Formální Metody a Specifikace
Cvičení 3a (101, 102)

17. březen 2011

1 Exercise 5

In the following, "to prove something in a certain theory" means that you can from the beginning assume all axioms from this theory as known facts.

1. In the theory of lists, prove

$$\forall l, x, y \,.\, l = \mathsf{cons}(x, \mathsf{cons}(y, \mathsf{empty}())) \Rightarrow \mathsf{first}(\mathsf{rest}(l)) = y$$

2. In the theory of arrays, prove

$$[\exists x, y \,.\, x \neq y] \Rightarrow \exists a, i, j, x \,.\, \mathsf{write}(a, i, x)[j] \neq x.$$

3. In the theory of arrays, prove

$$\forall a, i, x \,.\, \mathsf{write}(a, i, x)[j] = x \Rightarrow [i = j \vee a[j] = x]$$

4. Prove $\forall x \,.\, 0x = 0$ from the Peano axioms *without* the induction axiom, using induction as you learnt it in school. You may assume that, in addition to the Peano axioms, also the axiom $\forall x \,.\, 0 + x = x$ holds.

5. Prove $\forall x \,.\, 0x = 0$ from the Peano axioms plus the axiom $\forall x \,.\, 0+x = x$, using the induction axiom.

(5 points)

2 Exercise 6

Write down the constraint Φ_P defining the transition relation (=přechodová relace) of the following program P:

1: $i \leftarrow 1$
2: **if** $i < 10$ **then**
3:     **input** x
4:     $a[i] \leftarrow x$
5:     $i \leftarrow i + 1$
6:     **goto 2**
7: **return**

(2 points)

3 Exercise 7

Extend the definition of the notion "transition relation" from the lecture with the case that $s(pc)$ points to a program line corresponding to (the beginning or end) of a while loop. Here, handle the loop directly, and do *not* translate it to an if-then-goto construction.

(2 points)