Assignment 8: Forced, Damped Vibrations Equation Problem (Runge-Kutta Method)

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1 Problem

An instrument is supported by a spring and viscous damper in parallel so that only linear motion in the vertical direction occurs. Briefly derive an expression for the force transmitted to the support through the spring and damper, if the instrument generates an harmonic disturbing force $F \sin(vt)$ in the vertical direction as shown in Figure 1. Determine value of u at $t = 120s$

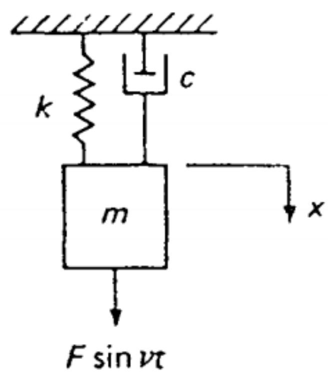

Figure 1: Forced, Damped Vibrations Problem.

Where :

Mass of Instrument (m)	$7\ kg$
Damping Coefficient	0.5
Constant of Spring	$2\ N/m$
Displacement at $t = 0$	1
Damping Velocity at $t = 0$	$0\ m/s$
Acceleration of Grafity	$9.81\ m/s^2$
frequency of the disturbing force f	$50\ Hz$

Hence

$$mu'' + f(u') + s(u) = F(t)$$

$$7u'' + 0.5u' + 2u = m.g.\sin(vt)$$

with

$$v = 2\pi f$$

So

$$7u'' + 0.5u' + 2u = 68.67\sin(100\pi t)$$

2 Solution By Runge-Kutta Method

The most popular Runge-Kutta methods are fourth order. The following is the most commonly used form, and we therefore call it the classical fourth-order RK method:

$$u_{n+1} = u_n + \frac{h}{6}(k_1 + 2k_2 + 2k_3 + k_4)$$

where:
$k_1 = f(t_n, u_n)$
$k_2 = f(t_n + \frac{h}{2}, u_n + h\frac{k_1}{2})$
$k_3 = f(t_n + \frac{h}{2}, u_n + h\frac{k_2}{2})$
$k_4 = f(t_n + h, u_n + hk_3)$

First step is to transform equation from the probem into first derivative equation. That equation becomes:

$$u' = \frac{du}{dt} = v$$

$$u'' = \frac{d^2u}{dt^2} = \frac{dv}{dt}$$

so, The equations will be:

$u' = v$
$v' = -0.0714v - 0.1429u + 9.81\sin(100\pi t_n)$

Next step is set first derivative equations to be numerical method equation.

The First equation becomes:

$$u_{n+1} = u_n + \frac{h}{6}(k_{u1} + 2k_{u2} + 2k_{u3} + k_{u4})$$

with value of
$k_{u1} = f(v_n)$
$k_{u2} = f(v_n + h\frac{k_{u1}}{2})$
$k_{u3} = f(v_n + h\frac{k_{u2}}{2})$
$k_{u4} = f(v_n + hk_{u3})$

While, The second equation becomes:

$$v_{n+1} = v_n + \frac{h}{6}(k_{v1} + 2k_{v2} + 2k_{v3} + k_{v4})$$

With value of
$k_{v1} = f(v_n, u_n, t_n)$
$k_{v2} = f(v_n + h\frac{k_{v1}}{2}, u_n + h\frac{k_{v1}}{2}, t_n + \frac{h}{2})$
$k_{v3} = f(v_n + h\frac{k_{v2}}{2}, u_n + h\frac{k_{v2}}{2}, t_n + \frac{h}{2})$
$k_{v4} = f(v_n + hk_{v3}, u_n + hk_{v3}, t_n + h)$

