Progress Report

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1 Finite Volume Method for 2D flows

1.1 Troubleshooting to reduce error of numerical solution

The code was modified to use maximum value of pressure and velocity when testing convergence of the solution.

Velocity profiles are presented here at a fixed set conditions for the three different velocity schemes. The analysis was repeated with an increasing number of grids and at different Reynolds numbers.

1.2 Vertical grid points, $ny = 10$ Reynolds number, $Re = 10$

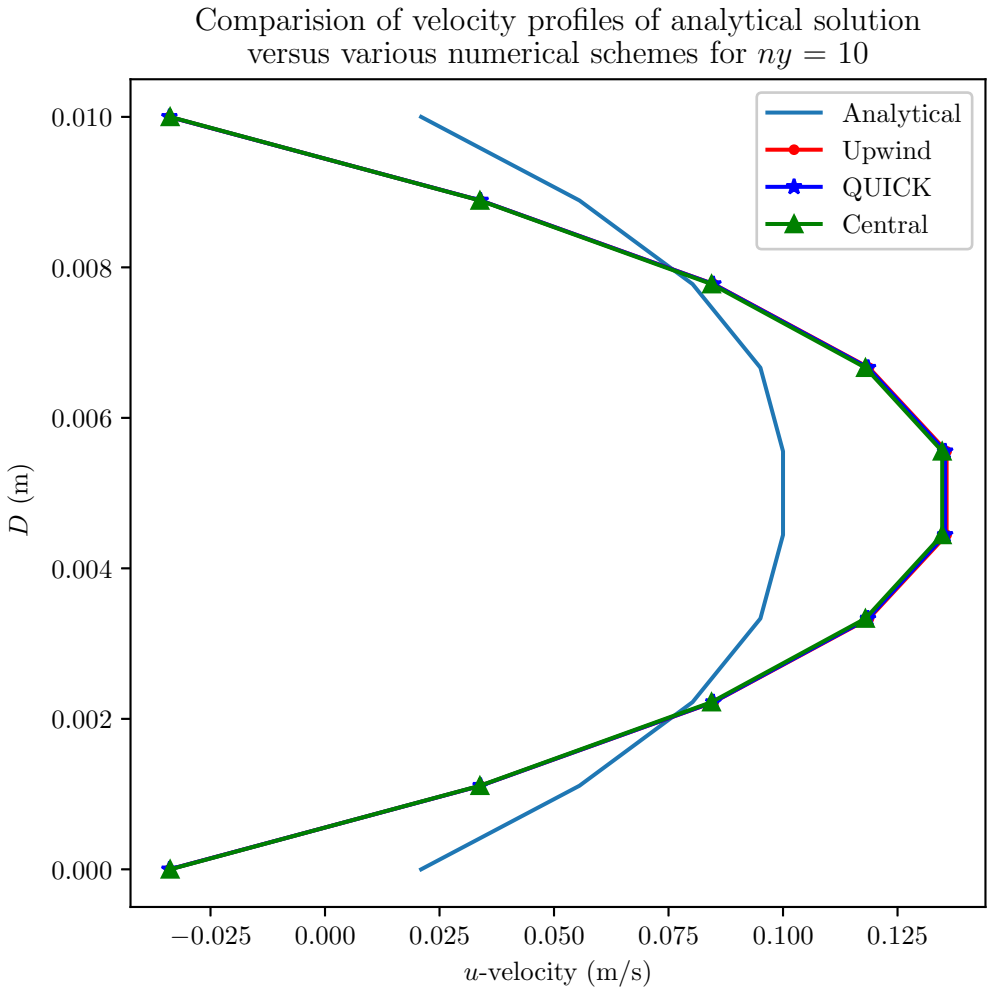

Figure 1: Velocity profiles of numerical solutions using different velocity schemes is compared with that of analytical solution, for 10 vertical grid points at $Re = 10$.

1.3 Vertical grid points, $ny = 30$ Reynolds number, $Re = 10$

Figure 2: Velocity profiles of numerical solutions using different velocity schemes is compared with that of analytical solution, for 30 vertical grid points at $Re = 10$.

1.4 Vertical grid points, $ny = 10$ Reynolds number, $Re = 100$

Figure 3: Velocity profiles of numerical solutions using different velocity schemes is compared with that of analytical solution, for 10 vertical grid points at $Re = 100$.

1.5 Substitutions for velocities in the convective terms

A simple mean is used for the velocities u_n, u_s, v_e and v_w,

$$u_n = u_{i,j} + \frac{u_{i,j+1} - u_{i,j}}{(y_{j+2} - y_j)/2} \qquad v_e = v_{i,j} + \frac{v_{i+1,j} - v_{i,j}}{(x_{i+2} - x_i)/2}$$
$$u_s = u_{i,j} - \frac{u_{i,j} - u_{i,j-1}}{(y_{j+1} - y_{j-1})/2} \qquad v_w = v_{i,j} - \frac{v_{i,j} - v_{i-1,j}}{(x_{i+1} - x_{i-1})/2}$$

For rest of the velocities in the convective terms, namely u_e, u_w, v_n and v_s, the following three schemes were used:

1. Upwind scheme
2. Central difference scheme
3. Quadratic Upwind Interpolation for Convective Kinematics (QUICK)

Upwind scheme
For positive velocities,

$$u_e = u_{i,j} \qquad v_n = v_{i,j}$$
$$u_w = u_{i-1,j} \qquad v_s = v_{i,j-1}$$

For negative velocities,

$$u_e = u_{i+1,j} \qquad v_n = v_{i,j+1}$$
$$u_w = u_{i,j} \qquad v_s = v_{i,j}$$

Central scheme

$$u_e = \frac{u_{i,j} + u_{i+1,j}}{2} \qquad v_n = \frac{v_{i,j} + v_{i,j+1}}{2}$$
$$u_w = \frac{u_{i-1,j} + u_{i+1,j}}{2} \qquad v_s = \frac{v_{i,j-1} + v_{i,j}}{2}$$

QUICK scheme

For positive velocities,

$$u_e = \frac{6}{8}u_{i,j} + \frac{3}{8}u_{i+1,j} - \frac{1}{8}u_{i-1,j} \qquad v_n = \frac{6}{8}v_{i,j} + \frac{3}{8}v_{i,j+1} - \frac{1}{8}v_{i,j-1}$$
$$u_w = \frac{6}{8}u_{i-1,j} + \frac{3}{8}u_{i,j} - \frac{1}{8}u_{i-2,j} \qquad v_s = \frac{6}{8}v_{i,j-1} + \frac{3}{8}v_{i,j} - \frac{1}{8}v_{i,j-2}$$

For negative velocities,

$$u_e = \frac{6}{8}u_{i+1,j} + \frac{3}{8}u_{i,j} - \frac{1}{8}u_{i+2,j} \qquad v_n = \frac{6}{8}v_{i,j+1} + \frac{3}{8}v_{i,j} - \frac{1}{8}v_{i,j+2}$$
$$u_w = \frac{6}{8}u_{i,j} + \frac{3}{8}u_{i-1,j} - \frac{1}{8}u_{i+1,j} \qquad v_s = \frac{6}{8}v_{i,j} + \frac{3}{8}v_{i,j-1} - \frac{1}{8}v_{i,j+1}$$