# Progress Report

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## Solving the pressure term for solution of Navier-Stokes equations using Finite Volume Method

Continuity equation:

$$\nabla \cdot \mathbf{u} = 0 \tag{1}$$

Navier-Stokes equation:

$$\frac{\partial \mathbf{u}}{\partial t} + \nabla \cdot (\mathbf{u}\mathbf{u}) = -\frac{1}{\rho}\nabla p + \nu \nabla^2 \mathbf{u} + \mathbf{f} \quad . \tag{2}$$

### Projection method

Ignoring the body force, projection method is used to decompose Navier-Stokes equation into

$$\frac{\mathbf{u}^* - \mathbf{u}^n}{\Delta t} = -\nabla \cdot (\mathbf{u}\mathbf{u}) + \nu \nabla^2 \mathbf{u} \tag{3}$$

$$\frac{\mathbf{u}^{n+1} - \mathbf{u}^*}{\Delta t} = -\frac{1}{\rho}\nabla p \tag{4}$$

where $n$ is the time level index.

### Poisson equation of pressure

The pressure term can be solved using

$$\nabla^2 p = \frac{\rho}{\Delta t}\nabla \cdot \mathbf{u}^* \quad , \tag{5}$$

which can be represented as

$$\frac{\partial^2 p}{\partial x^2} + \frac{\partial^2 p}{\partial y^2} = \frac{1}{\Delta t}\left(\frac{\partial u^*}{\partial x} + \frac{\partial v^*}{\partial y}\right) \quad . \tag{6}$$

Integrating it twice, discretizing and rearranging leads to

$$\begin{aligned} p_{i,j} =& \frac{1}{\left[-\frac{Dy}{Dx1} - \frac{Dy}{Dx2} - \frac{Dx}{Dy1} - \frac{Dx}{Dy2}\right]} \\ &\times \left[-\frac{Dy}{Dx1} p_{i+1,j} - \frac{Dy}{Dx2} p_{i-1,j} - \frac{Dx}{Dy1} p_{i,j+1} - \frac{Dx}{Dy2} p_{i,j-1}\right. \\ &\left. + \frac{1}{\Delta t}\left\{\left(u^*_{i,j} - u^*_{i-1,j}\right)\Delta y + \left(v^*_{i,j} - v^*_{i,j-1}\right)\Delta x\right\}\right] \quad . \end{aligned} \tag{7}$$

Using successive over-relaxation method (SOR),

$$\begin{aligned} p_{i,j} =& (1-\omega)\, p_{i,j} + \omega \left[\frac{1}{\left(-\frac{Dy}{Dx1} - \frac{Dy}{Dx2} - \frac{Dx}{Dy1} - \frac{Dx}{Dy2}\right)}\right. \\ &\times \left\{-\frac{Dy}{Dx1} p_{i+1,j} - \frac{Dy}{Dx2} p_{i-1,j} - \frac{Dx}{Dy1} p_{i,j+1} - \frac{Dx}{Dy2} p_{i,j-1}\right. \\ &\left.\left. + \frac{1}{\Delta t}\left(\left(u^*_{i,j} - u^*_{i-1,j}\right)\Delta y + \left(v^*_{i,j} - v^*_{i,j-1}\right)\Delta x\right)\right\}\right] \quad , \end{aligned} \tag{8}$$

where the relaxation factor, $\omega = 1.8$.

Finally, the correct velocity can be found using

$$\mathbf{u}^{n+1} = \mathbf{u}^* - \frac{\Delta t}{\rho} \cdot \nabla p \quad , \tag{9}$$

which is, for $u$- and $v$-components,

$$u^{n+1}_{i,j} = u^*_{i,j} - \frac{\Delta t}{\rho} \cdot \frac{p_{i+1,j} - p_{i,j}}{\Delta x} \tag{10}$$

and

$$v^{n+1}_{i,j} = v^*_{i,j} - \frac{\Delta t}{\rho} \cdot \frac{p_{i,j+1} - p_{i,j}}{\Delta y} \tag{11}$$