Progress Report

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Numerical differentiation using arrays for the first derivative

Arrays were utilized in an algorithm coded in C++ for numerical differentiation using three different commonly used schemes. The function $\sin(3x)$ was differentiated using forward difference, backward difference and central difference methods.

Forward difference method

$$f'(x) = \frac{f(x_{i+1}) - f(x_i)}{x_{i+1} - x_i} \tag{1}$$

Backward difference method

$$f'(x) = \frac{f(x_i) - f(x_{i-1})}{x_i - x_{i-1}} \tag{2}$$

Central difference method

$$f'(x) = \frac{f(x_{i+1}) - f(x_{i-1})}{x_{i+1} - x_{i-1}} \tag{3}$$

The results are shown in the figures below.

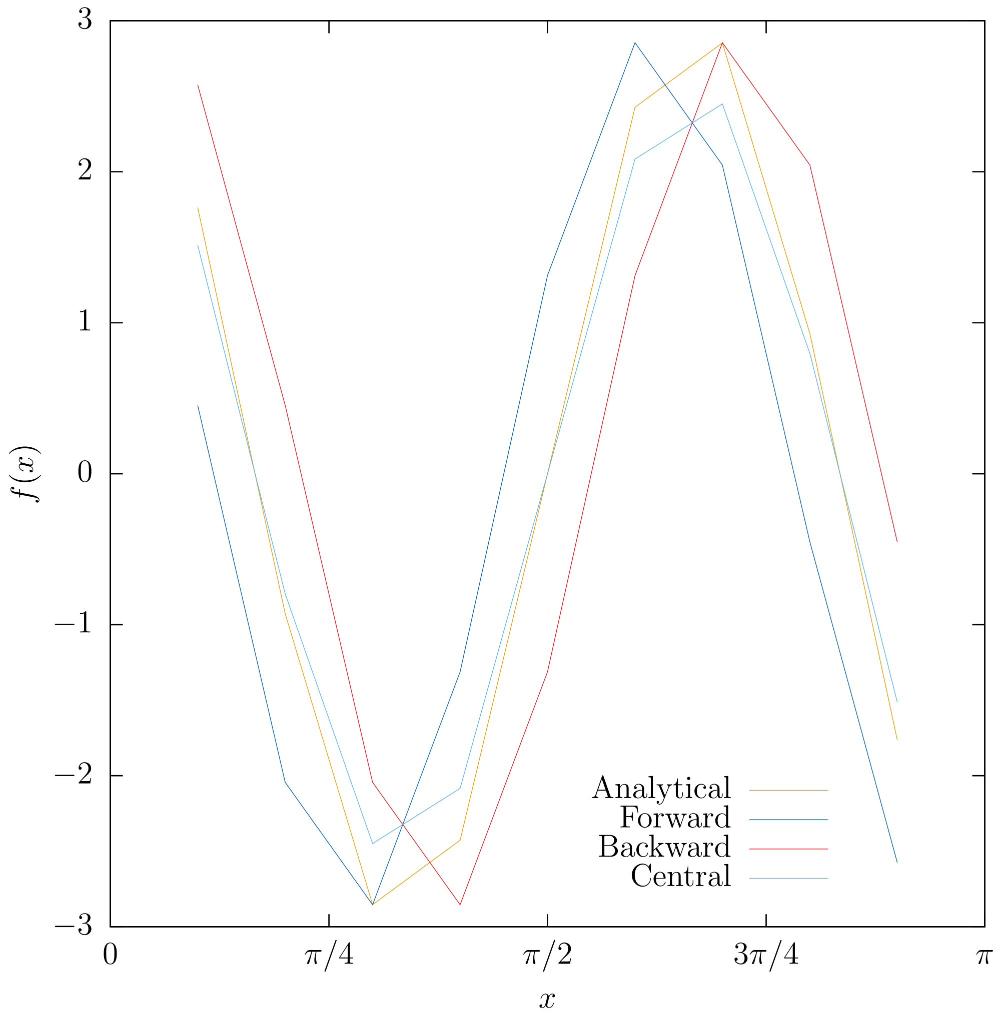

Figure 1: Comparison of numerical differentiation of $\sin(3x)$ using 10 intervals for the first derivative

