Heat equation

The heat equation is in carthesian coordinates is derived from Fourier's law of heat conduction and is written as

$$\frac{\partial}{\partial \mathbf{x}}\left(k\frac{\partial T}{\partial \mathbf{x}}\right) + \dot{e}_{gen} = \rho c\frac{\partial T}{\partial t} \tag{1}$$

where T is the variable temperature, $\dot{e}_{gen}$ is the constant rate of heat generated per unit volume, k is the material thermal conductivity, ρ is the matrial density and c is the specific heat. For a constant material conductivity a parameter $\alpha = \frac{k}{\rho c}$ is usualy introduced and it describes the thermal diffusivity.

Boundary conditions

Deriving exact solutions

For simple geometries and assumptions, exact solutions can be derived.

A finite difference approach

A second order scheme for the one dimensional heat equation with constant can be written as

$$\frac{T_{i+1}^{n} - 2T_{i}^{n} + T_{i-1}^{n}}{\Delta t^2} = \frac{1}{\alpha}\frac{T_{i}^{n+1} - T_{i}^{n-1}}{2\Delta t} \tag{2}$$