Progress Report

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2016.11.07

Numerical integration of sin(x) from 0 to π using two methods

The algorithms for numerical integration have been explored and the following two are utilized to calculate the result of integration of $\sin(x)$ from 0 to π:

1. Trapezoidal rule
2. Simpson's 1/3 rule

Both of these are part of the Newton-Cotes formulas. They involve using an easy approximating function in place of a complicated function or tabulated data.

The algorithm for both the methods was programmed in C++ and run using GNU C on Cygwin. The interval between the limits was divided into n number of segments with n ranging from 10 to 1000.

The results are compared with the analytical solution and graphs are used to display the log of ratio of difference ($\log(D)$ between the results versus the log of number of segments ($\log(N)$).

Analytical solution

$$I_A = \int_0^\pi \sin(x) = [-\cos(x)]_0^\pi = 2 \tag{1}$$

The analytical solution for this problem is 2.

Trapezoidal rule

$$I_N = (b-a)\frac{f(x_0) + 2\sum_{i=1}^{n-1} f(x_i) + f(x_n)}{2n} \tag{2}$$

Simpson's 1/3 rule

$$I_N = (b-a)\frac{f(x_0) + 4\sum_{i=1,3,5...}^{n-1} f(x_i) + 2\sum_{i=2,4,6...}^{n-2} f(x_i) + f(x_n)}{3n} \tag{3}$$

Conclusion

Simpson's 1/3 rule is found to converge to the analytical solution faster and is more accurate.