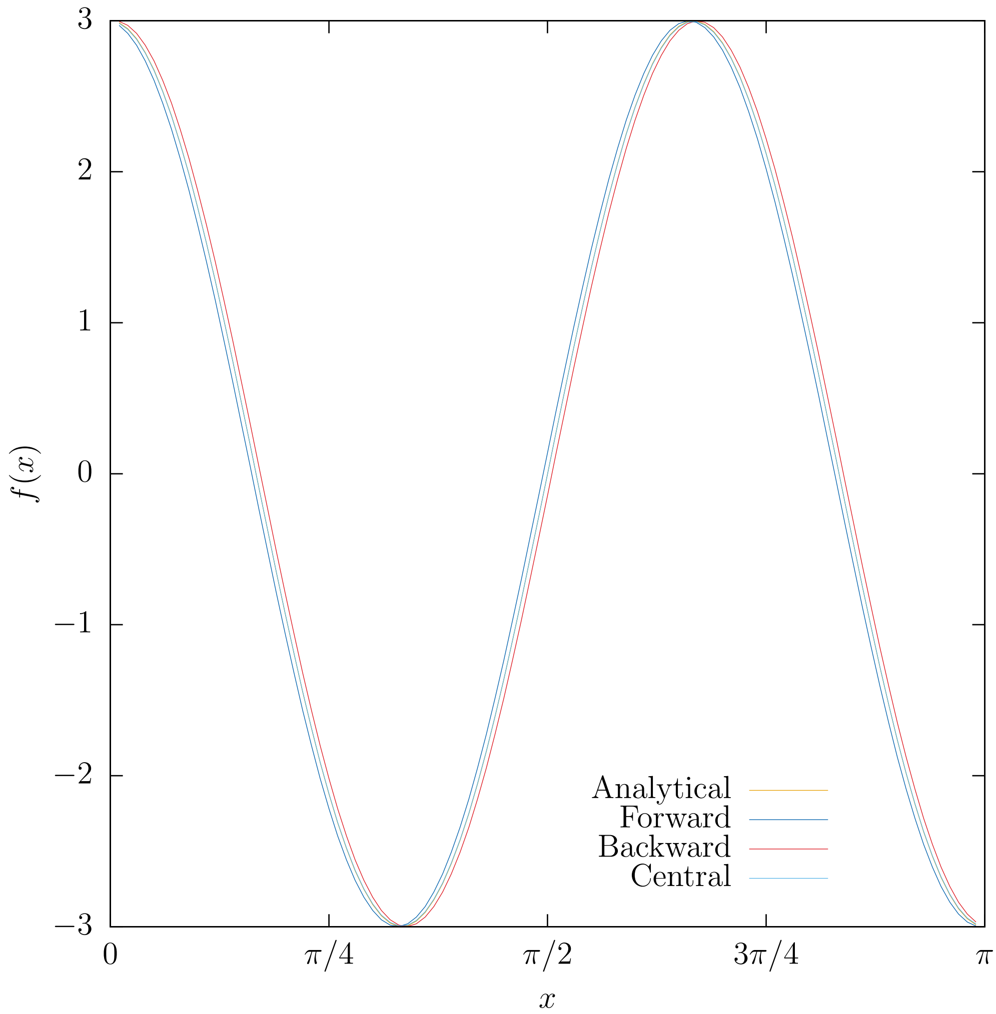

Figure 2: Comparison of numerical differentiation of $\sin(3x)$ using 100 intervals for the first derivative

Numerical differentiation using arrays for the second derivative

C++ arrays were also used for numerical differentation of the second derivative using schemes similar to those for first derivative. The function $\sin(3x)$ was differentiated twice using forward difference, backward difference and central difference methods.

Forward difference method for second derivative

$$f''(x) = \frac{f(x_{i+2}) - 2f(x_{i+1}) + f(x_i)}{h^2} \tag{4}$$

Backward difference method for second derivative

$$f''(x) = \frac{f(x_i) - 2f(x_{i-1}) + f(x_{i-2})}{h^2} \tag{5}$$

Central difference method for second derivative

$$f''(x) = \frac{f(x_{i+1}) - 2f(x) + f(x_{i-1})}{h^2} \tag{6}$$

The results for the second derivative are shown in the figures below.

