

PhD Thesis

Investigation of complex liquid-gas turbulent interfacial flows

A Numerical Study

Sagar Pal

April 22, 2020

Institut Jean le Rond ∂ 'Alembert
Sorbonne Université

Disclaimer

You can edit this page to suit your needs. For instance, here we have a no copyright statement, a colophon and some other information. This page is based on the corresponding page of Ken Arroyo Ohori's thesis, with minimal changes.

No copyright

© This book is released into the public domain using the CC0 code. To the extent possible under law, I waive all copyright and related or neighbouring rights to this work. To view a copy of the CC0 code, visit:

<http://creativecommons.org/publicdomain/zero/1.0/>

Colophon

This document was typeset with the help of KOMA-Script and L^AT_EX using the kaobook class.

The source code of this book is available at:

<https://github.com/fmarotta/kaobook>

(You are welcome to contribute!)

Publisher

First printed in May 2019 by Institut Jean le Rond d'Alembert
Sorbonne Université

The harmony of the world is made manifest in Form and
Number, and the heart and soul and all the poetry of
Natural Philosophy are embodied in the concept of
mathematical beauty.

– D'Arcy Wentworth Thompson

Contents

Contents	v
1 Introduction	1
NUMERICAL DEVELOPMENT	3
2 Methodology	4
2.1 Governing Equations	4
2.2 Interface Tracking	5
2.3 Time Marching	5
3 Artificial Atomization : The Falling Raindrop	6
3.1 Computational Setup	6
3.2 Exploration of Blowups	6
3.3 Origin of Numerical Instabilities	6
4 Consistent Mass-Momentum Transport	7
4.1 Principles of Momentum Consistent Schemes	7
4.2 Consistent Flux Computation	7
4.3 Reconstruction on Staggered Cells	8
4.4 Sub-Grid Strategy	8
4.5 Summary of Methods	9
5 Numerical Benchmarks	10
5.1 Static Droplet	10
5.2 Moving Droplet	14
5.3 Capillary Wave	16
5.4 Falling Raindrop	18
PHYSICS OF FRAGMENTATION	21
6 Ligament Mediated Paradigm	22
6.1 Mechanism of Drop Formation	22
6.2 Theories of Fragmentation	23
7 Droplet Generation in Corrugated Ligaments	24
7.1 Numerical Setup	24

7.2	Ligament Breakup	25
8	Statistics of Drop Sizes	27
8.1	Monte Carlo Approach to DNS	27
8.2	Millimeter Scale Ensembles	27
8.3	Exploration of Parameter Space Φ	28
	CONCLUSIONS & PERSPECTIVES	29
	APPENDIX	31
A	Heading on Level 0 (chapter)	32
A.1	Heading on Level 1 (section)	32
	Heading on Level 2 (subsection)	32
A.2	Lists	33
	Example for list (itemize)	33
	Example for list (enumerate)	33
	Example for list (description)	34
	Bibliography	35
	Notation	36

List of Figures

5.1	Schematic of the static droplet of dense fluid surrounded by a quiescent medium of lighter fluid. A 40×40 grid is employed to spatially discretize the domain.	11
5.2	M1 Decay of normalized spurious currents as a function of viscous dissipation time-scales for different density-ratios and Laplace numbers. The currents seem to initially decay quickly for all higher density-ratios, and relax to the numerical equilibrium curvature even within $0.2 \cdot T_\mu$. For combinations of large ρ_l/ρ_g and large La , the spurious currents seem to grow back to an order of magnitude (10^{-3}) which is quite far from that of machine precision (10^{-14}).	13
5.3	M2 Decay of normalized spurious currents as a function of viscous dissipation time-scales for different density-ratios and Laplace numbers. The currents seem to initially decay quickly for all higher density-ratios, and relax to the numerical equilibrium curvature even within $0.2 \cdot T_\mu$. For combinations of large ρ_l/ρ_g and large La , the spurious currents seem to grow back to an order of magnitude (10^{-3}) which is quite far from that of machine precision (10^{-14}). No considerable improvement is observed with respect to M1	13
5.4	M3 Decay of normalized spurious currents as a function of viscous dissipation time-scales for different density-ratios and Laplace numbers. The currents seem to decay very quickly in the case of higher density-ratios, and relax to the numerical equilibrium curvature even within $0.2 \cdot T_\mu$. For all combinations of ρ_l/ρ_g and the highest La number, the spurious currents first decay very quickly and then grow back to an order of magnitude 10^{-3} at a slower pace. All other combinations of density-ratio and Laplace numbers do seem to decay to an order of magnitude close to machine precision (10^{-14}). There is a slight improvement over M1 and M2 , especially for all except the highest Laplace number.	14
5.5	Schematic of the droplet of dense fluid advected in a surrounding medium of lighter fluid. A 64×64 grid is employed to spatially discretize the domain, which is spatially periodic in the direction of droplet advection.	15
5.6	Schematic of the initially perturbed planar interface separating two immiscible fluids of different densities and viscosities. A spatial resolution of 32×96 is used for spatial discretization (compared to 64×192 in Popinet [5]), with the width of the box corresponding to the size of the perturbed wavelength.	17
5.7	A 2D schematic of the numerical setup for the falling raindrop. We apply a uniform inflow velocity condition with $U_0(t)$ and an outflow velocity condition at the top which corresponds to zero gradient of the velocity at the boundary. Boundary conditions on the side walls correspond to those of free slip (no shear stress).	20

List of Tables

5.1	Parameter values used in the simulation of a falling water droplet in air.	19
-----	--	----

Multiphase Flows

Brief description of multiphase flows in nature Hello, here is some text without a meaning. This text should show what a printed text will look like at this place. If you read this text, you will get no information. Really? Is there no information? Is there a difference between this text and some nonsense like “Huardest gefburn”? Kjift – not at all! A blind text like this gives you information about the selected font, how the letters are written and an impression of the look. This text should contain all letters of the alphabet and it should be written in of the original language. There is no need for special content, but the length of words should match the language.

Surface tension dominated flows Hello, here is some text without a meaning. This text should show what a printed text will look like at this place. If you read this text, you will get no information. Really? Is there no information? Is there a difference between this text and some nonsense like “Huardest gefburn”? Kjift – not at all! A blind text like this gives you information about the selected font, how the letters are written and an impression of the look. This text should contain all letters of the alphabet and it should be written in of the original language. There is no need for special content, but the length of words should match the language.

Fragmentation

Brief description of atomization Hello, here is some text without a meaning. This text should show what a printed text will look like at this place. If you read this text, you will get no information. Really? Is there no information? Is there a difference between this text and some nonsense like “Huardest gefburn”? Kjift – not at all! A blind text like this gives you information about the selected font, how the letters are written and an impression of the look. This text should contain all letters of the alphabet and it should be written in of the original language. There is no need for special content, but the length of words should match the language.

Importance of drop size distributions Hello, here is some text without a meaning. This text should show what a printed text will look like at this place. If you read this text, you will get no information. Really? Is there no information? Is there a difference between this text and some nonsense like “Huardest gefburn”? Kjift – not at all! A blind text like this gives you information about the selected font, how the letters are written

and an impression of the look. This text should contain all letters of the alphabet and it should be written in of the original language. There is no need for special content, but the length of words should match the language.

Numerical Platforms

PARIS Simulator Hello, here is some text without a meaning. This text should show what a printed text will look like at this place. If you read this text, you will get no information. Really? Is there no information? Is there a difference between this text and some nonsense like “Huardest gefburn”? Kjift – not at all! A blind text like this gives you information about the selected font, how the letters are written and an impression of the look. This text should contain all letters of the alphabet and it should be written in of the original language. There is no need for special content, but the length of words should match the language.

Basilisk Hello, here is some text without a meaning. This text should show what a printed text will look like at this place. If you read this text, you will get no information. Really? Is there no information? Is there a difference between this text and some nonsense like “Huardest gefburn”? Kjift – not at all! A blind text like this gives you information about the selected font, how the letters are written and an impression of the look. This text should contain all letters of the alphabet and it should be written in of the original language. There is no need for special content, but the length of words should match the language.

NUMERICAL DEVELOPMENT

2.1 Governing Equations

2.1 Governing Equations	4
2.2 Interface Tracking	5
2.3 Time Marching	5

Conservative versus Non-Conservative Formulations Hello, here is some text without a meaning. This text should show what a printed text will look like at this place. If you read this text, you will get no information. Really? Is there no information? Is there a difference between this text and some nonsense like “Huardest gefburn”? Kjift – not at all! A blind text like this gives you information about the selected font, how the letters are written and an impression of the look. This text should contain all letters of the alphabet and it should be written in of the original language. There is no need for special content, but the length of words should match the language.

Description of Operators Hello, here is some text without a meaning. This text should show what a printed text will look like at this place. If you read this text, you will get no information. Really? Is there no information? Is there a difference between this text and some nonsense like “Huardest gefburn”? Kjift – not at all! A blind text like this gives you information about the selected font, how the letters are written and an impression of the look. This text should contain all letters of the alphabet and it should be written in of the original language. There is no need for special content, but the length of words should match the language.

