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# Recommendation systems

"what else to know about?"

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# Recommendation Systems

**Collaborative Filtering &  
Content-Based Recommendations**



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# Scarcity versus Abundance

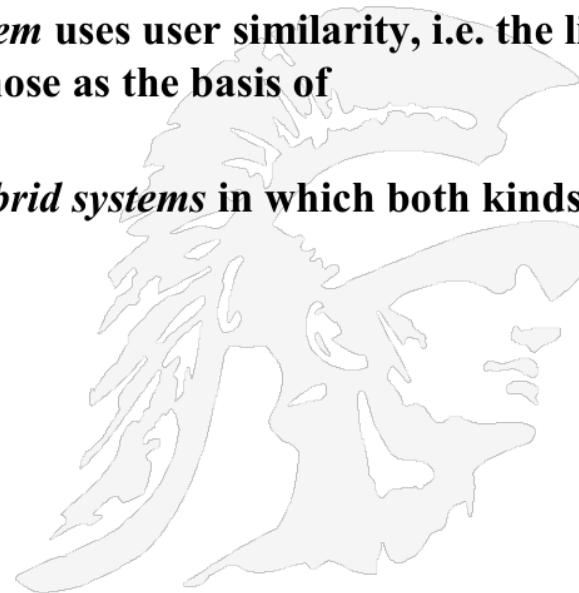
- Since online systems maintain large quantities of goods, systems that provide recommendations serve an important purpose
  - In some cases items sold from the long tail, (i.e. those not particularly popular) can cumulatively outweigh the initial portion of the graph, an in effect produce the majority of sales
- Recommendation systems are expected to increase diversity because they help us discover new products.
  - However, some algorithms may unintentionally do the opposite.
  - Because they recommend products based on past sales or ratings, they cannot usually recommend products with limited historical data. This can create a rich-get-richer effect for popular products
  - This bias toward popularity can prevent what are otherwise better consumer-product matches.
    - Several collaborative filtering algorithms have been developed to promote diversity and the long tail by recommending novel, unexpected, and serendipitous items
  - See [https://en.wikipedia.org/wiki/Collaborative\\_filtering](https://en.wikipedia.org/wiki/Collaborative_filtering)

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## Two Types of Recommendation Systems

- A *recommendation system* is any system which provides a recommendation/prediction/opinion to a user on items
- 1. A classic *content-based filtering system* uses item similarity/clustering to recommend items like ones you like
- 2. A classic *collaborative filtering system* uses user similarity, i.e. the links between users and the item they chose as the basis of recommendations
- Commonly many companies use *hybrid systems* in which both kinds of techniques are employed



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## Difference between Collaborative and Content-based Filtering-An Example

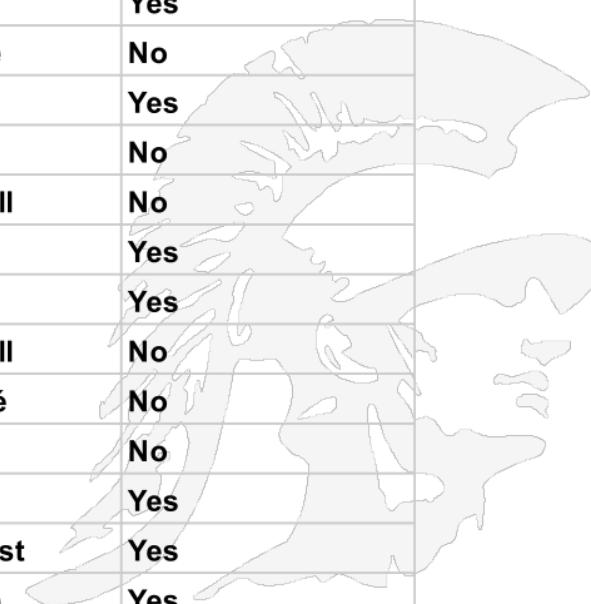
- *Here are two early systems that recommended music*
- *Last.fm* creates a "station" of recommended songs by observing what bands and individual tracks **the user** has listened to on a regular basis (*user similarity*) and comparing those against the listening behavior of other users.
  - Last.fm will play tracks that do not appear in the user's library, but are often played by other users with similar interests.
  - As this approach leverages the behavior of users, it is an example of a **collaborative filtering technique**.
- *Pandora* uses the **properties of a song** or artist (a subset of the 400 attributes provided by the Music Genome Project) to seed a "station" that plays music with similar properties (*item similarity*).
  - User feedback is used to refine the station's results, deemphasizing certain attributes when a user "dislikes" a particular song and emphasizing other attributes when a user "likes" a song.
  - This is an example of a **content-based technique**.