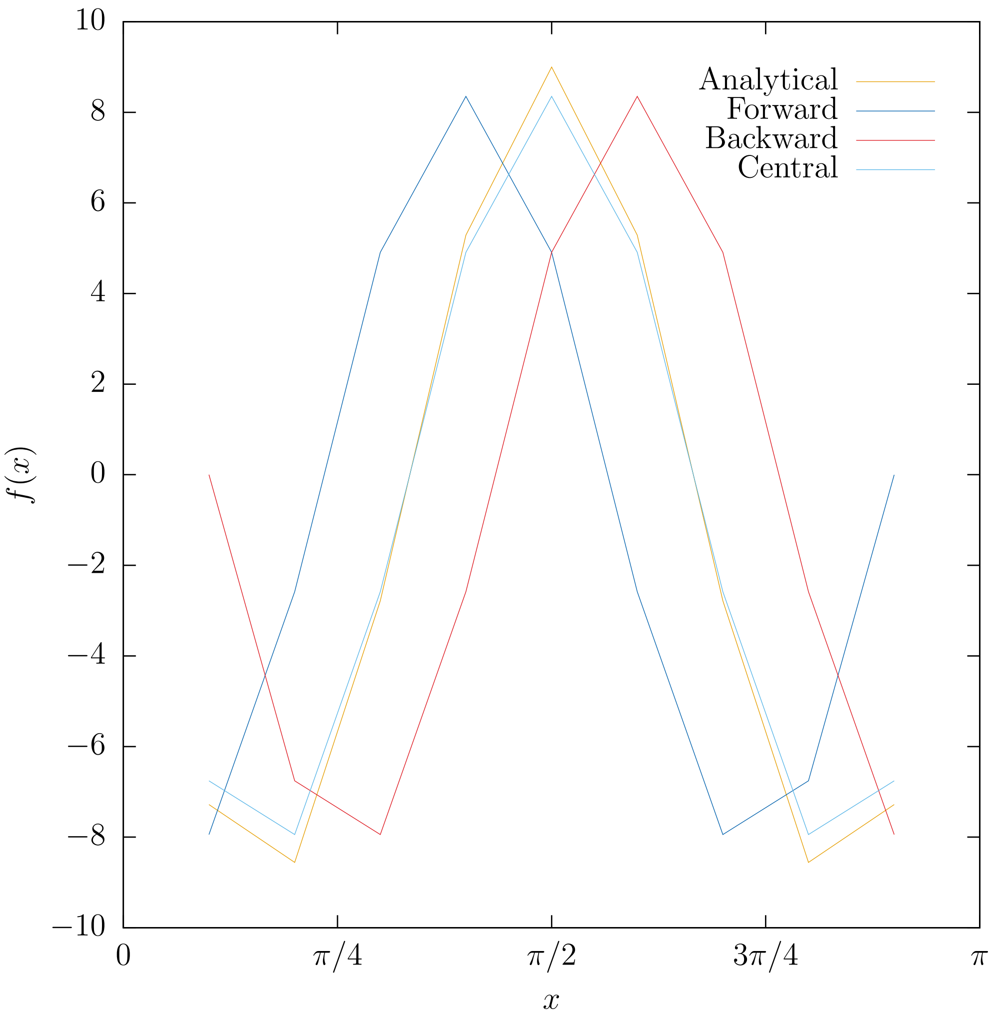

Figure 3: Comparison of numerical differentiation of second derivative of $\sin(3x)$ using 10 intervals

