

Seismic Site-Effects: Simplified numerical simulation



CentraleSupélec



POLITECNICO
MILANO 1863



Filippo Gatti^{1,2}

¹Laboratoire MSSMat - CentraleSupélec

²DICA - Politecnico di Milano

14 décembre 2016

The M_w 8.0 Mexico City earthquake (1985)

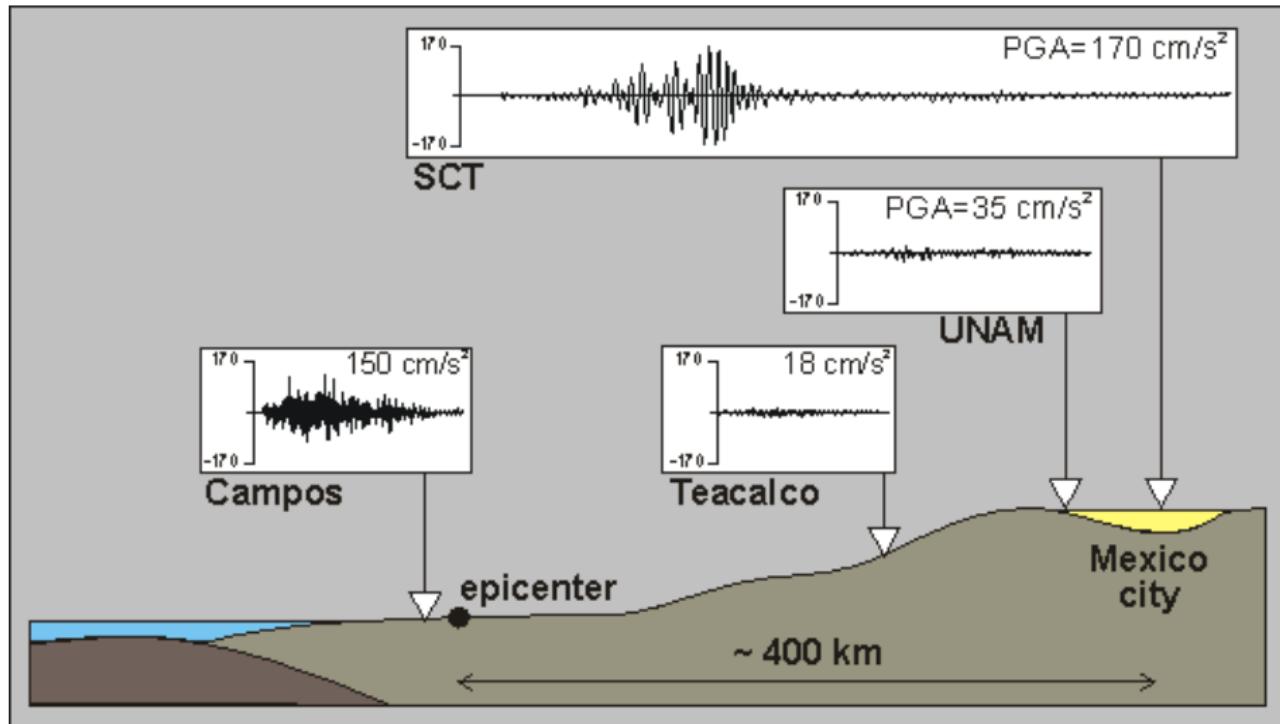


FIGURE 1 – From Wikipedia.

https://fr.m.wikipedia.org/wiki/Fichier:Site_effects_mexico_1985_recordings_v2.gif

From the source...

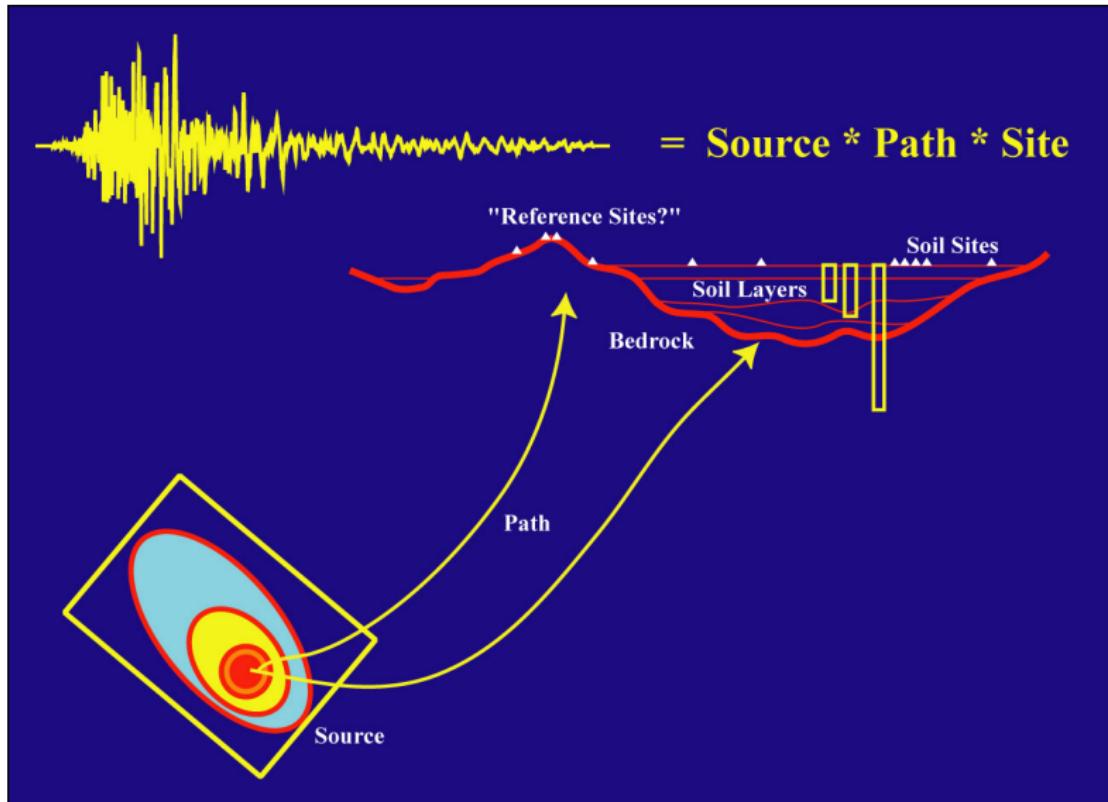
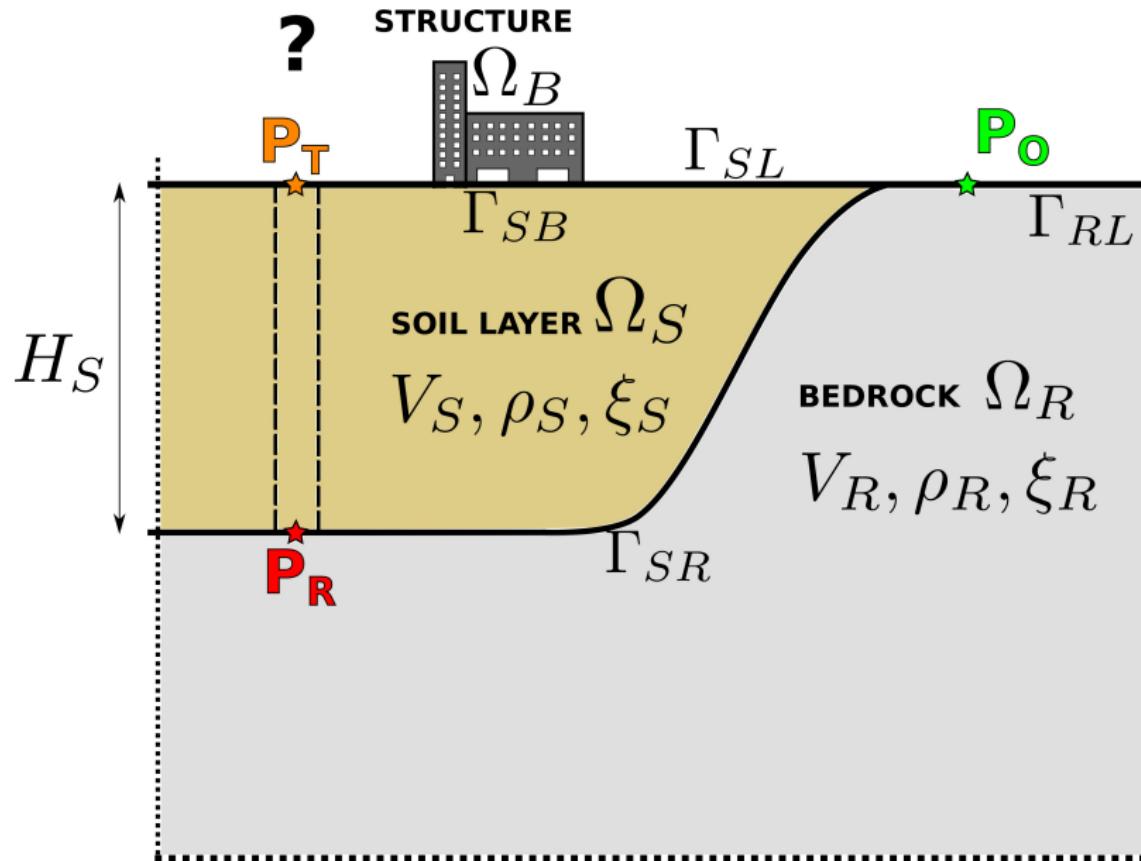
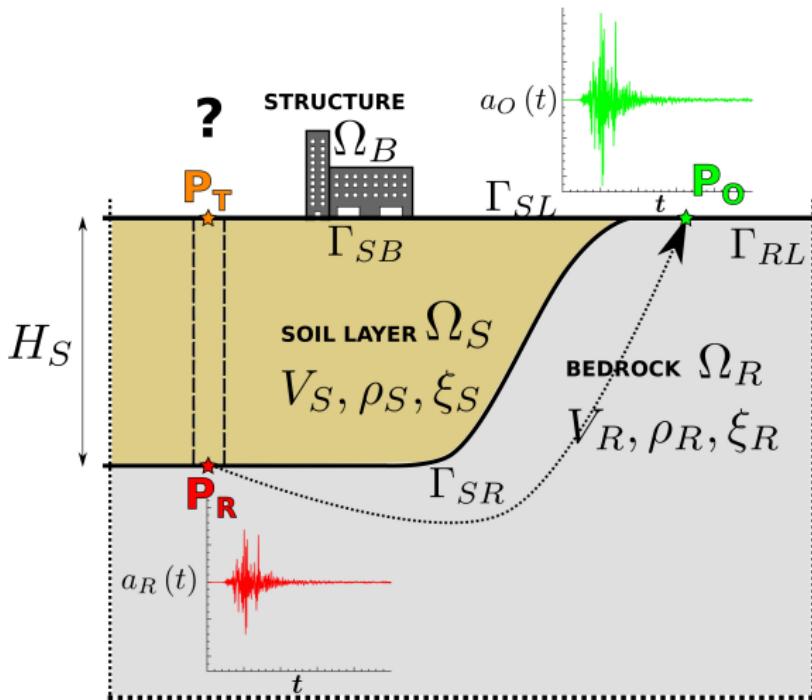


FIGURE 2 – After Boore (USGS).