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# Input

|       |              |     |
|-------|--------------|-----|
| Alice | Il Fornaio   | Yes |
| Bob   | Ming's       | No  |
| Cindy | Straits Café | No  |
| Dave  | Ming's       | Yes |
| Alice | Straits Café | No  |
| Estie | Zao          | Yes |
| Cindy | Zao          | No  |
| Dave  | Brahma Bull  | No  |
| Dave  | Zao          | Yes |
| Estie | Ming's       | Yes |
| Fred  | Brahma Bull  | No  |
| Alice | Mango Café   | No  |
| Fred  | Ramona's     | No  |
| Dave  | Homma's      | Yes |
| Bob   | Higashi West | Yes |
| Estie | Straits Café | Yes |



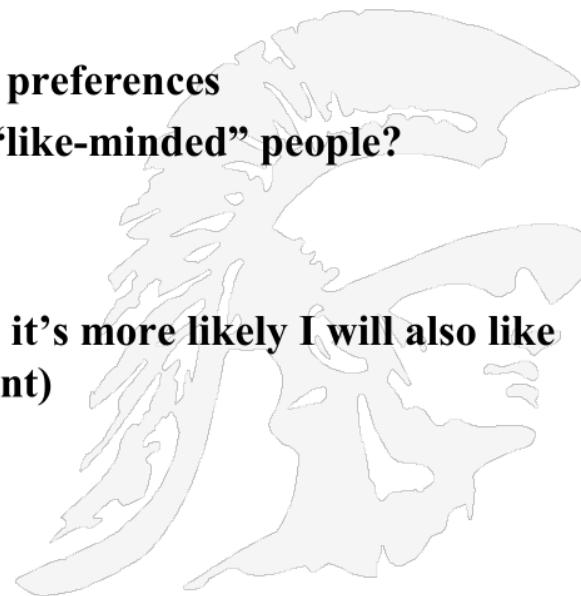
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## Algorithm 0

- **Strategy: Recommend to you the most popular restaurants**
  - say # positive votes minus # negative votes
- **But this ignores**
  - your culinary preferences
  - judgments of those with similar preferences
- **How can we exploit the wisdom of “like-minded” people?**
- **Basic assumption**
  - Preferences are not random
  - Assumption: if I like Il Fornaio, it's more likely I will also like Cenzo (another Italian restaurant)



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## Cast the Input as a Matrix