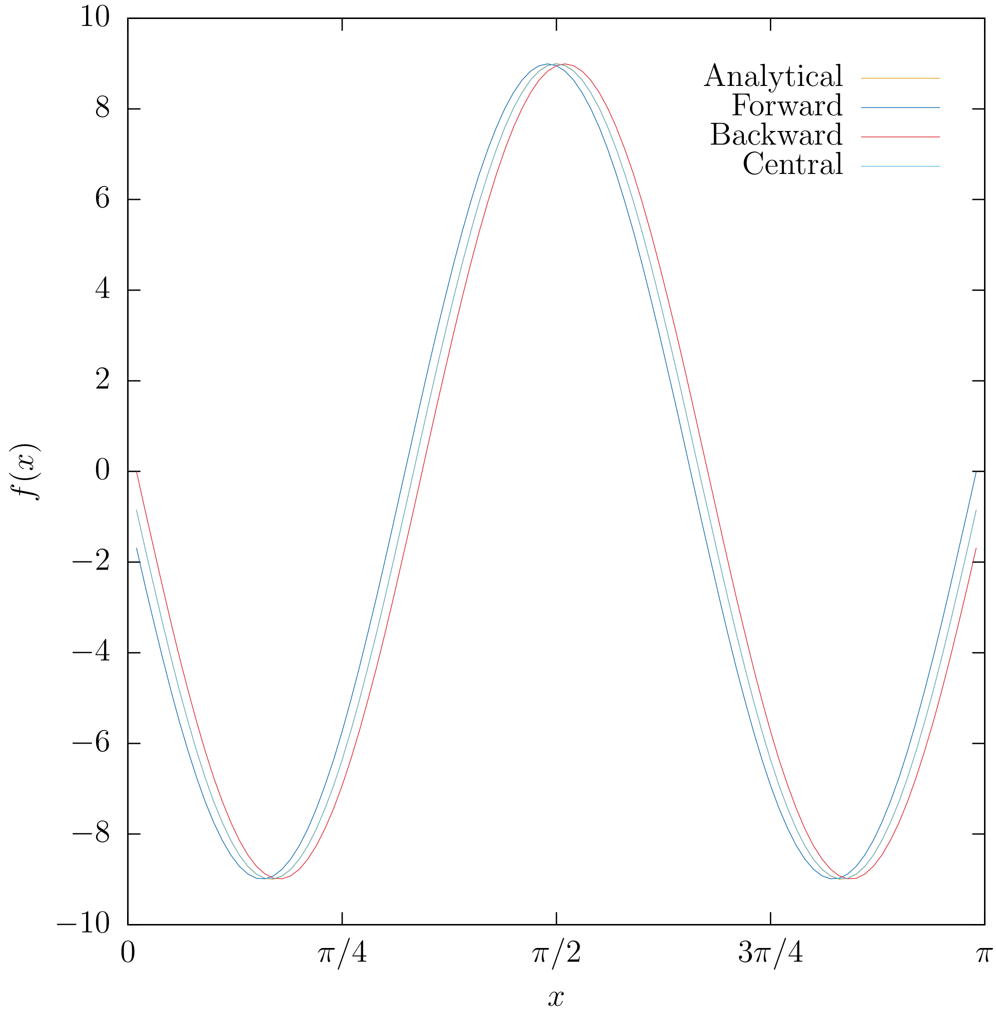

Figure 4: Comparison of numerical differentiation of second derivative of $\sin(3x)$ using 100 intervals

Determination of error for for first and second derivatives

The error for calculation of the first derivative was determined for each of the three differnt methods using the following formula:

$$E = \sqrt{\frac{\sum_{i=1}^{N} \left(\frac{A_N - A_a}{A_a}\right)^2}{N}} \tag{7}$$

The calculations were performed for intervals starting from 10 to 10,000 over the range 0 to pi. The plots are shown below.

The error for calculation of the second derivative was also determined for each of the three differnt methods using the same formula. The plot are as follows.

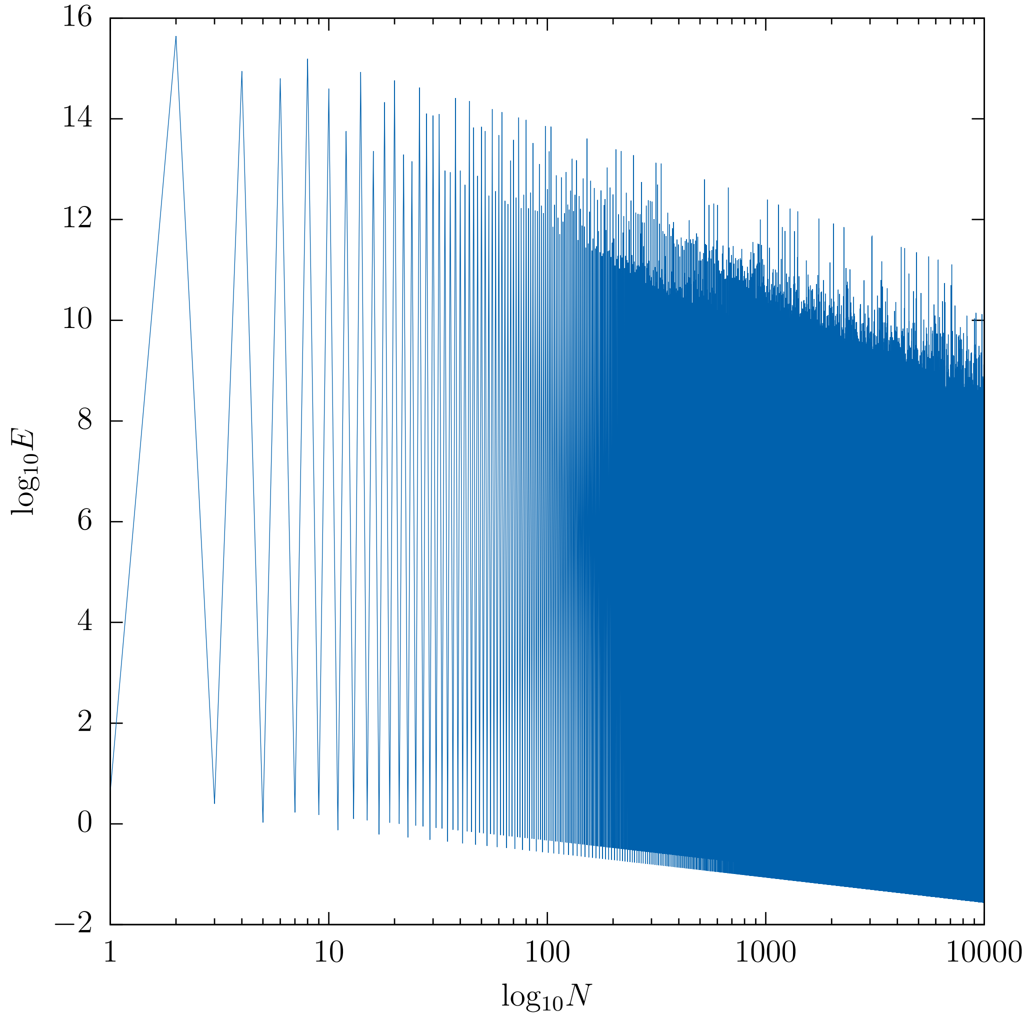

Figure 5: Plot of $\log_{10} E$ versus $\log_{10} N$ for numerical differentiation of first derivative using forward difference method

