NOMENCLATURE

English letter symbols

A	heat transfer or flow friction area
A_c	flow cross section area
a	radius of a circular duct, half width of rectangular duct, semi-major axis of the elliptical duct, half base width of triangular or sine duct, $a \geq b$ for rectangular and elliptical ducts with symmetric heating
a'	duct wall thickness
B_1, B_2	constants; see Eq. (76)
b	half spacing of parallel plates, half height of rectangular duct, semi-minor axis of elliptical duct, half height of triangular or sine duct, $b \leq a$ for rectangular and elliptical ducts with symmetric heating
b	amplitude of cosine heat flux variation around the periphery of a circular duct; see Fig. 7
C	flow stream capacity rate, Wc_p
c_1	a pressure gradient parameter, $(dp/dx)/(\mu/g_c)$
c_2	a temperature gradient parameter, $(\partial t/\partial x)/\alpha$
c_3	thermal energy source parameter, S/k
c_4	a parameter, $c_1 c_2$
c_5	a parameter, $c_3/c_4 a^2$
c_6	a parameter, $g_c(dp/dx)/\rho c_p(\partial t/\partial x)$
c_p	specific heat of the fluid at constant pressure
D_h	hydraulic diameter of the duct or flow passages, $D_h = 4r_h$
$E(m)$	complete elliptical integral of second kind
f	"Fanning" or "small" friction factor, for fully developed flow if no subscript, $\tau/(\rho u_m^2/2g_c)$, dimensionless

For the Newtonian fluid flowing through a circular duct, the local wall shear stress is given by [5]

$$\tau_x = \frac{\mu}{g_c}\left(\frac{\partial u}{\partial r}\right)_{r=a} \qquad (23)^7$$

The local wall shear stress for other duct geometries can be expressed similarly for the cartesian coordinate system. Except where more detailed information is needed, the local wall shear stress is consistently defined as average wall shear stress with respect to the perimeter of the duct; e.g. for the axisymmetric flow in a rectangular duct, Fig. 5, at any cross section x ,

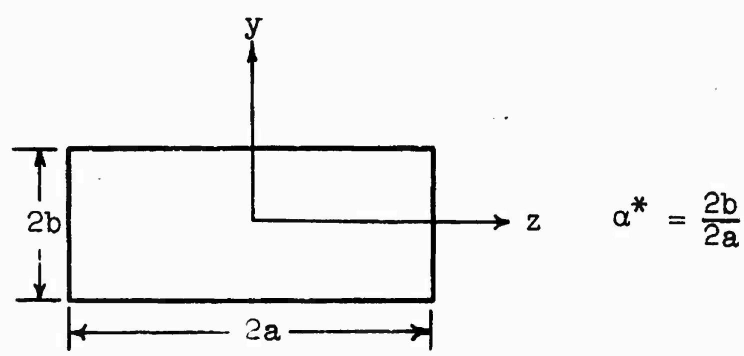

Fig. 5 A cross section of a rectangular duct

$$\tau_x = \frac{\mu}{2g_c(a+b)}\left[\int_{-a}^{a}\left(\frac{\partial u}{\partial y}\right)_{y=b} dz + \int_{-b}^{b}\left(\frac{\partial u}{\partial z}\right)_{z=a} dy\right] \qquad (24)$$

The local and average Fanning friction factors are subsequently determined. They are defined as

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[7] The dynamic viscosity coefficient μ defined here is the g_c times the usual fluid dynamics dynamic viscosity coefficient. Hence note that Newton's second law of motion is not invoked in Eq. (23), even though g_c appears in that equation.

3. RECTANGULAR DUCTS

For the rectangular duct fully developed fRe , Nu_{H1} , Nu_{H2} , and Nu_T as well as the Nusselt numbers for different wall boundary conditions on each wall have been determined. The hydrodynamic entry length problem has also been solved. However, there remains a need for the refinement for the thermal entry length solutions for the simultaneously developing flow, which so far were obtained by neglecting the transverse velocity components v and w . The experience with the circular duct, discussed in Section V.1.3.2 indicated that this neglect is not valid near the thermal entrance.