Evolution of phase-characteristic function Hello, here is some text without a meaning. This text should show what a printed text will look like at this place. If you read this text, you will get no information. Really? Is there no information? Is there a difference between this text and some nonsense like “Huardest gefburn”? Kjift – not at all! A blind text like this gives you information about the selected font, how the letters are written and an impression of the look. This text should contain all letters of the alphabet and it should be written in of the original language. There is no need for special content, but the length of words should match the language.

Material Properties Hello, here is some text without a meaning. This text should show what a printed text will look like at this place. If you read this text, you will get no information. Really? Is there no information? Is there a difference between this text and some nonsense like “Huardest gefburn”? Kjift – not at all! A blind text like this gives you information about the selected font, how the letters are written and an impression of the look. This text should contain all letters of the alphabet and it should be written in of the original language. There is no need for special content, but the length of words should match the language.

2.2 Interface Tracking

Volume-of-Fluid : PLIC Methodology Hello, here is some text without a meaning. This text should show what a printed text will look like at this place. If you read this text, you will get no information. Really? Is there no information? Is there a difference between this text and some nonsense like “Huardest gefburn”? Kjift – not at all! A blind text like this gives you information about the selected font, how the letters are written and an impression of the look. This text should contain all letters of the alphabet and it should be written in of the original language. There is no need for special content, but the length of words should match the language.

Flux Computation : CIAM , WY Hello, here is some text without a meaning. This text should show what a printed text will look like at this place. If you read this text, you will get no information. Really? Is there no information? Is there a difference between this text and some nonsense like “Huardest gefburn”? Kjift – not at all! A blind text like this gives you information about the selected font, how the letters are written and an impression of the look. This text should contain all letters of the alphabet and it should be written in of the original language. There is no need for special content, but the length of words should match the language.

2.3 Time Marching

Spatio-Temporal Discretization Hello, here is some text without a meaning. This text should show what a printed text will look like at this place. If you read this text, you will get no information. Really? Is there no information? Is there a difference between this text and some nonsense like “Huardest gefburn”? Kjift – not at all! A blind text like this gives you information about the selected font, how the letters are written and an impression of the look. This text should contain all letters of the alphabet and it should be written in of the original language. There is no need for special content, but the length of words should match the language.

Pressure-Projection Algorithm Hello, here is some text without a meaning. This text should show what a printed text will look like at this place. If you read this text, you will get no information. Really? Is there no information? Is there a difference between this text and some nonsense like “Huardest gefburn”? Kjift – not at all! A blind text like this gives you information about the selected font, how the letters are written and an impression of the look. This text should contain all letters of the alphabet and it should be written in of the original language. There is no need for special content, but the length of words should match the language.

Artificial Atomization : The Falling Raindrop

3

3.1 Computational Setup

Parameterization : Reynolds, Weber, Bond Hello, here is some text without a meaning. This text should show what a printed text will look like at this place. If you read this text, you will get no information. Really? Is there no information? Is there a difference between this text and some nonsense like “Huardest gefburn”? Kjift – not at all! A blind text like this gives you information about the selected font, how the letters are written and an impression of the look. This text should contain all letters of the alphabet and it should be written in of the original language. There is no need for special content, but the length of words should match the language.

3.2 Exploration of Blowups

Combinations of Advection Scheme & Flux Limiters Hello, here is some text without a meaning. This text should show what a printed text will look like at this place. If you read this text, you will get no information. Really? Is there no information? Is there a difference between this text and some nonsense like “Huardest gefburn”? Kjift – not at all! A blind text like this gives you information about the selected font, how the letters are written and an impression of the look. This text should contain all letters of the alphabet and it should be written in of the original language. There is no need for special content, but the length of words should match the language.

3.3 Origin of Numerical Instabilities

Un-physical Stagnation Pressures Hello, here is some text without a meaning. This text should show what a printed text will look like at this place. If you read this text, you will get no information. Really? Is there no information? Is there a difference between this text and some nonsense like “Huardest gefburn”? Kjift – not at all! A blind text like this gives you information about the selected font, how the letters are written and an impression of the look. This text should contain all letters of the alphabet and it should be written in of the original language. There is no need for special content, but the length of words should match the language.

3.1 Computational Setup	6
3.2 Exploration of Blowups	6
3.3 Origin of Numerical Instabilities	6

Consistent Mass-Momentum Transport

4

4.1 Principles of Momentum Consistent Schemes

Major Iterations in Literature Hello, here is some text without a meaning. This text should show what a printed text will look like at this place. If you read this text, you will get no information. Really? Is there no information? Is there a difference between this text and some nonsense like “Huardest gefburn”? Kjift – not at all! A blind text like this gives you information about the selected font, how the letters are written and an impression of the look. This text should contain all letters of the alphabet and it should be written in of the original language. There is no need for special content, but the length of words should match the language.

Overview of Methods Hello, here is some text without a meaning. This text should show what a printed text will look like at this place. If you read this text, you will get no information. Really? Is there no information? Is there a difference between this text and some nonsense like “Huardest gefburn”? Kjift – not at all! A blind text like this gives you information about the selected font, how the letters are written and an impression of the look. This text should contain all letters of the alphabet and it should be written in of the original language. There is no need for special content, but the length of words should match the language.

Our Strategies Hello, here is some text without a meaning. This text should show what a printed text will look like at this place. If you read this text, you will get no information. Really? Is there no information? Is there a difference between this text and some nonsense like “Huardest gefburn”? Kjift – not at all! A blind text like this gives you information about the selected font, how the letters are written and an impression of the look. This text should contain all letters of the alphabet and it should be written in of the original language. There is no need for special content, but the length of words should match the language.

4.2 Consistent Flux Computation

Schematic Hello, here is some text without a meaning. This text should show what a printed text will look like at this place. If you read this text, you will get no information. Really? Is there no information? Is there a difference between this text and some nonsense like “Huardest gefburn”? Kjift – not at all! A blind text like this gives you information about the selected font, how the letters are written and an impression of the look. This text should contain all letters of the alphabet and it should be written

4.1 Principles of Momentum Consistent Schemes	7
4.2 Consistent Flux Computation	7
4.3 Reconstruction on Staggered Cells	8
4.4 Sub-Grid Strategy	8
4.5 Summary of Methods	9

in of the original language. There is no need for special content, but the length of words should match the language.

Numerical Stencils Hello, here is some text without a meaning. This text should show what a printed text will look like at this place. If you read this text, you will get no information. Really? Is there no information? Is there a difference between this text and some nonsense like “Huardest gefburn”? Kjift – not at all! A blind text like this gives you information about the selected font, how the letters are written and an impression of the look. This text should contain all letters of the alphabet and it should be written in of the original language. There is no need for special content, but the length of words should match the language.

4.3 Reconstruction on Staggered Cells

Half-Fractions Method Hello, here is some text without a meaning. This text should show what a printed text will look like at this place. If you read this text, you will get no information. Really? Is there no information? Is there a difference between this text and some nonsense like “Huardest gefburn”? Kjift – not at all! A blind text like this gives you information about the selected font, how the letters are written and an impression of the look. This text should contain all letters of the alphabet and it should be written in of the original language. There is no need for special content, but the length of words should match the language.

Sub-Grid Method Hello, here is some text without a meaning. This text should show what a printed text will look like at this place. If you read this text, you will get no information. Really? Is there no information? Is there a difference between this text and some nonsense like “Huardest gefburn”? Kjift – not at all! A blind text like this gives you information about the selected font, how the letters are written and an impression of the look. This text should contain all letters of the alphabet and it should be written in of the original language. There is no need for special content, but the length of words should match the language.

4.4 Sub-Grid Strategy

Consistency and Conservation Hello, here is some text without a meaning. This text should show what a printed text will look like at this place. If you read this text, you will get no information. Really? Is there no information? Is there a difference between this text and some nonsense like “Huardest gefburn”? Kjift – not at all! A blind text like this gives you information about the selected font, how the letters are written and an impression of the look. This text should contain all letters of the alphabet and it should be written in of the original language. There is no need for special content, but the length of words should match the language.

Restriction and Prolongation Operators Hello, here is some text without a meaning. This text should show what a printed text will look like at this place. If you read this text, you will get no information. Really? Is there no information? Is there a difference between this text and some nonsense like "Huardest gefburn"? Kjift – not at all! A blind text like this gives you information about the selected font, how the letters are written and an impression of the look. This text should contain all letters of the alphabet and it should be written in of the original language. There is no need for special content, but the length of words should match the language.

4.5 Summary of Methods

Flowchart : Half-Fractions Method Hello, here is some text without a meaning. This text should show what a printed text will look like at this place. If you read this text, you will get no information. Really? Is there no information? Is there a difference between this text and some nonsense like "Huardest gefburn"? Kjift – not at all! A blind text like this gives you information about the selected font, how the letters are written and an impression of the look. This text should contain all letters of the alphabet and it should be written in of the original language. There is no need for special content, but the length of words should match the language.