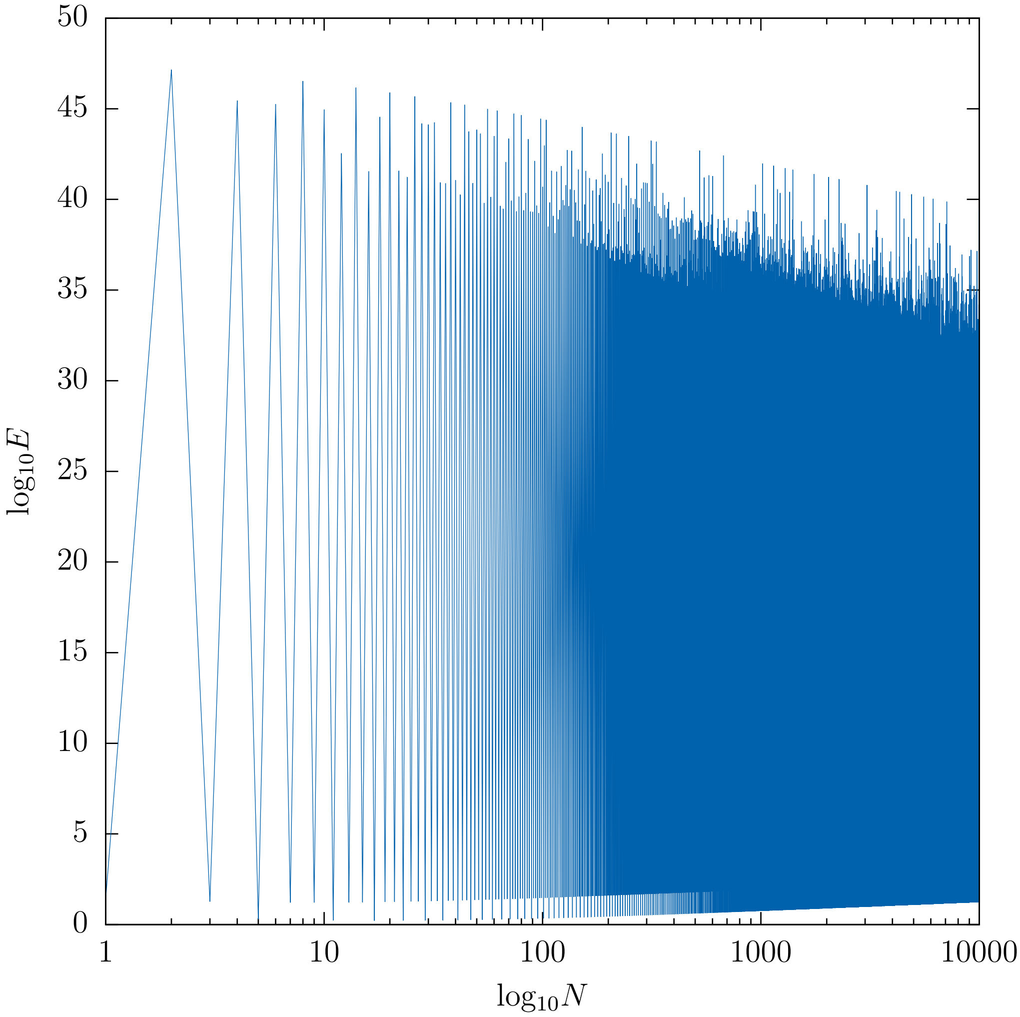

Figure 6: Plot of $\log_{10}E$ versus $\log_{10}N$ for numerical differentiation of first derivative using backward difference method

