An Overview of Model Predictive Control

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Abstract

Model predictive control is the family of controllers, makes the explicit use of model to obtain control signal. The reason for its popularity in industry and academia is its capability of operating without expert intervention for long periods. There are various control design methods based on model predictive control concepts. This paper provides review of the most commonly used methods that have been embedded in an industrial model predictive control. The most widely used strategies as Dynamic matrix control (DMC), Model algorithmic control (MAC), Predictive functional control (PFC), Extended prediction self-adaptive control (EPSAC), Extended horizon adaptive control(EHAC) and Generalized predictive control(GPC) have been described with history, basic idea, properties, and their controller formulation.

Keywords: *Model, Optimization, Prediction, Predictive control, Robustness, Stability.*

1. Introduction

Over the past two decades, there has been significant interest and developments by industry and academia in advanced process control. During 1960's, advanced control was taken in general as any algorithm that deviated from the classical three-term Proportional-Integral-Derivative (PID) controller. For quality product, safety and economic process, PID controller accounts for more than 80% of the installed automatic feedback control devices in the process industries [1].

For the purpose of automatic tuning, the automatic adjustment of PID, use of pattern recognition (Zeigler-Nichols) or transfer function derived from reaction curve are the technique used for consistent performance. The performance is better than randomly tuned manually settings of PID design. However, the basic design of PID is not very effective for the plant having dead time [2].

There are other methods being used as self tuning controllers like, Minimum variance (MV), Smith prediction, generalized minimum variance (GMV) (Clarke and Gawthrop, 1975, 1979), Pole placement (PP) (Wellstead 1979). With the proper choice of various design parameters these methods shows the excellent performance or control behavior.

Nevertheless, Pole placement is sensitive to model-order and it requires the solution of Diophantine equation, while MV/GMV to prior dead-time assumptions.

The limitation of Minimum variance (MV) is that it is unable to handle the non-minimum phase system and requires excessive control efforts for the non-minimum phase system. While generalized minimum variance (GMV) requires less control effort and can dealt with non-minimum phase system by adding a weighting on the control effort.

However, the use of GMV limits in minimizing a quadratic function of a single value of the output at time with delay time of the process and lacks of robustness with respect to variable or unknown dead-times.

Several modifications of GMV have led to the development of Model Predictive Control (MPC), which has been widely accepted in industry. MPC has attracted many researchers due to better performance and control of processes including non-minimum phase, long time delay or open-loop unstable characteristics over minimum variance (MV), generalized minimum variance (GMV) and pole placement (PP) techniques [3],[4]. The main advantages of MPC over structured PID controllers are its ability to handle constraints, non-minimum phase processes, changes in system parameters (robust control) and its straightforward applicability to large, multivariable processes [5],[6].

Model predictive control (MPC), also known as receding horizon control or moving horizon control, uses the range of control methods, making the use of an explicit dynamic plant model to predict the effect of future reactions of the manipulated variables on the output and the control signal obtained by minimizing the cost function [7]. The performance of the controller depends on how well the dynamics of the system being captured by the input–output model that is used for the design of the controller [8].

Model Predictive Control, MPC, usually contains the following three ideas,

1 Explicit use of a model to predict the process output along a future time horizon.
2 Calculation of a control sequence to optimize a performance index.
3 A receding horizon strategy, so that at each instant the horizon is moved towards the future, which involves the application of the first control signal of the sequence calculated at each step.

The MPC methodology is characterized by the strategy represented in figure 1 [4].

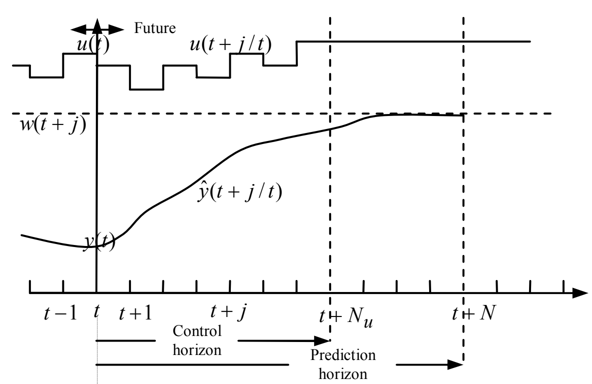

Figure 1. The moving horizon strategy of MPC

1. The process model calculates the predicted future outputs $\hat{y}(t+j/t)$, $j=1,\cdots,N-1$ for the prediction horizon (N) at each instant t. These depend upon the known values up to instance t (past inputs and outputs), including the current output (initial condition) $y(t)$ and on the future control signals $u(t+j/t)$, $j=0,\cdots,N-1$, to be calculated.