Flowchart : Sub-Grid Method Hello, here is some text without a meaning. This text should show what a printed text will look like at this place. If you read this text, you will get no information. Really? Is there no information? Is there a difference between this text and some nonsense like "Huardest gefburn"? Kjift – not at all! A blind text like this gives you information about the selected font, how the letters are written and an impression of the look. This text should contain all letters of the alphabet and it should be written in of the original language. There is no need for special content, but the length of words should match the language.

In the upcoming sections, we demonstrate the robustness and accuracy of our class of mass-momentum consistent numerical methods when applied to challenging high density-ratio flow configurations, primarily in comparison to the version of our method which does not maintain consistency between the mass and momentum advection. Most of the standard tests that exist in the current literature concerning numerical methods to tackle liquid-gas flows such as the decay of spurious currents in static and moving droplets, viscous damping of capillary waves etc., are carried out in the absence of any density jump (or viscosity jump) across the interface separating the fluids. In this chapter, we shall take a closer look in detail at the behavior of our methods when dealing with difficulties that arise due to the non-linear coupling between interfacial deformation/propagation, capillary and viscous forces, especially in the regime where the material properties across the interface are separated by orders of magnitude, particularly in which the flow features in question are poorly resolved.

In order to assess the performance of the different methods, we shall use an easier nomenclature to describe the different methods, which are as follows :

- **M1** Method with non-consistent momentum-mass transport.
- **M2** Method with consistent momentum-mass transport, but not conservative. Uses half-fractions strategy.
- **M3** Method with consistent and conservative momentum-mass transport. Uses sub-grid strategy.

5.1 Static Droplet

A popular numerical benchmark in the existing literature relevant to surface tension dominated flows is the case of a spherical droplet of the denser fluid immersed in a quiescent surrounding medium of the lighter fluid. In the hydrostatic limit of the Navier-Stokes equations, the droplet should stay in equilibrium, with a curvature induced pressure jump across the interface corresponding to Laplace's equilibrium. In practice however, numerically reproducing such a trivial equilibrium condition is not as straightforward, as there exists a slight difference between the initial numerical interface and the exact analytical shape of the sphere, thereby resulting in the generation of the well documented 'spurious' or 'parasitic' currents of varying intensity in the velocity field [1–3] . A lot of progress has been made since in the context of *well-balanced* surface tension formulations, that ensure consistency between the numerical stencils used for the discretization of the pressure gradient and the Heaviside approximation ($n\delta_s$) that projects the the surface force distribution onto the control volumes [4, 5]. A significant contribution to the interpretation of these parasitic currents within the well-balanced

5.1 Static Droplet	10
5.2 Moving Droplet	14
5.3 Capillary Wave	16
5.4 Falling Raindrop	18

[1]: Lafaurie et al. (1994), 'Modelling merging and fragmentation in multiphase flows with SURFER'

[2]: Harvie et al. (2006), 'An analysis of parasitic current generation in volume of fluid simulations'

[3]: Popinet et al. (1999), 'A front-tracking algorithm for accurate representation of surface tension'

framework was made by Popinet [5] which demonstrated that given sufficient time (of the order of viscous dissipation time-scales), a well-balanced method will relax to the ‘numerical’ equilibrium shape through the damping of the ‘physically consistent’ numerical capillary waves, therefore allowing us to recover the exact (to machine precision) Laplace equilibrium condition.

Setup

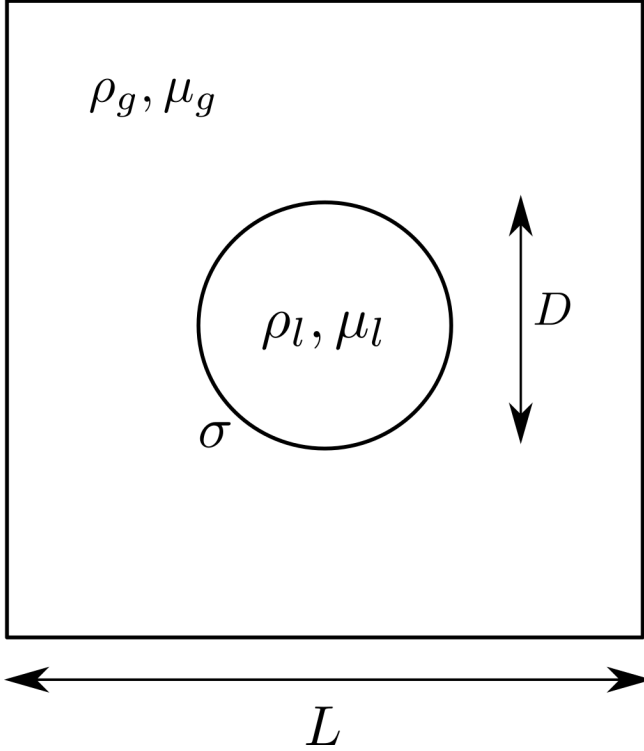


Figure 5.1: Schematic of the static droplet of dense fluid surrounded by a quiescent medium of lighter fluid. A 40×40 grid is employed to spatially discretize the domain.

The key difference in our implementation of this classic test case from that of Popinet [5] is that we consider the effect of density contrast across the interface separating the fluids. As we have previously discussed, a sharp density jump across the interface has an amplification effect on the numerical errors incurred as a result of interfacial reconstructions, curvature estimation and various other truncations, thereby rendering the method unstable. We demonstrate that in our framework of mass consistent momentum transport coupled with a well-balanced surface tension discretization, density-ratios as large as 1000 : 1 can be simulated without loss of numerical stability, in conjunction with the ability to recover the exact numerical equilibrium through the dissipation of spurious currents within relevant time-scales¹.

We consider a circular droplet of size D placed at the centre of a square domain of side L . The densities of the heavier and lighter phases are ρ_l and ρ_g respectively, likewise for the viscosities μ_l and μ_g , and σ being the surface tension coefficient (fig. 5.1). The ratio of the droplet size to the box is chosen as $D/L = 0.4$, coupled with a numerical resolution of $D/\Delta x = 12.8$ (where Δx is the grid size). As for boundary conditions, we use symmetry conditions on all sides of the square domain.

[5]: Popinet (2009), ‘An accurate adaptive solver for surface-tension-driven interfacial flows’

1: The viscous time-scale corresponding to the droplet length-scale is the most commonly used in literature.

The problem incorporates two natural time-scales, the capillary oscillation scale and the viscous dissipation scale, which are defined below :

$$T_\sigma = \left(\frac{\rho_l D^3}{\sigma} \right)^{1/2}, \quad T_\mu = \frac{\rho_l D^2}{\mu_l} \quad (5.1)$$

The ratio of these time-scales give us -

$$\frac{T_\mu}{T_\sigma} = \sqrt{\rho_l \sigma D} / \mu_l = \sqrt{La} \quad (5.2)$$

where La is the Laplace number based upon the heavier fluid. In the present study, we introduce the density-ratio ρ_l / ρ_g as another important parameter. In order to rescale our 'parasitic' velocity field, we define a velocity scale based on capillary oscillations as -

$$U_\sigma = \sqrt{\sigma / \rho_l D} \quad (5.3)$$

Additionally, the time-step in our numerical simulation must be smaller than the oscillation period corresponding to the grid wavenumber (fastest capillary wave with a time period $\sim (\rho_l \Delta x^3 / \sigma)^{1/2}$) as a stability criterion², as our surface tension model is explicit in time. For the scope of the present study, we shall not consider any viscosity contrast between the two fluids while varying the density-ratio, therefore $\mu_l / \mu_g = 1$.

2: Similar criteria are defined on the basis of the viscous and advection operators as well, with the smallest amongst the three selecting the numerical time-step

Decay of Spurious Currents

In figures 5.2 to 5.4, we illustrate the decay of the root-mean-square of the spurious currents as a function of time, in the case of four different density-ratios, with three different Laplace numbers for each ratio. The first figure (5.2) refers to simulations carried out without consistency between the momentum-mass transport (M1), the second (5.3) corresponds to that of the consistent but not conservative method (M2), and final one (5.4) refers to that of the consistent and conservative method (M3). The time is rescaled by the viscous dissipation scale, and the spurious currents by the capillary velocity scale. We can clearly observe that magnitude of the normalized spurious currents decay rapidly to the levels of machine precision, and that at much faster time-scales than T_μ for the higher density-ratio cases. It is important to note that the solver displays the desired functionality and robustness for density-ratios as well as Laplace numbers, both of which span several orders of magnitude.