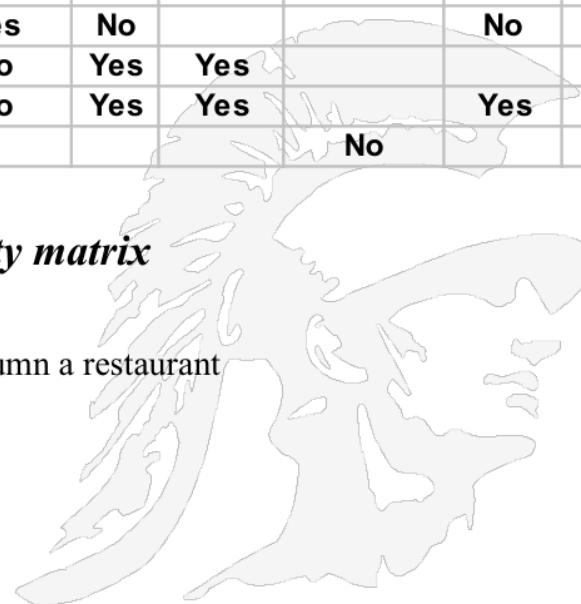
|       | Brahma Bull | Higashi West | Mango | Il Fornaio | Zao | Ming's | Ramona's | Straits | Homma's |
|-------|-------------|--------------|-------|------------|-----|--------|----------|---------|---------|
| Alice |             | Yes          | No    | Yes        |     |        |          | No      |         |
| Bob   |             | Yes          |       |            |     | No     |          | No      |         |
| Cindy |             |              |       | Yes        | No  |        |          | No      |         |
| Dave  | No          |              |       | No         | Yes | Yes    |          |         | Yes     |
| Estie |             |              |       | No         | Yes | Yes    |          | Yes     |         |
| Fred  | No          |              |       |            |     |        | No       |         |         |

Called a *utility matrix*

Each row represents an individual and each column a restaurant

In this example, entries are either yes/no;

In the more general case they can be any value

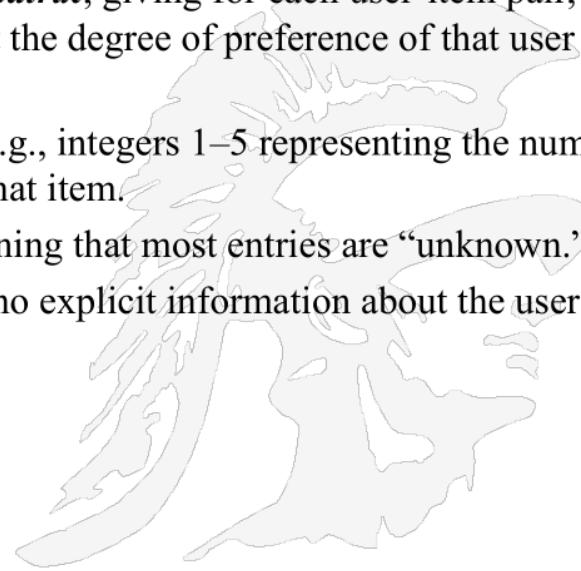


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# The Utility Matrix

- In a recommendation-system application there are two classes of entities, which we shall refer to as *users* and *items*
  - Users have preferences for certain items, and these preferences must be teased out of the data.
- The data itself is represented as a ***utility matrix***, giving for each user-item pair, a value that represents what is known about the degree of preference of that user for that item.
- *Values might come from an ordered set*, e.g., integers 1–5 representing the number of stars that the user gave as a rating for that item.
- We assume that the matrix is *sparse*, meaning that most entries are “unknown.”
- An unknown rating implies that we have no explicit information about the user’s preference for the item.



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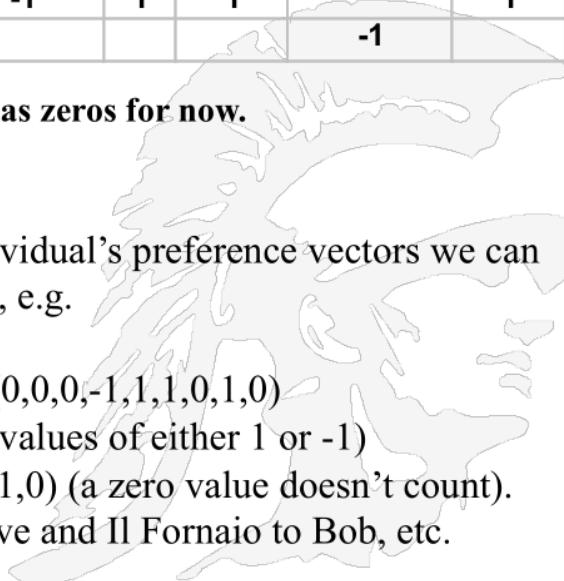


