Progress Report

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2018.03.21

1 Numerical solution of cavity flow problem using Finite Volume Method

1.1 Graphical results for cavity flow problem

The cavity flow problem was solved using Finite Volume Method by coding a solution in C++. The results are presented below.

1.2 Discretization of the convective and diffusive terms for 2D nonuniform grid

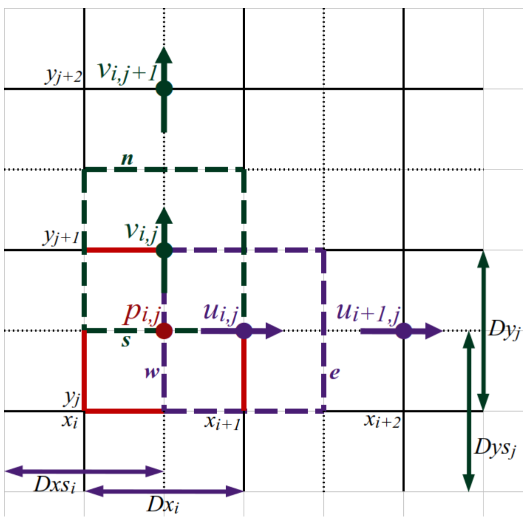

Figure 1: Visual representation of the staggered grid used for discretization in Finite Volume Method

Linear interpolation is used for the velocities u_n, u_s, v_e, and v_w. Let us analyze one of the velocities u_n.

$$\frac{u_n - u_{i,j}}{y_{j+1} - y_{j+1/2}} = \frac{u_{i,j+1} - u_{i,j}}{y_{j+3/2} - y_{j+1/2}}$$
$$\frac{u_n - u_{i,j}}{Dy_j/2} = \frac{u_{i,j+1} - u_{i,j}}{(Dy_{j+1}/2) + (Dy_j/2)}$$
$$\frac{u_n - u_{i,j}}{Dy_j/2} = \frac{u_{i,j+1} - u_{i,j}}{(Dy_{j+1} + Dy_j)/2}$$

Therefore, the terms reduce to the following:

$$u_n = u_{i,j} + \left(\frac{u_{i,j+1} - u_{i,j}}{Dys_{j+1}}\right)\left(\frac{Dy_j}{2}\right) \qquad v_e = v_{i,j} + \left(\frac{v_{i+1,j} - v_{i,j}}{Dxs_{i+1}}\right)\left(\frac{Dx_i}{2}\right)$$
$$u_s = u_{i,j-1} + \left(\frac{u_{i,j} - u_{i,j-1}}{Dys_j}\right)\left(\frac{Dy_{j-1}}{2}\right) \qquad v_w = v_{i-1,j} + \left(\frac{v_{i,j} - v_{i-1,j}}{Dxs_i}\right)\left(\frac{Dx_{i-1}}{2}\right)$$

1.2.1 Upwind scheme for velocities in the convective terms

For rest of the velocities in convective terms, the upwind scheme may be used. For positive velocities,

$$u_e = u_{i,j} \qquad u_w = u_{i-1,j} \qquad u_{e,v} = u_{i,j} \qquad u_{w,v} = u_{i-1,j}$$
$$v_n = v_{i,j} \qquad v_s = v_{i,j-1} \qquad v_{n,u} = v_{i,j} \qquad v_{s,u} = v_{i,j-1}$$

For negative velocities,

$$u_e = u_{i+1,j} \qquad u_w = u_{i,j} \qquad u_{e,v} = u_{i,j+1} \qquad u_{w,v} = u_{i-1,j+1}$$
$$v_n = v_{i,j+1} \qquad v_s = v_{i,j} \qquad v_{n,u} = v_{i+1,j} \qquad v_{s,u} = v_{i+1,j-1}$$

For the first time step, Euler scheme is used and for subsequent time steps, Adams-Bashforth scheme is utilized.

1.2.2 QUICK scheme for velocities in the convective terms

QUICK scheme is also incorporated in the code, with an option to switch between upwind (first order) or QUICK scheme (second order). For positive velocities,

$$u_e = \frac{u_i + u_{i+1}}{2} - \frac{Dx_{i+1}^2}{8Dxs_{i+1}}\left(\frac{u_{i+1} - u_i}{Dx_{i+1}} - \frac{u_i - u_{i-1}}{Dx_i}\right)$$

$$u_w = \frac{u_{i-1} + u_i}{2} - \frac{Dx_i^2}{8Dxs_i}\left(\frac{u_i - u_{i-1}}{Dx_i} - \frac{u_{i-1} - u_{i-2}}{Dx_{i-1}}\right)$$

$$v_n = \frac{v_j + v_{j+1}}{2} - \frac{Dy_{j+1}^2}{8Dys_{j+1}} \left(\frac{v_{j+1} - v_j}{Dy_{j+1}} - \frac{v_j - v_{j-1}}{Dy_j} \right)$$

$$v_s = \frac{v_{j-1} + v_j}{2} - \frac{Dy_j^2}{8Dys_j} \left(\frac{v_j - v_{j-1}}{Dy_j} - \frac{v_{j-1} - v_{j-2}}{Dy_{j-1}} \right)$$

For negative velocities,

$$u_e = \frac{u_i + u_{i+1}}{2} - \frac{Dx_{i+1}^2}{8Dxs_{i+2}} \left(\frac{u_{i+2} - u_{i+1}}{Dx_{i+2}} - \frac{u_{i+1} - u_i}{Dx_{i+1}} \right)$$

$$u_w = \frac{u_{i-1} + u_i}{2} - \frac{Dx_i^2}{8Dxs_{i+1}} \left(\frac{u_{i+1} - u_i}{Dx_{i+1}} - \frac{u_i - u_{i-1}}{Dx_i} \right)$$

$$v_e = \frac{v_j + v_{j+1}}{2} - \frac{Dy_{j+1}^2}{8Dys_{j+2}} \left(\frac{v_{j+2} - v_{j+1}}{Dy_{j+2}} - \frac{v_{j+1} - v_j}{Dy_{j+1}} \right)$$

$$v_w = \frac{v_{j-1} + v_j}{2} - \frac{Dy_j^2}{8Dys_{j+1}} \left(\frac{v_{j+1} - v_j}{Dy_{j+1}} - \frac{v_j - v_{j-1}}{Dy_j} \right)$$

1.3 Future work

- Generate figures for 3D grid
- Apply grid average and nodal value in QUICK scheme
- Use Smagorinsky-Lilly turbulent model for 2D case