...to the site

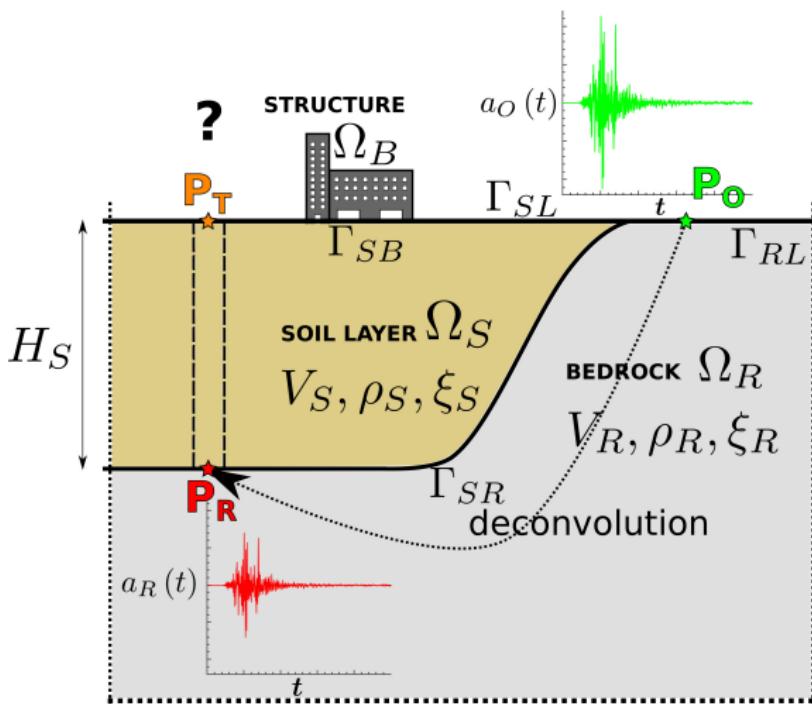


...to the site



- ▶ records available @ P_R (deep bedrock)
- ▶ propagation through the soil layer
 $P_R \rightarrow P_T$
- ▶ input motion for structural analysis

...to the site



- records available @ P_O (Outcrop Bedrock)
- deconvolution**
 $P_O \rightarrow P_R$
- propagation through the soil layer
 $P_R \rightarrow P_T$

Navier-Stokes equations

- ▶ **Crustal Rocks**

$$\underline{\nabla}_x \cdot \underline{\underline{\sigma}}_R (\underline{u}_R) (\underline{x}; t) + \rho_R (\underline{x}) \underline{g} = \rho_R (\underline{x}) \partial_{tt} \underline{u}_R (\underline{x}; t), \quad (\underline{x}; t) \in (\Omega_R, \mathbb{I}_t) \quad (1)$$

- ▶ **Soil deposit**

$$\underline{\nabla}_x \cdot \underline{\underline{\sigma}}_S (\underline{u}_S) (\underline{x}; t) + \rho_S (\underline{x}) \underline{g} = \rho_S (\underline{x}) \partial_{tt} \underline{u}_S (\underline{x}; t) \quad (\underline{x}; t) \in (\Omega_S, \mathbb{I}_t) \quad (2)$$

- ▶ **Foundation**

$$\underline{\nabla}_x \cdot \underline{\underline{\sigma}}_F (\underline{u}_F) (\underline{x}; t) + \rho_F (\underline{x}) \underline{g} = \rho_F (\underline{x}) \partial_{tt} \underline{u}_F (\underline{x}; t) \quad (\underline{x}; t) \in (\Omega_F, \mathbb{I}_t) \quad (3)$$

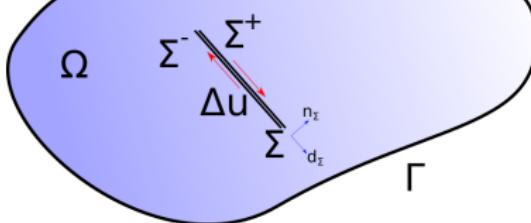
- ▶ **Structure**

$$\underline{\nabla}_x \cdot \underline{\underline{\sigma}}_B (\underline{u}_B) (\underline{x}; t) + \rho_B (\underline{x}) \underline{g} + \sum_i \underline{f}_i (\underline{x}; t) = \rho_B (\underline{x}) \partial_{tt} \underline{u}_B (\underline{x}; t) \quad (\underline{x}; t) \in (\Omega_B, \mathbb{I}_t) \quad (4)$$

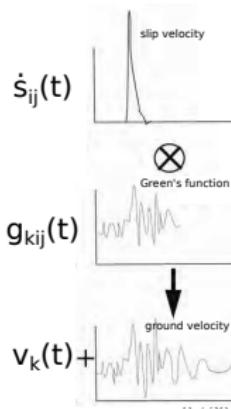
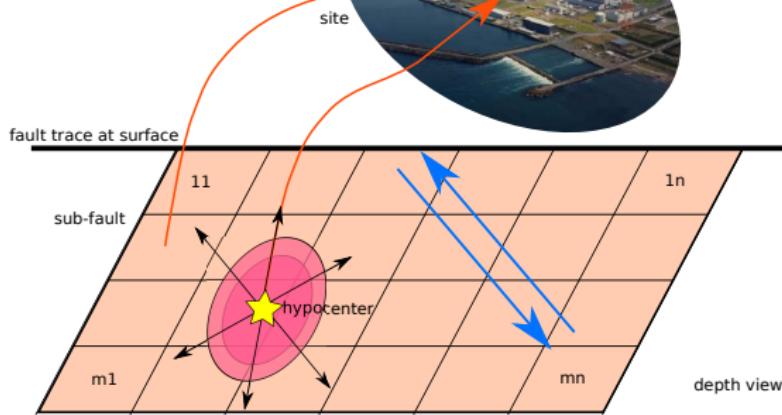
Soil dynamics : general framework

Boundary condition - Seismic Fault

$$\mathbf{f}_\Sigma(\mathbf{x}; t) = \nabla_\Sigma \cdot (\mathbb{D}^{el}(\mathbf{x}) : \boldsymbol{\varepsilon}(\mathbf{u}(\mathbf{x}; t))) = \nabla_\Sigma \cdot (\mathbb{D}^{el}(\mathbf{x}) : \Delta u(\mathbf{x}; t) \delta_\Sigma(\mathbf{x}) \mathbf{d}_\Sigma \otimes_s \mathbf{n}_\Sigma) \quad (5)$$



map-view

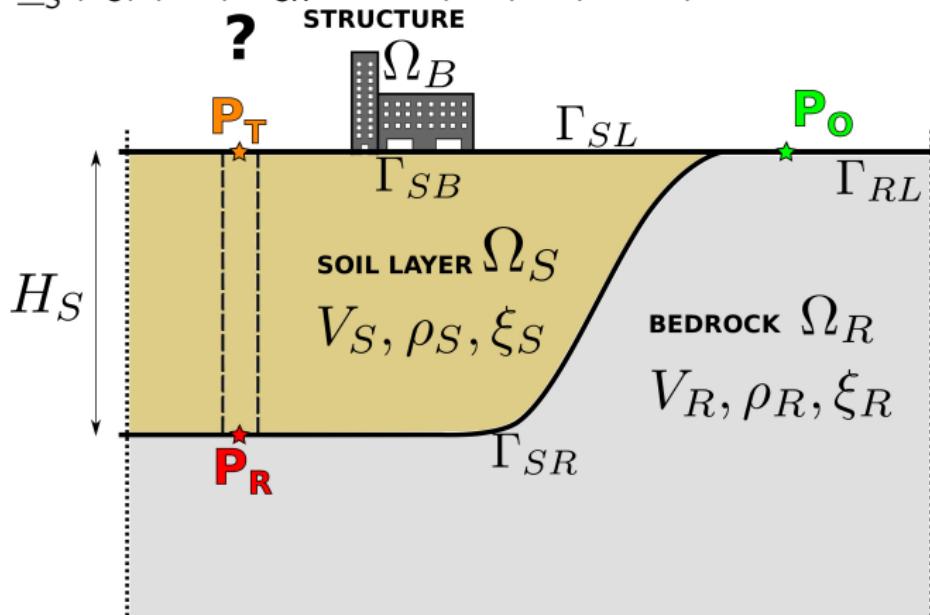


Boundary condition - Bedrock-Soil

$$\underline{u}_S(\underline{x}; t) = \underline{u}_R(\underline{x}; t), \quad (\underline{x}; t) \in (\Gamma_{SR}, \mathbb{I}_t)$$

$$\underline{\underline{\sigma}}_S(\underline{u}_S)(\underline{x}; t) \cdot \underline{n}_{SR} = \underline{\underline{\sigma}}_R(\underline{u}_R)(\underline{x}; t) \cdot \underline{n}_{SR}, \quad (\underline{x}; t) \in (\Gamma_{SR}, \mathbb{I}_t) \quad (6)$$

$$\underline{\underline{\sigma}}_S(\underline{u}_S)(\underline{x}; t) \cdot \underline{n}_{SR} = 0, \quad (\underline{x}; t) \in (\Gamma_{SL}, \mathbb{I}_t)$$



