3. RECTANGULAR DUCTS

For the rectangular duct fully developed fRe , Nu_{H1} , Nu_{H2} , and Nu_T as well as the Nusselt numbers for different wall boundary conditions on each wall have been determined. The hydrodynamic entry length problem has also been solved. However, there remains a need for the refinement for the thermal entry length solutions for the simultaneously developing flow, which so far were obtained by neglecting the transverse velocity components v and w . The experience with the circular duct, discussed in Section V.1.3.2 indicated that this neglect is not valid near the thermal entrance.

3.1 Fully Developed Flow

3.1.1. Velocity Profile and Friction Factors

Fully developed velocity profile for the rectangular ducts has been determined from the analogy with the stress function in theory of elasticity [2,31,33].

Consider the cross section of rectangular duct as shown in Fig. 5 with flow direction in x axis. The velocity profile, the solution of Eq. (3a) with boundary condition of Eq. (4), from Ref. [33] is

$$u = -\frac{16c_1a^2}{\pi^3}\sum_{n=1,3,\ldots}^{\infty}\frac{1}{n^3}(-1)^{\frac{n-1}{2}}\left[1-\frac{\cosh(n\pi y/2a)}{\cosh(n\pi b/2a)}\right]\cos\frac{n\pi z}{2a} \tag{147}$$

$$u_m = -\frac{c_1a^2}{3}\left[1-\frac{192}{\pi^5}\frac{a}{b}\sum_{n=1,3,\ldots}^{\infty}\frac{1}{n^3}\tanh\frac{n\pi b}{2a}\right] \tag{148}$$

and

$$fRe = -\frac{8c_1 a^2}{u_m(1 + \frac{a}{b})^2} \tag{149}$$

The velocity profile of Eq. (147) is in excellent agreement with the experimental results of Holmes and Vermeulen [210].

The friction factors were calculated from Eq. (149) on the Stanford IBM 360/67 computer using double precision and taking first 30 terms in the series and were checked against 25 terms in series. Seven digit accuracy was thus established. The results are presented in Table 13 and Fig. 19.

Table 13. Rectangular ducts fRe, $K(\infty)$, L^+_{hy}, Nu_T, Nu_{H1} and Nu_{H2} for fully developed laminar flow, when all four walls are transferring the heat.

α*	fRe	K(∞)[20]	L^+_{hy}[20]	Nu_T[211]	Nu_{H1}	Nu_{H2}[60]
1.000	14.22708	1.5515	0.0324	2.976	3.607949	3.091
0.900	14.26098				3.620452	
1/1.2	14.32808				3.645310	
0.800	14.37780				3.663823	
0.750	-	1.5203	0.0310		-	
1/1.4	14.56482			3.077	3.734193	
0.700	14.60538				3.749608	
2/3	14.71184			3.117[11]	3.790327	
0.600	14.97996				3.894556	
0.500	15.54806	1.3829	0.0255	3.391	4.123303	3.017
0.400	16.36810	1.2815	0.0217		4.471852	
1/3	17.08967			3.956	4.794796	
0.300	17.51209				4.989888	
0.250	18.23278	1.0759	0.0147	4.439	5.331064	2.930
0.200	19.07050				5.737689	
1/6	19.70220	0.9451	0.0110	5.137	6.049456	
1/7	20.19310				6.294041	
0.125	20.58464	0.8788	0.00938	5.597	6.490334	2.904
1/9	20.90385				6.651060	
0.100	21.16888	0.8392	0.00855		6.784947	
0.050	22.47701	0.7613	0.00709		7.450827	
0.000	24.00000	0.6857	0.00588	7.540716	8.235294	8.235