Spatial Convergence

Hello, here is some text without a meaning. This text should show what a printed text will look like at this place. If you read this text, you will get no information. Really? Is there no information? Is there a difference

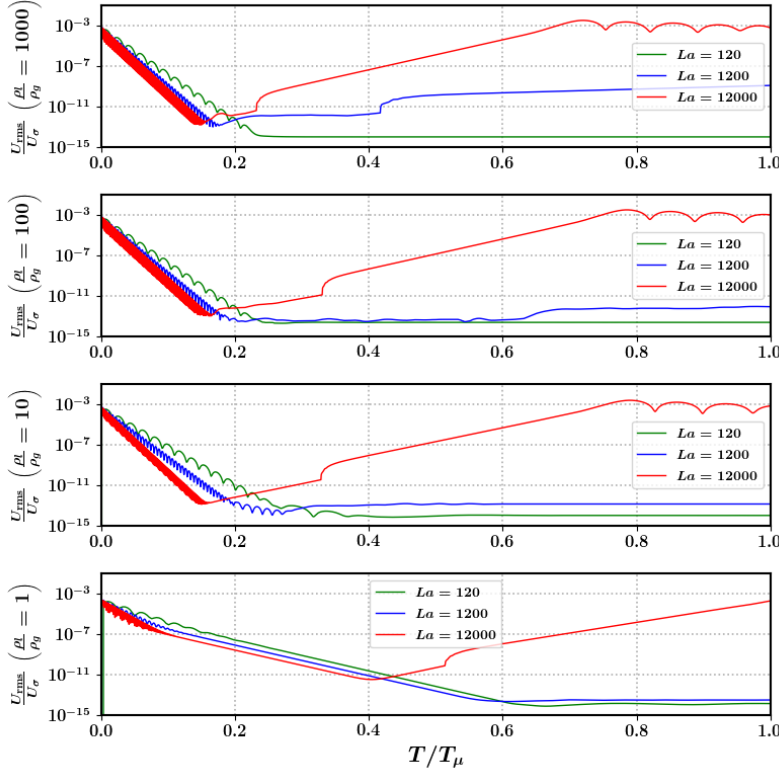


Figure 5.2: M1 Decay of normalized spurious currents as a function of viscous dissipation time-scales for different density-ratios and Laplace numbers. The currents seem to initially decay quickly for all higher density-ratios, and relax to the numerical equilibrium curvature even within $0.2 \cdot T_\mu$. For combinations of large ρ_l/ρ_g and large La , the spurious currents seem to grow back to an order of magnitude (10^{-3}) which is quite far from that of machine precision (10^{-14}).

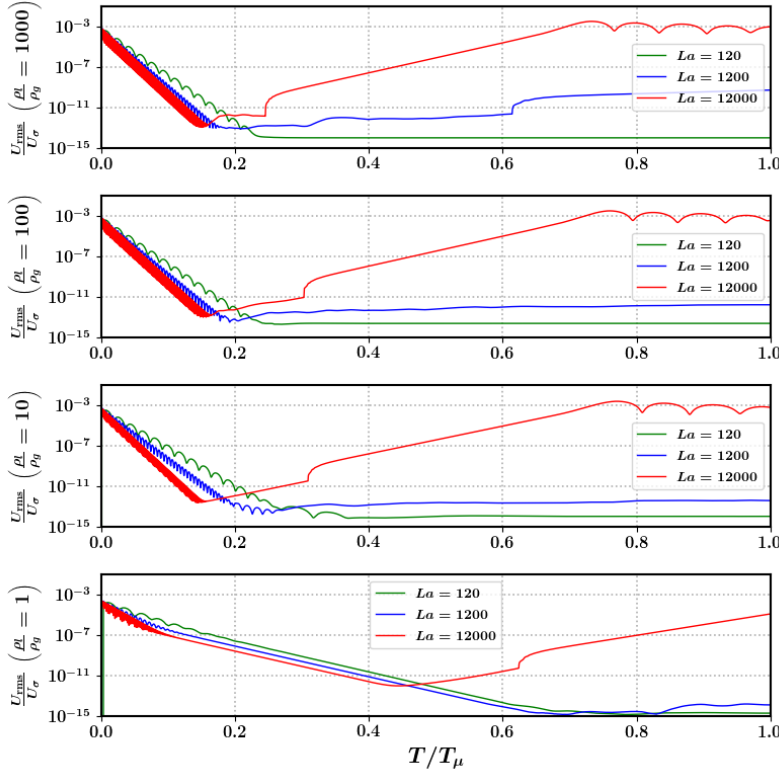


Figure 5.3: M2 Decay of normalized spurious currents as a function of viscous dissipation time-scales for different density-ratios and Laplace numbers. The currents seem to initially decay quickly for all higher density-ratios, and relax to the numerical equilibrium curvature even within $0.2 \cdot T_\mu$. For combinations of large ρ_l/ρ_g and large La , the spurious currents seem to grow back to an order of magnitude (10^{-3}) which is quite far from that of machine precision (10^{-14}). No considerable improvement is observed with respect to **M1**.

between this text and some nonsense like “Huardest gefburn”? Kjift – not at all! A blind text like this gives you information about the selected font, how the letters are written and an impression of the look. This text

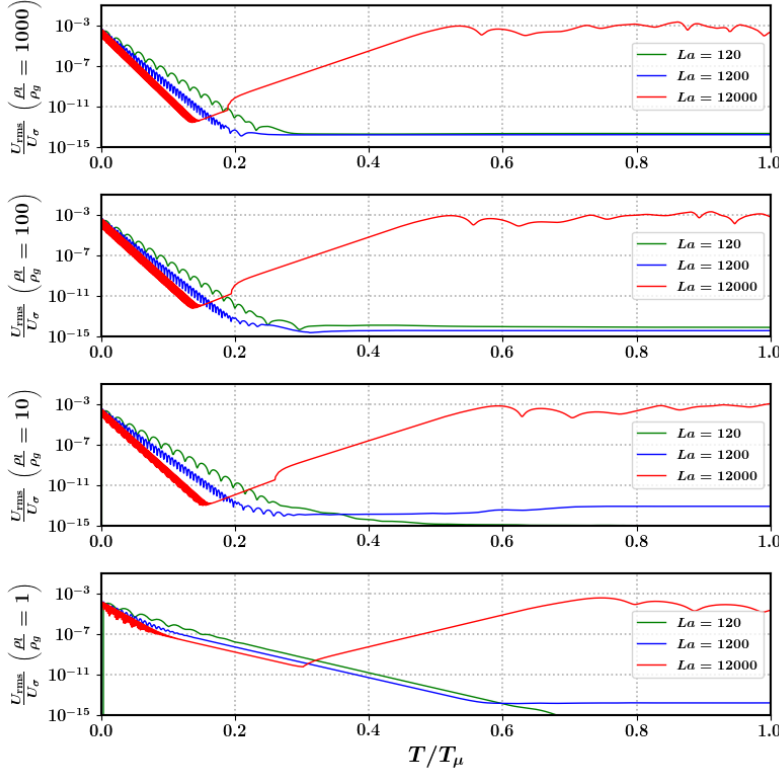


Figure 5.4: M3 Decay of normalized spurious currents as a function of viscous dissipation time-scales for different density-ratios and Laplace numbers. The currents seem to decay very quickly in the case of higher density-ratios, and relax to the numerical equilibrium curvature even within $0.2 \cdot T_\mu$. For all combinations of ρ_l/ρ_g and the highest La number, the spurious currents first decay very quickly and then grow back to an order of magnitude 10^{-3} at a slower pace. All other combinations of density-ratio and Laplace numbers do seem to decay to an order of magnitude close to machine precision (10^{-14}). There is a slight improvement over **M1** and **M2**, especially for all except the highest Laplace number.

should contain all letters of the alphabet and it should be written in of the original language. There is no need for special content, but the length of words should match the language.

5.2 Moving Droplet

An incisive numerical setup which evaluates the accuracy of the coupling between interfacial propagation and surface tension discretization was first proposed by Popinet [5], subsequently employed in the comparative study of Abadie et al. [6]. The manner in which this test differs from that of the static droplet is the presence of a uniform background velocity field, therefore serving as a better representation of droplets in complex surface tension dominated flows where they might be advected by the mean flow.