2. The sequence of future control signals is computed to optimize a performance criterion. Usually the control effort is included in the performance criterion.

3. Only the current control signal $u(t/t)$ is send to the process. At the next sampling instant $y(t+1)$ is measured and step 1 is repeated and all sequences brought up to date. Thus $u(t+1/t+1)$ is then calculated using the receding horizon concept.

In MPC, process models on which the control performance depends is of linear or nonlinear types, used to analyze the system behavior [9]. The future moves of the manipulated variables are determined by minimizing the predicted error or the objective function.

2. Historical Background

The MPC concept has a long history. In 1970's Engineers at Shell Oil developed their own dependent MPC technology with an initial application in 1973 [10].

In 1988, R. M. C. De Keyser et al., presented a comparative study of self-adaptive long range predictive control (LRPC) methods keeping focus on robustness with respect to unmodeled dynamics, parameter variations, process noise and varying dead-time [11].

The survey paper of Garcia et. al., (1989), includes MPC techniques. They pointed out the advantages in design and implementation and examined its relation to linear quadratic control. It includes effect on robustness and examined application of MPC to nonlinear systems [10].

R. Scattolini and S. Bittanti (1990) provided some simple rules stated in terms of the plant step response or impulse response for the selection of prediction horizon. Since it is fundamental for success, in LRPC, to guarantee the closed-loop stability [12].

According to D.W. Clarke and R. Scattolini (1991), constrained receding horizon predictive control optimizes a quadratic function over a costing horizon to stabilize general linear plants. However, the computation is more complex. The other way is to use finite-horizon methods, which are numerically highly sensitive [13].

S. Joe Qin and Thomas A. Badgwell provided an overview of commercially available MPC technology [14].

A. Bemporad and M. Morari presented an overview of robustness in MPC in 1999 [9] and proposed techniques for constraint handling, stability and performance.

David J. Sandoz et. al., (2000), presented a paper on long range (LR), long range quadratic programming (LRQP) and quadratic programming (QP) methods as options for use with MPC schemes to handle input and output constraints and constraint violations. LR method is robust, reliable and computationally efficient. LRQP method manages the input constraints. QP incorporates output constraints [15].

Bahram Kimiaghalam et. al., (2003), proposed the formulation suitable for indirect adaptive control algorithms. They presented it with real time application of MPC for enhancing the computational properties and demonstrated feasibility [16].

Gene Grim et. al., (2003), discussed that nonlinear system produce asymptotic instability in absence of robustness when the optimization problem contains constraints with short horizons. This requires MPC feedback law and value function are discontinuous at some point(s). This is apposite to the MPC for linear systems [17].

Adam L. Warren and Thomas E. Marlin proposed robust MPC formulation based upon a multi-region, closed-loop uncertainty description that is calculated off-line for stable systems to remain on an input constraint at steady state. This maintains the robust process outputs while handling output constraints and negative effects of input constraints [18].

Guang Li et. al., showed that the infinite horizon controller offers improved set-point tracking. However, if the constraints are active at the steady state then the controller produces an offset from the set point [19]. In 2006, Ali A. Jalali and Vahid Nadimi provided a review on Robust MPC with methods based on model uncertainties and disturbance uncertainties [9].

M. Abu-Ayyad and R. Dubay (2007) provided real time comparison of a number of predictive controllers [6].

3. Review of MPC methods

There are various control design methods based on model predictive control concepts. In this paper, an overview of the most commonly used six methods of MPC with history, basic idea, brief description of the principle and formulation is provided. The main features, advantages and disadvantages are discussed.

3.1 Dynamic matrix control

In 1979, Cutler and Ramaker of Shell oil Co. presented details of an unconstrained multivariable control algorithm, which they named Dynamic Matrix Control (DMC) [14]. It is evolved from a technique of representing process dynamics with a set of numerical coefficients [20]. The Dynamic matrix is used for projecting the future outputs. It is suitable for linear open loop stable process. The DMC technique is based on a step response model of the process.

The objective of the DMC controller is to drive the output to track the set point in the least squares sense including a penalty term on the input moves. This results in smaller computed input moves and a less aggressive output response [14].

