

# Transformace (2)

## příloha

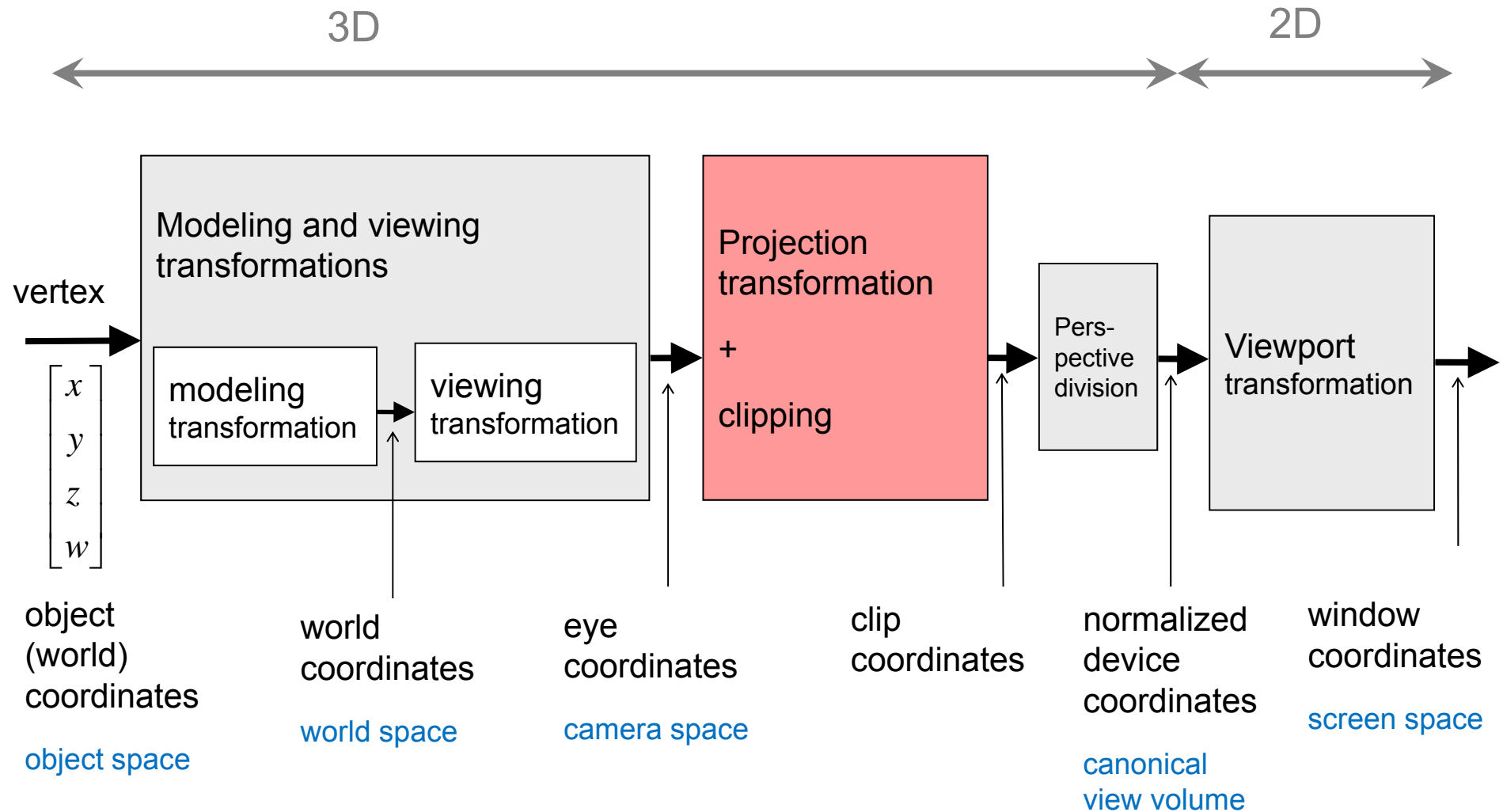
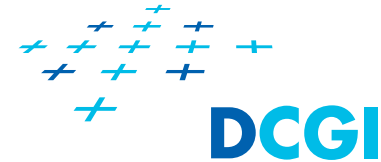
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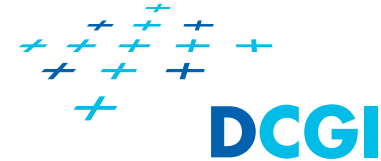
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S použitím materiálů Bohuslava Hudce, Jaroslava Sloupa a  
Vlastimila Havrana