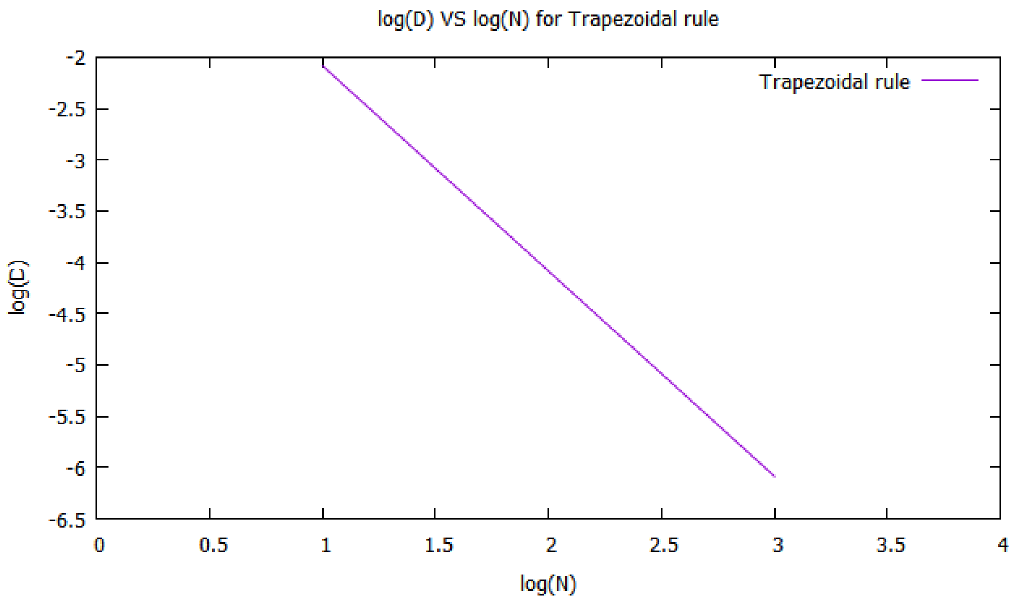

Figure 1: Plot of $\log(D)$ versus $\log(N)$ for trapezoidal rule

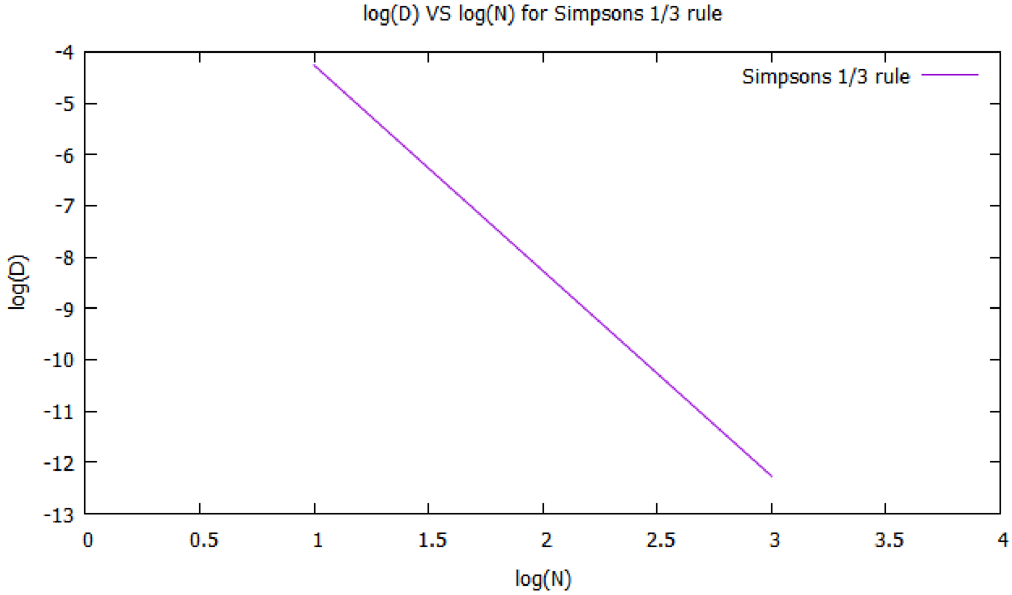

Figure 2: Plot of $\log(D)$ versus $\log(N)$ for Simpson's 1/3 rule

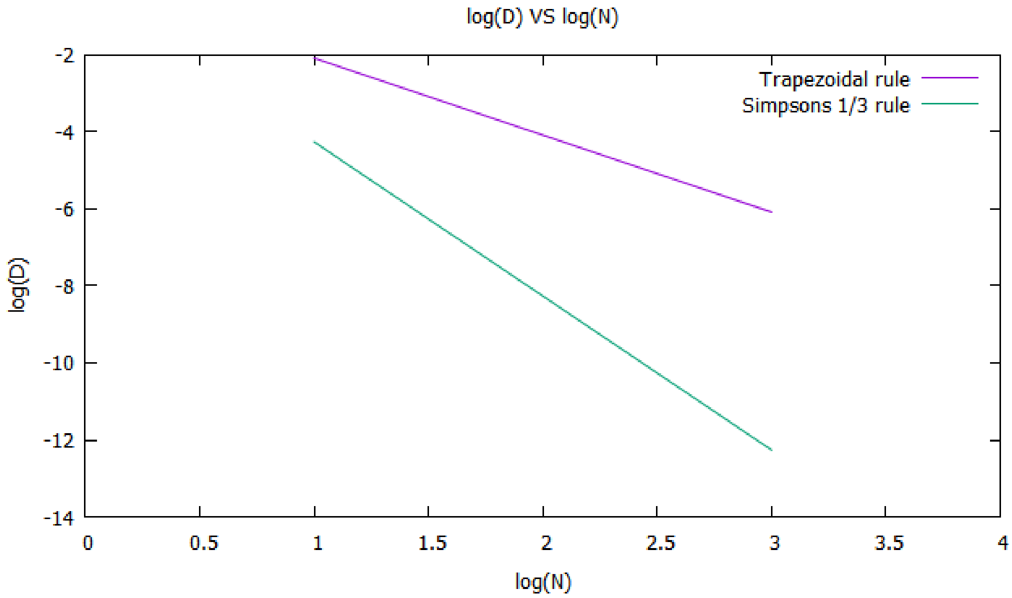

Figure 3: Plot of both algorithms together