Soil dynamics : general framework

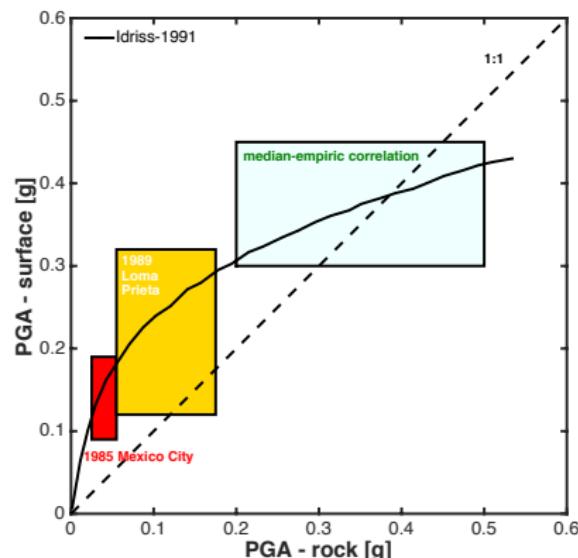
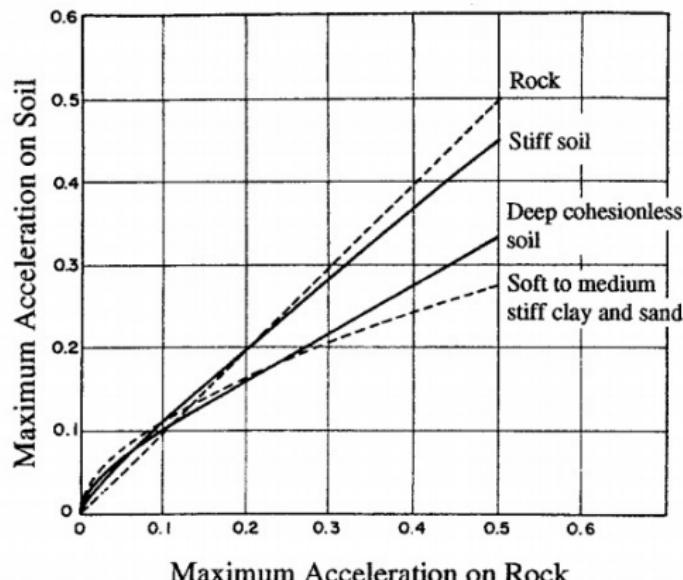
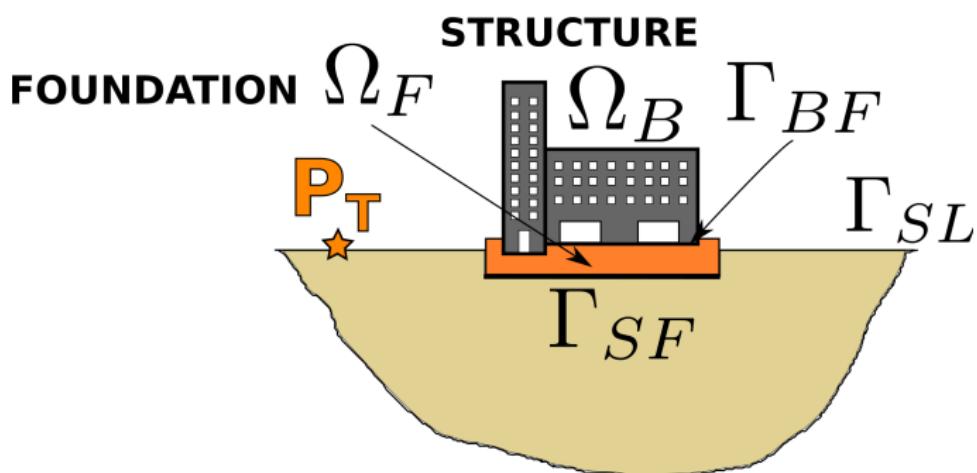


FIGURE 3 – Site effect amplification of the earthquake ground motion (after Beresnev and Wen (1996) [2] and Idriss (1991) [4])

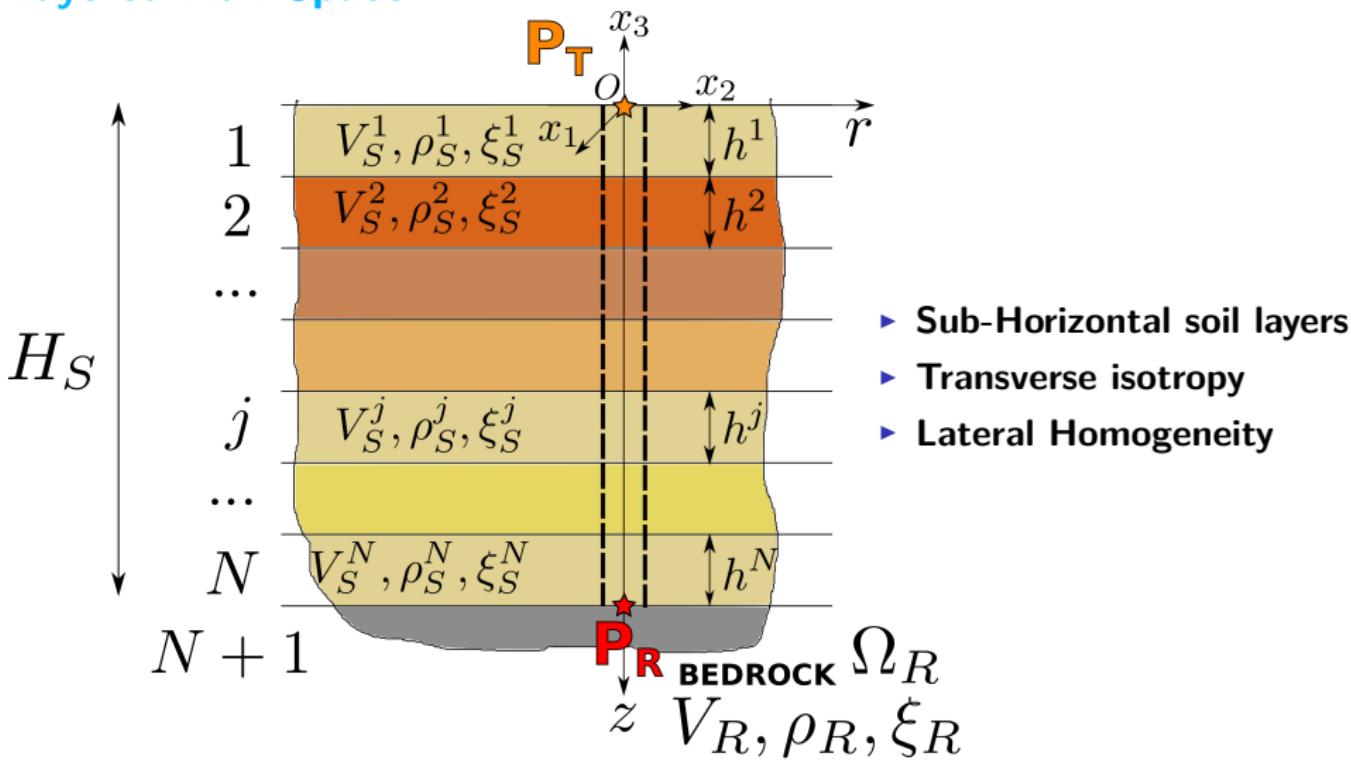
Boundary condition - Soil-Structure (SSI)

$$\begin{aligned}\underline{\mathbf{u}}_S(\underline{\mathbf{x}}; t) &= \underline{\mathbf{u}}_F(\underline{\mathbf{x}}; t), \quad (\underline{\mathbf{x}}; t) \in (\Gamma_{SF}, \mathbb{I}_t) \\ \underline{\mathbf{u}}_B(\underline{\mathbf{x}}; t) &= \underline{\mathbf{u}}_F(\underline{\mathbf{x}}; t), \quad (\underline{\mathbf{x}}; t) \in (\Gamma_{BF}, \mathbb{I}_t) \\ \underline{\underline{\sigma}}_S(\underline{\mathbf{u}}_S)(\underline{\mathbf{x}}; t) \cdot \underline{\mathbf{n}}_{SR} &= 0, \quad (\underline{\mathbf{x}}; t) \in (\Gamma_{SL}, \mathbb{I}_t)\end{aligned}\tag{6}$$

