Formální Metody a Specifikace - Cvičení 2

3. březen 2011

For the exercises below, use *only* the proof techniques from Lecture 3. Expecially, do *not use* truth tables, or equivalence rules or other short-cuts from Lecture 2. The only exception is the following: For a formula ϕ you may use the fact that ϕ and $\neg\neg\phi$ are equivalent and hence can be replaced by each other.

1 Exercise 3

Prove the following formulas:

- $[\neg p \Rightarrow p] \Rightarrow p$
- $\neg p \Rightarrow [p \Rightarrow q]$
- $[[p \vee q] \wedge \neg q] \Rightarrow p$
- $\neg[p \Rightarrow q] \Rightarrow [q \Rightarrow p]$
- $[p \Rightarrow q] \Rightarrow [\neg p \vee q]$
- $[p \Rightarrow [[q \vee r] \wedge \neg q \wedge \neg r]] \Rightarrow \neg p$
- $[\neg[[r \vee s] \Rightarrow q] \wedge [[r \vee s] \Rightarrow q]] \Rightarrow [[p \Rightarrow q] \wedge \neg[p \Rightarrow q]]$
- $q \Rightarrow [[p \wedge q] \vee [\neg p \wedge q]]$
- $\neg[p \wedge q] \Rightarrow \neg p \vee \neg q$
- $[[p \wedge q] \Rightarrow r] \Rightarrow [[p \Rightarrow r] \vee [q \Rightarrow r]]$
- $[p \wedge q] \Rightarrow \neg[\neg p \vee \neg q]$
- $[p \Rightarrow q] \vee [q \Rightarrow r]$

(12 points)

2 Exercise 4

Prove the following formulas (P and Q are unary predicates, and S is a 0-ary predicate):

- $[\forall x \,.\, P(x)] \Rightarrow [\forall y \,.\, P(y)]$
- $[\neg\forall x \,.\, P(x)] \Rightarrow [\exists x \,.\, \neg P(x)]$
- $[\forall x \,.\, P(x) \wedge Q(x)] \Rightarrow [[\forall x \,.\, P(x)] \wedge [\forall x \,.\, Q(x)]]$
- $[\exists x \,.\, P(x) \vee Q(x)] \Rightarrow [[\exists x \,.\, P(x)] \vee [\exists x \,.\, Q(x)]]$
- $[\exists x \,.\, S \Rightarrow Q(x)] \Rightarrow [S \Rightarrow \exists x \,.\, Q(x)]$
- $[[\exists x \,.\, P(x)] \Rightarrow S] \Rightarrow [\forall x \,.\, P(x) \Rightarrow S]$

(6 points)