

GP-based Model Predictive Control*

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Abstract—Describe topic and relevance in a few sentences so that the reader is motivated to read the whole paper.

I. INTRODUCTION

Model predictive control (MPC) is a popular control strategy which uses a dynamic plant model to obtain the control input that optimizes future reactions of the plant [3]. The performance of MPC depends highly on how well the model captures the dynamics of the plant [2]. But the identification of such an *a priori* model can be challenging and the dynamics of the plant could also change during the application [2], [6]. Therefore, a simple and fixed nominal model of the plant can be used in combination with a learned disturbance model. The disturbance model represents the error between the observed behaviour of the plant and the behaviour of the nominal model [6]. It can be modelled as a Gaussian Process (GP) regression which is a probabilistic, non-parametric model [3]. GPs have the advantage of characterizing the prediction uncertainties [3]. The mean estimate of a GP can also be used to model the full dynamics of the plant and not only the model error. This approach was applied to a cart pole swing-up environment and an autonomous racing task in [7]. To reduce high computational costs they choose sparse spectrum GPs. Kocijan et al. [3] use an offline-identified GP model instead. Another alternative is the generation of local GPs (LGPs) where for each subspace of the GP input space different GPs are identified. While Nguyen-Tuong et al. [5] and Meier et al. [4] identify many LGPs, Ostafew et al. [6] compute one single LGP based on data within a sliding window. Other applications of GPs in a MPC framework are .

In this report we present the results of our project within the course “Statistical Learning and Stochastic Control”. First, our literature research on GP-based MPC is summarized. Then a short introduction to the theory of MPC and GPs is given. Later, two examples which’s implementation was part of the project are explained.

Introduce topic and describe motivation and relevance of problem/topic.

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In this paper we give an introduction to the results presented in paper(s) [1]. We present the main results, discuss ideas and illustrate the results with simulations.

Notation. Define notation.

II. BACKGROUND

In this section a revision of GPs for our application and necessary background information on MPC are presented. The revision of GPs follows Kabzan et al. [2] as well as Rasmussen and Williams [8]

As described in the Introduction a GP is used to identify the disturbance d_{true} which describes the error between the nominal plant dynamics f_{nom} and the true dynamics f_{true} . Thus, the true system equations are given by

$$\begin{aligned} x_{k+1} &= f_{true}(x_k, u_k) \\ &= f_{nom}(x_k, u_k) + B_d(d_{true}(z_k) + w), \end{aligned} \quad (1)$$

with $z_k = B_z x_k$ and $w \sim \mathcal{N}(0, \Sigma_w)$, where $\Sigma_w = \text{diag}(\sigma_1^2, \dots, \sigma_{n_d}^2)$ the gaussian measurement noise. The disturbance $d = d_{true}(z_k) + w_k$ is identified using input and output data pairs $(z_k, y_k = d_{true}(z_k) + w_k)$ which are saved in a dictionary \mathcal{D} of length m

$$\begin{aligned} \mathcal{D} &= \{Z = [z_1, \dots, z_m] \in \mathbb{R}^{n_z \times m}, \\ &Y = [y_1, \dots, y_m] \in \mathbb{R}^{n_d \times m}\}. \end{aligned} \quad (2)$$

If $m > m_{max}$, the dictionary is updated by removing the oldest data pair.

Each output dimension $a \in \{1, \dots, n_d\}$ is treated as a different GP with the kernel k^a , which results in the posterior distribution mean $\mu^a(z)$ and variance $\Sigma^a(z)$

$$\mu^a(z) = k_{zz}^a (K_{ZZ}^a + I\sigma_a)^{-1} [Y]_{.,i}, \quad (3)$$

$$\Sigma^a(z) = k_{zz}^a - k_{zZ}^a (K_{ZZ}^a + I\sigma_a)^{-1} k_{Zz}^a \quad (4)$$

in dimension a for a test point z . The expressions k_{zz}^a , k_{zZ}^a , k_{Zz}^a and K_{ZZ}^a are compact notations for $k^a(z, z) \in \mathbb{R}$, $[k_{zZ}^a]_j = k^a(z, z_j)$, $[k_{Zz}^a]_j = k^a(z_j, z)$ and $[K_{ZZ}^a]_{ij} = k^a(z_i, z_j)$, respectively. It holds $k_{Zz}^a = (k_{zZ}^a)^T \in \mathbb{R}^m$.

For each output dimension a we make use of the squared exponential kernel given by

$$k^a(z, \bar{z}) = \sigma_{f,a}^2 \exp(-0.5(z - \bar{z})^T M^{-1}(z - \bar{z})), \quad (5)$$

where $\sigma_{f,a}^2$ and M are the squared output variance and the positive diagonal length scale matrix, respectively. The practical implementation of Gaussian process regression from page 19 of Rasmussen and Williams [8] which uses the

Cholesky factorization to address the matrix inversion is applied. This results in the following algorithm

$$L^a = \text{cholesky}(K(Z, Z) + \sigma_n(a, a)^2 I) \quad (6)$$

$$\alpha^a = L^T \backslash (L \backslash Y(:, a)), \quad (7)$$

where $[K(Z, Z)]_{ij} = k(z_i, z_j)$ with $z_i, z_j \in Z$. Thus, the predictive mean and variance are given by

$$\mu_y^a = K(z, Z)^T \alpha^a \quad (8)$$

$$\Sigma_y^a = k(z, z) - v^T v, \quad (9)$$

with $v = L/K(z, Z)$.

The model considered for control including the identified disturbance d_{GP} is given by

$$x_{k+1} = f_{nom}(x_k, u_k) + B_d d(z_k). \quad (10)$$

Model predictive control (MPC), receding horizon control or moving horizon control are all names for a control strategy which predicts the future dynamic behaviour within a finite prediction horizon and chooses the control input such that a performance functional is minimized (Allgöwer). Since the predicted behaviour is not equal to the system behaviour due to disturbances and model-plant mismatch, only the first input of the computed control input sequence is applied. Using the new measurement one sampling time later, the procedure is repeated to find a new control sequence within the receding horizon.

The mathematical formulation is summarized below.

III. MAIN RESULTS

Ideas, theorems, proofs and discussions

IV. EXAMPLES

Show and discuss simulation examples etc....

V. CONCLUSIONS

Summarize the main points (with more details than in the preceding introduction). The paper should not be between 4 and 8 pages.

APPENDIX

Appendixes should appear before the acknowledgment.

ACKNOWLEDGMENT

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