EARTHQUAKE AND NUCLEAR PLANT - IMPROVING AND SUSTAINING SAFETY

Seismic Site-Effects: Simplified numerical simulation

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8 décembre 2016

The M_W 8.0 Mexico City earthquake (1985)

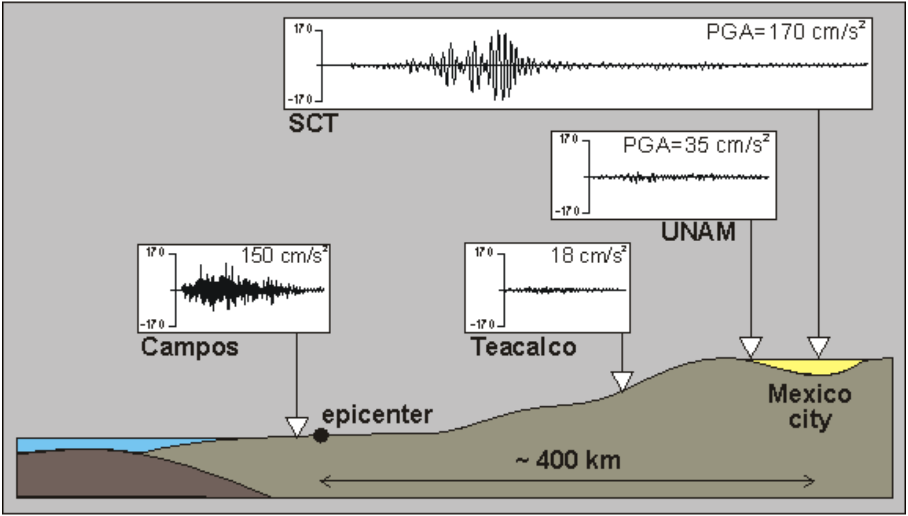


FIGURE 1 – From Wikipedia.
https://fr.m.wikipedia.org/wiki/Fichier:Site_effects_mexico_1985_recordings_v2.gif

From the source...

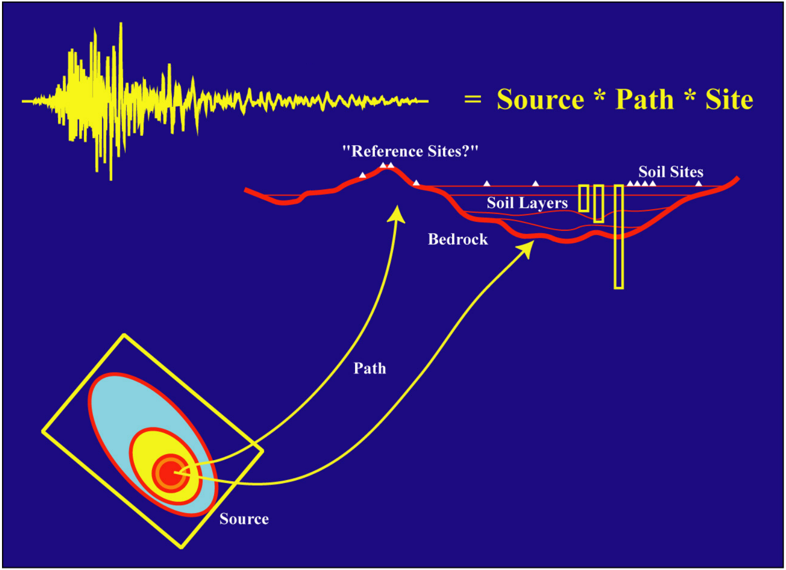
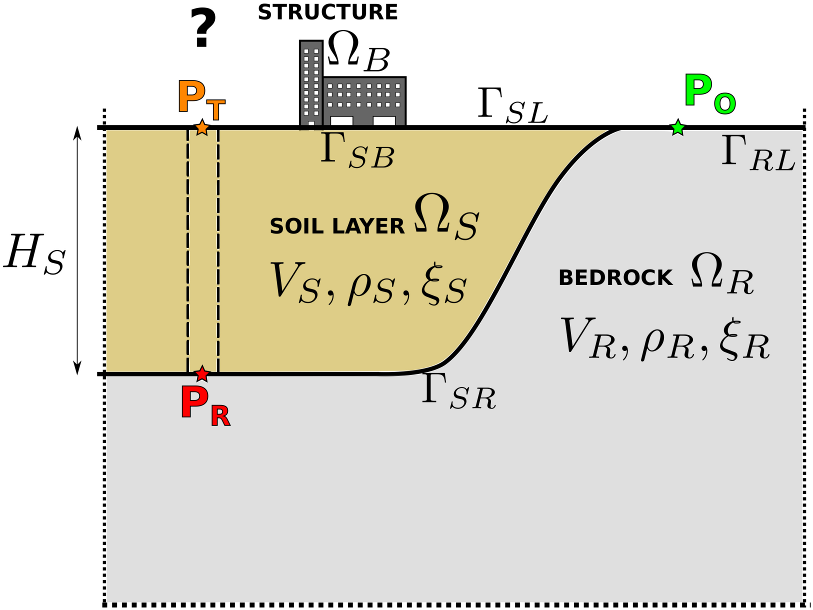
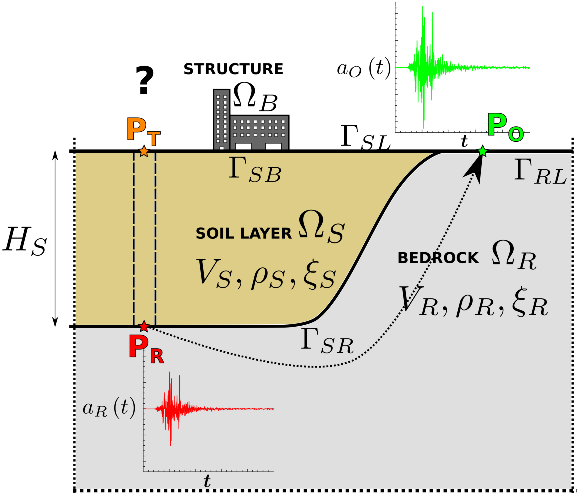


FIGURE 2 – After Boore (USGS).
http://daveboore.com/short_course_lectures_and_labs.html

...to the site

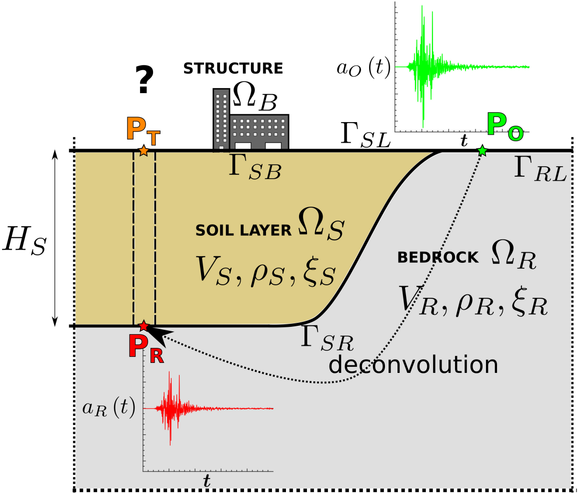

?
STRUCTURE
Ω_B
P_T
Γ_{SL}
P_O
Γ_{SB}
Γ_{RL}
H_S
SOIL LAYER Ω_S
V_S, ρ_S, ξ_S
BEDROCK Ω_R
V_R, ρ_R, ξ_R
Γ_{SR}
P_R

...to the site



- records available @P_R (deep bedrock)
- propagation through the soil layer $P_R \rightarrow P_T$
- input motion for structural analysis

...to the site



- records available @P_O (Outcrop Bedrock)
- **deconvolution** $P_O \rightarrow P_R$
- propagation through the soil layer $P_R \rightarrow P_T$

Soil dynamics : general framework

Navier-Stokes equations

- **Crustal Rocks**

$$\underline{\boldsymbol{\nabla}}_x \cdot \underline{\underline{\boldsymbol{\sigma}}}_R\left(\underline{\boldsymbol{u}}_R\right)\left(\underline{\boldsymbol{x}};t\right)+\rho_R\left(\underline{\boldsymbol{x}}\right)\underline{\boldsymbol{g}}=\rho_R\left(\underline{\boldsymbol{x}}\right)\partial_{tt}\underline{\boldsymbol{u}}_R\left(\underline{\boldsymbol{x}};t\right), \quad \left(\underline{\boldsymbol{x}};t\right)\in\left(\Omega_R,\mathbb{I}_t\right) \quad (1)$$