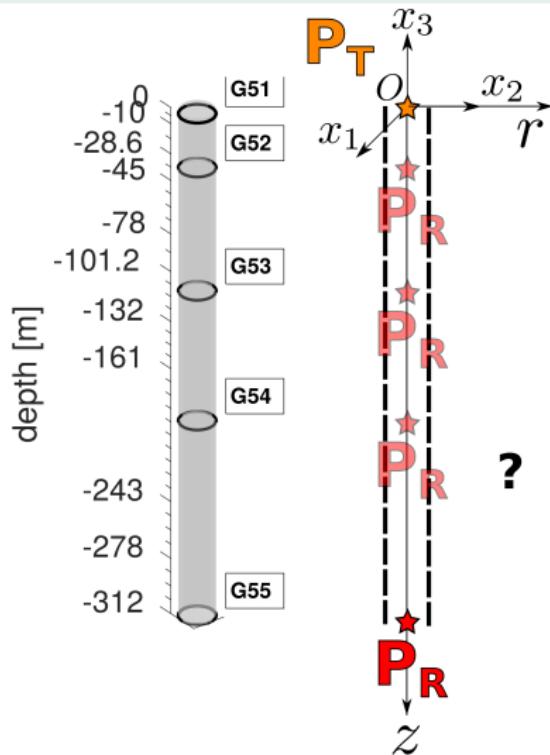


1D wave propagation : main assumptions

Layered Half-Space



1D wave propagation : main assumptions



1D wave propagation : main assumptions

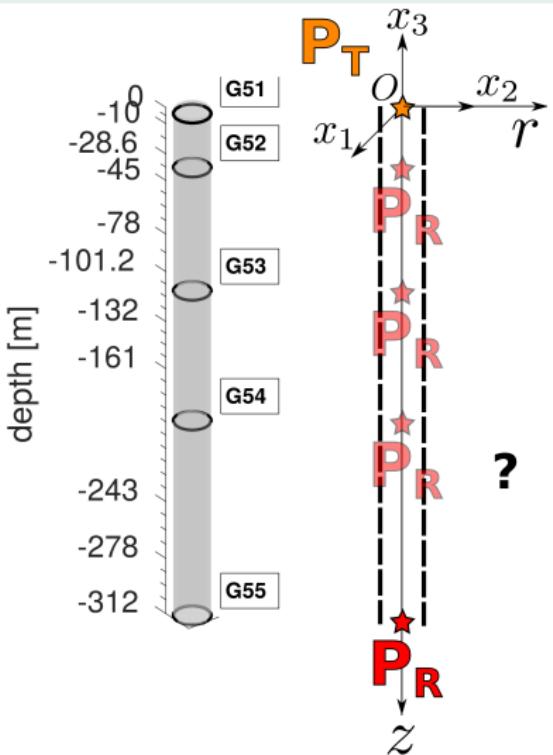
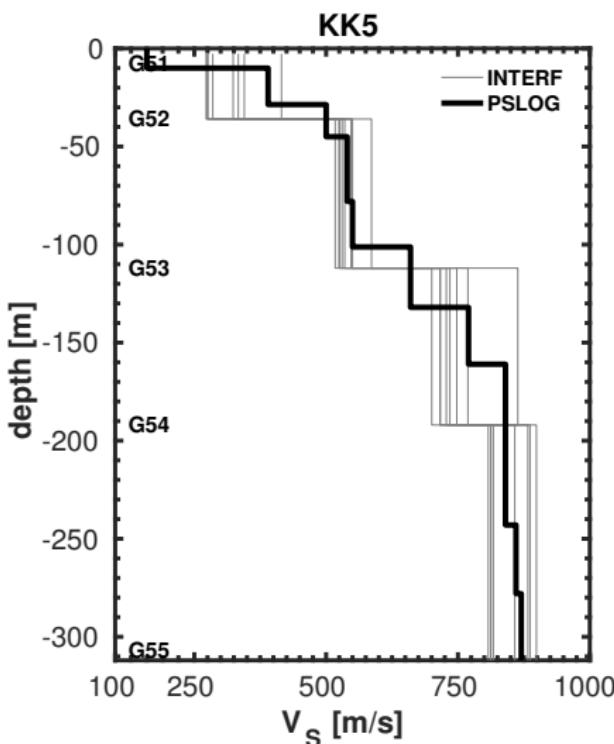


FIGURE 4 – V_S profile estimated via borehole interferometry at Kashiwazaki-Kariwa Nuclear Power Plant (Japon) (after Gatti et al. (2015) [3])



Space-Time Variable Separation

- $\underline{u}_S(\underline{x}; t) = \underline{U}_S(\underline{x}) \phi(t)$

Lateral Homogeneity

- $\underline{U}_S(\underline{x}) = \underline{U}_S(x_1 + v_1, x_2 + v_2, x_3), \forall (v_1, v_2) \in \mathbb{R}^2, (x_1, x_2, x_3) \in \mathbb{R}^2 \times]-H_S, 0[$
 $\rightarrow \underline{U}_S(\underline{x}) = \underline{U}_S(x_3)$

Pure SH Plane Wave Propagation

- $\frac{\partial(\underline{U}_S \cdot \underline{i}_3)}{\partial x_3}(x_3) = 0 \text{ } (\rightarrow \text{vertical settlements not considered})$
- $\underline{U}_S(x_3) \cdot \underline{i}_\theta = \underline{U}_S(x_3) \cdot \underline{i}_{\pi-\theta} = -\underline{U}_S(x_3) \cdot \underline{i}_\theta \rightarrow \underline{U}_S(x_3) \cdot \underline{i}_\theta = 0$
 $\rightarrow \underline{U}_S(\underline{x}) = U_{Sr}(z) \underline{i}_r$

Harmonic Solutions IN TIME

- $\underline{u}_S(\underline{x}; t) = \Re(\underline{U}_S(\underline{x}) \phi(t)) = \Re(U_{Sr}(z) e^{i\omega t}) \underline{i}_r$
- Strain Tensor : $\rightarrow \underline{\underline{\epsilon}}_x(\underline{x}; t) = (\underline{u}_S \otimes_s \underline{\nabla}_x)(\underline{x}; t) = \Re\left(\frac{\partial U_{Sr}}{\partial z} e^{i\omega t}(z)\right) \underline{i}_r \otimes \underline{i}_z = \Re(\gamma_{Srz}(z) e^{i\omega t}) \underline{i}_r \otimes \underline{i}_z$

1D Navier-Stokes equation $\frac{\partial \tau_{Szr}}{\partial z}(\gamma_{Srz}(z, t)) = \rho_S(z) \partial_{tt} \mathbf{u}_{Sr}(z, t)$

Space-Time Variable Separation

- ▶ $\underline{u}_S(\underline{x}; t) = \underline{U}_S(\underline{x}) \phi(t)$

Lateral Homogeneity

- ▶ $\underline{U}_S(\underline{x}) = \underline{U}_S(x_1 + v_1, x_2 + v_2, x_3), \forall (v_1, v_2) \in \mathbb{R}^2, (x_1, x_2, x_3) \in \mathbb{R}^2 \times]-H_S, 0[$
 $\rightarrow \underline{U}_S(\underline{x}) = \underline{U}_S(x_3)$

Pure SH Plane Wave Propagation

- ▶ $\frac{\partial(\underline{U}_S \cdot \underline{i}_3)}{\partial x_3}(x_3) = 0 \quad (\rightarrow \text{vertical settlements not considered})$
- ▶ $\underline{U}_S(x_3) \cdot \underline{i}_\theta = \underline{U}_S(x_3) \cdot \underline{i}_{\pi-\theta} = -\underline{U}_S(x_3) \cdot \underline{i}_\theta \rightarrow \underline{U}_S(x_3) \cdot \underline{i}_\theta = 0$
 $\rightarrow \underline{U}_S(\underline{x}) = U_{Sr}(z) \underline{i}_r$

Harmonic Solutions IN TIME

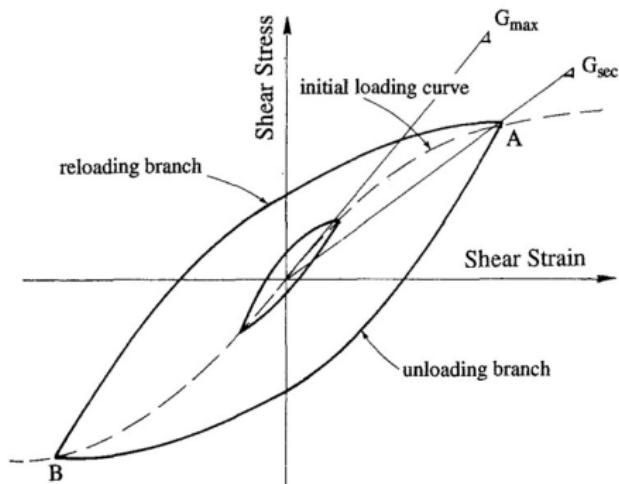
- ▶ $\underline{u}_S(\underline{x}; t) = \Re(\underline{U}_S(\underline{x}) \phi(t)) = \Re(U_{Sr}(z) e^{i\omega t}) \underline{i}_r$
- ▶ Strain Tensor : $\rightarrow \underline{\underline{\epsilon}}_x(\underline{x}; t) = (\underline{u}_S \otimes_s \underline{\nabla}_x)(\underline{x}; t) = \Re\left(\frac{\partial U_{Sr}}{\partial z} e^{i\omega t}(z)\right) \underline{i}_r \otimes \underline{i}_z = \Re(\gamma_{Srz}(z) e^{i\omega t}) \underline{i}_r \otimes \underline{i}_z$

How to compute the stress state ?

$$\underline{\underline{\sigma}}(\underline{u}_S)(\underline{x}; t) = \mathcal{F}\left(\underline{\underline{\epsilon}}_x(\underline{u}_S)(\underline{x}; t)\right)$$

Cyclic behaviour of geo-materials

FIGURE 5 – Typical stress-strain relationship of soil in cyclic shear deformation (adapted from Pecker, 1984 [6]). Initial loading curve has a hyperbolic form (dashed line). Subsequent unloading and reloading phases track a hysteretic path. Two hysteresis loops constructed according to Masing's rules [5] are shown, where A and B mark the reversal points of the loop



Cyclic behaviour of geo-materials

FIGURE 5 – Typical stress-strain relationship of soil in cyclic shear deformation (adapted from Pecker, 1984 [6]). Initial loading curve has a hyperbolic form (dashed line). Subsequent unloading and reloading phases track a hysteretic path. Two hysteresis loops constructed according to Masing's rules [5] are shown, where A and B mark the reversal points of the loop

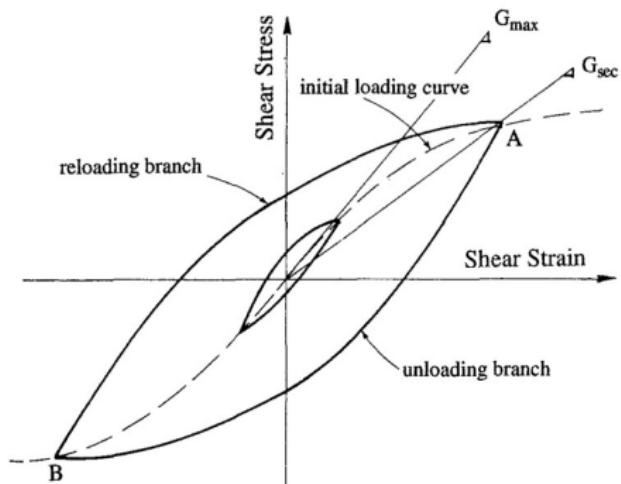
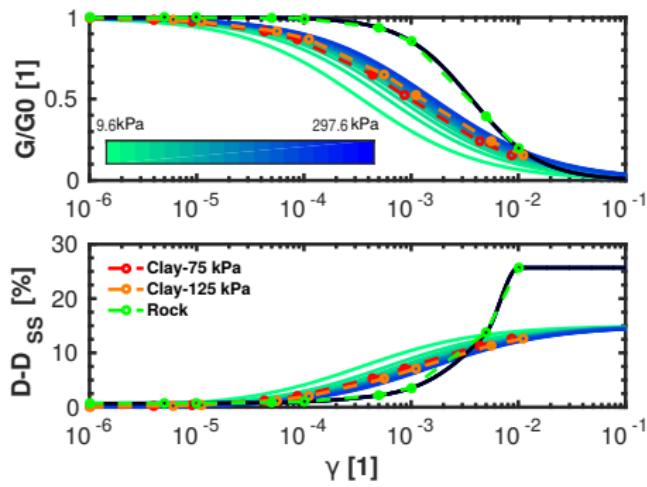


FIGURE 6 – Typical curves showing the dependence of shear modulus and damping ratio on strain amplitude of shallow subsoil at Kashiwazaki-Kariwa Nuclear Power Plant (after Gatti et al. 2015 [3]). Shear modulus G is normalized to its value at small strain (SS) G_{max} . Damping ratio D is subtracted from its small strain value D_{SS} .