Consider the single input single output (SISO) case, the step response model of the plant,

$$y(t) = \sum_{i=1}^{\infty} g_i \Delta u(t-i) \tag{1}$$

The disturbance at instant t along the horizon is,

$$\begin{aligned} \hat{x}(t+j/t) &= \hat{x}(t/t) = e(t) \\ e(t) &= y_m(t) - \hat{y}(t/t) \end{aligned} \tag{2}$$

The predicted value along the horizon will be:

$$\hat{y}(t+j/t) = \sum_{i=1}^{\infty} g_i \Delta u(t+j-i) + \hat{x}(t+j/t) \tag{3}$$

For constant disturbance the predicted value of output is,

$$\hat{y}(t+j/t) = \sum_{i=1}^{j} g_i \Delta u(t+j-i) + f(t+j) \tag{4}$$

The second term of equation (4) is the free response, which does not depend on the future control actions. The prediction along the prediction horizon p and m control actions ($j = 1, \cdots, p$) is [4],

$$\hat{y}(t+p/t) = \sum_{i=p-m+1}^{p} g_i \Delta u(t+p-i) + f(t+p) \tag{5}$$

Equation (4) can be written as,

$$\hat{y} = Gu + f \tag{6}$$

Equation (6) shows the relation between future output and control increments.

$$G=\begin{bmatrix} g_1 & 0 & \cdot & \cdot & \cdot & 0 \\ g_2 & g_1 & \cdot & \cdot & \cdot & 0 \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ g_m & g_{m-1} & \cdot & \cdot & \cdot & g_1 \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ g_p & g_{p-1} & \cdot & \cdot & \cdot & g_{p-m+1} \end{bmatrix}$$

The systems dynamic matrix G is made up of m (the control horizon) columns of the systems step response appropriately shifted down in order.
The predicted output with disturbances is,

$$\hat{y}_d = G_d u_d + f_d \tag{7}$$

The cost function to be minimized including control effort is [21],

$$J=\sum_{j=N_1}^{N_2}\delta(j)\left[\hat{y}(t+j/t)-w(t+j)\right]^2+\sum_{j=1}^{N_u}\lambda(j)\left[\Delta u(t+j-1)\right]^2 \tag{8}$$

Without constraints, cost functions is,

$$J = ee^T + \lambda uu^T \tag{9}$$

The solution to this cost function can be obtained by computing the derivative of J and equating it to zero,

$$u=(G^T G+\lambda I)^{-1}G^T(w-f) \tag{10}$$

In DMC control horizon and penalization factor are the tuning parameter. In least square formulation penalization factor is introduced to smoothen the control signal [11].

Standard QP at each sampling instant carries out the optimization. According to receding horizon concept, at time t, only the first input $(\Delta u(t))$ of the vector of future control increment or sequence (u) is actually applied to the plant. The remaining optimal inputs are discarded, and a new optimal control problem is solved at time $t+1$ [22].

When constraints are considered in input and output, the following equation must be added to the minimization [10].

$$\sum_{j=1}^{N} C_{yj}^i \hat{y}(t+j/t) + C_{uj}^i u(t+j-1) + c^i \le 0 \qquad \text{for } i=1,\cdots,N_c \tag{11}$$

While extending method from SISO to multi input multi output (MIMO) plant, the matter is of notations only. For multivariable case, the plant output equation can be written as,

$$y_j(t)=\sum_{k=1}^{n_u}\ \sum_{i=1}^{N_k} g_i^{kj} u^k(t-i) \tag{12}$$

The matrix G is $ny \times nu$ dimension for MIMO plant.

In DMC, the design of control is independent of the transport lag and it deals with constraints. In presence of disturbances, feed-forward can be easily implemented. It is robust to model error but the application is limited to open loop bounded input bounded output (BIBO) stable type of processes [23], [24]. For DMC algorithm to be closed loop stable, a long length of prediction horizon is required [14].

The performance of DMC is poor, for interactive multivariable plants, when ramp-like disturbances acting on plant output and changes in process dynamic becomes large.

In case, open loop time constant is much larger than the desired closed loop time constant then excessive numbers of step response coefficients are required to improve the feedback properties. However, this can be avoided by using general state space model [23], [24].

DMC is particularly successful in petrochemical industries with multivariable processes [14], [24].

3.2 Model Algorithmic Control

Model algorithmic control (MAC), whose software is called identification command (IDCOM), was initially called as Model Predictive Heuristic Control (MPHC). In 1978, Richalet described successful applications of MPHC [25]. It uses impulse response model. This method is very similar to DMC with following differences [4].

- Instead of the step response model involving Δu, an impulse response model involving u is employed. If the input u is penalized in the quadratic objective, then the controller does not remove offset. If the input u is not penalized then extremely awkward procedures are necessary to treat non-minimum phase systems.
- The number of input moves is not used for tuning.
- The disturbance estimate $\hat{x}(t+j/t)$ is filtered.