- **Soil deposit**

$$\underline{\boldsymbol{\nabla}}_x \cdot \underline{\underline{\boldsymbol{\sigma}}}_S\left(\underline{\boldsymbol{u}}_S\right)\left(\underline{\boldsymbol{x}};t\right)+\rho_S\left(\underline{\boldsymbol{x}}\right)\underline{\boldsymbol{g}}=\rho_S\left(\underline{\boldsymbol{x}}\right)\partial_{tt}\underline{\boldsymbol{u}}_S\left(\underline{\boldsymbol{x}};t\right) \quad \left(\underline{\boldsymbol{x}};t\right)\in\left(\Omega_S,\mathbb{I}_t\right) \quad (2)$$

- **Foundation**

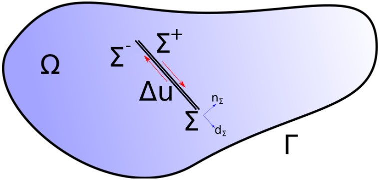
$$\underline{\boldsymbol{\nabla}}_x \cdot \underline{\underline{\boldsymbol{\sigma}}}_F\left(\underline{\boldsymbol{u}}_F\right)\left(\underline{\boldsymbol{x}};t\right)+\rho_F\left(\underline{\boldsymbol{x}}\right)\underline{\boldsymbol{g}}=\rho_F\left(\underline{\boldsymbol{x}}\right)\partial_{tt}\underline{\boldsymbol{u}}_F\left(\underline{\boldsymbol{x}};t\right) \quad \left(\underline{\boldsymbol{x}};t\right)\in\left(\Omega_F,\mathbb{I}_t\right) \quad (3)$$

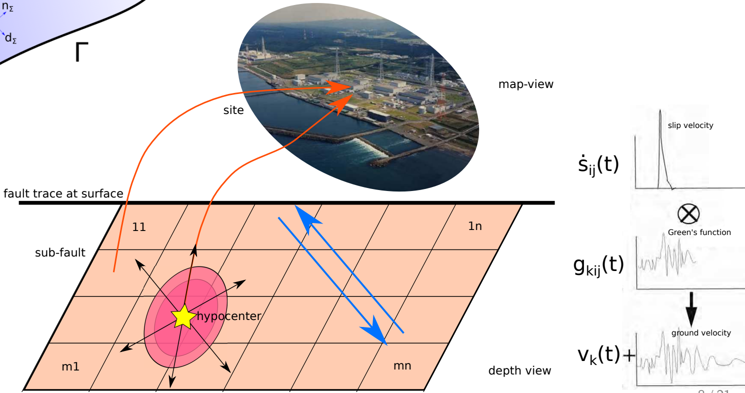
- **Structure**

$$\underline{\boldsymbol{\nabla}}_x \cdot \underline{\underline{\boldsymbol{\sigma}}}_B\left(\underline{\boldsymbol{u}}_B\right)\left(\underline{\boldsymbol{x}};t\right)+\rho_B\left(\underline{\boldsymbol{x}}\right)\underline{\boldsymbol{g}}+\Sigma_i\underline{\boldsymbol{f}}_i\left(\underline{\boldsymbol{x}};t\right)=\rho_B\left(\underline{\boldsymbol{x}}\right)\partial_{tt}\underline{\boldsymbol{u}}_B\left(\underline{\boldsymbol{x}};t\right) \quad \left(\underline{\boldsymbol{x}};t\right)\in\left(\Omega_B,\mathbb{I}_t\right) \quad (4)$$

Boundary condition - Seismic Fault

$$\underline{\boldsymbol{f}}_{\Sigma}\left(\underline{\boldsymbol{x}};t\right)=\underline{\boldsymbol{\nabla}}_{x}\cdot\left(\mathbb{D}^{el}_{R}\left(\underline{\boldsymbol{x}}\right):\Delta u\underline{\boldsymbol{d}}_{\Sigma}\otimes_{s}\delta_{\Sigma}\underline{\boldsymbol{n}}_{\Sigma}\right) \qquad (5)$$





Soil dynamics : general framework

Boundary condition - Bedrock-Soil

$$
\begin{aligned}
&\underline{\boldsymbol{u}}_S\left(\underline{\boldsymbol{x}};t\right) = \underline{\boldsymbol{u}}_R\left(\underline{\boldsymbol{x}};t\right), \quad \left(\underline{\boldsymbol{x}};t\right) \in \left(\Gamma_{SR}, \mathbb{I}_t\right) \\
&\underline{\underline{\boldsymbol{\sigma}}}_S\left(\underline{\boldsymbol{u}}_S\right)\left(\underline{\boldsymbol{x}};t\right).\underline{\boldsymbol{n}}_{SR} = \underline{\underline{\boldsymbol{\sigma}}}_R\left(\underline{\boldsymbol{u}}_R\right)\left(\underline{\boldsymbol{x}};t\right).\underline{\boldsymbol{n}}_{SR}, \quad \left(\underline{\boldsymbol{x}};t\right) \in \left(\Gamma_{SR}, \mathbb{I}_t\right) \\
&\underline{\underline{\boldsymbol{\sigma}}}_S\left(\underline{\boldsymbol{u}}_S\right)\left(\underline{\boldsymbol{x}};t\right).\underline{\boldsymbol{n}}_{SR} = 0, \quad \left(\underline{\boldsymbol{x}};t\right) \in \left(\Gamma_{SL}, \mathbb{I}_t\right)
\end{aligned}
\tag{6}
$$

Soil dynamics : general framework

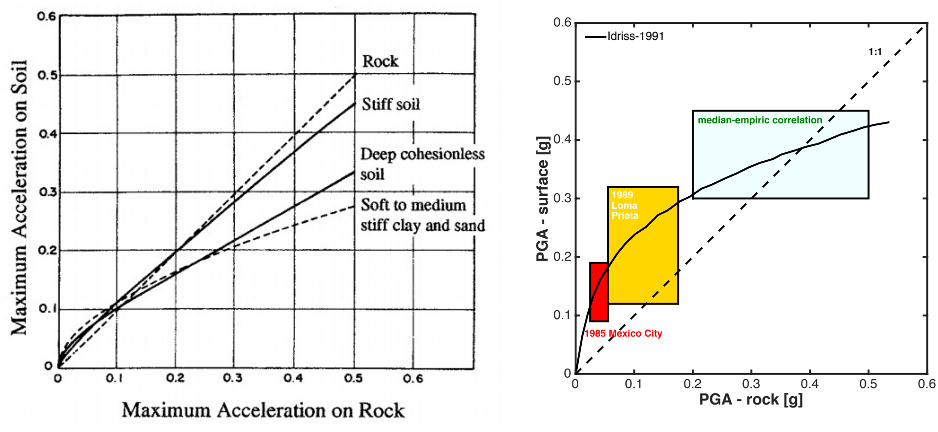
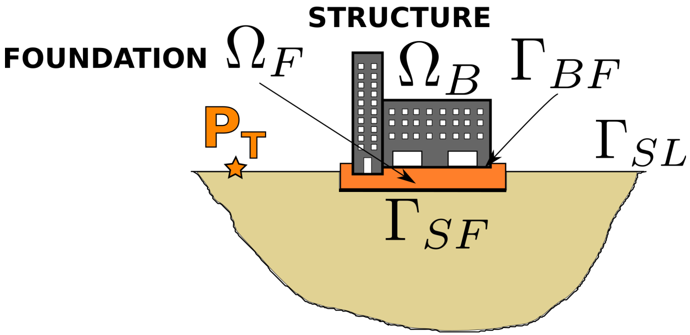


FIGURE 3 – Site effect amplification of the earthquake ground motion (after Beresnev and Wen (1996) [2] and Idriss (1991) [4]