► **Rheological model**

$$\tau_{Srz}(z, t) = G(\gamma_{Srz})(z, t) \gamma_{Srz}(z, t) + \eta(\gamma_{Srz}(z, t)) \dot{\gamma}_{Srz}(z, t)$$

$$\dot{\gamma}_{Srz}(z, t) = \frac{\partial^2 u_{Sr}}{\partial z \partial t}(z, t)$$

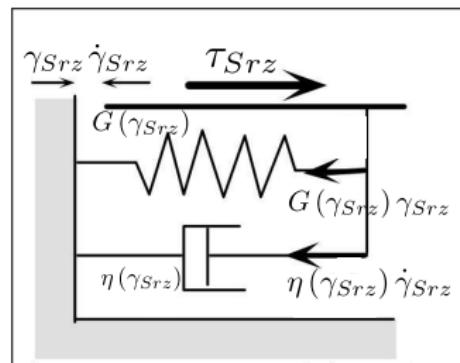
► **1D non-linear Navier-Stoke equation**

$$\frac{\partial}{\partial z} \left(G(\gamma_{Srz})(z, t) \frac{\partial u_{Sr}}{\partial z}(z, t) \right) + \frac{\partial}{\partial z} \left(\eta(\gamma_{Srz})(z, t) \frac{\partial^2 u_{Sr}}{\partial z \partial t}(z, t) \right) = \rho_S(z) \partial_{tt} u_{Sr}(z, t)$$

► **Harmonic Solution IN TIME**

$$\frac{\partial}{\partial z} \left(G(\gamma_{Srz})(z, \omega) \frac{\partial U_{Sr}}{\partial z}(z) \right) + i\omega \frac{\partial}{\partial z} \left(\eta(\gamma_{Srz})(z, \omega) \frac{\partial U_{Sr}}{\partial z}(z) \right) = -\rho_S(z) \omega^2 U_{Sr}(z)$$

Schematic representation of stress-strain model used in equivalent-linear model
(after Bardet et al. (2000) [1])



► **Rheological model**

$$\tau_{Srz}(z, t) = G(\gamma_{Srz})(z, t) \gamma_{Srz}(z, t) + \eta(\gamma_{Srz}(z, t)) \dot{\gamma}_{Srz}(z, t)$$

$$\dot{\gamma}_{Srz}(z, t) = \frac{\partial^2 u_{Sr}}{\partial z \partial t}(z, t)$$

► **1D non-linear Navier-Stoke equation**

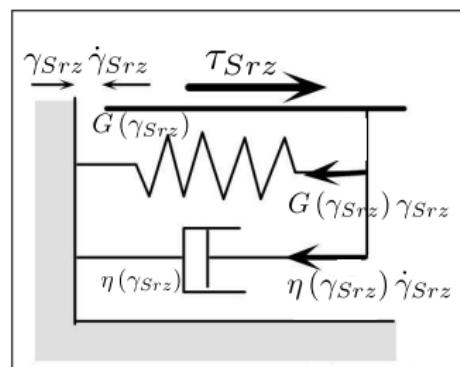
$$\frac{\partial}{\partial z} \left(G(\gamma_{Srz})(z, t) \frac{\partial u_{Sr}}{\partial z}(z, t) \right) + \frac{\partial}{\partial z} \left(\eta(\gamma_{Srz})(z, t) \frac{\partial^2 u_{Sr}}{\partial z \partial t}(z, t) \right) = \rho_S(z) \partial_{tt} u_{Sr}(z, t)$$

► **Harmonic Solution IN TIME**

$$\frac{\partial}{\partial z} \left(G(\gamma_{Srz})(z, \omega) \frac{\partial U_{Sr}}{\partial z}(z) \right) + i\omega \frac{\partial}{\partial z} \left(\eta(\gamma_{Srz})(z, \omega) \frac{\partial U_{Sr}}{\partial z}(z) \right) = -\rho_S(z) \omega^2 U_{Sr}(z)$$

Schematic representation of stress-strain model used in equivalent-linear model
(after Bardet et al. (2000) [1])

How EQLM solves non-linear Navier-Stokes equation ?



► **SH Plane Wave Time-Harmonic solution**

$$\Re(\gamma_{Srz}(z, t)) = \frac{\partial \Re(u_{Sr})}{\partial z}(z, t) = \frac{\partial U_{Sr}}{\partial z}(z) \cos(\omega t)$$

$$\Re(\dot{\gamma}_{Srz}(z, t)) = \Re\left(i\omega \frac{\partial U_{Sr}}{\partial z}(z) e^{i\omega t}\right) = -\omega \frac{\partial U_{Sr}}{\partial z}(z) \sin(\omega t)$$

► SH Plane Wave Time-Harmonic solution

$$\Re(\gamma_{Sr}(z, t)) = \frac{\partial \Re(u_{Sr})}{\partial z}(z, t) = \frac{\partial U_{Sr}}{\partial z}(z) \cos(\omega t)$$

$$\Re(\dot{\gamma}_{Sr}(z, t)) = \Re\left(i\omega \frac{\partial U_{Sr}}{\partial z}(z) e^{i\omega t}\right) = -\omega \frac{\partial U_{Sr}}{\partial z}(z) \sin(\omega t)$$

► Kelvin-Voigt model & Complex Shear Modulus

$$\Re(\tau_{Sr}(\gamma_{Sr}; \dot{\gamma}_{Sr})) = \Re\left((G(\gamma_{Sr}) + i\omega\eta(\gamma_{Sr})) \frac{\partial U_{Sr}}{\partial z} e^{i\omega t}\right) =$$

$$\Re(G^*(\gamma_{Sr}; \omega) e^{i\omega t}) \frac{\partial U_{Sr}}{\partial z} = (G(\gamma_{Sr}) \cos(\omega t) - \eta(\gamma_{Sr}) \omega \sin(\omega t)) \frac{\partial U_{Sr}}{\partial z}$$

► SH Plane Wave Time-Harmonic solution

$$\Re(\gamma_{Sr}(z, t)) = \frac{\partial \Re(u_{Sr})}{\partial z}(z, t) = \frac{\partial U_{Sr}}{\partial z}(z) \cos(\omega t)$$

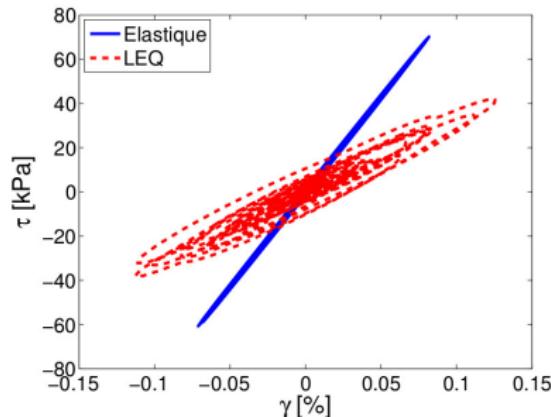
$$\Re(\dot{\gamma}_{Sr}(z, t)) = \Re(i\omega \frac{\partial U_{Sr}}{\partial z}(z) e^{i\omega t}) = -\omega \frac{\partial U_{Sr}}{\partial z}(z) \sin(\omega t)$$

► Kelvin-Voigt model & Complex Shear Modulus

$$\Re(\tau_{Sr}(\gamma_{Sr}; \dot{\gamma}_{Sr})) = \Re((G(\gamma_{Sr}) + i\omega\eta(\gamma_{Sr})) \frac{\partial U_{Sr}}{\partial z} e^{i\omega t}) =$$

$$\Re(G^*(\gamma_{Sr}; \omega) e^{i\omega t}) \frac{\partial U_{Sr}}{\partial z} = (G(\gamma_{Sr}) \cos(\omega t) - \eta(\gamma_{Sr}) \omega \sin(\omega t)) \frac{\partial U_{Sr}}{\partial z}$$

Parametric equations of an ellipse !



► SH Plane Wave Time-Harmonic solution

$$\Re(\gamma_{Sr}(z, t)) = \frac{\partial \Re(u_{Sr})}{\partial z}(z, t) = \frac{\partial U_{Sr}}{\partial z}(z) \cos(\omega t)$$

$$\Re(\dot{\gamma}_{Sr}(z, t)) = \Re\left(i\omega \frac{\partial U_{Sr}}{\partial z}(z) e^{i\omega t}\right) = -\omega \frac{\partial U_{Sr}}{\partial z}(z) \sin(\omega t)$$

► Kelvin-Voigt model & Complex Shear Modulus

$$\Re(\tau_{Sr}(\gamma_{Sr}; \dot{\gamma}_{Sr})) = \Re\left((G(\gamma_{Sr}) + i\omega\eta(\gamma_{Sr})) \frac{\partial U_{Sr}}{\partial z} e^{i\omega t}\right) =$$

$$\Re(G^*(\gamma_{Sr}; \omega) e^{i\omega t}) \frac{\partial U_{Sr}}{\partial z} = (G(\gamma_{Sr}) \cos(\omega t) - \eta(\gamma_{Sr}) \omega \sin(\omega t)) \frac{\partial U_{Sr}}{\partial z}$$

► Maximum strain (Hysteresis)

$$\mathcal{E}_{EL}(\gamma_{Sr}) = \oint_{\bar{\gamma}_{Sr}} \Re(\tau_{Sr}(\gamma_{Sr}(z, s))) \Re(\dot{\gamma}_{Sr}(z, s)) ds = \frac{1}{2} G(\gamma_{Sr}) \left(\frac{\partial U_{Sr}}{\partial z} \right)^2$$

► SH Plane Wave Time-Harmonic solution

$$\Re(\gamma_{Sr}(z, t)) = \frac{\partial \Re(u_{Sr})}{\partial z}(z, t) = \frac{\partial U_{Sr}}{\partial z}(z) \cos(\omega t)$$

$$\Re(\dot{\gamma}_{Sr}(z, t)) = \Re\left(i\omega \frac{\partial U_{Sr}}{\partial z}(z) e^{i\omega t}\right) = -\omega \frac{\partial U_{Sr}}{\partial z}(z) \sin(\omega t)$$