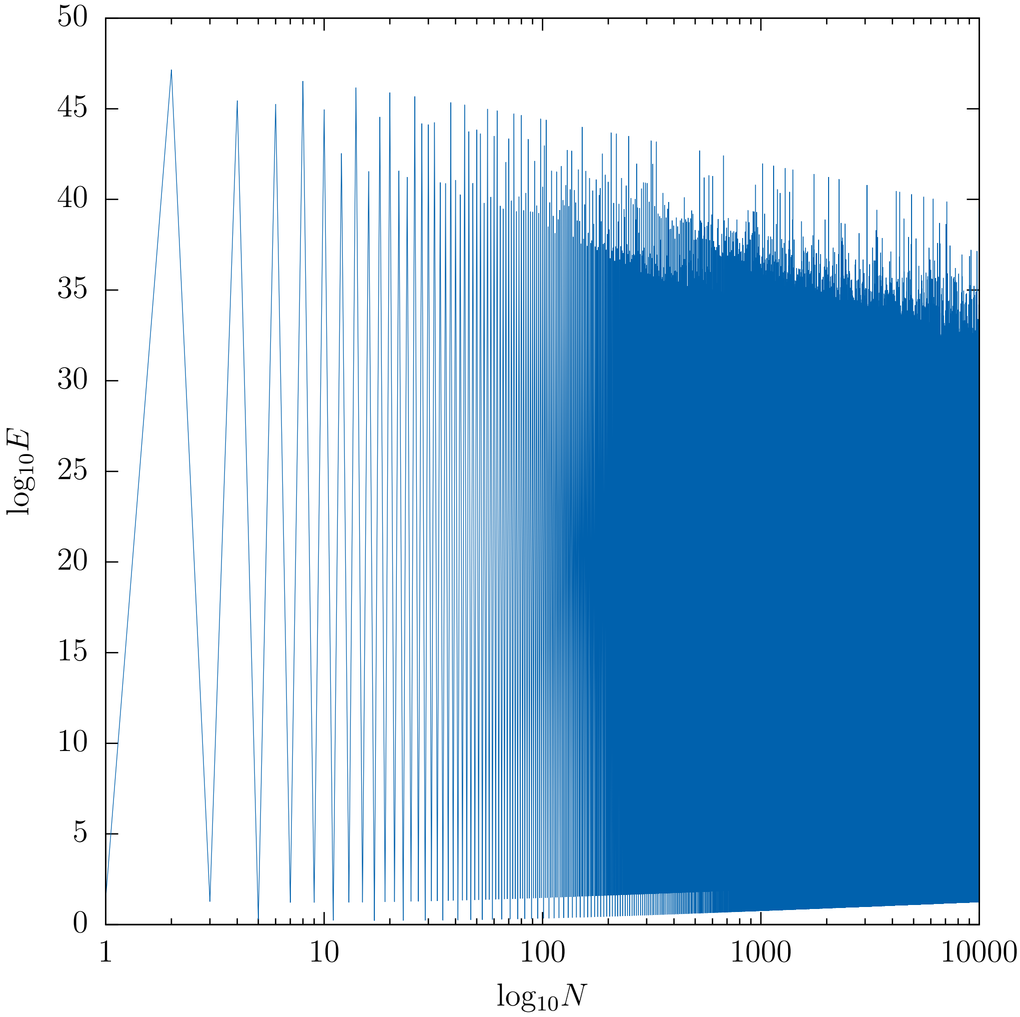

Figure 7: Plot of $\log_{10}E$ versus $\log_{10}N$ for numerical differentiation of first derivative using central difference method