Setup

In terms of the Laplace equilibrium, the hydrostatic solution is still valid in the frame of reference of the moving droplet. The point at which the solution in the moving reference frame diverges from that of the static droplet (5.1) is through the continuous injection of noise at the scale of the grid size. This ‘numerical’ noise emanates from the perturbations to the curvature estimates, which are in turn induced by the interfacial reconstructions carried out to propagate the interface. These fluctuating errors act as source terms for the momentum, thereby transforming the

[6]: Abadie et al. (2015), ‘On the combined effects of surface tension force calculation and interface advection on spurious currents within Volume of Fluid and Level Set frameworks’

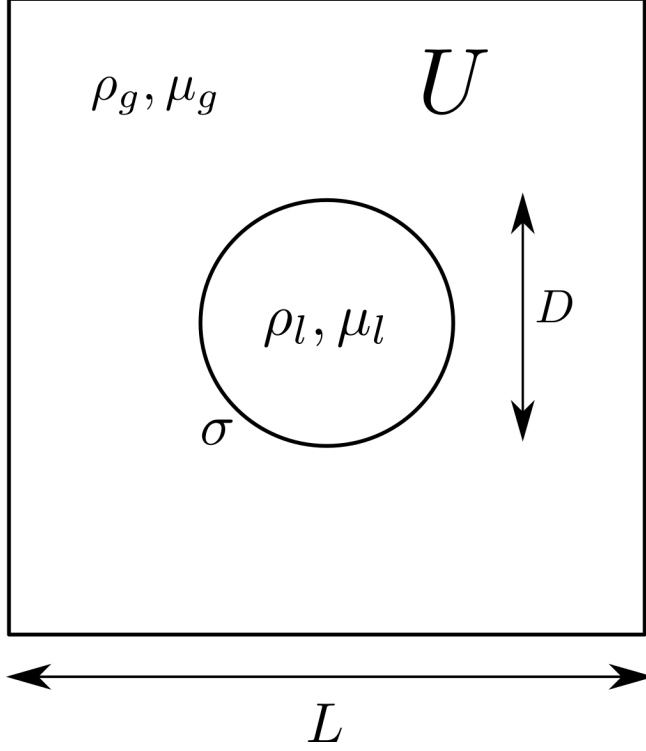


Figure 5.5: Schematic of the droplet of dense fluid advected in a surrounding medium of lighter fluid. A 64×64 grid is employed to spatially discretize the domain, which is spatially periodic in the direction of droplet advection.

problem into that of viscous dissipation in the presence of continuous forcing (in the moving reference frame).

In the present study, we evaluate our method using the advection of a droplet in a spatially periodic domain in the same setup as [5], but with the important difference of including sharp density jumps across the interface. As previously discussed (5.1), high density-ratios tend to rapidly amplify the fluctuations induced by the myriad numerical approximations (interface reconstruction, curvature estimation etc) involved in the algorithm, thereby leading to loss of numerical stability.

We again consider a circular droplet of size D placed at the centre of a square domain of side L . The densities of the heavier and lighter phases are ρ_l and ρ_g respectively, likewise for the viscosities μ_l and μ_g , and σ being the surface tension coefficient (fig. 5.1). A uniform velocity field U is initialized on the entire domain (horizontal direction). The ratio of the droplet size to the box is $D/L = 0.4$, with $D/\Delta x = 25.6$ (Δx being the grid size). As for boundary conditions, we use symmetry conditions on the top and bottom sides, and periodic boundary conditions on the horizontal direction (along which advection by U takes place).

We characterize by problem by introducing the following adimensional numbers (based on the heavier fluid) :

$$La = \frac{\rho_l \sigma D}{\mu_l^2} \quad , \quad We = \frac{\rho_l U^2 D}{\sigma} \quad (5.4)$$

In addition to the capillary and viscous time-scales for the static case (eqns. 5.1), we have an additional scale defined as :

$$T_U = D/U \quad (5.5)$$

Evolution of Spurious Currents

Hello, here is some text without a meaning. This text should show what a printed text will look like at this place. If you read this text, you will get no information. Really? Is there no information? Is there a difference between this text and some nonsense like “Huardest gefburn”? Kjift – not at all! A blind text like this gives you information about the selected font, how the letters are written and an impression of the look. This text should contain all letters of the alphabet and it should be written in of the original language. There is no need for special content, but the length of words should match the language.

Spatial Convergence

Hello, here is some text without a meaning. This text should show what a printed text will look like at this place. If you read this text, you will get no information. Really? Is there no information? Is there a difference between this text and some nonsense like “Huardest gefburn”? Kjift – not at all! A blind text like this gives you information about the selected font, how the letters are written and an impression of the look. This text should contain all letters of the alphabet and it should be written in of the original language. There is no need for special content, but the length of words should match the language.

Error Dependence : Laplace & Weber numbers

Hello, here is some text without a meaning. This text should show what a printed text will look like at this place. If you read this text, you will get no information. Really? Is there no information? Is there a difference between this text and some nonsense like “Huardest gefburn”? Kjift – not at all! A blind text like this gives you information about the selected font, how the letters are written and an impression of the look. This text should contain all letters of the alphabet and it should be written in of the original language. There is no need for special content, but the length of words should match the language.

5.3 Capillary Wave

One of fundamental features of immiscible multiphase flows involving interfaces are the presense and propagation of capillary waves. Therefore, a robust and accurate numerical method should not only be able to adequately resolve, but also accurately emulate the spatio-temporal evolution of such surface tension induced oscillations. A brief outline on the numerical implementations of capillary waves in existing literature is provided by Popinet in the comprehensive review [7] .

[7]: Popinet (2018), ‘Numerical models of surface tension’

Setup

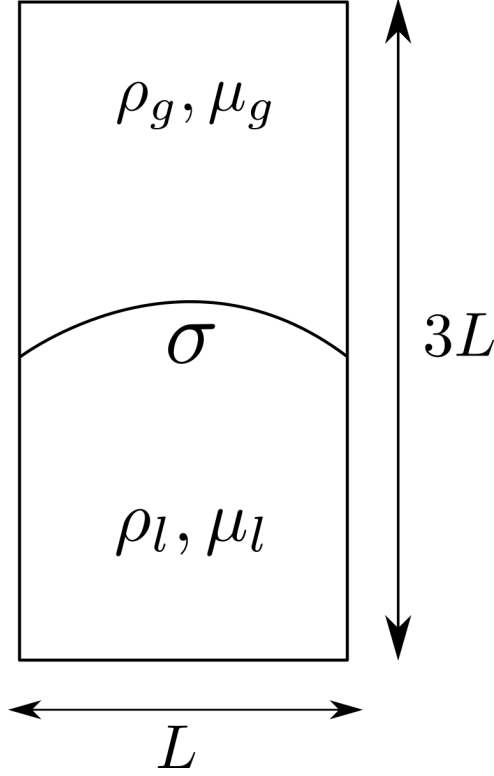


Figure 5.6: Schematic of the initially perturbed planar interface separating two immiscible fluids of different densities and viscosities. A spatial resolution of 32×96 is used for spatial discretization (compared to 64×192 in Popinet [5]), with the width of the box corresponding to the size of the perturbed wavelength.

Ideally, we would like to evaluate the accuracy of our method compared to the analytical solution of damped capillary oscillations. Analytical solutions exist for the case of extremely small initial perturbations, either in the inviscid limit (Lamb [8]) or the asymptotic limit of vanishing viscosity (Prosperetti [9, 10]). In the present study, we use the configuration of the viscosity-damped capillary oscillations of a planar interface, as was first implemented and popularized by Popinet & Zaleski [3].

We consider a rectangular domain of dimensions $L \times 3L$, where L corresponds to the wavelength of our initial perturbation. The densities of the heavier and lighter phases are ρ_l and ρ_g respectively, likewise for the viscosities μ_l and μ_g , and σ being the surface tension coefficient (fig. 5.6). An initial perturbation amplitude of $L/100$ is used, coupled with a numerical resolution given by $L/\Delta x = 32$ (Δx being the grid size). Symmetry conditions are applied on the top and bottom sides, with periodic conditions along the horizontal direction. We use the following adimensional parameters to characterize our problem :

$$T_0 = T\omega_0 \quad , \quad La = \frac{\rho_l \sigma L}{\mu_l^2} \quad (5.6)$$

where La is the Laplace number based on the heavier fluid, and ω_0 is defined using the dispersion relation [5] given as :

[8]: Lamb (1993), *Hydrodynamics*

[9]: Prosperetti (1980), 'Free oscillations of drops and bubbles: the initial-value problem'

[10]: Prosperetti (1981), 'Motion of two superposed viscous fluids'

$$\omega_0^2 = \frac{\sigma k^3}{2\rho_l} \quad , \quad \text{where} \quad k = \frac{2\pi}{L} \quad (5.7)$$

The dispersion relation is obtained via linear stability analysis at the inviscid limit [8]. In order to evaluate the influence of density-ratio on the performance of our method, we use 3 different numerical setups keeping the same Laplace number ($La = 3000$) as follows :

- ▶ $\rho_l/\rho_g = 1$, $\mu_l/\mu_g = 1$ (Popinet [5])
- ▶ $\rho_l/\rho_g = 10$, $\mu_l/\mu_g = 1$
- ▶ $\rho_l/\rho_g = 1000.0/1.2$, $\mu_l/\mu_g = 1.003 \cdot 10^{-3}/1.8 \cdot 10^{-5}$ (Air-Water)

Comparison with Prosperetti Solution

Hello, here is some text without a meaning. This text should show what a printed text will look like at this place. If you read this text, you will get no information. Really? Is there no information? Is there a difference between this text and some nonsense like “Huardest gefburn”? Kjift – not at all! A blind text like this gives you information about the selected font, how the letters are written and an impression of the look. This text should contain all letters of the alphabet and it should be written in of the original language. There is no need for special content, but the length of words should match the language.

