Progress Report

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Contents

1 Finite Volume Method for 2D flows

1.1 Comparison of velocity profiles of analytical versus numerical solution using different velocity schemes

The code was tested at a range of Reynolds numbers and similar results were obtained.

Velocity profiles are presented here at a fixed set conditions for the three different velocity schemes. The analysis was repeated with an increasing number of grids and at different Reynolds numbers.

1.2 Vertical grid points, $ny = 10$ Reynolds number, $Re = 10$

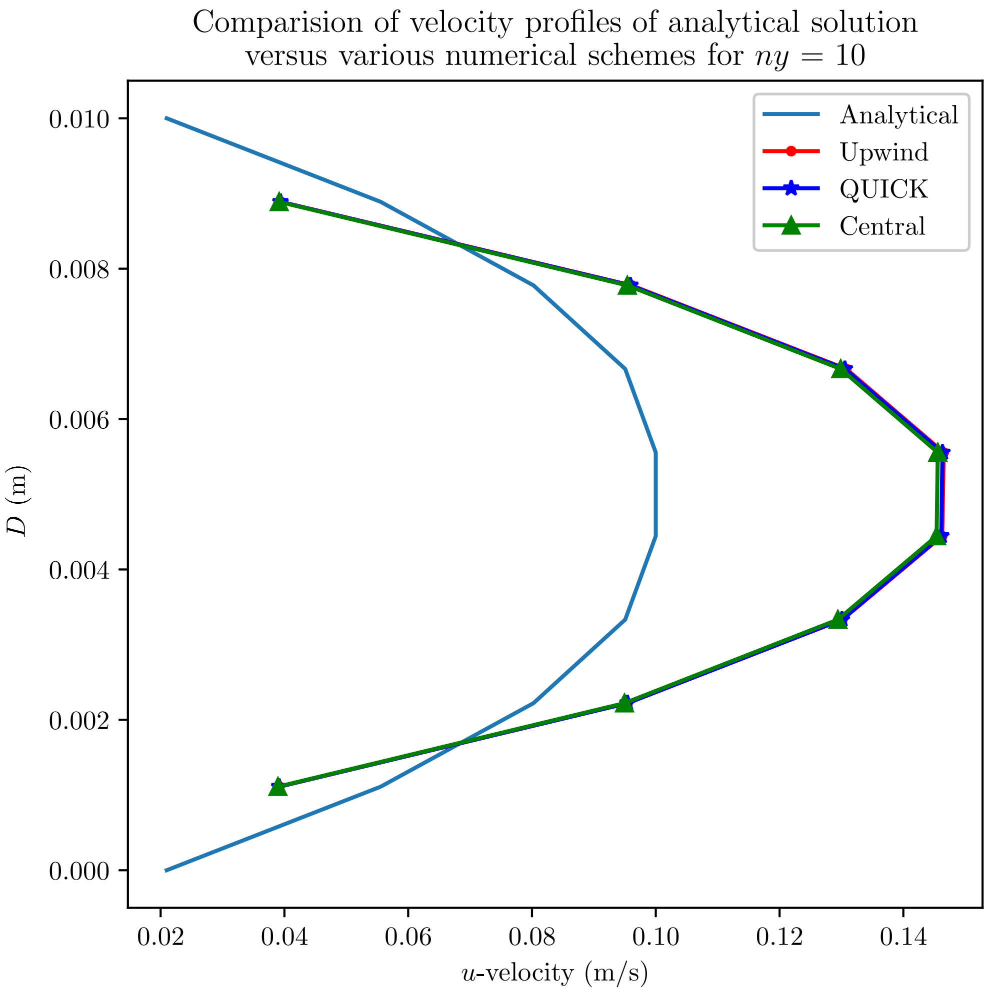

Figure 1: Velocity profiles of numerical solutions using different velocity schemes is compared with that of analytical solution, for 10 vertical grid points at $Re = 10$.