# Projection transformations



## Projection transformations (contd.)



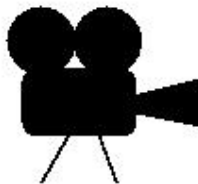
- the purpose of the projection transformation is to define a **viewing volume**, which is used in two ways
  - the viewing volume determines how an object is projected onto the screen (*that is, by using a perspective or an orthographic projection*)
  - it defines which objects or portions of objects are clipped out of the final image
- there are three types of projections supported by OpenGL
  - user defined (manually defined transformation matrix)
  - orthographic projection (parallel)
  - perspective projection
- note that projection transformation defines also so called clipping planes (6 planes – left, right, top, bottom, near, and far)

# Orthographic projection

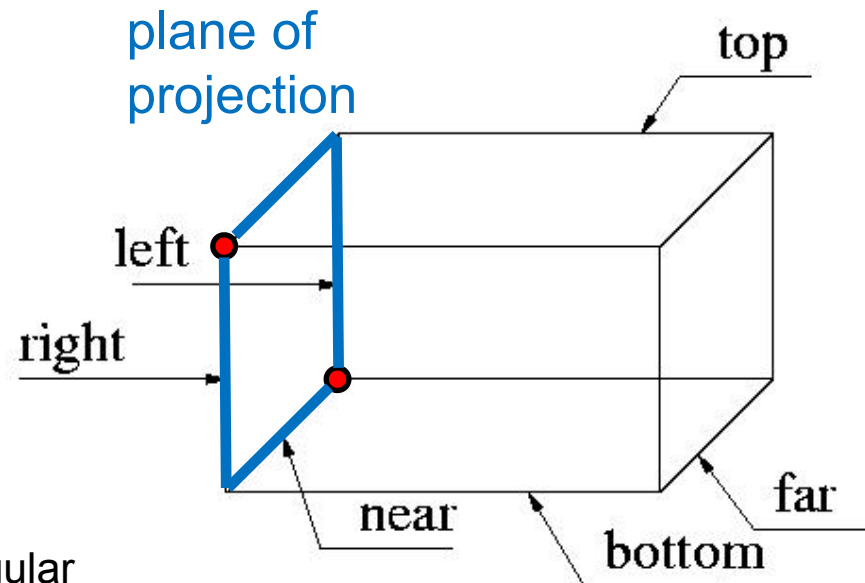


```
#include <glm/gtc/matrix_transform.hpp>
glm::mat4 glm::ortho(
    float left, float right,           /* range on x-axis */
    float bottom, float top,          /* range on y-axis */
    float near, float far );          /* range on z-axis */
```

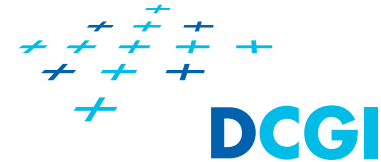
- creates a matrix for an orthographic parallel viewing volume
- [left, bottom, \*]** and **[right, top, \*]** are points on the near/far clipping plane that are mapped to the lower-left and upper-right corners of the viewport window



viewing volume = rectangular parallelepiped (box)