► Kelvin-Voigt model & Complex Shear Modulus

$$\Re(\tau_{Sr}(\gamma_{Sr}; \dot{\gamma}_{Sr})) = \Re\left((G(\gamma_{Sr}) + i\omega\eta(\gamma_{Sr})) \frac{\partial U_{Sr}}{\partial z} e^{i\omega t}\right) =$$

$$\Re(G^*(\gamma_{Sr}; \omega) e^{i\omega t}) \frac{\partial U_{Sr}}{\partial z} = (G(\gamma_{Sr}) \cos(\omega t) - \eta(\gamma_{Sr}) \omega \sin(\omega t)) \frac{\partial U_{Sr}}{\partial z}$$

► Maximum strain (Hysteresis)

$$\mathcal{E}_{EL}(\gamma_{Sr}) = \oint_{\bar{\gamma}_{Sr}} \Re(\tau_{Sr}(\gamma_{Sr}(z, s))) \Re(\dot{\gamma}_{Sr}(z, s)) ds = \frac{1}{2} G(\gamma_{Sr}) \left(\frac{\partial U_{Sr}}{\partial z} \right)^2$$

► Dissipated strain energy (Hysteresis)

$$\mathcal{D}_{KV}(\gamma_{Sr}; \omega) = \oint_{\bar{\gamma}_{Sr}} \Re(\tau_{Sr}(\gamma_{Sr}(z, s); \dot{\gamma}_{Sr}(z, s))) \Re(\dot{\gamma}_{Sr}(z, s)) ds = \pi \omega \eta \left(\frac{\partial U_{Sr}}{\partial z} \right)$$

► **SH Plane Wave Time-Harmonic solution**

$$\Re(\gamma_{Sr}(z, t)) = \frac{\partial \Re(u_{Sr})}{\partial z}(z, t) = \frac{\partial U_{Sr}}{\partial z}(z) \cos(\omega t)$$

$$\Re(\dot{\gamma}_{Sr}(z, t)) = \Re\left(i\omega \frac{\partial U_{Sr}}{\partial z}(z) e^{i\omega t}\right) = -\omega \frac{\partial U_{Sr}}{\partial z}(z) \sin(\omega t)$$

► **Kelvin-Voigt model & Complex Shear Modulus**

$$\Re(\tau_{Sr}(\gamma_{Sr}; \dot{\gamma}_{Sr})) = \Re\left((G(\gamma_{Sr}) + i\omega\eta(\gamma_{Sr})) \frac{\partial U_{Sr}}{\partial z} e^{i\omega t}\right) =$$

$$\Re(G^*(\gamma_{Sr}; \omega) e^{i\omega t}) \frac{\partial U_{Sr}}{\partial z} = (G(\gamma_{Sr}) \cos(\omega t) - \eta(\gamma_{Sr}) \omega \sin(\omega t)) \frac{\partial U_{Sr}}{\partial z}$$

► **Maximum strain (Hysteresis)**

$$\mathcal{E}_{EL}(\gamma_{Sr}) = \oint_{\bar{\gamma}_{Sr}} \Re(\tau_{Sr}(\gamma_{Sr}(z, s))) \Re(\dot{\gamma}_{Sr}(z, s)) ds = \frac{1}{2} G(\gamma_{Sr}) \left(\frac{\partial U_{Sr}}{\partial z} \right)^2$$

► **Dissipated strain energy (Hysteresis)**

$$\mathcal{D}_{KV}(\gamma_{Sr}; \omega) = \oint_{\bar{\gamma}_{Sr}} \Re(\tau_{Sr}(\gamma_{Sr}(z, s); \dot{\gamma}_{Sr}(z, s))) \Re(\dot{\gamma}_{Sr}(z, s)) ds = \pi \omega \eta \left(\frac{\partial U_{Sr}}{\partial z} \right)$$

► **Definition of Critical Damping Ratio**

$$\xi_S(\gamma_{Sr}; \omega) = \frac{\mathcal{D}_{KV}(\gamma_{Sr}; \omega)}{4\pi\mathcal{E}_{EL}(\gamma_{Sr})} = \frac{\omega\eta(\gamma_{Sr})}{2G(\gamma_{Sr})}$$

1D Multi-Layer SH Plane-Wave propagation

- ▶ Equivalent Linear Solution

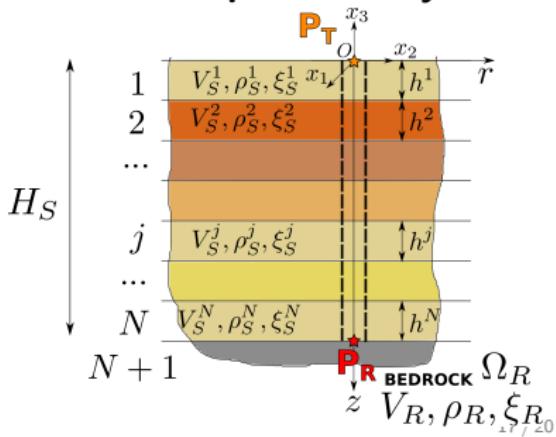
$$u_{Sr}(z, t) = \Re \left(\left(\sum_j^N U_{Sr}^j(z^j) \right) e^{i\omega t} \right), \quad z^j \in]0; h^j[$$

- ▶ 1D Navier Stokes equation

$$\left(G^j \left(\gamma_{Srz}^{j-\text{eff}} \left(\frac{h^j}{2} \right) \right) + i\omega \eta^j \left(\gamma_{Srz}^{j-\text{eff}} \left(\frac{h^j}{2} \right) \right) \right) \frac{\partial^2 U_{Sr}^j}{\partial z^{j2}}(z^j) = -\rho_S^j(z^j) \omega^2 U_{Sr}^j(z^j)$$

$$G^{j*} \left(\gamma_{Srz}^{j-\text{eff}} \left(\frac{h^j}{2} \right) \right) \frac{\partial^2 U_{Sr}^j}{\partial z^{j2}}(z^j) = -\rho_S^j \left(\frac{h^j}{2} \right) \omega^2 U_{Sr}^j(z^j)$$

$$\frac{\partial^2 U_{Sr}^j}{\partial z^{j2}}(z^j) + \frac{1}{V_S^{*j2}} U_{Sr}^j(z^j) = 0 \leftarrow \text{wave equation with complex velocity}$$



1D Multi-Layer SH Plane-Wave propagation

► Equivalent Linear Solution

$$u_{Sr}(z, t) = \Re \left(\left(\sum_j^N U_{Sr}^j(z^j) \right) e^{i\omega t} \right), \quad z^j \in]0; h^j[$$

► 1D Navier Stokes equation

$$\left(G^j \left(\gamma_{Srz}^{j-\text{eff}} \left(\frac{h^j}{2} \right) \right) + i\omega \eta^j \left(\gamma_{Srz}^{j-\text{eff}} \left(\frac{h^j}{2} \right) \right) \right) \frac{\partial^2 U_{Sr}^j}{\partial z^{j2}}(z^j) = -\rho_S^j(z^j) \omega^2 U_{Sr}^j(z^j)$$

$$G^{j*} \left(\gamma_{Srz}^{j-\text{eff}} \left(\frac{h^j}{2} \right) \right) \frac{\partial^2 U_{Sr}^j}{\partial z^{j2}}(z^j) = -\rho_S^j \left(\frac{h^j}{2} \right) \omega^2 U_{Sr}^j(z^j)$$

$$\frac{\partial^2 U_{Sr}^j}{\partial z^{j2}}(z^j) + \frac{1}{V_S^{*j2}} U_{Sr}^j(z^j) = 0 \leftarrow \text{wave equation with complex velocity}$$

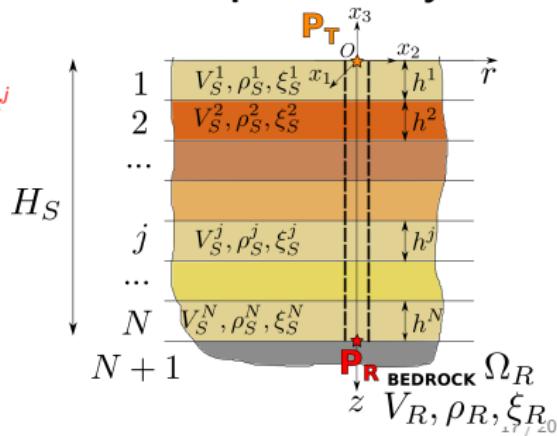
► Harmonic solution IN SPACE

$$U_{Sr}^j(z^j) = E_{Sr}^j \left(\frac{h^j}{2} \right) e^{ik^{*j} z^j} + F_{Sr}^j \left(\frac{h^j}{2} \right) e^{-ik^{*j} z^j}$$

$$k^{*j} = \frac{\omega}{V_S^{*j}}$$

$$E_{Sr}^j \left(\frac{h^j}{2} \right) \uparrow - F_{Sr}^j \left(\frac{h^j}{2} \right) \downarrow$$

2n + 1 unknowns (included downward propagating wave F_{Sr}^{N+1} into bedrock)



1D Multi-Layer SH Plane-Wave propagation

- ▶ **N Layer-to-Layer displacement continuity equations**

$$U_{Sr}^j(h^j) = U_{Sr}^{j+1}(0), \forall j \in [1, N] \cap \mathbb{N}$$

$$\rightarrow E_{Sr}^j\left(\frac{h^j}{2}\right) e^{ik^{*j} h^j} + F_{Sr}^j\left(\frac{h^j}{2}\right) e^{-ik^{*j} h^j} = E_{Sr}^{j+1}\left(\frac{h^{j+1}}{2}\right) + F_{Sr}^{j+1}\left(\frac{h^{j+1}}{2}\right)$$

- ▶ **N Layer-to-Layer traction continuity equations**

$$G^* \left(\gamma_{Sr}^{j-\text{eff}} \left(\frac{h^j}{2} \right) \right) \frac{\partial U_{Sr}^j}{\partial z^j} (h^j) = G^* \left(\gamma_{Sr}^{j+1-\text{eff}} \left(\frac{h^{j+1}}{2} \right) \right) \frac{\partial U_{Sr}^{j+1}}{\partial z^j} (0), \forall j \in [1, N] \cap \mathbb{N}$$

$$\rightarrow G^* \left(\gamma_{Sr}^{j-\text{eff}} \left(\frac{h^j}{2} \right) \right) k^{*j} E_{Sr}^j \left(\frac{h^j}{2} \right) e^{ik^{*j} h^j} - G^* \left(\gamma_{Sr}^{j-\text{eff}} \left(\frac{h^j}{2} \right) \right) k^{*j} F_{Sr}^j \left(\frac{h^j}{2} \right) e^{-ik^{*j} h^j} =$$

$$G^* \left(\gamma_{Sr}^{j+1-\text{eff}} \left(\frac{h^{j+1}}{2} \right) \right) k^{*j+1} E_{Sr}^{j+1} \left(\frac{h^{j+1}}{2} \right) -$$