1.3 Vertical grid points, $ny = 30$ Reynolds number, $Re = 10$

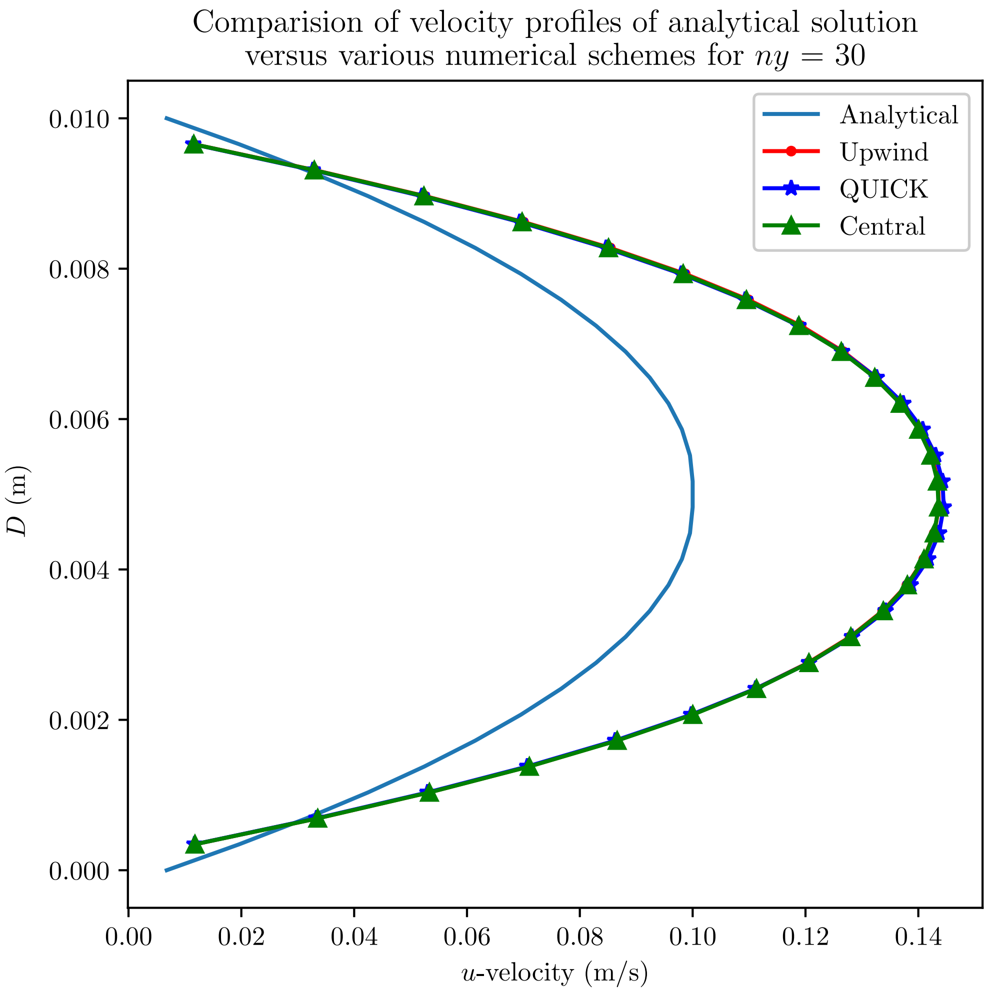

Figure 2: Velocity profiles of numerical solutions using different velocity schemes is compared with that of analytical solution, for 30 vertical grid points at $Re = 10$.