3.1 Fully Developed Flow

3.1.1. Velocity Profile and Friction Factors

Fully developed velocity profile for the rectangular ducts has been determined from the analogy with the stress function in theory of elasticity [2,31,33].

Consider the cross section of rectangular duct as shown in Fig. 5 with flow direction in x axis. The velocity profile, the solution of Eq. (3a) with boundary condition of Eq. (4), from Ref. [33] is

$$u = - \frac{16 c_1 a^2}{\pi^3} \sum_{n=1,3,\ldots}^{\infty} \frac{1}{n^3} (-1)^{\frac{n-1}{2}} \left[1 - \frac{\cosh(n\pi y/2a)}{\cosh(n\pi b/2a)} \right] \cos\frac{n\pi z}{2a} \tag{147}$$

$$u_m = - \frac{c_1 a^2}{3} \left[1 - \frac{192}{\pi^5} \frac{a}{b} \sum_{n=1,3,\ldots}^{\infty} \frac{1}{n^3} \tanh \frac{n\pi b}{2a} \right] \tag{148}$$

and

$$fRe = -\frac{8c_1 a^2}{u_m(1 + \frac{a}{b})^2} \qquad (149)$$

The velocity profile of Eq. (147) is in excellent agreement with the experimental results of Holmes and Vermeulen [210].

The friction factors were calculated from Eq. (149) on the Stanford IBM 360/67 computer using double precision and taking first 30 terms in the series and were checked against 25 terms in series. Seven digit accuracy was thus established. The results are presented in Table 13 and Fig. 19.

Table 13. Rectangular ducts fRe , $K(\infty)$, L^+_{hy} , Nu_T , Nu_{H1} and Nu_{H2} for fully developed laminar flow, when all four walls are transferring the heat.

α*	fRe	$K(\infty)$[20]	L^+_{hy}[20]	Nu_T[211]	Nu_{H1}	Nu_{H2}[60]
1.000	14.22708	1.5515	0.0324	2.976	3.607949	3.091
0.900	14.26098				3.620452	
1/1.2	14.32808				3.645310	
0.800	14.37780				3.663823	
0.750	-	1.5203	0.0310		-	
1/1.4	14.56482			3.077	3.734193	
0.700	14.60538				3.749608	
2/3	14.71184			3.117[11]	3.790327	
0.600	14.97996				3.894556	
0.500	15.54806	1.3829	0.0255	3.391	4.123303	3.017
0.400	16.36810	1.2815	0.0217		4.471852	
1/3	17.08967			3.956	4.794796	
0.300	17.51209				4.989888	
0.250	18.23278	1.0759	0.0147	4.439	5.331064	2.930
0.200	19.07050				5.737689	
1/6	19.70220	0.9451	0.0110	5.137	6.049456	
1/7	20.19310				6.294041	
0.125	20.58464	0.8788	0.00938	5.597	6.490334	2.904
1/9	20.90385				6.651060	
0.100	21.16888	0.8392	0.00855		6.784947	
0.050	22.47701	0.7613	0.00709		7.450827	
0.000	24.00000	0.6857	0.00588	7.540716	8.235294	8.235