Spatial Convergence

Hello, here is some text without a meaning. This text should show what a printed text will look like at this place. If you read this text, you will get no information. Really? Is there no information? Is there a difference between this text and some nonsense like “Huardest gefburn”? Kjift – not at all! A blind text like this gives you information about the selected font, how the letters are written and an impression of the look. This text should contain all letters of the alphabet and it should be written in of the original language. There is no need for special content, but the length of words should match the language.

5.4 Falling Raindrop

A flow configuration that combines the complexities of high density-ratios with the interaction between capillary, viscous and inertial stresses is that of a water droplet falling in air under the influence of gravitational acceleration.

Setup

The problem is characterized by a combination of Reynolds, Weber and Bond numbers, the definitions of which are as follows :

$$We = \frac{\rho_g U^2 d}{\sigma} \quad , \quad Re = \frac{\rho_g U d}{\mu_g} \quad , \quad Bo = \frac{(\rho_l - \rho_g) g d^2}{\sigma} \quad (5.8)$$

where the subscripts l and g refer to the liquid phase (water) and the gas phase (air), respectively. In our particular numerical setup, $We \simeq 3.2$, $Re \simeq 1455$ and $Bo \simeq 1.2$, thus corresponding to that of a $3mm$ diameter raindrop (a relatively large one) falling in air at an approximate terminal velocity of 8 m/s (interpolated from empirical data, refer to [11]). The parameters in the problem setup are given in Table 5.1, and the schematic diagram given by Fig. 5.7. The droplet is initially placed at the center of a cubic domain (3D), whose side is 4 times the diameter of the drop.

Table 5.1: Parameter values used in the simulation of a falling water droplet in air.

ρ_g (kg/m^3)	ρ_l (kg/m^3)	μ_g ($Pa \cdot s$)	μ_l ($Pa \cdot s$)	σ (N/m)	d (m)	g (m/s^2)
1.2	0.9982×10^3	1.98×10^{-5}	8.9×10^{-4}	0.0728	3×10^{-3}	9.81

In order to properly reproduce and analyse the dynamics of a relatively large drop (high Reynolds flow) such as in our case, the numerical method has to accurately resolve the thin boundary layers, the interaction of such layers with the capillary forces and finally the non-linear feedback of the complex 3D vortical structures present in the wake behind the droplet. Therefore, our objective behind the demonstration of this particular test case is not to develop a high fidelity model of a raindrop, but rather carry out a stringent evaluation of the robustness of our numerical method compared to the standard version of our solver which is not momentum consistent. For such a low Weber number the capillary forces dominate and the droplet should remain intact, and definitely not undergo any subsequent atomization.

Temporal Evolution : Kinetic Energy, Mass, Moment of Inertia

Hello, here is some text without a meaning. This text should show what a printed text will look like at this place. If you read this text, you will get no information. Really? Is there no information? Is there a difference between this text and some nonsense like “Huardest gefburn”? Kjift – not at all! A blind text like this gives you information about the selected font, how the letters are written and an impression of the look. This text should contain all letters of the alphabet and it should be written in of the original language. There is no need for special content, but the length of words should match the language.

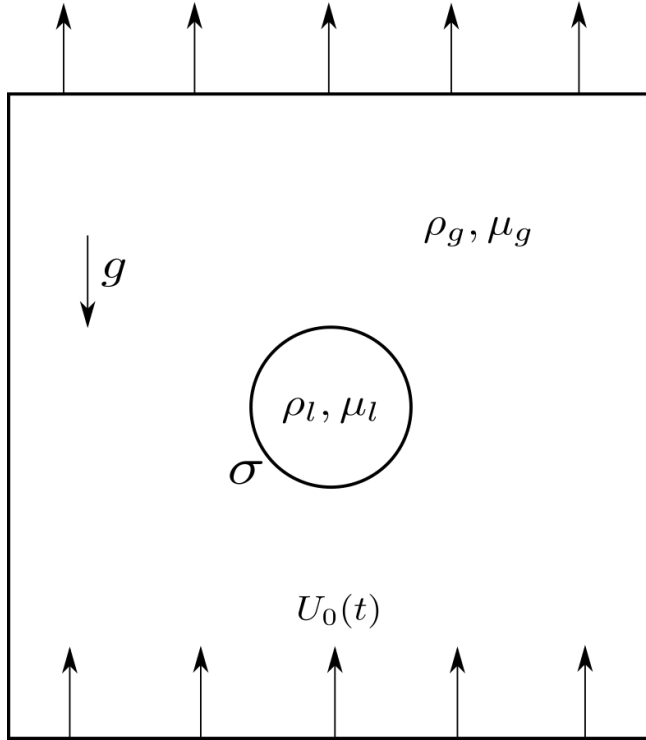


Figure 5.7: A 2D schematic of the numerical setup for the falling raindrop. We apply a uniform inflow velocity condition with $U_0(t)$ and an outflow velocity condition at the top which corresponds to zero gradient of the velocity at the boundary. Boundary conditions on the side walls correspond to those of free slip (no shear stress).

Convergence of Velocity & Acceleration

Hello, here is some text without a meaning. This text should show what a printed text will look like at this place. If you read this text, you will get no information. Really? Is there no information? Is there a difference between this text and some nonsense like “Huardest gefburn”? Kjift – not at all! A blind text like this gives you information about the selected font, how the letters are written and an impression of the look. This text should contain all letters of the alphabet and it should be written in of the original language. There is no need for special content, but the length of words should match the language.

PHYSICS OF FRAGMENTATION

6.1 Mechanism of Drop Formation

6.1 Mechanism of Drop Formation	22
6.2 Theories of Fragmentation	23

Disintegration of Jets & Shear Layers Hello, here is some text without a meaning. This text should show what a printed text will look like at this place. If you read this text, you will get no information. Really? Is there no information? Is there a difference between this text and some nonsense like “Huardest gefburn”? Kjift – not at all! A blind text like this gives you information about the selected font, how the letters are written and an impression of the look. This text should contain all letters of the alphabet and it should be written in of the original language. There is no need for special content, but the length of words should match the language.

Expansion of Sheets Hello, here is some text without a meaning. This text should show what a printed text will look like at this place. If you read this text, you will get no information. Really? Is there no information? Is there a difference between this text and some nonsense like “Huardest gefburn”? Kjift – not at all! A blind text like this gives you information about the selected font, how the letters are written and an impression of the look. This text should contain all letters of the alphabet and it should be written in of the original language. There is no need for special content, but the length of words should match the language.

Effervescent Atomization Hello, here is some text without a meaning. This text should show what a printed text will look like at this place. If you read this text, you will get no information. Really? Is there no information? Is there a difference between this text and some nonsense like “Huardest gefburn”? Kjift – not at all! A blind text like this gives you information about the selected font, how the letters are written and an impression of the look. This text should contain all letters of the alphabet and it should be written in of the original language. There is no need for special content, but the length of words should match the language.

Drop Impacts Hello, here is some text without a meaning. This text should show what a printed text will look like at this place. If you read this text, you will get no information. Really? Is there no information? Is there a difference between this text and some nonsense like “Huardest gefburn”? Kjift – not at all! A blind text like this gives you information about the selected font, how the letters are written and an impression of the look. This text should contain all letters of the alphabet and it should be written in of the original language. There is no need for special content, but the length of words should match the language.

6.2 Theories of Fragmentation

Cascade Mechanism : Log-Normal Distribution Hello, here is some text without a meaning. This text should show what a printed text will look like at this place. If you read this text, you will get no information. Really? Is there no information? Is there a difference between this text and some nonsense like “Huardest gefburn”? Kjift – not at all! A blind text like this gives you information about the selected font, how the letters are written and an impression of the look. This text should contain all letters of the alphabet and it should be written in of the original language. There is no need for special content, but the length of words should match the language.

Corrugation-Coalescence Mechanism : Gamma Distribution Hello, here is some text without a meaning. This text should show what a printed text will look like at this place. If you read this text, you will get no information. Really? Is there no information? Is there a difference between this text and some nonsense like “Huardest gefburn”? Kjift – not at all! A blind text like this gives you information about the selected font, how the letters are written and an impression of the look. This text should contain all letters of the alphabet and it should be written in of the original language. There is no need for special content, but the length of words should match the language.

Droplet Generation in Corrugated Ligaments

7

7.1 Numerical Setup

7.1 Numerical Setup 24

7.2 Ligament Breakup 25

Platform : Basilisk Hello, here is some text without a meaning. This text should show what a printed text will look like at this place. If you read this text, you will get no information. Really? Is there no information? Is there a difference between this text and some nonsense like “Huardest gefburn”? Kjift – not at all! A blind text like this gives you information about the selected font, how the letters are written and an impression of the look. This text should contain all letters of the alphabet and it should be written in of the original language. There is no need for special content, but the length of words should match the language.

