Formální Metody a Specifikace
Cvičení 4a (101,102)

31. březen 2011

From now on, when proving some predicate-logical formula, you may assume the axioms of all necessary theories. Moreover, in addition to the Peano axioms, you may assume all facts that intuitively hold for the natural numbers (e.g., associativity, commutativity of addition and multiplication) and all relevant definitions (e.g., the definition of division of natural numbers). Also, you may use the proof rules liberally. That is, you do not have to follow the proof rules in all details, as long as you are aware of how the missing details would look like.

1 Exercise 8

```
   x ∈ ℕ
1: while x ≤ 73 do
2:     x ← x + 1
3:     print 10/(10 − x)
```

Let

- $I :\Leftrightarrow pc = 1 \land x \leq 8$, and
- $O :\Leftrightarrow pc = 3 \Rightarrow x \neq 10$.

For each $\phi \in \{BMC(1), \ldots, BMC(10)\}$, prove either $\models \phi$, or $\models \neg\phi$. Some of the proofs will look similar. In such cases, it is not necessary to write down all of them. Instead, it suffices to explain the differences between them.

(3 points)

2 Exercise 9

$x \leftarrow a[2]$
$a \leftarrow \text{write}(a, 2, a[1])$
$a \leftarrow \text{write}(a, 1, x)$

Let

- $I :\Leftrightarrow pc = 1 \wedge \forall i \in \{1, \ldots 10\} \,.\, a[i] \leq 10$, and
- $O :\Leftrightarrow \forall i \in \{1, \ldots 10\} \,.\, a[i] \leq 10$.

For each $\phi \in \{BMC(1), BMC(2), BMC(3)\}$, prove either $\models \phi$, or $\models \neg\phi$.

(2 points)