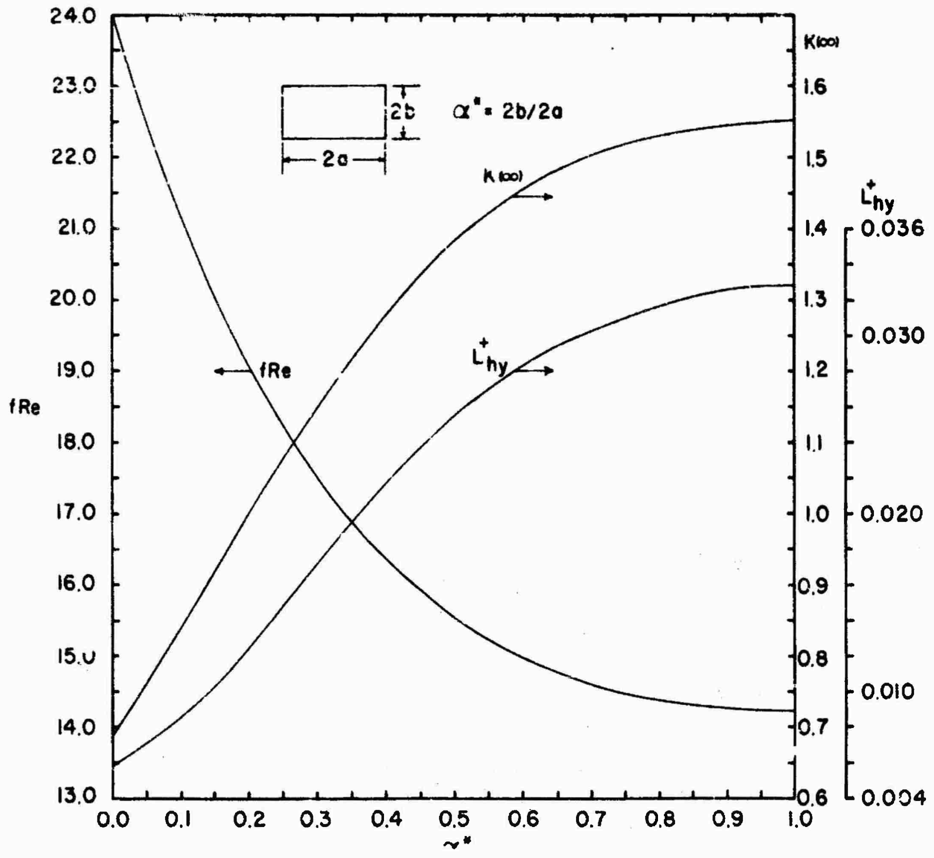

Fig. 19 <u>Rectangular ducts</u> fRe , K(∞) and L^+_{hy} for fully developed laminar flow.

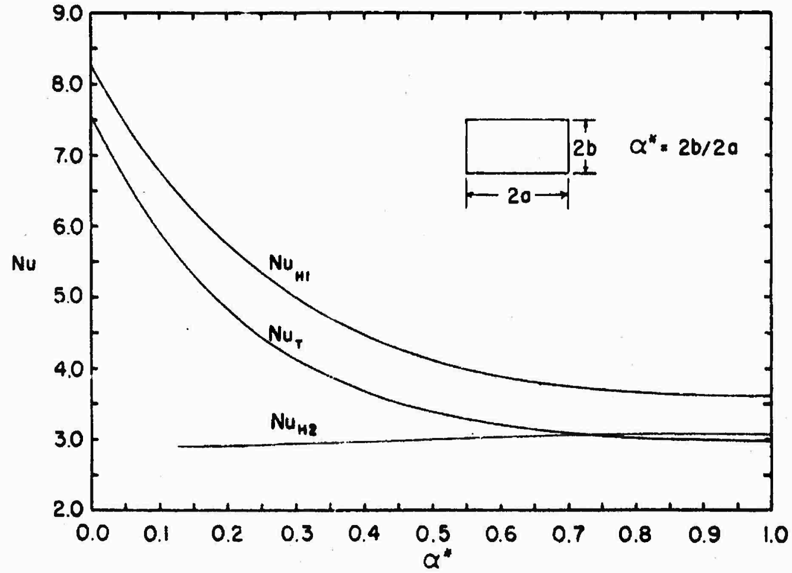

Fig. 20 <u>Rectangular ducts</u> Nu_T , Nu_{H1} and Nu_{H2} for fully developed laminar flow.