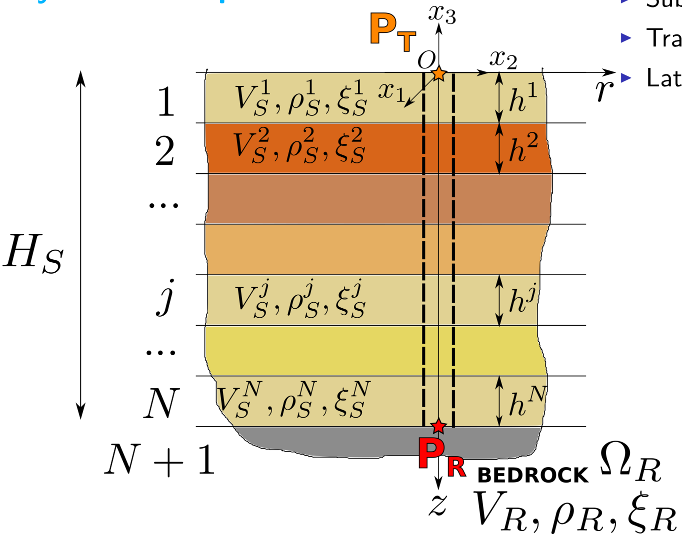
Boundary condition - Soil-Structure (SSI)

$$
\begin{aligned}
&\underline{\boldsymbol{u}}_S\left(\underline{\boldsymbol{x}};t\right) = \underline{\boldsymbol{u}}_F\left(\underline{\boldsymbol{x}};t\right), \quad \left(\underline{\boldsymbol{x}};t\right) \in \left(\Gamma_{SF}, \mathbb{I}_t\right)\\
&\underline{\boldsymbol{u}}_B\left(\underline{\boldsymbol{x}};t\right) = \underline{\boldsymbol{u}}_F\left(\underline{\boldsymbol{x}};t\right), \quad \left(\underline{\boldsymbol{x}};t\right) \in \left(\Gamma_{BF}, \mathbb{I}_t\right)\\
&\underline{\underline{\boldsymbol{\sigma}}}_S\left(\underline{\boldsymbol{u}}_S\right)\left(\underline{\boldsymbol{x}};t\right).\underline{\boldsymbol{n}}_{SR} = 0, \quad \left(\underline{\boldsymbol{x}};t\right) \in \left(\Gamma_{SL}, \mathbb{I}_t\right)
\end{aligned}
\tag{6}
$$



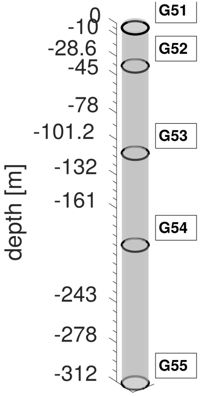
1D wave propagation : main assumptions

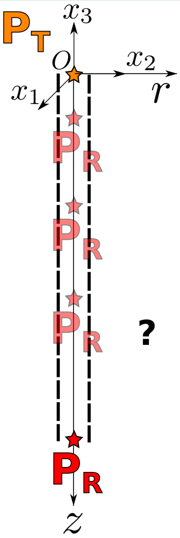
Layered Half-Space



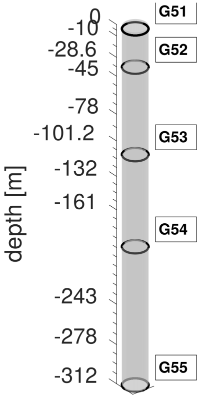
- Sub-Horizontal soil layers
- Transverse isotropy
- Lateral Homogeneity

1D wave propagation : main assumptions





1D wave propagation : main assumptions



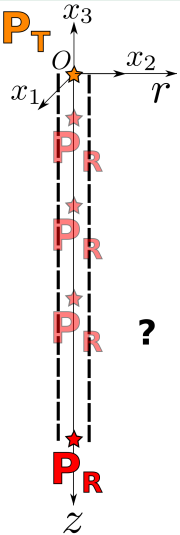
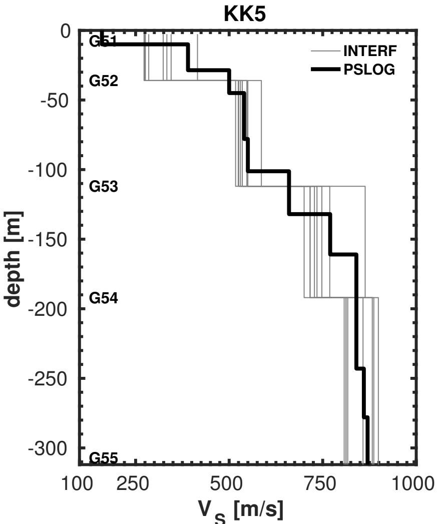


FIGURE 4 – V_S profile estimated via borehole interferometry at Kashiwazaki-Kariwa Nuclear Power Plant (Japon) (after Gatti et al. (2015) [3]



Space-Time Variable Separation

- $\underline{\boldsymbol{u}}_S(\underline{\boldsymbol{x}};t) = \underline{\boldsymbol{U}}_S(\underline{\boldsymbol{x}})\,\phi(t)$

Lateral Homogeneity

- $\underline{\boldsymbol{U}}_S(\underline{\boldsymbol{x}}) = \underline{\boldsymbol{U}}_S(x_1+v_1, x_2+v_2, x_3), \forall (v_1, v_2) \in \mathbb{R}^2, (x_1, x_2, x_3) \in \mathbb{R}^2 \times]-H_S, 0[$
  $\rightarrow \underline{\boldsymbol{U}}_S(\underline{\boldsymbol{x}}) = \underline{\boldsymbol{U}}_S(x_3)$

Pure SH Plane Wave Propagation

- $\frac{\partial(\underline{\boldsymbol{U}}_S.\underline{\boldsymbol{i}}_3)}{\partial x_3}(x_3) = 0$ ($\rightarrow$ vertical settlements not considered)
- $\underline{\boldsymbol{U}}_S(x_3).\underline{\boldsymbol{i}}_\theta = \underline{\boldsymbol{U}}_S(x_3).\underline{\boldsymbol{i}}_{\pi-\theta} = -\underline{\boldsymbol{U}}_S(x_3).\underline{\boldsymbol{i}}_\theta \rightarrow \underline{\boldsymbol{U}}_S(x_3).\underline{\boldsymbol{i}}_\theta = 0$
  $\rightarrow \underline{\boldsymbol{U}}_S(\underline{\boldsymbol{x}}) = U_{Sr}(z)\,\underline{\boldsymbol{i}}_r$

Harmonic Solutions IN TIME

- $\underline{\boldsymbol{u}}_S(\underline{\boldsymbol{x}};t) = \Re\left(\underline{\boldsymbol{U}}_S(\underline{\boldsymbol{x}})\,\phi(t)\right) = \Re\left(U_{Sr}(z)\,e^{i\omega t}\right)\underline{\boldsymbol{i}}_r$
- Strain Tensor : $\rightarrow \underline{\underline{\varepsilon}}_x(\underline{\boldsymbol{x}};t) = \left(\underline{\boldsymbol{u}}_S \otimes_s \underline{\boldsymbol{\nabla}}_x\right)(\underline{\boldsymbol{x}};t) = \Re\left(\frac{\partial U_{Sr}}{\partial z}e^{i\omega t}(z)\right)\underline{\boldsymbol{i}}_r \otimes \underline{\boldsymbol{i}}_z =$
  $\Re\left(\gamma_{Srz}(z)\,e^{i\omega t}\right)\underline{\boldsymbol{i}}_r \otimes \underline{\boldsymbol{i}}_z$

1D Navier-Stokes equation $\frac{\partial \tau_{Srz}}{\partial z}\left(\gamma_{Srz}(z,t)\right) = \rho_S(z)\,\partial_{tt}u_{Sr}(z,t)$

1D wave propagation : main assumptions

Space-Time Variable Separation

- $\underline{\boldsymbol{u}}_S(\underline{\boldsymbol{x}};t) = \underline{\boldsymbol{U}}_S(\underline{\boldsymbol{x}})\,\phi(t)$

Lateral Homogeneity

- $\underline{\boldsymbol{U}}_S(\underline{\boldsymbol{x}}) = \underline{\boldsymbol{U}}_S(x_1+v_1, x_2+v_2, x_3), \forall (v_1, v_2) \in \mathbb{R}^2, (x_1, x_2, x_3) \in \mathbb{R}^2 \times]-H_S, 0[$
  $\rightarrow \underline{\boldsymbol{U}}_S(\underline{\boldsymbol{x}}) = \underline{\boldsymbol{U}}_S(x_3)$

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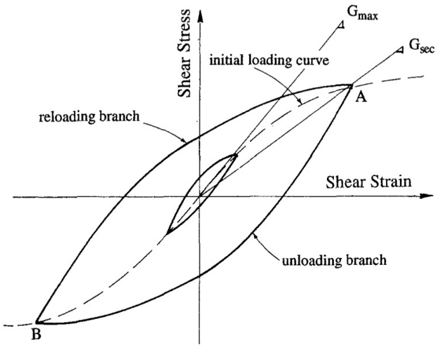
Harmonic Solutions IN TIME

- $\underline{\boldsymbol{u}}_S(\underline{\boldsymbol{x}};t) = \Re\left(\underline{\boldsymbol{U}}_S(\underline{\boldsymbol{x}})\,\phi(t)\right) = \Re\left(U_{Sr}(z)\,e^{i\omega t}\right)\underline{\boldsymbol{i}}_r$
- Strain Tensor : $\rightarrow \underline{\underline{\varepsilon}}_x(\underline{\boldsymbol{x}};t) = \left(\underline{\boldsymbol{u}}_S \otimes_s \underline{\boldsymbol{\nabla}}_x\right)(\underline{\boldsymbol{x}};t) = \Re\left(\frac{\partial U_{Sr}}{\partial z} e^{i\omega t}(z)\right)\underline{\boldsymbol{i}}_r \otimes \underline{\boldsymbol{i}}_z =$
  $\Re\left(\gamma_{Srz}(z)\,e^{i\omega t}\right)\underline{\boldsymbol{i}}_r \otimes \underline{\boldsymbol{i}}_z$