Computational Schematic Hello, here is some text without a meaning. This text should show what a printed text will look like at this place. If you read this text, you will get no information. Really? Is there no information? Is there a difference between this text and some nonsense like “Huardest gefburn”? Kjift – not at all! A blind text like this gives you information about the selected font, how the letters are written and an impression of the look. This text should contain all letters of the alphabet and it should be written in of the original language. There is no need for special content, but the length of words should match the language.

Random Surface Generation Hello, here is some text without a meaning. This text should show what a printed text will look like at this place. If you read this text, you will get no information. Really? Is there no information? Is there a difference between this text and some nonsense like “Huardest gefburn”? Kjift – not at all! A blind text like this gives you information about the selected font, how the letters are written and an impression of the look. This text should contain all letters of the alphabet and it should be written in of the original language. There is no need for special content, but the length of words should match the language.

Parameterization Hello, here is some text without a meaning. This text should show what a printed text will look like at this place. If you read this text, you will get no information. Really? Is there no information? Is there a difference between this text and some nonsense like “Huardest gefburn”? Kjift – not at all! A blind text like this gives you information about the selected font, how the letters are written and an impression of the look. This text should contain all letters of the alphabet and it should be written in of the original language. There is no need for special content, but the length of words should match the language.

7.2 Ligament Breakup

3D vs. 2D Simulations Hello, here is some text without a meaning. This text should show what a printed text will look like at this place. If you read this text, you will get no information. Really? Is there no information? Is there a difference between this text and some nonsense like “Huardest gefburn”? Kjift – not at all! A blind text like this gives you information about the selected font, how the letters are written and an impression of the look. This text should contain all letters of the alphabet and it should be written in of the original language. There is no need for special content, but the length of words should match the language.

Effect of Spatial Resolution Hello, here is some text without a meaning. This text should show what a printed text will look like at this place. If you read this text, you will get no information. Really? Is there no information? Is there a difference between this text and some nonsense like “Huardest gefburn”? Kjift – not at all! A blind text like this gives you information about the selected font, how the letters are written and an impression of the look. This text should contain all letters of the alphabet and it should be written in of the original language. There is no need for special content, but the length of words should match the language.

Effect of Droplet Removal Hello, here is some text without a meaning. This text should show what a printed text will look like at this place. If you read this text, you will get no information. Really? Is there no information? Is there a difference between this text and some nonsense like “Huardest gefburn”? Kjift – not at all! A blind text like this gives you information about the selected font, how the letters are written and an impression of the look. This text should contain all letters of the alphabet and it should be written in of the original language. There is no need for special content, but the length of words should match the language.

Effect of Corrugation Amplitude Hello, here is some text without a meaning. This text should show what a printed text will look like at this place. If you read this text, you will get no information. Really? Is there no information? Is there a difference between this text and some nonsense like “Huardest gefburn”? Kjift – not at all! A blind text like this gives you information about the selected font, how the letters are written and an impression of the look. This text should contain all letters of the alphabet and it should be written in of the original language. There is no need for special content, but the length of words should match the language.

Effect of Ohnesorge Number Hello, here is some text without a meaning. This text should show what a printed text will look like at this place. If you read this text, you will get no information. Really? Is there no information? Is there a difference between this text and some nonsense like “Huardest gefburn”? Kjift – not at all! A blind text like this gives you information about the selected font, how the letters are written and an impression of the look. This text should contain all letters of the alphabet

and it should be written in of the original language. There is no need for special content, but the length of words should match the language.

Effect of Cut-Off Wavenumber Hello, here is some text without a meaning. This text should show what a printed text will look like at this place. If you read this text, you will get no information. Really? Is there no information? Is there a difference between this text and some nonsense like "Huardest gefburn"? Kjift – not at all! A blind text like this gives you information about the selected font, how the letters are written and an impression of the look. This text should contain all letters of the alphabet and it should be written in of the original language. There is no need for special content, but the length of words should match the language.

Effect of Aspect Ratio Hello, here is some text without a meaning. This text should show what a printed text will look like at this place. If you read this text, you will get no information. Really? Is there no information? Is there a difference between this text and some nonsense like "Huardest gefburn"? Kjift – not at all! A blind text like this gives you information about the selected font, how the letters are written and an impression of the look. This text should contain all letters of the alphabet and it should be written in of the original language. There is no need for special content, but the length of words should match the language.

Quantization of Waves Hello, here is some text without a meaning. This text should show what a printed text will look like at this place. If you read this text, you will get no information. Really? Is there no information? Is there a difference between this text and some nonsense like "Huardest gefburn"? Kjift – not at all! A blind text like this gives you information about the selected font, how the letters are written and an impression of the look. This text should contain all letters of the alphabet and it should be written in of the original language. There is no need for special content, but the length of words should match the language.

8.1 Monte Carlo Approach to DNS

Characterization of Ligament Ensembles Hello, here is some text without a meaning. This text should show what a printed text will look like at this place. If you read this text, you will get no information. Really? Is there no information? Is there a difference between this text and some nonsense like “Huardest gefburn”? Kjift – not at all! A blind text like this gives you information about the selected font, how the letters are written and an impression of the look. This text should contain all letters of the alphabet and it should be written in of the original language. There is no need for special content, but the length of words should match the language.

8.1 Monte Carlo Approach to DNS	27
8.2 Millimeter Scale Ensembles	27
8.3 Exploration of Parameter Space	28
Φ	28

8.2 Millimeter Scale Ensembles

Diameter Distributions Hello, here is some text without a meaning. This text should show what a printed text will look like at this place. If you read this text, you will get no information. Really? Is there no information? Is there a difference between this text and some nonsense like “Huardest gefburn”? Kjift – not at all! A blind text like this gives you information about the selected font, how the letters are written and an impression of the look. This text should contain all letters of the alphabet and it should be written in of the original language. There is no need for special content, but the length of words should match the language.

Mass Distributions Hello, here is some text without a meaning. This text should show what a printed text will look like at this place. If you read this text, you will get no information. Really? Is there no information? Is there a difference between this text and some nonsense like “Huardest gefburn”? Kjift – not at all! A blind text like this gives you information about the selected font, how the letters are written and an impression of the look. This text should contain all letters of the alphabet and it should be written in of the original language. There is no need for special content, but the length of words should match the language.

Equivalent Diameters Hello, here is some text without a meaning. This text should show what a printed text will look like at this place. If you read this text, you will get no information. Really? Is there no information? Is there a difference between this text and some nonsense like “Huardest gefburn”? Kjift – not at all! A blind text like this gives you information about the selected font, how the letters are written and an impression of the look. This text should contain all letters of the alphabet

and it should be written in of the original language. There is no need for special content, but the length of words should match the language.

Local Distribution of Large Drop Sizes Hello, here is some text without a meaning. This text should show what a printed text will look like at this place. If you read this text, you will get no information. Really? Is there no information? Is there a difference between this text and some nonsense like “Huardest gefburn”? Kjift – not at all! A blind text like this gives you information about the selected font, how the letters are written and an impression of the look. This text should contain all letters of the alphabet and it should be written in of the original language. There is no need for special content, but the length of words should match the language.

8.3 Exploration of Parameter Space Φ

Bifurcation Parameter : Corrugation Amplitude Hello, here is some text without a meaning. This text should show what a printed text will look like at this place. If you read this text, you will get no information. Really? Is there no information? Is there a difference between this text and some nonsense like “Huardest gefburn”? Kjift – not at all! A blind text like this gives you information about the selected font, how the letters are written and an impression of the look. This text should contain all letters of the alphabet and it should be written in of the original language. There is no need for special content, but the length of words should match the language.

Scaling of D/W : Function of Parameter Space Hello, here is some text without a meaning. This text should show what a printed text will look like at this place. If you read this text, you will get no information. Really? Is there no information? Is there a difference between this text and some nonsense like “Huardest gefburn”? Kjift – not at all! A blind text like this gives you information about the selected font, how the letters are written and an impression of the look. This text should contain all letters of the alphabet and it should be written in of the original language. There is no need for special content, but the length of words should match the language.

To be added Hello, here is some text without a meaning. This text should show what a printed text will look like at this place. If you read this text, you will get no information. Really? Is there no information? Is there a difference between this text and some nonsense like “Huardest gefburn”? Kjift – not at all! A blind text like this gives you information about the selected font, how the letters are written and an impression of the look. This text should contain all letters of the alphabet and it should be written in of the original language. There is no need for special content, but the length of words should match the language.

CONCLUSIONS & PERSPECTIVES

Hello, here is some text without a meaning. This text should show what a printed text will look like at this place. If you read this text, you will get no information. Really? Is there no information? Is there a difference between this text and some nonsense like “Huardest gefburn”? Kjift – not at all! A blind text like this gives you information about the selected font, how the letters are written and an impression of the look. This text should contain all letters of the alphabet and it should be written in of the original language. There is no need for special content, but the length of words should match the language.