1.4 Vertical grid points, $ny = 10$ Reynolds number, $Re = 100$

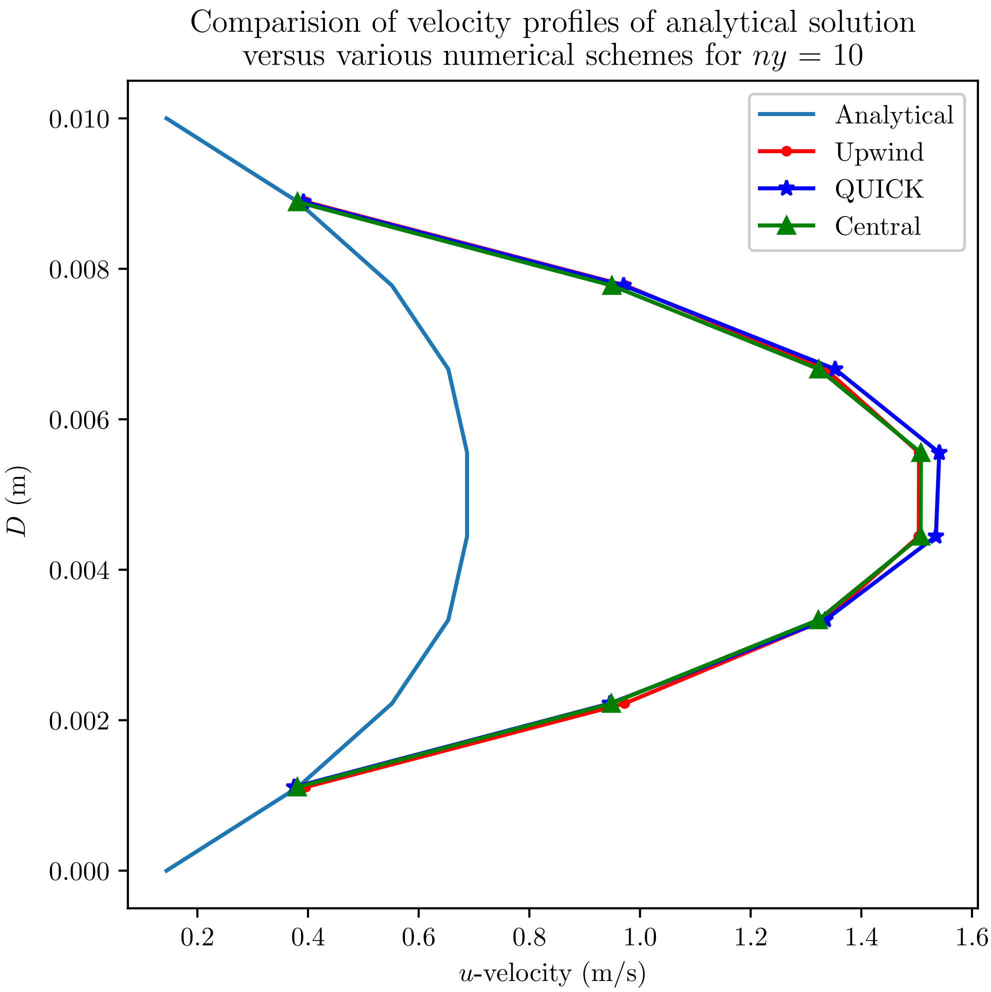

Figure 3: Velocity profiles of numerical solutions using different velocity schemes is compared with that of analytical solution, for 10 vertical grid points at $Re = 100$.

1.5 Substitutions for velocities in the convective terms

A simple mean is used for the velocities u_n, u_s, v_e and v_w,

$$u_n = u_{i,j} + \frac{u_{i,j+1} - u_{i,j}}{(y_{j+2} - y_j)/2} \qquad v_e = v_{i,j} + \frac{v_{i+1,j} - v_{i,j}}{(x_{i+2} - x_i)/2}$$
$$u_s = u_{i,j} - \frac{u_{i,j} - u_{i,j-1}}{(y_{j+1} - y_{j-1})/2} \qquad v_w = v_{i,j} - \frac{v_{i,j} - v_{i-1,j}}{(x_{i+1} - x_{i-1})/2}$$

For rest of the velocities in the convective terms, namely u_e, u_w, v_n and v_s, the following three schemes were used:

1. Upwind scheme
2. Central difference scheme
3. Quadratic Upwind Interpolation for Convective Kinematics (QUICK)

Upwind scheme
For positive velocities,

$$u_e = u_{i,j} \qquad v_n = v_{i,j}$$
$$u_w = u_{i-1,j} \qquad v_s = v_{i,j-1}$$

For negative velocities,

$$u_e = u_{i+1,j} \qquad v_n = v_{i,j+1}$$
$$u_w = u_{i,j} \qquad v_s = v_{i,j}$$

Central scheme

$$u_e = \frac{u_{i,j} + u_{i+1,j}}{2} \qquad v_n = \frac{v_{i,j} + v_{i,j+1}}{2}$$
$$u_w = \frac{u_{i-1,j} + u_{i+1,j}}{2} \qquad v_s = \frac{v_{i,j-1} + v_{i,j}}{2}$$

QUICK scheme
For positive velocities,

$$u_e = \tfrac{6}{8}u_{i,j} + \tfrac{3}{8}u_{i+1,j} - \tfrac{1}{8}u_{i-1,j} \qquad v_n = \tfrac{6}{8}v_{i,j} + \tfrac{3}{8}v_{i,j+1} - \tfrac{1}{8}v_{i,j-1}$$
$$u_w = \tfrac{6}{8}u_{i-1,j} + \tfrac{3}{8}u_{i,j} - \tfrac{1}{8}u_{i-2,j} \qquad v_s = \tfrac{6}{8}v_{i,j-1} + \tfrac{3}{8}v_{i,j} - \tfrac{1}{8}v_{i,j-2}$$

For negative velocities,

$$u_e = \tfrac{6}{8}u_{i+1,j} + \tfrac{3}{8}u_{i,j} - \tfrac{1}{8}u_{i+2,j} \qquad v_n = \tfrac{6}{8}v_{i,j+1} + \tfrac{3}{8}v_{i,j} - \tfrac{1}{8}v_{i,j+2}$$
$$u_w = \tfrac{6}{8}u_{i,j} + \tfrac{3}{8}u_{i-1,j} - \tfrac{1}{8}u_{i+1,j} \qquad v_s = \tfrac{6}{8}v_{i,j} + \tfrac{3}{8}v_{i,j-1} - \tfrac{1}{8}v_{i,j+1}$$

1.6 Work in progress

- Complete coding of Navier-Stokes equation for 3D flow
- Learn and use Paraview to represent the 3D plots