1D Navier-Stokes equation $\frac{\partial \tau_{Srz}}{\partial z}\left(\gamma_{Srz}(z,t)\right) = \rho_S(z)\,\partial_{tt} u_{Sr}(z,t)$

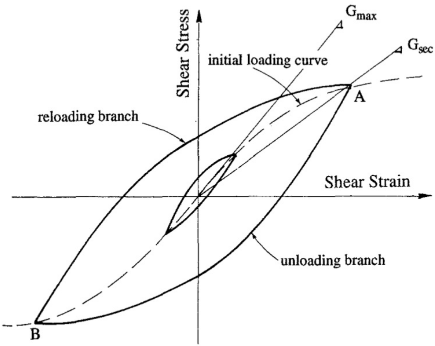
FIGURE 5 – Typical stress-strain relationship of soil in cyclic shear deformation (adapted from Pecker, 1984 [6]). Initial loading curve has a hyperbolic form (dashed line). Subsequent unloading and reloading phases track a hysteretic path. Two hysteresis loops constructed according to Masing's rules [5] are shown, where A and B mark the reversal points of the loop

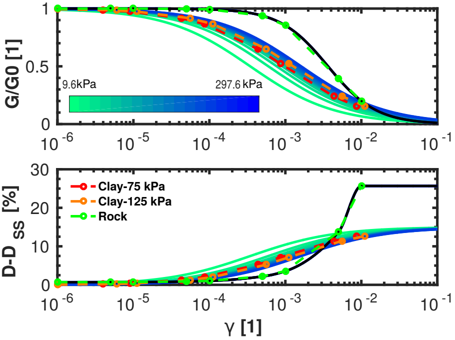


Cyclic behaviour of geo-materials

FIGURE 5 – Typical stress-strain relationship of soil in cyclic shear deformation (adapted from Pecker, 1984 [6]). Initial loading curve has a hyperbolic form (dashed line). Subsequent unloading and reloading phases track a hysteretic path. Two hysteresis loops constructed according to Masing's rules [5] are shown, where A and B mark the reversal points of the loop

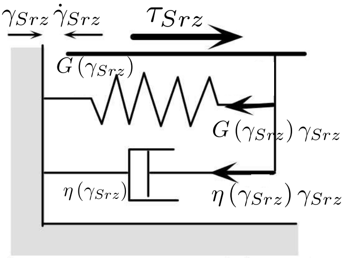
FIGURE 6 – Typical curves showing the dependence of shear modulus and damping ratio on strain amplitude of shallow subsoil at Kashiwazaki-Kariwa Nuclear Power Plant (after Gatti et al. 2015 [3]). Shear modulus G is normalized to its value at small strain (SS) G_{max}. Damping ratio D is subtracted from is small strain value D_{SS}.





EQuivalent Linear Model (EQLM)

FIGURE 7 – Schematic representation of stress-strain model used in equivalent-linear model (after Bardet et al. (2000) [1])



- **Rheological model**

  $\tau_{Srz}(z,t) =$
  $G(\gamma_{Srz})(z,t)\,\gamma_{Srz}(z,t) + \eta(\gamma_{Srz}(z,t))\,\dot{\gamma}_{Srz}(z,t)$

  $\dot{\gamma}_{Srz}(z,t) = \frac{\partial^2 u_{Sr}}{\partial z \partial t}(z,t)$

- **1D non-linear Navier-Stoke equation**

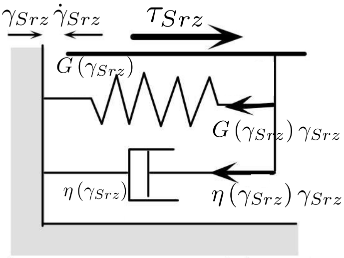
  $\frac{\partial}{\partial z}\left(G(\gamma_{Srz})(z,t)\,\frac{\partial u_{Sr}}{\partial z}(z,t)\right) +$
  $\frac{\partial}{\partial z}\left(\eta(\gamma_{Srz})(z,t)\,\frac{\partial^2 u_{Sr}}{\partial z \partial t}(z,t)\right) = \rho_S(z)\,\partial_{tt} u_{Sr}(z,t)$

- **Harmonic Solution IN TIME**

  $\frac{\partial}{\partial z}\left(G(\gamma_{Srz})(z,\omega)\,\frac{\partial U_{Sr}}{\partial z}(z)\right) +$
  $i\omega\frac{\partial}{\partial z}\left(\eta(\gamma_{Srz})(z,\omega)\,\frac{\partial U_{Sr}}{\partial z}(z)\right) = -\rho_S(z)\,\omega^2 U_{Sr}(z)$

EQuivalent Linear Model (EQLM)

FIGURE 7 – Schematic representation of stress-strain model used in equivalent-linear model (after Bardet et al. (2000) [1])



- **Rheological model**

  $\tau_{Srz}(z,t) =$
  $G(\gamma_{Srz})(z,t)\,\gamma_{Srz}(z,t) + \eta(\gamma_{Srz}(z,t))\,\dot{\gamma}_{Srz}(z,t)$

  $\dot{\gamma}_{Srz}(z,t) = \frac{\partial^2 u_{Sr}}{\partial z \partial t}(z,t)$

- **1D non-linear Navier-Stoke equation**

  $$\frac{\partial}{\partial z}\left(G(\gamma_{Srz})(z,t)\,\frac{\partial u_{Sr}}{\partial z}(z,t)\right) + \frac{\partial}{\partial z}\left(\eta(\gamma_{Srz})(z,t)\,\frac{\partial^2 u_{Sr}}{\partial z \partial t}(z,t)\right) = \rho_S(z)\,\partial_{tt} u_{Sr}(z,t)$$

- **Harmonic Solution IN TIME**

  $$\frac{\partial}{\partial z}\left(G(\gamma_{Srz})(z,\omega)\,\frac{\partial U_{Sr}}{\partial z}(z)\right) + i\omega\frac{\partial}{\partial z}\left(\eta(\gamma_{Srz})(z,\omega)\,\frac{\partial U_{Sr}}{\partial z}(z)\right) = -\rho_S(z)\,\omega^2 U_{Sr}(z)$$

How EQLM solves non-linear Navier-Stokes equation ?

EQLM Visco-Elastic approach

- **SH Plane Wave Time-Harmonic solution**
  $\Re\left(\gamma_{Srz}\left(z,t\right)\right) = \frac{\partial \Re(u_{Sr})}{\partial z}\left(z,t\right) = \frac{\partial U_{Sr}}{\partial z}\left(z\right)\cos\left(\omega t\right)$
  $\Re\left(\dot{\gamma}_{Srz}\left(z,t\right)\right) = \Re\left(i\omega \frac{\partial U_{Sr}}{\partial z}\left(z\right)e^{i\omega t}\right) = -\omega \frac{\partial U_{Sr}}{\partial z}\left(z\right)\sin\left(\omega t\right)$

EQLM Visco-Elastic approach

- **SH Plane Wave Time-Harmonic solution**
$\Re\left(\gamma_{Srz}\left(z,t\right)\right)=\frac{\partial\Re\left(u_{Sr}\right)}{\partial z}\left(z,t\right)=\frac{\partial U_{Sr}}{\partial z}\left(z\right)\cos\left(\omega t\right)$
$\Re\left(\dot{\gamma}_{Srz}\left(z,t\right)\right)=\Re\left(i\omega\frac{\partial U_{Sr}}{\partial z}\left(z\right)e^{i\omega t}\right)=-\omega\frac{\partial U_{Sr}}{\partial z}\left(z\right)\sin\left(\omega t\right)$

- **Kelvin-Voigt model & Complex Shear Modulus**
$\Re\left(\tau_{Srz}\left(\gamma_{Srz};\dot{\gamma}_{Srz}\right)\right)=\Re\left(\left(G\left(\gamma_{Srz}\right)+i\omega\eta\left(\gamma_{Srz}\right)\right)\frac{\partial U_{Sr}}{\partial z}e^{i\omega t}\right)=$
$\Re\left(G^{\star}\left(\gamma_{Srz};\omega\right)e^{i\omega t}\right)\frac{\partial U_{Sr}}{\partial z}=\left(G\left(\gamma_{Srz}\right)\cos\left(\omega t\right)-\eta\left(\gamma_{Srz}\right)\omega\sin\left(\omega t\right)\right)\frac{\partial U_{Sr}}{\partial z}$

