Construction of Modal Solution on a Heterogeneous Spherical Shell

A Chebfun script for creating a modal solution for linear elasticity on a radially heterogeneous spherical shell.

Readable Version of README.md

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pandoc README.md -o README.pdf
```

Synopsis

The purpose of the MATLAB Chebfun script `shell.m` is to construct a high-order approximation of a modal solution to a radially heterogeneous, isotropic elastic shell. If u_r, u_θ, u_ϕ are the displacements in a spherical coordinate system, then the solution is taken to be of the form $u_r = \cos(t)\phi(r)$ and $u_\phi = u_\theta = 0$.

The script specifies the material properties using the density ρ, Lamé's first parameter λ, and Lamé's second parameter μ (i.e., the shear modulus). The inner and outer radii of the shell are chosen so that the resulting boundary condition is traction free (free surface). Specifically, once $\phi(r)$ is determined the radii are chosen to be zeros of

$$\sigma_{rr} = (\lambda + 2\mu)\,\phi_{,r} + \frac{2}{r}\lambda\phi,$$

where the comma in the subscripts denotes a partial derivative with respect to the variable that follows.

The script outputs the inner and outer radii of the spherical shell (`R1` and `R2`) as well as information needed to build a polynomial interpolant (via barycentric interpolation) of the solution. The full output of the script is given in `shell_data.m`.

To convert from the spherical to Cartesian system the following transforms of the radial displacement u_r and the derivative $u_{r,r}$ can be used (from which the stresses and/or strains can be defined)

$$u_x = \frac{x}{r}u_r, \quad u_y = \frac{y}{r}u_r, \quad u_z = \frac{z}{r}u_r,$$

$$u_{x,x} = \frac{x^2 r u_{r,r} + y^2 u_r + z^2 u_r}{r^3}, \quad u_{x,y} = \frac{xy(r u_{r,r} - u_r)}{r^3}, \quad u_{x,z} = \frac{xz(r u_{r,r} - u_r)}{r^3},$$

$$u_{y,x} = \frac{yy(ru_{r,r} - u_r)}{r^3}, \quad u_{y,y} = \frac{y^2ru_{r,r} + x^2u_r + Z^2u_r}{r^3}, \quad u_{y,z} = \frac{yz(ru_{r,r} - u_r)}{r^3},$$

$$u_{z,x} = \frac{zy(ru_{r,r} - u_r)}{r^3}, \quad u_{z,y} = \frac{zy(ru_{r,r} - u_r)}{r^3}, \quad u_{z,z} = \frac{z^2ru_{r,r} + x^2u_r + y^2u_r}{r^3}.$$

Plots of the non-zero components of the solution and the material properties are shown in the figure

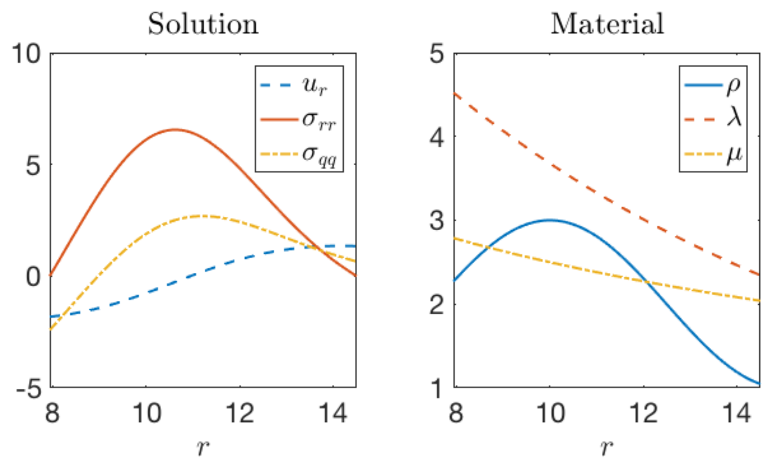

Figure 1: Solution and Material Properties

Dependencies

- MATLAB
- Chebfun

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