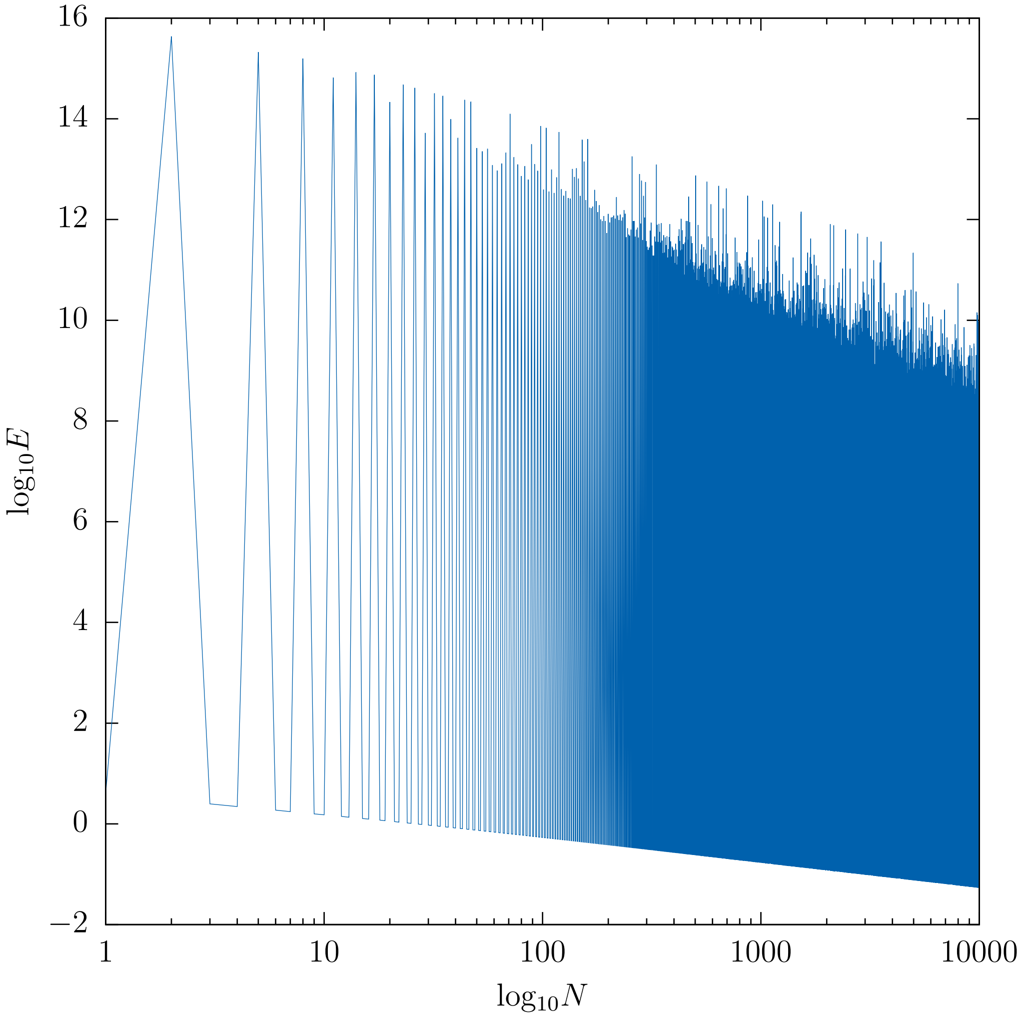

Figure 8: Plot of $\log_{10} E$ versus $\log_{10} N$ for numerical differentiation of second derivative using forward difference method