## Now That We Have a Matrix

|       | Brahma Bull | Higashi West | Mango | Il Fornaio | Zao | Ming's | Ramona's | Straits | Homma's |
|-------|-------------|--------------|-------|------------|-----|--------|----------|---------|---------|
| Alice |             | 1            | -1    | 1          |     |        |          | -1      |         |
| Bob   |             | 1            |       |            |     | -1     |          | -1      |         |
| Cindy |             |              |       | 1          | -1  |        |          | -1      |         |
| Dave  | -1          |              |       | -1         | 1   | 1      |          |         | 1       |
| Estie |             |              |       | -1         | 1   | 1      |          | 1       |         |
| Fred  | -1          |              |       |            |     |        | -1       |         |         |

View all other entries as zeros for now.

- To compute the similarity between individual's preference vectors we can use inner products as a good place to start, e.g.
  - Dave has similarity 3 with Estie,
    - e.g.  $(-1,0,0,-1,1,1,0,0,1)$  and  $(0,0,0,-1,1,1,0,1,0)$
    - (i.e. there are three matching values of either 1 or -1)
    - but -2 with Cindy  $(0,0,0,1,-1,0,0,-1,0)$  (a zero value doesn't count).
  - Perhaps recommend Straits Cafe to Dave and Il Fornaio to Bob, etc.



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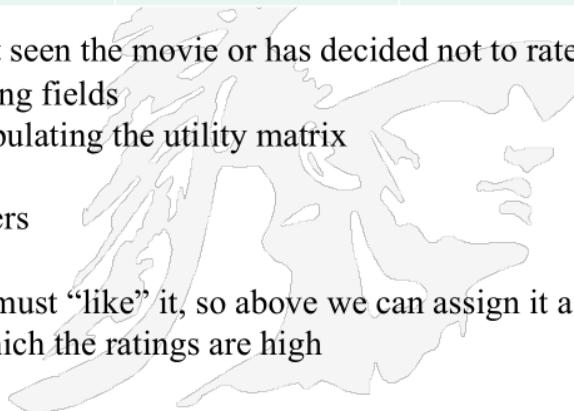
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## Another Utility Matrix Example - Movies

|       | Avatar | LOTR | MATRIX | PIRATES |
|-------|--------|------|--------|---------|
| ALICE | 1      |      | 0.2    |         |
| BOB   |        | 0.5  |        | 0.3     |
| CAROL | 0.2    |      | 1      |         |
| DAVID |        |      |        | 0.4     |

- The blank spaces indicate either the user has not seen the movie or has decided not to rate it
- Main issue: how to fill in the values in the missing fields
- In general there are two basic techniques for populating the utility matrix
  - Ask users to rate items  
E.g. movies, online stores from purchasers
  - Make inferences from user behaviors  
Assumption: Users who watch a movie must “like” it, so above we can assign it a 1;  
We are mostly interested in fields for which the ratings are high



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## Recommending documents can be Viewed as a Form of Recommendation System

- If the items we are considering are documents, then the profile will be the set of “important” words in the document
- How do we pick important words
  - We use the TF-IDF formulation seen earlier

Profile of a document  $d_j$  is the vector of weights  $w_{i,j}$ .  $Content(d_j) = (w_{1,j}, \dots, w_{k,j})$ .

$$w_{i,j} = TF_{i,j} \times IDF_i \quad TF_{i,j} = \frac{f_{i,j}}{\max_z f_{z,j}}, \quad IDF_i = \log \frac{N}{n_i}.$$

- **TF : Term Frequency, IDF : Inverse Document Frequency**
- **N : Number of the documents**
- **$n_i$  : How many times an element is seen in all of the documents**
- **$f_{i,j}$  : Number of times an element is seen in the document  $d_j$**

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## Example 1 Boolean Utility Matrix