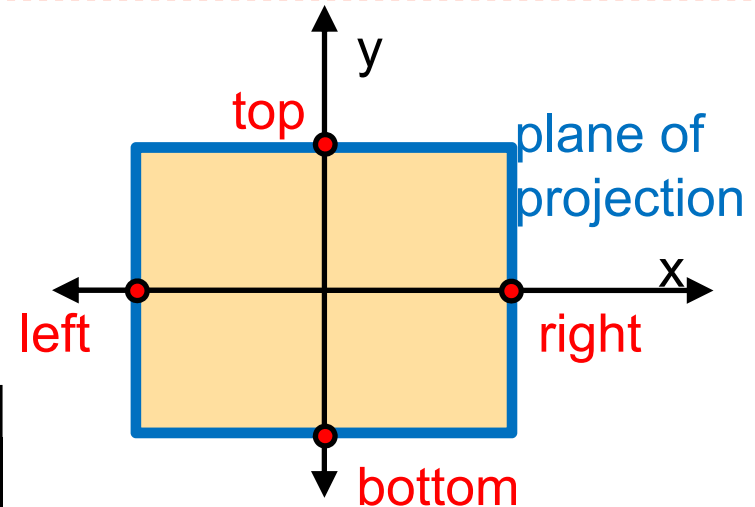


## Orthographic projection (contd.)



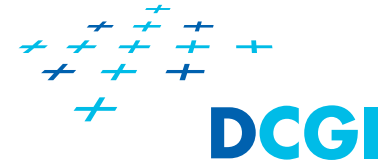
- plane of projection size  
(*the xy-plane cutout*)
- transformation matrix of  
orthographic projection

$$M_{\text{parallel}} = \begin{bmatrix} \frac{2}{\text{right-left}} & 0 & 0 & -\frac{\text{right+left}}{\text{right-left}} \\ 0 & \frac{2}{\text{top-bottom}} & 0 & -\frac{\text{top+bottom}}{\text{top-bottom}} \\ 0 & 0 & \frac{-2}{\text{far-near}} & -\frac{\text{far+near}}{\text{far-near}} \\ 0 & 0 & 0 & 1 \end{bmatrix}$$



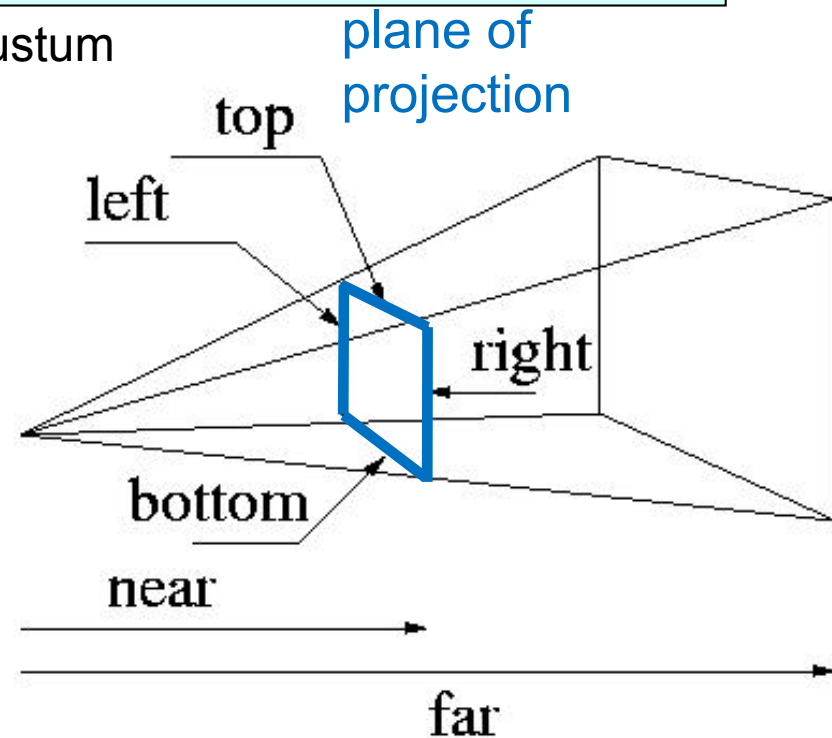
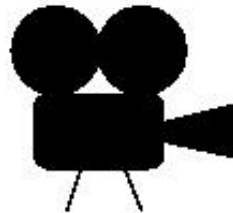
- camera may be inside the viewing volume (**e.g. `glm::ortho(-1,1,-1,1,-1,1)`**)  
⇒ the objects *behind the camera* are also displayed
- camera can be totally shifted (i.e. outside the viewing volume, *it's like using a periscope*)  
e.g. **`glm::ortho(-20,-10,-1,1,-1,1)`**

# Perspective projection

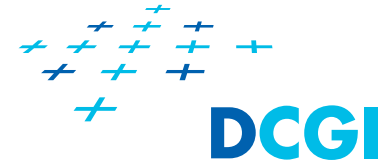


```
#include <glm/gtc/matrix_transform.hpp>
glm::mat4 glm::frustum(
    float left, float right,
    float bottom, float top,
    float near, float far );
```

