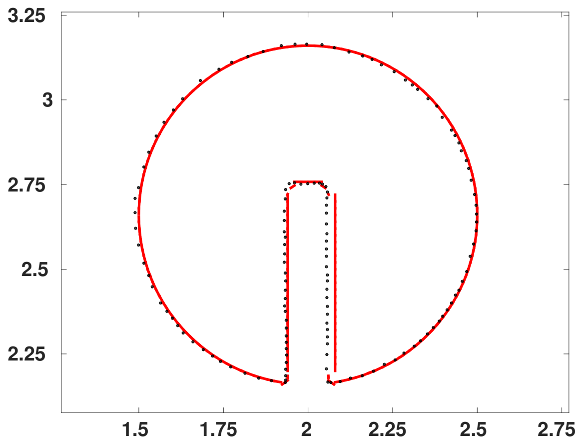


(b) After advecting 504 steps

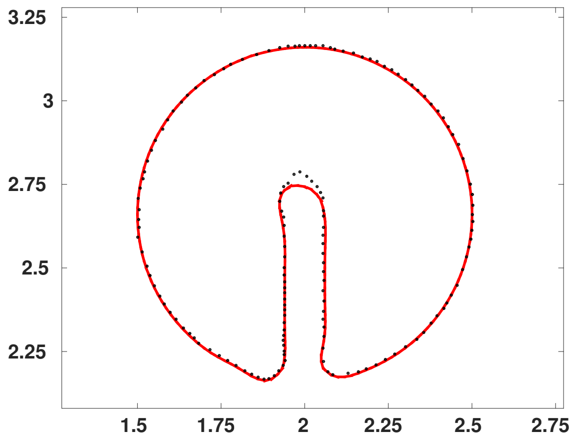
Figure 3: Advection test for velocity field U(2,1)

3 Advection test for solid body rotation

For solid body rotation test a slotted circle configuration is taken from [?], the center of slotted circle is at (2.0,2.75) and diameter is 1.0. The length and width of slot is 0.6 and 0.12 respectively.(Fig. 3(a)) The refinement of the domain which is $4.0X4.0$ is $200X200$. A curl free velocity field is given by $U(-\Omega(y-y_0), \Omega(x-x_0))$, where axis of rotation passes through the (x_0, y_0) and normal to the x-y plane. Ω is the angular velocity. Here $\Omega = 0.5$ and (x_0, y_0) is $(2, 2)$. The time step is 0.005.

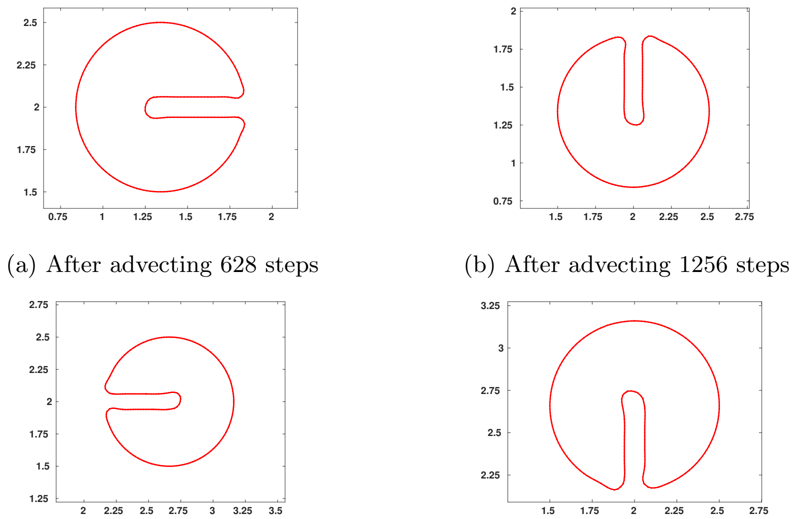


(a) Initial Condition for solid body rotation test.



(b) After full rotation

Figure 4: Comparison with [?] results. (Red LVIRA and Black [?] data)



(a) After advecting 628 steps

(b) After advecting 1256 steps

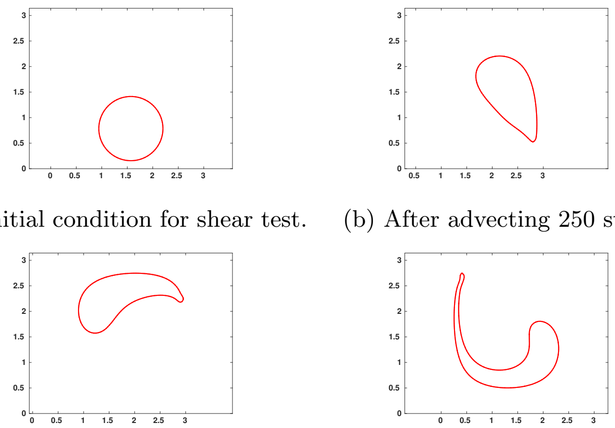
(c) After advecting 1885 steps

(d) After advecting 2513 steps.