- Items are movies, only feature is Actor
  - Item profile: vector with *0* or *1* for each actor
- Suppose user *X* has watched 5 movies
  - 2 movies featuring actor A (movies 1 and 3)
  - 3 movies featuring actor B (movies 2, 4, and 5)
- User profile = mean of item profiles
  - Feature A's weight =  $2/5 = 0.4$
  - Feature B's weight =  $3/5 = 0.6$

|        | ActA | ActB | ActC | ActD | ActE |
|--------|------|------|------|------|------|
| Movie1 | 1    | 0    | 0    | 0    | 0    |
| Movie2 | 0    | 1    | 0    | 0    | 0    |
| Movie3 | 1    | 0    | 0    | 0    | 0    |
| Movie4 | 0    | 1    | 0    | 0    | 0    |
| Movie5 | 0    | 1    | 0    | 0    | 0    |

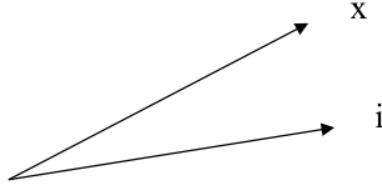
$$\text{ActA's weight} = \text{Sum(ActA)}/5$$

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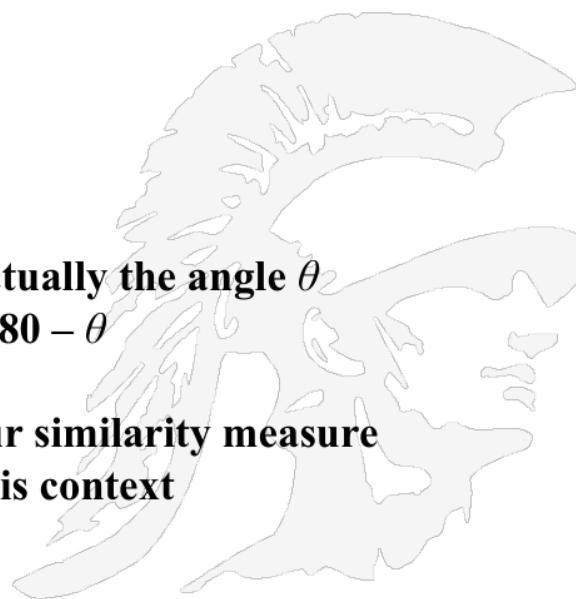


## Making Predictions

- Given user profile  $x$  (movies he/she watched) and item profile  $i$  (*movies with actor profiles*)
- Estimate the similarity of  $U(x,i) = \cos(\theta) = (x \cdot i) / (|x| |i|)$



- Technically the cosine distance is actually the angle  $\theta$  and the cosine similarity is the angle  $180 - \theta$
- For convenience we use  $\cos(\theta)$  as our similarity measure and call it the “cosine similarity” in this context

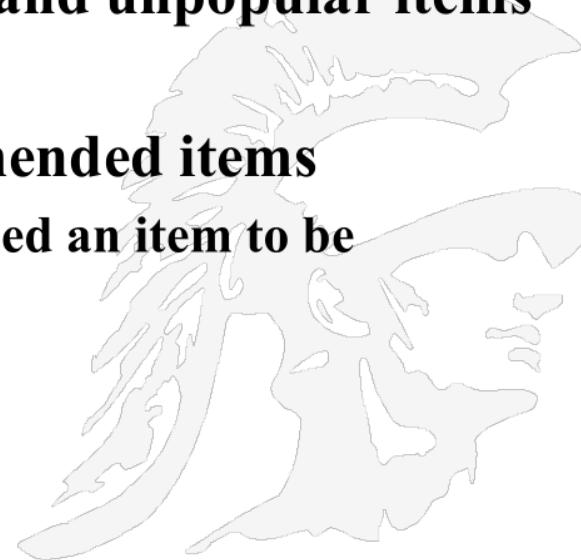


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## Pros: Content-based Approach

- No need for data on other users
- Able to recommend to users with unique tastes
- Able to recommend new and unpopular items
  - No first rater problem
- Explanations for recommended items
  - Content features that caused an item to be recommended





