Missouri University of Science & Technology	Department of Computer Science
Fall 2018	**CS 2500: Algorithms**

Homework 5: Graph Algorithms (Part II) & NP Completeness

Instructor: *Sid Nadendla* **Due:** *November 27, 2018*

In this homework, we will focus our attention to finding shortest-paths and maximum flow on graphs, and NP Completeness.

Problem 1: Shortest Path *40 points*

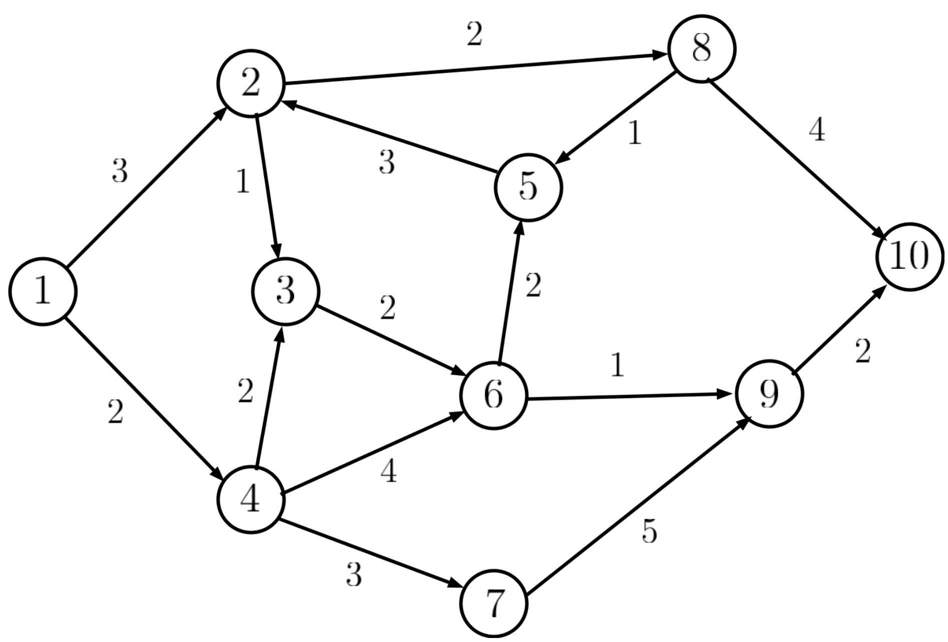
1. If $p = \{v_1, \cdots, v_n\}$ is the shortest path between v_1 and v_n, then prove that any subpath $p_{ij} = \{v_i, \cdots, v_j\}$ in p is the shortest path between v_i and v_j. (20 points)

2. Write the pseudocode to find a negative weight cycle in a directed graph $G = (V, E)$ with the weight function $w : E \to \mathbb{R}$. (20 points)

Bonus Problem (20 points):

1. Demonstate Dijkstra's algorithm on the following graph.

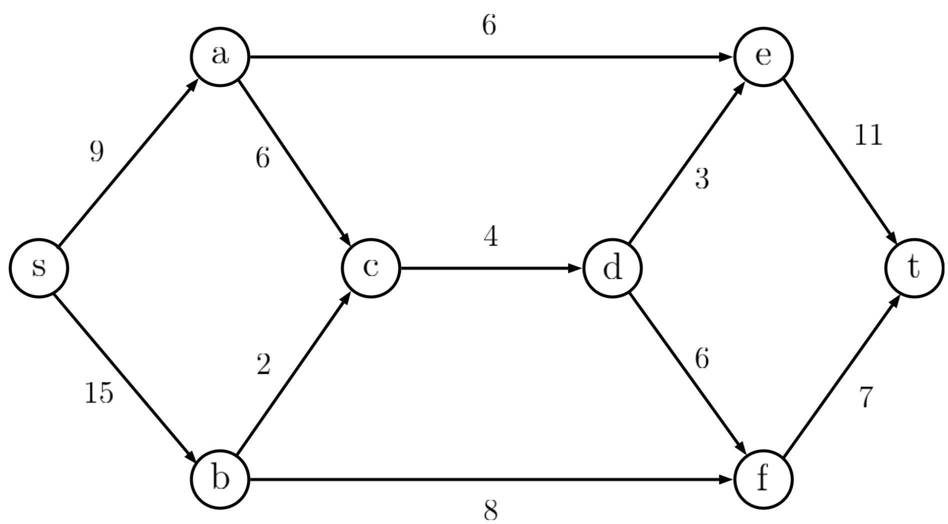
2. Implement Dijkstra's algorithm in Python, and validate your code on the following graph.



Problem 2: Flow Networks ***20 points***

1. Define slack (residual flow) in an edge $(u, v) \in E$ in a the residual graph of a given graph $G = (V, E)$. (10 points)

2. Demonstrate the Ford-Fulkerson algorithm on this following flow network, where each edge is labeled with its flow capacity. (10 points)



Bonus Problem (10 points):

Implement Edmonds-Karp algorithm in Python, and test your code on the given graph.

Problem 3: NP Completeness *40 points*

1. Prove that there are uncountable number of unsolvable binary decision problems. Furthermore, give an example of an unsolvable binary decision problem. (10 points)

2. Define NP, NP-Hard and NP-Complete classes, and give one problem in each of these complexity classes. (10 points)

3. Assuming that Hamiltonian circuit problem is NP-Complete, prove that traveling salesman problem is NP-Complete via reduction. (20 points)