The multivariable process to be controlled is represented by its impulse responses (constitutes internal model) used on line for prediction. This internal model is kept updated using plant operating data via Identification (Off-line identification is accurate for control purpose and On-line identification procedure used if the changes in the plant are rapid and random, but it is costly and complex) [26].

It introduces a reference trajectory as a first order system, which evolves from the actual output to the set point according to a determined time constant. The behavior of closed loop system is prescribed by the reference trajectory and it controls the aggressiveness of the algorithm. If the reference trajectory is much faster than the process then MPHC will not be efficient. Therefore, time constant of reference trajectory is the major parameter [4].

Controls are computed through an iterative procedure (involving several trials to calculate the best input for minimizing the tracking error without overstressing the actuators and the computational facilities of the process control computer [26], which is heuristic in the general case. Future inputs, when applied to the fast time internal predictive model, they induce outputs as close as possible to the desired reference trajectory [25].

Consider the output of LTI system with truncated impulse response [11],

$$y(t)=\sum_{i=1}^{\infty} h_i u(t-i) \tag{1}$$

The disturbance term is represented by

$$x(t)=\frac{C(z^{-1})}{\Delta}e(t) \tag{2}$$

Using this model a j-step ahead predictor can be written as

$$\hat{y}(t+j/t)=\sum_{i=1}^{\infty} h_i u(t+j-i)+\hat{x}(t+j/t) \tag{3}$$

Prediction without disturbance is,

$$\hat{y}(t+j/t) = \sum_{i=1}^{N} h_i u(t+j-i)$$
$$= h^T u(t+j) = u^T(t+j)h \tag{4}$$

At each sampling instant t, the sequence of operation is executed in two parts,
1) A model reference trajectory $w(t)$ of the first order form is computed over the prediction horizon N :

$$w(t+j) = \alpha w(t+j-1) + (1-\alpha) r(t+j) \qquad \text{for } j = 1,2,\cdots,N \tag{5}$$

2) The prediction over the prediction horizon is computed, assuming
a) No future input changes i. e.

$$\Delta u(t+j-1) = u(t+j-1) - u(t-1) = 0 \qquad \text{for } j = 1,2,\cdots,N \tag{6}$$
$$\hat{y}(t) = h^T u(t)$$

To deal with unknown load disturbances the predicted value is adjusted to the measured actual output $y(t)$ of the system.

$$y = \begin{bmatrix} \hat{y}(t+1) \\ \hat{y}(t+2) \\ . \\ . \\ \hat{y}(t+N) \end{bmatrix}$$

with

$$y(t+j/t) = \hat{y}(t+j/t) + (y_m(t) - \hat{y}(t/t)) \tag{7}$$

b) Changes in the input variable over the prediction horizon,

$$u(t+j-1) = \Delta u(t+j-1) - u(t-1) \qquad \text{for } j = 1,2,\cdots,N \tag{8}$$

The predictor can be written as

$$y = Hu + f \tag{9}$$

Control law of MAC (without constraint) is,

$$u = (H^T H + \lambda I)^{-1} H^T (w - f) \tag{10}$$

Where

$$H = \begin{bmatrix} h_1 & 0 & . & . & 0 \\ h_2 & h_1 & . & . & 0 \\ . & . & . & . & . \\ . & . & . & . & . \\ h_M & h_{M-1} & . & . & h_1 \end{bmatrix} \tag{11}$$

This law (equation 10) is simpler, compared to other formulations. It makes no use of control horizon concept [4].

The prediction horizon, penalization factor (λ) and α are the tuning parameters. α is selected as the main tuning parameter. $\alpha = 0$ gives faster control while λ is adjusted for smooth control signal.

In MAC, impulse response allows the enhancement of the robustness against identification errors and parameter perturbations effect of modeling error is less compared to state vector techniques. However, MAC is applicable for open loop stable processes.

The MAC is in use in Power plants, Glass furnaces, Steam generator, distillation column in an oil refinery and complete PVC plant etc. [25].

3.3 Predictive Functional Control

The principles of predictive functional control (PFC) were established in 1968 and the first applications took place in the early 70's.

Richalet at ADERSA Company developed it in late 80's for the application of fast processes. PFC can use any model, however due to its robustness characteristics, state space models are used most of the time [27] and allows for non-linear and unstable linear internal models.

However, use of State space method cannot meet the requirement of the practical control problem. Obtaining precise mathematical model of is very difficult, especially for non-linear 'uncertain' time-delay and time-varying processes.

