Mandatory assignment 2
MEK4250

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Exercise 1

Exercise 7.1

We are to prove the three conditions (7.14)-(7.16) in the lecture notes for $V_h = H_0^1$ and $Q_h = L^2$. I will use the Cauchy-Schwarz inequality (C-S)

$$| < u, v > | \leq ||u|| \, ||v|| \tag{1}$$

and the Poincare inequality (PC)

$$||u||_{L^2} \leq C||\nabla u||_{L^2} = C|u|_{H^1} \tag{2}$$

Condition (7.14):

$$\begin{aligned}
a(u_h, v_h) &= \int \nabla u_h : \nabla v_h dx \\
&\leq \left(\left(\int \nabla u_h : \nabla v_h \right)^2 \right)^{1/2} \\
&= |\langle \nabla u_h, \nabla v_h \rangle| \\
&= ||\nabla u_h : \nabla v_h||_{L^2} \\
&\overset{\text{C-S}}{\leq} C||\nabla u_h||_{L^2} ||\nabla v_h||_{L^2} \\
&= C|u_h|_{H^1} |v_h|_{H^1} \\
&\leq C||u_h||_{H^1} ||v_h||_{H^1}
\end{aligned} \tag{3}$$

In the last inequality I used that a semi norm will yield a lesser value than a full norm.

Condition (7.15):

$$
\begin{aligned}
b(u_h, q_h) &= \int q_n \nabla \cdot u_h dx \\
&\leq \left(\left(\int q_n \nabla \cdot u_h dx \right)^2 \right)^{1/2} \\
&= ||q_h \nabla \cdot u_h||_{L^2} \\
&= | < q_h, \nabla \cdot u_h > | \\
&\overset{\text{C-S}}{\leq} C||q_h||_{L^2}||\nabla \cdot u_h||_{L^2} \\
&= C||q_h||_{L^2}|u_h|_{H^1} \\
&\leq C||q_h||_{L^2}||u_h||_{H^1}
\end{aligned}
\tag{4}
$$

Condition (7.16):

$$
a(\tag{5}
$$

Exercise 2

Mathematical model

The equation set we are to solve are

$$\begin{aligned} -\mu\Delta u - \lambda\nabla\nabla\cdot u &= f \qquad \text{in} \quad \Omega = (0,1)^2 \\ u &= u_e \qquad \text{on} \quad \partial\Omega \end{aligned} \tag{6}$$

with $u_e = \left(\frac{\partial\phi}{\partial y}, -\frac{\partial\phi}{\partial x}\right)$ where $\phi = sin(\pi xy)$. This gives $\nabla\cdot u_e = 0$.

Part a

Calculating $u_e = (\pi x cos(\pi xy), -\pi y cos(\pi xy))$ and inserting in equation 6 gives

$$\begin{aligned} f &= -\mu\Delta u_e - \lambda\nabla\nabla\cdot u_e \\ &= \end{aligned} \tag{7}$$