Figure 5: Advection test result for solid body rotation

4 Shear Test

The real problems typically encounters the interface deformation, which includes merging and deformation. Hence the algorithm has to be tested for shear velocity field. The velocity field is given by $U(sinxcosy, -cosxsiny)$. The domain size is given by $\pi X\pi$ and refinement is $100X100$. A circle of radius $\pi/5$ and center at $(\pi/2, \pi/4)$ was advected in the velocity field for 1000 steps forward and 1000 steps backward by reversing the signs of velocity field.



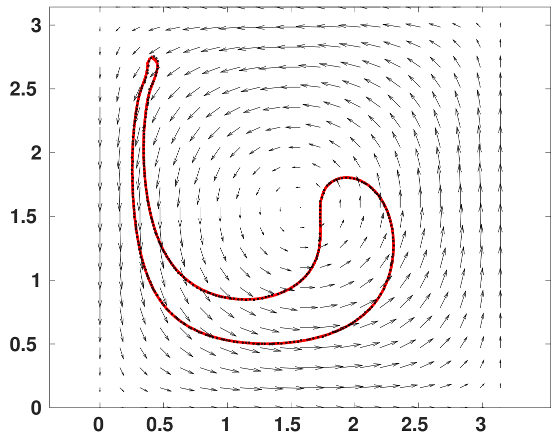
(a) Initial condition for shear test.

(b) After advecting 250 steps

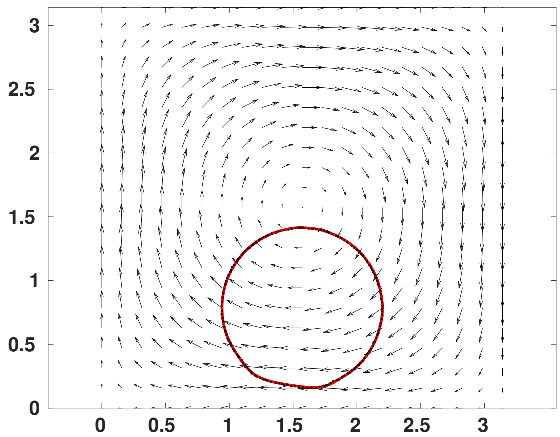
(c) After advecting 500 steps

(d) After advecting 1000 steps

Figure 6: Advection test result for shear velocity field



(a) After 1000 steps forward



(b) 1000 steps backward followed by 1000 steps forward

Figure 7: Comparison with [?] results. (Red LVIRA and Black [?] data)

5 Calculation of error

The above results can be quantified by defining the error by

$$E = \frac{\sum |V_{i,j}^n - V_{i,j}^e|}{\sum V_{i,j}^0} \tag{2}$$

where V^n is the solution of volume fraction field after n time steps by computation, V^e is the exact solution, and V^0 is the initial solution. The initial solution can be calculated by the initial volume fraction field, exact solution of fields for translational fields can be easily calculated by recreating the circle at the center which has moved with the velocity field. For solid body rotation the exact solution is equals to the initial solution after one full rotation. For shear after 1000 backward steps the final solution should also be equal to initial solution.

Table 1: Error for various tests

Test	SLIC	Hirt-Nichols	FCT-VOF	Youngs	LVIRA
Translational (V(1,0))	$1.30X10^{-2}$	$4.55X10^{-2}$	$1.28X10^{-2}$	$3.08X10^{-3}$	$1.5X10^{-3}$
Translational (V(2,1))	$9.18X10^{-2}$	$1.9X10^{-1}$	$3.99X10^{-2}$	$2.98X10^{-2}$	$1.05X10^{-2}$
Shear Flow	$4.59X10^{-2}$	$6.66X10^{-2}$	$3.14X10^{-2}$	$8.60X10^{-3}$	$6.90X10^{-3}$
Solid Body Rotation (Slotted circle)	$8.38X10^{-2}$	$9.62X10^{-2}$	$3.29X10^{-2}$	$1.09X10^{-2}$	$9.7X10^{-3}$