## Cons: Content-Based Approach

- **Finding the appropriate features is not always obvious**
  - E.g. movie features may be obvious but what about images and music
- **Overspecialization**
  - Never recommends items outside user's content profile
  - People might have multiple interests
  - Unable to exploit quality judgements of other users
- **Cold-start problem for new users**
  - How to build a user profile



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## Let's Switch Focus to Collaborative Filtering

- Collaborative filtering is a method of making automatic predictions (filtering) about the interests of a user by collecting preferences or taste information from many other users (collaborating).
  - The underlying assumption of the collaborative filtering approach is that if a person *A* has the same opinion as a person *B* on an issue, *A* is more likely to have *B*'s opinion on a different issue than that of a randomly chosen person



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## Similar Users and Jaccard Similarity

movies

|   | HP1 | HP2 | HP3 | TW | SW1 | SW2 | SW3 |
|---|-----|-----|-----|----|-----|-----|-----|
| A | 4   |     |     | 5  | 1   |     |     |
| B | 5   | 5   | 4   |    |     |     |     |
| C |     |     |     | 2  | 4   | 5   |     |
| D |     | 3   |     |    |     |     | 3   |

HP:Harry Potter  
SW:Star Wars  
TW:Twilight

4 users

- Consider users  $x$  and  $y$  with rating vectors  $r_x$  and  $r_y$ 
  - The rating vector of user B is  $(5,5,4,0,0,0,0)$
- We need a similarity metric  $\text{sim}(x,y)$  between rating vectors
- The metric should capture the intuition that  $\text{sim}(A,B) > \text{sim}(A,C)$ 
  - A and B both liked HP1, but A and C had very different opinions about TW and SW1
- Recall  $\text{sim}(A,B) = |\text{r}_a \text{ intersect } \text{r}_b| / |\text{r}_a \text{ union } \text{r}_b|$  try Jaccard Similarity
- $\text{Sim}(A,B) = 1/5$ ;  $\text{sim}(A,C) = 2/4$ . since A&B rated only one movie in common
  - But using Jaccard Similarity we get a result we don't want, namely
  - $\text{Sim}(A,B) < \text{Sim}(A,C)$
- Problem: ignores rating values

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# Cosine Similarity

|   | HP1 | HP2 | HP3 | TW | SW1 | SW2 | SW3 |
|---|-----|-----|-----|----|-----|-----|-----|
| A | 4   |     |     | 5  | 1   |     |     |
| B | 5   | 5   | 4   |    |     |     |     |
| C |     |     |     | 2  | 4   | 5   |     |
| D |     | 3   |     |    |     |     | 3   |

- Instead of using Jaccard similarity, which ignored the rating values, let's use cosine similarity, the angle between the vectors; now we treat the unknown values as zero
  - $\text{sim}(A, B) = \cos(r_a, r_b)$
- $\text{sim}(A, B) = 0.38$ ,  $\text{sim}(A, C) = 0.32$ 
  - Now  $\text{sim}(A, B) > \text{sim}(A, C)$ , which is what we wanted, but not by much
- Problem: by treating missing ratings as zero, we sort of got the result we wanted, but actually the similarity of A and C should be farther apart than what we computed; using zero was not a great idea

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## Centered Cosine Captures User Preferences

Solution: Normalize ratings by subtracting row mean

|   | HP1 | HP2 | HP3 | TW | SW1 | SW2 | SW3 |
|---|-----|-----|-----|----|-----|-----|-----|
| A | 4   |     |     | 5  | 1   |     |     |
| B | 5   | 5   | 4   |    |     |     |     |
| C |     |     |     | 2  | 4   | 5   |     |
| D |     | 3   |     |    |     |     | 3   |

|   | HP1 | HP2 | HP3  | TW   | SW1  | SW2 | SW3 |
|---|-----|-----|------|------|------|-----|-----|
| A | 2/3 |     |      | 5/3  | -7/3 |     |     |
| B | 1/3 | 1/3 | -2/3 |      |      |     |     |
| C |     |     |      | -5/3 | 1/3  | 4/3 |     |
| D |     | 0   |      |      |      |     | 0   |

