Gaussian Process based Model Predictive Control

Statistical Learning and Stochastic Control

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Given the GP Likelihood:

$$Y|X,\theta \sim \mathcal{N}(0, \underbrace{K(X,X) + \sigma_n^2 I}_{K_y}) \tag{1}$$

where $\theta = [\{M\}, \sigma_f^2, \sigma_n^2]$ is a vector containing all hyper-parameters.

Among many possible choices, we chose to parameterize the length-scale covariance matrix M as:

$$M = \begin{bmatrix} l_1 & & 0 \\ & \ddots & \\ 0 & & l_n \end{bmatrix} = I_{n\times n} \begin{bmatrix} l_1 \\ \vdots \\ l_n \end{bmatrix} \tag{2}$$

with $l_i \geq 0, \forall i \in 0, \ldots, n$

such that the hyperparameter vector becomes $\theta = [l_1, \ldots, l_n, \sigma_f^2, \sigma_n^2]$

Hyper-parameter Optimization

One may optimize the GP hyper-parameters by maximizing the log likehood:

$$log\, p(Y|X,\theta) = -\frac{1}{2}y^T K_y^{-1} y - \frac{1}{2}\log|K_y| - \frac{n}{2}\log(2\pi)$$

$$\theta = \arg\max_{\theta}\ log\, p(Y|X,\theta) \quad (3)$$

which allows local optimization of the hyper-parameters (one opt. for each output dimension).

Obs: Even gradient-free tools like `fmincon` from Matlab, showed to be efficient.

MPC Formulation

Show basic MPC formulation

Adaptive MPC

$$x_{k+1} = f_d(x_k, u_k) + B_d \, (d(z_k) + w) \tag{4}$$

where

$$w \sim \mathcal{N}(0, \sigma_n^2) \tag{5}$$

$$a \tag{6}$$

Propagation of uncertainty

Efficient MPC Formulation

Show how the inequality constraints can be removed
Show equation and plot of the Relaxed-Barrier function
Results =¿ unconstrained optimization problem, which can be solved very efficiently with nonlinear optimization solvers

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Efficient MPC Formulation

Show how the inequality constraints can be removed
Show equation and plot of the Relaxed-Barrier function
Results: unconstrained optimization problem, which can be solved very efficiently with nonlinear optimization solvers

Cost function and constraints

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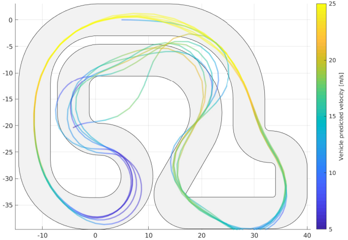
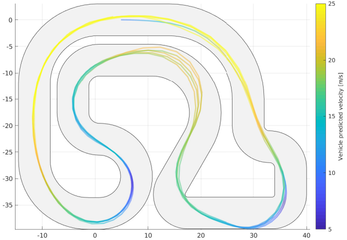


Adaptive GP MPC



Outlook and Conclusion

- GP can introduce high nonlinearities in the prediction, making more difficult for the optimizer to a find a good local optimum.
- Replacing the inequality constraints by relaxed barrier functions increase significantly the computational performance, while always ensuring feasibility.
- Hyper-parameter optimization (GP training) plays an important role in the final controller performance