EQLM Visco-Elastic approach

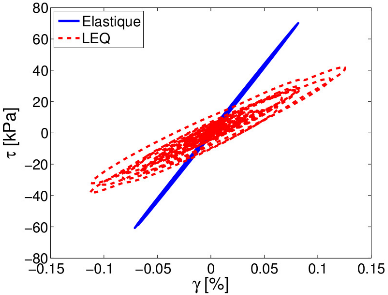
- **SH Plane Wave Time-Harmonic solution**

$$\Re\left(\gamma_{Srz}\left(z,t\right)\right) = \frac{\partial \Re(u_{Sr})}{\partial z}\left(z,t\right) = \frac{\partial U_{Sr}}{\partial z}\left(z\right)\cos\left(\omega t\right)$$
$$\Re\left(\dot{\gamma}_{Srz}\left(z,t\right)\right) = \Re\left(i\omega\frac{\partial U_{Sr}}{\partial z}\left(z\right)e^{i\omega t}\right) = -\omega\frac{\partial U_{Sr}}{\partial z}\left(z\right)\sin\left(\omega t\right)$$

- **Kelvin-Voigt model & Complex Shear Modulus**

$$\Re\left(\tau_{Srz}\left(\gamma_{Srz};\dot{\gamma}_{Srz}\right)\right) = \Re\left(\left(G\left(\gamma_{Srz}\right) + i\omega\eta\left(\gamma_{Srz}\right)\right)\frac{\partial U_{Sr}}{\partial z}e^{i\omega t}\right) =$$
$$\Re\left(G^{\star}\left(\gamma_{Srz};\omega\right)e^{i\omega t}\right)\frac{\partial U_{Sr}}{\partial z} = \left(G\left(\gamma_{Srz}\right)\cos\left(\omega t\right) - \eta\left(\gamma_{Srz}\right)\omega\sin\left(\omega t\right)\right)\frac{\partial U_{Sr}}{\partial z}$$

Parametric equations of an ellipse!



EQLM Visco-Elastic approach

- **SH Plane Wave Time-Harmonic solution**

$$\Re\left(\gamma_{Srz}\left(z,t\right)\right) = \frac{\partial \Re\left(u_{Sr}\right)}{\partial z}\left(z,t\right) = \frac{\partial U_{Sr}}{\partial z}\left(z\right)\cos\left(\omega t\right)$$

$$\Re\left(\dot{\gamma}_{Srz}\left(z,t\right)\right) = \Re\left(i\omega\frac{\partial U_{Sr}}{\partial z}\left(z\right)e^{i\omega t}\right) = -\omega\frac{\partial U_{Sr}}{\partial z}\left(z\right)\sin\left(\omega t\right)$$

- **Kelvin-Voigt model & Complex Shear Modulus**

$$\Re\left(\tau_{Srz}\left(\gamma_{Srz};\dot{\gamma}_{Srz}\right)\right) = \Re\left(\left(G\left(\gamma_{Srz}\right) + i\omega\eta\left(\gamma_{Srz}\right)\right)\frac{\partial U_{Sr}}{\partial z}e^{i\omega t}\right) =$$

$$\Re\left(G^{\star}\left(\gamma_{Srz};\omega\right)e^{i\omega t}\right)\frac{\partial U_{Sr}}{\partial z} = \left(G\left(\gamma_{Srz}\right)\cos\left(\omega t\right) - \eta\left(\gamma_{Srz}\right)\omega\sin\left(\omega t\right)\right)\frac{\partial U_{Sr}}{\partial z}$$

- **Maximum strain (Hysteresis)**

$$\mathcal{E}_{EL}\left(\gamma_{Srz}\right) = \oint_{\bar{\gamma}_{Srz}}\Re\left(\tau_{Srz}\left(\gamma_{Srz}\left(z,s\right)\right)\right)\Re\left(\dot{\gamma}_{Srz}\left(z,s\right)\right)ds = \frac{1}{2}G\left(\gamma_{Srz}\right)\left(\frac{\partial U_{Sr}}{\partial z}\right)^2$$

EQLM Visco-Elastic approach

- **SH Plane Wave Time-Harmonic solution**
$$\Re\left(\gamma_{Srz}\left(z,t\right)\right) = \frac{\partial \Re(u_{Sr})}{\partial z}\left(z,t\right) = \frac{\partial U_{Sr}}{\partial z}\left(z\right)\cos\left(\omega t\right)$$
$$\Re\left(\dot{\gamma}_{Srz}\left(z,t\right)\right) = \Re\left(i\omega\frac{\partial U_{Sr}}{\partial z}\left(z\right)e^{i\omega t}\right) = -\omega\frac{\partial U_{Sr}}{\partial z}\left(z\right)\sin\left(\omega t\right)$$

- **Kelvin-Voigt model & Complex Shear Modulus**
$$\Re\left(\tau_{Srz}\left(\gamma_{Srz};\dot{\gamma}_{Srz}\right)\right) = \Re\left(\left(G\left(\gamma_{Srz}\right)+i\omega\eta\left(\gamma_{Srz}\right)\right)\frac{\partial U_{Sr}}{\partial z}e^{i\omega t}\right) =$$
$$\Re\left(G^{\star}\left(\gamma_{Srz};\omega\right)e^{i\omega t}\right)\frac{\partial U_{Sr}}{\partial z} = \left(G\left(\gamma_{Srz}\right)\cos\left(\omega t\right)-\eta\left(\gamma_{Srz}\right)\omega\sin\left(\omega t\right)\right)\frac{\partial U_{Sr}}{\partial z}$$

- **Maximum strain (Hysteresis)**
$$\mathcal{E}_{EL}\left(\gamma_{Srz}\right) = \oint_{\bar{\gamma}_{Srz}}\Re\left(\tau_{Srz}\left(\gamma_{Srz}\left(z,s\right)\right)\right)\Re\left(\dot{\gamma}_{Srz}\left(z,s\right)\right)ds = \frac{1}{2}G\left(\gamma_{Srz}\right)\left(\frac{\partial U_{Sr}}{\partial z}\right)^2$$

- **Dissipated strain energy (Hysteresis)**
$$\mathcal{D}_{KV}\left(\gamma_{Srz};\omega\right) = \oint_{\bar{\gamma}_{Srz}}\Re\left(\tau_{Srz}\left(\gamma_{Srz}\left(z,s\right);\dot{\gamma}_{Srz}\left(z,s\right)\right)\right)\Re\left(\dot{\gamma}_{Srz}\left(z,s\right)\right)ds = \pi\omega\eta\left(\frac{\partial U_{Sr}}{\partial z}\right)$$

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EQLM Visco-Elastic approach

- **SH Plane Wave Time-Harmonic solution**

$$\Re\left(\gamma_{Srz}\left(z,t\right)\right)=\frac{\partial\Re(u_{Sr})}{\partial z}\left(z,t\right)=\frac{\partial U_{Sr}}{\partial z}\left(z\right)\cos\left(\omega t\right)$$
$$\Re\left(\dot{\gamma}_{Srz}\left(z,t\right)\right)=\Re\left(i\omega\frac{\partial U_{Sr}}{\partial z}\left(z\right)e^{i\omega t}\right)=-\omega\frac{\partial U_{Sr}}{\partial z}\left(z\right)\sin\left(\omega t\right)$$

- **Kelvin-Voigt model & Complex Shear Modulus**

$$\Re\left(\tau_{Srz}\left(\gamma_{Srz};\dot{\gamma}_{Srz}\right)\right)=\Re\left(\left(G\left(\gamma_{Srz}\right)+i\omega\eta\left(\gamma_{Srz}\right)\right)\frac{\partial U_{Sr}}{\partial z}e^{i\omega t}\right)=$$
$$\Re\left(G^{\star}\left(\gamma_{Srz};\omega\right)e^{i\omega t}\right)\frac{\partial U_{Sr}}{\partial z}=\left(G\left(\gamma_{Srz}\right)\cos\left(\omega t\right)-\eta\left(\gamma_{Srz}\right)\omega\sin\left(\omega t\right)\right)\frac{\partial U_{Sr}}{\partial z}$$

- **Maximum strain (Hysteresis)**

$$\mathcal{E}_{EL}\left(\gamma_{Srz}\right)=\oint_{\bar{\gamma}_{Srz}}\Re\left(\tau_{Srz}\left(\gamma_{Srz}\left(z,s\right)\right)\right)\Re\left(\dot{\gamma}_{Srz}\left(z,s\right)\right)ds=\frac{1}{2}G\left(\gamma_{Srz}\right)\left(\frac{\partial U_{Sr}}{\partial z}\right)^{2}$$