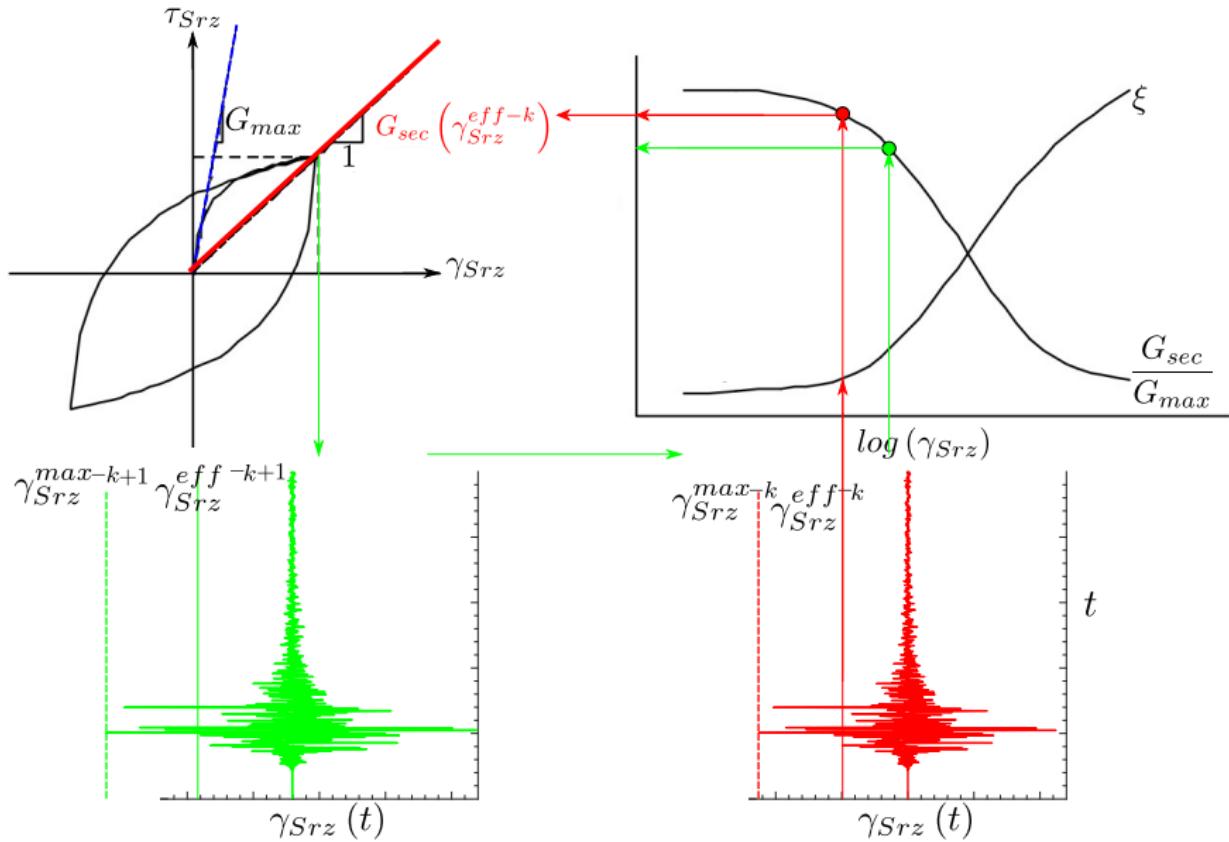
$$G^* \left(\gamma_{Sr}^{j+1-\text{eff}} \left(\frac{h^{j+1}}{2} \right) \right) k^{*j+1} F_{Sr}^{j+1} \left(\frac{h^{j+1}}{2} \right)$$

$$\gamma_{Sr z} (h^j, t) = ik^{*j} \left(E_{Sr}^j \left(\frac{h^j}{2} \right) e^{ik^{*j} h^j} - F_{Sr}^j \left(\frac{h^j}{2} \right) e^{-ik^{*j} h^j} \right)$$

- ▶ **Impedance ratios**

$$\alpha^{*j} (h^j) = \frac{G^* \left(\gamma_{Sr}^{j-\text{eff}} \left(\frac{h^j}{2} \right) \right) k^{*j}}{G^* \left(\gamma_{Sr}^{j+1-\text{eff}} \left(\frac{h^{j+1}}{2} \right) \right) k^{*j+1}}$$

1D Seismic Site Response by EQLM



1D Multi-Layer SH Plane-Wave propagation

- ▶ **$2N$ Interface recursive conditions**

$$E_{Sr}^{j+1}\left(\frac{h^{j+1}}{2}\right) = \frac{1}{2} E_{Sr}^j\left(\frac{h^j}{2}\right) (1 + \alpha^{*j}(h^j)) e^{ik^{*j} h^j} + \frac{1}{2} F_{Sr}^j\left(\frac{h^j}{2}\right) (1 - \alpha^{*j}(h^j)) e^{-ik^{*j} h^j}$$
$$F_{Sr}^{j+1}\left(\frac{h^{j+1}}{2}\right) = \frac{1}{2} E_{Sr}^j\left(\frac{h^j}{2}\right) (1 - \alpha^{*j}(h^j)) e^{ik^{*j} h^j} + \frac{1}{2} F_{Sr}^j\left(\frac{h^j}{2}\right) (1 + \alpha^{*j}(h^j)) e^{-ik^{*j} h^j}$$

- ▶ **System of $2N + 1$ equations in $2N + 1$ unknowns**

$$\underline{\mathbf{V}}_{Sr}^{j+1}\left(\frac{h^{j+1}}{2}\right) = \underline{\underline{\mathbf{P}}}^j(k^{*j}; \alpha^{*j}(h^j)) \underline{\mathbf{V}}_{Sr}^j\left(\frac{h^j}{2}\right)$$

$$\underline{\mathbf{V}}_{Sr}^{N+1} = \underline{\underline{\mathbf{P}}}(k^{*1, \dots, N}; \alpha^{*j, \dots, N}(h^{1, \dots, N})) \underline{\mathbf{V}}_{Sr}^1\left(\frac{h^1}{2}\right)$$

$$\rightarrow \underline{\underline{\mathbf{P}}}(k^{*1, \dots, N}; \alpha^{*j, \dots, N}(h^{1, \dots, N})) = \prod_{j=N}^1 \underline{\underline{\mathbf{P}}}^j(k^{*j}; \alpha^{*j}(h^j))$$

1D Multi-Layer SH Plane-Wave propagation

- ▶ **$2N$ Interface recursive conditions**

$$E_{Sr}^{j+1}\left(\frac{h^{j+1}}{2}\right) = \frac{1}{2} E_{Sr}^j\left(\frac{h^j}{2}\right) (1 + \alpha^{*j}(h^j)) e^{ik^{*j} h^j} + \frac{1}{2} F_{Sr}^j\left(\frac{h^j}{2}\right) (1 - \alpha^{*j}(h^j)) e^{-ik^{*j} h^j}$$
$$F_{Sr}^{j+1}\left(\frac{h^{j+1}}{2}\right) = \frac{1}{2} E_{Sr}^j\left(\frac{h^j}{2}\right) (1 - \alpha^{*j}(h^j)) e^{ik^{*j} h^j} + \frac{1}{2} F_{Sr}^j\left(\frac{h^j}{2}\right) (1 + \alpha^{*j}(h^j)) e^{-ik^{*j} h^j}$$

- ▶ **System of $2N + 1$ equations in $2N + 1$ unknowns**

$$\underline{\mathbf{V}}_{Sr}^{j+1}\left(\frac{h^{j+1}}{2}\right) = \underline{\underline{\mathbf{P}}}^j(k^{*j}; \alpha^{*j}(h^j)) \underline{\mathbf{V}}_{Sr}^j\left(\frac{h^j}{2}\right)$$

$$\underline{\mathbf{V}}_{Sr}^{N+1} = \underline{\underline{\mathbf{P}}}(k^{*1, \dots, N}; \alpha^{*j, \dots, N}(h^{1, \dots, N})) \underline{\mathbf{V}}_{Sr}^1\left(\frac{h^1}{2}\right)$$

$$\rightarrow \underline{\underline{\mathbf{P}}}(k^{*1, \dots, N}; \alpha^{*j, \dots, N}(h^{1, \dots, N})) = \prod_{j=N}^1 \underline{\underline{\mathbf{P}}}^j(k^{*j}; \alpha^{*j}(h^j))$$

Last equation to close the system ?