This is the second paragraph. Hello, here is some text without a meaning. This text should show what a printed text will look like at this place. If you read this text, you will get no information. Really? Is there no information? Is there a difference between this text and some nonsense like “Huardest gefburn”? Kjift – not at all! A blind text like this gives you information about the selected font, how the letters are written and an impression of the look. This text should contain all letters of the alphabet and it should be written in of the original language. There is no need for special content, but the length of words should match the language.

And after the second paragraph follows the third paragraph. Hello, here is some text without a meaning. This text should show what a printed text will look like at this place. If you read this text, you will get no information. Really? Is there no information? Is there a difference between this text and some nonsense like “Huardest gefburn”? Kjift – not at all! A blind text like this gives you information about the selected font, how the letters are written and an impression of the look. This text should contain all letters of the alphabet and it should be written in of the original language. There is no need for special content, but the length of words should match the language.

After this fourth paragraph, we start a new paragraph sequence. Hello, here is some text without a meaning. This text should show what a printed text will look like at this place. If you read this text, you will get no information. Really? Is there no information? Is there a difference between this text and some nonsense like “Huardest gefburn”? Kjift – not at all! A blind text like this gives you information about the selected font, how the letters are written and an impression of the look. This text should contain all letters of the alphabet and it should be written in of the original language. There is no need for special content, but the length of words should match the language.

Hello, here is some text without a meaning. This text should show what a printed text will look like at this place. If you read this text, you will get no information. Really? Is there no information? Is there a difference between this text and some nonsense like “Huardest gefburn”? Kjift – not at all! A blind text like this gives you information about the selected font, how the letters are written and an impression of the look. This text should contain all letters of the alphabet and it should be written in of the original language. There is no need for special content, but the length of words should match the language.

APPENDIX



Heading on Level 0 (chapter)

Hello, here is some text without a meaning. This text should show what a printed text will look like at this place. If you read this text, you will get no information. Really? Is there no information? Is there a difference between this text and some nonsense like “Huardest gefburn”? Kjift – not at all! A blind text like this gives you information about the selected font, how the letters are written and an impression of the look. This text should contain all letters of the alphabet and it should be written in of the original language. There is no need for special content, but the length of words should match the language.

A.1 Heading on Level 1 (section)

Hello, here is some text without a meaning. This text should show what a printed text will look like at this place. If you read this text, you will get no information. Really? Is there no information? Is there a difference between this text and some nonsense like “Huardest gefburn”? Kjift – not at all! A blind text like this gives you information about the selected font, how the letters are written and an impression of the look. This text should contain all letters of the alphabet and it should be written in of the original language. There is no need for special content, but the length of words should match the language.

Heading on Level 2 (subsection)

Hello, here is some text without a meaning. This text should show what a printed text will look like at this place. If you read this text, you will get no information. Really? Is there no information? Is there a difference between this text and some nonsense like “Huardest gefburn”? Kjift – not at all! A blind text like this gives you information about the selected font, how the letters are written and an impression of the look. This text should contain all letters of the alphabet and it should be written in of the original language. There is no need for special content, but the length of words should match the language.

Heading on Level 3 (subsubsection)

Hello, here is some text without a meaning. This text should show what a printed text will look like at this place. If you read this text, you will get no information. Really? Is there no information? Is there a difference between this text and some nonsense like “Huardest gefburn”? Kjift –

not at all! A blind text like this gives you information about the selected font, how the letters are written and an impression of the look. This text should contain all letters of the alphabet and it should be written in of the original language. There is no need for special content, but the length of words should match the language.

Heading on Level 4 (paragraph) Hello, here is some text without a meaning. This text should show what a printed text will look like at this place. If you read this text, you will get no information. Really? Is there no information? Is there a difference between this text and some nonsense like “Huardest gefburn”? Kjift – not at all! A blind text like this gives you information about the selected font, how the letters are written and an impression of the look. This text should contain all letters of the alphabet and it should be written in of the original language. There is no need for special content, but the length of words should match the language.

A.2 Lists

Example for list (itemize)

- ▶ First item in a list
- ▶ Second item in a list
- ▶ Third item in a list
- ▶ Fourth item in a list
- ▶ Fifth item in a list

Example for list (4*itemize)

- ▶ First item in a list
 - First item in a list
 - * First item in a list
 - First item in a list
 - Second item in a list
 - * Second item in a list
 - Second item in a list
- ▶ Second item in a list

Example for list (enumerate)

1. First item in a list
2. Second item in a list
3. Third item in a list
4. Fourth item in a list
5. Fifth item in a list

Example for list (4*enumerate)

1. First item in a list
 - a) First item in a list
 - i. First item in a list
 - A. First item in a list
 - B. Second item in a list
 - ii. Second item in a list
 - b) Second item in a list
2. Second item in a list

Example for list (description)

First item in a list
Second item in a list
Third item in a list
Fourth item in a list
Fifth item in a list

Example for list (4*description)

First item in a list
 First item in a list
 First item in a list
 First item in a list
 Second item in a list
 Second item in a list
 Second item in a list
Second item in a list

Bibliography

Here are the references in citation order.

- [1] Bruno Lafaurie et al. 'Modelling merging and fragmentation in multiphase flows with SURFER'. In: *Journal of Computational Physics* 113.1 (1994), pp. 134–147 (cited on page 10).
- [2] Dalton JE Harvie, MR Davidson, and Murray Rudman. 'An analysis of parasitic current generation in volume of fluid simulations'. In: *Applied mathematical modelling* 30.10 (2006), pp. 1056–1066 (cited on page 10).
- [3] Stéphane Popinet and Stéphane Zaleski. 'A front-tracking algorithm for accurate representation of surface tension'. In: *International Journal for Numerical Methods in Fluids* 30.6 (1999), pp. 775–793 (cited on pages 10, 17).
- [4] Marianne M Francois et al. 'A balanced-force algorithm for continuous and sharp interfacial surface tension models within a volume tracking framework'. In: *Journal of Computational Physics* 213.1 (2006), pp. 141–173 (cited on page 10).
- [5] Stéphane Popinet. 'An accurate adaptive solver for surface-tension-driven interfacial flows'. In: *Journal of Computational Physics* 228.16 (2009), pp. 5838–5866 (cited on pages 10, 11, 14, 15, 17, 18).
- [6] Thomas Abadie, Joelle Aubin, and Dominique Legendre. 'On the combined effects of surface tension force calculation and interface advection on spurious currents within Volume of Fluid and Level Set frameworks'. In: *Journal of Computational Physics* 297 (2015), pp. 611–636 (cited on page 14).
- [7] Stéphane Popinet. 'Numerical models of surface tension'. In: *Annual Review of Fluid Mechanics* 50 (2018), pp. 49–75 (cited on page 16).
- [8] Horace Lamb. *Hydrodynamics*. Cambridge university press, 1993 (cited on pages 17, 18).
- [9] Andrea Prosperetti. 'Free oscillations of drops and bubbles: the initial-value problem'. In: *Journal of Fluid Mechanics* 100.2 (1980), pp. 333–347 (cited on page 17).
- [10] Andrea Prosperetti. 'Motion of two superposed viscous fluids'. In: *The Physics of Fluids* 24.7 (1981), pp. 1217–1223 (cited on page 17).
- [11] Ross Gunn and Gilbert D Kinzer. 'The terminal velocity of fall for water droplets in stagnant air'. In: *Journal of Meteorology* 6.4 (1949), pp. 243–248 (cited on page 19).

Notation

The next list describes several symbols that will be later used within the body of the document.

c Speed of light in a vacuum inertial frame

h Planck constant

Greek Letters with Pronunciation

Character	Name	Character	Name
α	alpha <i>AL-fuh</i>	ν	nu <i>NEW</i>
β	beta <i>BAY-tuh</i>	ξ, Ξ	xi <i>KSIGH</i>
γ, Γ	gamma <i>GAM-muh</i>	\omicron	omicron <i>OM-uh-CRON</i>
δ, Δ	delta <i>DEL-tuh</i>	π, Π	pi <i>PIE</i>
ϵ	epsilon <i>EP-suh-lon</i>	ρ	rho <i>ROW</i>
ζ	zeta <i>ZAY-tuh</i>	σ, Σ	sigma <i>SIG-muh</i>
η	eta <i>AY-tuh</i>	τ	tau <i>TOW (as in cow)</i>
θ, Θ	theta <i>THAY-tuh</i>	υ, Υ	upsilon <i>OOP-suh-LON</i>
ι	iota <i>eye-OH-tuh</i>	ϕ, Φ	phi <i>FEE, or FI (as in hi)</i>
κ	kappa <i>KAP-uh</i>	χ	chi <i>KI (as in hi)</i>
λ, Λ	lambda <i>LAM-duh</i>	ψ, Ψ	psi <i>SIGH, or PSIGH</i>
μ	mu <i>MEW</i>	ω, Ω	omega <i>oh-MAY-guh</i>

Capitals shown are the ones that differ from Roman capitals.