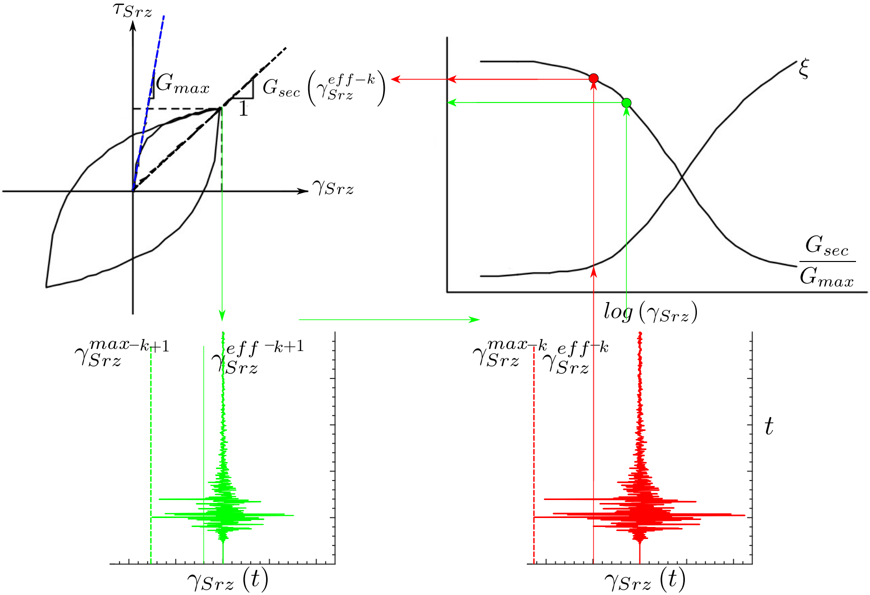
- **Dissipated strain energy (Hysteresis)**

$$\mathcal{D}_{KV}\left(\gamma_{Srz};\omega\right)=\oint_{\bar{\gamma}_{Srz}}\Re\left(\tau_{Srz}\left(\gamma_{Srz}\left(z,s\right);\dot{\gamma}_{Srz}\left(z,s\right)\right)\right)\Re\left(\dot{\gamma}_{Srz}\left(z,s\right)\right)ds=\pi\omega\eta\left(\frac{\partial U_{Sr}}{\partial z}\right)$$

- **Definition of Critical Damping Ratio**

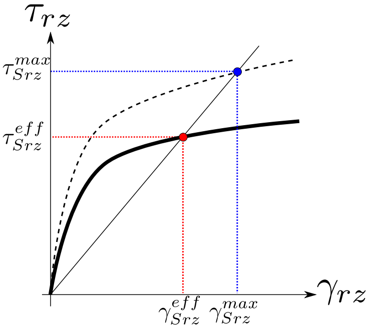
$$\xi_{S}\left(\gamma_{Srz};\omega\right)=\frac{\mathcal{D}_{KV}(\gamma_{Srz};\omega)}{4\pi\mathcal{E}_{EL}(\gamma_{Srz})}=\frac{\omega\eta(\gamma_{Srz})}{2G(\gamma_{Srz})}$$

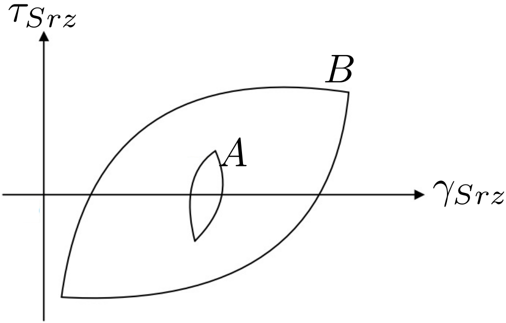
EQLM : Iterative solution



EQLM : further aspects

- **Strain softening excluded (ill-posed problem)**
  $\rightarrow \frac{\partial \tau_{Srz}}{\partial \gamma_{Srz}} = G\left(\gamma_{Srz}\right) + \frac{\partial G}{\partial \gamma_{Srz}} \gamma_{Srz} \geq 0$
- $\mathbf{\frac{G}{G_{max}}} - \gamma -$ **D curves numerical discretization :**
  $\rightarrow -\frac{\Delta G}{G_{max}}\left(\gamma^n_{Srz}\right) \leq \frac{\Delta \gamma^n_{Srz}}{\gamma^n_{Srz}} \frac{G(\gamma^n_{Srz})}{G_{max}}, \Delta G\left(\gamma^n_{Srz}\right) < 0 \Leftrightarrow \Delta \gamma^n_{Srz} > 0$
- **Convergence achieved :**
  $\Delta \gamma^n_{Srz} \nearrow \rightarrow \left|\Delta G\left(\gamma^n_{Srz}\right)\right| \searrow$





1D Multi-Layer SH Plane-Wave propagation

- **Equivalent Linear Solution**

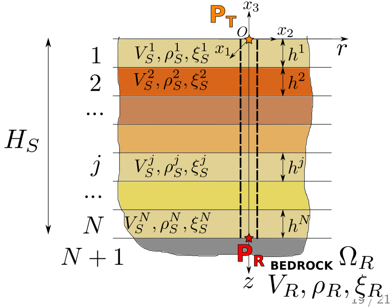
$$u_{Sr}(z,t) = \Re\left(\left(\sum_j^N U_{Sr}^j\left(z^j\right)\right)e^{i\omega t}\right), \quad z^j \in \left]0; h^j\right[$$

- **1D Navier Stokes equation**

$$\left(G^j\left(\gamma_{Srz}^{j-eff}\left(\frac{h^j}{2}\right)\right) + i\omega\eta^j\left(\gamma_{Srz}^{j-eff}\left(\frac{h^j}{2}\right)\right)\right)\frac{\partial^2 U_{Sr}^j}{\partial z^{j2}}\left(z^j\right) = -\rho_S^j\left(z^j\right)\omega^2 U_{Sr}^j\left(z^j\right)$$

$$G^{j\star}\left(\gamma_{Srz}^{j-eff}\left(\frac{h^j}{2}\right)\right)\frac{\partial^2 U_{Sr}^j}{\partial z^{j2}}\left(z^j\right) = -\rho_S^j\left(\frac{h^j}{2}\right)\omega^2 U_{Sr}^j\left(z^j\right)$$

$\frac{\partial^2 U_{Sr}^j}{\partial z^{j2}}\left(z^j\right) + \frac{1}{V_S^{\star j2}}U_{Sr}^j\left(z^j\right) = 0 \leftarrow$ **wave equation with complex velocity**



1D Seismic Site Response by EQLM

1D Multi-Layer SH Plane-Wave propagation

- **Equivalent Linear Solution**

  $u_{Sr}(z,t) = \Re\left(\left(\sum_j^N U^j_{Sr}\left(z^j\right)\right)e^{i\omega t}\right), \quad z^j \in \left]0; h^j\right[$

- **1D Navier Stokes equation**

  $$\left(G^j\left(\gamma^{j-eff}_{Srz}\left(\frac{h^j}{2}\right)\right) + i\omega\eta^j\left(\gamma^{j-eff}_{Srz}\left(\frac{h^j}{2}\right)\right)\right)\frac{\partial^2 U^j_{Sr}}{\partial z^{j2}}\left(z^j\right) = -\rho^j_S\left(z^j\right)\omega^2 U^j_{Sr}\left(z^j\right)$$

  $$G^{j\star}\left(\gamma^{j-eff}_{Srz}\left(\frac{h^j}{2}\right)\right)\frac{\partial^2 U^j_{Sr}}{\partial z^{j2}}\left(z^j\right) = -\rho^j_S\left(\frac{h^j}{2}\right)\omega^2 U^j_{Sr}\left(z^j\right)$$

  $\frac{\partial^2 U^j_{Sr}}{\partial z^{j2}}\left(z^j\right) + \frac{1}{V^{\star j2}_S}U^j_{Sr}\left(z^j\right) = 0 \leftarrow$ **wave equation with complex velocity**

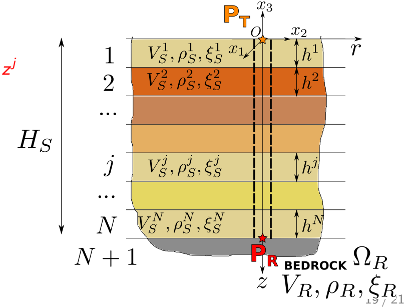
- **Harmonic solution IN SPACE**

  $U^j_{Sr}\left(z^j\right) = E^j_{Sr}\left(\frac{h^j}{2}\right)e^{ik^{\star j}z^j} + F^j_{Sr}\left(\frac{h^j}{2}\right)e^{-ik^{\star j}z^j}$

