Formální Metody a Specifikace - Cvičení 2b (103)

10. březen 2011

For Exercises 3 and 4 below, use *only* the proof techniques from Lecture 3. Expecially, do *not use* truth tables, or equivalence rules or other shortcuts from Lecture 2. The only exception is the following: For a formula ϕ you may use the fact that ϕ and $\neg\neg\phi$ are equivalent and hence can be replaced by each other.

1 Exercise 3

Prove the following formulas:

1. $[\neg p \Rightarrow p] \Rightarrow p$
2. $\neg p \Rightarrow [p \Rightarrow q]$
3. $[[p \vee q] \wedge \neg q] \Rightarrow p$
4. $\neg[p \Rightarrow q] \Rightarrow [q \Rightarrow p]$
5. $\neg[p \vee q] \Rightarrow [p \Rightarrow q]$
6. $[p \Rightarrow [[q \vee r] \wedge \neg q \wedge \neg r]] \Rightarrow \neg p$
7. $[[p \Rightarrow q] \wedge \neg[p \Rightarrow q]] \Rightarrow \neg[[r \vee s] \Rightarrow q] \wedge [[r \vee s] \Rightarrow q]$
8. $q \Rightarrow [[p \wedge q] \vee [\neg p \wedge q]]$
9. $\neg[p \wedge q] \Rightarrow [\neg p \vee \neg q]$
10. $[[p \wedge q] \Rightarrow r] \Rightarrow [[p \Rightarrow r] \vee [q \Rightarrow r]]$
11. $\neg[\neg p \vee \neg q] \Rightarrow [p \wedge q]$
12. $[p \Rightarrow q] \vee [q \Rightarrow r]$

(12 points)

2 Exercise 4

Prove the following formulas (P and Q are unary predicates, and S is a 0-ary predicate):

1. $[\exists x \ . \ P(x)] \Rightarrow [\exists y \ . \ P(y)]$
2. $[\neg\exists x \ . \ P(x)] \Rightarrow [\forall x \ . \ \neg P(x)]$
3. $[[\forall x \ . \ P(x)] \wedge [\forall x \ . \ Q(x)]] \Rightarrow [\forall x \ . \ P(x) \wedge Q(x)]$
4. $[[\exists x \ . \ P(x)] \vee [\exists x \ . \ Q(x)]] \Rightarrow [\exists x \ . \ P(x) \vee Q(x)]$
5. $[S \Rightarrow \exists x \ . \ Q(x)] \Rightarrow \exists x \ . \ [S \Rightarrow Q(x)]$
6. $[[\forall x \ . \ P(x)] \Rightarrow S] \Rightarrow [\exists x \ . \ P(x) \Rightarrow S]$

(6 points)

3 Exercise 5

In the following, "to prove something in a certain theory" means that you can from the beginning assume all axioms from this theory as known facts.

1. In the theory of lists, prove $\forall x \ . \ \mathsf{cons}(x, \mathsf{empty}()) \neq \mathsf{empty}())$.
2. In the theory of arrays, prove $\forall a, i \ . \ \mathsf{write}(a, i, a[i]) = a$.
3. In the theory of arrays, prove $\exists a, i, j, x \ . \ \mathsf{write}(a, i, x)[j] = x$.
4. Prove $\forall x \ . \ 0 + x = x$ from the Peano axioms *without* the induction axiom, using induction as you learnt it in school.
5. Prove $\forall x \ . \ 0+x = x$ from the Peano axioms using the induction axiom.

(5 points)