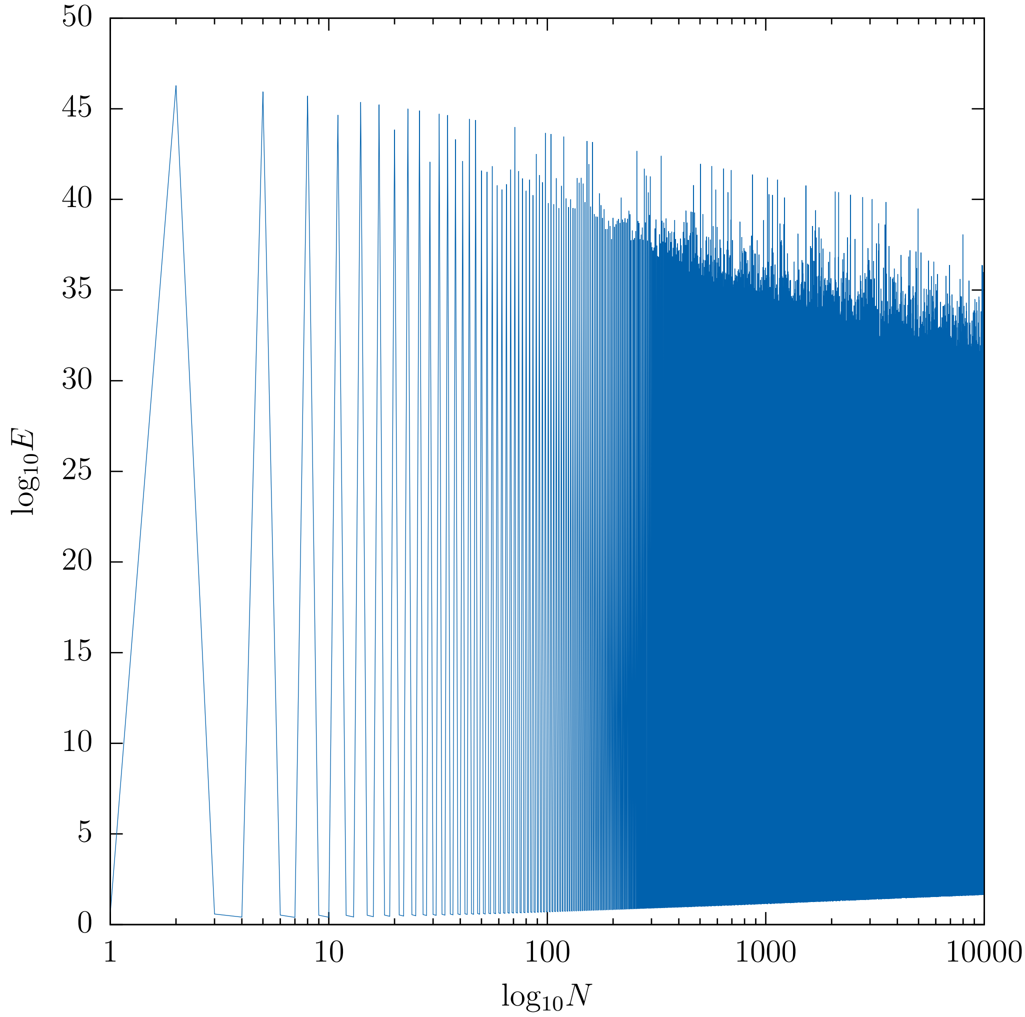

Figure 9: Plot of $\log_{10} E$ versus $\log_{10} N$ for numerical differentiation of second derivative using backward difference method

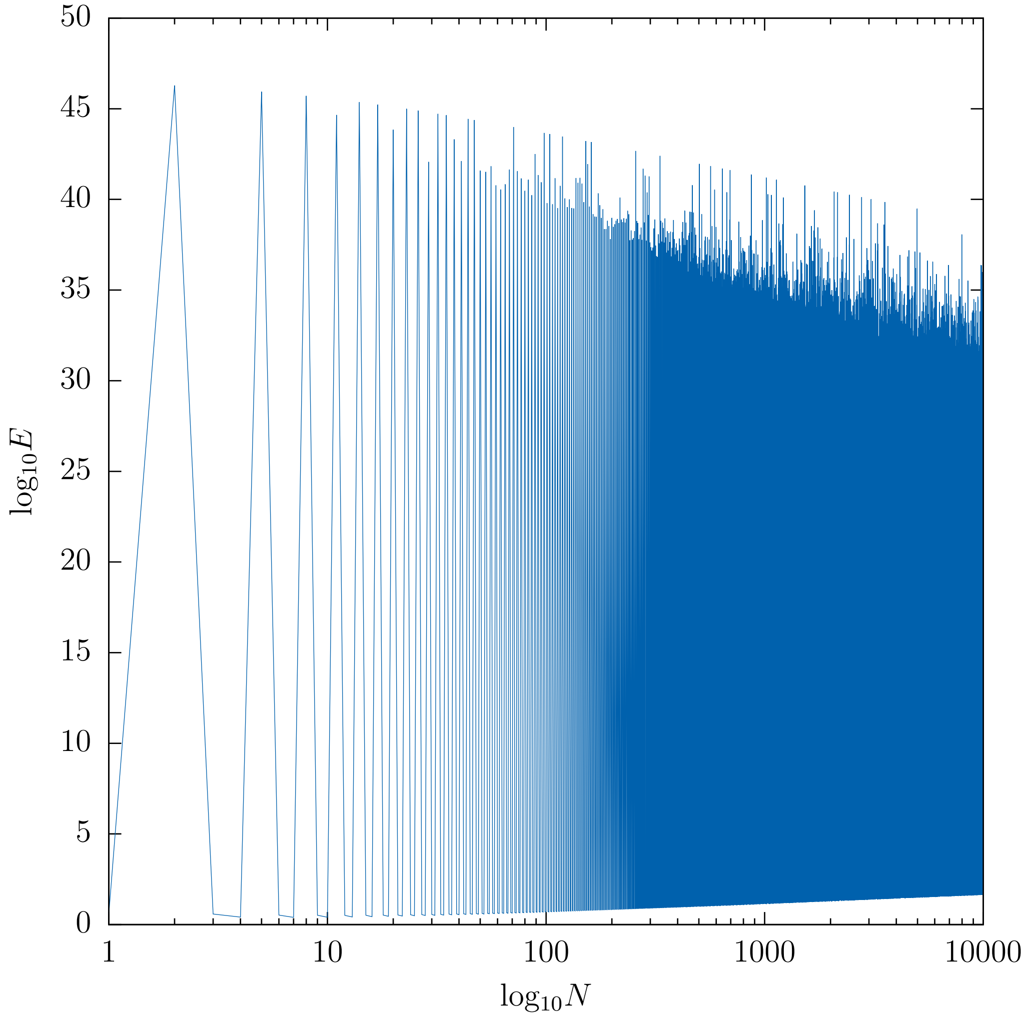

Figure 10: Plot of $\log_{10} E$ versus $\log_{10} N$ for numerical differentiation of second derivative using central difference method

Numerical solution for 1D heat transfer

Forward time center in space (FTCS) method was used with the following formula:

$$T_i^{n+1} = T_i^n + \Delta t(\alpha \frac{\partial^2 T}{\partial x^2}) \tag{8}$$