  $k^{\star j} = \frac{\omega}{V^{\star j}_S}$

  $E^j_{Sr}\left(\frac{h^j}{2}\right)\uparrow$-$F^j_{Sr}\left(\frac{h^j}{2}\right)\downarrow$

  $2n+1$ unknowns (included downward propagating wave F^{N+1}_{Sr} into bedrock)



1D Multi-Layer SH Plane-Wave propagation

- **N Layer-to-Layer displacement continuity equations**
  $U^j_{Sr}\left(h^j\right) = U^{j+1}_{Sr}\left(0\right), \forall j \in [1, N] \cap \mathbb{N}$
  $\rightarrow E^j_{Sr}\left(\frac{h^j}{2}\right) e^{ik^{\star j} h^j} + F^j_{Sr}\left(\frac{h^j}{2}\right) e^{-ik^{\star j} h^j} = E^{j+1}_{Sr}\left(\frac{h^{j+1}}{2}\right) + F^{j+1}_{Sr}\left(\frac{h^{j+1}}{2}\right)$
- **N Layer-to-Layer traction continuity equations**
  $G^\star\left(\gamma^{j-eff}_{Sr}\left(\frac{h^j}{2}\right)\right)\frac{\partial U^j_{Sr}}{\partial z^j}\left(h^j\right) = G^\star\left(\gamma^{j+1-eff}_{Sr}\left(\frac{h^{j+1}}{2}\right)\right)\frac{\partial U^{j+1}_{Sr}}{\partial z^j}\left(0\right), \forall j \in [1, N] \cap \mathbb{N}$
  $\rightarrow$
  $G^\star\left(\gamma^{j-eff}_{Sr}\left(\frac{h^j}{2}\right)\right) k^{\star j} E^j_{Sr}\left(\frac{h^j}{2}\right) e^{ik^{\star j} h^j} - G^\star\left(\gamma^{j-eff}_{Sr}\left(\frac{h^j}{2}\right)\right) k^{\star j} F^j_{Sr}\left(\frac{h^j}{2}\right) e^{-ik^{\star j} h^j} =$
  $G^\star\left(\gamma^{j+1-eff}_{Sr}\left(\frac{h^{j+1}}{2}\right)\right) k^{\star j+1} E^{j+1}_{Sr}\left(\frac{h^{j+1}}{2}\right) -$
  $G^\star\left(\gamma^{j+1-eff}_{Sr}\left(\frac{h^{j+1}}{2}\right)\right) k^{\star j+1} F^{j+1}_{Sr}\left(\frac{h^{j+1}}{2}\right)$
  $\gamma_{Srz}\left(h^j, t\right) = ik^{\star j}\left(E^j_{Sr}\left(\frac{h^j}{2}\right) e^{ik^{\star j} h^j} - F^j_{Sr}\left(\frac{h^j}{2}\right) e^{-ik^{\star j} h^j}\right)$
- **Impedance ratios**
  $\alpha^{\star j}\left(h^j\right) = \frac{G^\star\left(\gamma^{j-eff}_{Sr}\left(\frac{h^j}{2}\right)\right) k^{\star j}}{G^\star\left(\gamma^{j+1-eff}_{Sr}\left(\frac{h^{j+1}}{2}\right)\right) k^{\star j+1}}$

1D Multi-Layer SH Plane-Wave propagation

- $2N$ **Interface recursive conditions**
$$E_{Sr}^{j+1}\left(\frac{h^{j+1}}{2}\right)=\frac{1}{2}E_{Sr}^{j}\left(\frac{h^{j}}{2}\right)\left(1+\alpha^{\star j}\left(h^{j}\right)\right)e^{ik^{\star j}h^{j}}+\frac{1}{2}F_{Sr}^{j}\left(\frac{h^{j}}{2}\right)\left(1-\alpha^{\star j}\left(h^{j}\right)\right)e^{-ik^{\star j}h^{j}}$$
$$F_{Sr}^{j+1}\left(\frac{h^{j+1}}{2}\right)=\frac{1}{2}E_{Sr}^{j}\left(\frac{h^{j}}{2}\right)\left(1-\alpha^{\star j}\left(h^{j}\right)\right)e^{ik^{\star j}h^{j}}+\frac{1}{2}F_{Sr}^{j}\left(\frac{h^{j}}{2}\right)\left(1+\alpha^{\star j}\left(h^{j}\right)\right)e^{-ik^{\star j}h^{j}}$$
- **System of** $2N+1$ **equations in** $2N+1$ **unknowns**
$$\underline{\boldsymbol{V}}_{Sr}^{j+1}\left(\frac{h^{j+1}}{2}\right)=\underline{\underline{\boldsymbol{P}}}^{j}\left(k^{\star j};\alpha^{\star j}\left(h^{j}\right)\right)\underline{\boldsymbol{V}}_{Sr}^{j}\left(\frac{h^{j}}{2}\right)$$
$$\underline{\boldsymbol{V}}_{Sr}^{N+1}=\underline{\underline{\mathbb{P}}}\left(k^{\star 1,\dots,N};\alpha^{\star j,\dots,N}\left(h^{1,\dots,N}\right)\right)\underline{\boldsymbol{V}}_{Sr}^{1}\left(\frac{h^{1}}{2}\right)$$
$$\rightarrow\underline{\underline{\mathbb{P}}}\left(k^{\star 1,\dots,N};\alpha^{\star j,\dots,N}\left(h^{1,\dots,N}\right)\right)=\prod_{j=N}^{1}\underline{\underline{\boldsymbol{P}}}^{j}\left(k^{\star j};\alpha^{\star j}\left(h^{j}\right)\right)$$

1D Multi-Layer SH Plane-Wave propagation

- $2N$ **Interface recursive conditions**

$$E_{Sr}^{j+1}\left(\frac{h^{j+1}}{2}\right)=\frac{1}{2}E_{Sr}^{j}\left(\frac{h^{j}}{2}\right)\left(1+\alpha^{\star j}\left(h^{j}\right)\right)e^{ik^{\star j}h^{j}}+\frac{1}{2}F_{Sr}^{j}\left(\frac{h^{j}}{2}\right)\left(1-\alpha^{\star j}\left(h^{j}\right)\right)e^{-ik^{\star j}h^{j}}$$

$$F_{Sr}^{j+1}\left(\frac{h^{j+1}}{2}\right)=\frac{1}{2}E_{Sr}^{j}\left(\frac{h^{j}}{2}\right)\left(1-\alpha^{\star j}\left(h^{j}\right)\right)e^{ik^{\star j}h^{j}}+\frac{1}{2}F_{Sr}^{j}\left(\frac{h^{j}}{2}\right)\left(1+\alpha^{\star j}\left(h^{j}\right)\right)e^{-ik^{\star j}h^{j}}$$

- **System of** $2N+1$ **equations in** $2N+1$ **unknowns**

$$\underline{\boldsymbol{V}}_{Sr}^{j+1}\left(\frac{h^{j+1}}{2}\right)=\underline{\underline{\boldsymbol{P}}}^{j}\left(k^{\star j};\alpha^{\star j}\left(h^{j}\right)\right)\underline{\boldsymbol{V}}_{Sr}^{j}\left(\frac{h^{j}}{2}\right)$$

$$\underline{\boldsymbol{V}}_{Sr}^{N+1}=\underline{\underline{\mathbb{P}}}\left(k^{\star 1,\ldots,N};\alpha^{\star j,\ldots,N}\left(h^{1,\ldots,N}\right)\right)\underline{\boldsymbol{V}}_{Sr}^{1}\left(\frac{h^{1}}{2}\right)$$

$$\rightarrow\underline{\underline{\mathbb{P}}}\left(k^{\star 1,\ldots,N};\alpha^{\star j,\ldots,N}\left(h^{1,\ldots,N}\right)\right)=\prod_{j=N}^{1}\underline{\underline{\boldsymbol{P}}}^{j}\left(k^{\star j};\alpha^{\star j}\left(h^{j}\right)\right)$$

Last equation to close the system ?