PFC deals with the quick tracking control problems and an effective control method for rapid processes [28]. Flexibility and efficiency are appreciated because of decomposition principle. The PFC algorithm requires an online optimizing method. A Quadratic Performance (QP) index may be adopted in PFC.

The coincidence points and basis function are the two characteristics of PFC [4]. The coincidence point is used to simplify the calculation by considering only subset of points in the prediction horizon. The desired and the predicted future outputs are required to coincide only at the subset of points in the prediction horizon and not in the whole prediction horizon.

The selection of basis functions depends on the characteristics of process and the desired set point. This specifies the input profile over a long horizon using small number of parameters.

Consider the state space model [4].

$$x(t) = Ax(t-1) + Bu(t-1)$$
$$y(t) = Cx(t) \tag{1}$$

The prediction is obtained by adding auto compensation term,

$$\hat{y}(t+j/t) = y(t+j) + \hat{e}(t+j/t) \tag{2}$$

The future control signal is structured as a linear combination of the basis functions,

$$u(t+j) = \sum_{i=1}^{N_B} \mu_i(t) B_i(j) \tag{3}$$

The cost function to be minimized is [3],

$$J = \sum_{i=1}^{N_H} \left[\hat{y}(t+h_i) - w(t+h_i)\right]^2 \tag{4}$$

h_i = total number of the coincidence point (This nomenclature is limited for PFC algorithm only).

$$w(t+j) = r(t+j) - \alpha^j (r(t) - y(t)) \tag{5}$$

A quadratic factor of the form $\lambda[\Delta u(j)]^2$ can be added to the cost function in order to get a smooth control signal [29].

Prediction model output of PFC includes two parts: free output and forced output. Free output depends on past input and output but not on current and future input. The forced output is the response to the input from current time [30].

The output of prediction model is,

$$y(t+j) = CA^j x(t) + \sum_{i=1}^{N_B} y_{B_i}(j) \mu_i(t) \tag{6}$$

Future control action can be obtained by minimizing the sum of squares between the predicted output and reference trajectory at coincidence points.

$$J = \sum_{i=1}^{N_H} \left[y_B(h_i)\mu - d(t+h_i) \right]^2 \tag{7}$$

The control signal is given by

$$u(t) = \sum_{i=1}^{N_B} \mu_i(t) B_i(0) \tag{8}$$

Only the first value of control signal sequence is executed. This algorithm can only be used for stable models [31].

PFC uses time constant of the reference trajectory as the main tuning parameter. Smaller time constants demand more aggressive control, while larger time constants result in less aggressive action. It improves the relative stability of the closed-loop system, Means the controller will be able to tolerate model mismatch, which a pure set point cannot tolerate [28].

The PFC algorithms provide an additional level of flexibility by allowing both linear & quadratic terms in the steady state objective function. They also include hard input & output constraints [14].

An analytical approach is used for handling input constraints. While output and state constraints are handled by logical approach [31].

It is robust to modeling errors, over and under parameterization and it overcome the unclear control input law, which exists in other MPC [32], [33]. Simplicity of tuning and ease of maintenance are the advantages of PFC [27].

It would be out of work when the model changed greatly. Fixed model PFC cannot guarantee good quality when applied to serious time varying plant [32], [33].

It is used in processes like nonage robot, rocket, object dogging, reactor and heater, mechanical servo application etc. [28]. It is also used in Steel and aluminum industry, Defense Automotive etc.

3.4 Extended Prediction Self-Adaptive Control

De Keyser and Van Cauwenberghe developed extended prediction self-adaptive control (EPSAC) in 1985 [3]. It uses a discrete (z-transform) transfer function to model the process and proposes a constant control signal starting from the present moment while using a sub-optimal predictor instead of solving a Diophantine equation [4], [11], [34].

For the prediction, process is modeled by the T.F with disturbance:

$$A(z^{-1})y(t) = B(z^{-1})u(t-d) + x(t) \tag{1}$$

$$x(t) = C(z^{-1})e(t)$$

The parameter vector is estimated by means of the recursive (extended) least squares method:

$$\Delta y(t) = \phi^T(t).\hat{\theta}(t) + \eta(t) \tag{2}$$

Then the one-step ahead predictor is,

$$\Delta \hat{y}(t+1/t) = \hat{y}(t+1/t) - y(t) \tag{3}$$

For the j step ahead prediction ($j = 2,3,\cdots,N$), the process output (without disturbances) is predicted by means of the multi-step predictor:

$$A(z^{-1})[\hat{y}(t+j/t) - \hat{y}(t+j-1/t) = B(z^{-1})\Delta u(t+j-d) \tag{4}$$