- creates a matrix for a perspective-view frustum
- the viewing volume is a truncated pyramid whose top has been cut off by a plane parallel to its base
- objects that are closer to the viewpoint appear larger because they occupy a proportionally larger amount of the viewing volume



## Perspective projection (contd.)



- transformation matrix of perspective projection

$$M_{\text{frustum}} = \begin{bmatrix} \frac{2 \cdot \text{near}}{\text{right} - \text{left}} & 0 & \frac{\text{right} + \text{left}}{\text{right} - \text{left}} & 0 \\ 0 & \frac{2 \cdot \text{near}}{\text{top} - \text{bottom}} & \frac{\text{top} + \text{bottom}}{\text{top} - \text{bottom}} & 0 \\ 0 & 0 & -\frac{(\text{far} + \text{near})}{\text{far} - \text{near}} & -\frac{2 \cdot \text{far} \cdot \text{near}}{\text{far} - \text{near}} \\ 0 & 0 & -1 & 0 \end{bmatrix}$$

- viewing volume can be asymmetric



`glm::frustum(-1,1,-1,1,1,3.5)`

`glm::frustum(-1,1,-1,1,1.6,3.5)`

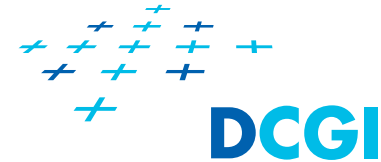


PGR



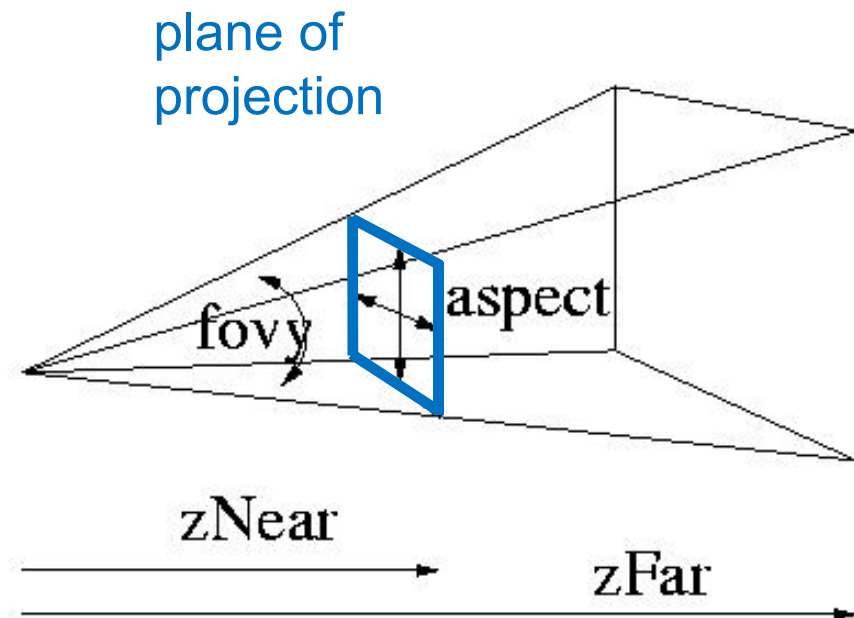
`glm::frustum(-0.1,1,0.1,1,1,3.5)`

## Perspective projection (contd.)



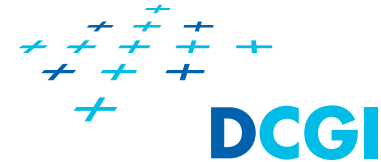
```
#include <glm/gtc/matrix_transform.hpp>
glm::mat4 glm::perspective(
    float fovy, float aspect,
    float near, float far );
```

- creates a matrix for a symmetric perspective-view frustum
- **fovy** is the angle of the field of view in the x-z plane, it must be in the range  $[0.0, 180.0]$
- **aspect** is the aspect ratio of the frustum, its width divided by its height
- near and far values should always be positive ( $\text{near} > 0$ )