1D Multi-Layer SH Plane-Wave propagation

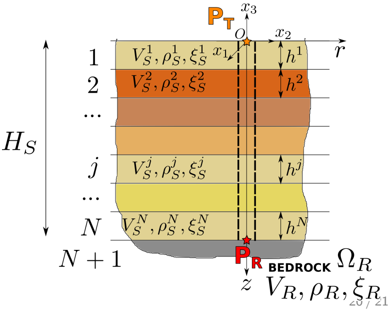
- $2N$ **Interface recursive conditions**
  $E_{Sr}^{j+1}\left(\frac{h^{j+1}}{2}\right) = \frac{1}{2}E_{Sr}^{j}\left(\frac{h^{j}}{2}\right)\left(1+\alpha^{\star j}\left(h^{j}\right)\right)e^{ik^{\star j}h^{j}} + \frac{1}{2}F_{Sr}^{j}\left(\frac{h^{j}}{2}\right)\left(1-\alpha^{\star j}\left(h^{j}\right)\right)e^{-ik^{\star j}h^{j}}$
  $F_{Sr}^{j+1}\left(\frac{h^{j+1}}{2}\right) = \frac{1}{2}E_{Sr}^{j}\left(\frac{h^{j}}{2}\right)\left(1-\alpha^{\star j}\left(h^{j}\right)\right)e^{ik^{\star j}h^{j}} + \frac{1}{2}F_{Sr}^{j}\left(\frac{h^{j}}{2}\right)\left(1+\alpha^{\star j}\left(h^{j}\right)\right)e^{-ik^{\star j}h^{j}}$
- **System of** $2N+1$ **equations in** $2N+1$ **unknowns**
  $\underline{\boldsymbol{V}}_{Sr}^{j+1}\left(\frac{h^{j+1}}{2}\right) = \underline{\underline{\boldsymbol{P}}}^{j}\left(k^{\star j};\alpha^{\star j}\left(h^{j}\right)\right)\underline{\boldsymbol{V}}_{Sr}^{j}\left(\frac{h^{j}}{2}\right)$
  $\underline{\boldsymbol{V}}_{Sr}^{N+1} = \underline{\underline{\mathbb{P}}}\left(k^{\star 1,\dots,N};\alpha^{\star j,\dots,N}\left(h^{1,\dots,N}\right)\right)\underline{\boldsymbol{V}}_{Sr}^{1}\left(\frac{h^{1}}{2}\right)$
  $\rightarrow \underline{\underline{\mathbb{P}}}\left(k^{\star 1,\dots,N};\alpha^{\star j,\dots,N}\left(h^{1,\dots,N}\right)\right) = \prod_{j=N}^{1}\underline{\underline{\boldsymbol{P}}}^{j}\left(k^{\star j};\alpha^{\star j}\left(h^{j}\right)\right)$

1D Multi-Layer SH Plane-Wave propagation

- 1 **free-surface condition**
  $G^{\star}\left(\gamma_{Sr}^{1-eff}\left(\frac{h^{1}}{2}\right)\right)\frac{\partial U_{Sr}^{1}}{\partial z^{1}}\left(h^{1}\right) = 0$
  $\rightarrow E_{Sr}^{1}\left(\frac{h^{1}}{2}\right) = F_{Sr}^{1}\left(\frac{h^{1}}{2}\right) \rightarrow \underline{\boldsymbol{V}}_{Sr}^{1}\left(\frac{h^{1}}{2}\right) = E_{Sr}^{1}\left(\frac{h^{1}}{2}\right)\underline{\mathbf{1}}$

1D Multi-Layer SH Plane-Wave propagation

- **Wave-amplitude unknown at every layer but the bedrock**
  $E_{Sr}^{1,\dots,N}\left(\frac{h^{1,\dots,N}}{2}\right), F_{Sr}^{1,\dots,N}\left(\frac{h^{1,\dots,N}}{2}\right) = ?$
  $E_{Sr}^{1}\left(\frac{h^{1}}{2}\right)\underline{\mathbf{1}} = \underline{\underline{\mathbb{P}}}^{-1}\left(k^{\star 1,\dots,N};\alpha^{\star j,\dots,N}\left(h^{1,\dots,N}\right)\right)\underline{\boldsymbol{V}}_{Sr}^{N+1}\ ?$
- **Use of Transfer Functions** $T^{ij}(\omega) = \frac{U_{Sr}^{i}}{U_{Sr}^{j}} = \frac{E_{Sr}^{i}+F_{Sr}^{i}}{E_{Sr}^{j}+F_{Sr}^{j}}$
- **Need to introduce the Input Ground Motion** $@P_R$
  E_{Sr}^{N+1} : incident wave-field
  F_{Sr}^{N+1} : unknown reflected wave-field



1D Multi-Layer SH Plane-Wave propagation

- **Wave-amplitude unknown at every layer but the bedrock**

$$E_{Sr}^{1,\dots,N}\left(\frac{h^{1,\dots,N}}{2}\right), F_{Sr}^{1,\dots,N}\left(\frac{h^{1,\dots,N}}{2}\right) = ?$$

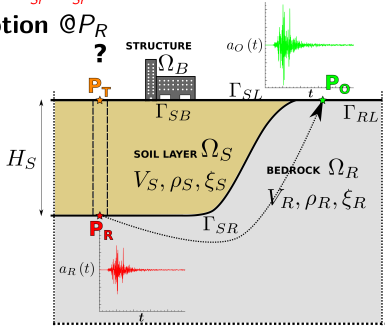
$$E_{Sr}^{1}\left(\frac{h^{1}}{2}\right)\underline{\mathbf{1}} = \underline{\underline{\mathbb{P}}}^{-1}\left(k^{\star 1,\dots,N}; \alpha^{\star j,\dots,N}\left(h^{1,\dots,N}\right)\right)\underline{\boldsymbol{V}}_{Sr}^{N+1}\,?$$

- **Use of Transfer Functions** $T^{ij}(\omega) = \frac{U_{Sr}^{i}}{U_{Sr}^{j}} = \frac{E_{Sr}^{i}+F_{Sr}^{i}}{E_{Sr}^{j}+F_{Sr}^{j}}$

- **Need to introduce the Input Ground Motion @P_R**

E_{Sr}^{N+1} : incident wave-field

F_{Sr}^{N+1} : unknown reflected wave-field



Within Condition

$$U_{Rr} = E_{Sr}^{N+1} + F_{Sr}^{N+1}$$

$$T^{ij}(\omega) = \frac{E_{Sr}^{i}+F_{Sr}^{i}}{E_{Sr}^{N+1}+F_{Sr}^{N+1}}(\omega)$$

$$\rightarrow U_{Sr}^{i}(\omega) = T^{ij}(\omega)\,U_{Rr}$$

1D Multi-Layer SH Plane-Wave propagation

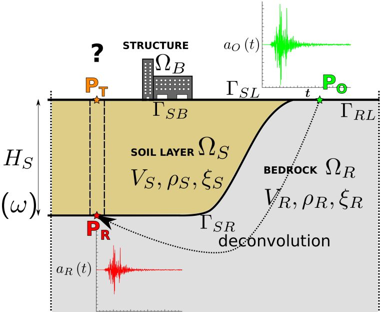
- **Wave-amplitude unknown at every layer but the bedrock**
  $E_{Sr}^{1,\dots,N}\left(\frac{h^{1,\dots,N}}{2}\right), F_{Sr}^{1,\dots,N}\left(\frac{h^{1,\dots,N}}{2}\right) = ?$
  $E_{Sr}^{1}\left(\frac{h^{1}}{2}\right)\underline{\mathbf{1}} = \underline{\underline{\mathbb{P}}}^{-1}\left(k^{\star 1,\dots,N};\alpha^{\star j,\dots,N}\left(h^{1,\dots,N}\right)\right)\underline{\boldsymbol{V}}_{Sr}^{N+1}$?
- **Use of Transfer Functions** $T^{ij}(\omega) = \frac{U_{Sr}^{i}}{U_{Sr}^{j}} = \frac{E_{Sr}^{i}+F_{Sr}^{i}}{E_{Sr}^{j}+F_{Sr}^{j}}$
- **Need to introduce the Input Ground Motion @**P_R
  E_{Sr}^{N+1} : incident wave-field
  F_{Sr}^{N+1} : unknown reflected wave-field

Outcrop Condition

$U_{Rr} = 2E_{Sr}^{N+1}$

$T^{ij}(\omega) = \frac{E_{Sr}^{i}+F_{Sr}^{i}}{E_{Sr}^{N+1}+F_{Sr}^{N+1}}(\omega)\,,\; T^{RO}(\omega) = \frac{E_{Sr}^{N+1}+F_{Sr}^{N+1}}{2E_{Sr}^{N+1}}(\omega)$

$\rightarrow U_{Sr}^{i}(\omega) = T^{ij}(\omega)\,T^{RO}(\omega)\,U_{Rr}$



GRAZIE DELL'ATTENZIONE
MERCI DE VOTRE ATTENTION
THANKS FOR THE ATTENTION

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`https://github.com/FilLTP89`

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The work carried out under the SINAPS@ project has benefited from French funding managed by the National Research Agency under the program "Future Investments" - SINAPS@ reference No. ANR-11-RSNR-0022.