While computing the control action, the fact is that the prediction values depend on the postulated future control policy.
Considering,

$$\Delta u(t+j) = 0 \qquad \text{for} \quad j > 0$$

The control signal value is obtained to minimize the cost function:

$$J = \sum_{j=d}^{N} \delta(j)\left[w(t+j) - P(z^{-1})\hat{y}(t+j/t)\right]^2 \tag{5}$$

$P(z^{-1})$ = design polynomial with unit static gain
The control law that minimizes the cost is,

$$u(t) = \frac{\sum_{j=d}^{N} h_j \delta(j)[w(t+j) - P(z^{-1})\hat{y}(t+j/t)]}{\sum_{j=d}^{N} \delta(j) h_j^2} \tag{6}$$

This shows that control law structure is very simple and the calculation is reduced to one single value $u(t)$.

The possible tuning parameters are prediction horizon, the weighting factor and the filter polynomial. However, on-line tuning of the prediction horizon influence the structure of the multi-step predictor and control structure.

The open loop zeros appear also as closed-loop zeros. Because the process zeros are not cancelled, the long-range predictive control strategy can tackle non-minimum phase processes. A constant set point w is tracked without error [11], [34].

3.5 Extended Horizon Adaptive Control

B. E. Ydstie developed extended horizon adaptive control (EHAC) in 1984. It uses a parametric process model. This approach is called EHAC because it allows a longer time to drive the process output to its desired value instead of using a fixed delay time and short interval [4], [35]. EHAC leads to a multi-step policy since the final output depends on current input as well as on the inputs that are implemented in the future.

The process modeled by T.F without a disturbance model is [4], [11],

$$A(z^{-1})y(t) = B(z^{-1})u(t-d) \tag{1}$$

The basic idea of EHAC is to compute, a sequence of inputs at each sampling instant $[u(t), u(t+1), \cdots, u(t+N-d)]$, to minimize the discrepancy between the model and the reference at instant $(t+N)$:

$$\hat{y}(t+N/t) - w(t+N) = 0 \qquad \text{with} \quad N \geq d \tag{2}$$

Another strategy is to assume the constant control $u(t) = u(t+1) = \cdots = u(t+N-d)$, over the interval, $[t, t+N-d]$ or to minimize the control effort i.e. to compute $u(t) \cdots u(t+N-d)$, to minimize the cost,

$$J = \sum_{j=0}^{N-d} u^2(t+j) \qquad \text{for} \quad N \geq d \tag{3}$$

The cost function, which dealt with the disturbances in the load [11] is,

$$J = \sum_{j=0}^{N-d} \Delta u^2(t+j) \tag{4}$$

Where

$\Delta u(t+j) = u(t+j) - u(t+j-1)$

The incremental N step ahead predictor is,

$$\hat{y}(t+N/t) = y(t) + F(z^{-1})\Delta y(t) + E(z^{-1})B(z^{-1})\Delta u(t+N-d) \quad (5)$$

The first element of control signal that minimizes the cost index (4) is,

$$u(t) = u(t-1) + \frac{\alpha_0(w(t+N) - \hat{y}(t+N/t)}{\sum_{j=0}^{N-d} \alpha_j^2} \quad (6)$$

α_j is the coefficient of $\Delta u(t+j)$ in the prediction equation.

Thus, the control law only depends on the process parameter and only tuning parameter in the EHAC method is the prediction horizon N. In EHAC, only the deviation at one moment in the future is taken into account hence it is easy to implement. With proper choice of time horizon, certain non-minimum systems can be stabilized.

However, for $\alpha_0 = 0$, no control is obtained. Certain frequencies cannot be eliminated because it is not possible to ponder the control effort at each point and extension of control horizon will result in slower control [4].

The use of EHAC is extended to multivariable open loop systems in 1986. The algorithm does not require knowledge of system interactor matrix and it tolerates the effect of output disturbances [36]. However, multivariable controllers based on a one-step ahead criterion are sensitive to the choice of delay structure. The cyclic behavior is avoided using receding horizon control [37].

3.6 Generalized Predictive Control

Generalized predictive control (GPC) is one of the most popular predictive control algorithms developed by D. W. Clarke in 1987 [38].

GPC retains the design flexibility and performance of GMV/PP technique. It also caters for offsets (since it uses integrated controlled auto regressive moving average (CARIMA) model), feed-forward signals, and multivariable plant without detailed prior knowledge of structural indices. The main difference between GPC and DMC is the model used to describe the plant and the formulation of the dynamic matrix.

