Progress Report

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Comparison of analytical and numerical solution of Laplace equation using SOR method

The following Laplace equation was considered:

$$\frac{\partial^2\phi}{\partial x^2} + \frac{\partial^2\phi}{\partial y^2} = 0 \tag{1}$$

The equation was discretized as follows:

$$\frac{\phi_{i+1,j} - 2\phi_{i,j} + \phi_{i-1,j}}{(\Delta x)^2} + \frac{\phi_{i,j+1} - 2\phi_{i,j} + \phi_{i,j-1}}{(\Delta y)^2} = 0 \tag{2}$$

This equation can be re-arranged as follows:

$$\phi_{i,j} = \left(\frac{\phi_{i+1,j} + \phi_{i-1,j}}{(\Delta x)^2} + \frac{\phi_{i,j+1} + \phi_{i,j-1}}{(\Delta y)^2}\right)\left(\frac{(\Delta x)^2(\Delta y)^2}{2[(\Delta x)^2 + (\Delta y)^2]}\right) \tag{3}$$

Jacobi Method

For Jacobi method, the above equation would take the following form:

$$\phi_{i,j}^{m+1} = \left(\frac{\phi_{i+1,j}^{m} + \phi_{i-1,j}^{m}}{(\Delta x)^2} + \frac{\phi_{i,j+1}^{m} + \phi_{i,j-1}^{m}}{(\Delta y)^2}\right)\left(\frac{(\Delta x)^2(\Delta y)^2}{2[(\Delta x)^2 + (\Delta y)^2]}\right) \tag{4}$$

Gauss-Seidel Method

In Gauss-Seidel method, the values of ϕ are utilized as soon as they are calculated and the equation takes the following form:

$$\phi_{i,j}^{m+1} = \left(\frac{\phi_{i+1,j}^{m} + \phi_{i-1,j}^{m+1}}{(\Delta x)^2} + \frac{\phi_{i,j+1}^{m} + \phi_{i,j-1}^{m+1}}{(\Delta y)^2}\right)\left(\frac{(\Delta x)^2(\Delta y)^2}{2[(\Delta x)^2 + (\Delta y)^2]}\right) \tag{5}$$

SOR Method

The equation changes for Successive Over-Relaxation method because of the relaxation factor:

$$\phi_{i,j}^{m+1} = (1-\omega)\phi_{i,j}^{m} + \omega \left(\frac{\phi_{i+1,j}^{m} + \phi_{i-1,j}^{m+1}}{(\Delta x)^2} + \frac{\phi_{i,j+1}^{m} + \phi_{i,j-1}^{m+1}}{(\Delta y)^2} \right) \left(\frac{(\Delta x)^2 (\Delta y)^2}{2[(\Delta x)^2 + (\Delta y)^2]} \right) \tag{6}$$

This equation was solved using $\omega = 1.8$ for a 50×50 grid. The error between the numerical solution and the exact solution is shown below in the figure for various tolerance values.

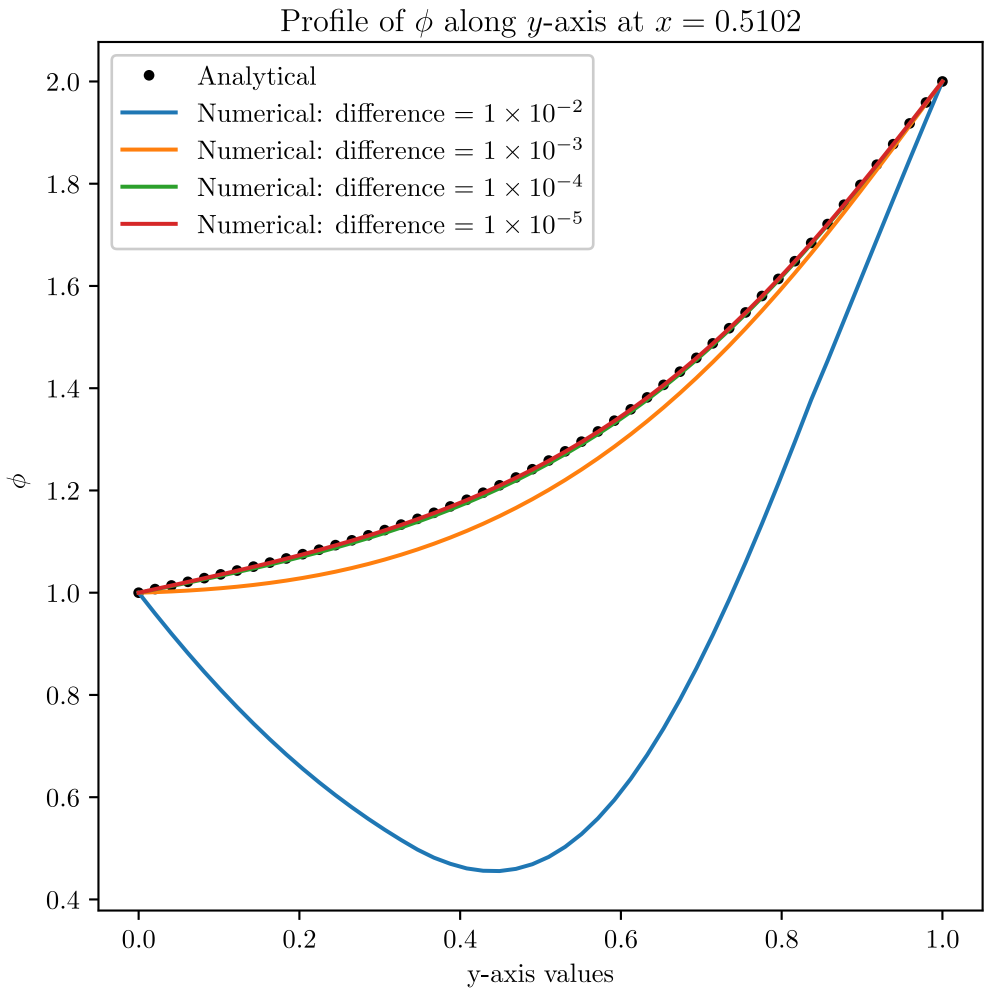

Figure 1: Comparison of numerical solution and exact solution of Laplace equation for various runs of SOR method using different tolerance (difference) values.