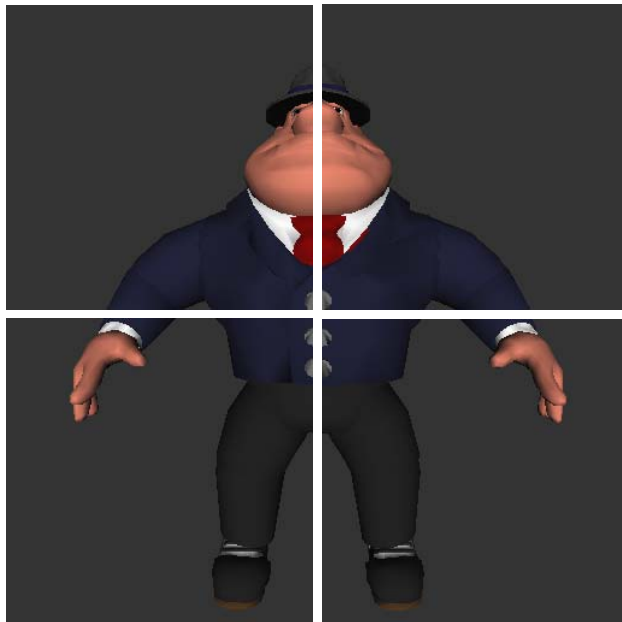
## Perspective projection (contd.)



How to get image in double the maximal resolution?

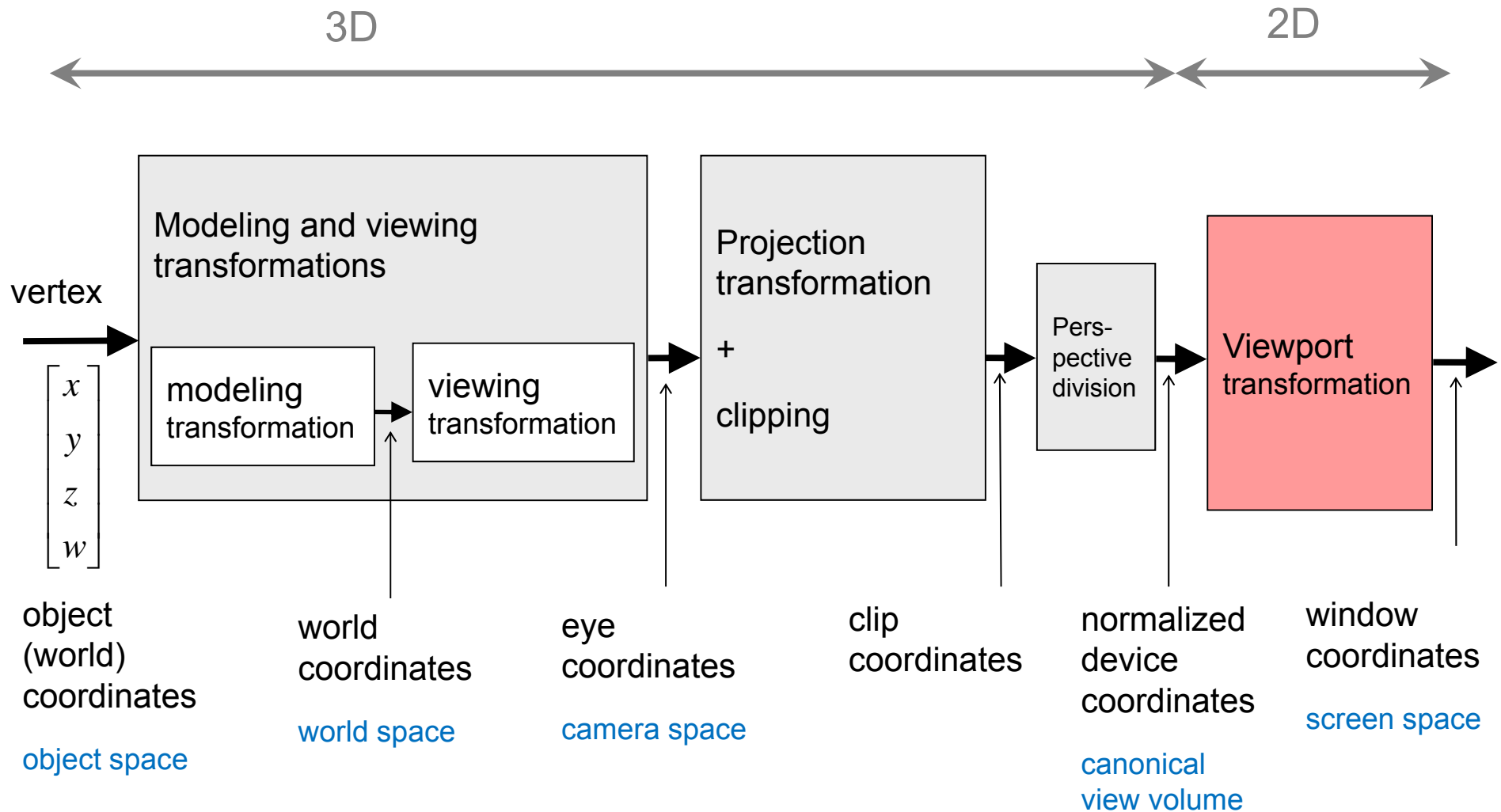
- draw the scene four times setting the projections as shown in the table
- save the rendered images
- glue these images together (e.g. in Photoshop or Gimp)