1D Multi-Layer SH Plane-Wave propagation

- ▶ 1 free-surface condition

$$G^* \left(\gamma_{Sr}^{1-\text{eff}} \left(\frac{h^1}{2} \right) \right) \frac{\partial U_{Sr}^1}{\partial z^1} (h^1) = 0$$

$$\rightarrow E_{Sr}^1 \left(\frac{h^1}{2} \right) = F_{Sr}^1 \left(\frac{h^1}{2} \right) \rightarrow \underline{\mathbf{V}}_{Sr}^1 \left(\frac{h^1}{2} \right) = E_{Sr}^1 \left(\frac{h^1}{2} \right) \underline{\mathbf{1}}$$

Input ground motion definition

1D Multi-Layer SH Plane-Wave propagation

- ▶ Wave-amplitude unknown at every layer but the bedrock

$$E_{Sr}^{1,\dots,N}\left(\frac{h^1,\dots,N}{2}\right), F_{Sr}^{1,\dots,N}\left(\frac{h^1,\dots,N}{2}\right) = ?$$

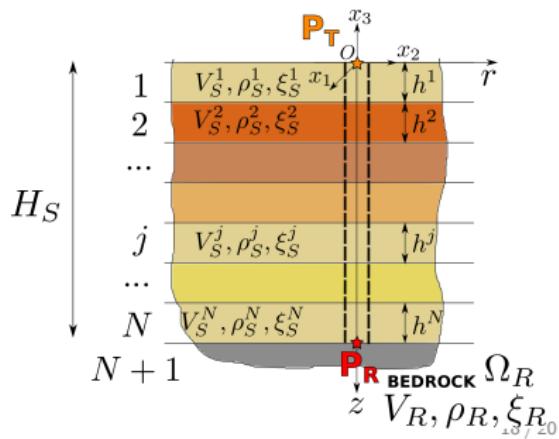
$$E_{Sr}^1\left(\frac{h^1}{2}\right) \underline{\mathbf{1}} = \underline{\underline{\mathbb{P}}}^{-1}\left(k^{*1,\dots,N}; \alpha^{*j,\dots,N}(h^1,\dots,N)\right) \underline{\mathbf{V}}_{Sr}^{N+1} ?$$

- ▶ Use of Transfer Functions $T^{ij}(\omega) = \frac{U_{Sr}^i}{U_{Sr}^j} = \frac{E_{Sr}^i + F_{Sr}^i}{E_{Sr}^j + F_{Sr}^j}$

- ▶ Need to introduce the Input Ground Motion @ P_R

E_{Sr}^{N+1} : incident wave-field

F_{Sr}^{N+1} : unknown reflected wave-field



Input ground motion definition

1D Multi-Layer SH Plane-Wave propagation

- ▶ Wave-amplitude unknown at every layer but the bedrock

$$E_{Sr}^{1,\dots,N}\left(\frac{h^1,\dots,N}{2}\right), F_{Sr}^{1,\dots,N}\left(\frac{h^1,\dots,N}{2}\right) = ?$$

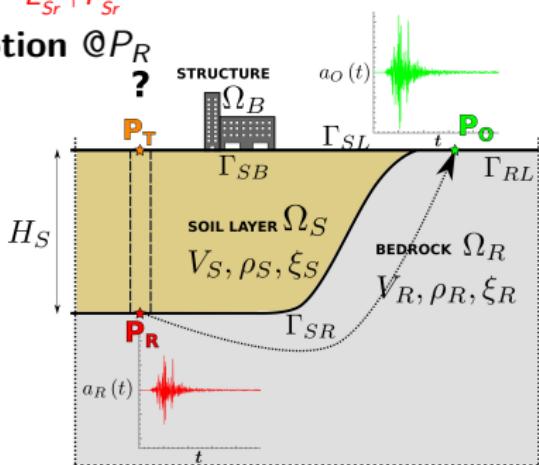
$$E_{Sr}^1\left(\frac{h^1}{2}\right) \underline{\mathbf{1}} = \underline{\underline{\mathbb{P}}}^{-1}\left(k^{*1,\dots,N}; \alpha^{*j,\dots,N}(h^1,\dots,N)\right) \underline{\mathbf{V}}_{Sr}^{N+1} ?$$

- ▶ Use of Transfer Functions $T^{ij}(\omega) = \frac{U_{Sr}^i}{U_{Sr}^j} = \frac{E_{Sr}^i + F_{Sr}^i}{E_{Sr}^j + F_{Sr}^j}$

- ▶ Need to introduce the Input Ground Motion @ P_R

E_{Sr}^{N+1} : incident wave-field

F_{Sr}^{N+1} : unknown reflected wave-field



Within Condition

$$U_{Rr} = E_{Sr}^{N+1} + F_{Sr}^{N+1}$$

$$T^{iR}(\omega) = \frac{E_{Sr}^i + F_{Sr}^i}{E_{Sr}^{N+1} + F_{Sr}^{N+1}}(\omega)$$

$$\rightarrow U_{Sr}^i(\omega) = T^{iR}(\omega) U_{Rr}$$

Input ground motion definition

1D Multi-Layer SH Plane-Wave propagation

- ▶ Wave-amplitude unknown at every layer but the bedrock
 $E_{Sr}^{1,\dots,N}\left(\frac{h^1,\dots,N}{2}\right), F_{Sr}^{1,\dots,N}\left(\frac{h^1,\dots,N}{2}\right) = ?$
- ▶ Use of Transfer Functions $T^{ij}(\omega) = \frac{U_{Sr}^i}{U_{Sr}^j} = \frac{E_{Sr}^i + F_{Sr}^i}{E_{Sr}^j + F_{Sr}^j}$
- ▶ Need to introduce the Input Ground Motion @ P_R

E_{Sr}^{N+1} : incident wave-field

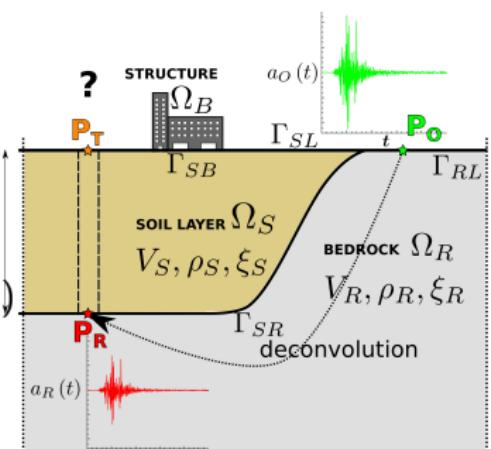
F_{Sr}^{N+1} : unknown reflected wave-field

Outcrop Condition

$$U_{Rr} = 2E_{Sr}^{N+1}$$

$$T^{iR}(\omega) = \frac{E_{Sr}^i + F_{Sr}^i}{E_{Sr}^{N+1} + F_{Sr}^{N+1}}(\omega), T^{RO}(\omega) = \frac{E_{Sr}^{N+1} + F_{Sr}^{N+1}}{2E_{Sr}^{N+1}}(\omega)$$

$$\rightarrow U_{Sr}^i(\omega) = T^{iR}(\omega) T^{RO}(\omega) U_{Rr}$$



- Strain softening excluded (ill-posed problem)

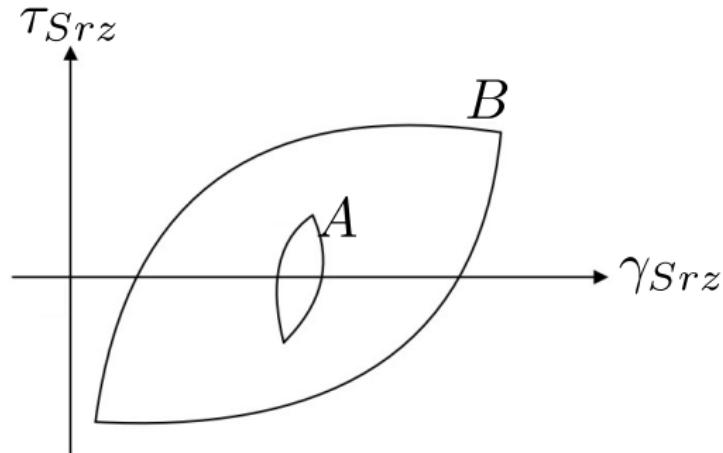
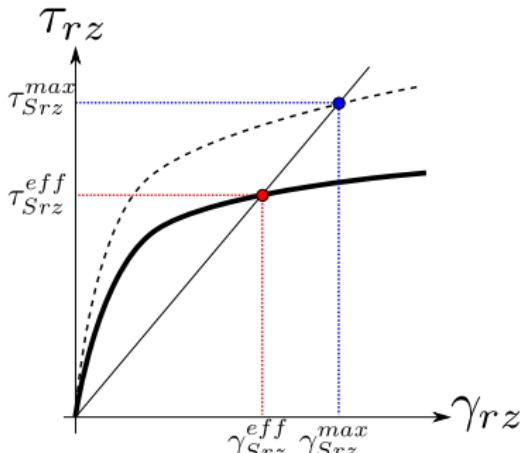
$$\rightarrow \frac{\partial \tau_{Srz}}{\partial \gamma_{Srz}} = G(\gamma_{Srz}) + \frac{\partial G}{\partial \gamma_{Srz}} \gamma_{Srz} \geq 0$$

- $\frac{G}{G_{max}} - \gamma - D$ curves numerical discretization :

$$\rightarrow -\frac{\Delta G}{G_{max}}(\gamma_{Srz}^n) \leq \frac{\Delta \gamma_{Srz}^n}{\gamma_{Srz}^n} \frac{G(\gamma_{Srz}^n)}{G_{max}}, \Delta G(\gamma_{Srz}^n) < 0 \Leftrightarrow \Delta \gamma_{Srz}^n > 0$$

- Convergence achieved :

$$\Delta \gamma_{Srz}^n \nearrow \rightarrow |\Delta G(\gamma_{Srz}^n)| \searrow$$



**GRAZIE DELL'ATTENZIONE
MERCI DE VOTRE ATTENTION
THANKS FOR THE ATTENTION**

Filippo Gatti[3, 7]

**filippo.gatti@centralesupelec.fr
filippo.gatti@polimi.it**



<https://github.com/FillLTP89>

REFERENCES & ACKNOWLEDGEMENTS

JP Bardet, K Ichii, and CH Lin.

EERA - A computer program for Equivalent-linear Earthquake site Response Analyses of layered soil deposits.

University of Southern California, August 2000.

I.A. Beresnev and Kuo-liang Wen.

Nonlinear Soil Response-A Reality?

Bulletin of the Seismological Society of America, 86(6), 1964–1978, 1996.

Filippo Gatti, Fernando Lopez-Caballero, and Didier Clouteau.

One-Dimensional Seismic Response at the Nuclear Power Plant of Kashiwazaki-Kariwa during the 2007 Niigata-Chuetsu-Oki Earthquake.

In J. Kruis, Y. Tsompanakis, and R.H.W. Topping, editors, *Proceedings of the Fifteenth International Conference on Civil, Structural and Environmental Engineering Computing*, number 157, pages 1–14, Stirlingshire, UK., 2015. Civil-Comp Press.

I. M. Mriss.

Procedures for Selecting Earthquake Ground Motions at Rock Sites.

Technical report, 1991.

G Masing.



On Heyn's hardening theory of metals due to inner elastic stresses.

Wiss. Veröff. Siemens-Konzern, 3 :231–239, 1923.

Alain Pecker.

Dynamique des Sols.

Presse de l'École Nationale des Ponts et Chaussées, 1984.

C. Smerilli, F. Gatti, and R. Paolucci.

An artificial neural network approach to support physics-based generation of broadband earthquake ground motions, Septembre 2016.



The work carried out under the SINAPS® project has benefited from French funding managed by the National Research Agency under the program "Future Investments" - SINAPS® reference No. ANR-11-RSNR-0022.