For satisfying the control objectives, it makes the use of a CARIMA model and various horizons. This model is more appropriate in industrial applications where disturbances are non-stationary.

A CARIMA model is used to obtain good output predictions and optimize a sequence of future control signals to minimize a multistage cost function defined over a prediction horizon. The inclusion of disturbance is necessary to deduce the correct controller structure.

$$A(z^{-1})y(t) = B(z^{-1})u(t-1) + C(z^{-1})\frac{e(t)}{\Delta} \quad (1)$$

The derivation of optimal prediction can be obtained by recursion of Diophantine equation [4].

In GPC, the predictions are posed in terms of increments in control $\{\Delta u(j); j \geq t\}$. These assumptions are the cornerstone of the GPC approach [39].

The best prediction of $y(t+j)$ is,

$$\hat{y}(t+j/t) = G_j(z^{-1})\Delta u(t+j-1) + F_j(z^{-1})y(t) \quad (2)$$

The predicted output depends on previous values of output and previous and future values of the control signal.

The prediction of $y(t+j)$ with future error is:

$$\hat{y}(t+j/t) = G_j(z^{-1})\Delta u(t+j-1) + F_j(z^{-1})y(t) + E_j(z^{-1})e(t+j) \tag{3}$$

Where

$G_j(z^{-1}) = E_j(z^{-1})B(z^{-1})$

$G_j(z^{-1}) = B_j(z^{-1})[1 - z^{-j}F(z^{-1})]/A(z^{-1})\Delta$

Polynomials E_{j+1} and F_{j+1} are obtained from the values of E_j and F_j for the range of j 's.

The polynomial $E_{j+1}(z^{-1})$ is given by,

$$E_{j+1}(z^{-1}) = E_j(z^{-1}) + e_{j+1,j}\, z^{-j} \tag{4}$$

and

$$G_{j+1}(z^{-1}) = B(z^{-1})E_{j+1}(z^{-1}) \tag{5}$$

Where

$e_{j+1,j} = f_{j,0}$

$f_{j+1,j} = f_{j,j+1} - \tilde{a}_{j+1}e_{j+1,j}$

$\tilde{a}_{j+1}$ is the $(j+1)^{\text{th}}$ coefficient of the polynomial.

To generate a set of predicted outputs $\hat{y}(t+j/t)$, the prediction model equation (3) is used. The value of $\hat{y}(t+j/t)$ for $j > t$ depends on future control signals $u(t+j)$. These control signals are used to achieve the objective in GPC by minimizing the cost function given as:

$$J(N_1, N_2, N_u) = \sum_{j=N_1}^{N_2} \delta(j)[\hat{y}(t+j/t) - w(t+j)]^2 + \sum_{j=1}^{N_u} \lambda(j)[\Delta u(t+j-1)]^2 \tag{6}$$

Assuming, in open loop, the control signal is calculated ignoring the future noise sequence. Then the optimal prediction can be:

$$\hat{y}(t+N/t) = G_N\Delta u(t) + F_N y(t) \tag{7}$$

This equation can be written as:

$$y = Gu(t) + F(z^{-1})y(t) + G'(z^{-1})\Delta u(t-1) \tag{8}$$

The last two terms depend on the past. The equation (8) can be written as,

$$y = Gu + f \tag{9}$$

The expected output sequence is equal to the first column of matrix G. If the plant dead time, $d > 1$, the first $d-1$ rows of G will be null. When $N_1 = d$, the leading element is non-zero. The expected cost function can be written as:

$$J_1 = \{J(1,N)\} = \{(y-w)^T(y-w) + \lambda u^T u\} \tag{10}$$

$$J_1 = \{(Gu+f-w)^T(Gu+f-w) + \lambda u^T u\} \tag{11}$$

$$J = \frac{1}{2}u^T Hu + b^T u + f_0 \tag{12}$$

Where

$H = 2(G^T G + \lambda I)$

$b^T = 2(f-w)^T G$

$f_0 = (f-w)^T(f-w)$

For no constraints, the future control for minimization of cost is,

$$u = -H^{-1}b = (G^T G + \lambda I)^{-1} G^T (w - f) \tag{13}$$

The first element of the control signal u is,

$$\Delta u(t) = K(w - f) \tag{14}$$

Where

K is the first row of matrix $(G^T G + \lambda I)^{-1} G^T$

The current control is,

$$u(t) = u(t-1) + K(w - f) \tag{15}$$

For $w - f = 0$, there is no control move.