x=[ left, right] y=[bottom, top]	
x=[-1, 0] y=[ 0, 1]	x=[ 0, 1] y=[ 0, 1]
x=[-1, 0] y=[-1, 0]	x=[ 0, 1] y=[-1, 0]

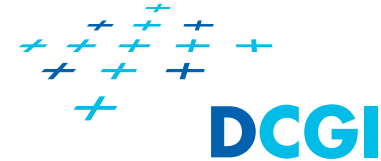


```
glm::mat4 matrix;
/* upper left part */
matrix = glm::frustum(-1,0,0,1, 1.6,3.5);
RenderModel();
/* upper right part */
matrix = glm::frustum(0,1,0,1, 1.6,3.5);
RenderModel();
/* lower left part */
matrix = glm::frustum(-1,0,-1,0, 1.6,3.5);
RenderModel();
/* lower right part */
matrix = glm::frustum(0,1,-1,0, 1.6,3.5);
RenderModel();
```

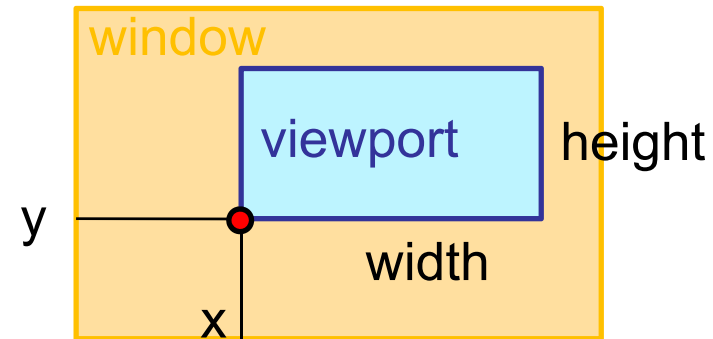
# Viewport transformation



## Viewport transformation (contd.)



- the viewport is the rectangular region of the window where the image is drawn



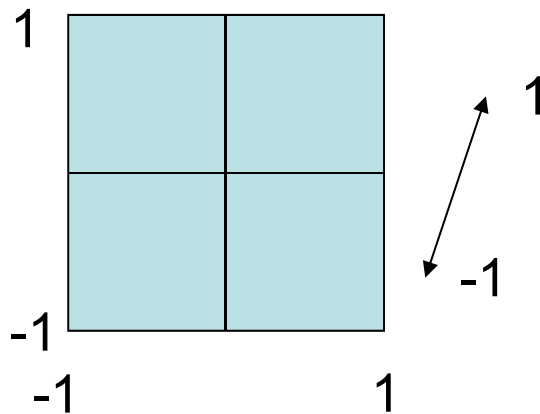
```
void glViewport(  
    GLint x, GLint y, GLsizei width, GLsizei height);
```