|   | HP1 | HP2 | HP3  | TW   | SW1  | SW2 | SW3 |
|---|-----|-----|------|------|------|-----|-----|
| A | 2/3 |     |      | 5/3  | -7/3 |     |     |
| B | 1/3 | 1/3 | -2/3 |      |      |     |     |
| C |     |     |      | -5/3 | 1/3  | 4/3 |     |
| D |     | 0   |      |      |      |     | 0   |

- The average rating for A is 10/3;
- The average rating for B is 14/3;
- The solution is to subtract the average rating for each user's score; e.g. user A and HP1 we get  $4 - (10/3) = 2/3$

The resulting matrix

$$\sim(A, B) = \cos(r_a r_b) = 0.09; \\ \sim(A, C) = -0.56$$

Result: A and C are very DISSimilar

- $\sim(A, B) > \sim(A, C)$
- Captures intuition better
  - Missing ratings treated as average
  - Handles "tough raters" and "easy raters"

Note: Summing the rows for any user gives zero, so positive ratings means they liked the movie  
 Another name for centered cosine is Pearson Correlation

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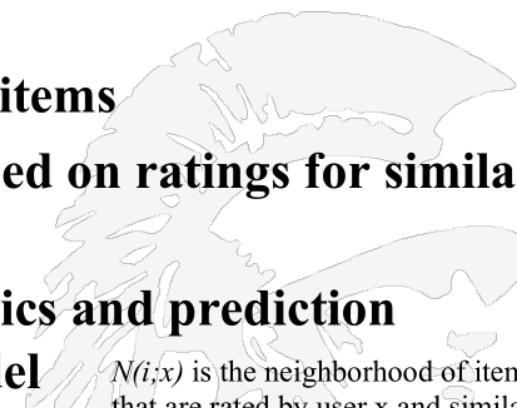
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## Item-Item Collaborative Filtering

- So far: we have used user-user collaborative filtering
- Another view: item-item
  - For item  $i$ , find other similar items
  - Estimate rating for item  $i$  based on ratings for similar items
  - Can use same similarity metrics and prediction functions as in user-user model



$N(i;x)$  is the neighborhood of items  
that are rated by user  $x$  and similar to item  $i$

$$r_{xi} = \frac{\sum_{j \in N(i;x)} s_{ij} \cdot r_{xj}}{\sum_{j \in N(i;x)} s_{ij}}$$

$s_{ij}$ ... similarity of items  $i$  and  $j$

$r_{xj}$ ... rating of user  $x$  on item  $j$

$N(i;x)$ ... set items rated by  $x$  similar to  $i$

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**Let's Do An Example**  
**Item – Item CF ( $|N| = 2$ )**

**Goal:** Estimate rating of movie 1 by user 5

N is 2, looking at the two nearest neighbors

Conclusion: user 5 will like movie 1: 2.6

Here we use Pearson correlation as similarity:  
 1) Subtract mean rating  $m_i$  from each movie  $i$   
 $m_1 = (1+3+5+4)/5 = 3.6$   
 row 1: [-2.6, 0, -0.6, 0, 0, 1.4, 0, 0, 1.4, 0, 0.4, 0]  
 2) Compute cosine similarities between rows

Movie 3 and 6 are the two recommendations that are most similar to movie 1  
 Compute similarity weights  
 $S_{13} = 0.41$   $S_{16} = 0.59$   
 Use the weighted average to compute  
 $R_{15} = (0.41*2 + 0.59*3)/(0.41+0.59) = 2.6$

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## Item-Item vs. User-User

- In theory user-user and item-item are dual approaches
- In practice, item-item outperforms user-user in many use cases
- Items are “simpler” than users
  - Items belong to a small set of “genres”, users have varied tastes
  - *Item* similarity is more meaningful than *user* similarity