The third step is to create a code to solve the problem. The code for this problem is shown in the Figure 2.

```
#include <iostream>
#include <iomanip>
#include <cmath>
#include <fstream>
#include <sstream>


using namespace std;
double f1(double v)
{
    double a= v;
    return a;
}
double f2(double v, double u, double m, double b, double k, double f, double ph, double t, double g)
{
    double c= (sin(2 * f * ph * t) * g ) - (b/m * v )-(k/m * u);
    return c;
}
int main()
{

    double v0, m, b, k, v, u0, t0, tn, I[11], h , u, g, f, ph;
    double ku1, ku2, ku3, ku4, kv1, kv2, kv3, kv4;
    int j, n;
    cout.precision(6);
    cout.setf(ios::fixed);
    ifstream IDN;
    IDN.open("IDN.txt");
    string row1;
    int i=0 ;
    while (getline(IDN,row1))
    {
        stringstream ssIDN(row1);
        ssIDN >> I[i];
        i++;
    }

    cout<<"\nValue of v0            :"<<I[0]<<"\n";
    cout<<"\nValue of m             :"<<I[1]<<"\n";
    cout<<"\nValue of b             :"<<I[2]<<"\n";
    cout<<"\nValue of k             :"<<I[3]<<"\n";
    cout<<"\nFind Value of u if t   :"<<I[4]<<"\n";
    cout<<"\nWith delta t           :"<<I[5]<<"\n";
    cout<<"\nValue of g             :"<<I[8]<<"\n";
    cout<<"\nValue of frequency     :"<<I[9]<<"\n";
    cout<<"\nValue of phi           :"<<I[10]<<"\n";
```

```
    cout<<"\nInitial Value of u      :"<<I[6]<<"\n";
    cout<<"\nInitial Value of t      :"<<I[7]<<"\n";
    cout<<"\nInitial Value of v      :"<<I[0]<<"\n";
    v0  = I[0];
    m   = I[1];
    b   = I[2];
    k   = I[3];
    tn  = I[4];
    h   = I[5];
    u0  = I[6];
    t0  = I[7];
    g   = I[8];
    f   = I[9];
    ph = I[10];
    n   = (tn-t0) / h;
    ofstream outputFinalScore;
    outputFinalScore.open("Resultrungekutta.txt");
    for ( j = 0; j < n ; j++)
    {
        ku1 = f1 (v0);
        kv1 = f2 (v0, u0, m, b, k, f, ph, t0, g);

        ku2 = f1 (v0+(kv1*h*0.5));
        kv2 = f2 ((v0+(kv1*h* 0.5)), (u0+(ku1*h*0.5)), m, b, k, f, ph, t0 + (h* 0.5), g );

        ku3 = f1 (v0+(kv2*h*0.5));
        kv3 = f2 ((v0+(kv2*h* 0.5)), (u0+(ku2*h*0.5)), m, b, k, f, ph, t0 + (h* 0.5), g );

        ku4 = f1 (v0+(kv3*h));
        kv4 = f2 ((v0+(kv3*h)), (u0+(ku3*h)), m, b, k, f, ph, t0 + h, g );

        v   = v0 + (h / 6 * (kv1 + (2*kv2) + (2*kv3) + kv4 ));
        u   = u0 + (h / 6 * (ku1 + (2*ku2) + (2*ku3) + ku4 ));
        cout<<t0<<setw(16)<<v0<<setw(16)<<u0<<setw(16)<<v<<setw(16)<<u<<endl;
        outputFinalScore << t0 << "\t" "\t";
        outputFinalScore << u0 << "\t" "\t" << endl;
        t0  = t0 + h;
        u0  = u;
        v0  = v;
    }
    cout<<t0<<setw(16)<<v0<<setw(16)<<u0<<setw(16)<<v<<setw(16)<<u<<endl;
    outputFinalScore << t0 << "\t" "\t";
    outputFinalScore << u0 << "\t" "\t" << endl;
    return 0;
}
```

Figure 2: Runge-Kutta Method's Code.

Interval of t in this code is 0.005. Based on this program value of Desplacement u at $t = 120$ is 0.00687709.

The result of the problem from Analytical Solution is:

$$u = e^{-\frac{1}{28}t}(cos(0.533328t) + 0.0358sin(0.533328t)) + 1.319910^{-6}\sin(100\pi t)$$

for $t = 120$ the value of u is 0.00592369.A comparation of two solutions shown in Figure 3.

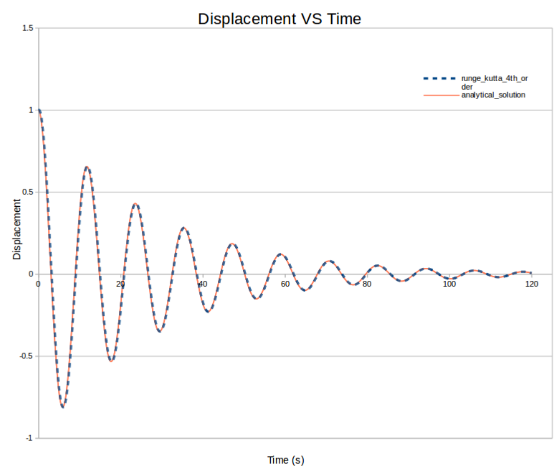

Figure 3: Comparation Between Runge-Kutta Method and Analitycal Solution.