- defines a pixel rectangle in the window into which the final image is mapped
- viewport is usually set in the reshape callback
- the aspect ratio of a viewport should generally equal the aspect ratio of the viewing volume, if these two ratios are different, the projected image will be distorted when mapped into the viewport
- your application should detect window resize events and modify the viewport appropriately

# Viewport transformation (contd.)



normalized device  
coordinates  $[x_d, y_d, z_d]$



$$x_w = (w / 2) x_d + o_x$$

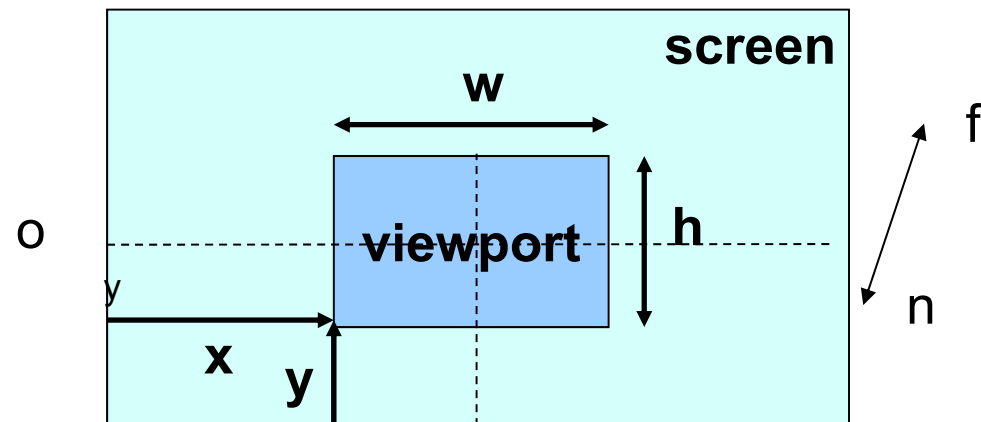
$$y_w = (h / 2) y_d + o_y$$

$$z_w = [(f-n) / 2] z_d + (n+f) / 2$$

$z_w$  visibility testing (Z-buffer)

window coordinates

$[x_w, y_w, z_w]$



$$o_x = x + w/2$$

$$o_y = y + h/2$$

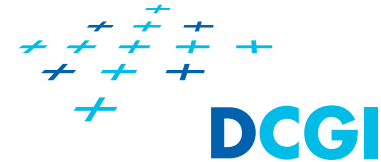
**glDepthRange(n, f)**

Set depth range  $\rightarrow$  clamp(n,f)

near near clipping plane 0.0

far far clipping plane 1.0

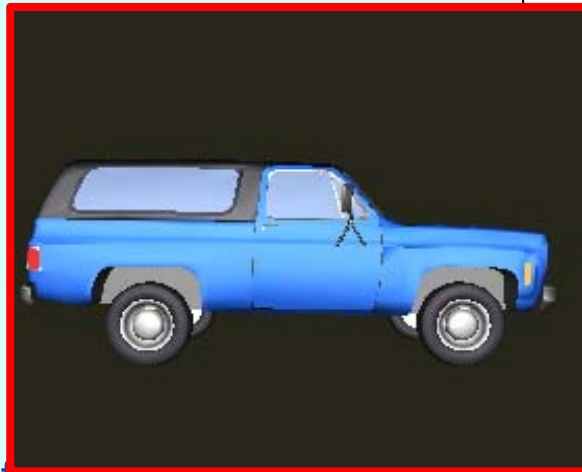
## Viewport transformation (contd.)



Example: Rendering into two different viewports.

```
glm::mat4 matrix = glm::mat4(1.0);  
glm::mat4 projectionMatrix = glm::perspective(  
    60, winW/(2.0*winH), 0.1, 10 );  
glViewport( 0, 0, winW/2, winH ); /* left viewport */  
passMatrixToVertexShader( projectionMatrix );  
passMatrixToVertexShader( matrix );  
drawModel();
```

winW = 600... window width  
winH = 300 ... window height



```
/* right viewport */  
matrix = glm::rotate(  
    matrix, 45, glm::vec3(1.0f, 1.0f, 0.0f)),  
glViewport( winW/2, 0, winW/2, winH );  
passMatrixToVertexShader( matrix );  
drawModel();
```