From the Diophantine equation,

$$\begin{aligned} F_j(z^{-1})y(t) &= (1 - F_j'(z^{-1})\Delta)y(t) \\ &= y(t) - F_j'\Delta y(t) \end{aligned} \tag{16}$$

If $y(t)$ is constant so that $\Delta y(t) = 0$ then $F_j(z^{-1})y(t)$ reduces to constant. Equation (15) and (16) ensures offset free behavior by integral action.

GPC depends on the integration of assumption of a CARIMA plant model, use of LRPC, recursion of Diophantine equation, consideration of weighting of control increments in cost function and the choice of a control horizon [38].

GPC is applicable to non-minimum phase, open loop unstable and having variable dead time. It is capable of considering both constant and varying future set points. It is unaffected (unlike pole-placement strategies) if the plant model is over parameterized. However, GPC has limitations with minimum phase processes for some of the most obvious choices of its design parameters [40].

GPC shows better performance in cement mill, a spray-drying tower and compliant robot arms [39].

4. Conclusion

From the review of the above methods of MPC, we observed that,

1) DMC uses step response model for open loop BIBO stable processes. It is robust and dealt with constraints however; performance is poor for ramp like disturbances and interactive multivariable plants. Control horizon and penalization factors are the tuning parameters.

2) MAC is easy to implement and better for multivariable but for stable processes only. It uses impulse response model. It is robust against identification errors and parameter perturbations. It limits its use for slow reference trajectory. The major tuning parameter is α

3) PFC uses state space model, operates fast processes and provides robustness to modeling errors, over and under parameterization. It is simple, requires less calculation and gives high control precision. It has ability to handle constraint. Application limits to stable models only. The performance is obtained by tuning time constant of the reference trajectory.

4) EPSAC with discrete transfer function has simple control law. It automatically includes feed forward action and handles measurable disturbances. It tracks constant set point and tackles non-minimum phase systems. On-line tuning of the prediction horizon influence the structure of the multi-step predictor and control structure. The possible tuning parameters are prediction horizon, the weighting factor and the filter polynomial.

5) Transfer function model is used in EHAC. It is easy to implement since prediction horizon is the only tuning parameter. It provides stability for certain non-minimum phase

system. It operates multivariable but open loop systems also performs slowly for higher control horizon.

6) Using CARIMA model GPC provides offset free response. It has wide application area compared with other methods. It tracks both varying and constant future set points. Nevertheless, for prediction, recursion of Diophantine equation is necessary. Proper choice of prediction and control horizon, weighting and penalization factor gives satisfactory performance.

The GPC is most easy and suitable still there is good scope to improve its performance and increase the use of it.

Nomenclatures

$y(t)$ = plant output
$u(t)$ = plant input
g_i = step response matrix coefficient
i = truncation order
$\hat{y}$ = vector of system predictions
G = systems dynamic matrix
u = vector of control increments
$\hat{y}(t+j/t)$ = predicted value of output
$\hat{y}_d$ = Predicted disturbances
f_d = Part of response that does not depend on disturbance
J = cost function
e = vector of future error
$\lambda(j)$ = control weighted coefficients
w = reference trajectory
G_d = Matrix similar to G containing coefficients of the system response
u_d = Vector of disturbance increments
N_1 = minimum costing horizon
N_2 = maximum costing horizon
N_u = control horizon
$N = N_1 - N_2$ = Prediction horizon
α_j = coefficient of $\Delta u(t+j)$ in the prediction equation.
T_r = Time constant of reference trajectory,
r = set point
μ_i = coefficients of base functions linear combination
B_i = basis function
y_{B_i} = system response to the basis function B_i
N_B = number of base function
h_i (for other than PFC method) = coefficient of impulse response
$\phi(t)$ = measurement vector
$\theta(t)$ = parameter vector
$P(z^{-1})$ = design polynomial with unit static gain and

$\delta(j)$ = error weighted coefficients

h_j = discrete time impulse response

d = dead time

N_c = Number of constraints,

$C_{yi}^{j}, C_{ui}^{j}, c^{j}$ = constant matrices.

$\Delta = 1 - z^{-1}$ is the backward shift operator.

A, B, C are the polynomials in backward shift operator z^{-1}

g_i^{kj} = response of output to j to step in input k

$\alpha = \exp(-T / T_r)$,

T = Sampling period

$y_m(t)$ = measured value of output

$\hat{y}(t/t)$ = output value estimated by the model.

$\hat{x}(t + j/t)$ is the disturbance at instant t along the horizon.

$r(t + j)$ = future set point

$w(t + j)$ = first order approach to the known reference

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